ICSE MATH

Made Simple

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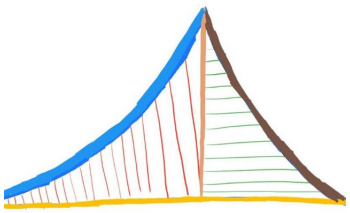

Contents

Introduction

This book links high school coordinate geometry to linear algebra and matrix analysis through solved problems.

Chapter 1

Linear Forms

Chapter 2

Circles

2.1. 2022

2.1.1. 10

1. $ABCD$ is a cyclic quadrilateral. If $\angle BAD = (2x + 5)^\circ$ and $\angle BCD = (x + 10)^\circ$ then x is equal to:

   (a) 65°

   (b) 45°

   (c) 55°

   (d) 5°

2. In the given figure O is the centre of the circle. PQ and PR are tangents and $\angle QPR = 70°$. Calculate:

   (a) $\angle QOR$

   (b) $\angle QSR$

3. Two chords AB and CD of a circle intersect extenally at E. if $EC = 2cm$, $EA = 3cm$ and $AB = 5cm$, find the length of CD.

4. In the given figure A, B, C and D are points on the circle with centre O. Given $\angle ABS = 62°$. Find:

   (a) $\angle ADC$

   (b) $\angle CAB$

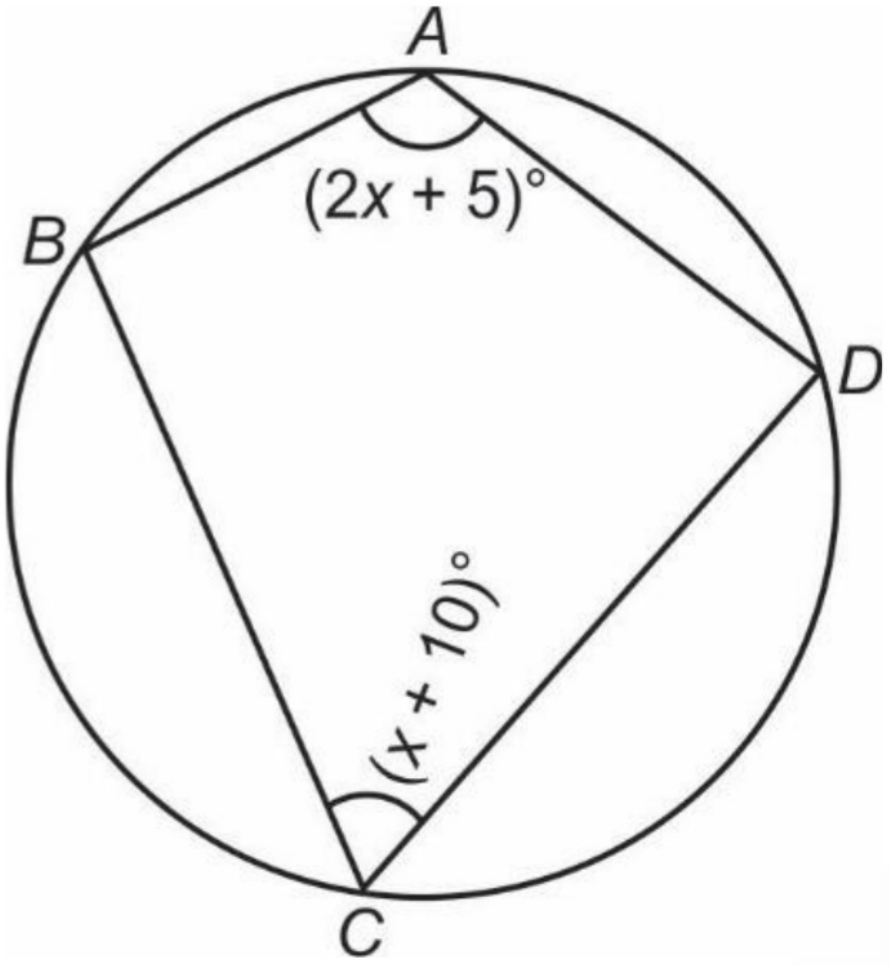

Figure 2.1:

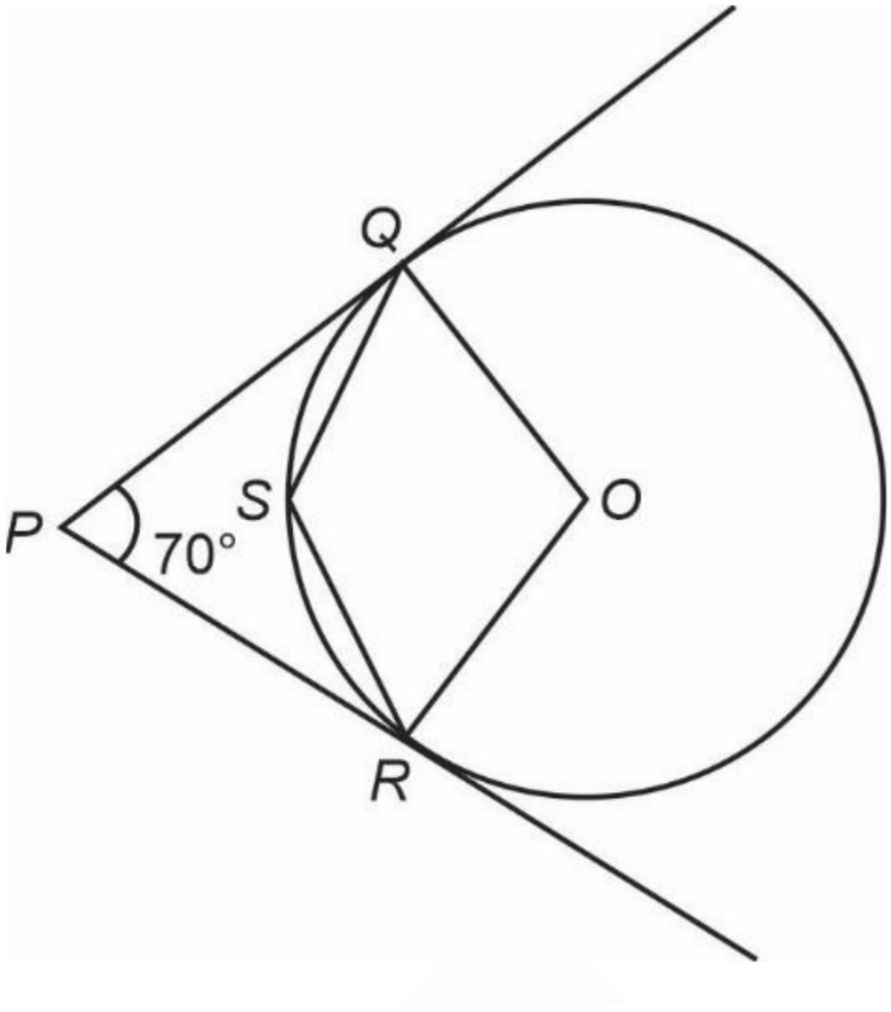

Figure 2.2:

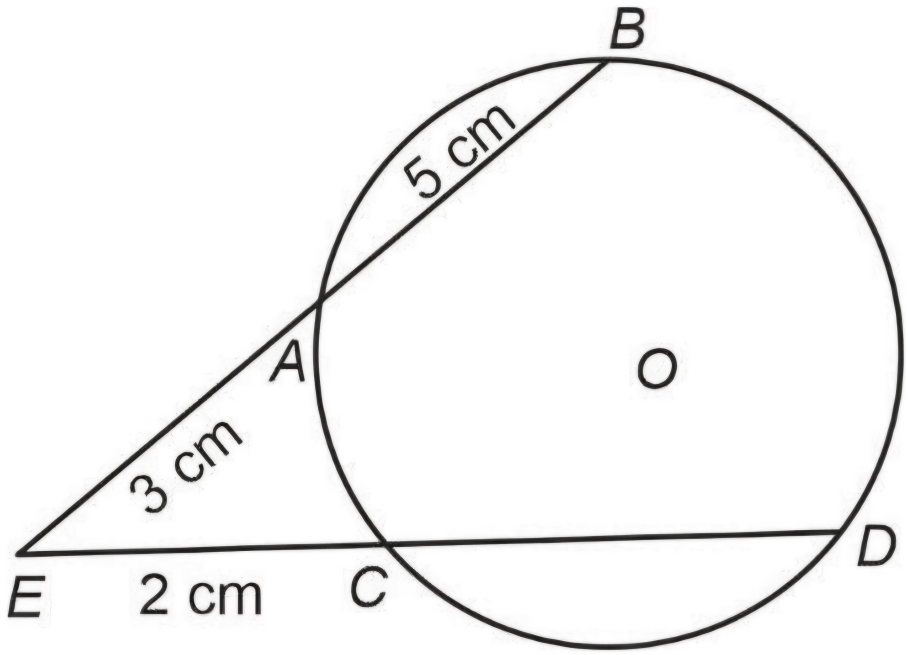

Figure 2.3:

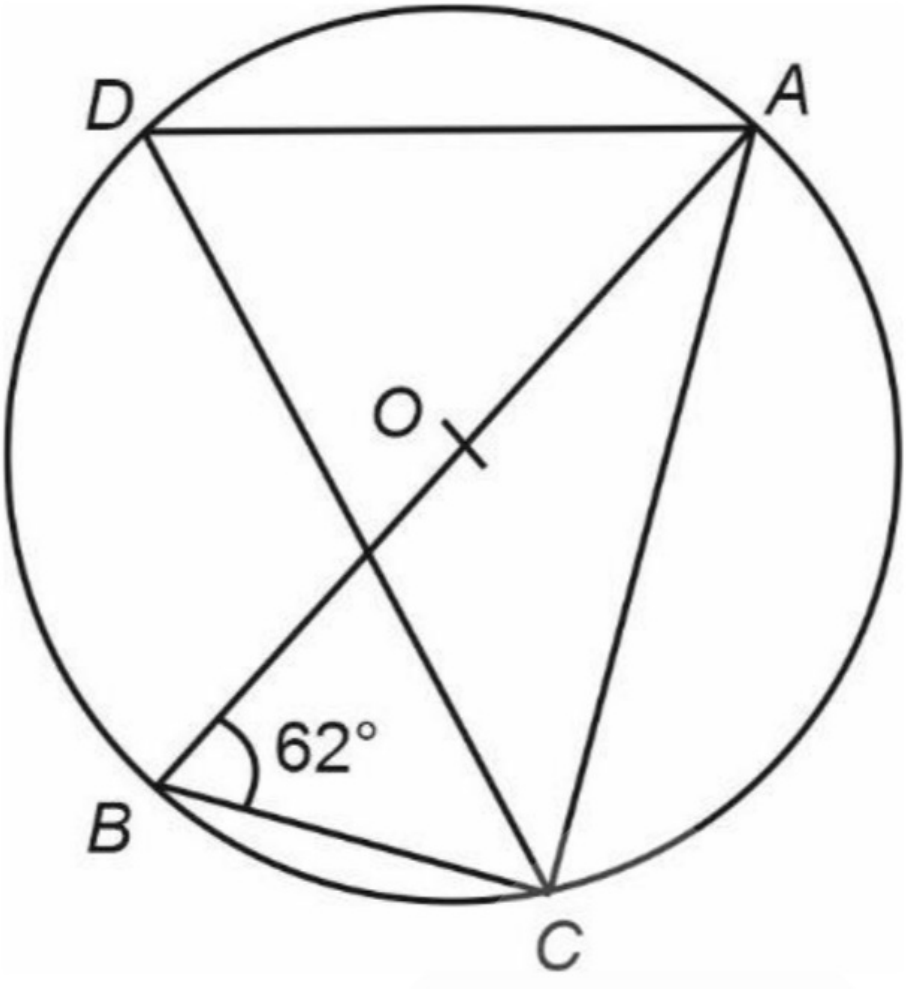

Figure 2.4:

Chapter 3

Intersection of Conics

Chapter 4

Probability

4.1. 2019

4.1.1. 10

1. In a class of 40 students ,marks obtained by the students in a class test ($out of 10$) are given below:

Marks	1	2	3	4	5	6	7	8	9	10
Number of students	1	2	3	3	6	10	5	4	3	3

   Calculate the following for the given distribution:

   (a) Median

   (b) Mode

2. The data on the number of patients attending a hospital in a month are given below. Find the average ($mean$) number of patients attending the hospital in a month by using the shortcut method. Take the assumed

mean as 45.Give your answer correct to 2 decimal places.

Number of patients	10-20	20-30	30-40	40-50	50-60	60-70
Number of Days	5	2	7	9	2	5

3. There are 25 discs numbered 1 to 25. They are put in a closed box and shaken thoroughly. A disc is drawn at random from the box.
   Find the probability that the number on the disc is:

   (a) an odd number

   (b) divisible 2 and 3 both.

   (c) a number is less than 16.

4. Use graph for this question.
   The marks obtained by 120 students in a English test are given below:

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No.of students	5	9	16	22	26	18	11	6	4	3

   Draw the ogive and hence,estimate:

   (a) the median marks.

   (b) the number of students who did not pass the pass percentage was 50.

   (c) the upper quartile marks.

4.2. 2022

4.2.1. 10

1. The probability of getting a number divisible by 3 in throwing a dice is:

   (a) $\frac{1}{6}$

   (b) $\frac{1}{3}$

   (c) $\frac{1}{2}$

   (d) $\frac{2}{3}$

2. A bag contains 5 white, 2 red and 3 black balls. A ball is drawn at random. What is the probability that the ball drawn is a red ball?

3. A letter of the word "$SECONDARY$", is selected at random. What is the probability that the letter selected is not a vowel?

Chapter 5

Construction

Chapter 6

Optimization

6.1. 2019

6.1.1. 10

1. A solid metallic sphere of radius 6cm is melted and made into a solid cylinder of height 32cm.Find the:

   (a) radius of the cylinder

   (b) curved surface area of the cylinder

2. A hemispherical and a conical hole is scooped out of a solid wooden cylinder. Find the volume of the remaining solid where the measurements are as follows: The height of the solid cylinder is 7cm, radius of each of hemisphere, cone and cylinder is 3cm.Height of cone is 3cm. Give your answer correct to the nearest whole number. Take $\pi = \frac{22}{7}$.

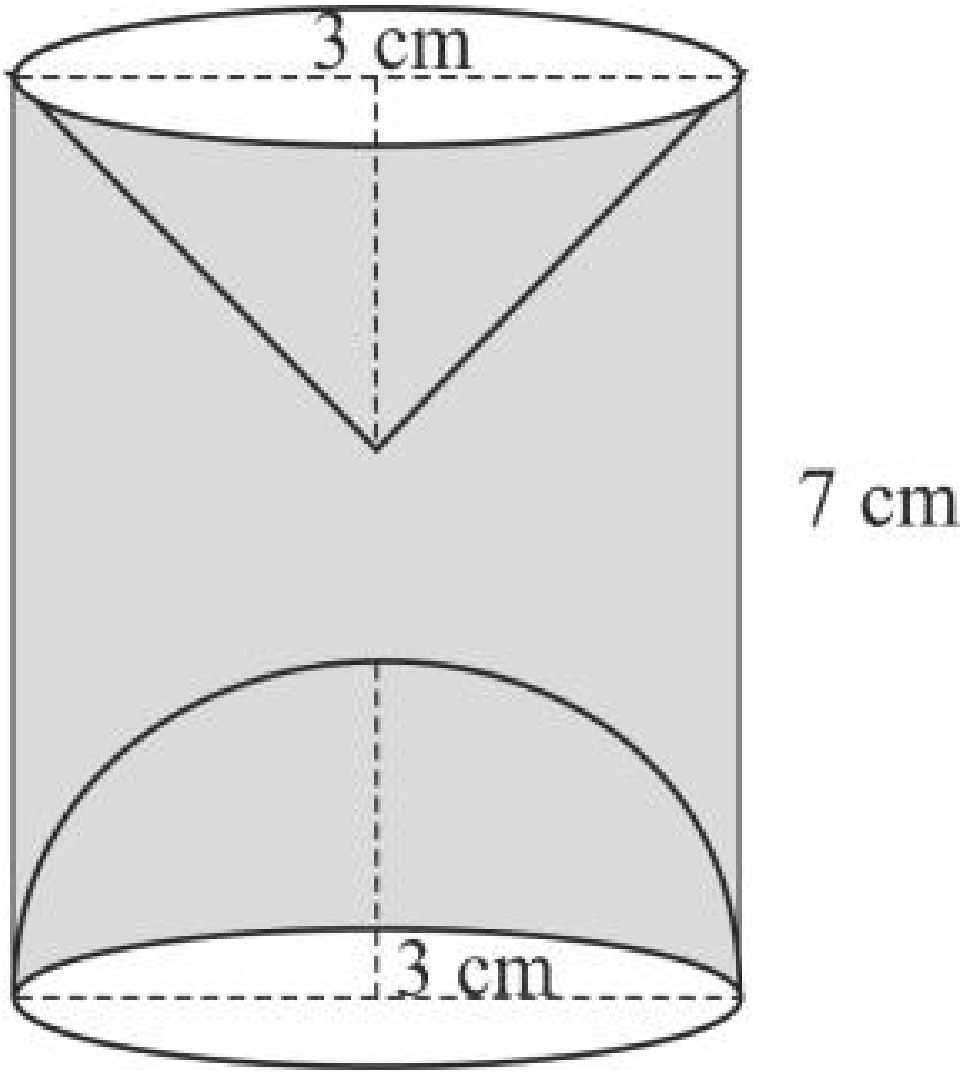

Figure 6.1:

Chapter 7

Algebra

7.1. 2019

7.1.1. 10

1. Solve the following inequation and write down the solution set:

$$11x - 4 < 15x + 4 \leq 13x + 14, x \in W \tag{7.1}$$

   Represent the solution on a real number line.

2. Solve for x the quadratic equation $x^2 - 4x - 8 = 0$.

   Give your answer correct to three significant figures.

3. Using the factor theorem, show that $(x - 2)$ is a factor of $x^3 + x^2 - 4x - 4$.

   Hence factorise the polynomial completely.

4. Using the Remainder Theorem find the remainders obtained when $x^3 + (kx + 8)\,x + k$ is divided by $x + 1$ and $x - 2$.

Hence find **k** if the sum of the two remainders is 1.

5. In an Arithmetic Progression ($A.P.$) the fourth and sixth terms are 8 and 14 respectively.Find the:

   (a) first term

   (b) common difference

   (c) sum of the first 20 terms.

6. The first and last term of a Geometrical Progression ($G.P.$) are 3 and 96 respectively. If the common ratio is 2, find:

   (a) **n'** the number of terms of the G.P.

   (b) Sum of the **n** terms.

7. The sum of the first three terms of an Arithmetic Progression ($A.P.$) is 42 and the product of the first and third term is 52. Find the first term and the common difference.

8. Using properties of proportion solve for$x$,given

$$\frac{\sqrt{5x}+\sqrt{2x-6}}{\sqrt{5x}-\sqrt{2x-6}}=4 \tag{7.2}$$

9. The product of two consecutive natural numbers which are multiples of 3 is equal to 810.Find the two numbers.

Chapter 8

Geometry

8.1. 2019

8.1.1. 10

1. M and N are two points on the X axis and Y axis respectively.
   P $(3, 2)$ divides the line segment MN in the ratio $2 : 3$.
   Find.

   (a) the coordinates of M and N

   (b) slope of the line MN

2. The vertices of a $\triangle ABC$ are $A\,(3, 8)$,$B\,(-1, 2)$ and $C\,(6, -6)$.Find:

   (a) Slope of BC

   (b) Equation of a line perpendicular to BC and passing through A.

3. Use ruler and compass only for answering this question. Draw a circle of radius 4cm. Mark the centre as O. Mark a point P outside the circle at a distance of 7cm from the centre. Construct two tangents to the

circle from the external point P. Measure and write down the length of any one tangent.

4. Using ruler and a compass only, construct a semi-circle with diameter BC =7cm. Locate a point A on the circumference of the semicircle such that A is equidistant from B and C. Complete the cyclic quadrilateral $ABCD$, such that D is equidistant from AB and BC. Measure $\angle ADC$ and write it down.

5. In the given figure AC is a tangent to the circle with center O. If $\angle ADB = 55°$,find x and y.Give reasons for your answers.

6. In the given figure ,$ABCDE$ is a pentagon inscribed in a circle such that AC is a diameter and side BC // AE.If $\angle BAC$=50°,find giving reasons:

   (a) $\angle ACB$

   (b) $\angle EDC$

   (c) $\angle BEC$

   Hence prove that BE is also a diameter

7. In the given figure ,$\angle PQR$=$\angle PST$=90°,PQ =5cm.and PS =2cm.

   (a) Prove that $\triangle PQR \sim \triangle PST$.

   (b) Find Area of $\triangle PQR$: Area of quadrilateral $SRQT$.

8.2. 2022

8.2.1. 10

1. A lighthouse is $80m$ high. The angle of elevation of its top from a point $80m$ away from its foot along the same horizontal line is:

   (a) $60°$

   (b) $45°$

   (c) $30°$

   (d) $90°$

2. Two lamp posts AB and CD each of height $100m$ are on either side of the road. P is a point on the road between the two lamp posts. The angle of elevation of the top of the lamp posts from te point P are $60°$ and $40°$. Finf the distances PB and CD.

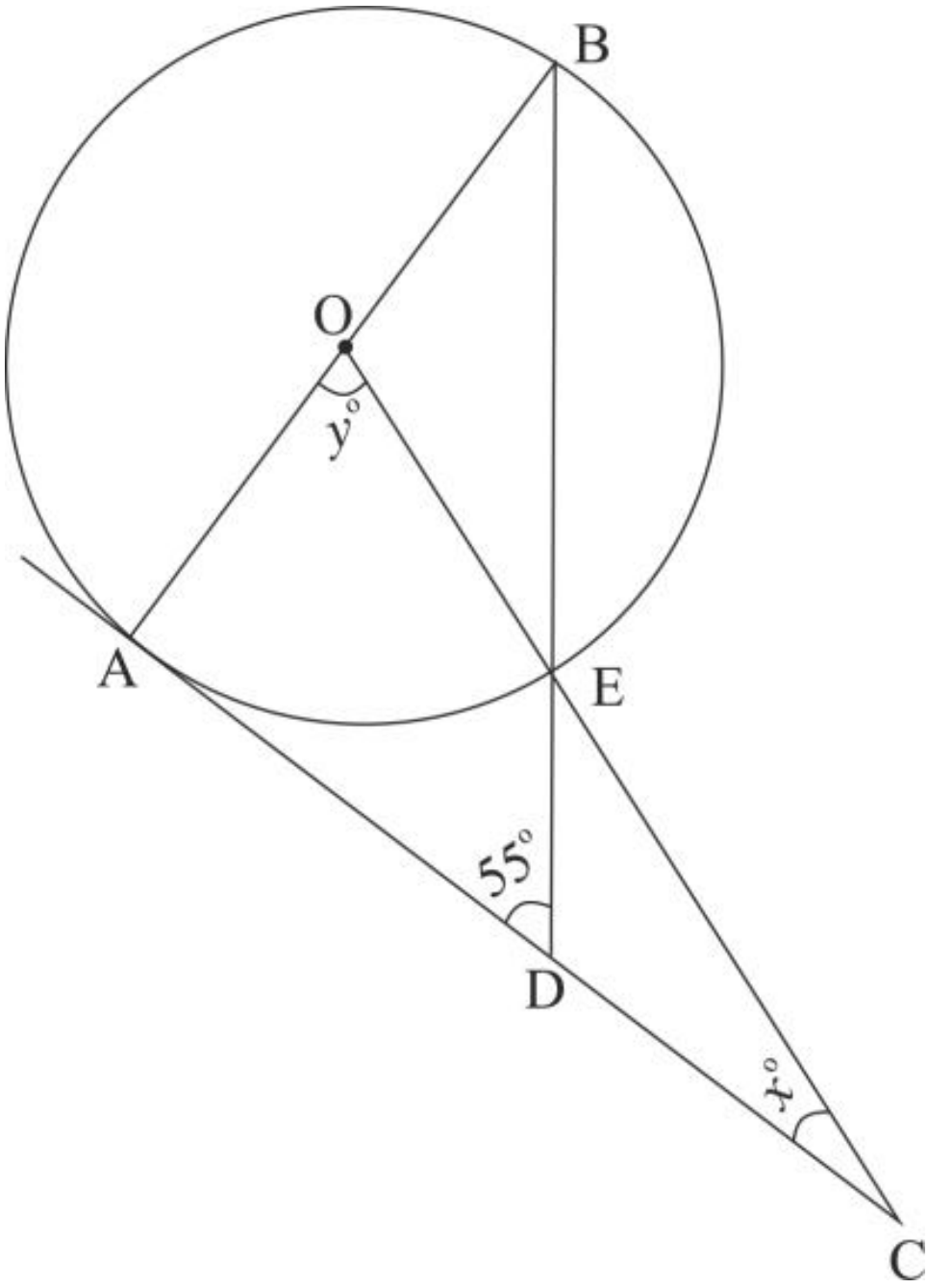

Figure 8.1:

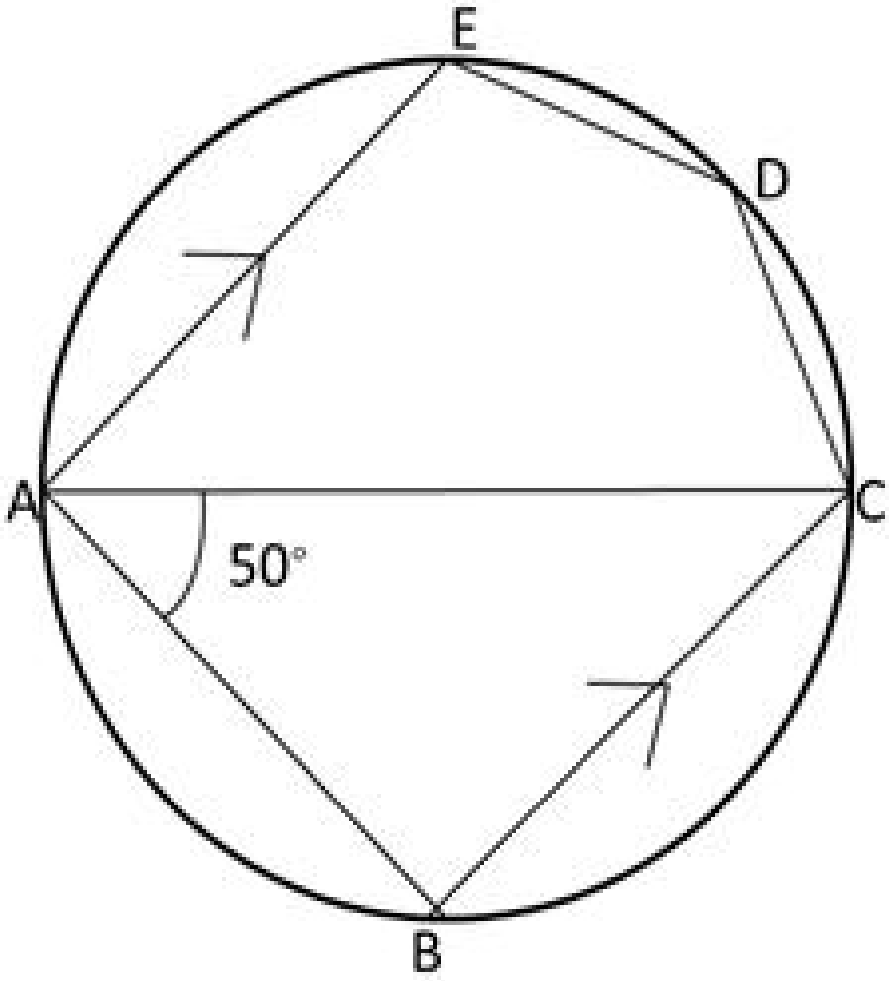

Figure 8.2:

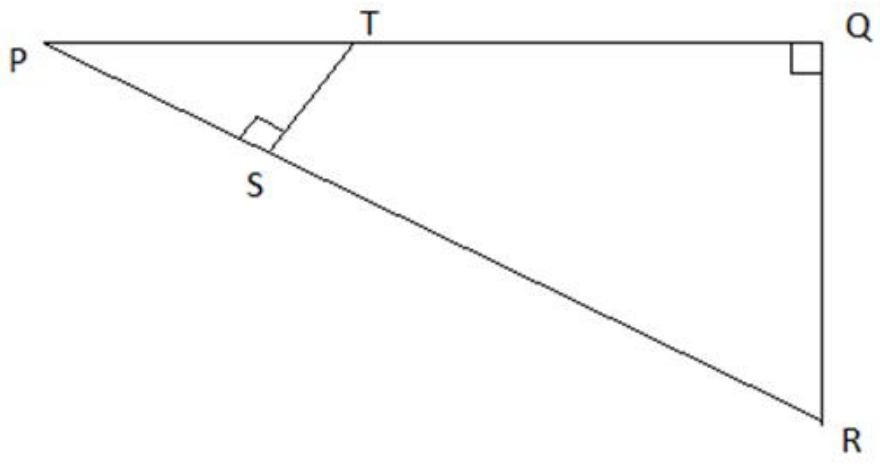

Figure 8.3:

Chapter 9

Coordinate Geometry

9.1. 2022

9.1.1. 10

1. If two lines are perpendicular to one another then the relation between their slopes m_1 and m_2 is:

   (a) $m_1 = m_2$

   (b) $m_1 = \frac{1}{m_2}$

   (c) $m_1 = -m_2$

   (d) $m_1 \times m_2 = -1$

2. The coordinates of the point $P(-3, 5)$ on reflecting on the x-axis are:

   (a) $(3, 5)$

   (b) $(-3, -5)$

   (c) $(3, -5)$

   (d) $(-3, 5)$

3. $A(1,4), B(4,1)$ and $C(x,4)$ are the vertices of $\triangle ABC$. If the centroid of the triangle is $G(4,3)$ then x is equal to:

   (a) 2

   (b) 1

   (c) 7

   (d) 4

4. Find $'a'$, if $A(2a+2,3,\ B(7,4)$ ad $C(2a+5,2)$ are collinear.

5. Find a point P which divides internally the line segment joining the points $A(-3,9)$ and $B(1,-3)$ in the ratio $1:3$.

6. Use a graph paper for this question. Take $2cm = 1$ unit along both the axes

   (a) Plot the points $A(0,4), B(2,2), C(5,2)$ and $D(4,0)$. $E(0,0)$ is the origin.

   (b) Reflect B, C, D on the y-axis and name them as B', C', D' respectively.

   (c) Join the points $ABCDD'C'B'$ and A in order and give a geometrical name to the closed figure.

7. Find the equation of a line parallel to the line $2x+y-7=0$ and passi ng through the intersection of the lines $x+y-4=0$ and $2x-y=8$.

8. Line AB is perpendicular to CD. Coordinates of $B, CandD$ are respectively $(4,0), (0,-1)$ and $(4,3)$. Find:

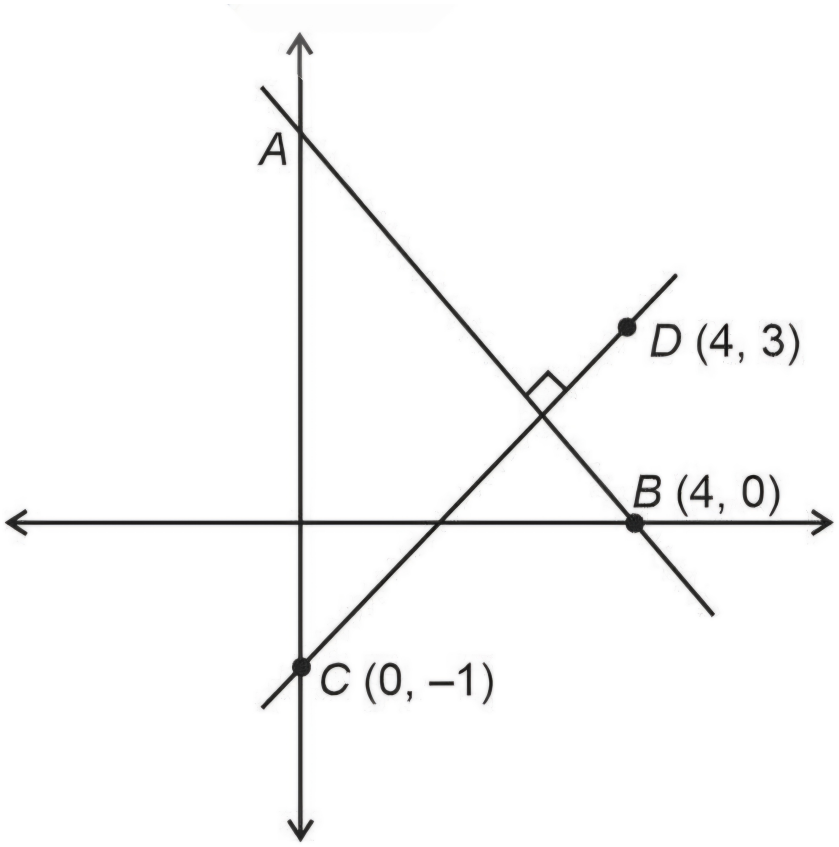

Figure 9.1:

(a) Slope of CD

(b) Equation of AB

Chapter 10

3D-Geometry

10.1. 2022

10.1.1. 10

1. The volume of a conical tent is $462m^3$ and the area of the base is $154m^2$. The height of the cone is:

   (a) $15m$

   (b) $12m$

   (c) $9m$

   (d) $24m$

2. The radius of a roller $100cm$ long is $14cm$. The curved surface area of the roller is: $(Take\pi = \frac{22}{7})$

   (a) $13200cm^2$

   (b) $15400cm^2$

   (c) $4400cm^2$

(d) $8800cm^2$

3. A solid cone of radius $5cm$ and height $9cm$ is melted and made into small cyliders of radius of $0.5cm$ and height $1.5cm$. Find the number of cylinders so formed.

4. A solid wooden cylinder is of radius $6cm$ and height $16cm$. Two cones ech of radius $2cm$ and height $6cm$ are drilled out of the cylinder. Find the volume of the remainiing solid.

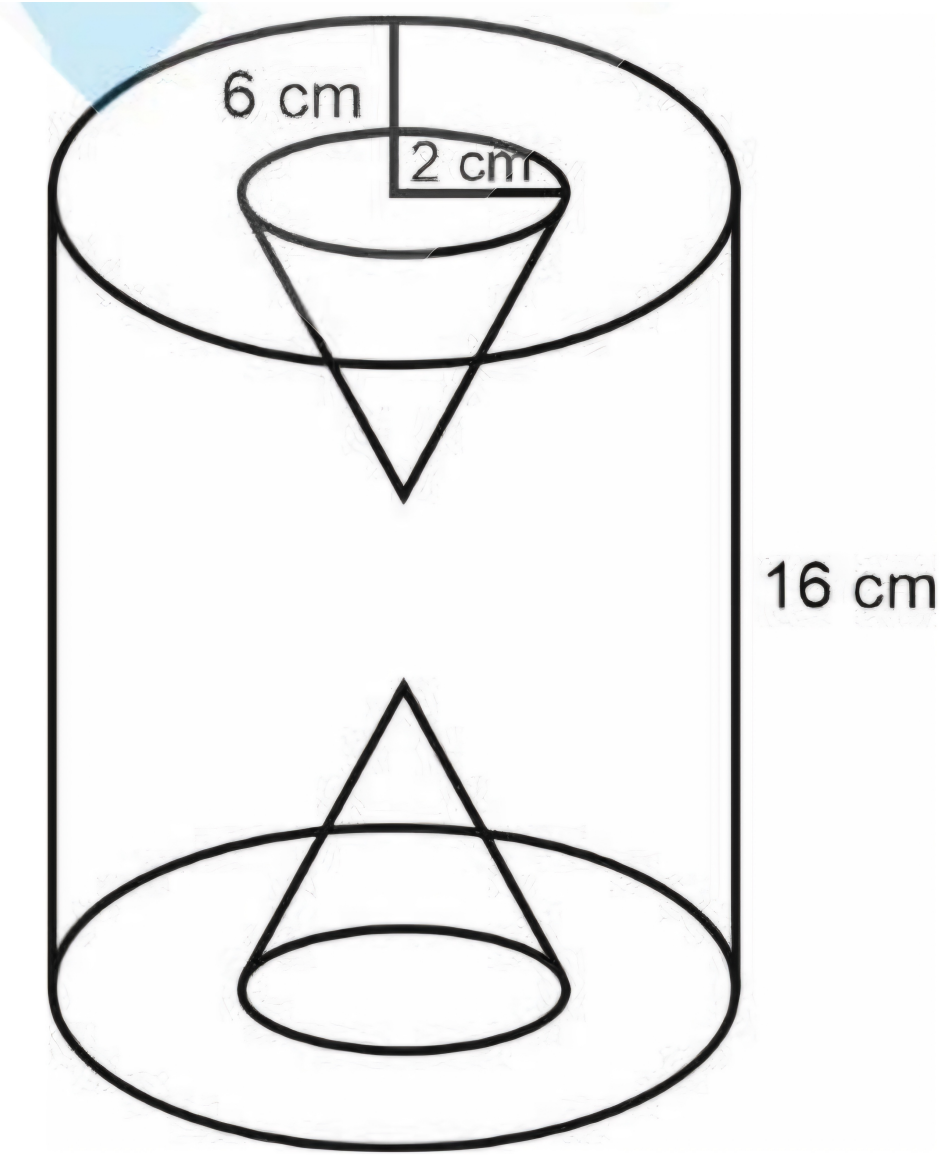

Figure 10.1:

Chapter 11

Discrete

Chapter 12

Number Systems

Chapter 13

Differentiation

Chapter 14

Integration

Chapter 15

Functions

Chapter 16

Matrices

Chapter 17

Data-Handling

17.1. 2019

17.1.1. 10

1. A man invests ₹ 4500 in shares of a company which is paying 7.5 % dividend. If ₹ 100 shares are available at a discount of 10 %. Find:

   (a) Number of shares he purchases.

   (b) His annual income.

2. Sachin invests ₹ 8500 in 10 %, ₹ 100 shares at ₹ 170. He sells the shares when the price of each share rises by ₹ 30. He invests the proceeds in 12 % ₹ 100 shares at ₹ 125.Find:

   (a) the sale proceeds.

   (b) the number of ₹ 125 shares he buys.

   (c) the change in his annual income.

3. Rekha opened a recurring deposit account for 20months.The rate of

interest is 9 % per annum and Rekha receives ₹ 441 as interest at the time of maturity. Find the amount Rekha deposited each month.

17.2. 2022

17.2.1. 10

1. The median class for the given distribution is:

Class Interval	Frequency
0-10	2
10-20	4
20-30	3
30-40	5

   (a) $0 - 10$

   (b) $10 - 20$

   (c) $20 - 30$

   (d) $30 - 40$

2. The modal class of a given distribution always corresponds to the:

   (a) interval with highest frequency

   (b) interval with lowest frequency

   (c) the first interval

   (d) the last interval

3. Calculate the mean of the following frequency distribution.

Class Interval	Frequency
5-15	2
15-25	6
25-35	4
35-45	8
45-55	4

4. Marks obtained by 100 students in an examination are given below.

Marks	No. of students
0-10	5
10-20	15
20-30	20
30-40	28
40-50	20
50-60	12

Draw a histogram for the given data using a graph paper and find the mode. Take $2cm = 10$ marks along one axis and $2cm = 10$ students along the other axis.

5. The mean of the following distribution is 50. Find the unknown frequency.

Class Interval	Frequency
0-20	6
20-40	f
40-60	8
60-80	12
80-100	8

6. Marks obtained by 40 students in an examination are given below.

Marks	No. of students
10-20	3
20-30	8
30-40	14
40-50	9
50-60	4
60-70	2

Using graph paper draw an ogive and estimate the median marks.

Take $2cm = 10$ marks along one axis and $2cm = 5$ students along the other axis.

Chapter 18

Trignometry

18.1. 2019

18.1.1. 10

1. Prove that:

$$(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1 \tag{18.1}$$

2. Simplify

$$\sin A \begin{bmatrix} \sin A & -\cos A \\ \cos A & \sin A \end{bmatrix} + \cos A \begin{bmatrix} \cos A & \sin A \\ -\sin A & \cos A \end{bmatrix}$$

3. A man observes the angle of elevation of the top of the tower to be 45°. He walks towards it in a horizontal line through its base. On covering 20m the angle of elevation changes to 60°.Find the height of the tower correct to 2 significant figures.

18.2. 2022

18.2.1. 10

1. Prove that:

   $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$

2. Prove that:

   $\frac{(1+\sin\theta)^2+(1-\sin\theta)^2}{2\cos^2\theta} = \sec^2\theta + \tan^2\theta$

3. Prove that:

   $1 + \frac{\tan^2\theta}{1+\sec\theta} = \sec\theta$