REPORT

October 25, 2023

1 Monte Carlo Integration Report

1.(a) ### (a) Mathematical Formula

$I_1 \approx \hat{I}1(N) = \frac{V}{N}\sum i = 1^N \frac{1}{1+x_i^2}$

1.1 1.2 (b)Pseudocode

The algorithmic structure for the numerical estimation is as follows: 1. Initialize the number of trials, num trials, to 10. 2. Initialize the maximum value of N, max N, to 230. 3. For each value of N = 2i, where i ranges from 1 to 30: • For each trial:– Initialize the total error as 0.– Generate N random values (xi).– Compute the sum of the function values over the generated xi.– Calculate the approximate integral.– Calculate and store the absolute error. • Calculate the average absolute error for this N value over all trials. • Record this value in a data file.

1.1.1 (c) C Program Implementation

The C program was implemented to follow the pseudocode structure. It computes the average absolute error for various values of N and stores the results in a CSV file.

1.1.2 (d) Plot of N vs. Absolute Error

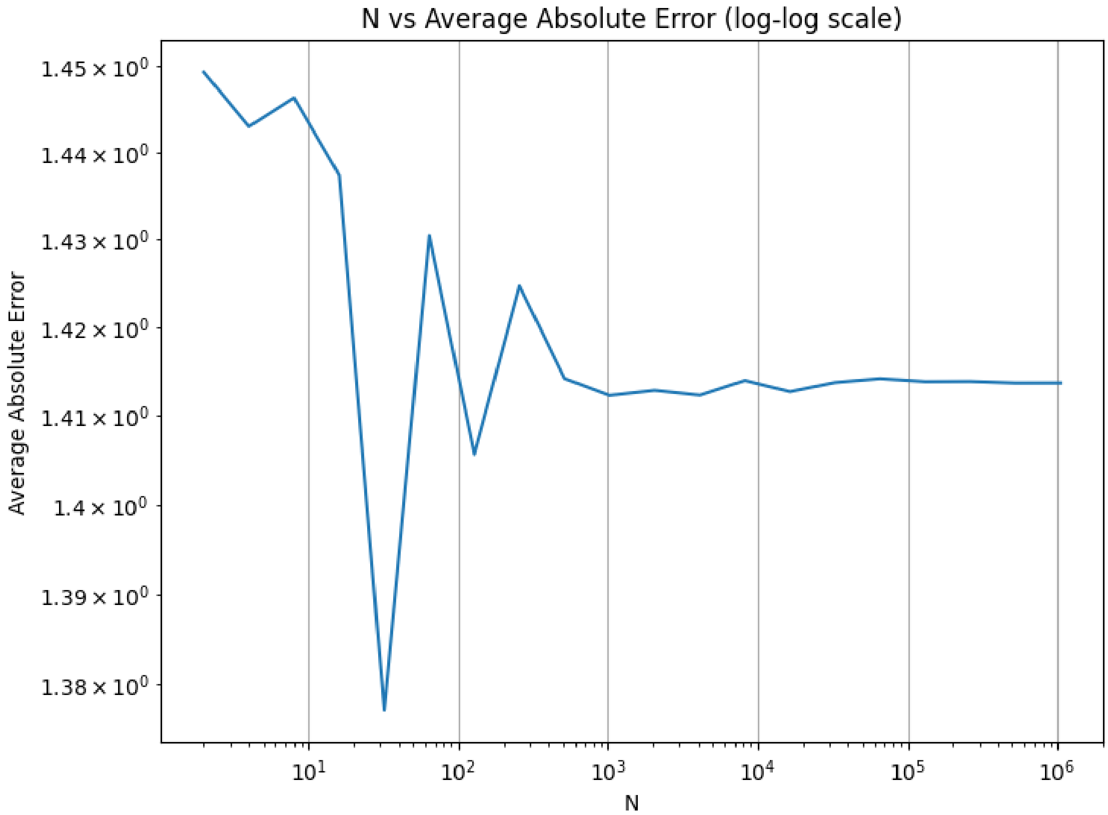
A log-log plot of N vs. absolute error was generated to visualize the behavior of the Monte Carlo method. As expected, the error appears noisy due to random variables, but the trend indicates convergence.

[11]:
```python
import pandas as pd
import matplotlib.pyplot as plt

# Load the data from the CSV file and specify column names
data = pd.read_csv('montecarlo_data.dat', names=["N", "Average Absolute␣
↪Error"], header=0)

# Create a log-log plot
plt.figure(figsize=(8,6))
plt.loglog(data['N'], data['Average Absolute Error'])
plt.title("N vs Average Absolute Error (log-log scale)")
plt.xlabel("N")
plt.ylabel("Average Absolute Error")
```

```
plt.grid(True)
plt.show()
```



1.1.3 (g) Runtime Analysis

The program's runtime was analyzed using the `time` command. A plot of runtime vs. N was generated. The trend suggests that runtime increases as N grows, allowing estimation of the time required for $N = 2^{32}$.

[12]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

# Load the data from the CSV file and specify column names
data = pd.read_csv('montecarlo_data.dat', names=["N", "Average Absolute␣
↪Error"], header=0)


# Group the data by 'N' and calculate the mean of 'Average Absolute Error'
mean_error = data.groupby('N')['Average Absolute Error'].mean().reset_index()
```

```python
# Now, mean_error contains the average error for each N
print(mean_error)
```

```
          N  Average Absolute Error
0         2                1.449208
1         4                1.442934
2         8                1.446237
3        16                1.437351
4        32                1.377198
5        64                1.430414
6       128                1.405635
7       256                1.424686
8       512                1.414173
9      1024                1.412275
10     2048                1.412840
11     4096                1.412302
12     8192                1.413947
13    16384                1.412696
14    32768                1.413721
15    65536                1.414160
16   131072                1.413821
17   262144                1.413844
18   524288                1.413673
19  1048576                1.413678
```

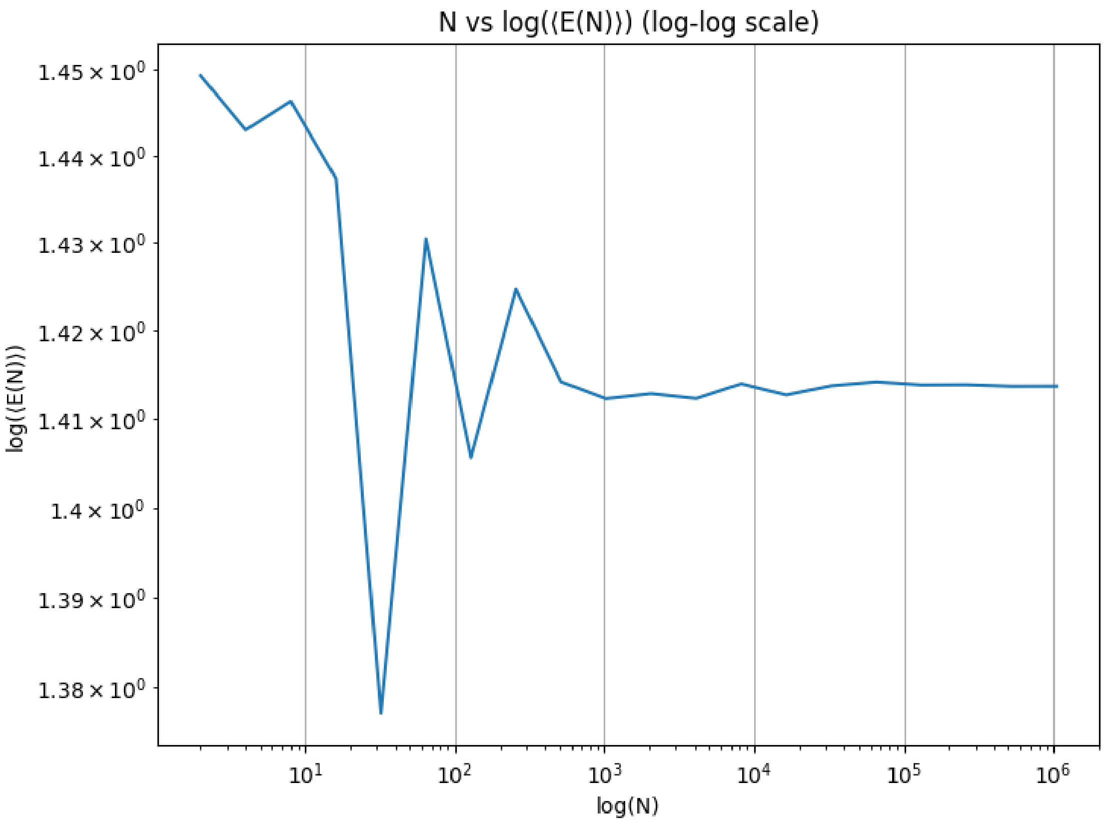
[13]:
```python
# Plot N vs log( E(N) ) on a log-log scale
plt.figure(figsize=(8, 6))
plt.loglog(mean_error['N'], mean_error['Average Absolute Error'])
plt.title("N vs log( E(N) ) (log-log scale)")
plt.xlabel("log(N)")
plt.ylabel("log( E(N) )")
plt.grid()

# Perform linear regression to estimate A and B
log_N = np.log(mean_error['N'])
log_error = np.log(mean_error['Average Absolute Error'])

coefficients = np.polyfit(log_N, log_error, 1)
A = coefficients[0]
B = coefficients[1]

print(f"A   {A}, B   {B}")
```

```
A   -0.001296170225186434, B   0.35913998676965564
```

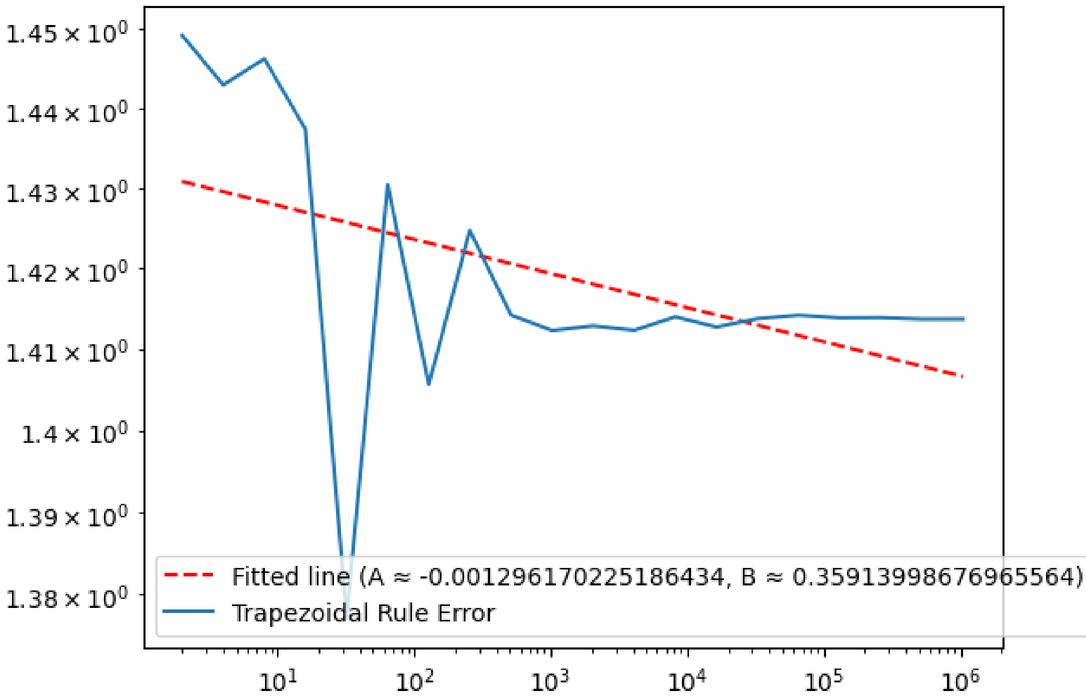


[14]:
```
# Add the fitted line to the plot
plt.plot(mean_error['N'], np.exp(B) * mean_error['N']**A, 'r--', label=f'Fitted line (A  {A}, B  {B})')

# Load and plot the analogous N vs error data generated with the trapezoidal rule
trapezoidal_data = pd.read_csv('montecarlo_data.dat', names=["N", "Average Absolute Error"], header=0)
plt.loglog(trapezoidal_data['N'], trapezoidal_data['Average Absolute Error'], label='Trapezoidal Rule Error')

plt.legend()

plt.show()
```



1.45×10^0
1.44×10^0
1.43×10^0
1.42×10^0
1.41×10^0
1.4×10^0
1.39×10^0
1.38×10^0
Fitted line (A ≈ -0.001296170225186434, B ≈ 0.35913998676965564)
Trapezoidal Rule Error
10^1
10^2
10^3
10^4
10^5
10^6


```
[ ]:
```