Monte Carlo Integration Report

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1 Problem 1: Estimating the Integral of $1/(1 + x^2)$

1.1 (a) Mathematical Formula

The mathematical formula for approximating the integral of the function $\frac{1}{1+x^2}$ is given as:

$$I_1 \approx \hat{I}_1(N) = V \frac{1}{N} \sum_{i=1}^{N} \frac{1}{1 + x_i^2}$$

Where:

- $V = 2$ (the volume of the region)
- x_i are random variables drawn from a uniform distribution on the interval $[-1, 1]$.

1.2 (b) Pseudocode

The algorithmic structure for the numerical estimation is as follows:

1. Initialize the number of trials, `num_trials`, to 10.
2. Initialize the maximum value of N, `max_N`, to 2^{30}.
3. For each value of $N = 2^i$, where i ranges from 1 to 30:
   - For each trial:
     - Initialize the total error as 0.
     - Generate N random values (x_i).
     - Compute the sum of the function values over the generated x_i.
     - Calculate the approximate integral.
     - Calculate and store the absolute error.
   - Calculate the average absolute error for this N value over all trials.
   - Record this value in a data file.

1.3 (c) C Program Implementation

The C program was implemented to follow the pseudocode structure. It computes the average absolute error for various values of N and stores the results in a CSV file.

1.4 (d) Plot of N vs. Absolute Error

A log-log plot of N vs. absolute error was generated to visualize the behavior of the Monte Carlo method. As expected, the error appears noisy due to random variables, but the trend indicates convergence.

1.5 (e) Automated Reruns

The program was automated to run 10 times, ensuring reproducibility and reliability of results.

1.6 (f) Mean of the Error

The mean of the error ($\langle E(N) \rangle$) over 10 trials was computed for each N value and plotted on a log-log scale. The results indicate that the error follows a logarithmic relationship with N.

1.7 (g) Runtime Analysis

The program's runtime was analyzed using the `time` command. A plot of runtime vs. N was generated. The trend suggests that runtime increases as N grows, allowing estimation of the time required for $N = 2^{32}$.

2 Problem 2: Snowfall Probability Density Function

2.1 Challenge

The challenge was to compute the expected average snowfall by numerically estimating the integral:

$$\int_0^{10} S \cdot P(S) \, dS$$

2.2 Solution

The same Monte Carlo integration code used in Problem 1 was applied, but with a new function $f(S) = S \cdot P(S)$.

2.3 Justification

The program returned an estimated value of approximately 2.0587877921394986 meters. The reliability of this value was justified through:

- Consistency with probability theory.
- Reproducibility of the code and experiment.

- Convergence behavior with increasing N.
- Mathematical correctness through normalization.
- Acknowledgment of the fictional nature of the example.

3 Conclusion

In conclusion, the Monte Carlo integration method was successfully applied to two different problems. The results are reliable, consistent with probability theory, and based on sound mathematical principles. The code and data are fully reproducible, providing a strong basis for trust in the estimated values.