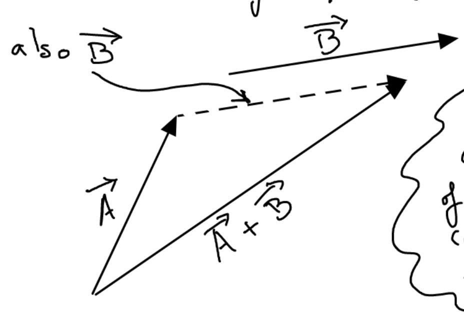
Force Vectors

Remember the common mistake students make

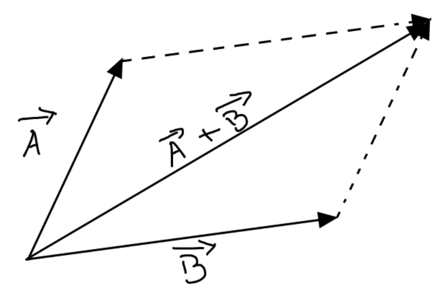
- Vector Addition

Triangle Addition

Parallelogram Addition



Use either law of sines or cosines to solve



- When 2 vectors are joined head to tail.
- Sum goes from tail of first to the head of second.

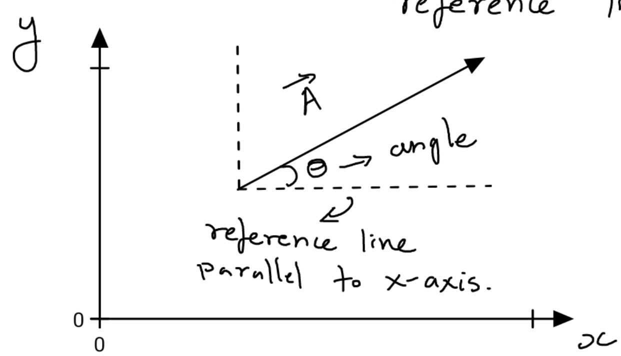
- When 2 vectors are joined ~~tail~~ to tail.
- Sum is the diagonal of the parallelogram formed by the two vectors as adjacent sides.

When I say that $\vec{A}$ and $\vec{B}$ are given to you, what do I mean?

- 2 pieces of information about both $\vec{A}$ and $\vec{B}$ are given.

These 2 pieces could be:

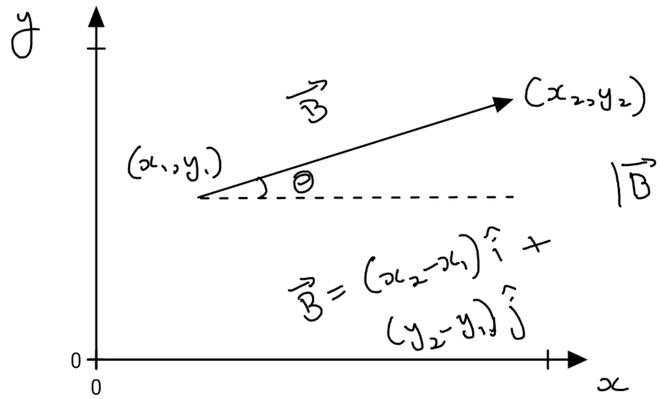
- Magnitude & An angle the vector makes with a reference line



Magnitude is the scalar number that conveys intensity or effect of the vector.

E.g., $|\vec{A}| = 10\ m$

- X and Y coordinate pair. (2D).



Here,

Magnitude

$$|\vec{B}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Direction

$$\tan\theta = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

This format is called the cartesian vector Format.

- Vector addition in a triangle or parallelogram is carried out through law of sines or cosines.

- Vector addition when cartesian vector format of 2 vectors are given.

$$\vec{A} = a_x\hat{i} + a_y\hat{j}$$

$$\vec{B} = b_x\hat{i} + b_x\hat{j}$$

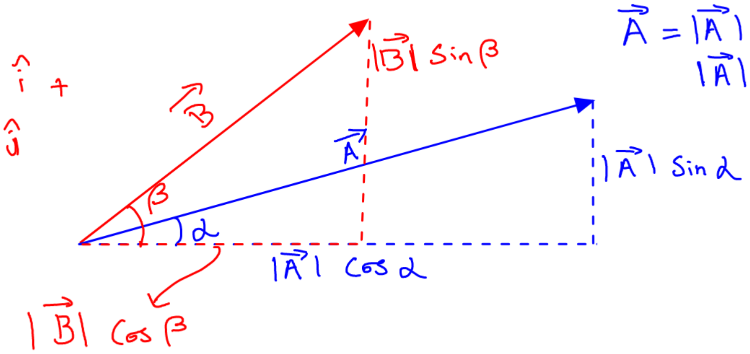
$$\vec{A} + \vec{B} = (a_x + b_x)\hat{i} + (a_y + b_y)\hat{j}$$

Just add the coefficients in $\hat{i}$, $\hat{j}$ directions independently.

- Vector addition when "magnitude and direction" format is given.

  - Resolve into components
  - Add independently X & Y components.

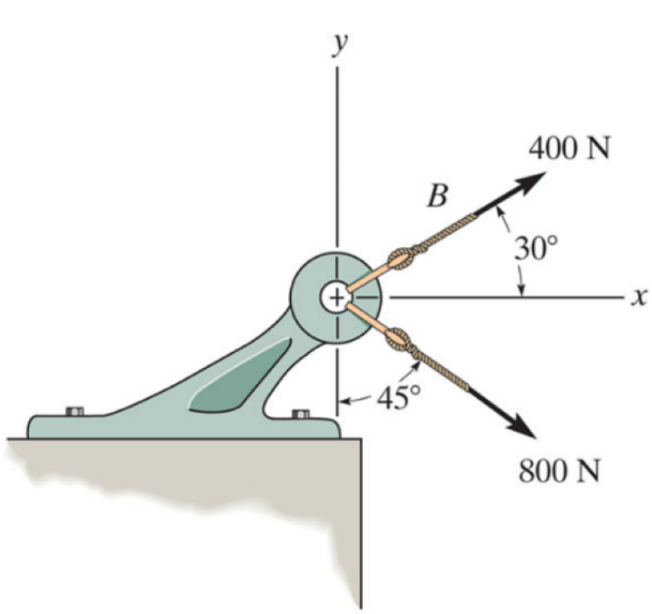
$\vec{B} = |\vec{B}| \cos\beta \, \hat{i} + |\vec{B}| \sin\beta \, \hat{j}$



$\vec{A} = |\vec{A}| \cos\alpha \, \hat{i} + |\vec{A}| \sin\alpha \, \hat{j}$

	X	Y
$\vec{A}$	$\lvert\vec{A}\rvert \cos\alpha$	$\lvert\vec{A}\rvert \sin\alpha$
$\vec{B}$	$\lvert\vec{B}\rvert \cos\beta$	$\lvert\vec{B}\rvert \sin\beta$
$\vec{A}+\vec{B}$	$\downarrow$ add up	$\downarrow$ add up.

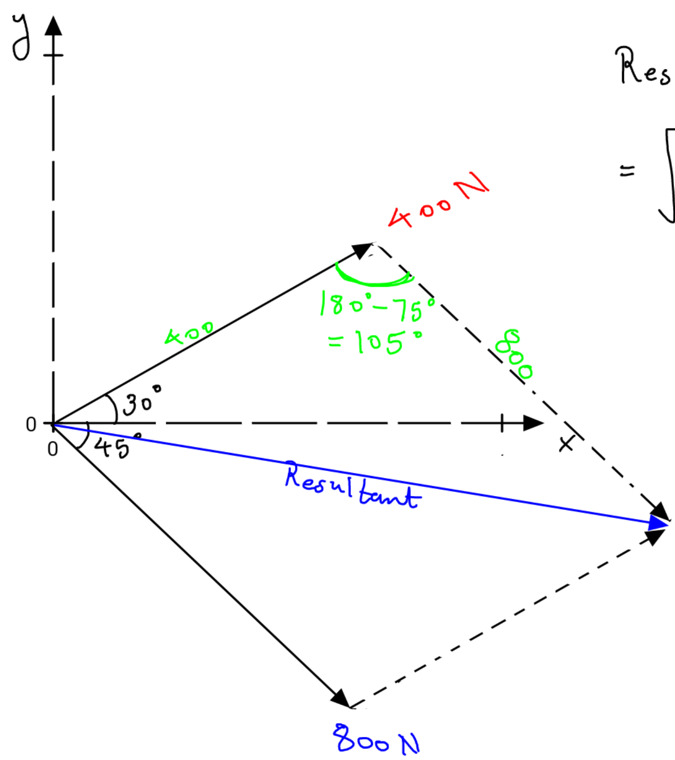
- And finally, why do we need vector addition in this course?
  - To add up all forces acting on a body to find the net force.



Problems
P-33.

We will approach this in 2 different ways.

First by parallelogram law.



Resultant, Cosine law

$$= \sqrt{400^2 + 800^2 - 2(400)(800)\cos(105^\circ)}$$

$$= \underline{\underline{983 \text{ N}}}$$

Direction? Sine law

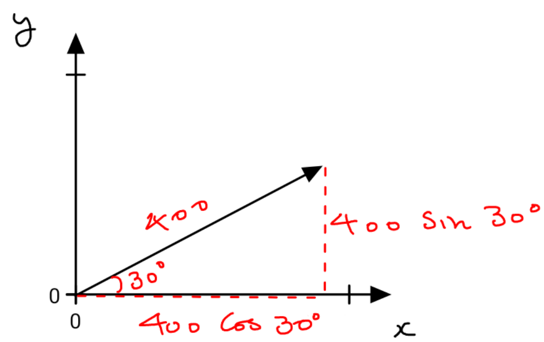
$$\frac{800}{\sin\alpha} = \frac{983}{\sin 105}$$

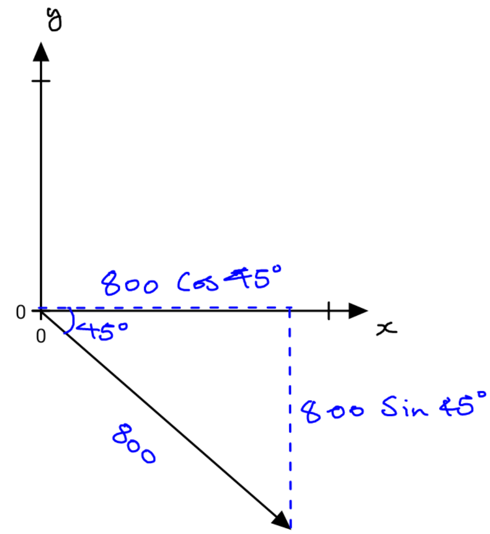
$$\sin\alpha = 0.786$$

$$\alpha = \underline{\underline{51.8^\circ}}$$

$$\therefore \theta = \alpha - 30^\circ = \underline{\underline{21.8^\circ}}$$

Next by resolving and adding.

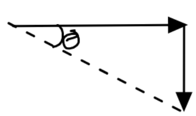




	A	B	R	
X	$400 \cos 30^\circ$	$800 \cos 45^\circ$	912.1	$\rightarrow$
Y	$400 \sin 30^\circ$	$-800 \sin 45^\circ$	365.69	$\downarrow$

$$\text{Magnitude} = \sqrt{(912.1)^2 + (365.69)^2} = 983\ \text{N}$$

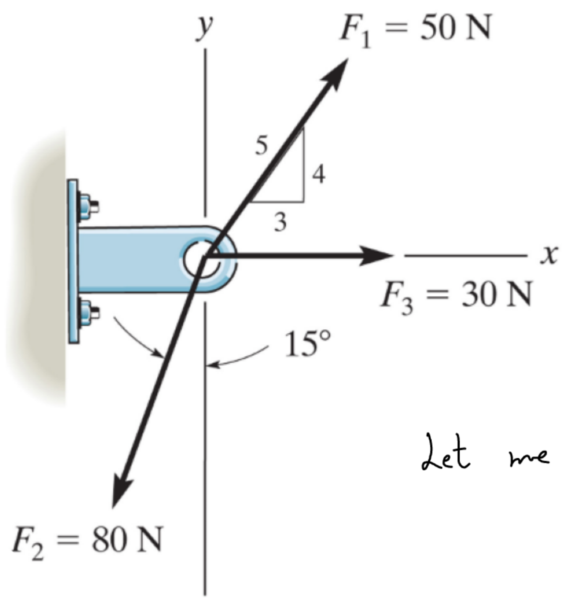
Direction $\Rightarrow$



$$\tan\theta = \frac{365.69}{912.1}$$

$$\theta = 21.8^\circ$$

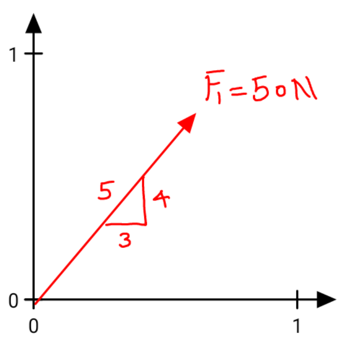
P. 2-38



Parallelogram law of addition will take a long time when you have 3 or more vectors to add.

I am going to use my favorite method :- Resolve & Add.

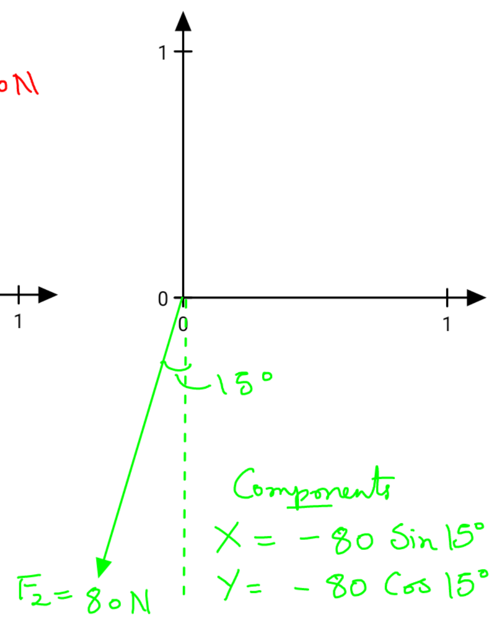
Let me pick each vector one by one.



Components

$X = 50\left(\frac{3}{5}\right)$

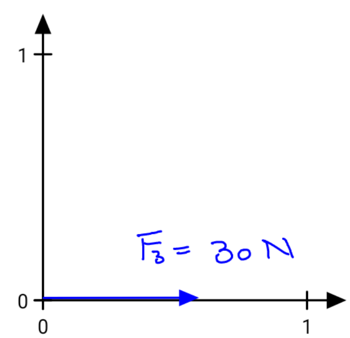
$Y = 50\left(\frac{4}{5}\right)$



Components

$X = -80 \sin 15°$

$Y = -80 \cos 15°$



Components

$X = 30$ N

$Y = 0$ N

Form ⇒ $X\hat{i} + Y\hat{j}$

Resultant ⇒ $\left[50\left(\frac{3}{5}\right) - 80 \sin 15° + 30\right]\hat{i} + \left[50\left(\frac{4}{5}\right) - 80 \cos 15° + 0\right]\hat{j}$ Newtons

Simplify yourselves.

Force Vectors (cont...)

Powerful!

- Unit vector — Position vector
- Right handed system
- Direction cosines
- Addition of 3D vectors
- Multiplication (Dot) —— Projection = Effect.

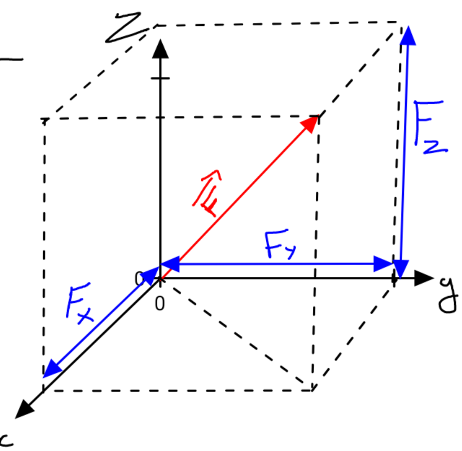
Angle between 2 vectors.

Force Vectors (cont...)

Review Cartesian Vector Form

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{j}$$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$$



What is the "right" way to draw a 3-D coordinate system?

You have to use a lot of imagination to visualize 3-D vectors.

Direction Angles,

$$\cos\alpha = \frac{F_x}{F} \; ; \; \cos\beta = \frac{F_y}{F} \; ; \; \cos\gamma = \frac{F_z}{F}$$

But fear not, because unit vectors are here to help to keep track.

$$\vec{u}_F = \frac{\vec{F}}{|\vec{F}|}$$

$$= \frac{F_x}{F}\hat{i} + \frac{F_y}{F}\hat{j} + \frac{F_z}{F}\hat{k}$$

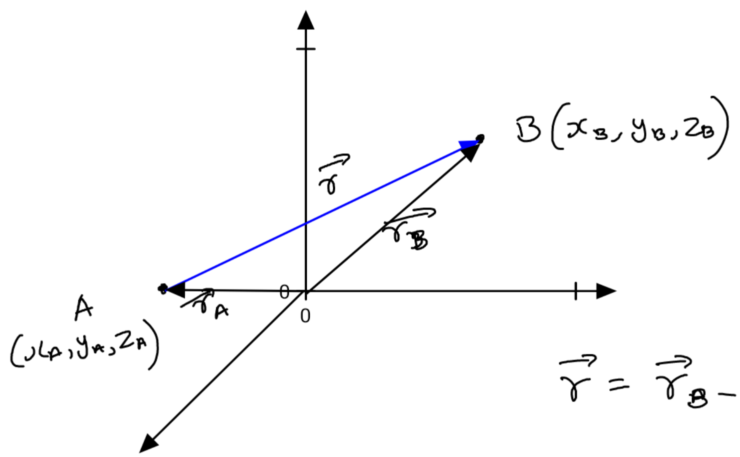
Why the name "unit" vector?

$$|\vec{u}_F| = 1 = \cos^2\alpha + \cos^2\beta + \cos^2\gamma$$

Independent of physics! Stores the direction information like a memory chip.

Position vectors are excellent candidates to determine unit vectors !

$$\vec{r} = (x_B - x_A)\,\hat{i} + (y_B - y_A)\,\hat{j} + (z_B - z_A)\,\hat{k}$$



From triangle addition.

$$\vec{r} = \vec{r}_B - \vec{r}_A$$

Trust the math !

Force directed along a line,

$$\vec{F} = F\hat{u}_r = F\left(\frac{\vec{r}}{|\vec{r}|}\right)$$

Dot product (Scalar produc

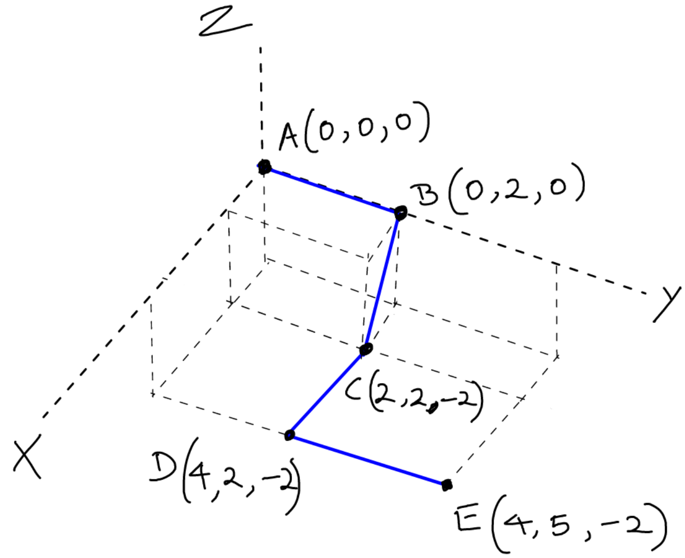
$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$$

$$\overbrace{\vec{A} \cdot \vec{B}}^{\text{Vector}} = \overbrace{A_x B_x + A_y B_y + A_z B_z}^{\text{Scalar}}$$

Application :
- Angle between 2 vectors
- Projections.

Problems.

2-113



$\vec{F}$ goes from E to B.

$$\vec{r}_{EB} = (0-4)\hat{i} + (2-5)\hat{j} + (0-(-2))\hat{k}$$

$$= -4\hat{i} - 3\hat{j} + 2\hat{k}$$

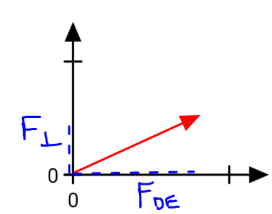
$$\hat{u}_{AB} = \frac{-4\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{4^2+3^2+2^2}} = \underbrace{-0.743\hat{i} - 0.557\hat{j} + 0.371\hat{k}}_{\text{Direction Cosines.}}$$

$$\vec{F} = 600\left(\hat{u}_{AB}\right)$$

$$= -445.67\hat{i} - 334.25\hat{j} + 222.83\hat{k}$$

$$\vec{r}_{DE} = 0\hat{i} + 3\hat{j} + 0\hat{k} = 3\hat{j}$$

$$\hat{u}_{DE} = \hat{j}$$



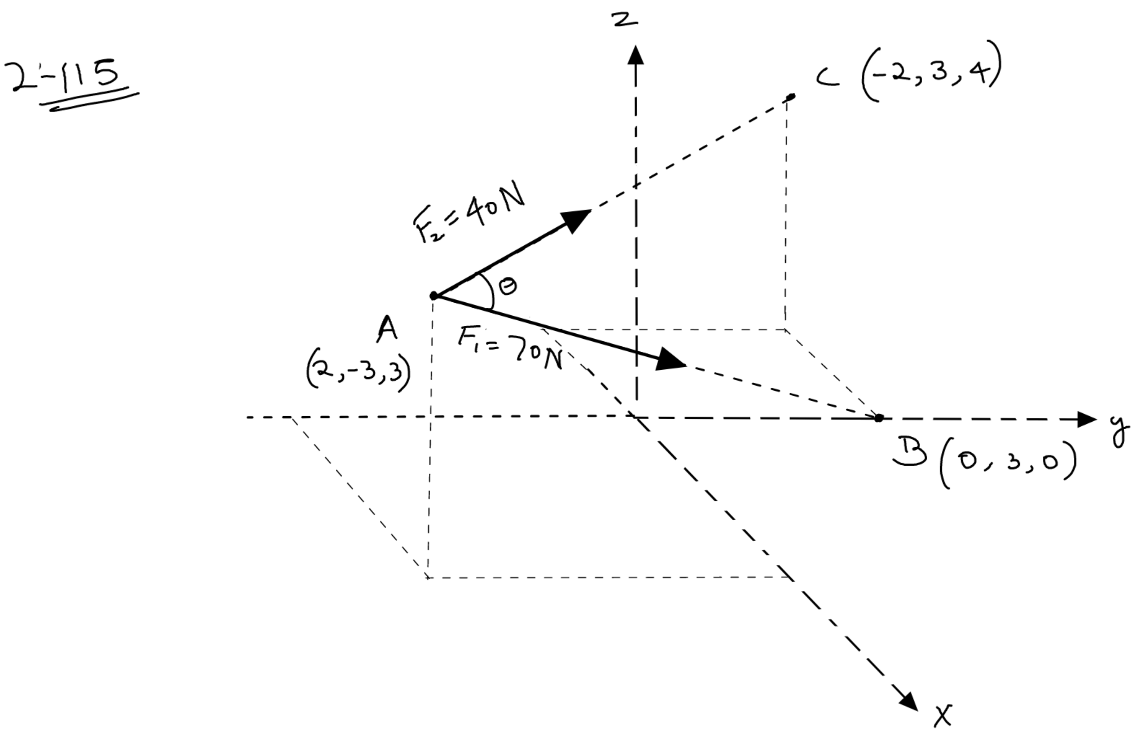
Parallel $\Rightarrow$ $\vec{F} \cdot \hat{u}_{DE} = -334.25 \ \underline{\underline{N}}$

$$\vec{F}_{DE} = -334.25\hat{j}$$

Perpendicular $\Rightarrow$ $\vec{F}_{\perp} = \vec{F} - \vec{F}_{DE} = -445.67\hat{i} + 222.83\hat{k}$

$$F_{\perp} = \sqrt{445.67^2 + 222.83^2}$$

$$= \underline{\underline{498.27 \text{ N}}}$$



$$\hat{u}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = \frac{(-2)\hat{i} + (6)\hat{j} + (-3)\hat{k}}{\sqrt{4 + 36 + 9}} = -\frac{2}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{3}{7}\hat{k}$$

check $|\hat{u}_{AB}| = 1$

$$F_1 = 70\left(-\frac{2}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{3}{7}\hat{k}\right) = -20\hat{i} + 60\hat{j} - 30\hat{k}$$

$$\hat{u}_{AC} = \frac{\vec{r}_{AC}}{|\vec{r}_{AC}|} = \frac{(-4)\hat{i} + (6)\hat{j} + (1)\hat{k}}{\sqrt{16 + 36 + 1}} = -0.549\hat{i} + 0.824\hat{j} + 0.137\hat{k}$$

$$\text{Projection} = F_1 \cdot \hat{u}_{AC}$$

$$= 10.98\hat{i} + 49.44\hat{j} - 4.11\hat{k}$$

$$\text{Magnitude} = \sqrt{10.98^2 + 49.44^2 + 4.11^2} = \underline{\underline{50.8 \text{ N}}}$$

Equilibrium of a Particle.

Static Equilibrium

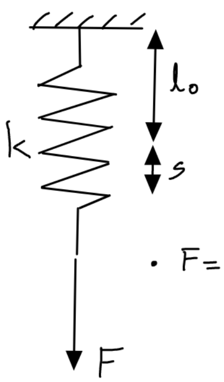
Under static equilibrium, the net force acting on a body is zero.

$$\sum F = 0 .$$

$$\text{Equilibrium} \Rightarrow \sum F = 0$$

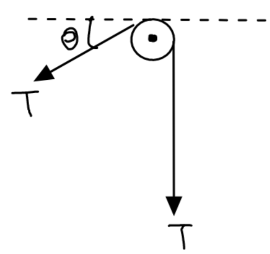
$$\sum F = 0 \Rightarrow \text{Equilibrium}$$

Springs



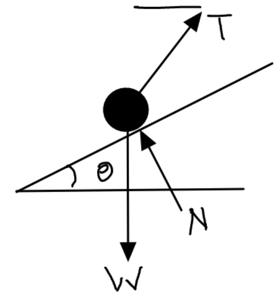
- $F = ks$

Cables & Pulleys



- T along rope is the same

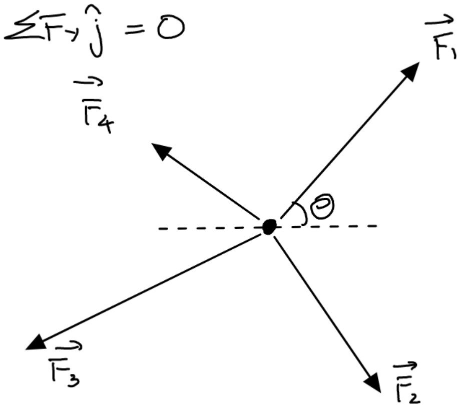
Smooth Contact



- Weight acts down
- Normal force acts $\perp$.

$$\sum \vec{F} = 0$$

$$\Rightarrow \quad \sum F_x \hat{i} + \sum F_y \hat{j} = 0$$



$$F_{1x} + F_{2x} + F_{3x} + F_{4x} = 0$$
$$F_{1y} + F_{2y} + F_{3y} + F_{4y} = 0$$

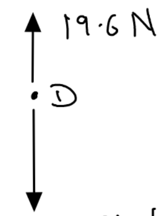
$$F_{1x} = F \cos\theta$$
$$F_{1y} = F \sin\theta$$

Isolating a point as shown above with all forces acting on it displayed is called a Free Body Diagram (FBD)

Problems

3-14

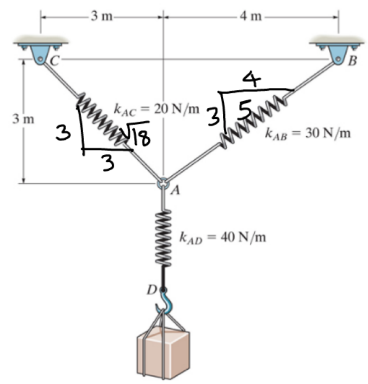
FBD.

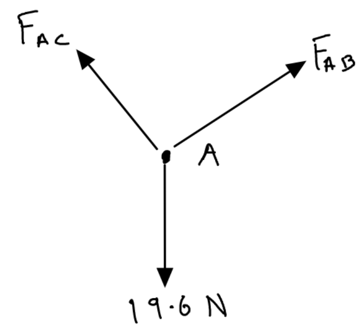


$2kg \times 9.81 m/s^2$

$= \underline{\underline{19.6\ N}}$

Problems 14-15





	$F_{19.6}$	F_{AB}	F_{AC}	
X	0	$F_{AB}\left(\frac{4}{5}\right)$	$-F_{AC}\left(\frac{3}{\sqrt{18}}\right)$	$= 0$
Y	-19.6	$F_{AB}\left(\frac{3}{5}\right)$	$F_{AC}\left(\frac{3}{\sqrt{18}}\right)$	$= 0$

Solve to get $F_{AC} = 15.86\ N$

$F_{AB} = 14.01\ N$

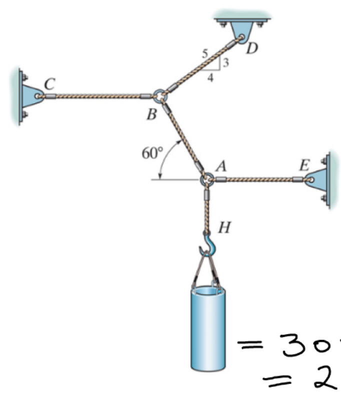
$$s_{AD} = \frac{19.6}{40} = 0.4905\ m$$

$$s_{AC} = \frac{15.86}{20} = 0.793\ m$$

$$s_{AB} = \frac{14.01}{30} = \underline{\underline{0.467\ m}}$$

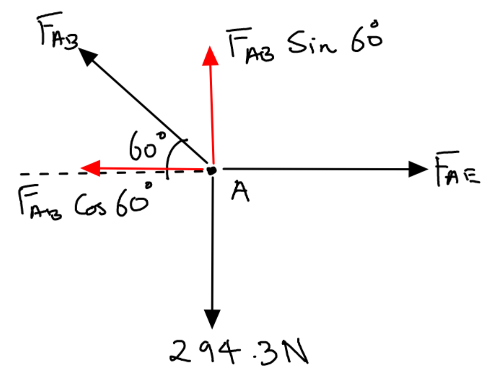
3-26

Problems 26-27



$= 30 \times 9.81$
$= 294.3\ N$

FBD

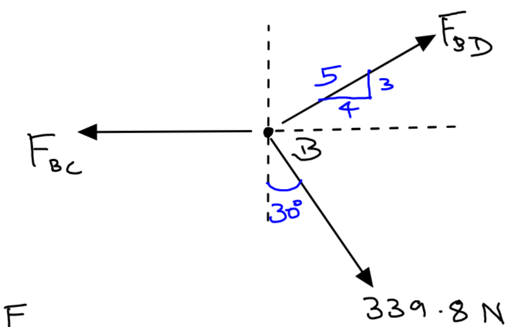


$F_{AB}\sin 60° = 294.3\ N$

$F_{AB} = \underline{\underline{339.8}}\ N$

$339.8\cos 60° = F_{AE}$

$F_{AE} = \underline{\underline{169.9}}\ N$



	F 339.8	F_{BC}	F_{BD}	
X	$339.8(\sin 30°)$	$-F_{BC}$	$F_{BD}\left(\frac{4}{5}\right)$	$= 0$
Y	$-339.8(\cos 30°)$	0	$F_{BD}\left(\frac{3}{5}\right)$	$= 0$

Solving, $F_{BD} = 490\ N$
$F_{BC} = 562\ N$