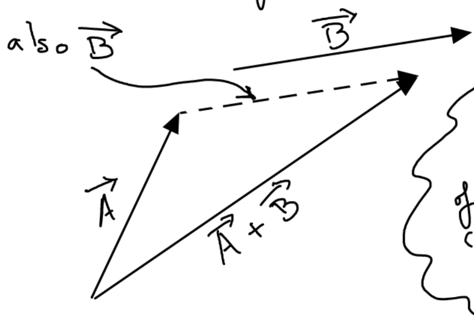
Force Vectors

Remember the common mistake students make

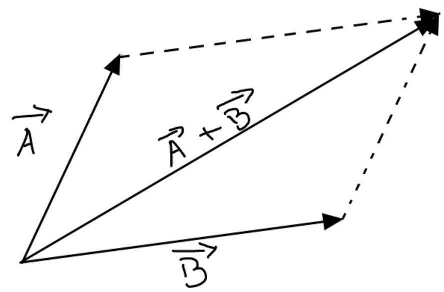
- Vector Addition

Triangle Addition



Use either law of sines or cosines to solve

Parallelogram Addition



- When 2 vectors are joined head to tail.
- Sum goes from tail of first to the head of second.

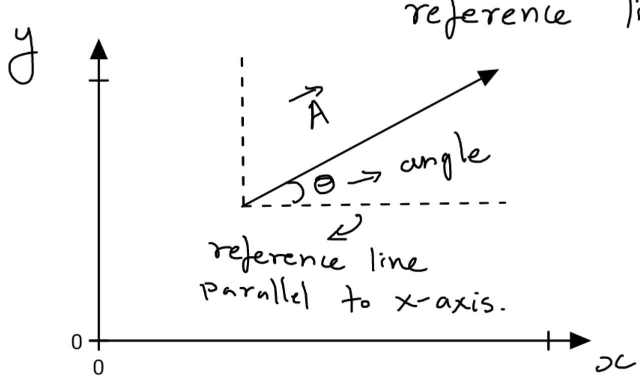
- When 2 vectors are joined ~~tail~~ tail to tail.
- Sum is the diagonal of the parallelogram formed by the two vectors as adjacent sides.

When I say that $\vec{A}$ and $\vec{B}$ are given to you, what do I mean?

- 2 pieces of information about both $\vec{A}$ and $\vec{B}$ are given.

These 2 pieces could be:

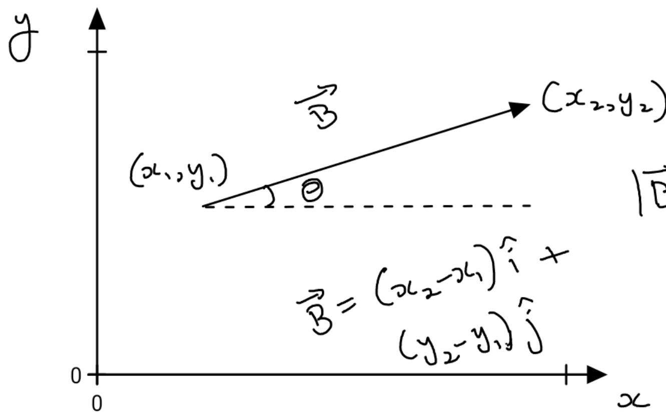
- Magnitude & An angle the vector makes with a reference line



Magnitude is the scalar number that conveys intensity or effect of the vector.

E.g., $|\vec{A}| = 10$ m

- X and Y coordinate pair. (2D).



Here,

Magnitude

$$|\vec{B}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Direction

$$\tan \theta = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

This format is called the cartesian vector format.

- Vector addition in a triangle or parallelogram is carried out through law of sines or cosines.

- Vector addition when cartesian vector format of 2 vectors are given.

$$\vec{A} = a_x \hat{i} + a_y \hat{j}$$

$$\vec{B} = b_x \hat{i} + b_x \hat{j}$$

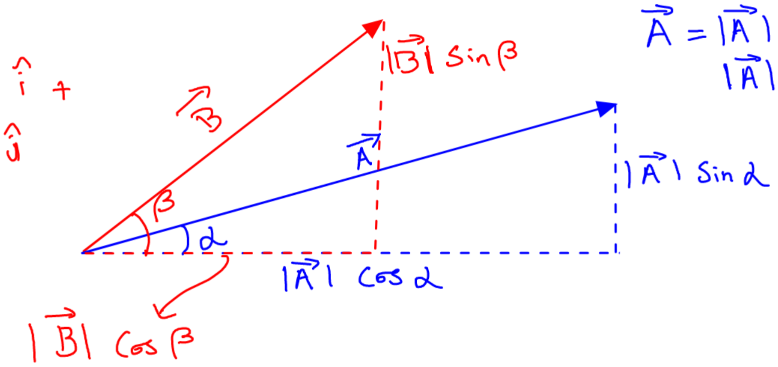
$$\vec{A} + \vec{B} = (a_x + b_x)\hat{i} + (a_y + b_y)\hat{j}$$

Just add the coefficients in $\hat{i}, \hat{j}$ directions independently.

- Vector addition when "magnitude and direction" format is given.
  - Resolve into components
  - Add independently X & Y components.

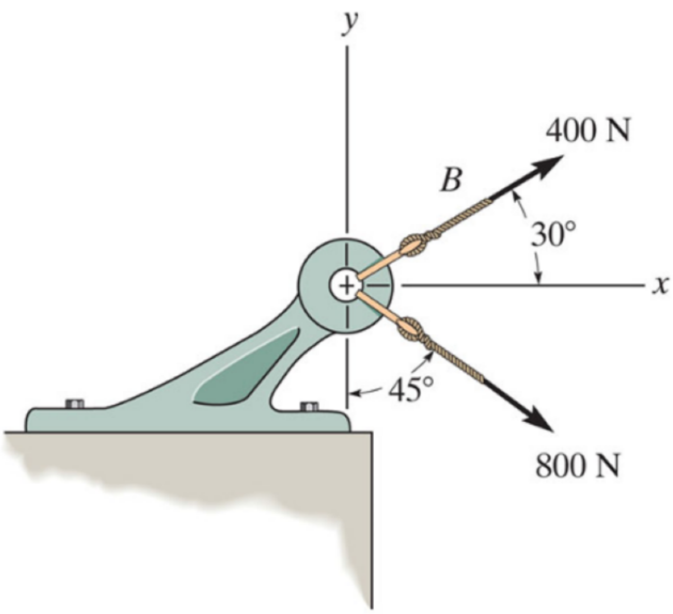
$\vec{B} = |\vec{B}| \cos\beta \, \hat{i} + |\vec{B}| \sin\beta \, \hat{j}$

$\vec{A} = |\vec{A}| \cos\alpha \, \hat{i} + |\vec{A}| \sin\alpha \, \hat{j}$



	X	Y
$\vec{A}$	$\lvert\vec{A}\rvert \cos\alpha$	$\lvert\vec{A}\rvert \sin\alpha$
$\vec{B}$	$\lvert\vec{B}\rvert \cos\beta$	$\lvert\vec{B}\rvert \sin\beta$
$\vec{A}+\vec{B}$	↓ add up	↓ add up.

- And finally, why do we need vector addition in this course?
  - To add up all forces acting on a body to find the net force.



Problems

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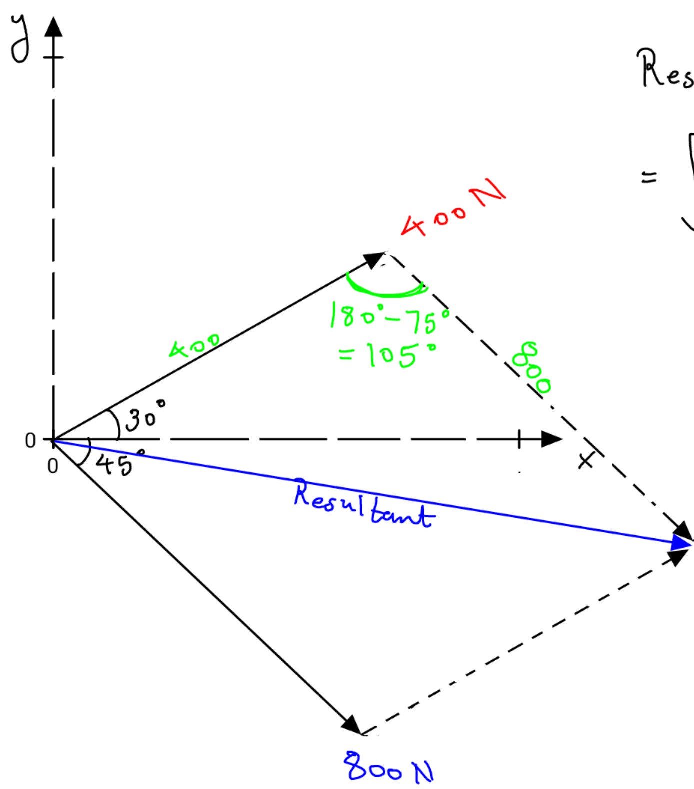
We will approach this in 2 different ways.

First by parallelogram law.



Resultant, Cosine law

$$= \sqrt{400^2 + 800^2 - 2(400)(800)\cos(105^\circ)}$$

$$= \underline{\underline{983\ N}}$$

Direction? Sine law

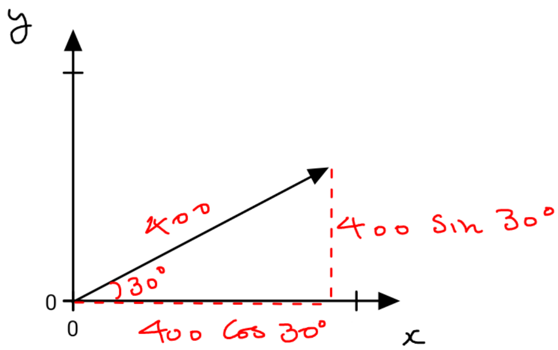
$$\frac{800}{\sin\alpha} = \frac{983}{\sin 105}$$

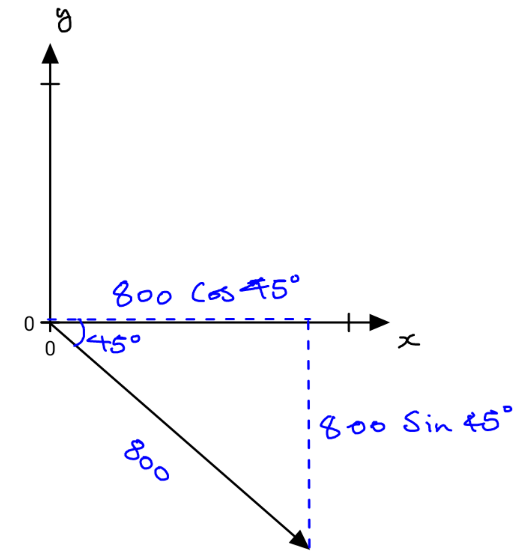
$$\sin\alpha = 0.786$$

$$\alpha = \underline{\underline{51.8^\circ}}$$

$$\therefore \theta = \alpha - 30^\circ = \underline{\underline{21.8^\circ}}$$

Next by resolving and adding.

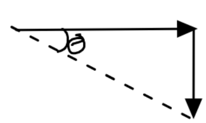




	A	B	R	
X	$400 \cos 30^\circ$	$800 \cos 45^\circ$	912.1	$\rightarrow$
Y	$400 \sin 30^\circ$	$-800 \sin 45^\circ$	365.69	$\downarrow$

$$\text{Magnitude} = \sqrt{(912.1)^2 + (365.69)^2} = \underline{\underline{983 \text{ N}}}$$

Direction $\Rightarrow$



$$\tan\theta = \frac{365.69}{912.1}$$

$$\theta = \underline{\underline{21.8^\circ}}$$

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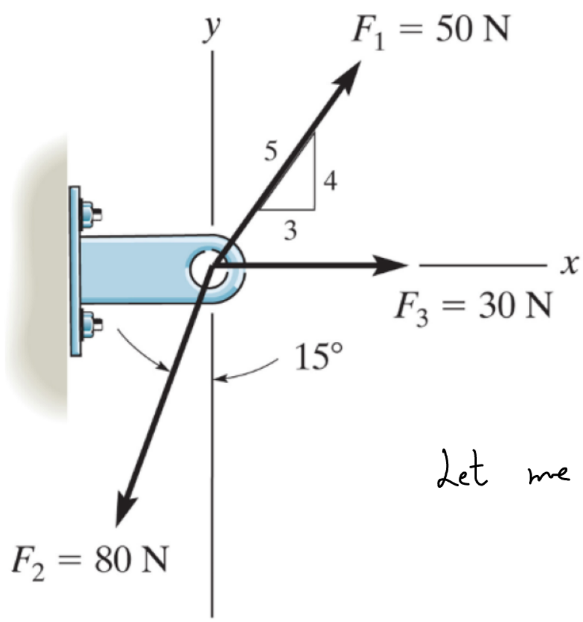
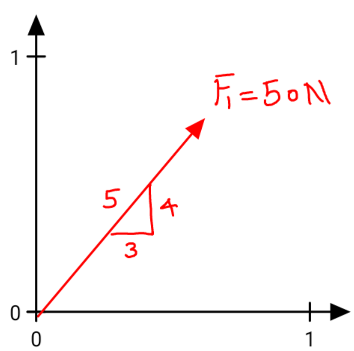


Parallelogram law of addition will take a long time when you have 3 or more vectors to add.

I am going to use my favorite method :- Resolve & Add.

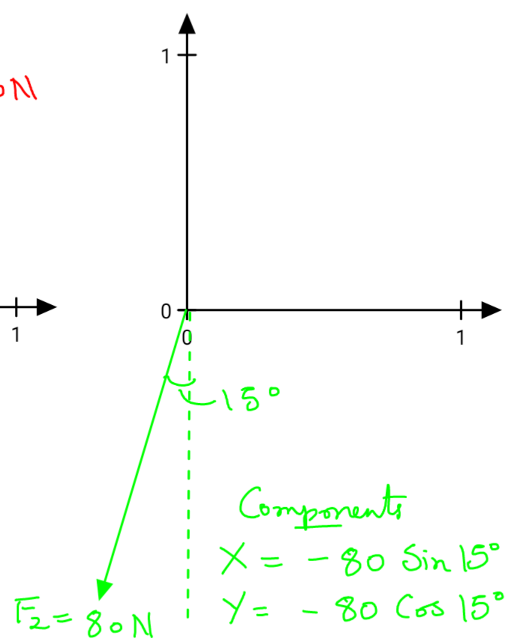
Let me pick each vector one by one.



Components

$X = 50\left(\frac{3}{5}\right)$

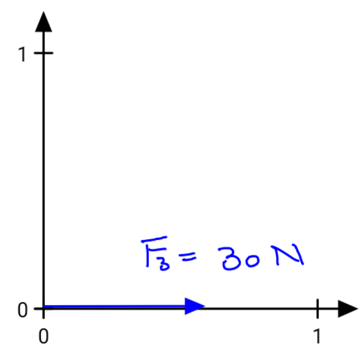
$Y = 50\left(\frac{4}{5}\right)$



Components

$X = -80 \sin 15°$

$Y = -80 \cos 15°$



Components

$X = 30$ N

$Y = 0$ N

Form $\Rightarrow \quad X\hat{i} + Y\hat{j}$

Resultant $\Rightarrow \left[50\left(\frac{3}{5}\right) - 80 \sin 15° + 30\right]\hat{i}$ +

$\left[50\left(\frac{4}{5}\right) - 80 \cos 15° + 0\right]\hat{j}$ Newtons

Simplify yourselves.

Force Vectors (cont...)

Powerful!

- Unit vector — Position vector
- Right handed system
- Direction cosines
- Addition of 3D vectors
- Multiplication (Dot) Projection = Effect.

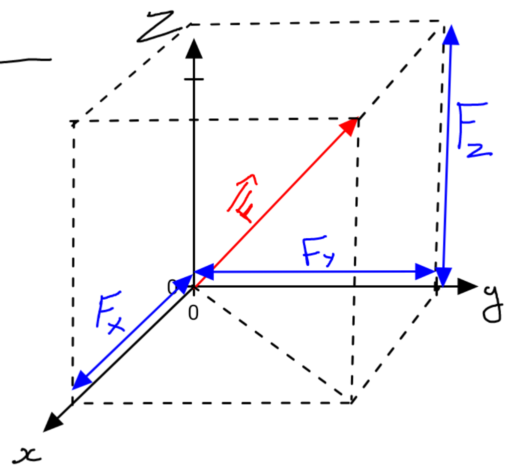
Angle between 2 vectors.

Force Vectors (cont...)

Review Cartesian Vector Form

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{j}$$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$$



What is the "right" way to draw a 3-D coordinate system?

You have to use a lot of imagination to visualize 3-D vectors.

Direction Angles,

$$\cos\alpha = \frac{F_x}{F} \;;\; \cos\beta = \frac{F_y}{F} \;;\; \cos\gamma = \frac{F_z}{F}$$

But fear not, because unit vectors are here to help to keep track.

$$\vec{u}_F = \frac{\vec{F}}{|\vec{F}|}$$

$$= \frac{F_x}{F}\hat{i} + \frac{F_y}{F}\hat{j} + \frac{F_z}{F}\hat{k}$$

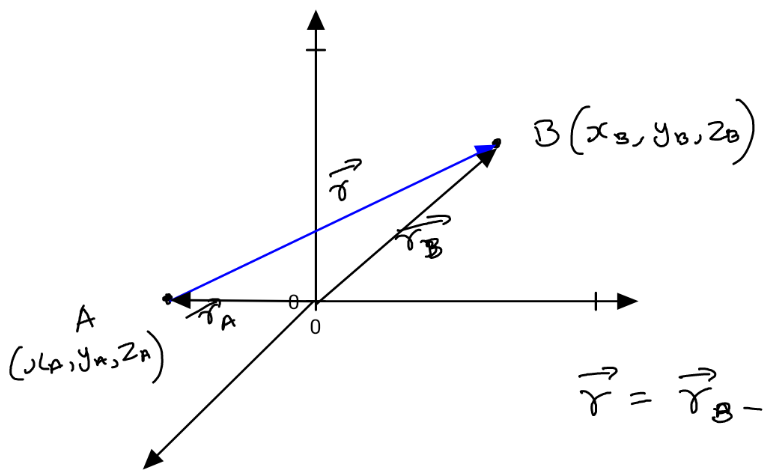
Why the name "unit" vector?

$$|\vec{u}_F| = 1 = \cos^2\alpha + \cos^2\beta + \cos^2\gamma$$

Independent of physics! Stores the direction information like a memory chip.

Position vectors are excellent candidates to determine unit vectors!

$$\vec{r} = (x_B - x_A)\hat{i} + (y_B - y_A)\hat{j} + (z_B - z_A)\hat{k}$$



From triangle addition.

$$\vec{r} = \vec{r}_B - \vec{r}_A$$

Force directed along a line,

$$\vec{F} = F\hat{u}_r = F\left(\frac{\vec{r}}{|\vec{r}|}\right)$$

Trust the math!

Dot product (Scalar produc

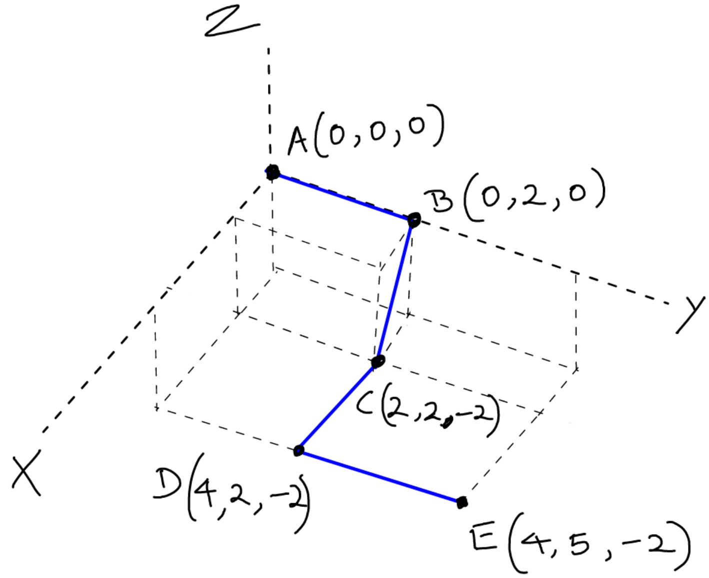
$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$$

$$\overbrace{\vec{A} \cdot \vec{B}}^{\text{Vector}} = \overbrace{A_x B_x + A_y B_y + A_z B_z}^{\text{Scalar}}$$

Application:
- Angle between 2 vectors
- Projections.

Problems.

2-113



$\vec{F}$ goes from E to B.

$$\vec{r}_{EB} = (0-4)\hat{i} + (2-5)\hat{j} + (0-(-2))\hat{k}$$

$$= -4\hat{i} - 3\hat{j} + 2\hat{k}$$

$$\hat{u}_{AB} = \frac{-4\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{4^2+3^2+2^2}} = \underbrace{-0.743\hat{i} - 0.557\hat{j} + 0.371\hat{k}}_{\text{Direction Cosines.}}$$

$$\vec{F} = 600\,(\hat{u}_{AB})$$

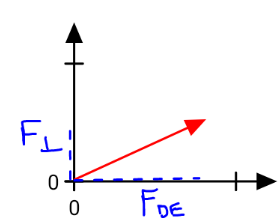
$$= -445.67\hat{i} - 334.25\hat{j} + 222.83\hat{k}$$

$$\vec{r}_{DE} = 0\hat{i} + 3\hat{j} + 0\hat{k} = 3\hat{j}$$

$$\hat{u}_{DE} = \hat{j}$$

Parallel $\Rightarrow$ $\vec{F}\cdot\hat{u}_{DE} = -334.25$ N

$$\vec{F}_{DE} = -334.25\hat{j}$$

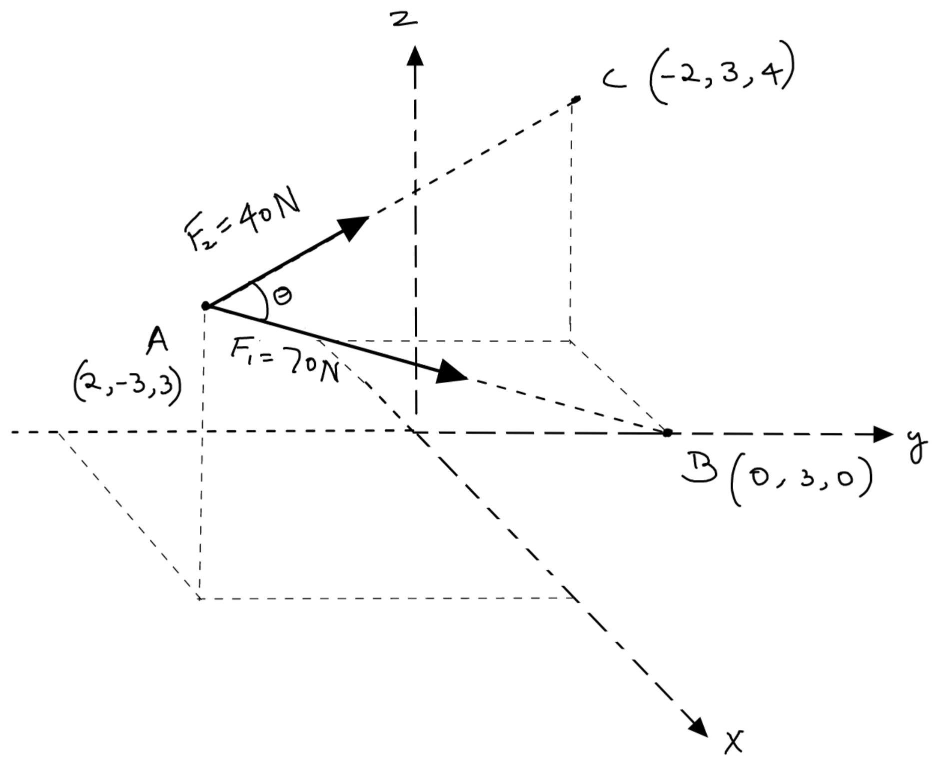


Perpendicular $\Rightarrow$ $\vec{F}_{\perp} = \vec{F} - \vec{F}_{DE} = -445.67\hat{i} + 222.83\hat{k}$

$$F_{\perp} = \sqrt{445.67^2 + 222.83^2}$$

$$= \underline{\underline{498.27\ N}}$$

2-115



$$\hat{u}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = \frac{(-2)\hat{i} + (6)\hat{j} + (-3)\hat{k}}{\sqrt{4 + 36 + 9}} = -\frac{2}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{3}{7}\hat{k}$$

check $|\hat{u}_{AB}| = 1$

$$F_1 = 70\left(-\frac{2}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{3}{7}\hat{k}\right) = -20\hat{i} + 60\hat{j} - 30\hat{k}$$

$$\hat{u}_{AC} = \frac{\vec{r}_{AC}}{|\vec{r}_{AC}|} = \frac{(-4)\hat{i} + (6)\hat{j} + (1)\hat{k}}{\sqrt{16 + 36 + 1}} = -0.549\hat{i} + 0.824\hat{j} + 0.137\hat{k}$$

$$\text{Projection} = F_1 \cdot \hat{u}_{AC}$$

$$= 10.98\hat{i} + 49.44\hat{j} - 4.11\hat{k}$$

$$\text{Magnitude} = \sqrt{10.98^2 + 49.44^2 + 4.11^2} = \underline{\underline{50.8\ N}}$$

Equilibrium of a Particle.

Static Equilibrium

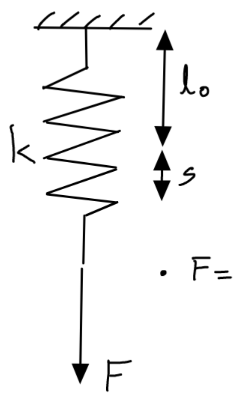
Under static equilibrium, the net force acting on a body is zero.

$$\Sigma F = 0 .$$

$$\text{Equilibrium} \Rightarrow \Sigma F = 0$$

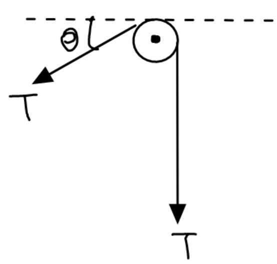
$$\Sigma F = 0 \Rightarrow \text{Equilibrium}$$

Springs



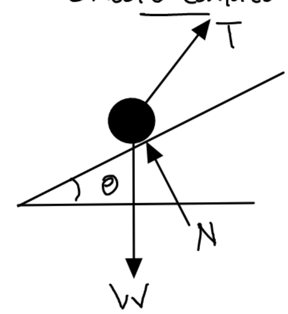
- $F = ks$

Cables & Pulleys



- T along rope is the same

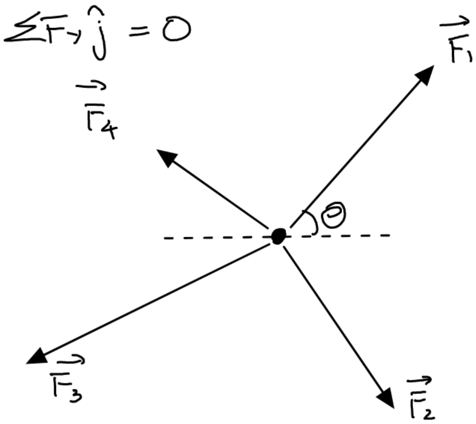
Smooth Contact



- Weight acts down
- Normal force acts ⊥.

$$\Sigma \vec{F} = 0$$

$$\Rightarrow \quad \Sigma F_x \hat{i} + \Sigma F_y \hat{j} = 0$$



$$F_{1x} + F_{2x} + F_{3x} + F_{4x} = 0$$
$$F_{1y} + F_{2y} + F_{3y} + F_{4y} = 0$$

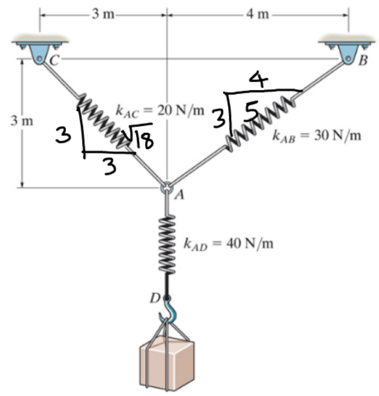
$$F_{1x} = F \cos\theta$$
$$F_{1y} = F \sin\theta$$

Isolating a point as shown above with all forces acting on it displayed is called a Free Body Diagram (FBD)

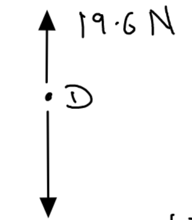
Problems

3-14

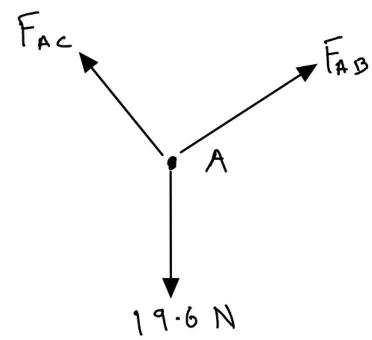
Problems 14-15



FBD.



$2\,kg \times 9.81\,m/s^2$
$= 19.6\ N$



	$F_{19.6}$	F_{AB}	F_{AC}	
X	0	$F_{AB}\left(\frac{4}{5}\right)$	$-F_{AC}\left(\frac{3}{\sqrt{18}}\right)$	$= 0$
Y	-19.6	$F_{AB}\left(\frac{3}{5}\right)$	$F_{AC}\left(\frac{3}{\sqrt{18}}\right)$	$= 0$

Solve to get $F_{AC} = 15.86\ N$
$F_{AB} = 14.01\ N$

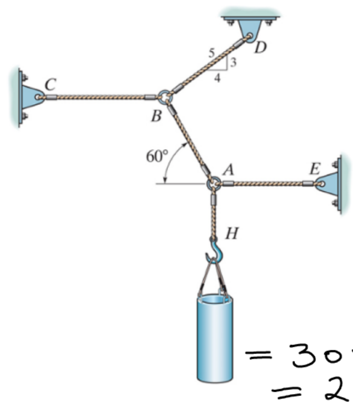
$$S_{AD} = \frac{19.6}{40} = 0.4905\ m$$

$$S_{AC} = \frac{15.86}{20} = 0.793\ m$$

$$S_{AB} = \frac{14.01}{30} = 0.467\ m$$

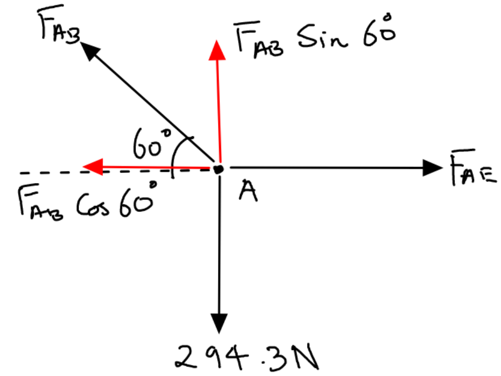
3-26

Problems 26-27



$= 30 \times 9.81$
$= 294.3\ N$

FBD.



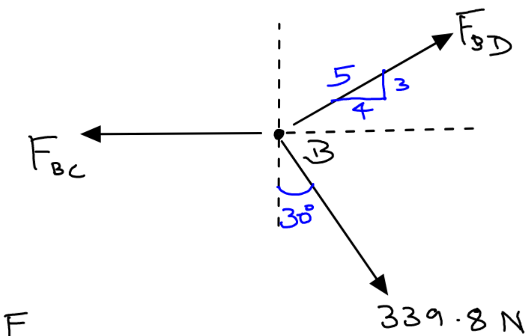
$\uparrow \sum F_y = 0$

$F_{AB} \sin 60° = 294.3\ N$

$F_{AB} = \underline{\underline{339.8\ N}}$

$339.8 \cos 60° = F_{AE}$

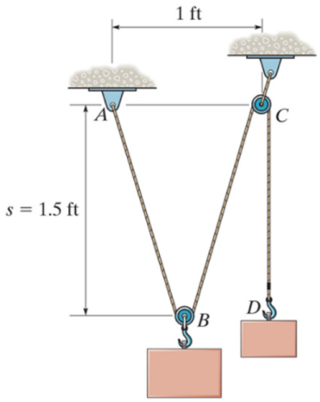
$F_{AE} = \underline{\underline{169.9\ N}}$



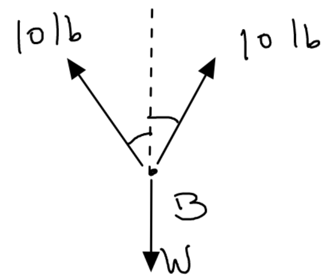
	F 339.8	F_{BC}	F_{BD}	
$\rightarrow \sum F_x = 0$ X	$339.8(\sin 30°)$	$-F_{BC}$	$F_{BD}\left(\frac{4}{5}\right)$	$= 0$
$\uparrow \sum F_y = 0$ Y	$-339.8(\cos 30°)$	0	$F_{BD}\left(\frac{3}{5}\right)$	$= 0$

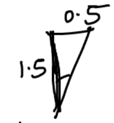
Solving, $F_{BD} = 490\ N$
$F_{BC} = \underline{\underline{562\ N}}$

3-42



FBD.





$\tan\theta = \frac{0.5}{1.5}$

$= 1/3$

Only Y-components needed for solution.

$\uparrow \Sigma F_y = 0$

$$W = 2 \times 10 \times \underbrace{\frac{1.5}{\sqrt{10}}}_{\text{cosine}}$$

$$= 9.49 \text{ lb}$$

Force System Resultants
Moments

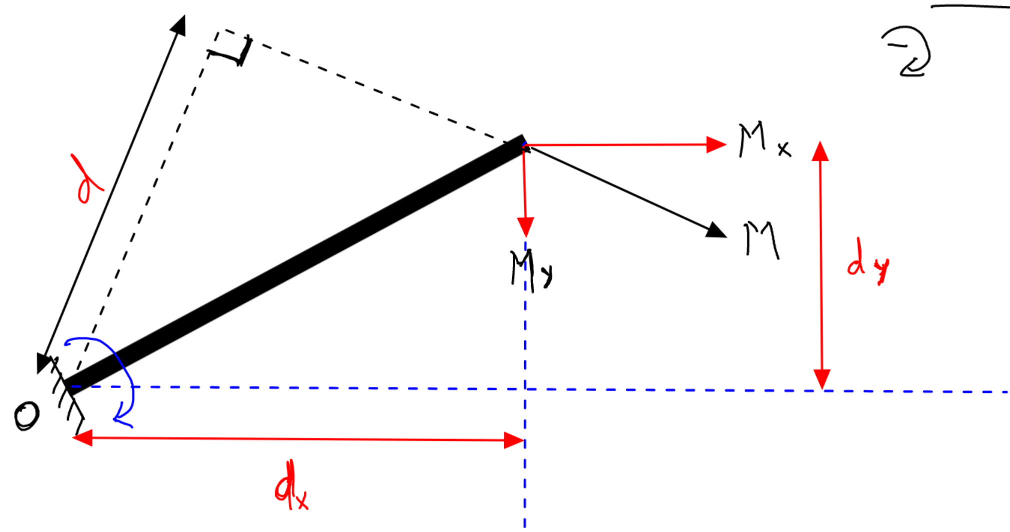
Until now you solved force systems when the forces act on a single point. (Concentric)

What happens when they don't? (Eccentric)
- Things tend to rotate.
- The cause of this rotation ⇒ Moment.

$$\underline{\underline{M = F d}}$$

Sign convention

$\circlearrowleft -$ $\quad$ $\circlearrowright +$

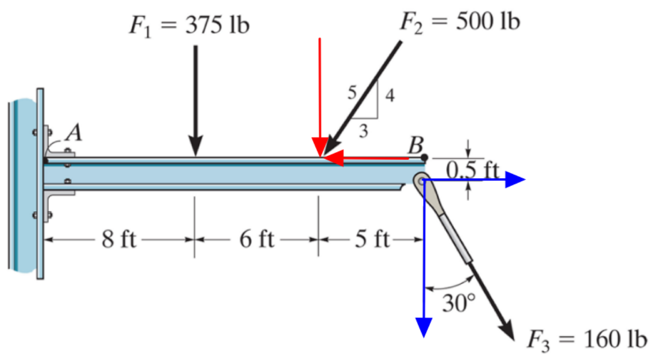


$$\boxed{M_O = M d = M_x (d_y) + M_y (d_x)}$$

Principle of Moments

Problems

4-5 //



$\underline{F_1}$

$$M_{B_1} = 375 \times 11$$
$$= 4125 \text{ lb-ft}$$
$$= 4.13 \text{ k-ft} \circlearrowleft$$

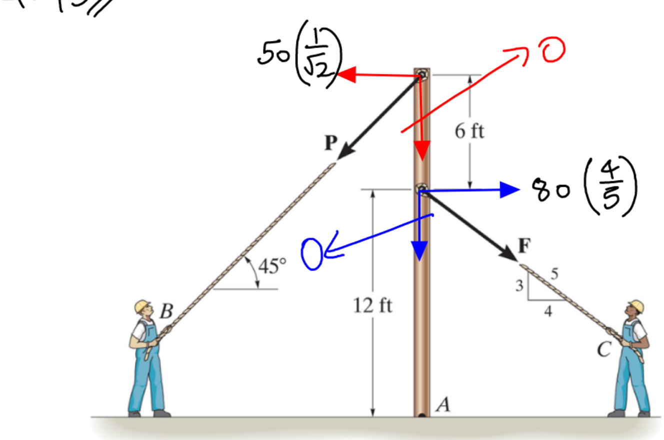
$\underline{F_2}$

$$M_{B_2} = 500\left(\frac{4}{5}\right)(5) + 0$$
$$= 2000 \text{ lb-ft}$$
$$= \underline{\underline{2 \text{ k ft}}} \circlearrowleft$$

$\underline{F_3}$

$$-160 \sin 30° (0.5) + 0$$
$$= \underline{\underline{40 \text{ lb-ft}}} \circlearrowright$$

4-15 //



$$M_A = -80\left(\frac{4}{5}\right)(12) + 50\left(\frac{1}{\sqrt{2}}\right)(18)$$
$$= -768 + 636$$
$$= -132 \text{ lb-ft}$$
$$= \underline{\underline{-ve}}$$

$\therefore$ $\circlearrowright$ Clockwise.

Moment of a Force - Vector formulation

Last class, we talked about moment. $M = F.d$

- There were some rules like distance has to be perpendicular to the force. This is because we were using scalar analysis.
- Can we generalize the concept of Moment? Beauty of vectors?

Cross-Product.

$$\vec{A} \times \vec{B} = (|\vec{A}||\vec{B}| \sin\theta)\, \hat{u}_c$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A} \qquad \vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

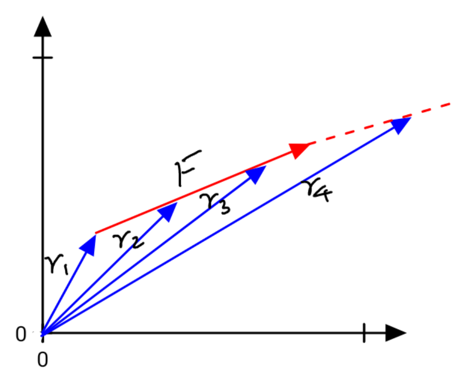
$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \qquad \vec{A} \cdot \vec{B} = a_x b_x + a_y b_y + a_z b_z$$

- Remember position vector?

$$\vec{r} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k}$$

$$\vec{M} = \vec{r} \times \vec{F}$$

$$\vec{M} = \vec{r}_1 \times \vec{F} = \vec{r}_2 \times \vec{F} = \vec{r}_3 \times \vec{F} = \ldots$$



Principle of moments still apply.

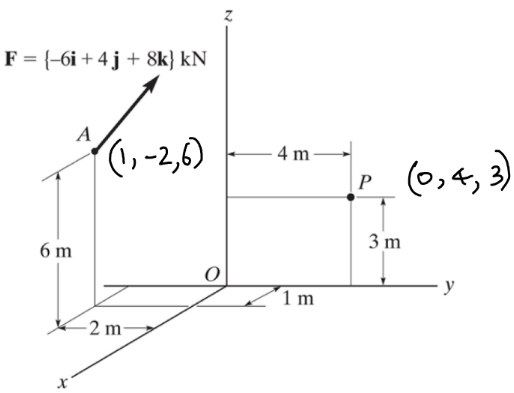
$$\hat{i} \times \hat{j} = \hat{k} \qquad \hat{i} \times \hat{i} = 0$$
$$\hat{j} \times \hat{k} = \hat{i} \qquad \hat{j} \times \hat{j} = 0$$
$$\hat{k} \times \hat{i} = \hat{j} \qquad \hat{k} \times \hat{k} = 0$$

Moment of a force about an axis.

$$M_{axis} = \hat{u}_{axis} \cdot (\vec{r} \times \vec{F}) = \begin{vmatrix} u_x & u_y & u_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Problems

4-28



$$\vec{r} = (1-0)\,\hat{i} + (-2-4)\,\hat{j} + (6-3)\,\hat{k}$$
$$= \hat{i} - 6\hat{j} + 3\hat{k}$$

$$\vec{M} = \vec{r} \times \vec{F}$$

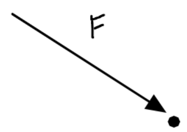
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -6 & 3 \\ -6 & 4 & 8 \end{vmatrix}$$

$$\hat{i}\,(-48-12) - \hat{j}\,(8+18) + \hat{k}\,(4-36)$$

$$(-60\hat{i} - 26\hat{j} - 32\hat{k}) \text{ kN m}$$

Moment of a Couple

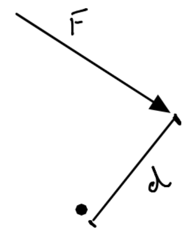
- Say, we have a particle of interest.



A net force acting along the point tends to move the particle.

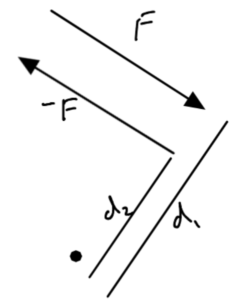
Translation only

$M = Fd$



The same net force acting at some distance d tends to move and rotate the particle.

Translation & Rotation.



What if the same force acts at a distance d_1, but an equal and opposite force acts at d_2.

Rotation only.

- This equal & opposite system of forces is known as a couple.

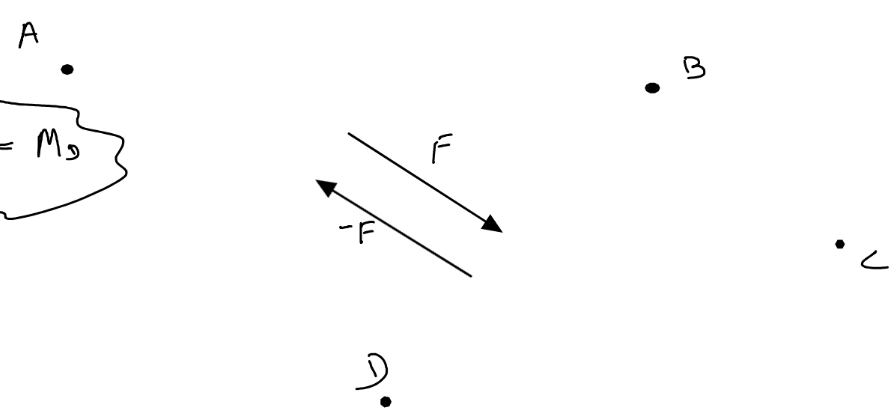
- $M = Fd_1 + (-Fd_2)$

$= F(d_1 - d_2) = F \times (\text{Distance between})$

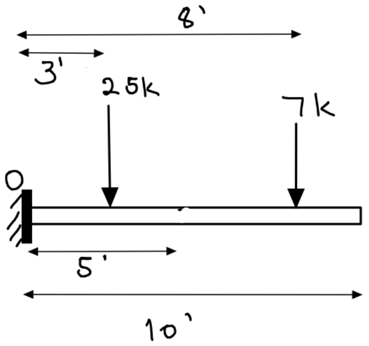
a.k.a Lever arm

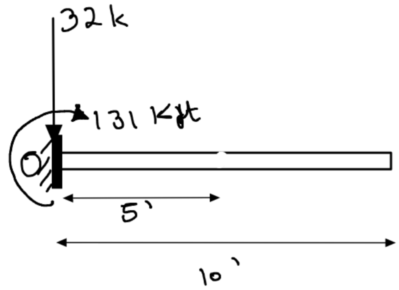
- This means, it doesn't matter where our point of interest is. A couple system always impart the same effect regardless.

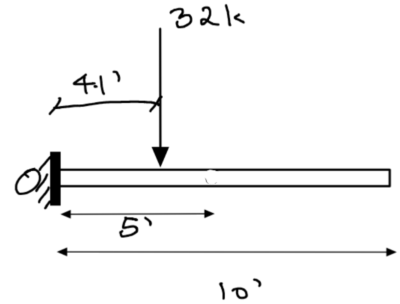
$M_A = M_B = M_C = M_D$



Consider 3 problems.







$M = -131$ kft
$F = -32$ k

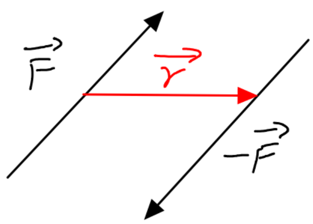
$M = 131$ kft
$F = -32$ k

$M = -131$ kft
$F = -32$ k

All systems are equivalent.

Insofar as point O is concerned, all systems have equal effects.

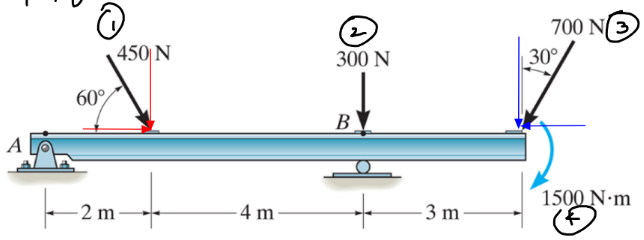
Vector analysis also apply.



$$\vec{M} = \vec{r} \times \vec{F} = -\vec{r} \times -\vec{F}$$

Problems

4-118



Where force acts from B ?

In these problems, only absence applied forces & moments & point B.

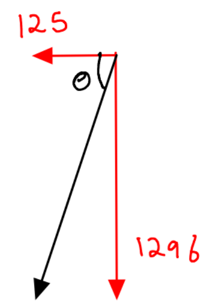
We couldn't care less what they are applied on.

First lets reduce this to one force & one moment.

Force = sum of all forces.
Moment = sum of all moments + sum of all moments due to forces.

$\xrightarrow{+} \sum F_x$

$450 \cos 60^\circ - 700 \sin 30^\circ = -125\ N$



$+\uparrow \sum F_y$

$-450 \sin 60^\circ - 300 - 700 \cos 30^\circ = -1296\ N$

$|F| = \sqrt{125^2 + 1296^2} = 1302\ N$

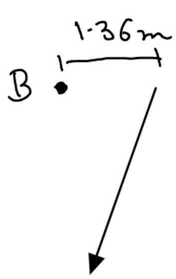
$\tan\theta = \left(\dfrac{1296}{125}\right) = \underline{\underline{84.5^\circ}}$

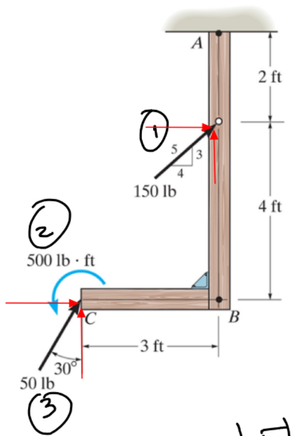
$\circlearrowleft + \sum M_z$

$-1500 + 450 \sin 60 (4) - 700 \cos 30^\circ (3)$

$= \underline{\underline{-1760\ Nm}}$

or





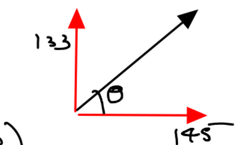
$\rightarrow \sum F_x$

$$150\left(\frac{4}{5}\right) + 50(\sin 30^\circ) = 145 \text{ lb}$$

$\uparrow \sum F_y$

$$150\left(\frac{3}{5}\right) + 50(\cos 30^\circ) = 133.3 \text{ lb}$$

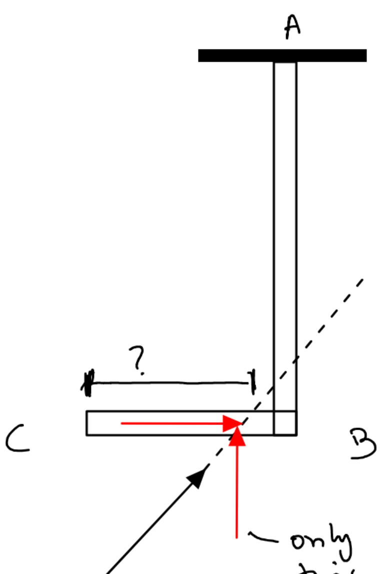
$$|F| = \sqrt{133.3^2 + 145^2} = \underline{\underline{197 \text{ lb}}}$$



$$\tan\theta = \left(\frac{133}{145}\right)$$
$$\theta = 42.5^\circ$$

$\circlearrowleft \underline{\underline{M_{zc}}}$

$$500 - 150\left(\frac{4}{5}\right)(4) + 150\left(\frac{3}{5}\right)(3)$$
$$= \underline{\underline{290}} \text{ lb ft}$$

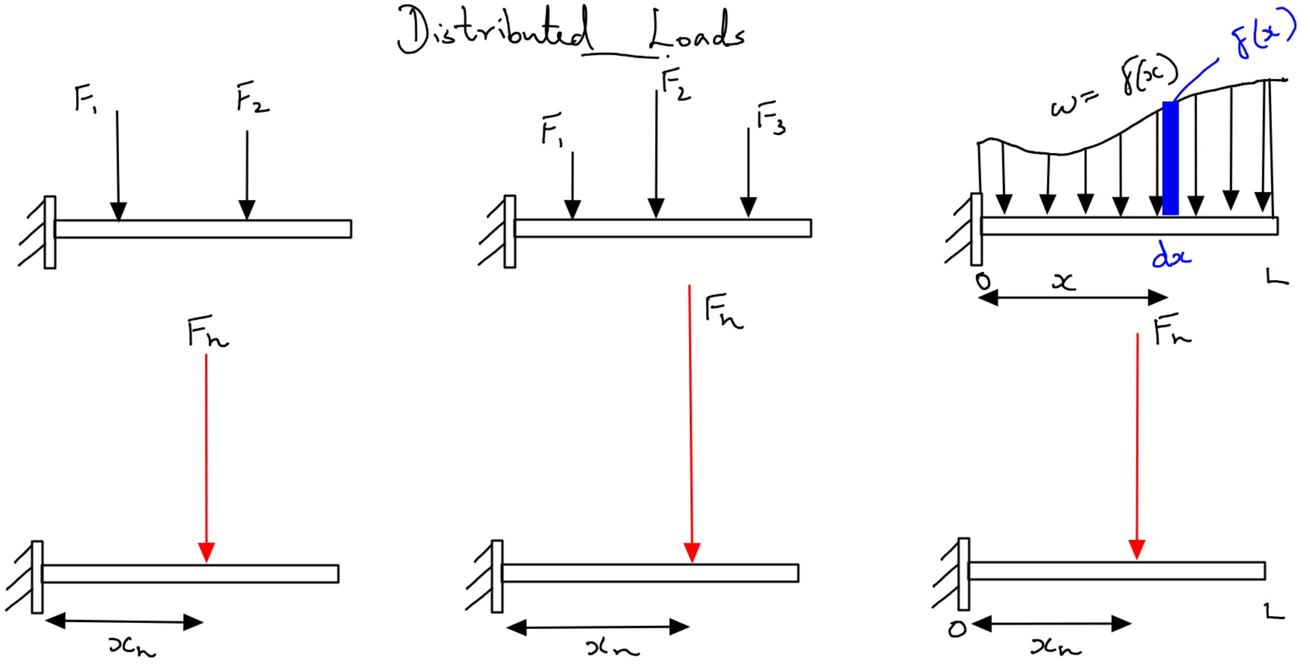


only this component causes moment.

$$d = \frac{290}{133.3} = \underline{\underline{2.18}} \text{ ft from C}$$

$$\therefore \text{ from B} \Rightarrow 3 - 2.18$$
$$= \underline{\underline{0.82 \text{ ft}}}$$

Distributed Loads



$F_R = F_1 + F_2$

$M_R = M_1 + M_2$

$x_R = \dfrac{M_R}{F_R}$

$F_R = F_1 + F_2 + F_3$

$M_R = M_1 + M_2 + M_3$

$x_R = \dfrac{M_R}{F_R}$

$dF = f(x)\,dx$

$F_R = \displaystyle\int_0^L f(x)\,dx = \text{Area}$

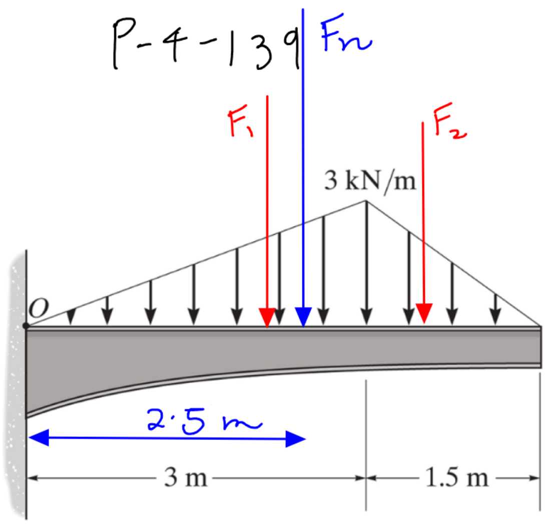
$dM = x f(x)\,dx$

$\therefore\ M_R = \displaystyle\int_0^L x f(x)\,dx = 1^{st}\text{ moment}$

$x_R = \dfrac{M_R}{F_R} = \dfrac{\int_0^L x f(x)\,dx}{\int_0^L f(x)\,dx}$

- For regular shapes, it is sufficient to know the
  - Area $\Rightarrow$ Magnitude
  - Centroid $\Rightarrow$ Location.

P-4-139



$$F_1 = -\frac{1}{2}(3)(3) = 4.5\ \text{kN}$$

$$F_2 = -\frac{1}{2}(1.5)(3) = 2.25\ \text{kN}$$

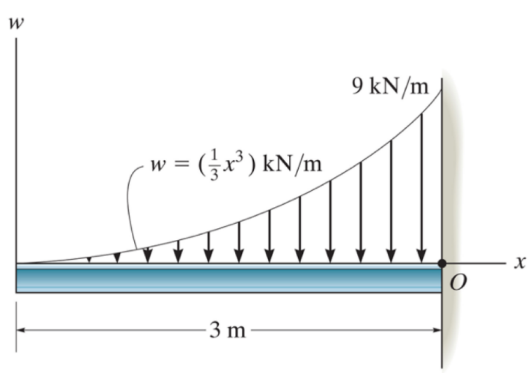
$$F_n = -6.75\ \text{kN}$$

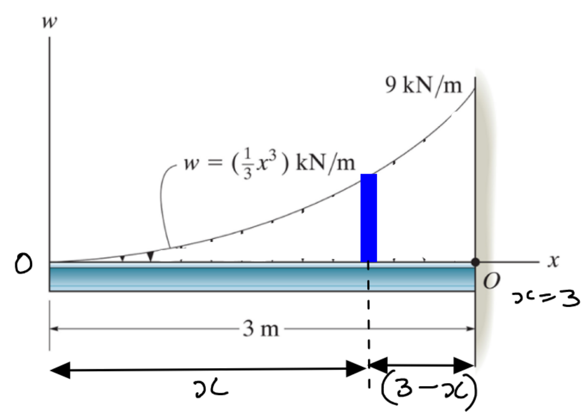
$$M_n = -4.5\left(\frac{2}{3}\right)(3) - 2.25\left(3+\frac{1}{3}(1.5)\right)$$

$$= -16.875\ \text{kNm}$$

$$x_n = \frac{16.875}{6.75} = \underline{\underline{2.5\ \text{m}}}$$

P 4-157





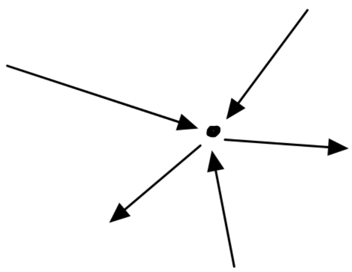
$$F_n = -\int_0^3 \left(\frac{1}{3}x^3\right) dx = \underline{\underline{-6.75\ \text{kN}}}$$

$$M_n = \int_0^3 \frac{1}{3}x^3(3-x)\,dx = \underline{\underline{4.05\ \text{kNm}}} \quad \circlearrowleft$$

$$x_n = \frac{4.05}{6.75} = \underline{\underline{0.6\ \text{m}}}$$

Equilibrium of a Rigid Body

Particle



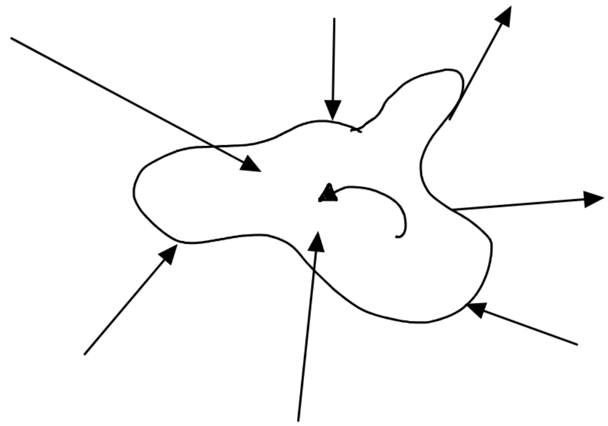
$\sum F = 0 \iff$ Equilibrium

2D Equations (2)

$\xrightarrow{+} \sum F_x = 0$

$+\uparrow \sum F_y = 0$

Rigid Body



Equilibrium $\iff \sum F = 0$ &

$\sum M_F + \sum M_m = 0$

2D Equations (3)

$\xrightarrow{+} \sum F_x = 0$

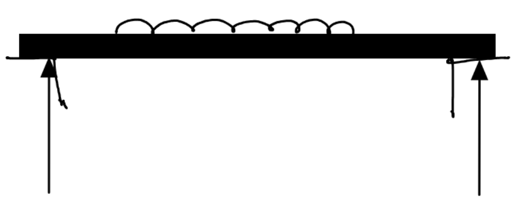
$+\uparrow \sum F_y = 0$

$(+\circlearrowleft) \sum M_z = 0$

This has to happen at any point in a FBD.

Free Body Diagrams

- Perhaps the most important concept in this course.
- When a diagram shows all the forces acting on a body.
- When FBD is in equilibrium $\iff$ 3 equations apply.
- Generate FBD by isolating the part of a system and drawing the effect of the rest of the system on that body.



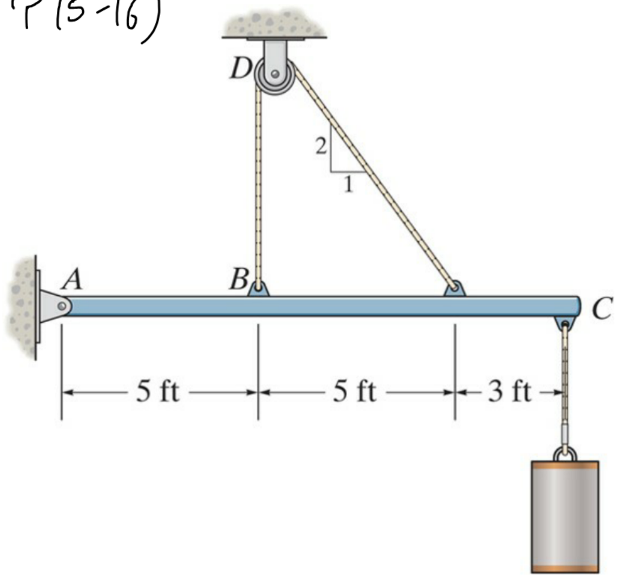
What is keeping this beam in equilibrium?

Support reactions

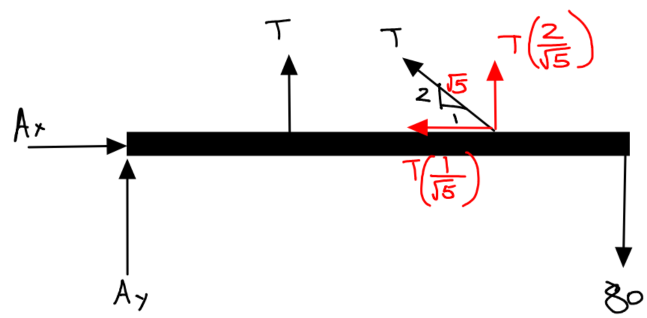
Reactions develop when motion is restricted $\Rightarrow$ Mantra!

Problems.

P(5-16)



FBD.



$\circlearrowleft^{+} \Sigma M_A = 0$

$$T(5) + T\left(\frac{2}{\sqrt{5}}\right)(10) - 80(13) = 0$$

$$T = \underline{\underline{74.6 \text{ lb}}}$$

$\xrightarrow{+} \Sigma F_x = 0$

$$A_x - 74.6\left(\frac{1}{\sqrt{5}}\right) = 0$$

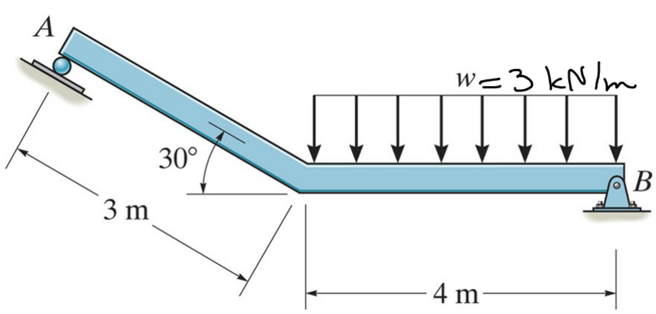
$$A_x = \underline{\underline{33.4 \text{ lb}}}$$

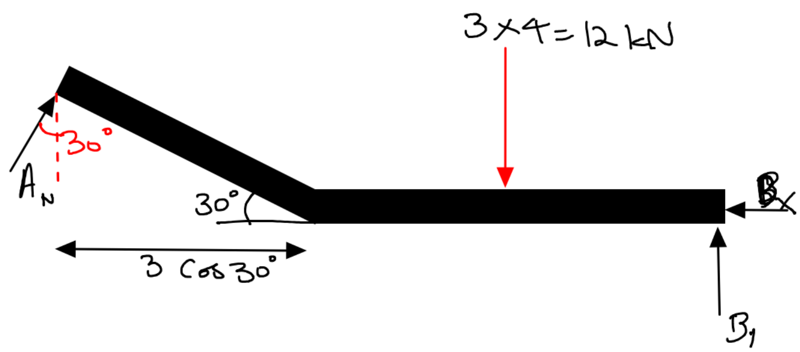
$+\uparrow \Sigma F_y = 0$

$$A_y + 74.6 + 74.6\left(\frac{2}{\sqrt{5}}\right) - 80 = 0$$

$$A_y = \underline{\underline{61.3 \text{ lb}}}$$

5-22





$\circlearrowleft^{+} \Sigma M_B = 0$

$$-A_N(\cos 30°)(3\cos 30° + 4) - A_N(\sin 30°)(3\sin 30°) + 12(2) = 0$$

$$-A_N(5.71) - A_N(0.75) + 24 = 0$$

$$A_N = \underline{\underline{3.72 \text{ kN}}}$$

$\xrightarrow{+} \Sigma F_x = 0$

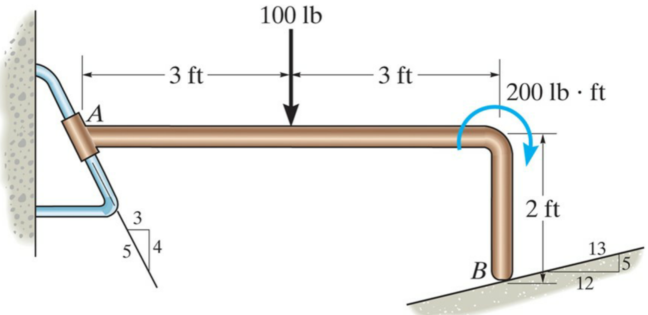
$$B_x = 3.72 \sin 30$$
$$= 1.86 \text{ kN} \;//$$

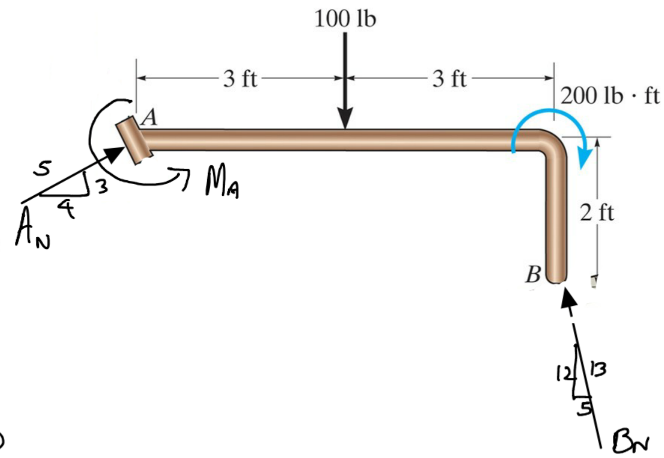
$\uparrow \Sigma F_y = 0$

$$3.72 \cos 30° - 12 + B_y = 0$$

$$B_y = 8.78 \text{ kN} \;//$$

5-25





$\xrightarrow{+} \sum F_x = 0$

$$A_N\left(\frac{4}{5}\right) - B_N\left(\frac{5}{13}\right) = 0$$

$+\uparrow \sum F_y = 0$

$$A_N\left(\frac{3}{5}\right) + B_N\left(\frac{12}{13}\right) - 100 = 0$$

Solving $A_N = 39.7$ lb

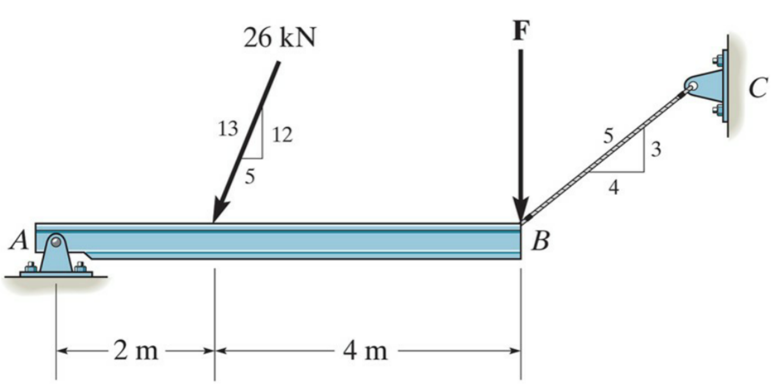
$B_N = 82.5$ lb

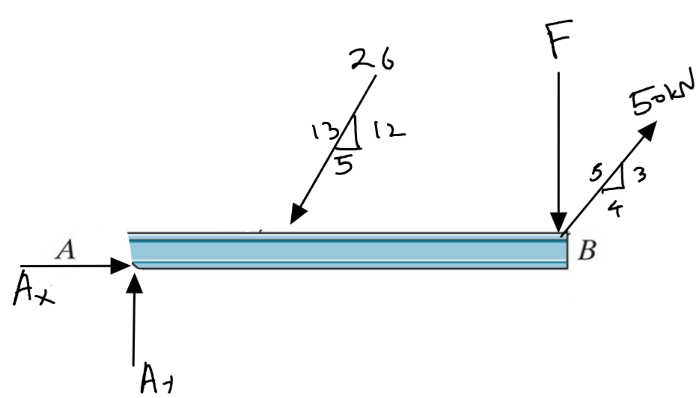
$(+\circlearrowleft) \sum M_A = 0$

$$M_A - 100(3) - 200 + 82.5\left(\frac{12}{13}\right)(6) - 82.5\left(\frac{5}{13}\right)(2) = 0$$

$$M_A = 106 \text{ lb ft}$$

5-48



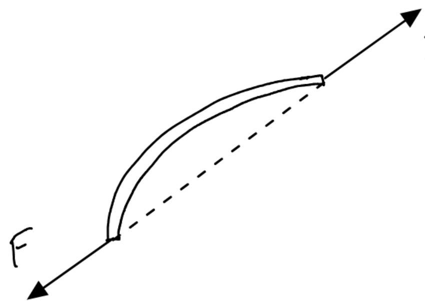


$(+\circlearrowleft) \sum M_A = 0$

$$-26\left(\frac{12}{13}\right)(2) - F(6) + 50\left(\frac{3}{5}\right)6 = 0$$

$$F = 22 \text{ kN}$$

Two & Three Force Members

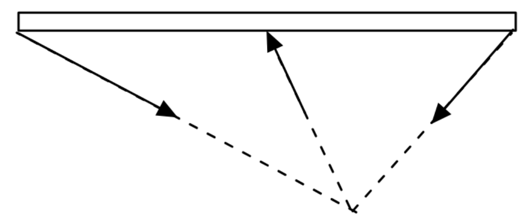


- If a member is acted upon by 2 and only 2 forces, it is a 2-force member.

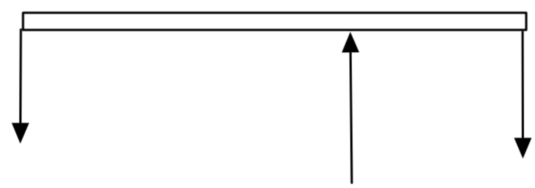
- The two forces have to be equal and opposite.

- The only way they can be equal and opposite is if they act along the same line of action.

- If you identify a two force member in a problem, you essentially reduce the number of unknowns you solve.



or



- If 3 and only 3 forces act on a member, it is called a three force member.

- The forces have to be either concurrent or parallel.

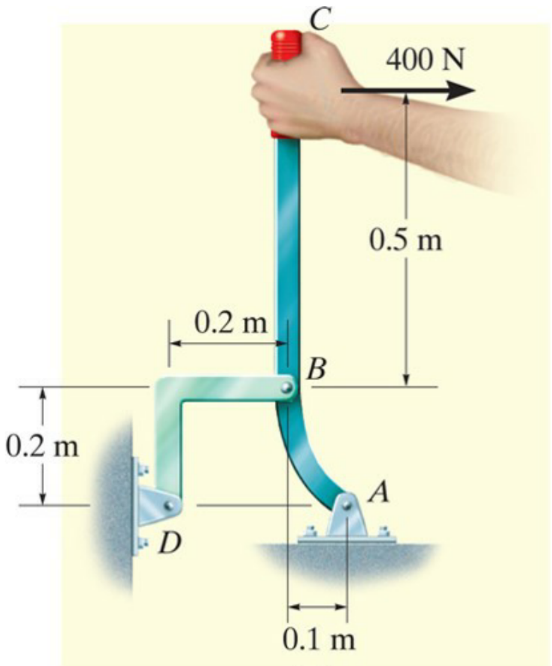
- If concurrent, you already know the directions of the forces in the problem.

- If parallel, you know that one force is the sum of the other 2.

<u>Proof</u>:

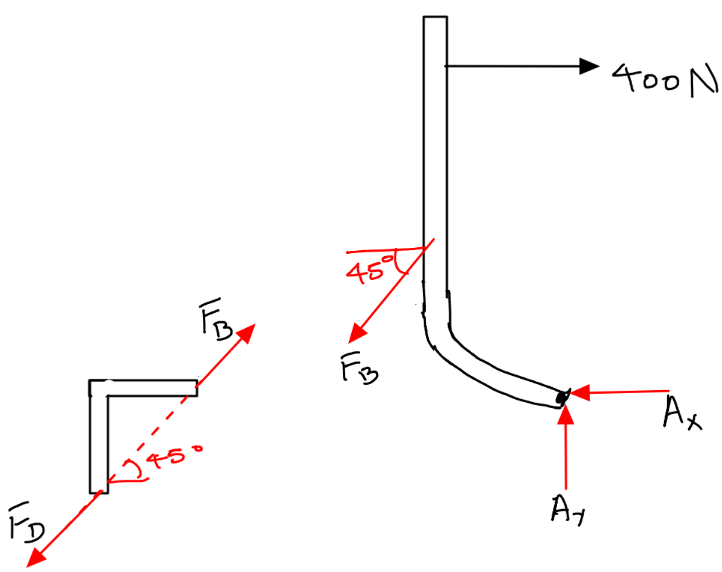
If in 2 or 3 force members, the forces had not followed the above rules, you will not have had equilibrium.

Example 13



(a)

FBD



$F_D = F_B$

$\circlearrowleft + \sum M_A = 0$

$$F_B \cos 45^\circ (0.2) + F_B \sin 45^\circ (0.1) - 400(0.7) = 0$$

$$\Rightarrow F_B = 1320 \text{ N}$$

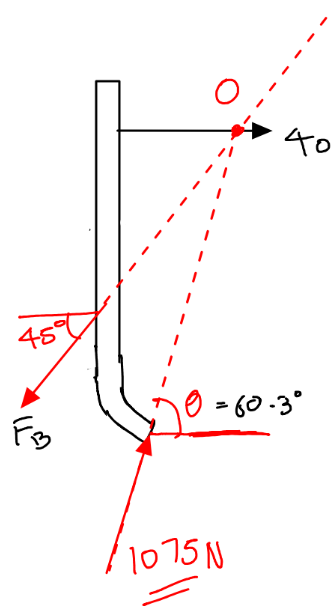
$A_x = -F_B \cos 45^\circ + 400 = -533 \text{ N}$ $\leftarrow$

$A_y = F_B \sin 45^\circ = 933 \text{ N}$ $\uparrow$

$F_A = \sqrt{533^2 + 933^2} = 1075 \text{ N}$

$\tan\theta = \dfrac{933}{533}$

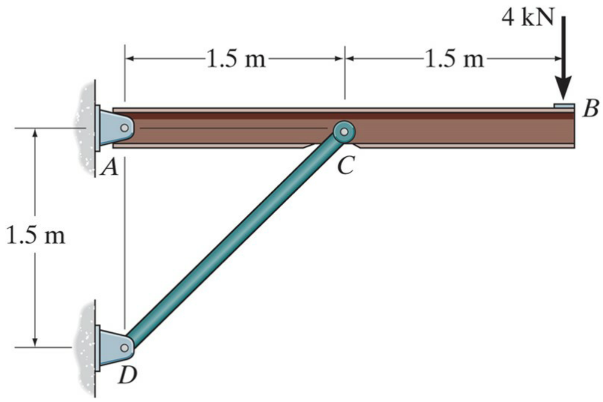
$\theta = 60.3^\circ$



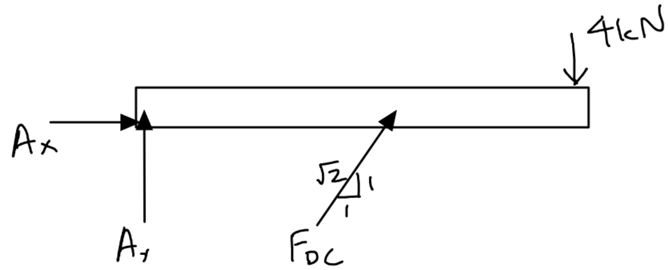
or.

You could have also taken ABC as a three force member.

Solution this way is in the text book.



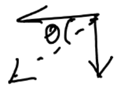
DC is a two-force member



$$\stackrel{+}{\circlearrowleft} \Sigma M_A = 0$$

$$F_{DC}\left(\frac{1}{\sqrt{2}}\right)(1.5) - 4(3) = 0$$

$$F_{DC} = \underline{\underline{11.3}} \text{ kN} \nearrow$$



$$A_x = -F_{DC}\left(\frac{1}{\sqrt{2}}\right) = -8 \text{ kN}$$

$$A_y = -F_{DC}\left(\frac{1}{\sqrt{2}}\right) + 4 = -4 \text{ kN}$$

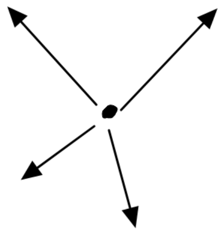
$$R_A = \sqrt{8^2 + 4^2}$$

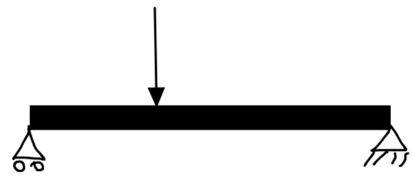
$$= \underline{\underline{8.9}} \text{ kN}$$

$$\tan\theta = \frac{4}{8} \Rightarrow \theta = \underline{\underline{26.6^\circ}}$$

Trusses.

Concepts required from previous chapters:





How to solve a 2D particle equilibrium problem

$\Rightarrow \sum F_x = 0$

$\sum F_y = 0$

How to solve for support reactions

$\Rightarrow \sum F_x = 0$

$\sum F_y = 0$

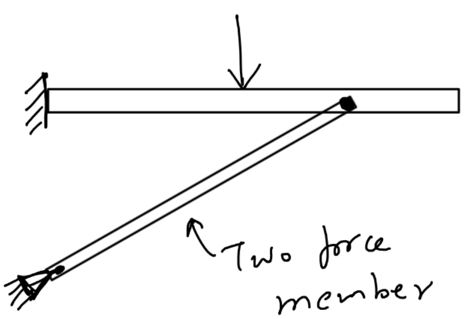
$\sum M = 0$

&

FBD of all kinds.

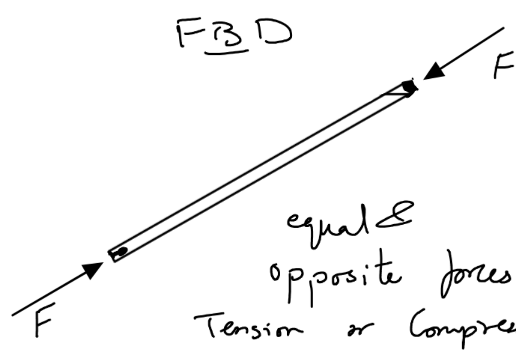
Remember two-force members?

- Often the two ends are connected by pins.



$\Rightarrow$

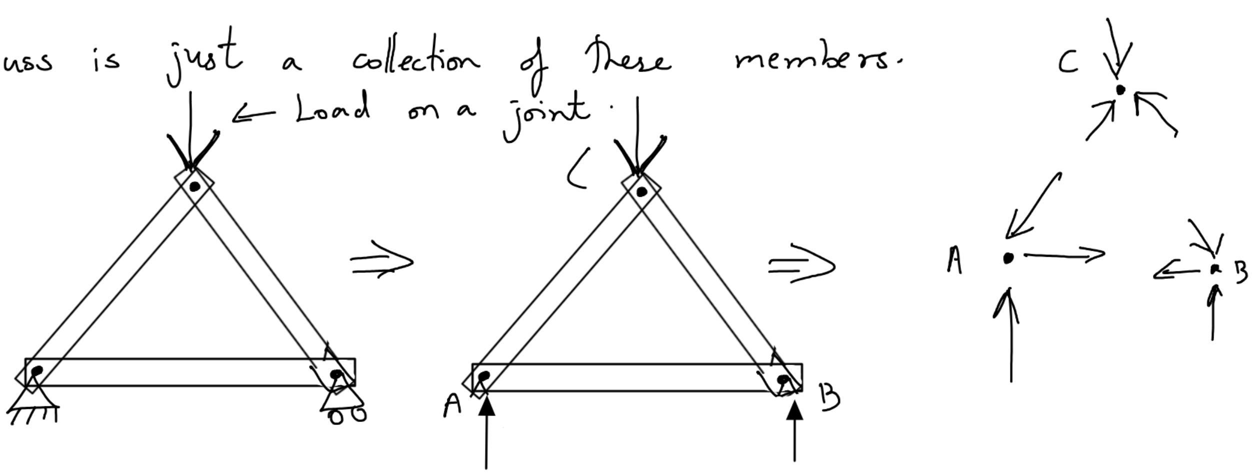
FBD



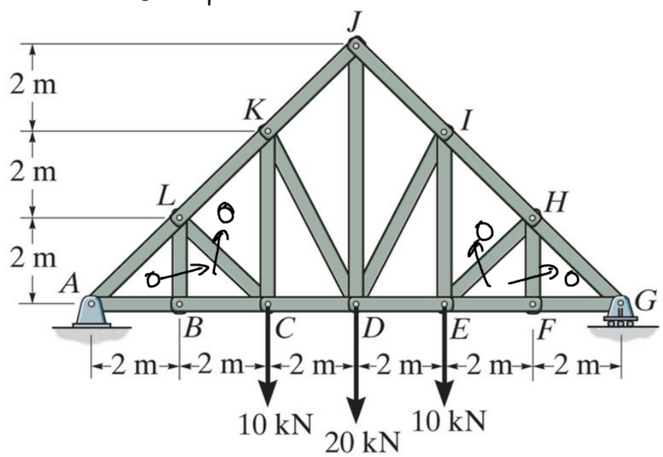
equal & opposite forces

Tension or Compression

- A truss is just a collection of these members.

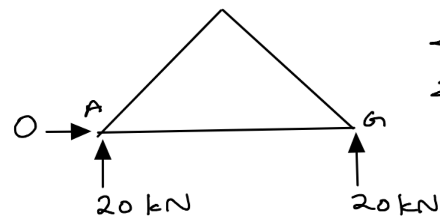


6–11



- Use symmetry.
- Eliminate zero force members.
- Find reactions.
- FBD of each point to solve 2D particle equilibrium.

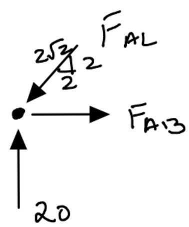
FBD 1. (Reactions)



$\Sigma M_G = 0$

$\Sigma F_y = 0$

FBD 2 (A)



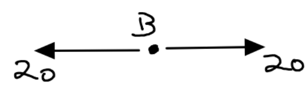
$$-F_{AL}\left(\frac{2}{2\sqrt{2}}\right) + 20 = 0$$

$$\boxed{F_{AL} = 28.28 \text{ kN (C)}}$$

$$F_{AB} - 28.28\left(\frac{2}{2\sqrt{2}}\right) = 0$$

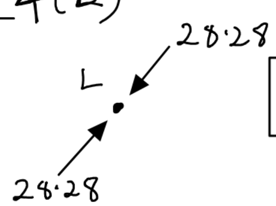
$$\boxed{F_{AB} = 20 \text{ kN (T)}}$$

FBD 3 (B)



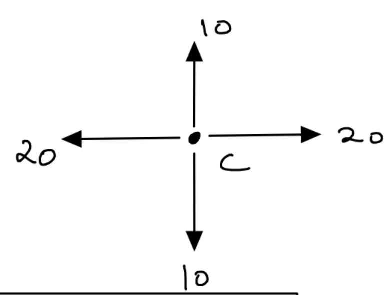
$$\boxed{F_{BC} = 20 \text{ kN (T)}}$$

FBD 4 (L)



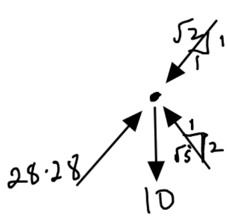
$$\boxed{F_{LK} = 28.28 \text{ kN (C)}}$$

FBD 5 (C)



$$\boxed{\begin{aligned} F_{CK} &= 10 \text{ kN (T)} \\ F_{CD} &= 20 \text{ kN (T)} \end{aligned}}$$

FBD 6 (K)



$\Sigma F_x = 0$

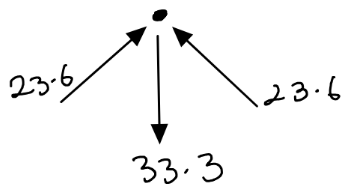
$$28.28\left(\frac{1}{\sqrt{2}}\right) - F_{KJ}\left(\frac{1}{\sqrt{2}}\right) - F_{DK}\left(\frac{1}{\sqrt{5}}\right) = 0$$

$\Sigma F_y = 0$

$$-10 + 28.28\left(\frac{1}{\sqrt{2}}\right) + F_{DK}\left(\frac{2}{\sqrt{5}}\right) - F_{KJ}\left(\frac{1}{\sqrt{2}}\right) = 0$$

$$\boxed{\begin{aligned} F_{KD} &= 7.45\ \text{kN}\ (C) \\ F_{KJ} &= 23.6\ \text{kN}\ (C) \end{aligned}}$$

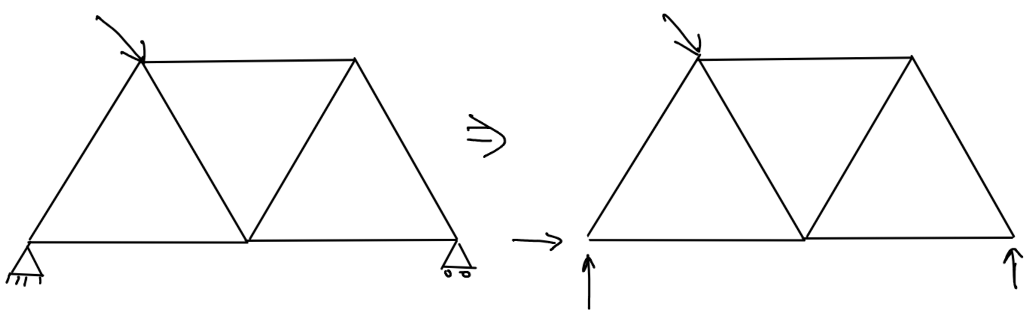
FBD 7 (J)



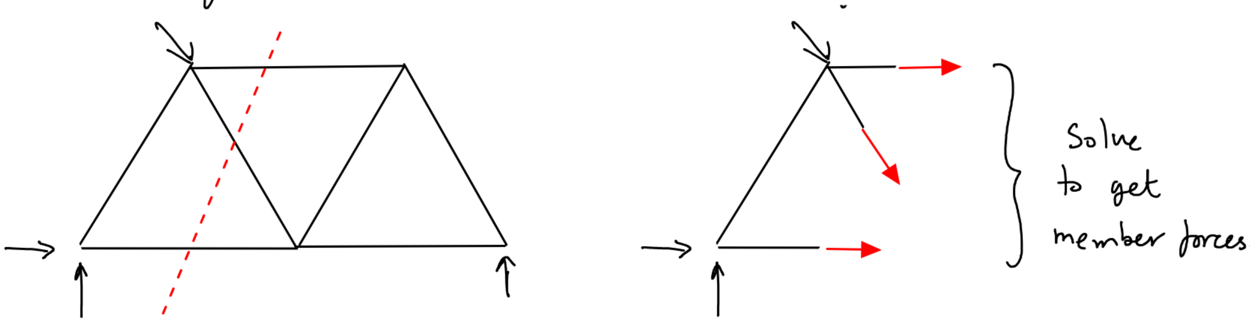
$$\boxed{F_{JD} = 33.3\ \text{kN}\ (T)}$$

Method of Sections

- We used method of joints when we isolated points by drawing their FBD.
- At a point, given equilibrium, there are 2 equations of equilibrium.
- There is an alternate way of solving truss problems.
- The concept is already known to you. You have used it previously.
- Here we introduce it officially.



- You solved the reactions by solving 3 equilibrium equations.
- Remember, if a body is in equilibrium, any portion should also be in equilibrium.

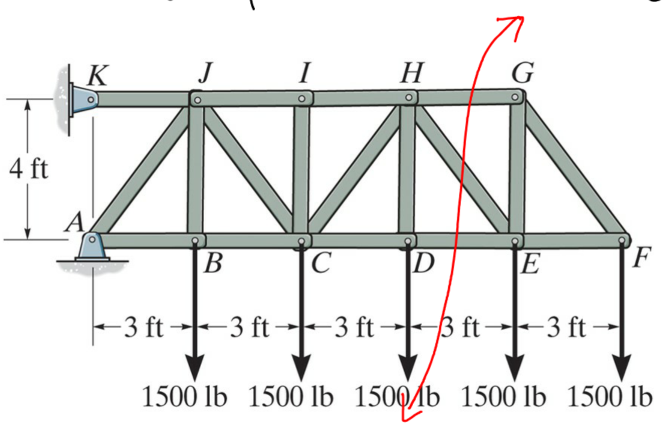


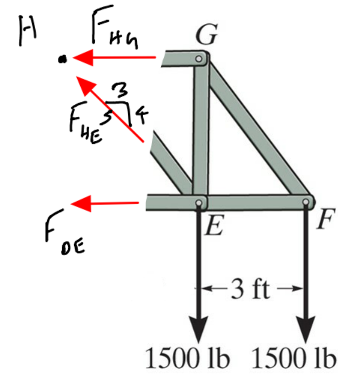
Full system

Partial with internal forces.

6-29

Did not need to find reactions.





$\Sigma M_E = 0$

$$F_{HG}(4) - 1500(3) = 0$$
$$F_{HG} = 1125 \text{ lb (T)}$$

2 Moment eqs + 1 Force eq. also valid.

$\Sigma M_H = 0$

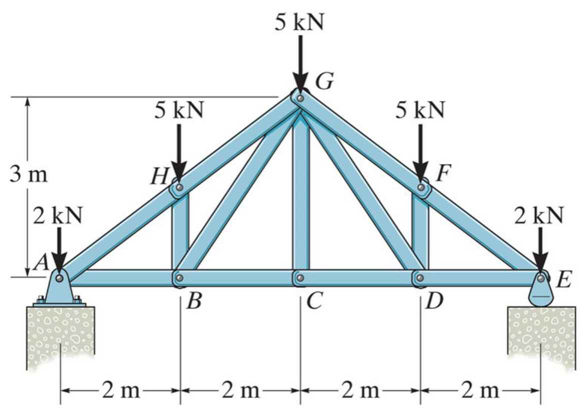
$$F_{DE}(4) - 1500(6) - 1500(3) = 0$$
$$F_{DE} = 3375 \text{ lb (C)}$$

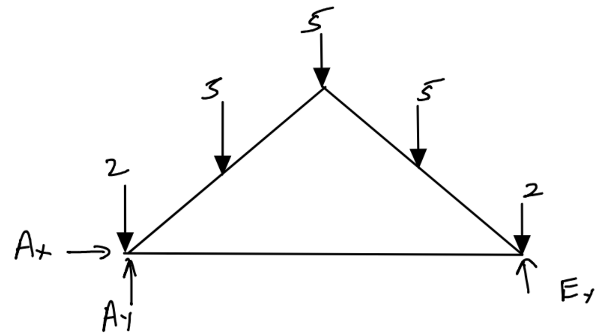
$+\uparrow \Sigma F_y = 0$

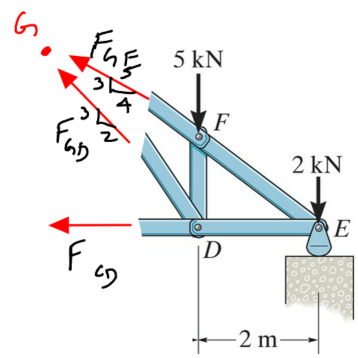
$$F_{HE}\left(\frac{4}{5}\right) - 1500 - 1500 = 0$$

$$F_{EH} = 3750 \text{ lb (T)}$$

6-33







$\Sigma M_A = 0$

$$E_y(8) - 2(8) - 5(6) - 5(4) - 5(2) = 0$$

$$E_y = 9.5 \text{ kN}$$

$\Sigma M_D = 0$

$$-\frac{4}{5}(F_{GF})(1.5) - 2(2) + 9.5(2) = 0$$

$$F_{GF} = 12.5 \text{ kN } (C)$$

$\Sigma M_G = 0$

$$9.5(4) - 2(4) - 5(2) - F_{CD} = 6.67 \text{ kN } (T)$$

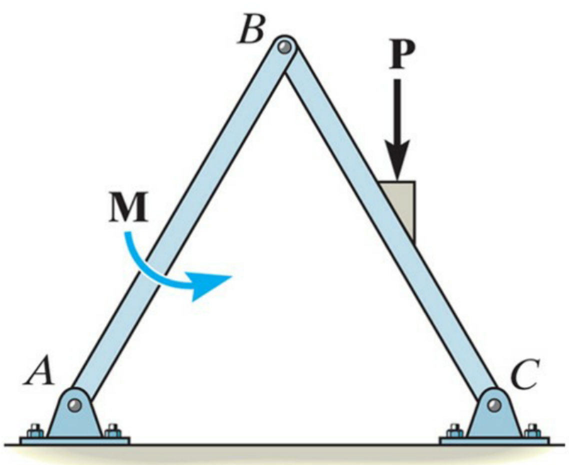
$F_{GC} = 0$ from equilibrium at joint C.

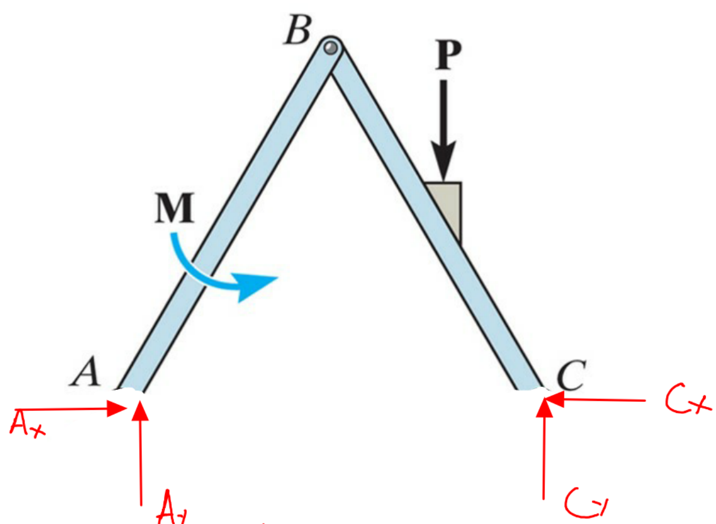
Frames and Machines

- By this point, you should be able to solve for unknown forces given any free body diagram.

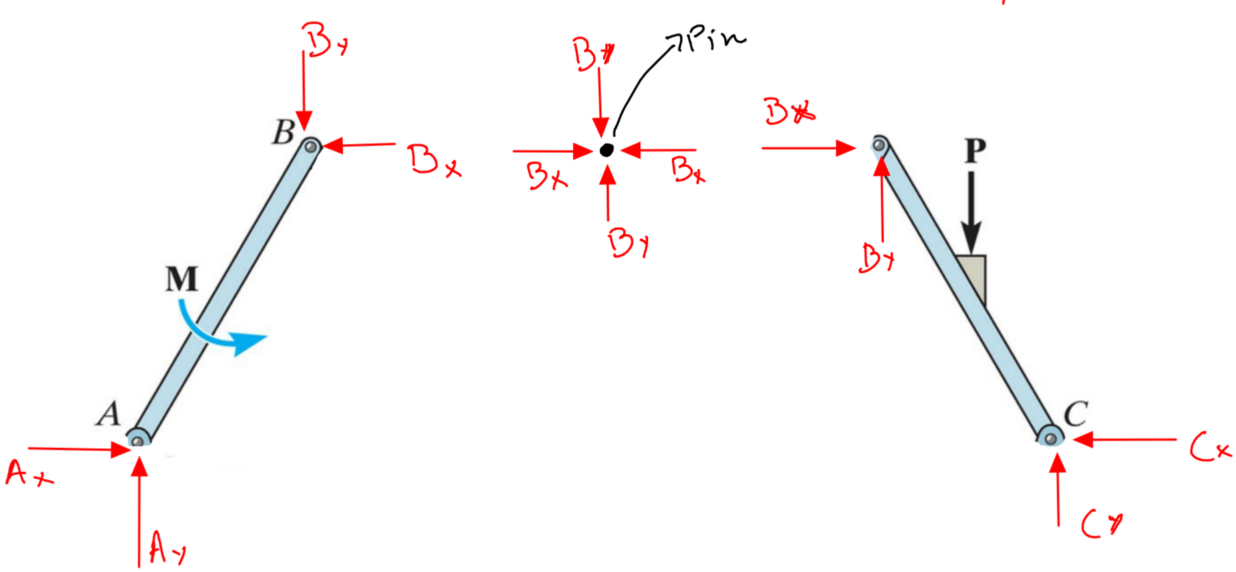
$$\xrightarrow{+} \sum F_x = 0$$
$$+\uparrow \sum F_y = 0$$
$$\circlearrowleft + \sum M_{\text{any point}} = 0$$

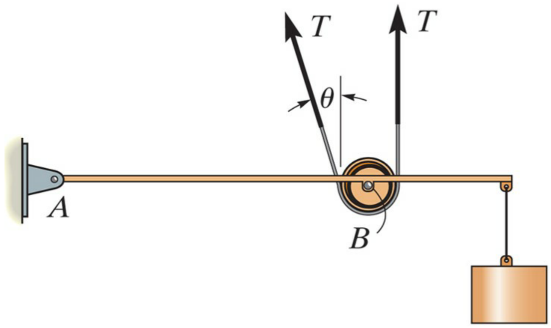
- Today we will just have fun coming up with FBD that are useful and accurate.

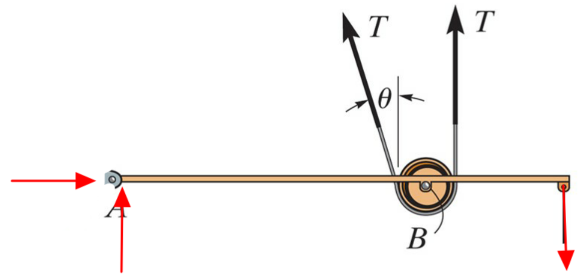


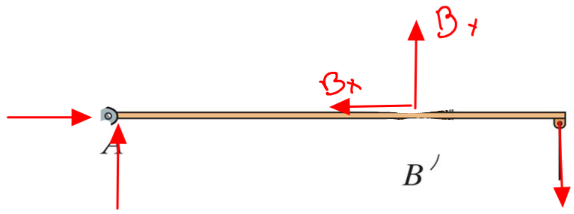


- 4 unknowns.
- cannot solve.
- Break into parts.

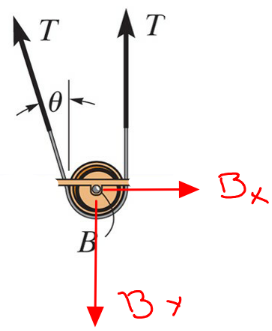


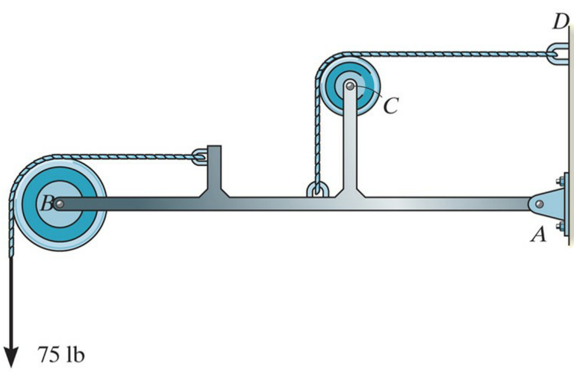




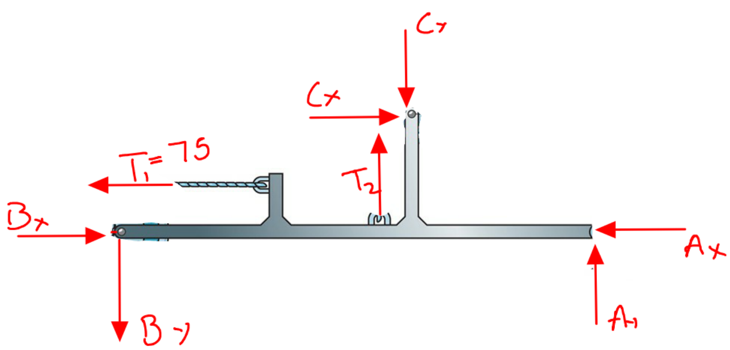


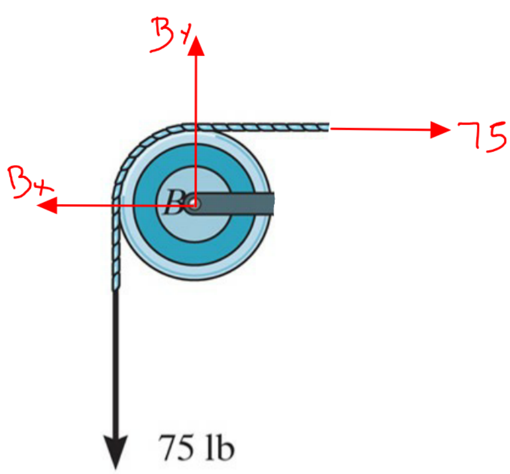
Only the pin is in contact with the rod. Not the pulley.

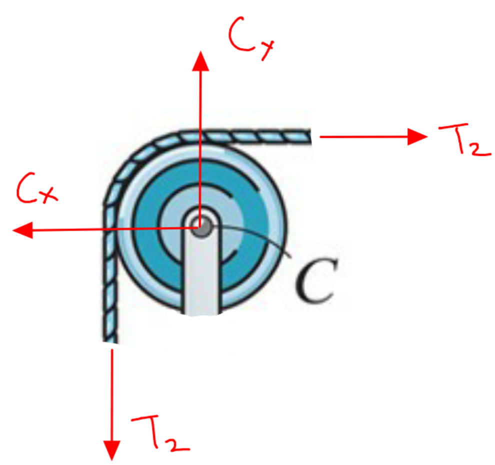


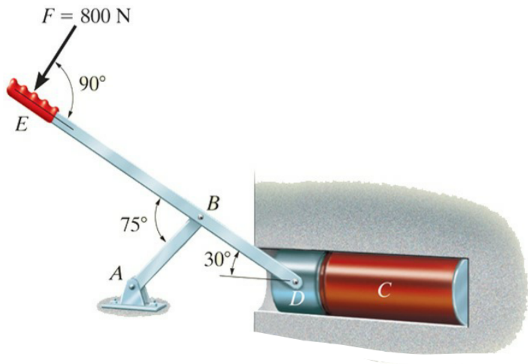


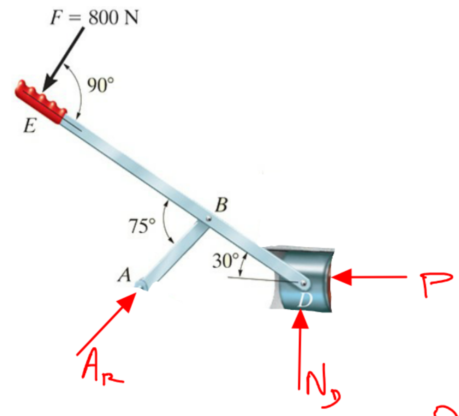
(a)

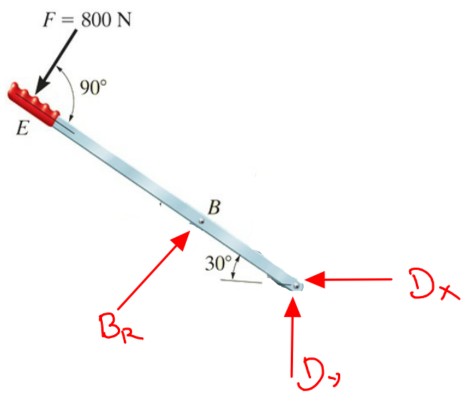


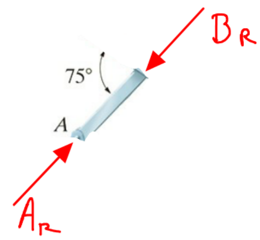


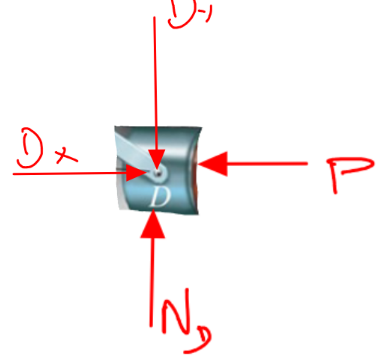


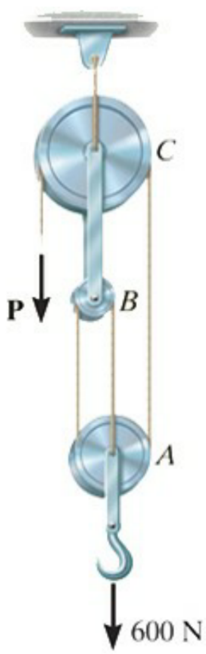

F = 800 N
90°
E
75°
B
30°
A
D
C

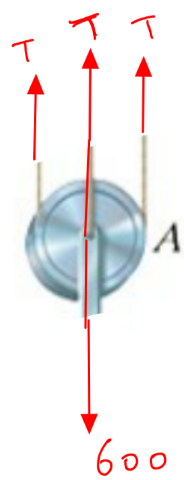

F = 800 N
90°
E
75°
B
30°
A
D
P
A_R
N_D


F = 800 N
90°
E
B
30°
B_R
Dx
Dy

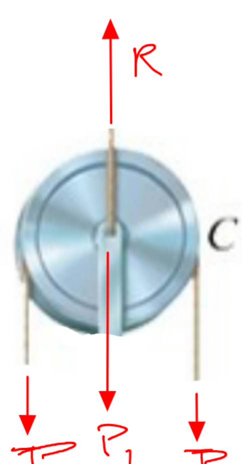

B_R
75°
A
A_R

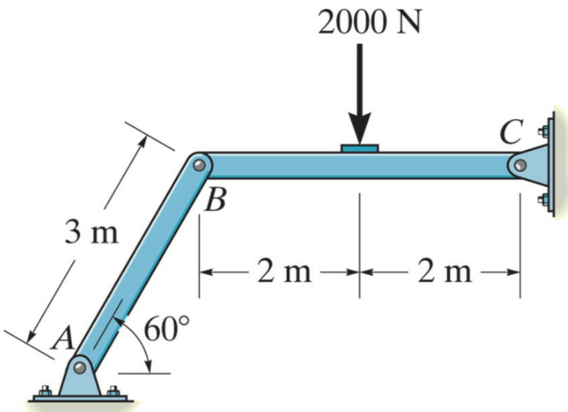

Dy
Dx
D
P
N_D

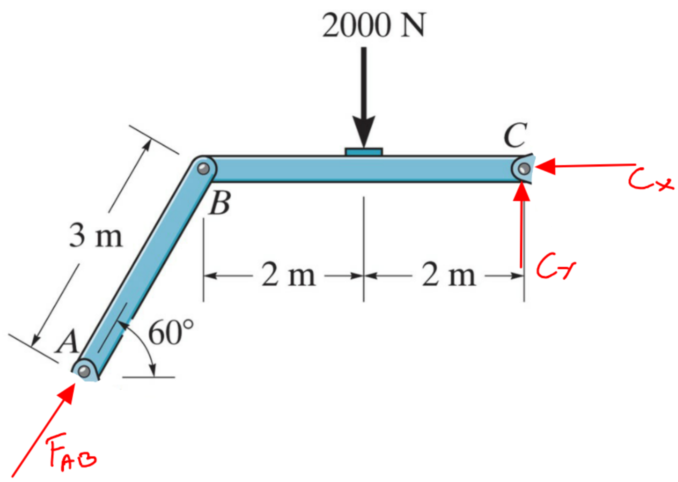

C
P
B
A
600 N

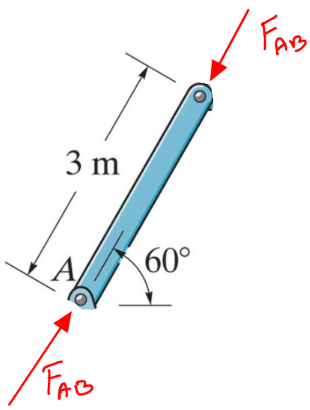

T
T
T
A
600

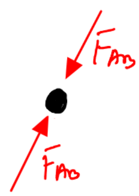

P1
T
T

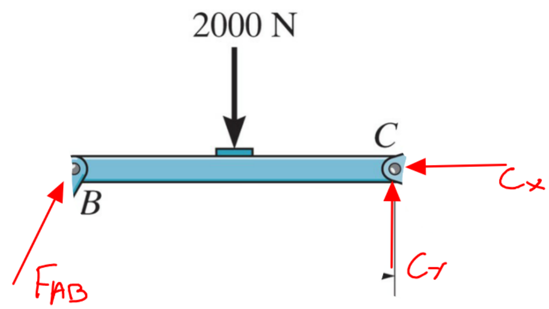

R
C
P
P1
P

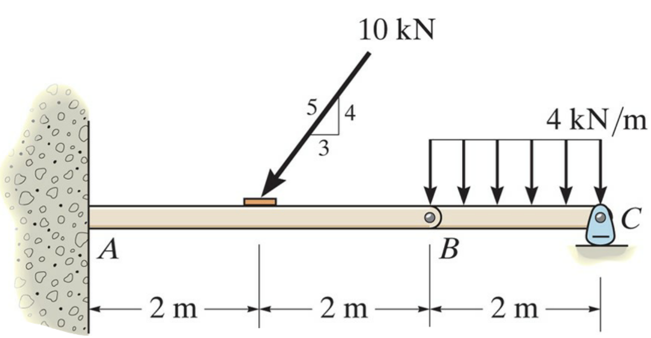

2000 N
C
B
3 m
2 m
2 m
A
60°

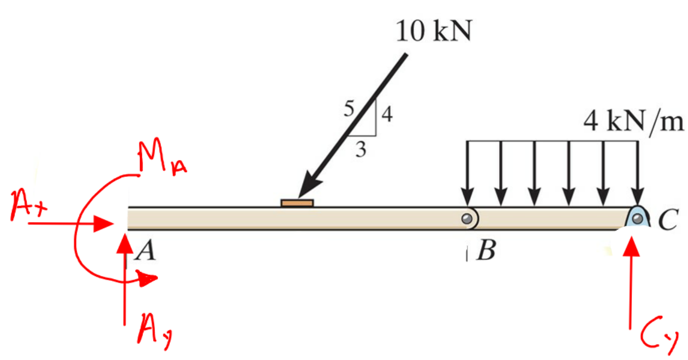

2000 N
C
C_x
B
3 m
2 m
2 m
C_y
A
60°
F_{AB}

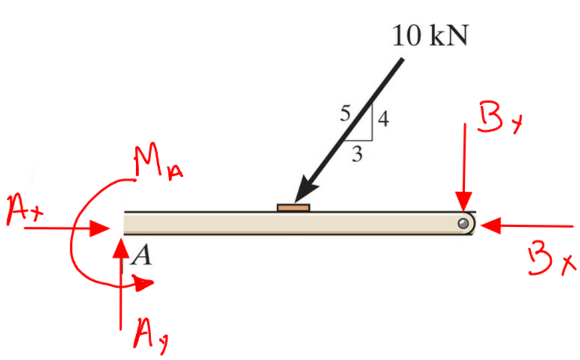

F_{AB}
3 m
A
60°
F_{AB}

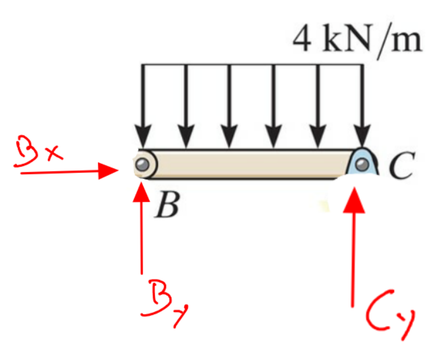

F_{AB}
F_{AB}

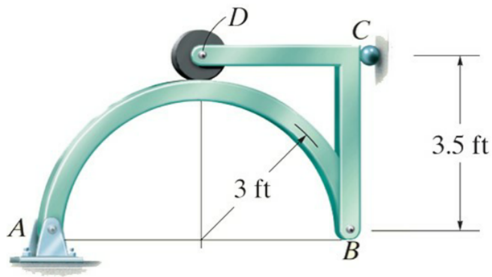

2000 N
C
C_x
B
C_y
F_{AB}

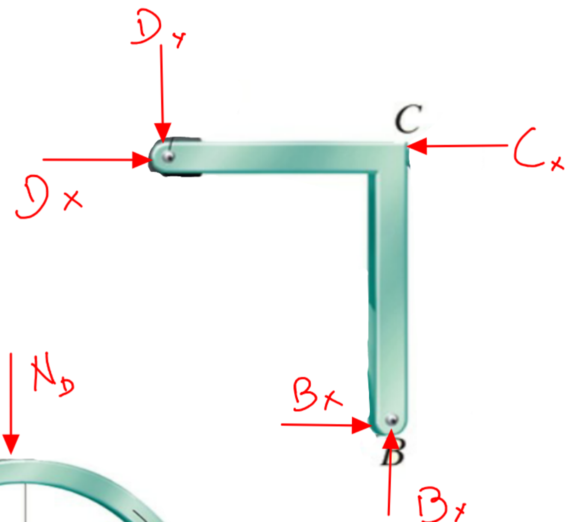

10 kN
5
4
3
4 kN/m
C
A
B
2 m
2 m
2 m

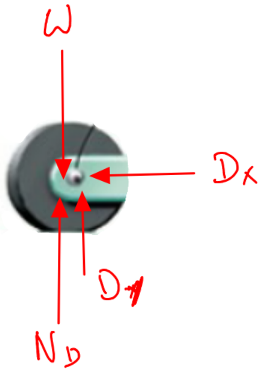

10 kN
5
4
3
4 kN/m
M_A
A_x
C
A
B
A_y
C_y

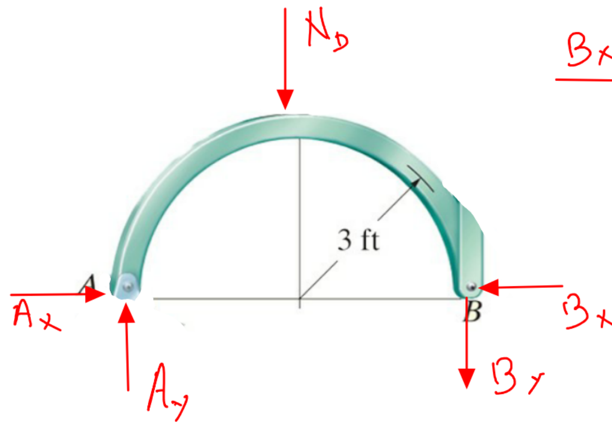

10 kN
5
4
3
B_y
M_A
A_x
A
B_x
A_y

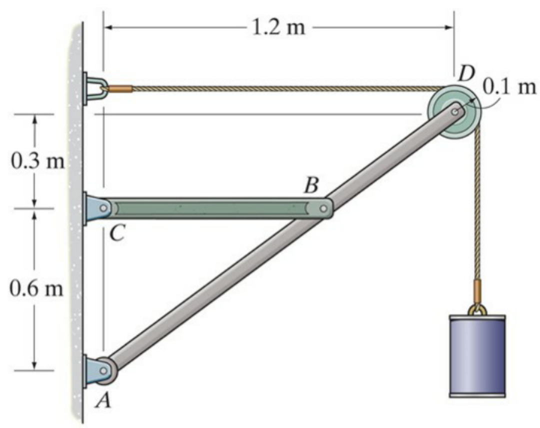

4 kN/m
B_x
C
B
B_y
C_y

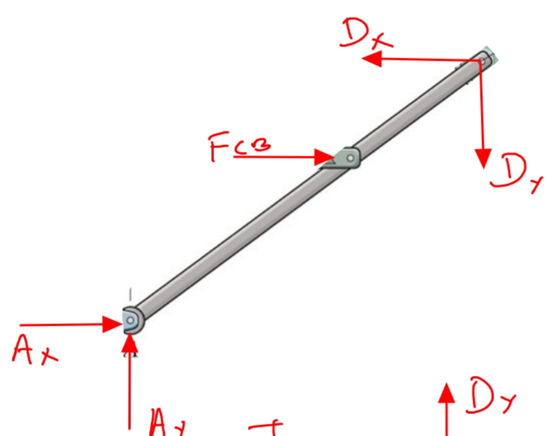

D
C
3.5 ft
3 ft
A
B

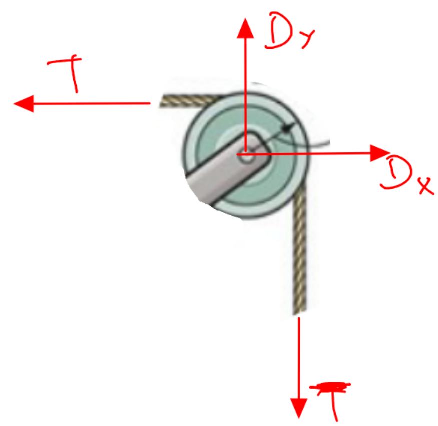

Dy
C
Dx
Cx
Bx
B
By


W
Dx
Dy
ND


ND
3 ft
A
Ax
Ay
B
Bx
By

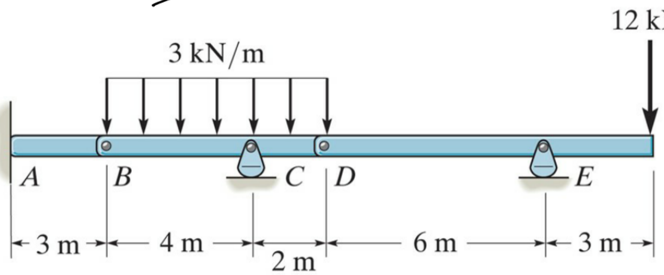

1.2 m
D
0.1 m
0.3 m
B
C
0.6 m
A

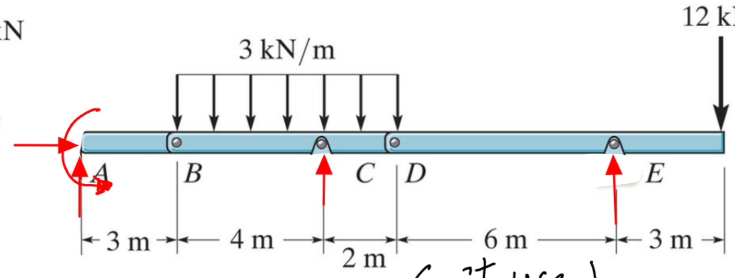

Dx
FCB
Dy
Ax
Ay


Dy
T
Dx
T

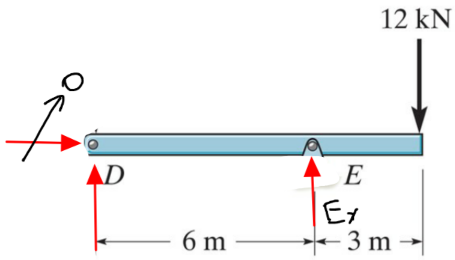

B
FBC
C
FBC

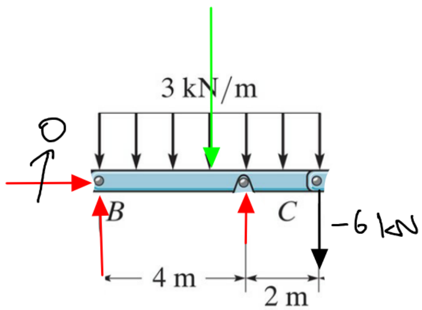
6-71

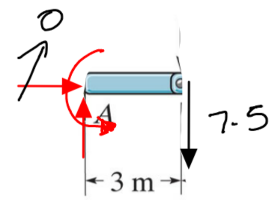




Can't use! Too many unknowns.







$\Sigma M_D = 0$

$-12(9) + E_y(6) = 0$

$E_y = 18\ kN$

$\therefore D_y = -6\ kN$

$\Sigma M_B = 0$

$-3 \times 6 \times 3 + C_y(4) - (-6)6 = 0$

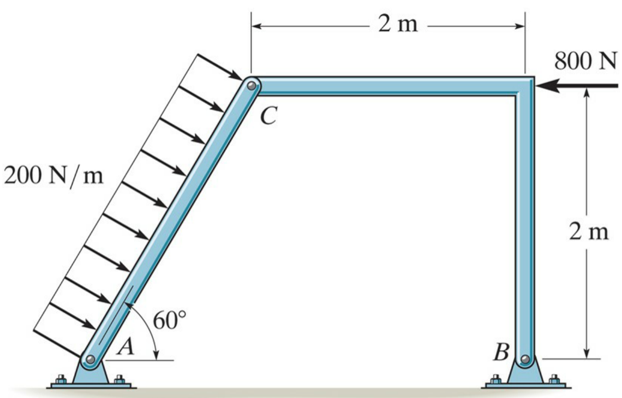
$C_y = 4.5\ kN$

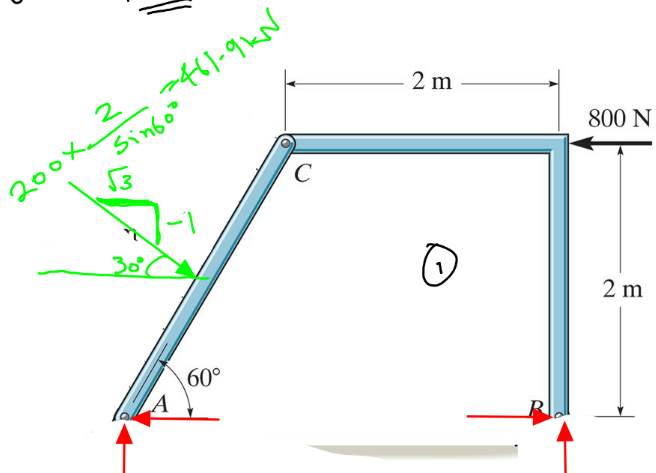
$B_y = 7.5\ kN$

$A_y = 7.5\ kN$

$M_A = 7.5 \times 3$

$= 22.5\ kNm$



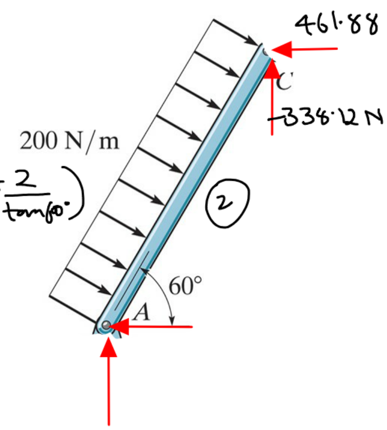


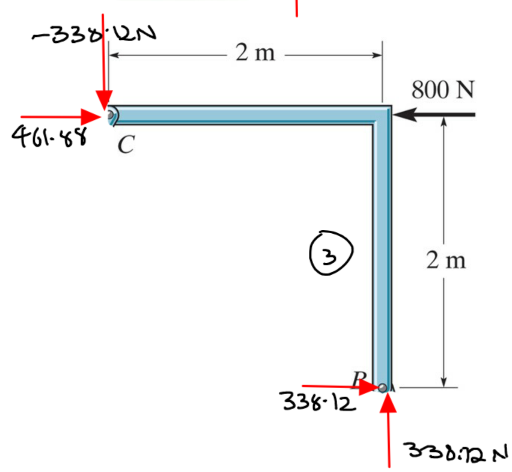
In FBD 1,

$\Sigma M_A = 0$

$-461.9\left(\frac{1}{\sin 60°}\right) + 800(2) + B_y\left(2 + \frac{2}{\tan 60°}\right) = 0$

$B_y = -338.12\ N$





In FBD (2),

$\sum M_A = 0$,

$$-200\left(\frac{2}{\sin 60^\circ}\right)\left(\frac{1}{\sin 60^\circ}\right) - 338.12\left(\frac{2}{\tan 60^\circ}\right) + C_x(2) = 0$$

$$C_x = 461.88\ N$$

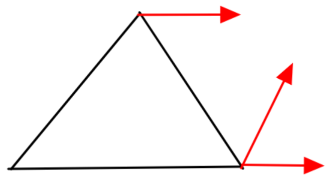
$A_x = -61.88\ N$ //

$A_y = 569.06\ N$ //

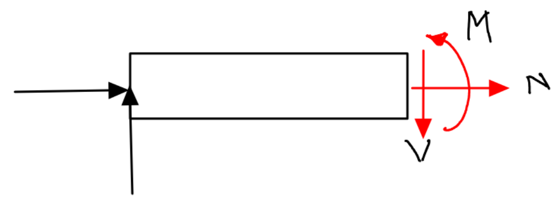
Internal Loadings

- Review method of sections and relate it to internal loadings.
- Clarify external and internal loading terminology.
- Sign conventions.

Trusses

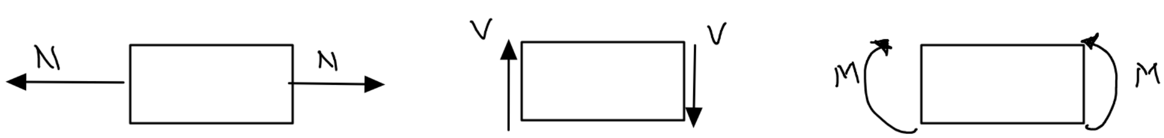


Beams



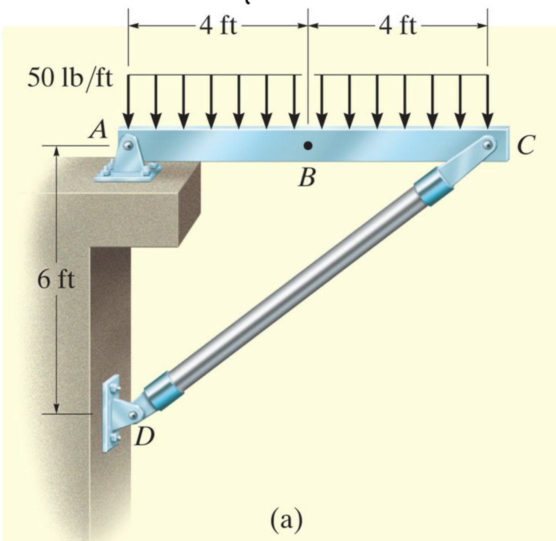
N - normal force
V - Shear force
M - Bending moment

- When you draw the FBD of entire system, all loads shown are External.
- When you draw the FBD of sections, forces that get released are internal.



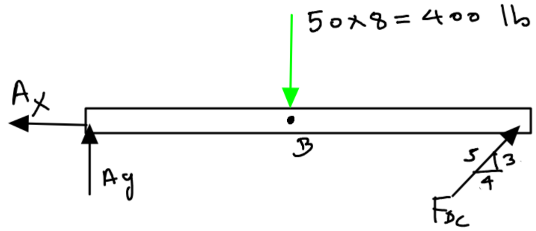
Positive sign convention.

Example 3



(a)

Find reactions first.

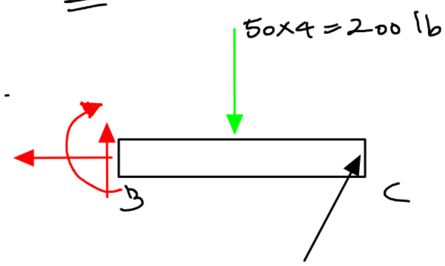


$\underline{\Sigma M_A = 0}$

$$F_{DC}\left(\frac{3}{5}\right)(8) - 400(4) = 0$$

$$F_{DC} = 333.\underline{\underline{3}}\ \text{lb}$$

Cut section at B.



$\underline{\Sigma M_B = 0}$

$$-M_B - 200(2) + 333.3\left(\frac{3}{5}\right)(4) = 0$$

$$M_B = 400\ \underline{\underline{\text{lb}}}\ \text{ft}$$

$\underline{\underline{\Sigma F_x = 0}}$

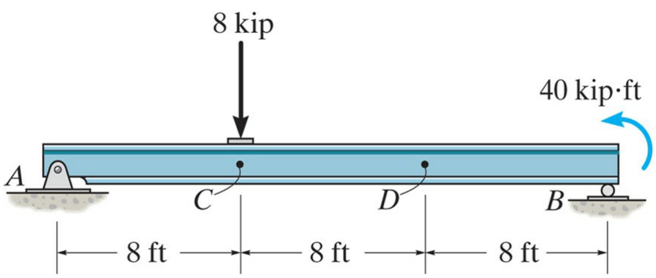
$$-N_B + 333.3\left(\frac{4}{5}\right) = 0 \quad \Rightarrow \quad N_B = 267\ \text{lb.}$$

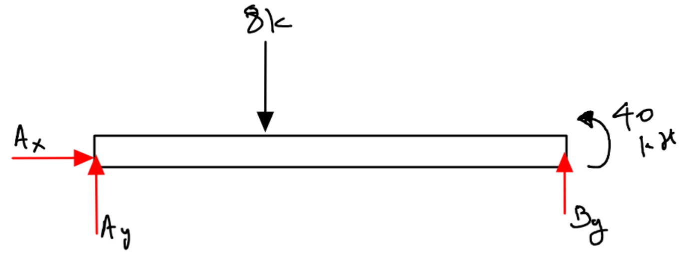
$\underline{\Sigma F_y = 0}$

$$V_B - 200 + 333.3\left(\frac{3}{5}\right) = 0$$

$$\Rightarrow V_B = \underline{\underline{0}}$$

7-2



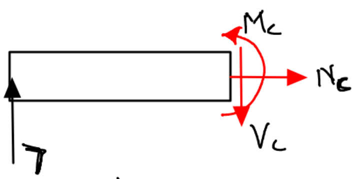


$\Sigma M_A = 0$

$-8(8) + B_y(24) + 40 = 0$

$B_y = \underline{\underline{1k}}$

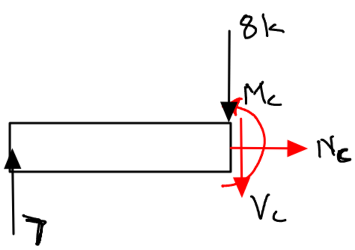
$\therefore A_y = 7k, \quad A_x = 0.$



$V_c = 7k$

$N_c = 0$

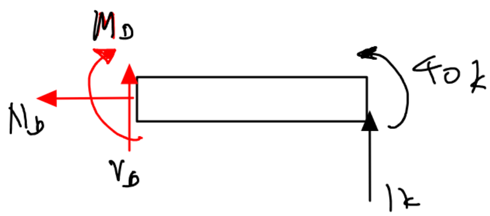
$M_c = \underline{\underline{56}}\ k\,ft$



$N_c = 0$

$M_c = 56\ k\,ft$

$V_c = 7 - 8 = \underline{\underline{-1k}}$



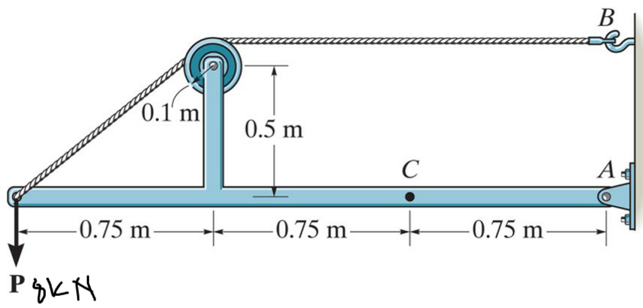
$N_D = 0$

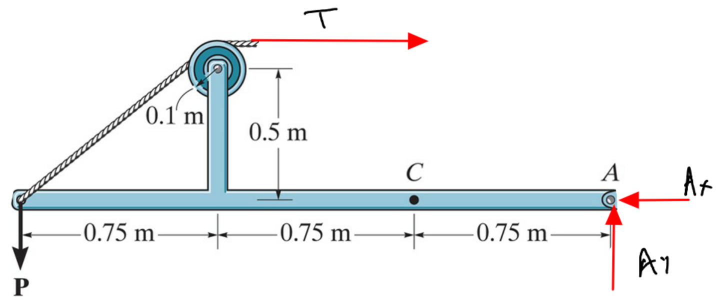
$V_D = -1k$

$M_D = 40 + 1(8)$

$= \underline{\underline{48}}\ k\,ft$

7-9

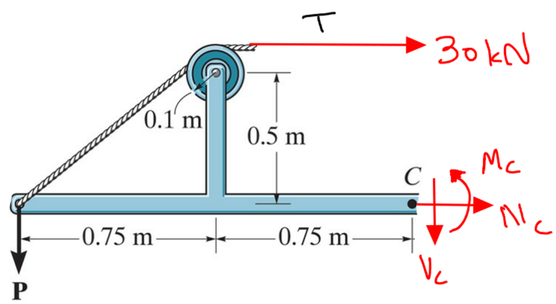




$\Sigma M_A = 0$

$8(2.25) - T(0.6) = 0$

$T = 30 \text{ kN}$



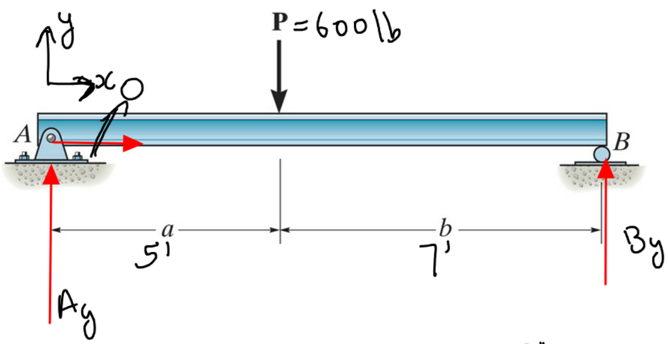
$N_c = -30 \text{ kN}$

$V_c = -8 \text{ kN}$

$8(1.5) - 30(0.6) + M_c = 0$

$M_c = 6 \underline{\underline{\text{kNm}}}$

7-47

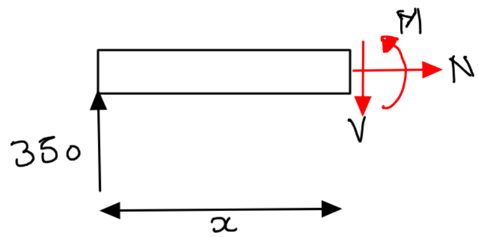


$\circlearrowleft^{+} \sum M_A = 0$

$$-600(5) + B_y(12) = 0$$

$$B_y = 250 \text{ lb}$$

$$\therefore A_y = 350 \text{ lb.}$$



$\sum F_y = 0$

$V = 350$ lb.

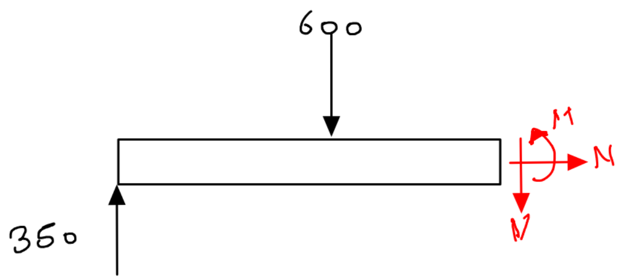
$\sum F_x = 0$

$N = 0$

$\sum M_o = 0$

$-350(x) + M = 0$

$M = 350x$ lb ft.



$N = 0$

$\sum F_y = 0$

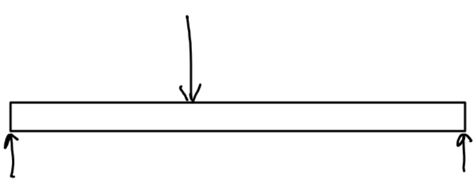
$V = 350 - 600 = -250.$

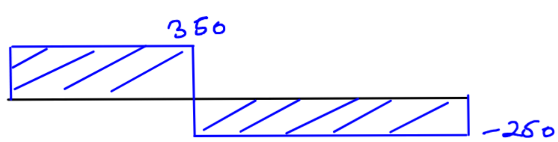
$\sum M_o = 0$

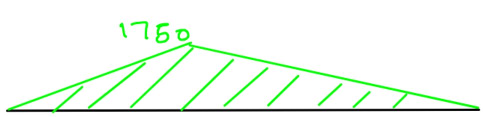
$-350x + 600(x - 5) + M = 0$

$M = 350x - 600x + 3000$

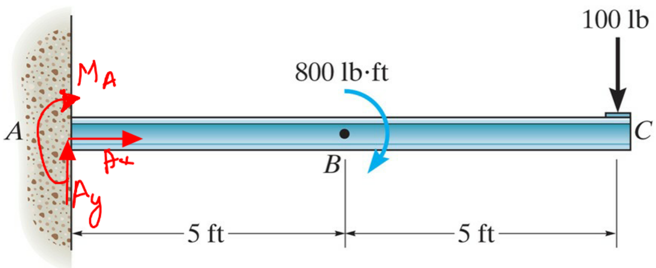
$= -250x + 3000$







7-48.

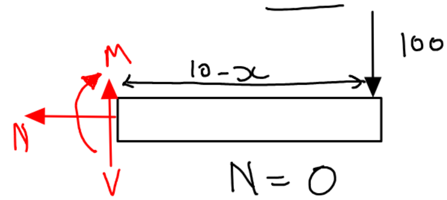


$A_x = 0$

$A_y = 100 \text{ lb}$

$M_A = 800 + 100(10)$

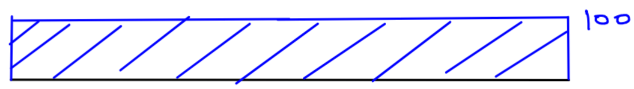
$= \underline{\underline{1800 \text{ lb ft}}}$

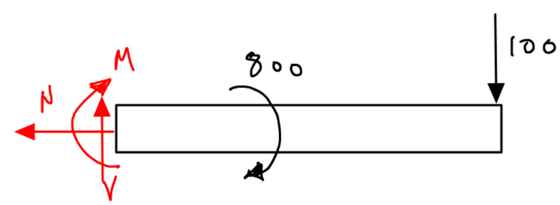


$N = 0$

$V = 100 \text{ lb}$

$M = -100(10 - x) = 100x - 1000$



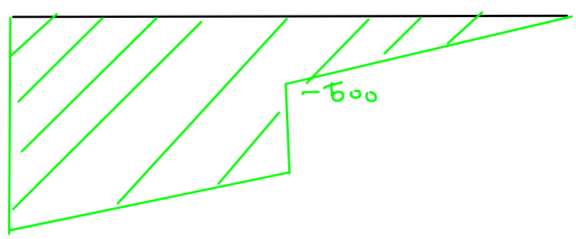


$N = 0$

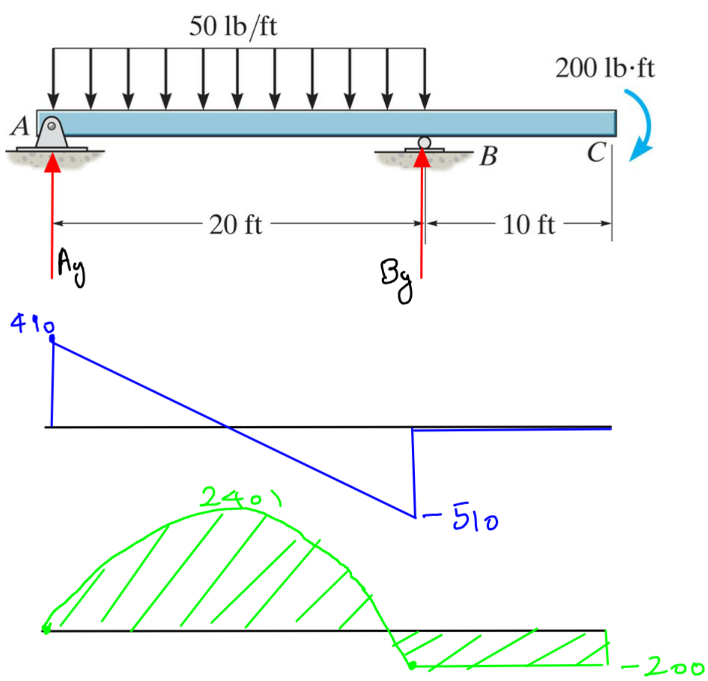
$V = 100 \text{ lb}$

$-100(10 - x) - 800 - M = 0$

$M = 100x - 1800 \text{ lb ft}$



7-53

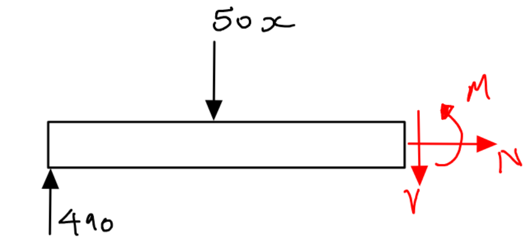


$\Sigma M_A = 0$

$-50(20)(10) + B_y(20) - 200 = 0$

$B_y = 510 \text{ lb}$

$A_y = 50 \times 20 - 510 = 490 \text{ lb}$



$V = 490 - 50x \text{ lb}$

$-490x + 50\frac{x^2}{2} + M = 0$

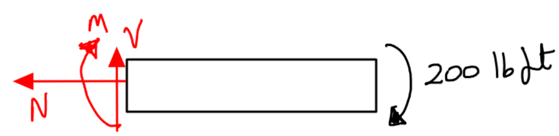
$M = 490x - 25x^2 \text{ lb ft}$

$\frac{dM}{dx} = 490 - 50x = 0$

$x = 9.8$

$M_{max} = 490(9.8) - 25(9.8)^2$

$= 2401 \text{ lb ft}$

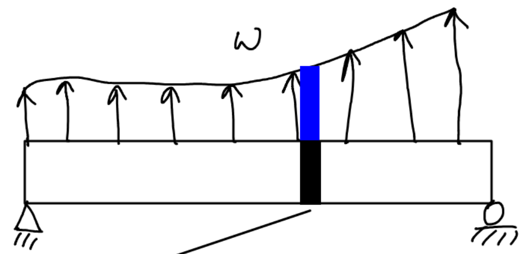


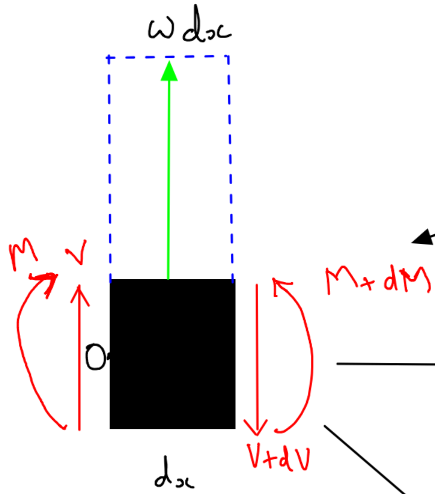
$N = 0$

$V = 0$

$M = -200 \text{ lb ft}$

Relationships between w, V, M.





$$+\uparrow \sum F_y = 0$$

$$\cancel{V} + w\,dx - \cancel{V} + dV = 0$$

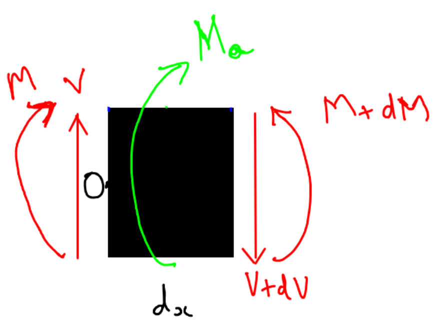
$$w = \frac{dV}{dx} \quad \text{or} \quad dV = w\,dx$$

$$\circlearrowleft \sum M_o = 0$$

$$-\cancel{M} + \cancel{w\,dx\,\frac{dx}{2}}^{\approx 0} + \cancel{M} + dM - (V+dV)\,dx = 0$$

$$dM - V\,dx - \cancel{dV\,dx}^{\approx 0} = 0$$

$$\frac{dM}{dx} = V \quad \text{or} \quad dM = V\,dx.$$



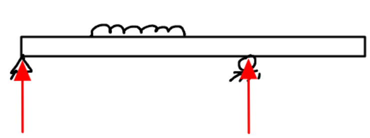
$$\circlearrowleft \sum M_o = 0$$

assume 0. when no shear force.

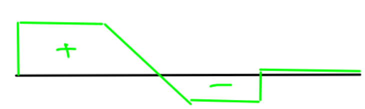
$$-\cancel{M} - M_a - \cancel{(V+dV)}\,dx + \cancel{M} + dM = 0$$

$$dM = M_o$$

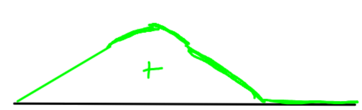
Typically.



SFD



BMD

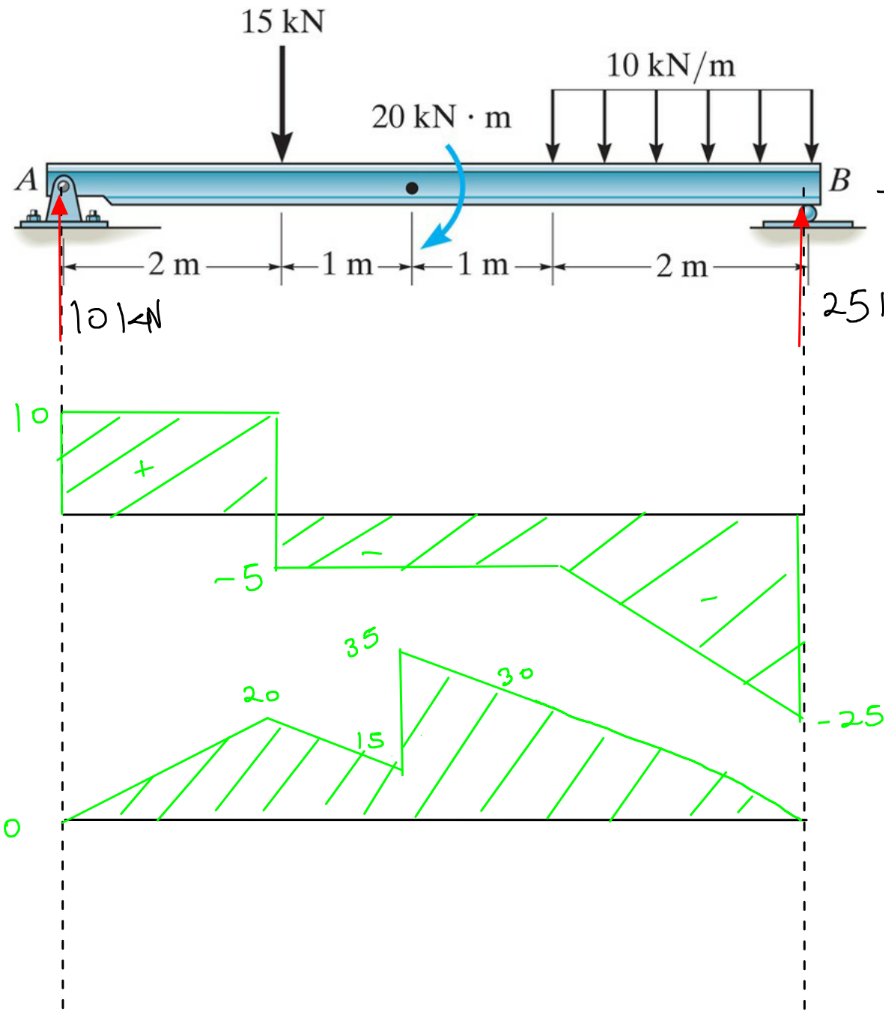


7-76.

$\Sigma M_A = 0$

$-15(2) - 20 - 10(2)(5) + B_y(6) = 0$

$B_y = 25\ \text{kN}$



$\Sigma M_B = 0$

$-A_y(6) + 15(4) - 20 + 10(2)(1) = 0$

$A_y = 10\ \text{kN}$

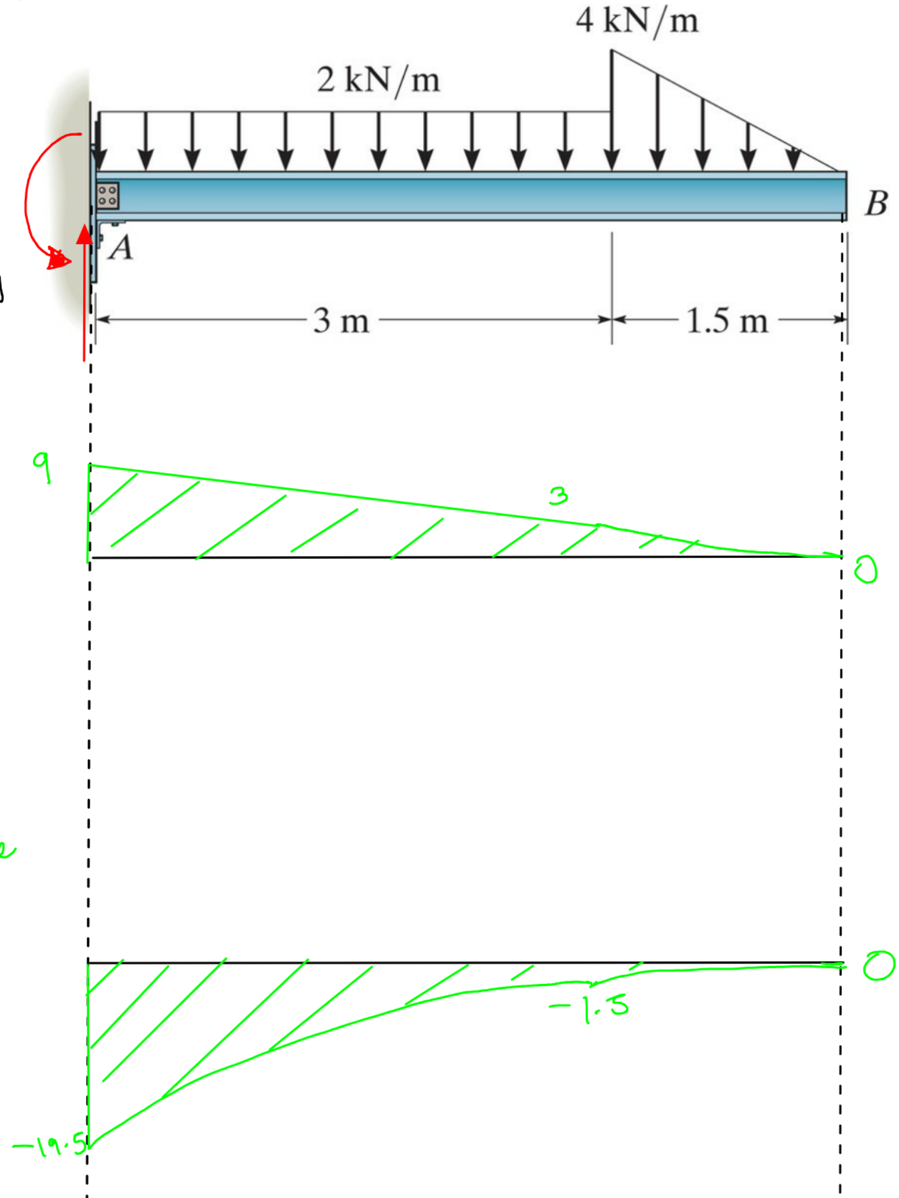
7-87

$\Sigma F_y = 0$

$A_y = 2(3) + \frac{1}{2}(4)(1.5) = 9 \text{ kN}$

$\Sigma M_A = 0$

$M_A - 2(3)(1.5) - \frac{1}{2}(4)(1.5) \times \left(3 + \frac{1}{3}(1.5)\right) = 19.5 \text{ kN m}$



$M = \int V\,dx$

$\neq 18$

$\frac{dM}{dx} = V$ +ve

7-77

$\Sigma M_A = 0.$

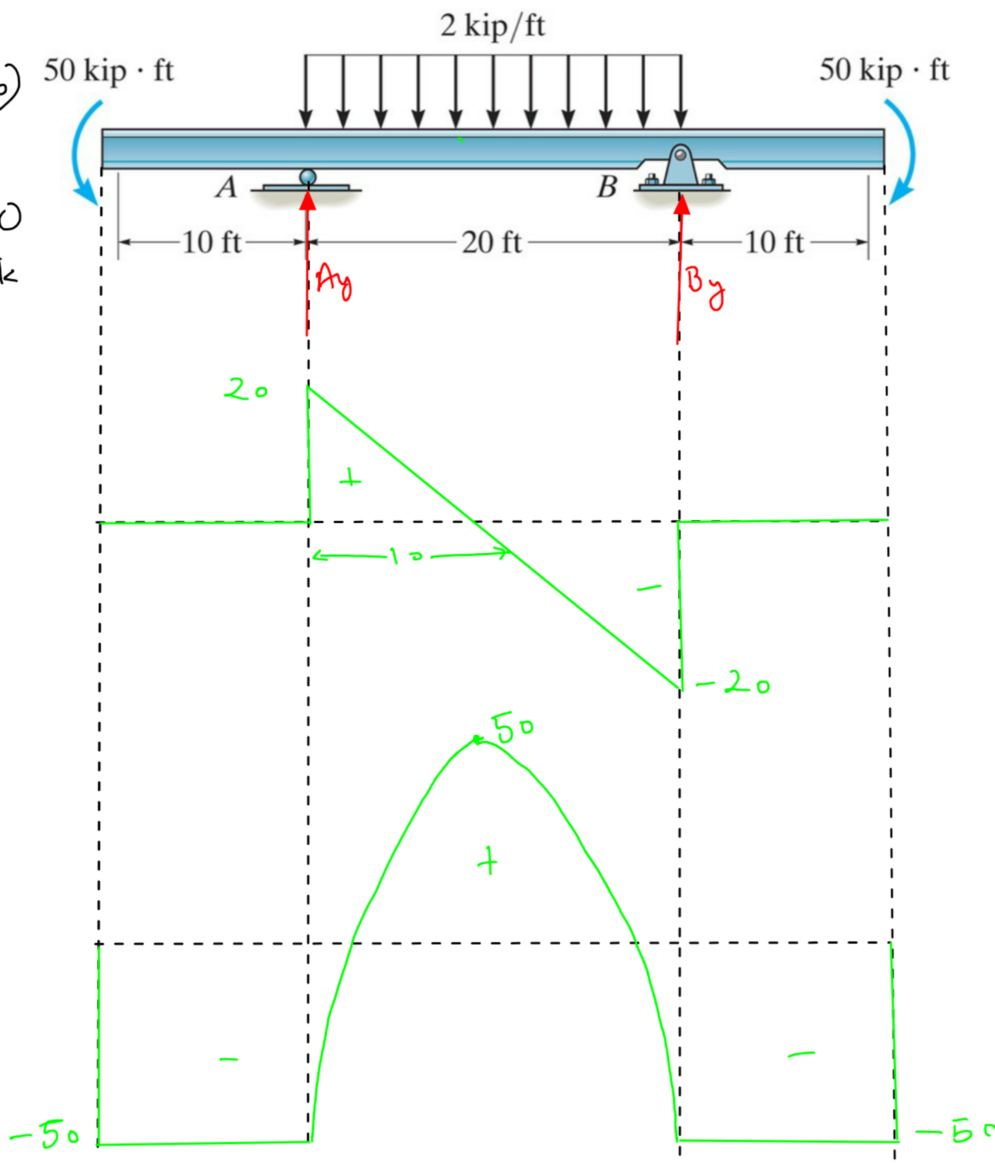
$\cancel{5}0 - 2(20)(10) + B_y(20) - \cancel{5}0 = 0$

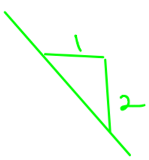
$\underline{\underline{B_y = 20\ k}}$

$\Sigma F_y = 0$

$-2(20) + 20 + A_y = 0$

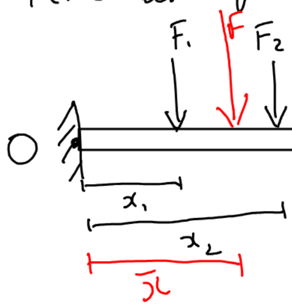
$\underline{\underline{A_y = 20\ k}}$

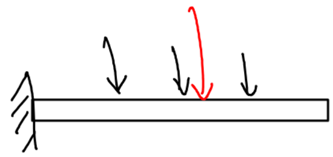


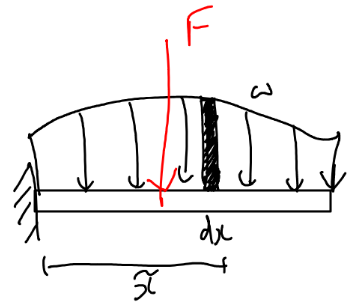


Centers of Mass, Gravity, and Areas.

Remember finding equivalent single force?







ΣM_O

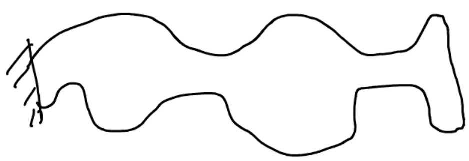
$$F\bar{x} = x_1 F_1 + x_2 F_2$$

$$\bar{x} = \frac{x_1 F_1 + x_2 F_2}{F_1 + F_2}$$

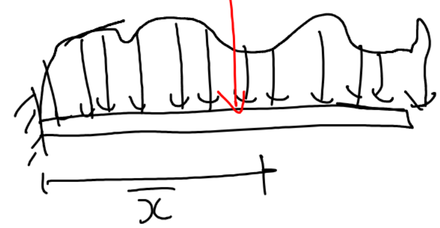
$$\bar{x} = \frac{x_1 F_1 + x_2 F_2 + x_3 F_3}{F_1 + F_2 + F_3}$$

$$\bar{x} = \frac{\int_0^L \tilde{x}\, \omega\, dx}{\int_0^L \omega\, dx}$$

The distributed load could also happen as a result of irregular mass density.



$\Rightarrow$

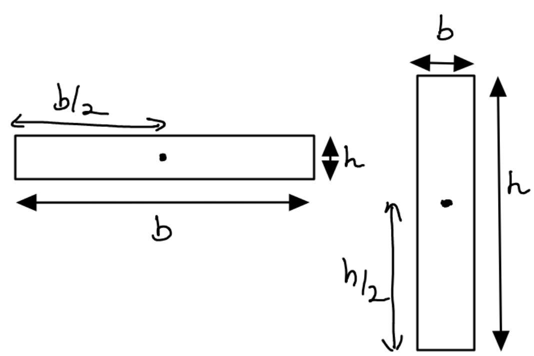


It is the weighted average.

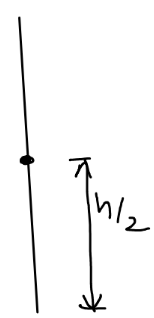
center of Gravity or center of Mass } same for now.

Centroid.

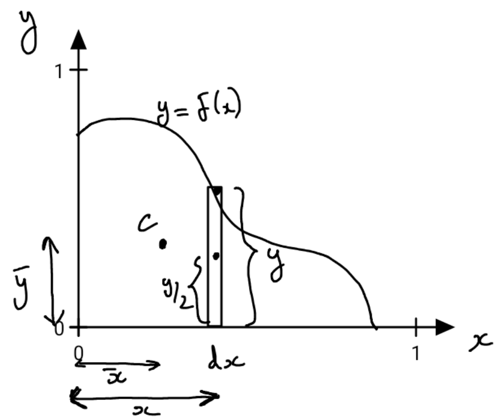
Rectangle :

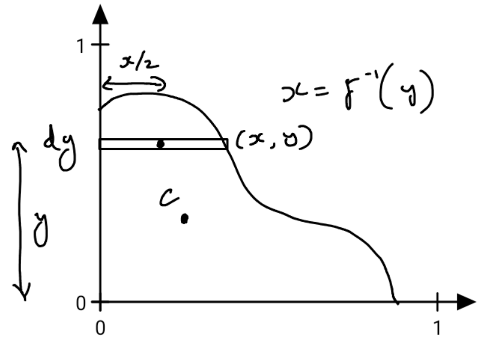


limiting case :



Always fix an origin.





$$dA = y\,dx.$$

$$\int dA \cdot \bar{x} = \int x\,dA$$

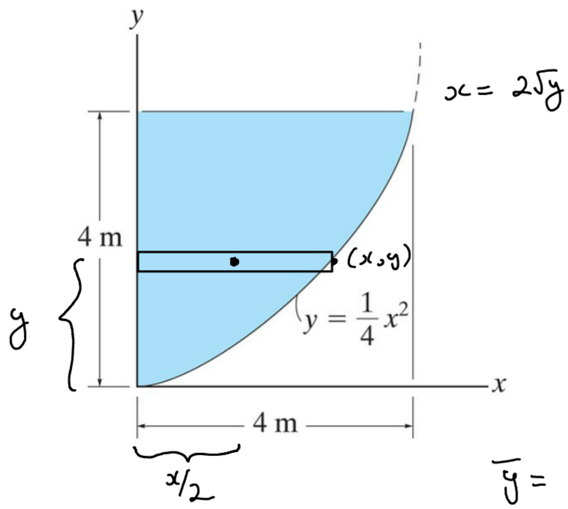
$$\bar{x} = \frac{\int x\,dA}{\int dA}$$

$$\int dA \cdot \bar{y} = \int \frac{y}{2} \cdot dA$$

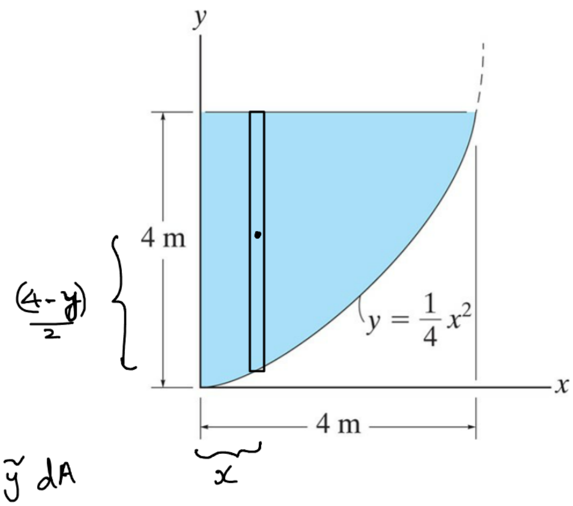
$$\bar{y} = \frac{\int \frac{y}{2}\,dA}{\int dA}$$

weighting factor is the area.

9-10



$$\bar{y} = \frac{\int \tilde{y}\, dA}{\int dA}$$



$dA = x\,dy$

$$\int dA = \int_0^4 x\,dy = \int_0^4 2\sqrt{y}\,dy$$

$$= 2\left[\frac{2}{3} y^{3/2}\right]_0^4 = \frac{4}{3}[8] = \frac{32}{3} =$$

$$= 10.67\ m^2$$

$$\int \tilde{y}\,dA = \int_0^4 y(2\sqrt{y})\,dy = 2\int_0^4 y^{3/2}\,dy$$

$$= 2\left[\frac{2}{5} y^{5/2}\right]_0^4 = 25.6\ m^3$$

$$\bar{y} = \frac{25.6}{10.7} = 2.4\ m$$

$dA = (4-y)\,dx$

$$\int dA = \int_0^4 (4-y)\,dx = \int_0^4 \left(4 - \frac{x^2}{4}\right)dx$$

$$= \int_0^4 4\,dx - \int_0^4 \frac{x^2}{4}\,dx = 4[x]_0^4 - \left[\frac{x^3}{12}\right]_0^4 = 16 - 5.33$$

$$= 10.67\ m^2$$

$$\int \tilde{y}\,dA = \int_0^4 \left(y + \frac{(4-y)}{2}\right)(4-y)\,dx$$

$$= \int_0^4 y(4-y) + \frac{(4-y)^2}{2}\,dx$$

$$= \int_0^4 \cancel{4y} - y^2 + \frac{4^2}{2} + \frac{y^2}{2} - \cancel{\frac{8y}{2}}\,dx$$

$$= \int_0^4 8 - \frac{y^2}{2}\,dx$$

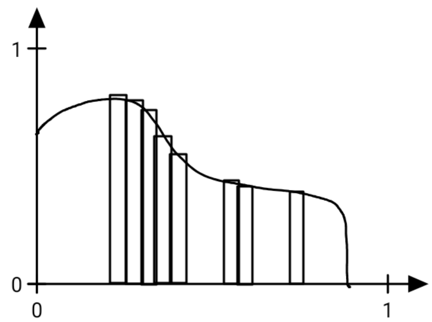
$$= \int_0^4 8\,dx - \int_0^4 \frac{x^4}{32}\,dx$$

$$= 8[x]_0^4 - \left[\frac{x^5}{160}\right]_0^4$$

$$= 25.6\ m^3$$

Centroids of Composite Bodies

- Last class we used calculus to calculate centroid.

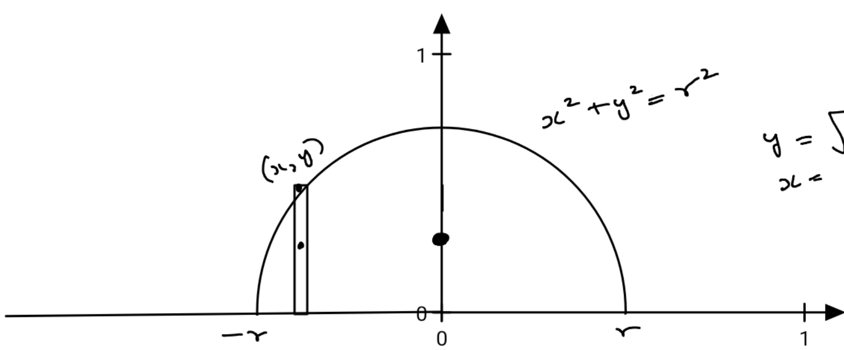


- divided the area into, many rectangles.

- Why rectangles? Because we know where its centroid is.

- In equation of centroid, numerator was sum of Area of rectangles, weighted by the distance from the origin. Denominator was just the sum of areas.

- We can extend this to composite bodies which can be divided into areas we know.



$y = \sqrt{r^2-x^2}$

$x = \sqrt{r^2-y^2}$

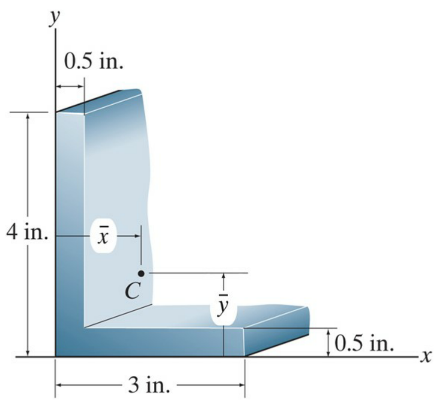
$dA = \sqrt{r^2-x^2}\,dx$

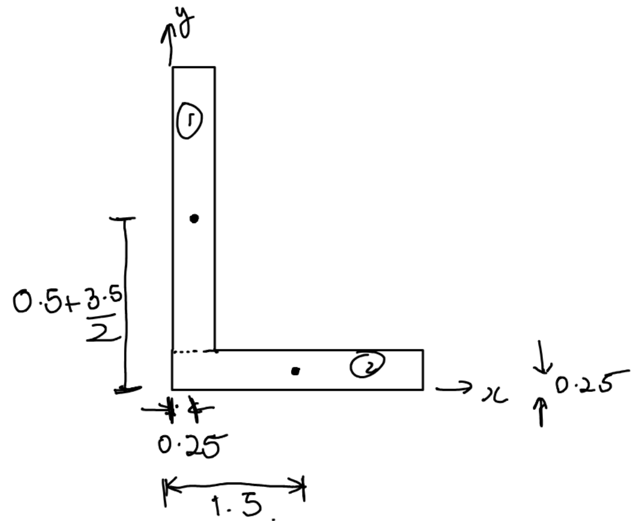
$$\bar{y} = \frac{\int y\,dA}{\int dA} = \frac{\int_{-r}^{r} \frac{(r^2-x^2)}{\cancel{2}}\,dx}{\frac{\pi r^2}{\cancel{2}}} = \frac{r^2x\Big|_{-r}^{r} - \frac{x^3}{3}\Big|_{-r}^{r}}{\pi r^2}$$

$$= \frac{\left[r^3-(-r^3)\right] - \left[\frac{r^3}{3} - \left(-\frac{r^3}{3}\right)\right]}{\pi r^2}$$

$$= \frac{2r^3 - \frac{2r^3}{3}}{\pi r^2} = \frac{4\cancel{r^3}^{\,r}}{3} \times \frac{1}{\pi \cancel{r^2}} = \frac{4r}{3\pi}$$

F 9-10





	Area	$\tilde{x}$	$\tilde{y}$	$A\tilde{x}$	$A\tilde{y}$
A_1	$3.5(0.5)$	0.25"	2.25"	0.4375	3.9375
A_2	$3(0.5)$	1.5"	0.25"	2.25	0.375
	3.25			2.6875	4.3125

$\bar{x} = 0.83''$

$\bar{y} = 1.33''$

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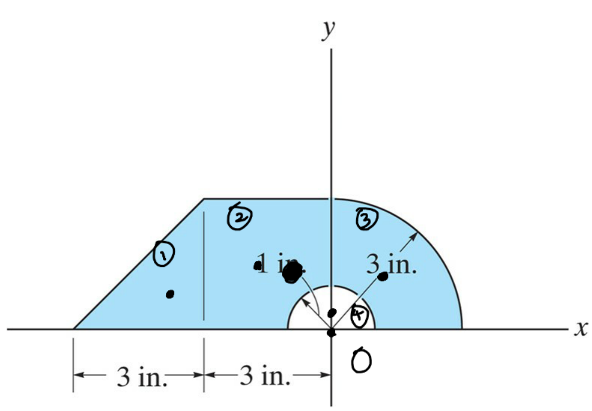


	Area	$\tilde{x}$	$\tilde{y}$	$A\tilde{x}$	$A\tilde{y}$
A_1	4.5	−4	1	−18	4.5
A_2	9	−1.5	1.5	−13.5	13.5
A_3	7.07	1.27	1.27	8.979	8.979
A_4	−1.57	0	0.42	0	−0.659
	19			−22.52	26.32

$\bar{x} = -1.185''$

$\bar{y} = 1.385''$

Moments of Inertia

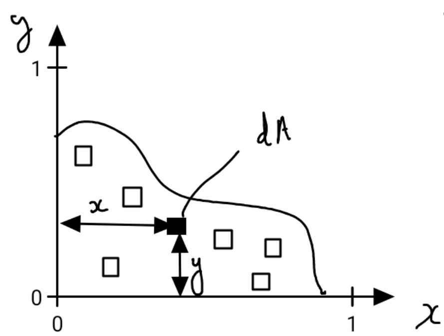
→ The numerator term to find centroid ⇒ $\int y\,dA$ or $\int x\,dA$

→ This term is called the first moment of area.

→ Analogous to moment of a force.

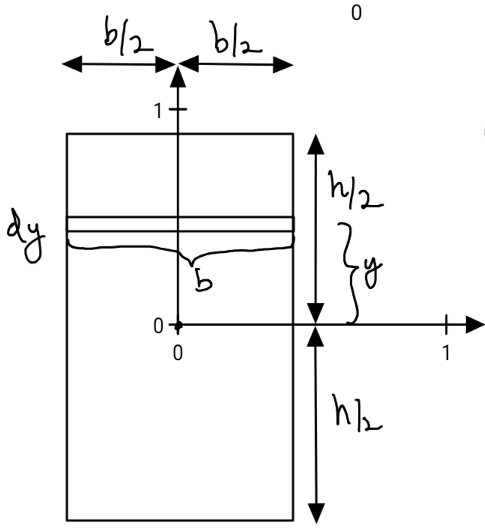
→ There is also what is called second moment of area.

$$\int y^2\,dA \quad \text{or} \quad \int x^2\,dA.$$



$I_x = \int y^2\,dA$

$I_y = \int x^2\,dA$



Calculate I_x. about x-axis

$dA = b\,dy$

$$I_x = \int_{-h/2}^{h/2} y^2\, b\,dy$$

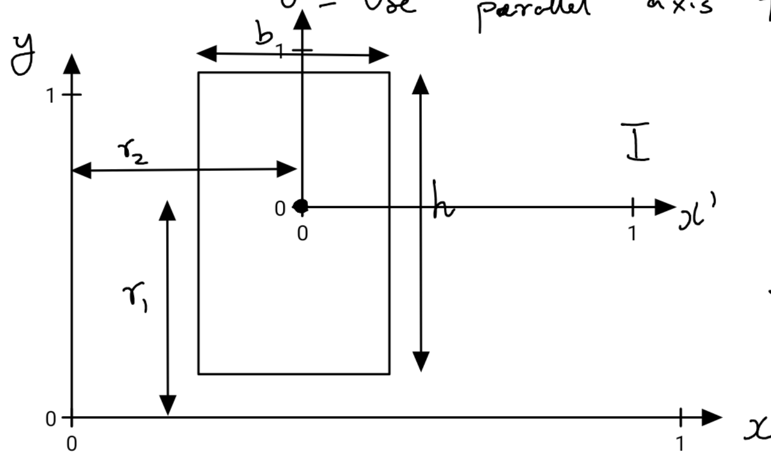
$$= b\left[\frac{y^3}{3}\right]_{-h/2}^{h/2}$$

$$= b\,\frac{\left[\frac{h^3}{8} + \frac{h^3}{8}\right]}{3} = \frac{1}{12}bh^3 \quad m^4$$

For a rectangle, the moment of inertia about its centroidal axis is $\frac{1}{12}bh^3$.

What about a different axis?

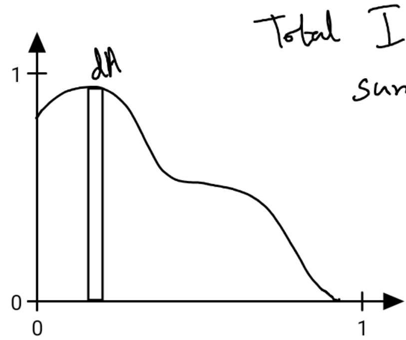
– Use parallel axis theorem.



$$I_x = I_{x'} + A r_1^2$$

$$= \frac{1}{12} b h^3 + A r_1^2$$

$$I_y = \frac{1}{12} h b^3 + A r_2^2$$



Total I_x for random area is the sum of dI_x for all the rectangular dAs.

$\frac{\cancel{x} + 1 - \cancel{x}}{2}$

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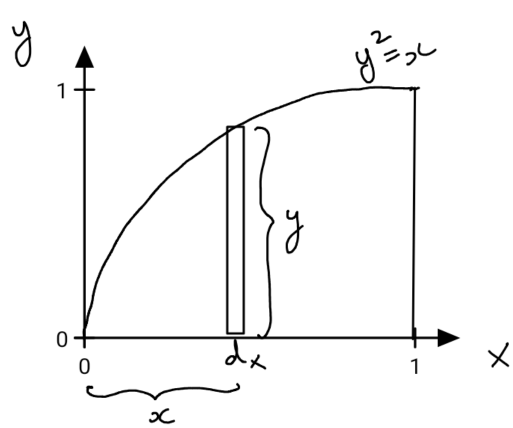
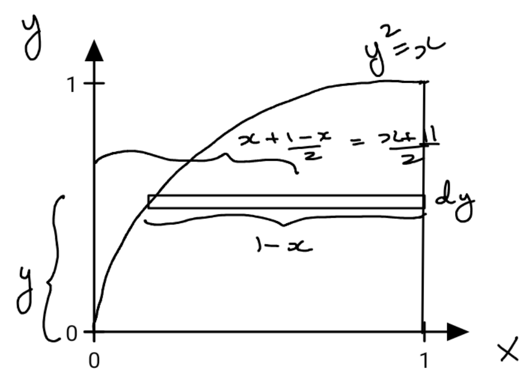
Find I_y





$$dI_y = dI_{y'} + y\,dx\,(x^2)$$

$$= \frac{1}{12}\cancel{y\,dx^3}^{\,0} + x^{5/2}\,dx$$

$$I_y = \int_0^1 x^{5/2}\,dx$$

$$= \frac{2x^{7/2}}{7}\Big|_0^1 = \frac{2}{7}\ m^4$$

$$dI_y = \frac{1}{12}(1-x)^3 dy + (1-x)\frac{(x+1)^2}{2^2} dy$$

$=$

I_x for the same

$= \frac{2}{15}\ m^4$ //