Introduction to Cryptography

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Word of Warning

- This is a short, simplified introduction to cryptography
- We will only introduce what is needed for systems security discussions

What is cryptography (alt. cryptology)?

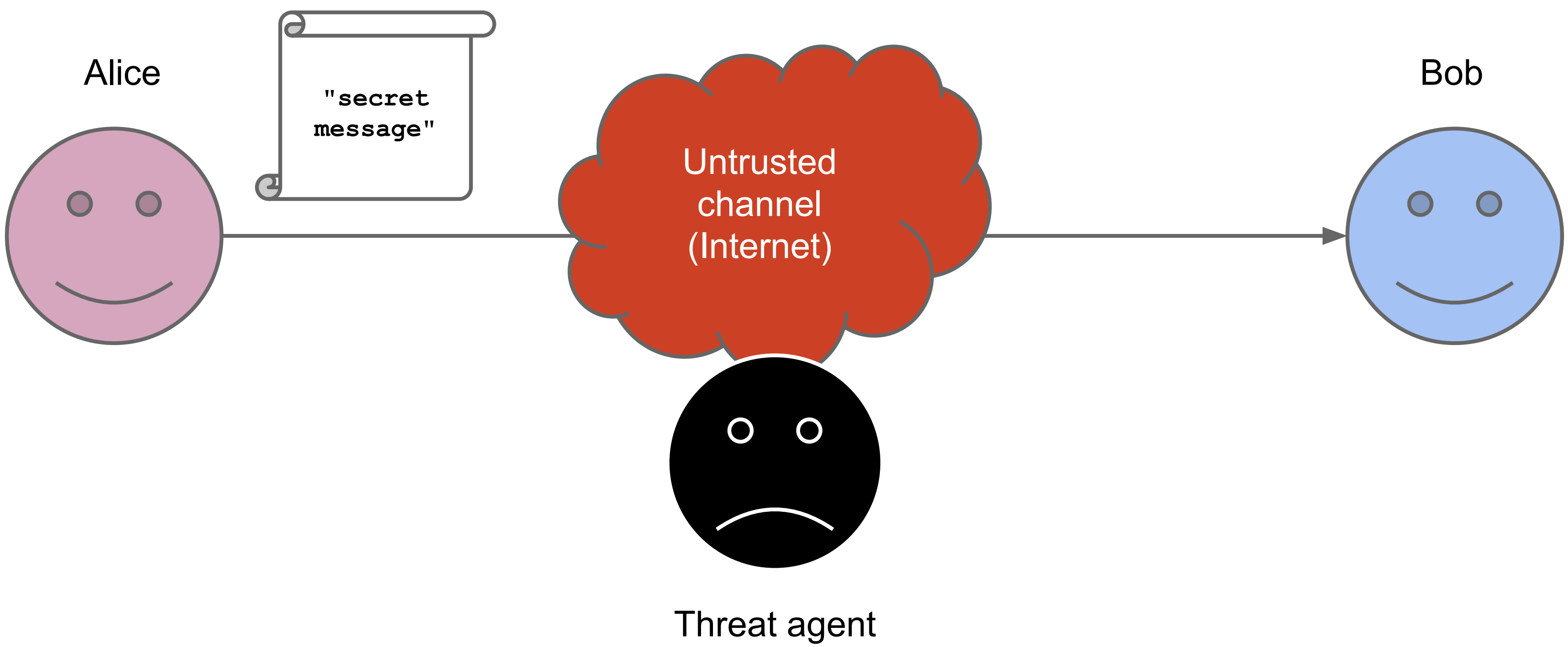
Definition

- The study of techniques to allow secure communication and data storage in presence of attackers

Features provided

- **Confidentiality**: data can be accessed only by chosen entities
- **Integrity/freshness**: detect/prevent tampering or replays
- **Authenticity**: data and their origin are certified
- **Non-repudiation**: data creator cannot repudiate created data
- **Advanced features**: proofs of knowledge/computation

The Problem to Solve: Confidentiality and Integrity



A Brief History of Cryptography

- From Greek: *kryptos*, hidden, and *graphein*, to write (i.e., *“art of secret writing”*)
- Ancient history: writing itself was already a “secret technique”.
- Cryptography born in Greek society, when writing became more common, and hidden writing became a need.

Cryptographic prehistory

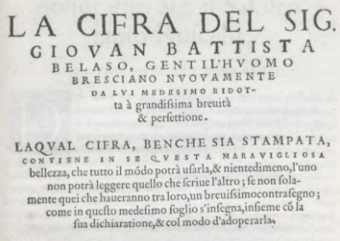
As old as written communication

- Born for commercial (recipe for lacquer on clay tablets) or military (Spartans) uses
- Designed by humans, for human computers
- Algorithms computed by hand, with pen and paper

Ancient view

Original approach

- A battle of wits between
  - cryptographers: ideate a secret method to obfuscate a text
  - cryptanalysts: figure out the method, break the "cipher"
- Bellaso (1553) [1] separates the encryption method from the key


LA CIFRA DEL SIG.
GIOVAN BATTISTA
BELASO, GENTIL'HVOMO
BRESCIANO NVOVAMENTE
DA LVI MEDESIMO RIDOT-
ta à grandiſsima breuità
& perfettione.

LAQVAL CIFRA, BENCHE SIA STAMPATA,
CONTIENE IN SE QVESTA MARAVIGLIOSA
bellezza, che tutto il mõdo potrà uſarla,& nientedimeno,l'uno
non potrà leggere quello che ſcriue l'altro ; ſe non ſola-
mente quei che haueranno tra loro,un breuiſsimocontraſegno ;
come in queſto medeſimo foglio s'inſegna,inſieme cõ la
ſua dichiaratione,& col modo d'adoperarla.

A Brief History of Cryptography

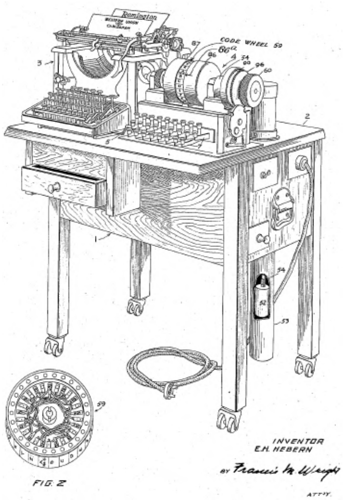
- Medieval and renaissance studies
  - **Gabriele de Lavinde**, who wrote a manual in 1379, copy available at the Vatican archives.
  - The mirror writing of **Leonardo da Vinci**.
- Mostly a *military interest*
  - Italian Army General **Luigi Sacco** wrote a famous *“Nozioni di crittografia”* book in 1925, one of the last “non-formalized” exercises in cryptography.

1883 - Kerchoff's six principles for a good cipher (apparatus)

1. It must be practically, if not mathematically, unbreakable
2. It should be possible to make it public, even to the enemy
3. The key must be communicable without written notes and changeable whenever the correspondants want
4. It must be applicable to telegraphic communication
5. It must be portable, and should be operable by a single person
6. Finally, given the operating environment, it should be easy to use, it shouldn't impose excessive mental load, nor require a large set of rules to be known

The advent of the machines

- Mechanical computation changes cryptography
  - First rotor machine in 1917 by Ed Hebern
  - Design “popularized” in WWII by German Enigma
- Cryptanalysist at Bletchley park (Turing among them) credited for a decisive effort in winning the war by Eisenhower



When Math Won a War

- During WWII, **Alan Turing** worked at Bletchley Park to **break** Axis ciphers, in particular the **Enigma cipher**.

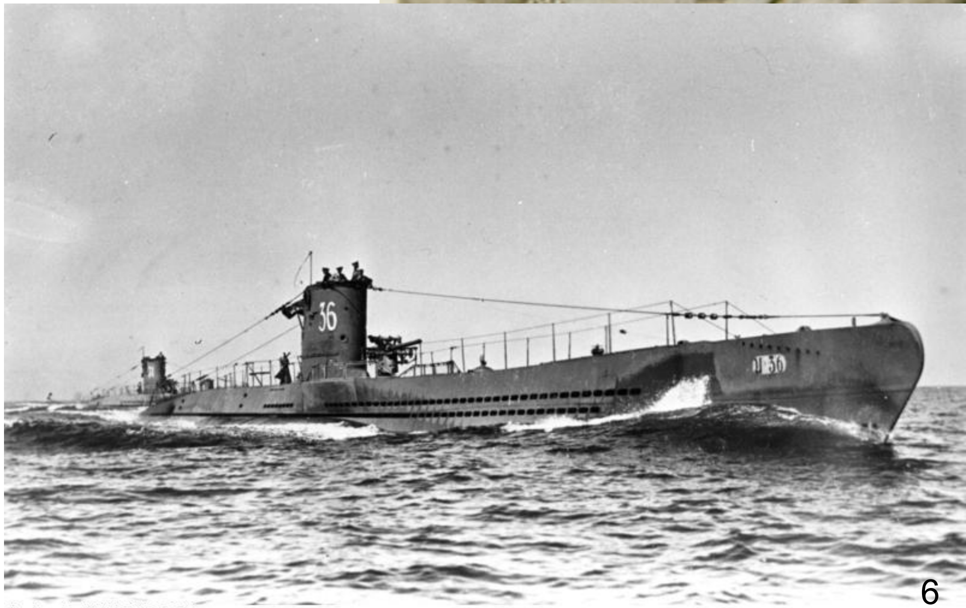


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Foto: o.Ang. | 1936/1939 ca.

When Math Won a War

- During WWII, **Alan Turing** worked at Bletchley Park to **break** Axis ciphers, in particular the **Enigma cipher**.
- Birth of the first universal computers was stimulated by this effort.

THE BOMBE (replica)

Moving into the modern age

An end to the battle of wits

- Shannon (1949) [4] - Proves that a mathematically unbreakable cipher exists
- Nash (1955) [2] - Argues that **computationally secure** ciphers are ok
    - Considers a cipher with a finite, λ bit long, key
    - Conjecture: if "*parts of the key interact complexly [...] in the determination of their effects on the ciphertext*", the attacker effort to break the cipher would be $\mathcal{O}(2^{\lambda})$
    - The owner of the key takes $\mathcal{O}(\lambda^2)$ to compute the cipher
    - The computational gap is unsurmountable for large λ

Key Concepts in Cryptography

- First formalized by Claude Shannon in his 1949 paper *"Communication theory of secrecy systems"*.
- **Cryptosystem**: a system that takes in input a message (known as **plaintext**) and transforms it into a **ciphertext** with a reversible function that usually takes a **key** as a further input.
- The use of *"text"* is historical, and today we mean *"string of bits"*.

Kerckhoffs' Principle

- *The security of a cryptosystem relies only on the secrecy of the key, and never on the secrecy of the algorithm.*
  - Auguste Kerckhoffs, *“La criptographie militaire”*, 1883
- This means that:
  - In a secure cryptosystem we cannot retrieve the plaintext from the ciphertext without the key.
  - Also, we cannot retrieve the key from analyzing ciphertext-plaintext pairs.
  - Algorithms must always be assumed known to the attacker, no secret sauce!

Outline of the topics

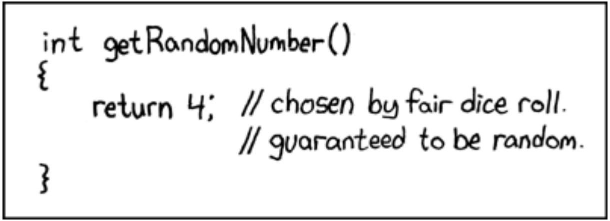
In this course

- Definitions of ciphers as components with functionalities
- How to obtain confidentiality, integrity, data/origin authentication
- An overview of protocols (combinations of ciphers)
- Goal: be able to use cryptographic components properly

In Cryptography and Architectures for Computer Security

- Design and cryptanalysis techniques for ciphers
- Cryptographic protocols and their inner workings
- Efficient, side channel attack resistant implementations
- Goal: be able to engineer cryptographic components

Before we start...



https://xkcd.com/221/

A word on randomness

- Randomness (in this course) characterizes a generative process
- Stating: “00101 is a random string” actually makes little sense

Definitions

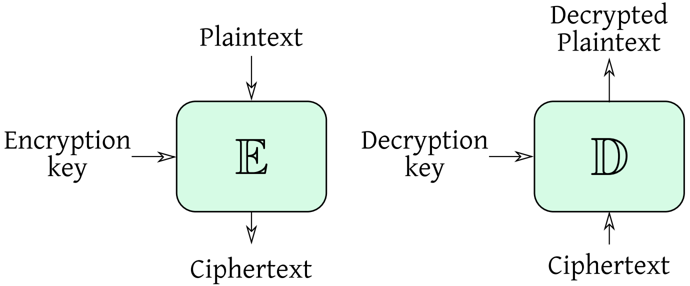
Data

- Plaintext space **P**: set of possible messages $\texttt{ptx} \in \mathbf{P}$
  - Old times: words in some human-readable alphabet, modern times $\{0,1\}^l$
- Ciphertext space **C**: set of possible ciphertext $\texttt{ctx} \in \mathbf{C}$
  - Usually $\{0,1\}^{l'}$, not necessarily $l = l'$ (ciphertexts may be larger)
- Key space **K**: set of possible keys
  - $\{0,1\}^{\lambda}$, keys with special formats are derived from bitstrings

Definitions

Functions

- Encryption function $\mathbb{E} : \mathbf{P} \times \mathbf{K} \rightarrow \mathbf{C}$
- Decryption function $\mathbb{D} : \mathbf{C} \times \mathbf{K} \rightarrow \mathbf{P}$
  - Correctness: for all $\mathtt{ptx} \in \mathbf{P}$, we need $k, k' \in \mathbf{K}$ s.t. $\mathbb{D}(\mathbb{E}(\mathtt{ptx}, k), k') = \mathtt{ptx}$



Providing confidentiality

Goal

- Prevent anyone not authorized from being able to understand data

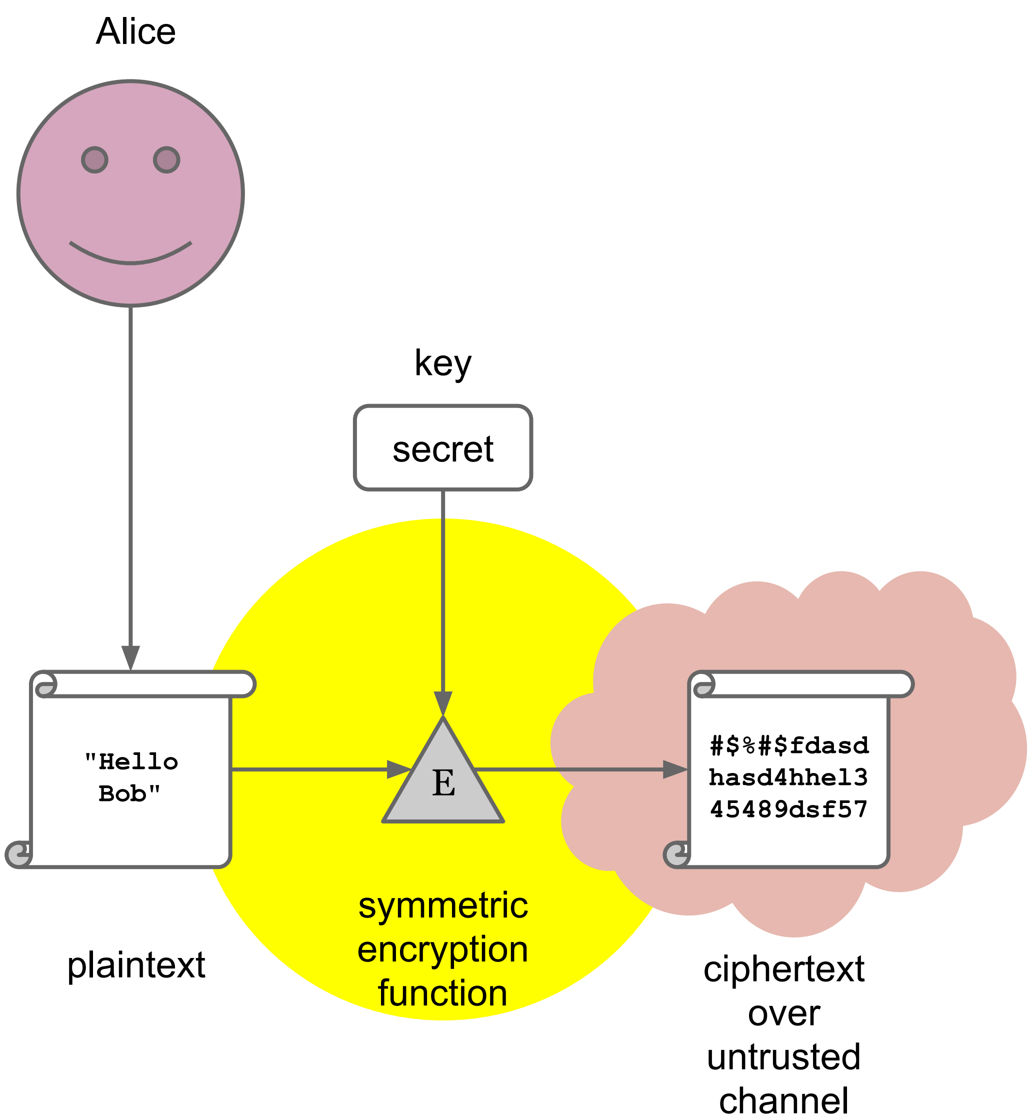
Possible attacker models

- The attacker simply eavesdrops (ciphertext only attack)
- The attacker knows a set of possible plaintexts
  - Limit case: the attacker chooses the set of plaintexts
- The attacker may tamper with the data and observe the reactions of a decryption-capable entity
  - Limit case: the attacker sees the actual decrypted value

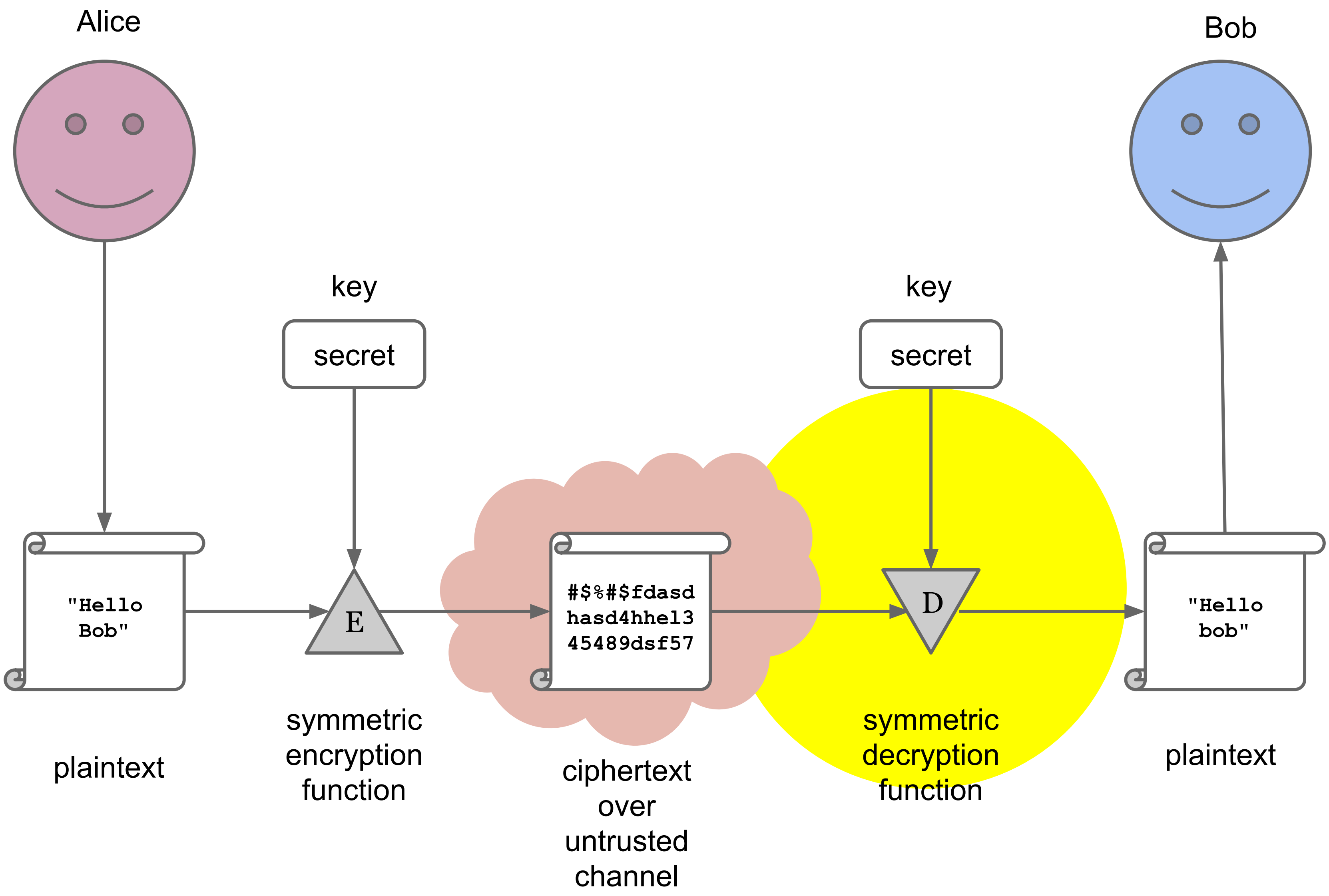
Symmetric Encryption

Confidentiality

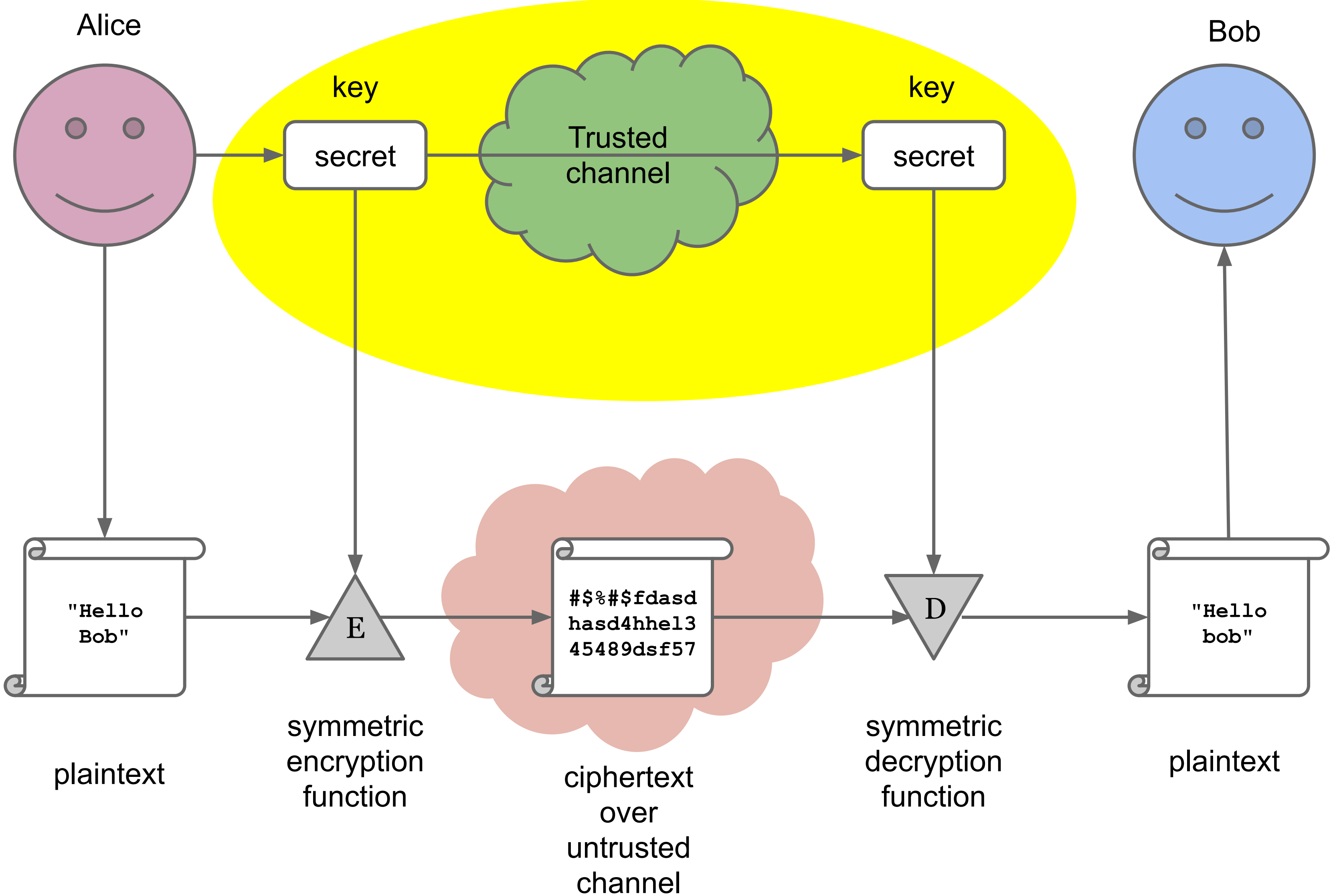
Symmetric Encryption



Symmetric Encryption



Symmetric Encryption



Symmetric Encryption

- The basic idea of encryption
  - Use key K to ***encrypt*** plaintext in ciphertext
  - Use same key K to ***decrypt*** ciphertext in plaintext
- Synonyms: shared key encryption, secret key encryption
- **Issue:** how do we agree on the key?
  - Cannot send key on same channel as message!
  - Off-band transmission mechanism needed
- **Issue:** scalability
- A symmetric algorithm is a cocktail...

First ingredient: substitution

Substitution: “replacing each byte with another”

Toy example (Caesar cipher)

- replace each letter in a sentence with the one following it by K positions in the alphabet
- Example: “**`SECURE`**” becomes “**`VHFXUH`**” with $K = 3$

Many issues (it’s a toy example!)

First ingredient: substitution

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Many issues (it’s a toy example!):

- if cipher known, with 25 attempts at most, 13 on average, we have the key: **keyspace too small**

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Many issues (it's a toy example!)**:**

- if cipher known, with 25 attempts at most, 13 on average, we have the key: **keyspace too small**.
- **repetitions** and **structure "**visible" in ciphertext: monoalphabetic ciphers are weak (frequency analysis - CPTX only attack) https://en.wikipedia.org/wiki/Letter_frequency

Polyalphabetic ciphers (Vigenere Cipher)

m=SECURE

k=SECRET

c=?

-- PLAINTEXT --

KEY	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V

Polyalphabetic ciphers (Vigenere Cipher)

m=SECURE

k=SECRET

c=K

-- PLAINTEXT --

KEY	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	[illegible]	[illegible]	[illegible]	[illegible]	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	[illegible]	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V

Polyalphabetic ciphers (Vigenere Cipher)

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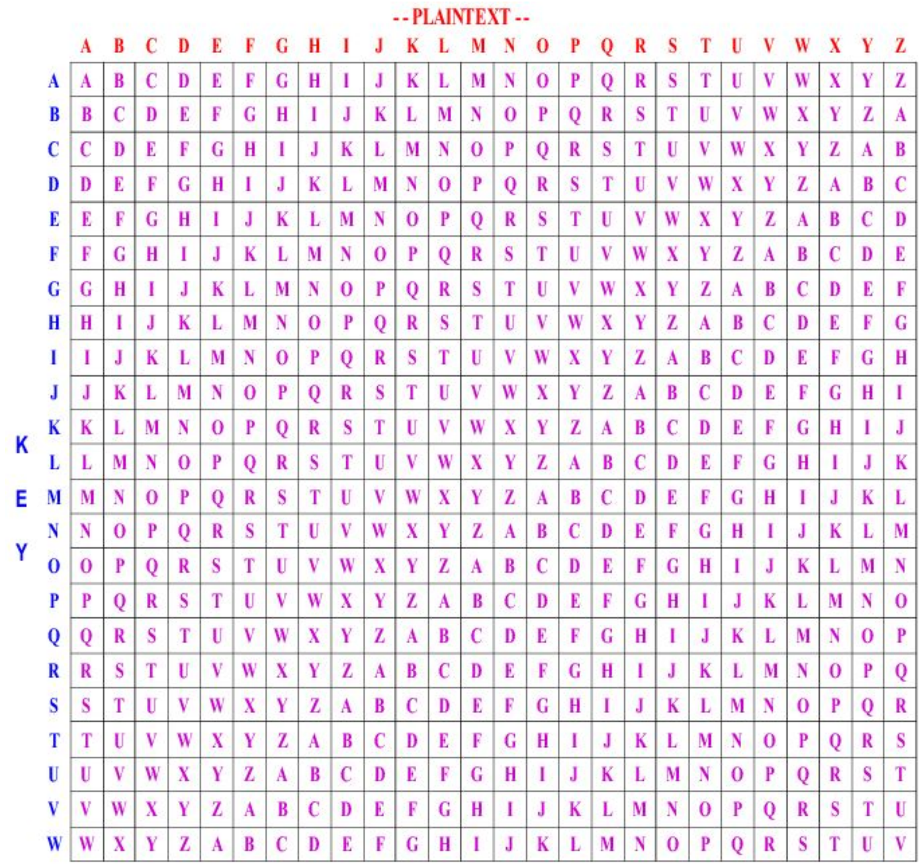
c=KIELVX



--PLAINTEXT--

KEY

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V

Second ingredient: transposition

Transposition (or diffusion) means "swapping the values of given bits"

Toy example (matrix):

- Write by rows, read by columns
- Key: $K = (R, C)$ with $\mathrm{R} * \mathrm{C} \sim \mathrm{len(msg)}$

plaintext

H	A	L	L	O
	E	V	E	R
Y	O	N	E	!

ciphertext

Many issues (it's a toy example!)

Example - Diffusion

H	A	L	L	O
	E	V	E	R
Y	O	N	E	!

m= HALLO EVERYONE!

k=(3,5)

c=H YAEOLVNLEEOR!

Example - Diffusion

H	**A**	**L**	**L**	**O**

m= HALLO

k=(3,5)

c=H A L L O

R * C >> len(msg)

15 >> 4

Second ingredient: transposition

Transposition (or diffusion) means “swapping the values of given bits”

plaintext

H	**A**	**L**	**L**	**O**
	E	**V**	**E**	**R**
Y	**O**	**N**	**E**	

ciphertext

Toy example (matrix):

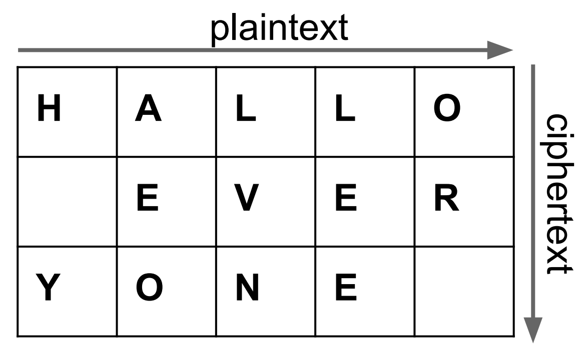
- Write by rows, read by columns
- Key: $K = (R, C)$ with $\mathrm{R} * \mathrm{C} \sim \mathrm{len(msg)}$

Many issues (it’s a toy example!):

- Keyspace still relatively small

Second ingredient: transposition

Transposition (or diffusion) means "swapping the values of given bits"



Toy example (matrix):

- Write by rows, read by columns
- Key: $K = (R, C)$ with $\mathrm{R} * \mathrm{C} \sim \mathrm{len(msg)}$

Many issues (it's a toy example!):

- Keyspace still relatively small

But **repetitions** and **structure** gone

- We now really need to test all possible structures

Perfectly secure cipher

Definition

- In a *perfect cipher*, for all $\mathtt{ptx} \in \mathbf{P}$ and $\mathtt{ctx} \in \mathbf{C}$, $\Pr(\text{ptx sent} = \mathtt{ptx}) = \Pr(\text{ptx sent} = \mathtt{ptx} \mid \text{ctx sent} = \mathtt{ctx})$
- In other words: seeing a ciphertext $c \in \mathbf{C}$ gives us *no information* on what the plaintext corresponding to c could be

Question

- The definition is not constructive! Does a perfect cipher exist?
  - If yes, what does it look like?

Perfectly secure cipher

Theorem (Shannon 1949)

Any symmetric cipher $\langle \mathbf{P}, \mathbf{K}, \mathbf{C}, \mathbb{E}, \mathbb{D} \rangle$ with $|\mathbf{P}| = |\mathbf{K}| = |\mathbf{C}|$ is perfectly secure if and only if

- every key is used with probability $\frac{1}{|\mathbf{K}|}$
- a unique key maps a given plaintext into a given ciphertext: $\forall(\mathtt{ptx}, \mathtt{ctx}) \in \mathbf{P} \times \mathbf{C},\ \exists! k \in \mathbf{K} \text{ s.t. } \mathbb{E}(\mathtt{ptx}, k) = \mathtt{ctx}$

Perfectly secure cipher

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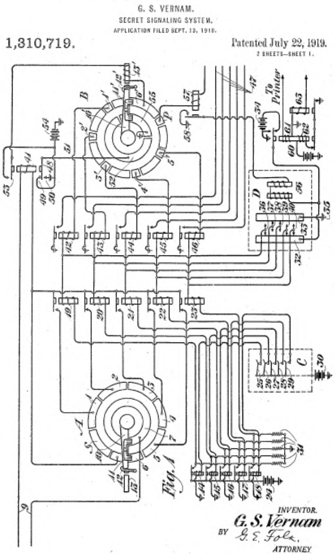
A simple working example

- Assume $\mathbf{P}, \mathbf{K}, \mathbf{C}$ to be set of binary strings. The encryption function draws a *uniformly random, fresh* key k out of $\mathbf{K}$ each time it is called and computes $\mathtt{ctx} = \mathtt{ptx} \oplus k$

Anticausal implementations

A concrete apparatus

- Gilbert Vernam actually patented a telegraphic machine implementing $\mathtt{ptx} \oplus k$ on Baudot code in 1919
- Joseph Mauborgne suggested the use of a random tape containing k
- Using Vernam's encrypting machine with Mauborgne's suggestion implements a perfect cipher



The One Time Pad: Perfect Cipher

- XOR of a message m and a random key k of the same size of m: $len(k) = len(m)$
  - The key is pre-shared and consumed while writing. Can never be re-used again!
- The OTP is a **minimal perfect cipher**
  - Minimal because $|K| = |M|$

Plain text: T H I S I S S E C R E T
OTP-Key : X V H E U W N O P G D Z

Ciphertext: Q C P W C O F S R X H S
In groups : QCPWC OFSRX HS

Perfectly secure $\neq$ perfectly usable

Key storage/management

- storing key material and changing keys is a nightmare
- perfect cipher broken in practice due to key theft/reuse
- generating random keys was also an issue (and caused breaks)

Photo courtesy of Cryptomuseum.com

Imperfections and Brute Force

- Real-world algorithms are not perfect (|K|<|M|), and so can be broken
  - each ciphertext-plaintext pair leaks a small amount of information (because the key is re-used)
- Only thing unknown is the key (Kerckhoffs)
  - Remember: the algorithm itself is known!
- Brute forcing is possible for any <u>real world cipher</u>
  - Try all possible keys, until **one** produces an output that "*makes sense*".
- <u>**Perfect ciphers (one time pads) are not vulnerable to Brute Force**</u>
  - because trying all the (random) keys will yield all the (possible) plaintexts, which are all equally likely (= no clue)

Non Perfect Cipher Example

M = UGO (21 7 15)

K=+3

C = XJR (24 10 18)

Bruteforcing: k=-1-2-3.....-26 For each k, he/she shifts all letters...

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K=-3 ... M3 = UGO

“Toy” Perfect Cipher Example

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K=+1+2+3

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-1-1-2,.....-26-26-26

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Bruteforcing...For each letter try all k=-1-1-1, -1-1-2,.....-26-26-26

M1=ADA

M2=XKR

M3=ELM

M4=UGO

Cryptanalysis: Breaking Ciphers

A real (non perfect) cryptosystem is **broken** if there is a way to break it that is **faster** than brute forcing.

Types of attacks:

- **Ciphertext attack**: analyst has only ciphertexts
- **Known plaintext attack:** analyst has a set of pairs plain-ciphertext
- **Chosen plaintext attack:** analyst can choose plaintexts and obtain their respective ciphertexts

Example: can you break this?

- I give you a ZIP-compressed file encrypted with a (secret) 4-bytes key
- I tell you how I encrypted it -- algorithm should not be secret (by Kerchoffs):
  - *C* = *K* xor *M*
  - Example:
    - K(hex) = `AA BB CC DD` (repeat the key)
    - M(hex) = `50 4B 03 04 BA DA 55 55 .. .. .. (and so on)`

      XOR
    - C(hex) = `FA F0 CF D9 10 61 99 88 .. .. .. .. .. .. ..`
- I give you a ZIP file encrypted with a key: can you recover the key w/o bruteforcing?

The Zip Example

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__PERFECT OR NOT ?__

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<u>NOT PERFECT -> len(k) < len(M) -> k is reused</u>

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- *C* = *K* xor *M*
- *K = M xor C*
- *K* = *X X X X.... xor FA F0 CF D9....*

```
michele@starkiller ~/Desktop
➤ xxd test.zip | head
00000000: 504b 0304 1400 0808 0800 bb74 3150 0000  PK.........t1P..
00000010: 0000 0000 0000 0000 0000 1400 0000 6974  ..............it
00000020: 2d36 3931 3439 3678 3038 3931 3635 2e70  -691496x089165.p
00000030: 6466 ccbb 6554 5ccb ba36 8abb bbbb 5be3  df..eT\..6....[.
00000040: eeee ee6e 8dbb bb6b 20b8 8510 9ce0 ee2e  ...n...k .......
00000050: 0182 bb3b c125 b806 bb49 d65e 7baf 7dce  ...;.%...I.^{.}.
00000060: face ddf7 8cef c71d 73d4 acaa 77be 56d2  ........s...w.V.
00000070: b3bb 9ef1 34a5 b2b8 2423 0b13 3b1c e5b7  ....4...$#..;...
00000080: 9dc9 5938 0e12 6612 4753 1b12 387e 7e38  ..Y8..f.GS..8~~8
00000090: 80ba b713 9004 20e1 e526 a5e6 66e2 0684  ...... ..&..f...
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```
michele@starkiller ~/Desktop
➤ xxd test2.zip | head
00000000: 504b 0304 1400 0808 0000 c051 3050 0000  PK.........Q0P..
00000010: 0000 0000 0000 0000 0000 3200 0000 5241  ..........2...RA
00000020: 4d53 4553 2046 494e 414c 2052 6576 6965  MSES FINAL Revie
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- *K = M xor C*
- *K* = **50 4B 03 04** *xor* FA F0 CF D9 = **AA BB CC DD ->**
  - **ATTACK?**

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- *K = M xor C*
- *K =* `50 4B 03 04` *xor* `FA F0 CF D9 = AA BB CC DD ->`
  - `KNOWN PLAINTEXT ATTACK`

Computationally secure cryptography

A more practical assumption

- Build a cipher so that a successful attack is also able to solve a hard computational problem efficiently
  - Solve a generic nonlinear Boolean simultaneous equation set
  - Factor large integers / find discrete logarithms
  - Decode a random code/find shortest lattice vector

Can we avoid assumptions?

- Open question: prove an exponential lower bound for the time taken to solve a hard problem, which has efficiently verifiable solution
  - it would shift from a computational security assumption to a theorem
  - ... and prove $P \neq NP$ as a corollary

Factor Large Integers (hints)

If p and q are two ***large primes***:

- computing $n = p * q$ is ***easy***
- but given n it is painfully ***slow*** to get p and q
  - quadratic sieve field, basically "try all primes until you get to the smaller between p and q"

Here "slow/difficult" means *"computationally very intensive"*, for all practical purposes the problem requires bruteforce over all possible values of x

Discrete logarithm (hints)

- If $y = a^x$ then $x = log_a y$ (Math 101)
- given x, a, p,
  - it is **easy** to compute $y = a^x \bmod p$,
  - but knowing y, it is **difficult** to compute x

Different problem than factorization, but it can be shown that they are related

Proving computational security

Outline of the method

1. Define the ideal attacker behaviour
2. Assume a given computational problem is hard
3. Prove that any non ideal attacker solves the hard problem

How to represent attacker and properties?

- Attacker represented as a program able to call given libraries
- Libraries implement the cipher at hand
- Define the security property as answering to a given question
- The attacker wins the game if it breaks the security property more often than what is possible through a random guess

Cryptographically Safe Pseudorandom Number Generators

Motivation and assumption

- We want to use a finite-length key and a Vernam cipher
  - We somehow need to "expand" the key
- We assume that the attacker can only perform poly(λ) computations

Definition

A CSPRNG is a deterministic function $\textsc{prng}: \{0,1\}^{\lambda} \to \{0,1\}^{\lambda+l}$ whose output cannot be distinguished from an uniform random sampling of $\{0,1\}^{\lambda+l}$ in $\mathcal{O}(\text{poly}(\lambda))$. l is the CSPRNG stretch.

CSPRNGs in practice

Existence

- In practice, we have only candidate CSPRNGs
  - We have no proof that a function PRNG exists
  - Proving that a CSPRNG exists implies directly P$\neq$NP

Practical constructions

- Building a CSPRNG "from scratch" is possible, but it is not the way they are commonly built (not efficient)
- Practically built with another building block: PseudoRandom Permutations (PRPs)
  - defined starting from PseudoRandom Functions (PRFs)

Random Functions (RFs)

Randomly drawing a function

- Consider the set $\mathbf{F} = \{f : \{0,1\}^{in} \rightarrow \{0,1\}^{out}; in, out \in \mathbb{N}\}$
- A uniformly randomly sampled $f \xleftarrow{\$} \mathbf{F}$ can be encoded by a 2^{in} entries table, each entry *out* bit wide. $|\mathbf{F}| = (2^{out})^{2^{in}}$

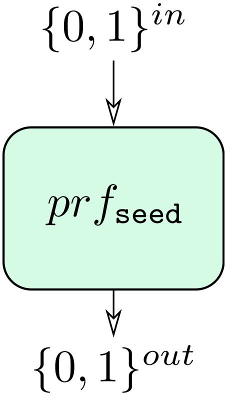
Toy example $in = 2$, $out = 1$

- $\mathbf{F} = \{f : \{0,1\}^2 \rightarrow \{0,1\}^1\}$ is the set of the 16 Boolean functions w/ two inputs
- Each one is represented by a 4-entry truth table
- Intuitively, the functions are 16 as there are $2^4 = 16 = (2^1)^{2^2}$ tables

Pseudorandom Functions (PRFs)

Definition

- A function $prf_{\mathsf{seed}} : \{0,1\}^{in} \to \{0,1\}^{out}$ taking an input and a λ bits seed.
- The entire prf_{seed} is described by the value of the seed
- It cannot be told apart from a random $f \in \{f : \{0,1\}^{in} \to \{0,1\}^{out}\}$ in poly(λ)
- That is, if they give you $a \in \{f : \{0,1\}^{in} \to \{0,1\}^{out}\}$, you can't tell which one of the following is true
  - $a = prf_{\mathsf{seed}}(\cdot)$ with seed$\xleftarrow{\$}\{0,1\}^{\lambda}$
  - $b \xleftarrow{\$} \mathbf{F}$, where $\mathbf{F} = \{f : \{0,1\}^{in} \to \{0,1\}^{out}\}$



Pseudorandom Permutations (PRPs)

Pseudorandom Permutation definition

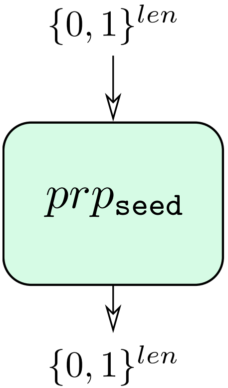
- A bijective PRF: $prf_{\texttt{seed}} : \{0,1\}^{len} \to \{0,1\}^{len}$

Wrapping your mind around it

- It is uniquely identified by the value of the seed
- It is not possible to tell apart in poly(λ) from a RF
- It's a permutation of all the possible $\{0,1\}^{len}$ strings

Operatively speaking

- acts on a block of bits outputs another one of the same size
- the output "looks unrelated" to the input
- its action is fully identified by the seed
  - Useful to think of the seed as a key



Real world PRPs

The issue

- No formally proven PRP exists, yet
  - again, its existence would imply P$\neq$NP

Typical construction

1. Compute a small bijective Boolean function f of input and key
2. Compute f again between the previous output and the key
3. Repeat 2 until you're satisfied

In the real world...

Practical solution: public scrutiny

- Modern PRPs are the outcome of public contests
- Cryptanalytic techniques provide ways (=poly(λ) tests) to detect biases in their outputs: good designs are immune

PRPs a.k.a. Block ciphers

- Concrete PRPs go by the historical name of block ciphers
- Considered broken if, with less than 2^λ operations, they can be told apart from a PRP, e.g., via:
  - Deriving the input corresponding to an output without the key
  - Deriving the key identifying the PRP, or reducing the amount of plausible ones
  - Identifying non-uniformities in their outputs
- The key length λ is chosen to be large enough so that computing 2^λ guesses is not practically feasible

Keyspace and Brute Forcing

Keyspace generally measured in bits

- Attack time exponential on the number of bits (i.e., 33 bits need twice the time of 32)

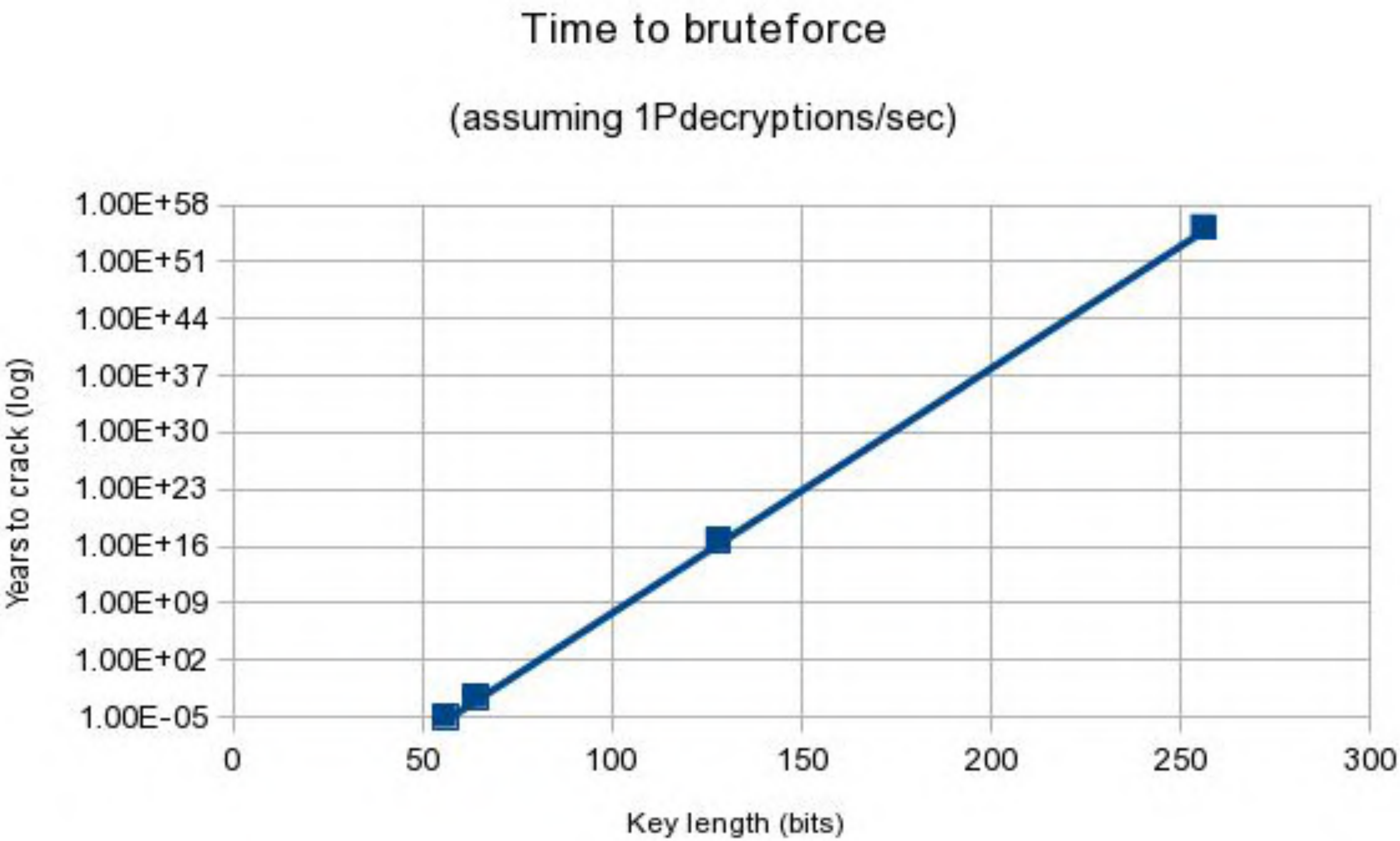
Basic Solution ?

Keyspace and Brute Forcing

Keyspace generally measured in bits

- Attack time exponential on the number of bits (i.e., 33 bits need twice the time of 32)
- Need to balance computational power vs key length.

Keyspace *vs.* Time for Brute Forcing



Quantifying computational unfeasibility

Boolean operations vs. energy to bring from 20 °C → 100 °C

- 2^{65} op.s $\approx$ an Olympic swimming pool
- 2^{80} op.s $\approx$ the annual rainfall on the Netherlands
- 2^{114} op.s $\approx$ all water on Earth

Practically acceptable unfeasibility

- Legacy level security: at least 2^{80} Boolean operations
- 5 to 10 years security: at least 2^{128} Boolean operations
- Long term security: at least 2^{256} Boolean operations

Widespread block ciphers

Advanced Encryption Standard (AES)

- 128 bit block, three key lengths: 128, 192 and 256 bits
- Selected after a 3 years public contest in 2000-10-2 by NIST out of 15 candidates, re-standardized by ISO/IEC
- ARMv8 and AMD64 include dedicated instructions accelerating its computation (hitting 3+ GB/s)

Data Encryption Algorithm (DEA, a.k.a. DES)

- Legacy standard by NIST (1977), the key is too short (56b)
- Patch via triple encryption, $\lambda = 112$ equivalent security
- Still found in some legacy systems, officially deprecated

Case Study: DES

Originally designed by IBM (1973-1974)

Its core function is an S-box (a key-dependent substitution)

It uses a 56 bit key (2^{56} keyspace)

Case Study: DES vs. NSA

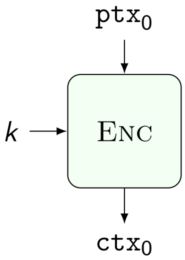
In 1976 it becomes a US standard; its S-boxes are "redesigned" by the NSA

- **Late 1980s:** *differential cryptanalysis* discovered
- **1993**: shown that the original S-boxes would have made DES vulnerable to the differential cryptanalysis, whereas the NSA-designed S-boxes were specifically immune to that.
- **Wait!** Wasn't differential cryptanalysis unknown until late 1980s? *Mmmmmmmaybe* the NSA knew about differential crypto in the 70s.

An Electronic CodeBook (ECB)

A first attempt to encryption with PRPs

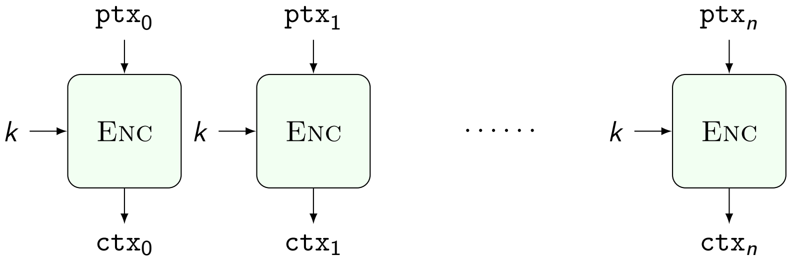
- It's ok to encrypt a plaintext $\leqslant$ block size with a block cipher



An Electronic CodeBook (ECB)

A first attempt to encryption with PRPs

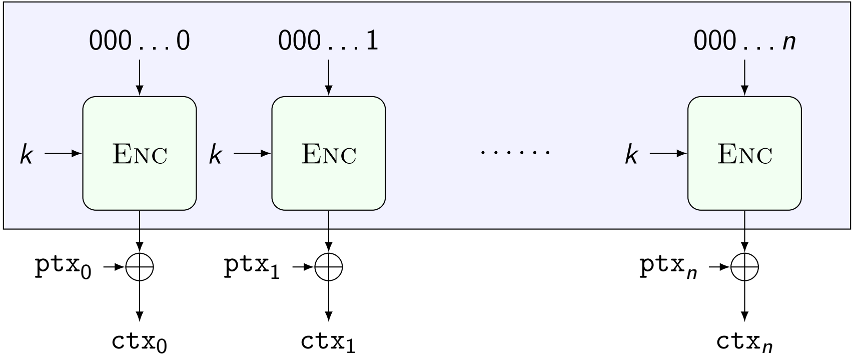
- It's ok to encrypt a plaintext $\leqslant$ block size with a block cipher
- An extension to multiple blocks could be split-and-encrypt
  - Is it good (equivalent to Vernam fed with a CSPRNG)?



Counter (CTR) mode

Getting it right

- The boxed construction is provably a PRNG if Enc is a PRP
- There is nothing special in the starting point of the counters



Raising the requirements

Confidentiality achieved ... for CoA

- Up to now, the attacker knew only ciphertext material

Confidentiality against Chosen Plaintext Attacks (CPAs)

- Our attacker knows a set of plaintexts which can be encrypted
- He wants to understand which one is being encrypted
- Ideal attacker: cannot tell which plaintext was encrypted out of two *he* chose (having the same length)
- Feels strange, but it happens with:
  - management data packets in network protocols (e.g., ICMP)
  - telling apart a encrypted commands to a remote host

Achieving CPA Security

No deterministic encryption

- The CTR mode of operation is insecure against CPA
  - The encryption is deterministic: same ptxs → same ctx

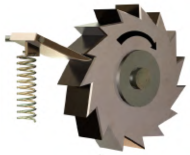
Decryptable nondeterministic encryption

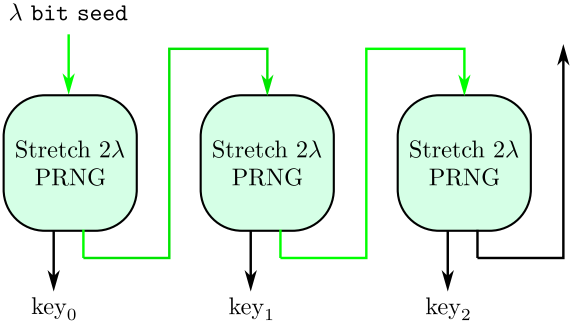
1. Rekeying: change the key for each block with a ratchet
2. Randomize the encryption: add (removable) randomness to the encryption (change mode of employing PRP)
3. Numbers used ONCE (NONCEs): in the CTR case, pick a NONCE as the counter starting point. NONCE is public

Symmetric ratcheting

Getting it right

- The construction takes the name from the mechanical component: it is not possible to roll-back the procedure once you delete the value carried by green arrows

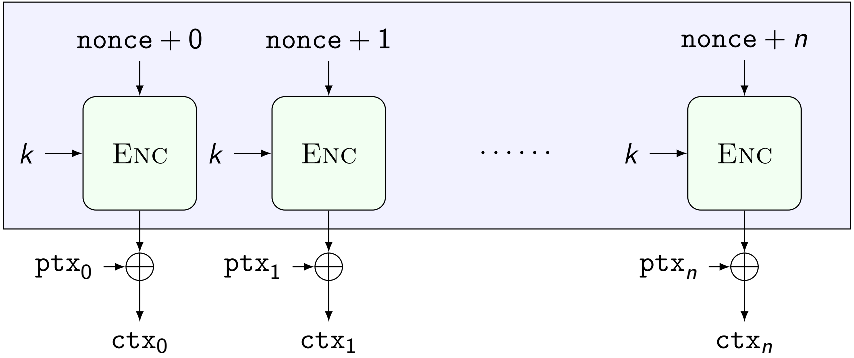




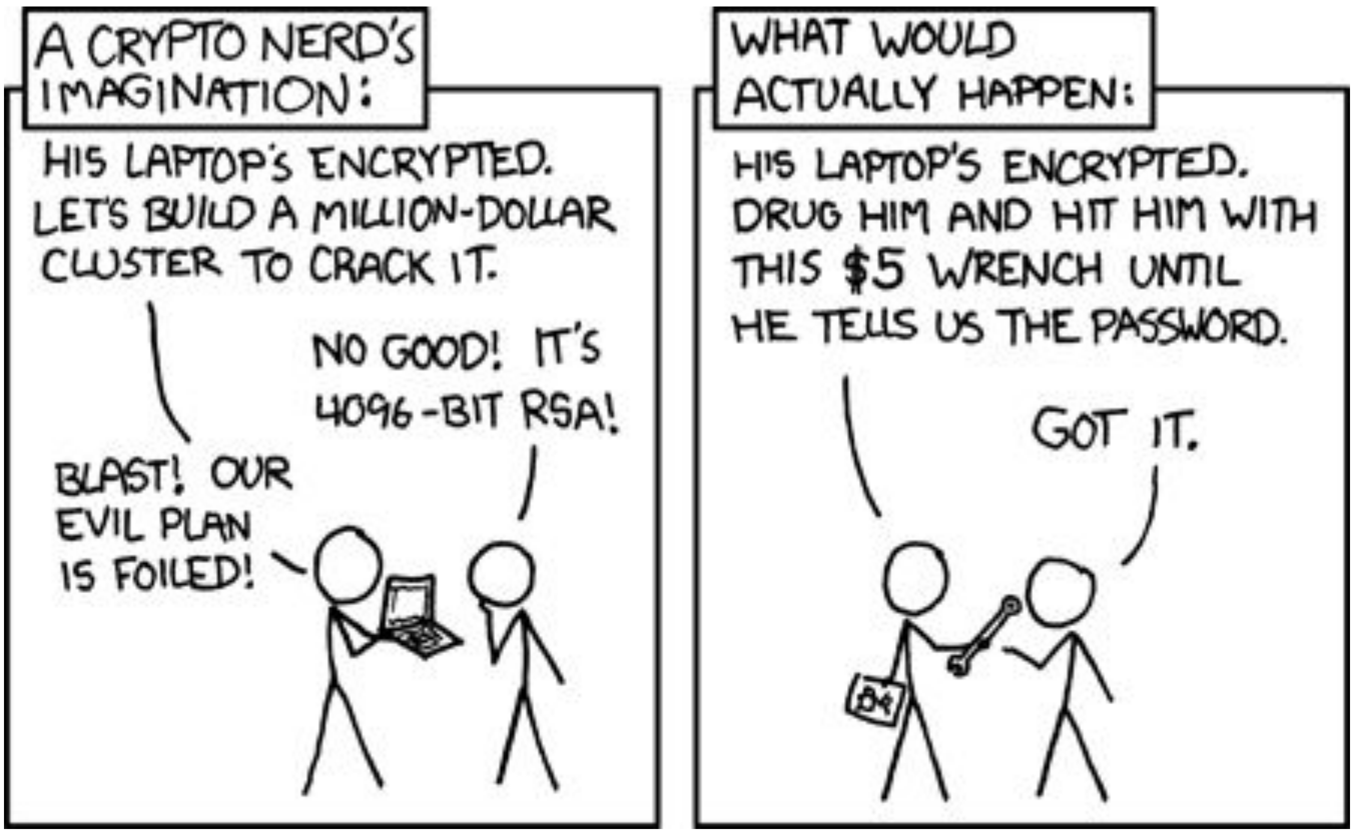
CPA-Secure Counter (CTR) mode

Getting it right

- Picking the counter start as a NONCE generates different bitstreams to be `xor`-ed with the `ptx` each time
- The same plaintext encrypted twice is turned into two different, random-looking, ciphertexts



A good point to remember...



Malleability and active attackers

Malleability

- Making changes to the ciphertext (not knowing the key) maps to predictable changes in the plaintext
  - Think about AES-CTR and AES-ECB
- Can be creatively abused to build decryption attacks
- Can be turned into a feature (homomorphic encryption)

How to avoid malleability

- Design an intrinsically non malleable scheme (non trivial)
- Add a mechanism ensuring data integrity (against attackers)

Providing data integrity

Confidentiality $\not\Rightarrow$ Integrity

- Up to now our encryption schemes provide confidentiality
- Changes in the ciphertext are undetected (at best)

Message Authentication Codes (MAC)

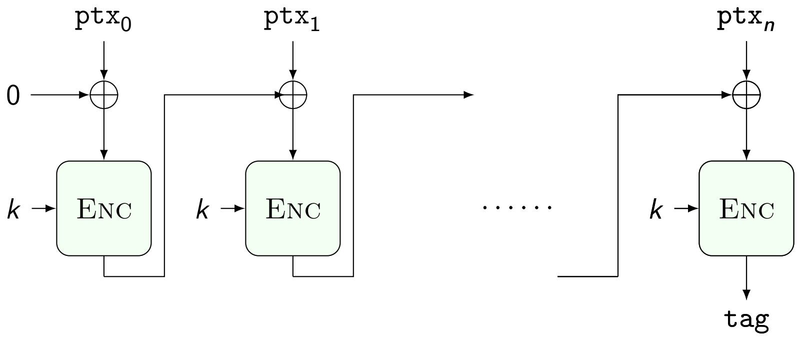
- Add a small piece of information (tag) allowing us to test for the message integrity of the encrypted message itself
  - Adding it to the plaintext and then encrypting is not good
- Nomenclature misleads: MACs do not provide data authentication

MACs

Definition

- A MAC is constituted by a pair of functions:
  - COMPUTE_TAG(string,key): returns the tag for the input string
  - VERIFY_TAG(string,tag,key): returns `true` or `false`
- Ideal attacker model:
  - knows as many message-tag pairs as he wants
  - cannot forge a valid tag for a message for which he does not know it already
  - forgery also includes tag splicing from valid messages
- N.B. the tag creating entity and the verifying entity must both know the same secret key
  - The tag verifier is able to create a valid tag too
  - ... and there goes the non-repudiation property

How to build a MAC? the CBC-MAC



Building a MAC with a PRP (block cipher)

- The CBC-MAC is secure for prefix free messages (why?)
- Encrypting the tag once more fixes (provably) the issue

Practical MAC uses

Browser cookies

- HTTP cookies are a "note to self" for the HTTP server[a]
- The note should not be tampered between server reads
- Solution: server runs COMPUTE_TAG(cookie,k) and stores both the (cookie,tag)

[a] You can find a two slides cookies refresher at slides 40-41 of `https://polimi365-my.sharepoint.com/:b:/r/personal/10032133_polimi_it/Documents/FCI/2-Livello_Applicativo_v2020.pdf`

Later in the course

- Mitigating SYN-based denial of service attacks (SYN Cookies)
- Time-based two-factor authentication mechanisms (TOTP/HOTP)

Compressing for the sake of efficiency

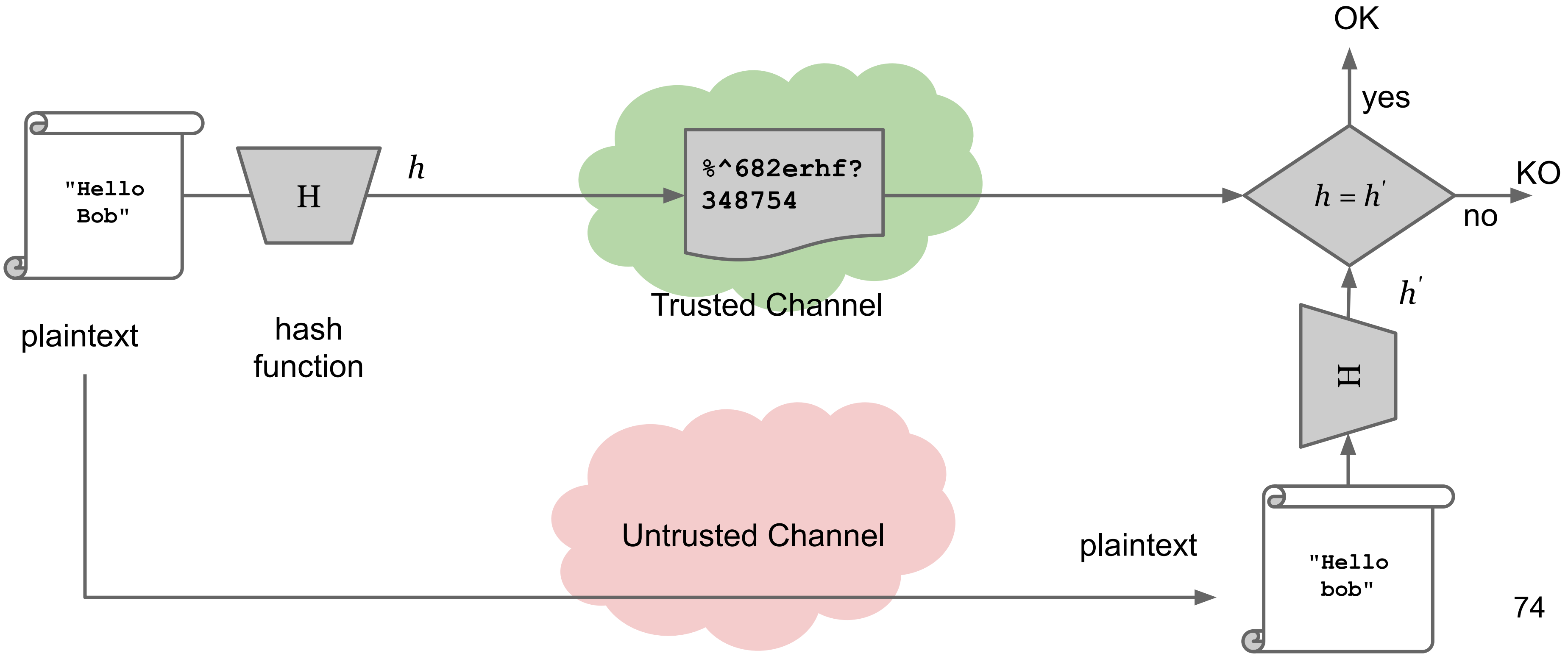
Testing integrity

- Testing the integrity of a file requires us to compare it bit by bit with an intact copy or read it entirely to compute a MAC
- It would be fantastic to test only short, fixed length strings independently from the file size, representing the file itself
  - Major roadblock: there is a lower bound to the number of bits to encode a given content without information loss
- Can we build something close to the ideal scenario?

What is a Hash Function

A function $H(\ )$ that maps arbitrary-length input x on fixed-length output, h *(digest)*

- Need to be **Fast**



What is a Hash Function

A function $H(\ )$ that maps arbitrary-length input x on fixed-length output, h

- Need to be **Fast**
- **Collisions:** codomain “smaller” than domain.

Computationally infeasible to find:

- input x such that $H(x) = h$ (given a specific hash)
  - **preimage attack resistance**
- input y s.t. $y \neq x$ and $H(y) = H(x)$, with a given x
  - **second preimage attack resistance**
- couples of inputs $\{x, y\}$ s.t. $H(x) = H(y)$
  - **collision resistance**

Attacks to Hash Functions (1)

Hash functions may be *broken*.

1. Arbitrary collision or (1st or 2nd) preimage attack:

Given a specific hash h, the attacker can find

$$x \text{ such that } H(x) = h$$

or, equivalently, given a specific input x can find

$$y \text{ such that } y \neq x \text{ and } H(y) = H(x)$$

<u>faster than brute forcing</u>.

With a n-sized hash function, *random* collisions can happen in (2^{n-1}) cases.

Attacks to Hash Functions (2)

2. Simplified collision attack:

The attacker can generate colliding couples

$$\{x, y\} \text{ s.t. } H(x) = H(y)$$

<u>faster than brute forcing.</u>

Random collisions can happen in ($2^{n/2}$) cases because of the birthday paradox:

- given *n* randomly chosen people, some pairs will have same birthday
  - probability = 100% if ***n* = 367**
  - probability = 99.9% if ***n* = 70** people
  - probability = 50% if *n* = 23 people, and so on...
- vs. very low chances that some of you are born on a specific date

Cryptographic hashes

A pseudo-unique labeling function

- A cryptographic hash is a function $H : \{0,1\}^* \rightarrow \{0,1\}^l$ for which the following problems are computationally hard
  1. given $d = H(s)$ find s (1st preimage)
  2. given $s, d = H(s)$ find $r \neq s$ with $H(r) = d$ (2nd preimage)
  3. find $r, s; r \neq s$, with $H(s) = H(r)$ (collision)
- Ideal behaviour of a concrete cryptographic hash:
  1. finding 1st preimage takes $\mathcal{O}(2^d)$ hash computations guessing s
  2. finding 2nd preimage takes $\mathcal{O}(2^d)$ hash comp.s guessing r
  3. finding a collision takes $\approx \mathcal{O}(2^{\frac{d}{2}})$ hash computations
- The output bitstring of a hash is known as a digest

Concrete hash functions

What to use

- SHA-2 was privately designed (NSA), $d \in \{256, 384, 512\}$
- SHA-3 followed a public design contest (similar to AES), selected among ≈ 60 candidates, $d \in \{256, 384, 512\}$
- Both currently unbroken and widely standardized (NIST, ISO)

What not to use

- SHA-1: $d = 160$, collision-broken [6] (obtainable in $\approx 2^{61}$ op.s)
- MD-5: horribly broken [7]. Collisions in 2^{11}, public tools online [5]
  - In particular, collisions with arbitrary input prefixes in $\approx 2^{40}$

Uses for hash functions

Pseudonymized match

- Store/compare hashes instead of values (e.g., Signal contact discovery)

MACs

- Building MACs: generate tag hashing together the message and a secret string, verify tag recomputing the same hash
- A field-proven way of combining message and secret is HMAC
  - Standardized (RFC 2104, NIST FIPS 198)
  - Uses a generic hash function as a plug-in, combination denoted as HMAC-hash_name
  - HMAC-SHA1 (!), HMAC-SHA2 and HMAC-SHA3 are ok

Forensic use

- Write down only the hash of the disk image you obtained in official documents

Game changing ideas

Features we would like to have

- Agreeing on a short secret over a public channel
- Confidentially sending a message over a public authenticated channel without sharing a secret with the recipient
- Actual data authentication

Solution: asymmetric cryptosystems

- Before 1976: rely on human carriers / physical signatures
- DH key agreement (1976) / Public key encryption (1977)
- Digital signatures (1977)

Asymmetric Cryptosystems

Confidentiality (plus something more)

The Diffie-Hellman key agreement

Goal

- Make two parties share secret value w/ only public messages

Attacker model

- Can eavesdrop anything, but not tamper
- The Computational Diffie-Hellman assumption should hold

CDH Assumption

- Let $(\mathbf{G}, \cdot) \equiv \langle g \rangle$ be a finite cyclic group, and two numbers a, b sampled unif. from $\{0, \ldots, |\mathbf{G}| - 1\}$ $(\lambda = len(a) \approx \log_2 |\mathbf{G}|)$
- given g^a, g^b finding g^{ab} costs more than $\text{poly}(\log |\mathbf{G}|)$
- Best current attack approach: find either b or a (discrete log problem)

Structure

Key agreement between Alice and Bob

- Alice: picks $a \xleftarrow{\$} \{0, \ldots, |\mathbf{G}| - 1\}$, sends g^a to Bob
- Bob: picks $b \xleftarrow{\$} \{0, \ldots, |\mathbf{G}| - 1\}$, sends g^b to Alice
- Alice: gets g^b from Bob and computes $(g^b)^a$
- Bob: gets g^a from Bob and computes $(g^a)^b$
- $(\mathbf{G}, \cdot)$ is commutative $\rightarrow (g^b)^a = (g^a)^b$, we're done!

Groups used in practice

- A subgroup $(\mathbf{G}, \cdot)$ of $(\mathbb{Z}_n^*, \cdot)$ (integers mod n), breaking CDH takes $\min\left(\mathcal{O}\left(e^{k(\log(n))^{\frac{1}{3}}(\log(\log(n)))^{\frac{2}{3}}}\right), \mathcal{O}(2^{\frac{\lambda}{2}})\right)$
- EC points w/ dedicated addition, breaking CDH takes $\mathcal{O}(2^{\frac{\lambda}{2}})$

Example: Diffie-Hellman Agreement

- Used by Alice and Bob to agree on a **secret** over an **insecure channel**
  - two people talk in the middle of the classroom, everybody hears them, but at the end only those two people know a secret, and nobody else
- **One-way trapdoor/hard problem**: **discrete logarithm**
  - If $y = a^x$ then $x = log_a y$ (Math 101)
  - given x, a, p, it is easy to compute $y = a^x \bmod p$, but knowing y, it is difficult to compute x
  - Here "difficult" means *"computationally very intensive"*, for all practical purposes the problem requires bruteforce over all possible values of x

How does D-H work (1) - Example

Pick p prime, a primitive root of p, public

Primitive root: a number a such that raising it to any number between 1 and $(p - 1)$, $\mathrm{mod}\ p$, we obtain each number between 1 and $(p - 1)$

- Example: 3 is a primitive root of 7 because
  - $3^1 \bmod 7 = 3$ $3^2 \bmod 7 = 2$ $3^3 \bmod 7 = 6$
  - $3^4 \bmod 7 = 4$ $3^5 \bmod 7 = 5$ $3^6 \bmod 7 = 1$

So let $a = 3$, $p = 7$ known to everyone in the system

How does D-H work (2) - Keys

Secret number (undisclosed)**:** They pick a number *X* in *[1, 2, ..., (p-1)]*

- Alice X_A
- Bob X_B

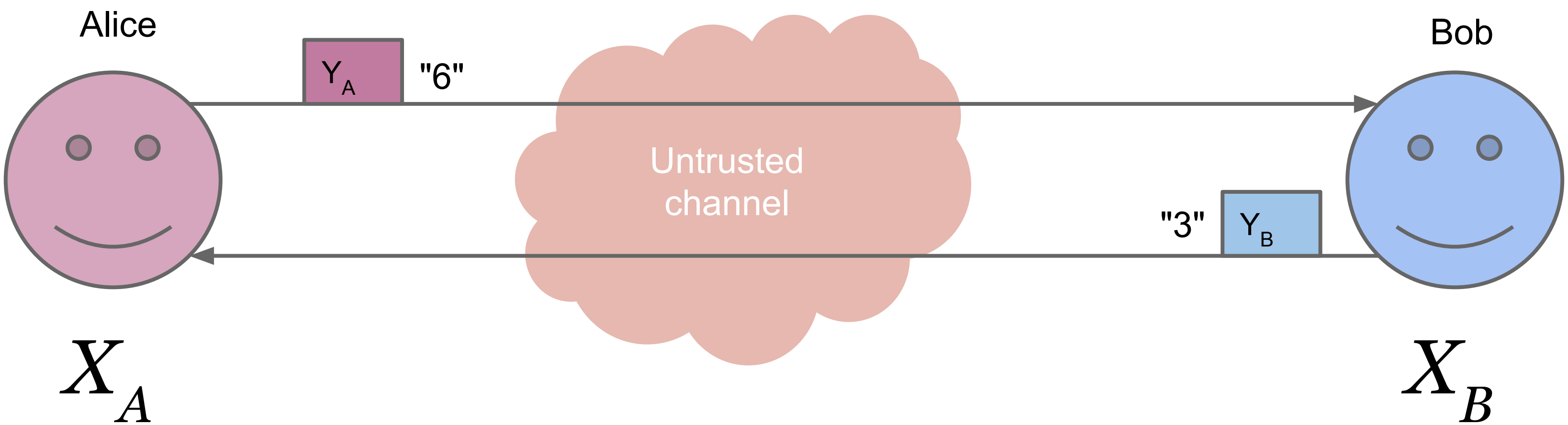
$X_A = 3$
$X_B = 1$

Public number (disclosed to everyone): They compute:

- Alice $Y_A = a^{X_A} \mod p$
- Bob $Y_B = a^{X_B} \mod p$

$Y_A = 3^3 \bmod 7 = 6$
$Y_B = 3^1 \bmod 7 = 3$

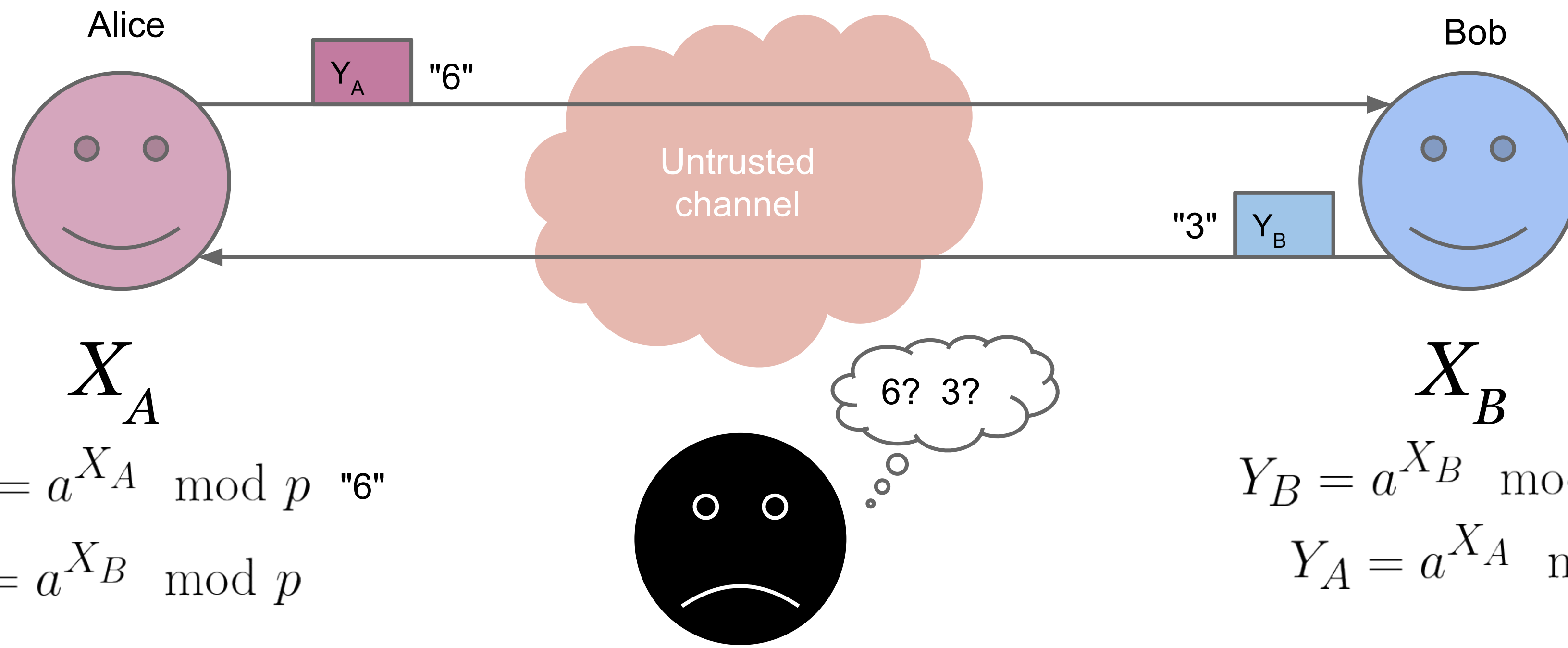
How does D-H work (3)



$Y_A = a^{X_A} \mod p$ "6"

$Y_B = a^{X_B} \mod p$ "3"

How does D-H work (3)



$Y_A = a^{X_A} \mod p$ "6"

$Y_B = a^{X_B} \mod p$

$Y_B = a^{X_B} \mod p$ "3"

$Y_A = a^{X_A} \mod p$

How does D-H work (4) - Secret

At this point, they can compute a ***secret*** K

- Since $Y_A^{X_B} \mod p = (a^{X_A} \mod p)^{X_B} \mod p = (a^{X_A})^{X_B} \mod p = (a^{X_B})^{X_A} \mod p = (a^{X_B} \mod p)^{X_A} \mod p = Y_B^{X_A} \mod p$

- Alice $K_A = Y_B^{X_A} \mod p = 3^3 \mod 7 = 6$

$$\boxed{K_B = K_A = K}$$

- Bob $K_B = Y_A^{X_B} \mod p = 6^1 \mod 7 = 6$

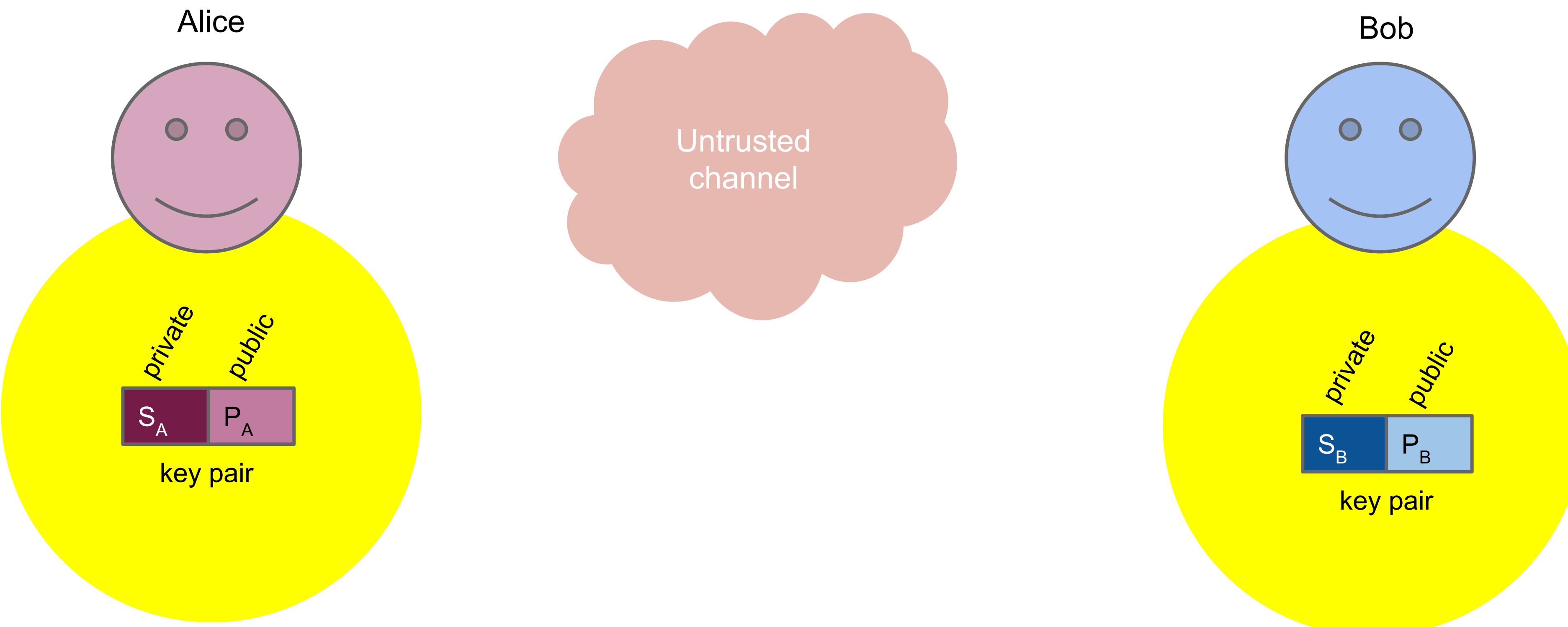
Anybody else can listen, but cannot compute K

- Because they miss the secret.

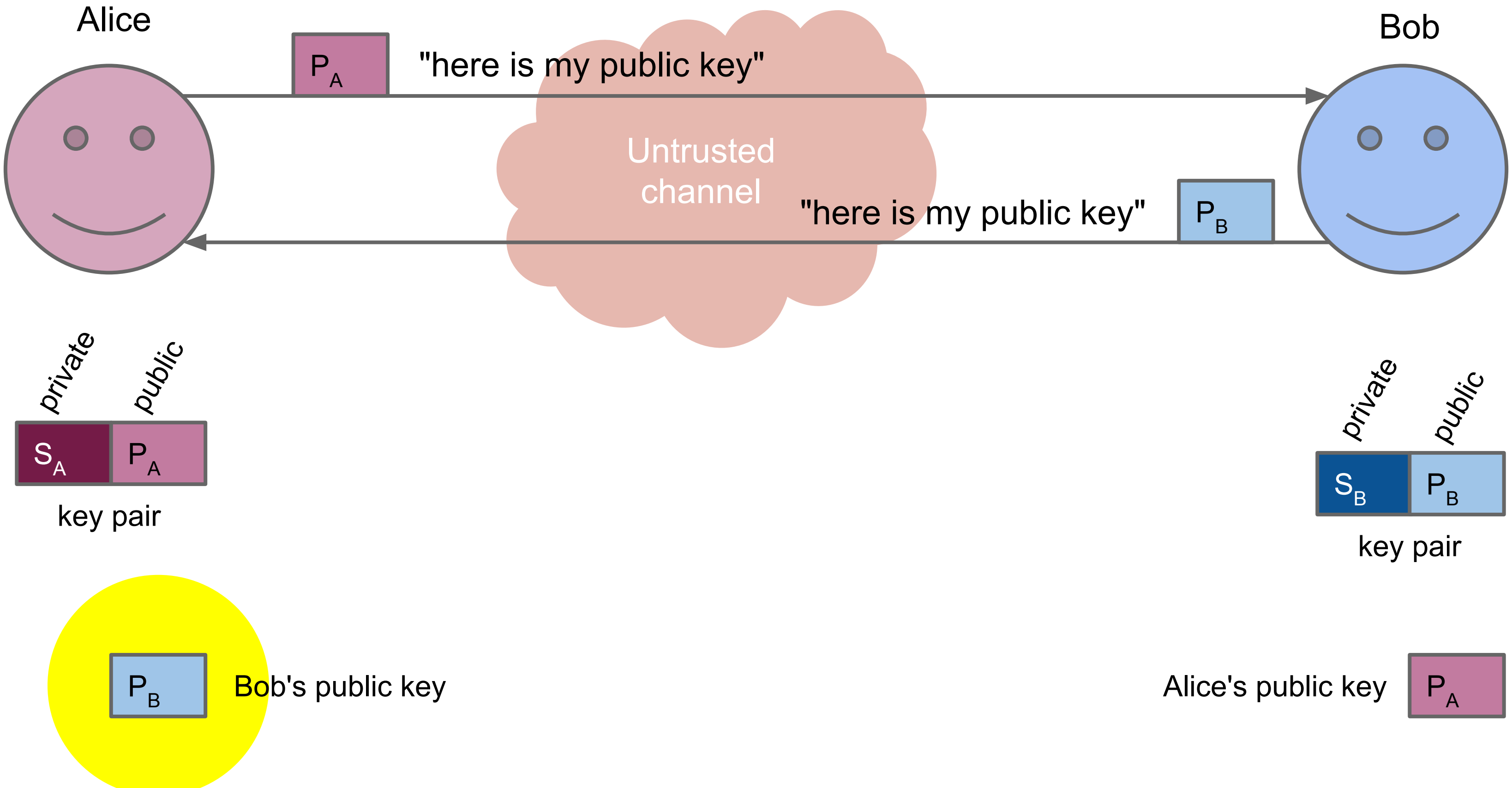
Public key encryption

- **Concept:** a cipher that uses two keys
  - What is encrypted with **key1** can be decrypted only with **key2** (and not with key1 itself), and viceversa.
- Also called *"public key cryptography"*
  - **Idea:** one of the two keys is kept **private** by the subject, and the other can be **publicly** disclosed.
- It solves the problem of **key exchange**
- We will not describe their maths in depth
  - They use a **one-way function with a trapdoor**
    - The private key "cannot" be retrieved from the public key
    - It should be easy to compute the public from the private
  - They are usually computational-intensive

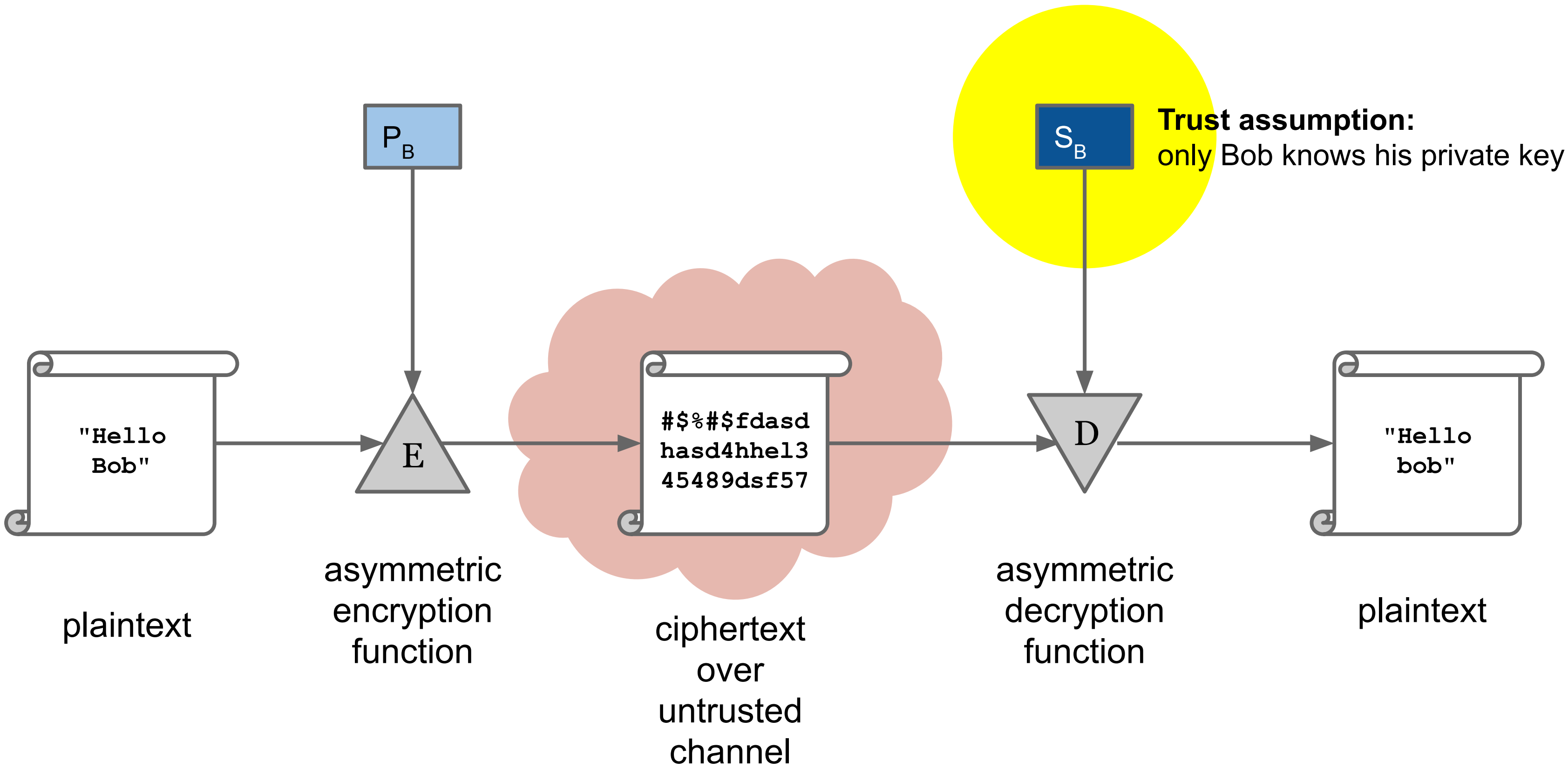
Public key encryption: Key Exchange



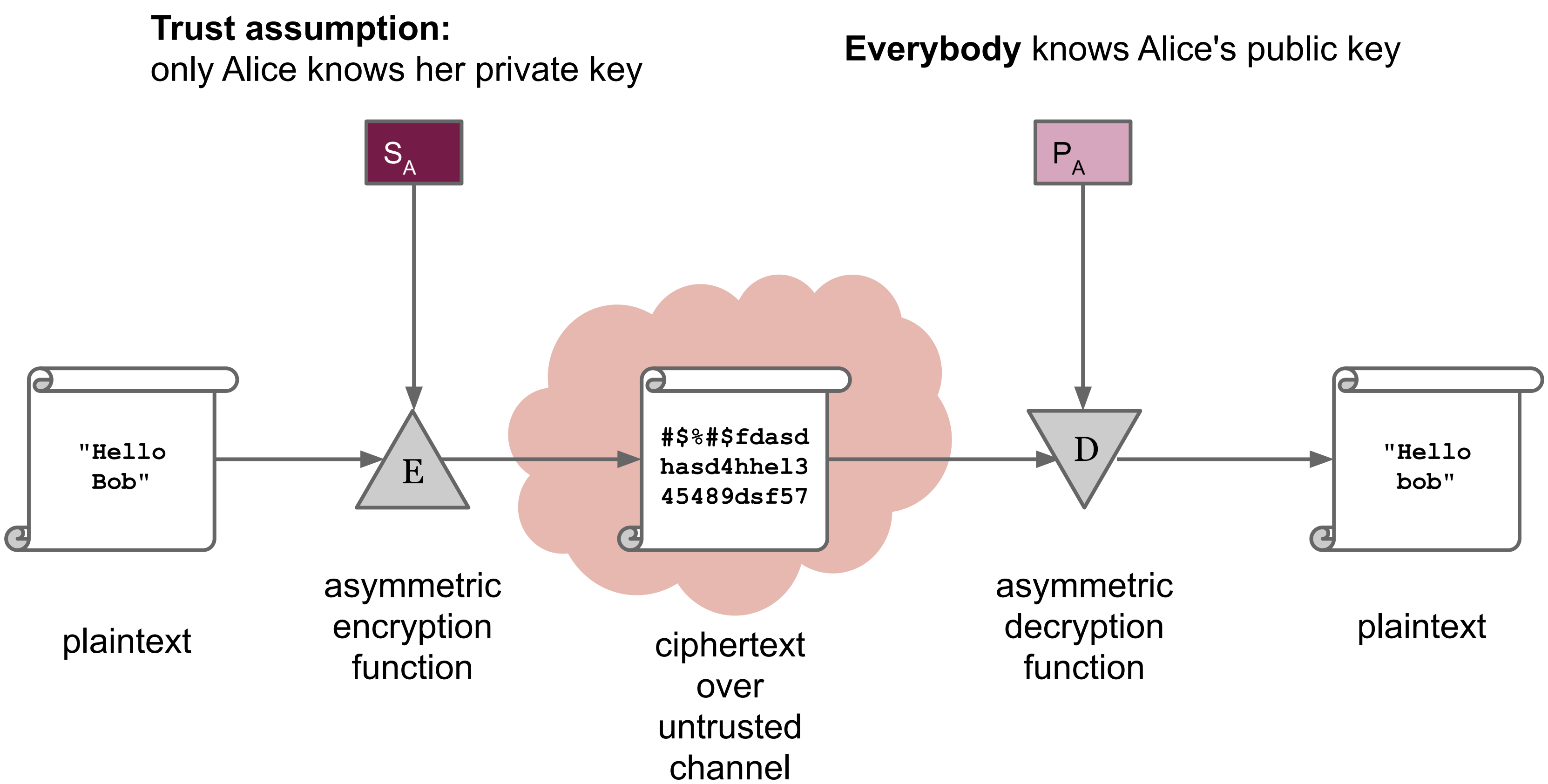
Public key encryption: Key Exchange



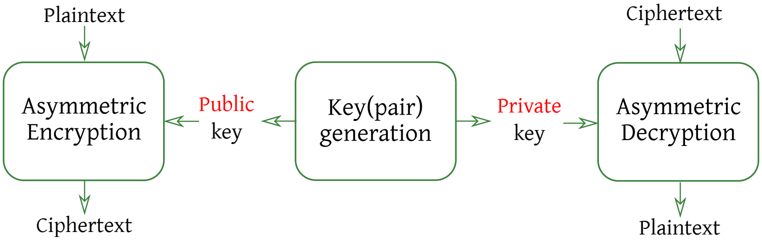
Public key encryption: Confidentiality



Exercise: what is this instead?



Public Key Encryption



Components

- *Different* keys are employed in encryption/decryption
- It is computationally hard to:
  - Decrypt a ciphertext without the *private* key
  - Compute the *private* key given only the *public* key

Widespread Asymmetric encryption ciphers

Rivest, Shamir, Adleman (RSA), 1977

- 2048 to 4096 bit message-and key-sizes
- Patented after the invention, patent now expired
- No ciphertext expansion
- Incidentally, the encryption with a fixed key is a PRP

Elgamal encryption scheme, 1985

- Either kbit range keys, or 100's of bits keys, depending on the variant
- Not encumbered by patents
- The ciphertext is twice the size of the plaintext
- Widely used as an RSA alternative where patents were a concern

The RSA Algorithm (hints) - 1

Hard problem: **factorization**

If p and q are two ***large primes***:

- computing $n = p * q$ is ***easy***
- but given n it is painfully ***slow*** to get p and q
- quadratic sieve field, basically "try all primes until you get to the smaller between p and q"
- Different problem than mod-log (D-H), but it can be shown that they are related

The RSA Algorithm (hints) - 2

- Factoring n is exponential in the number of bits of n
- Computation time for encryption grows linearly in the number of bits of n
    - square-and-multiply algorithm in hardware
- At the moment of writing:
    - 512-bit RSA factored within 4 hours on EC2 for < $100: http://seclab.upenn.edu/projects/faas/faas.pdf
    - No demonstration of practical factoring of anything larger than 700 bits
    - key sizes > 1024 are safe
    - 2048 or 4096 typical choices

Computational hardness

- Up to now, enumeration of the secret parameter was the best possible attack
- This is ok for modern block ciphers $\rightarrow$ best attack: $\mathcal{O}(2^\lambda)$
- Asymmetric cryptosystems rely on hard problems for which bruteforcing the secret parameter is not the best attack
  - Factoring a λ bit number takes $\mathcal{O}\left(e^{k(\lambda)^{\frac{1}{3}}(\log(\lambda))^{\frac{2}{3}}}\right)$
- Comparing bit-sizes of the security parameters instead of actual complexities is really wrong
- Concrete bit-sizes for λ depending on the cipher: `www.keylength.org`

Key Lengths: caveat

- Key length measured in bits both in **symmetric** and **asymmetric** algorithms
- However, they measure different things
  - Symmetric: number of ***decryption attempts***
  - Asymmetric: number of ***key-breaking attempts***
- Therefore:
  - You can compare symmetric algorithms based on the key (e.g., CAST-128 bit “weaker” than AES-256)
  - You **cannot** <u>**directly**</u> **compare** asymmetric algorithms based on key length.
  - More importantly, **never compare directly** asymmetric vs. symmetric key lengths!
  - https://www.keylength.com/en/4/

Key Encapsulation

Assumption

- A public channel between Alice and Bob is available
- For the moment, the attacker model is "eavesdrop only"

Sharing a secret without agreement

- Alice: generates a keypair (k_{pri}, k_{pub}), sends to Bob
- Bob: gets $s \xleftarrow{\$} \{0,1\}^{\lambda}$, encrypts it with k_{pub}, sends ctx to Alice
- Alice: decrypts ctx with k_{pri}, recovers s
- Note: Bob alone decides the value of the shared secret s
  - Repeat the procedure with swapped roles and combine the two secrets to achieve similar guarantees to a key agreement

Efficiency considerations

Can't I skip a step?

- Employing an asymmetric cryptosystem Bob encrypts a text for Alice without the need of sharing a secret beforehand
- In principle, Bob and Alice could employ *only* an asymmetric cryptosystem to communicate

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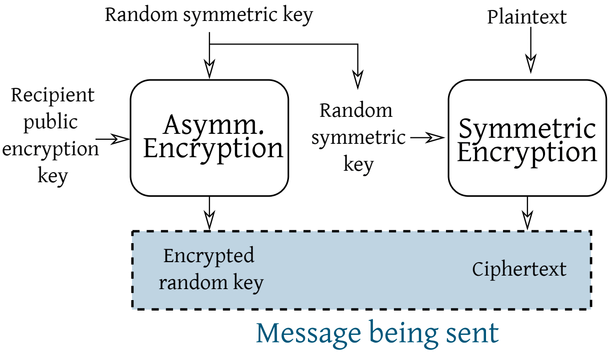
Can't I skip a step?

- Employing an asymmetric cryptosystem Bob encrypts a text for Alice without the need of sharing a secret beforehand
- In principle, Bob and Alice could employ *only* an asymmetric cryptosystem to communicate
- In practice this approach would be extremely inefficient
  - Asymmetric cryptosystems are from $10\times$ to $1000\times$ slower than their symmetric counterparts

Best of both worlds



Hybrid encryption schemes

- Asymmetric schemes to provide key transport/agreement
- Symmetric schemes to encrypt the bulk of the data
- All modern secure transport protocols built around this idea

Authenticating data

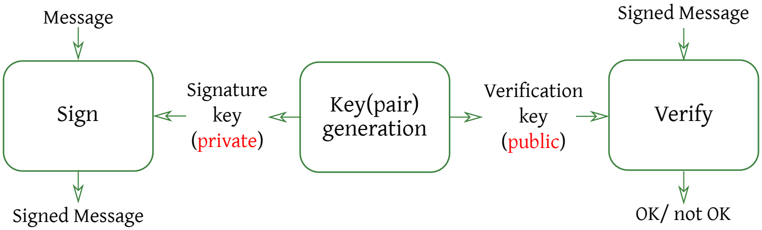
Motivations

- To build a secure hybrid encryption scheme we need to be sure that the public key the sender uses is the one of the recipient
- We'd like to be able to verify the authenticity of a piece of data without a pre-shared secret

Digital signatures

- Provide strong evidence that data is bound to a specific user
- No shared secret is needed to check (validate) the signature
- Proper signatures cannot be repudiated by the user
- They are asymmetric cryptographic algorithms
  - formally proven that you cannot get non repudiation otherwise

Digital signatures



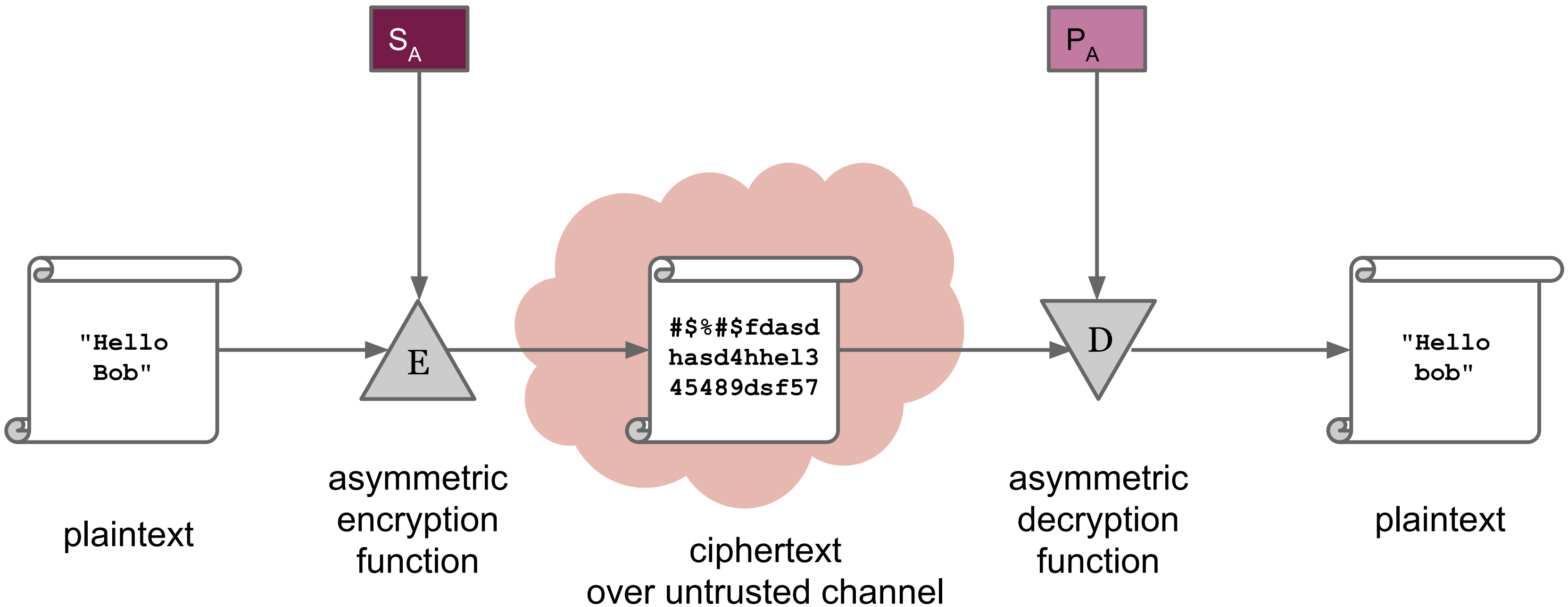
Computationally hard problems

- Sign a message without the signature key
  - this includes splicing signatures from other messages
- Compute the signature key given only the verification key
- Derive the signature key from signed messages

Message Authentication

Trust assumption:
only Alice knows her private key

Everybody knows Alice's public key



Digital signature: Authentication and Integrity

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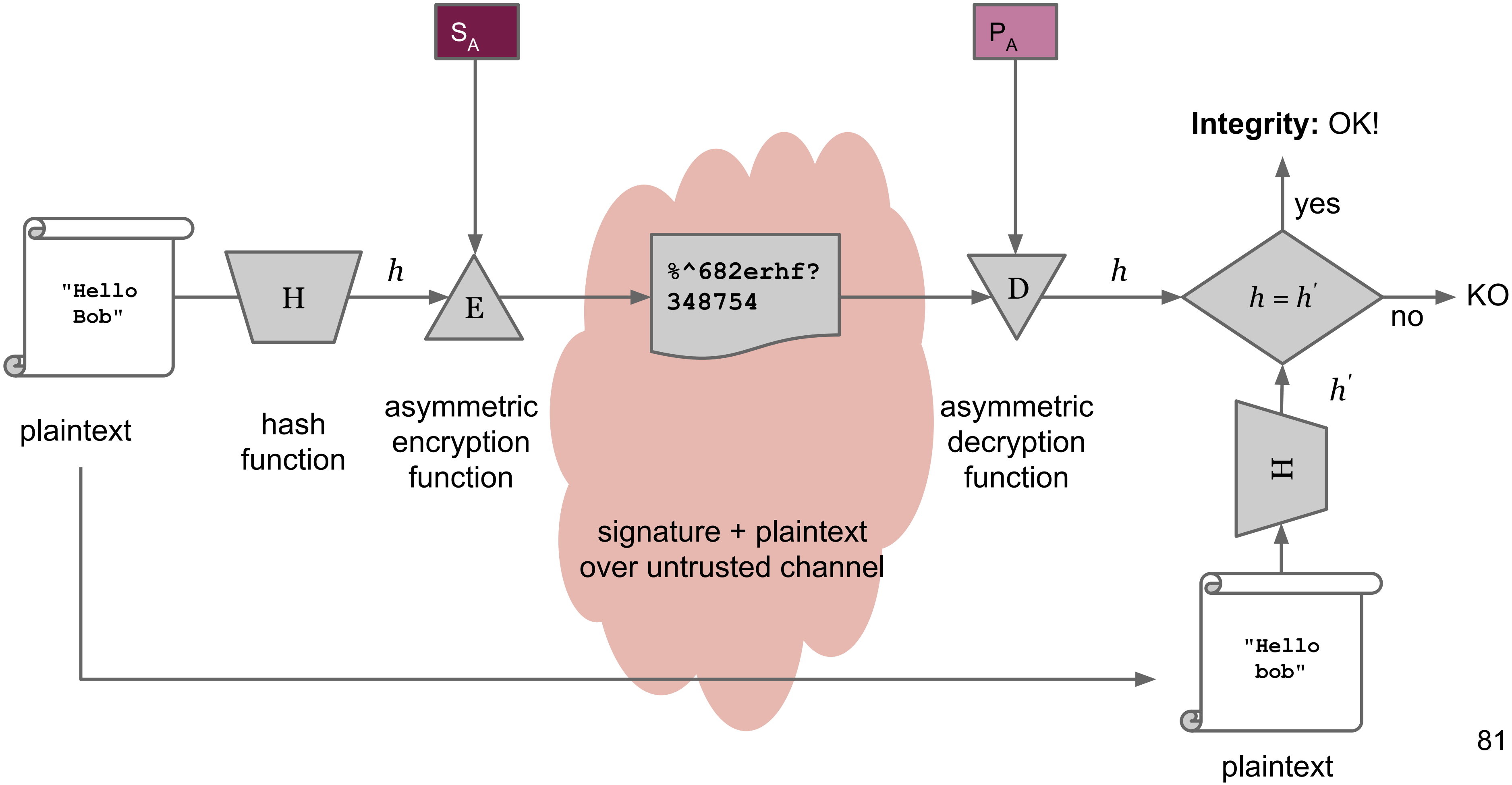

Widespread Signature schemes

Rivest, Shamir, Adleman (RSA), 1977

- Unique case: the same hard-to-invert function to build an asymmetric encryption scheme and a signature (different message processing!)
- Signing definitely slower than verification ($\approx 300\times$)
- Standardized in NIST DSS (FIPS-184-4)

Digital Signature Standard (DSA)

- Derived from tweaking signature schemes by Schnorr and Elgamal
- Also standardized in NIST DSS (FIPS-184-4)
- Signature and verification take roughly the same time

Digital signature uses

Authenticating digital documents

- For performance reasons, sign the hash of the document instead of the document
  - Signature properties now guaranteed only if both signature and hash algorithms are not broken

Authenticating users

- Alternative to password-based login to a system
  - The server has the user's public verification key (e.g. deposited at account creation)
  - The server asks the client to sign a long randomly generated bitstring (challenge)
  - If the client returns a correctly signed challenge, it has proven its identity to the server

The public key binding problem

Cautionary note

- Both in asymmetric encryption and digital signatures, the public key must be bound to the correct user identity
- If public keys are not authentic:
  - A MITM attack is possible on asymmetric encryption
  - Anyone can produce a signature on behalf of anyone else
- The public key authenticity is guaranteed with... another signature
  - We need someone to sign the public-key/identity pair
  - We need a format to distribute signed pairs

A case of identity

- A digital signature ensures that plaintext was authored by someone.

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- Ditto for using public key for encryption!

A case of identity

- A digital signature ensures that plaintext was authored by someone.
- **Not really!** It ensures it was encrypted with a certain key...it says nothing about “who” is using that private key
- Ditto for using public key for encryption!
- Exchange of public keys **must be secured** (either out of band, or otherwise)
- PKI (Public Key Infrastructure) associates keys with identity on a wide scale

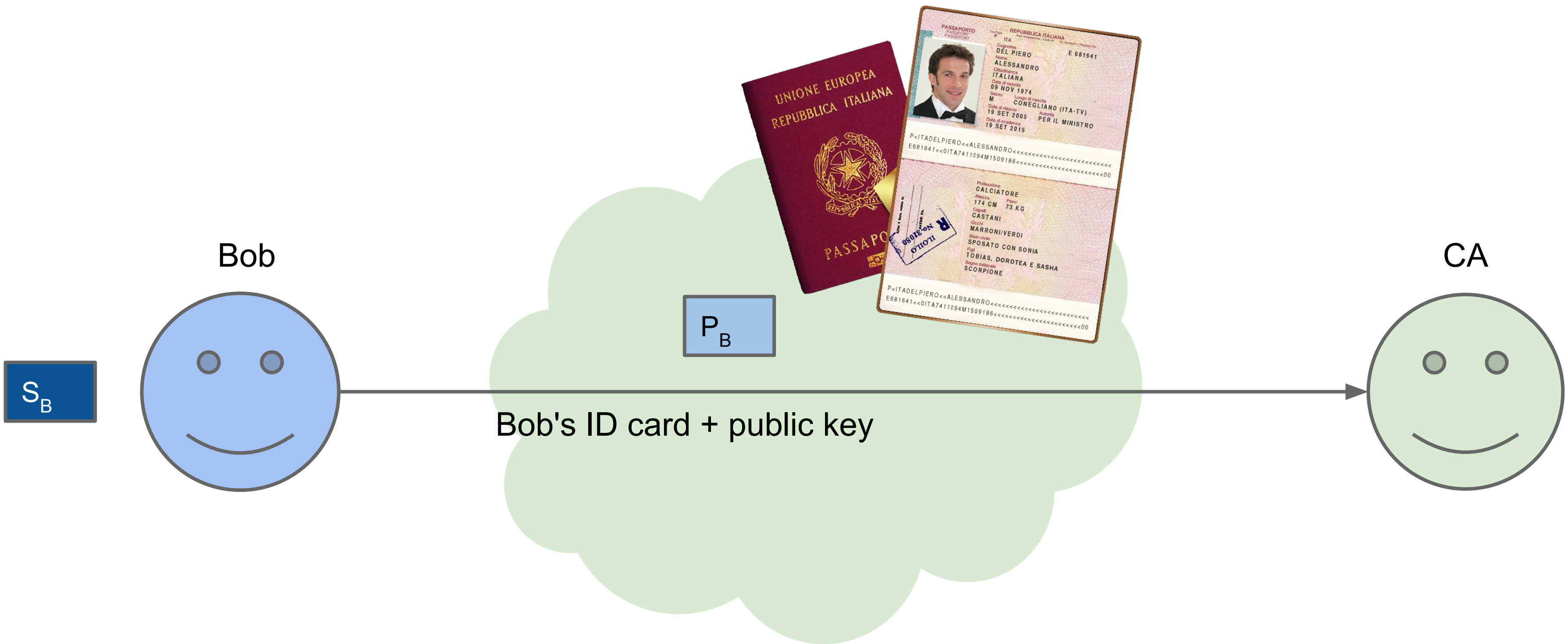
PKI

- A PKI uses a trusted third party called a **certification authority** (CA)
- The CA **digitally signs** files called **digital certificates**, which bind an identity to a public key
  - Identity = “Distinguished Name (DN)”
  - As defined in the X.509 standard (most used one)
- Now we can recognize a number of subjects...

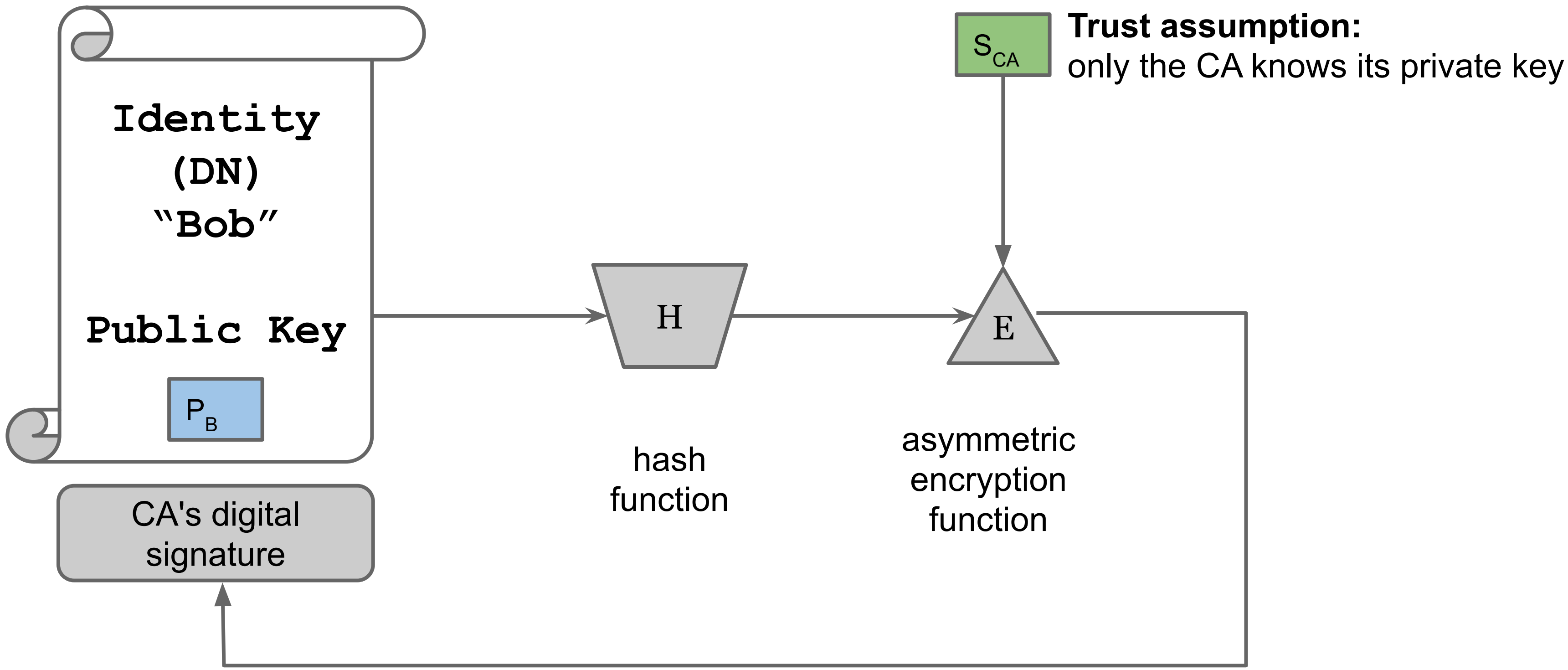
Digital certificates

- They bind a public key to a given identity, which is:
  - for humans: an ASCII string
  - for machines: either the CNAME or IP address
- They specify the intended use for the public key contained
  - Avoids ambiguities when a key format is ok for both an encryption and a signature algorithm
- They contain a time interval in which they are valid
- Most widely deployed format is described in ITU X.509

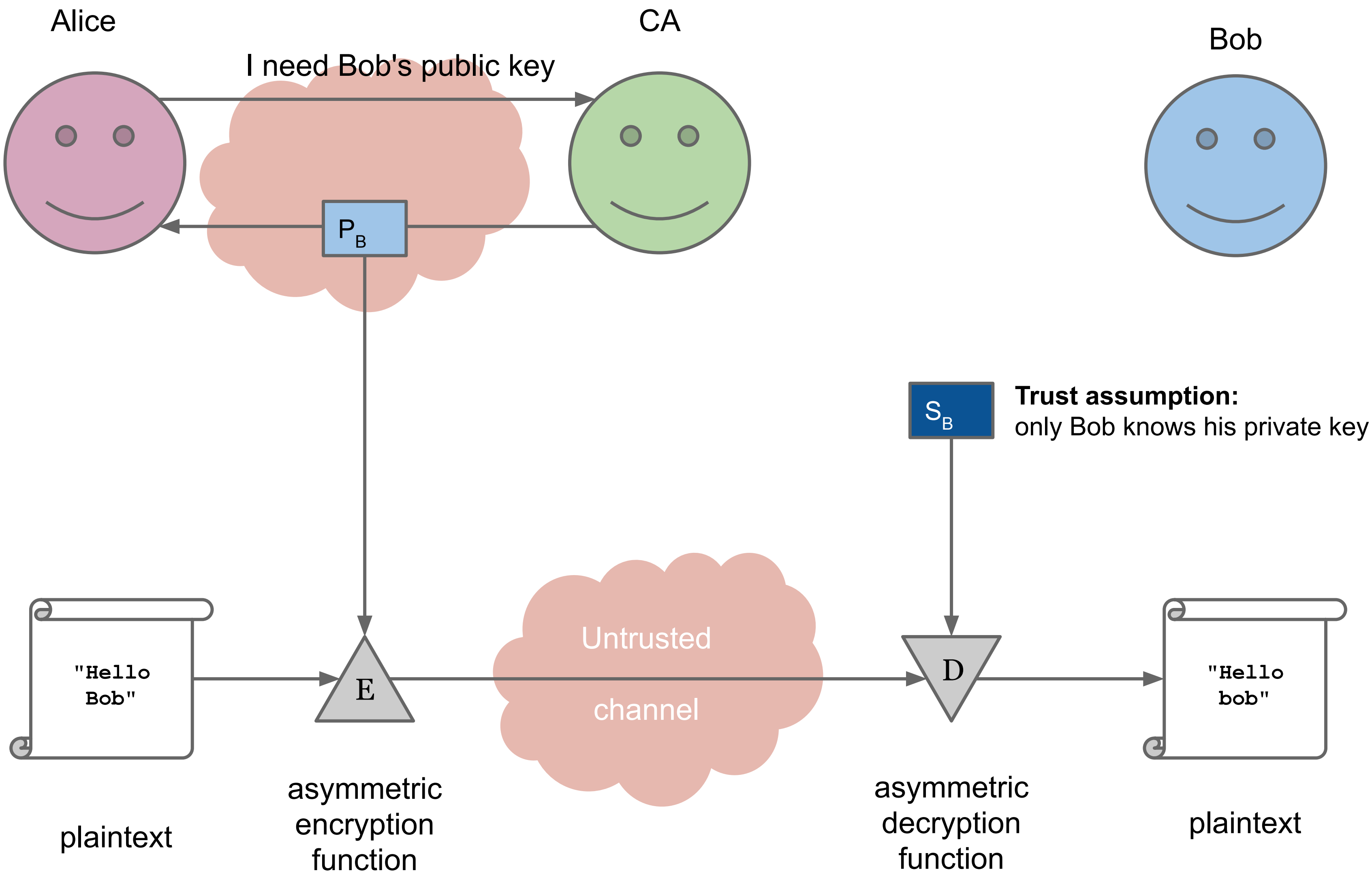
Bob's Digital Certificate (1)



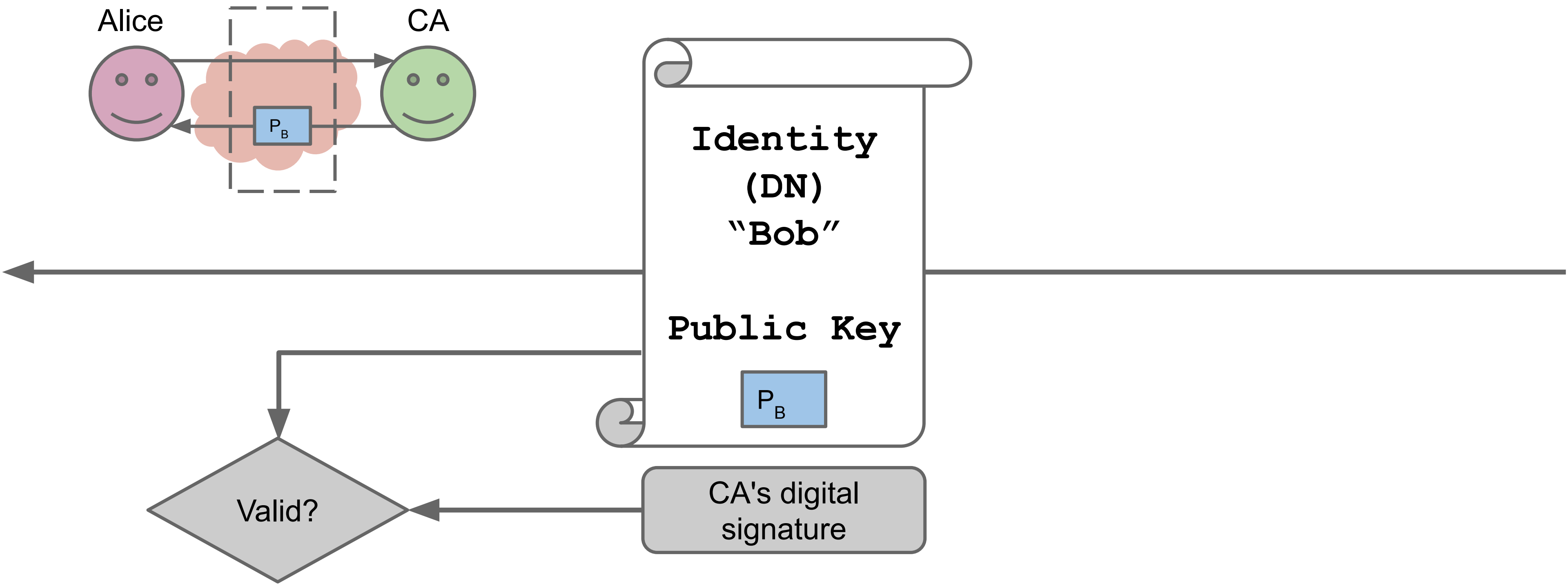
Bob's Digital Certificate (2)



Retrieving Bob's Certificate



Zoom in: Is the public key valid?



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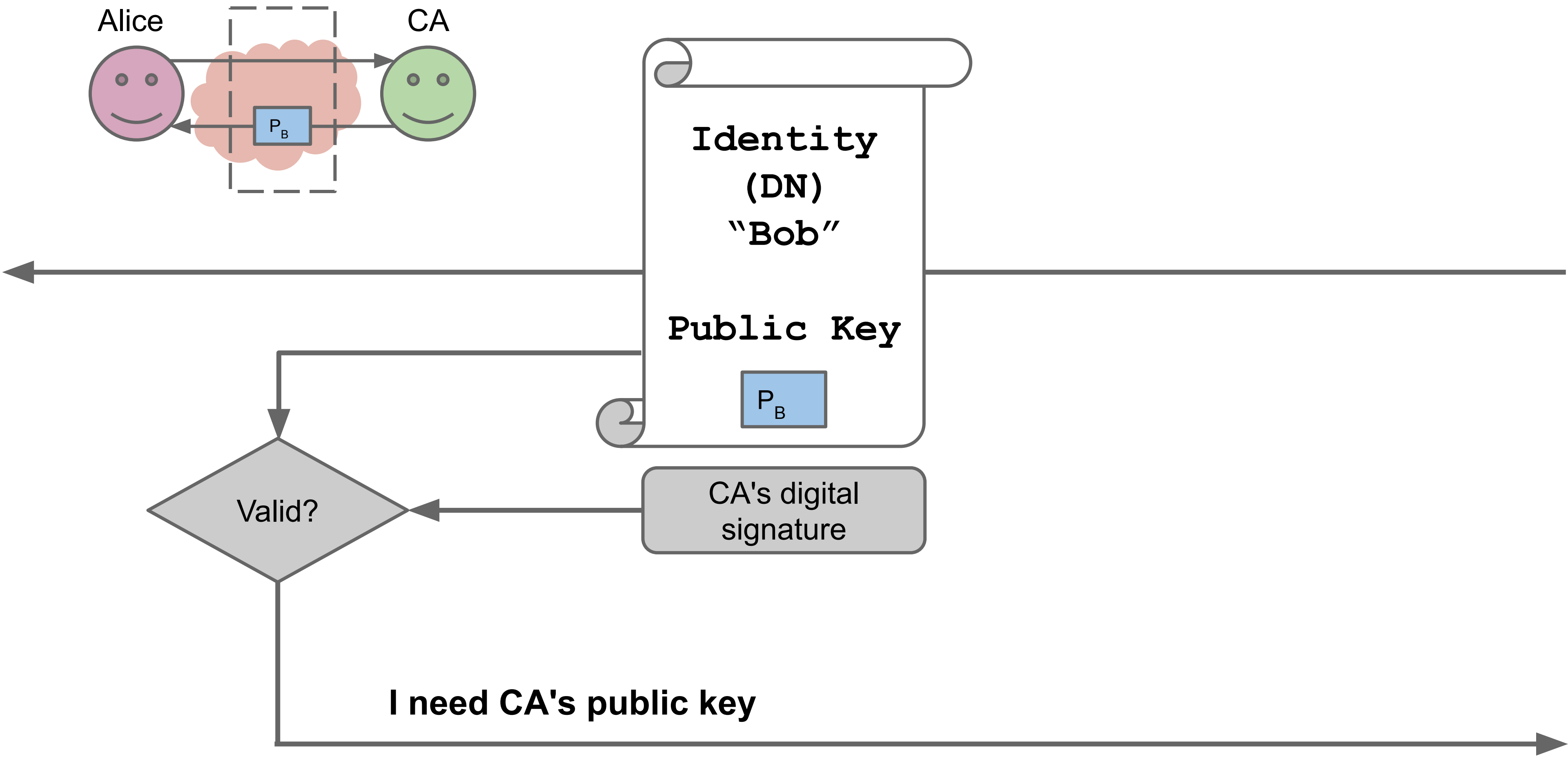
    - As defined in the X.509 standard (most used one)
- Now we can recognize a number of subjects...provided that we can obtain the **public key** of the CA

Zoom in: Is the public key valid?



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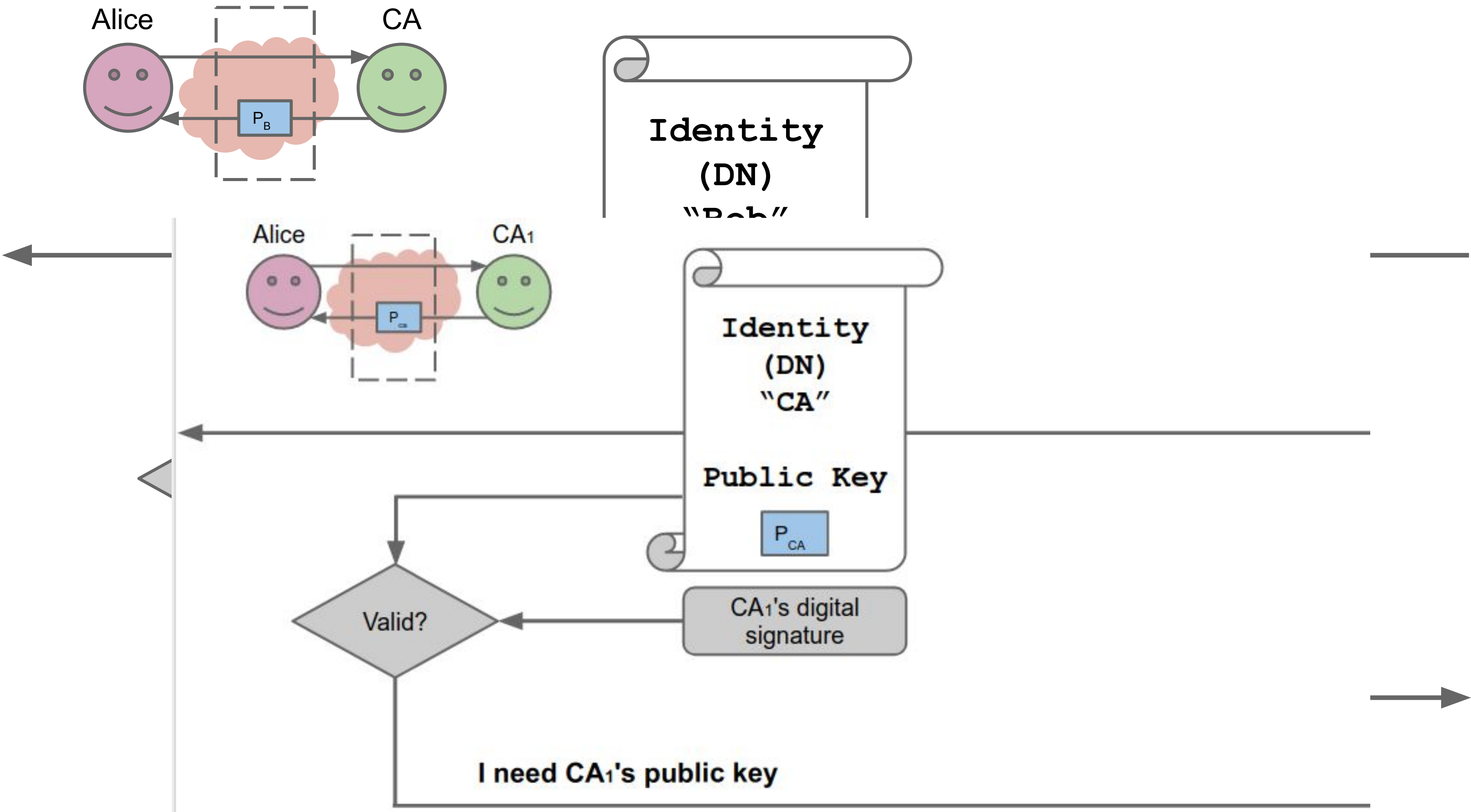

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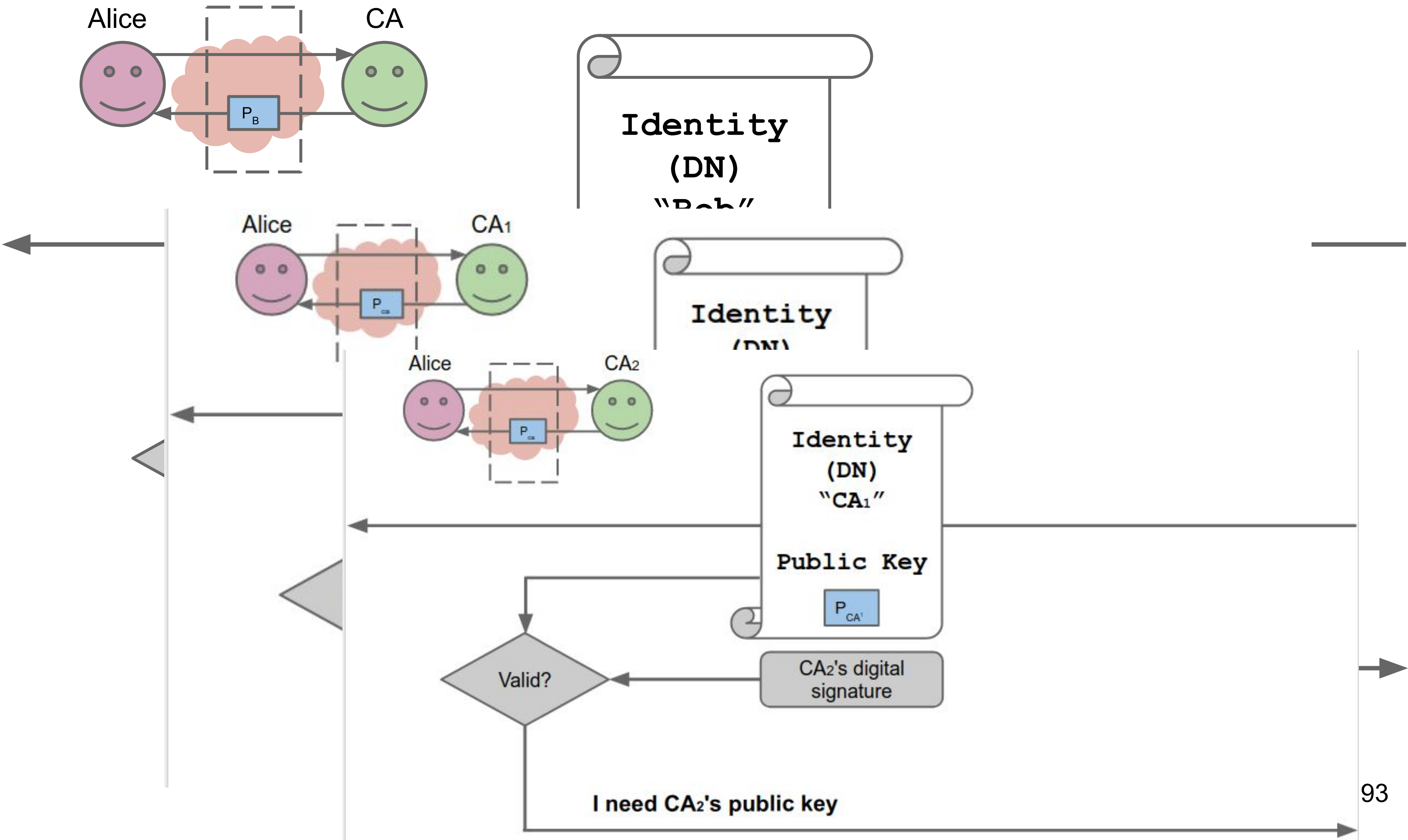

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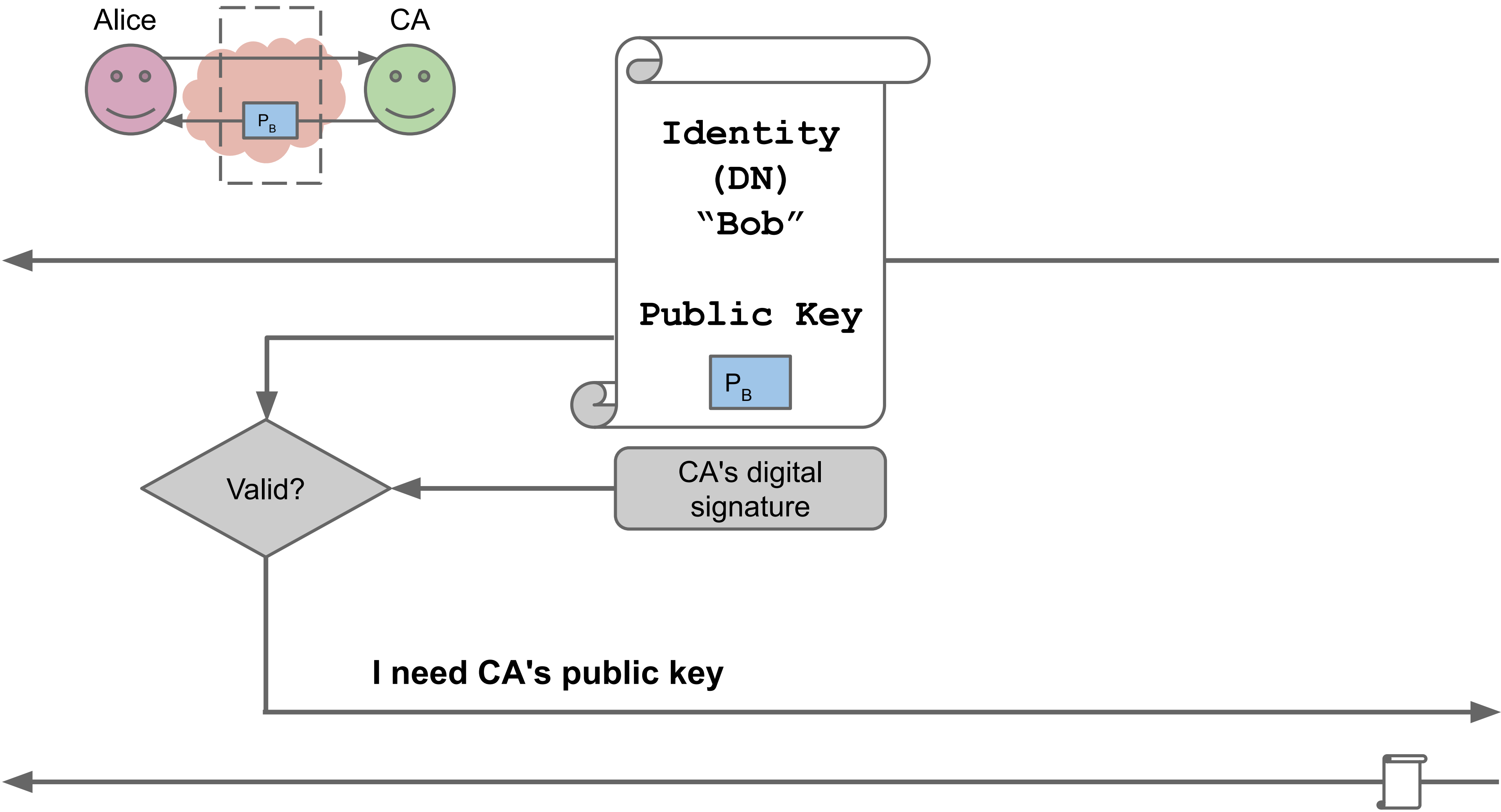


Where is this going to end?

The Certificate Chain

"Quis custodiet custodes?"

The CA needs a *private key* to sign a certificate

- The *public key*...must be in a certificate.

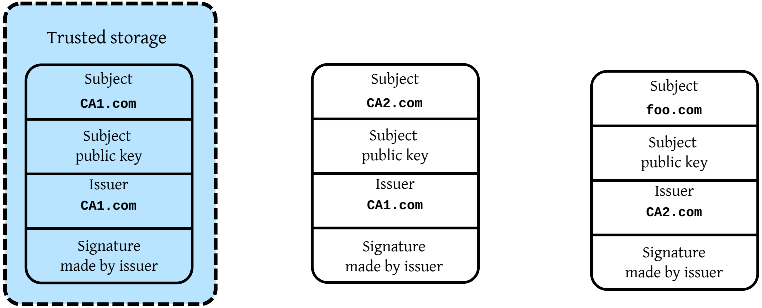
Someone else needs to sign **that** certificate

- And so on…at some point this needs to stop!

Certification authorities

Who signs the certificates

- The certificate signer is a trusted third party, the CA
- The CA public key is authenticated... with another certificate
- Up to a self-signed certificate which has to be trusted a priori



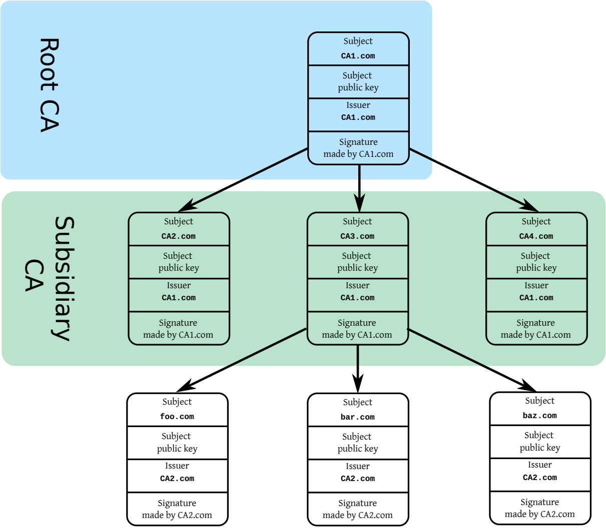
We Need a Trusted Element: Root of trust

Top-level CA (root CA, source CA)

Uses a self-signed certificate

- cannot be verified: it's a **trusted element**
- Basically a document that says “I am myself”

Certification authorities hierarchy

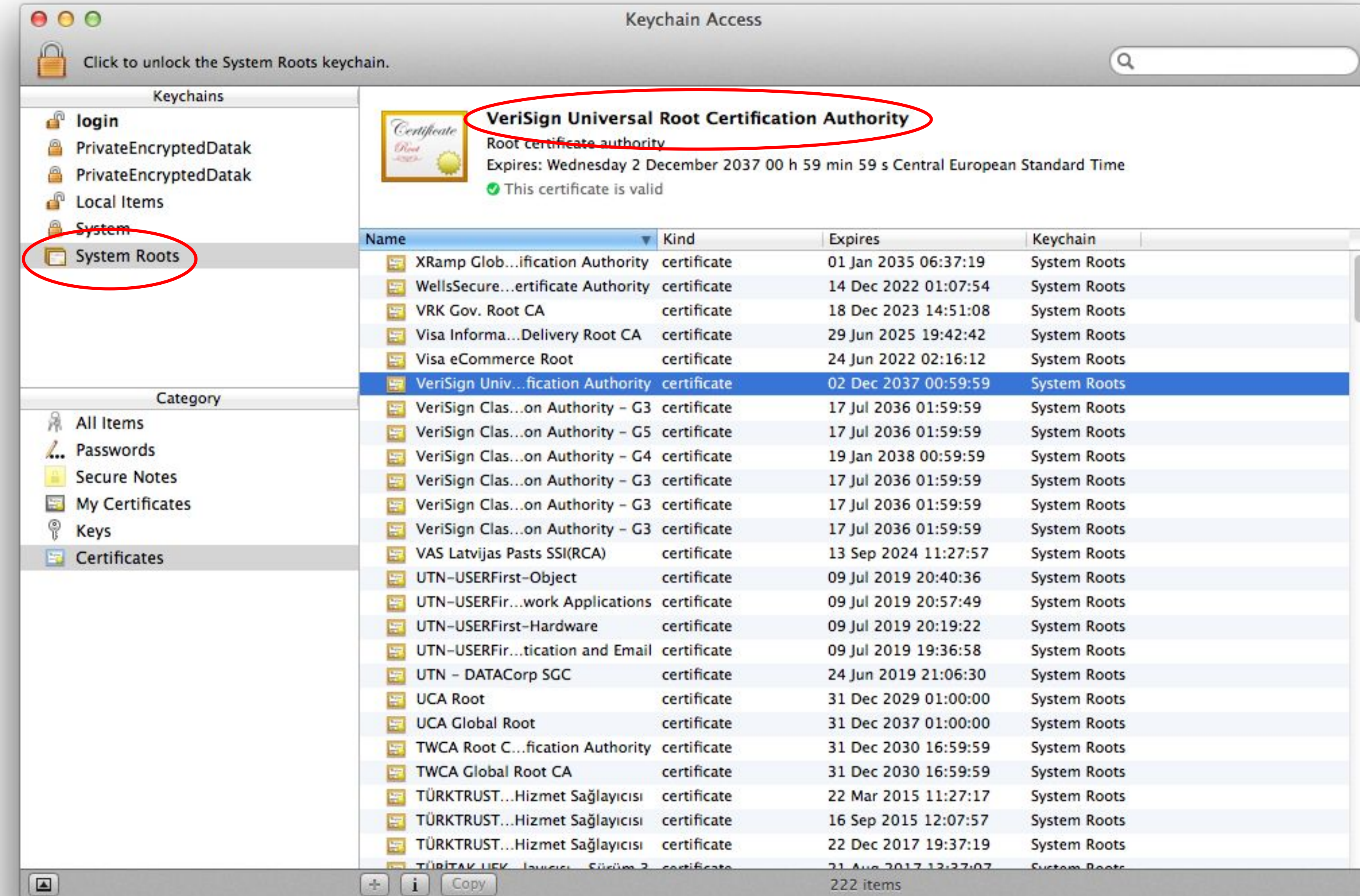


How to distribute the trusted element?

An **authority** releases it

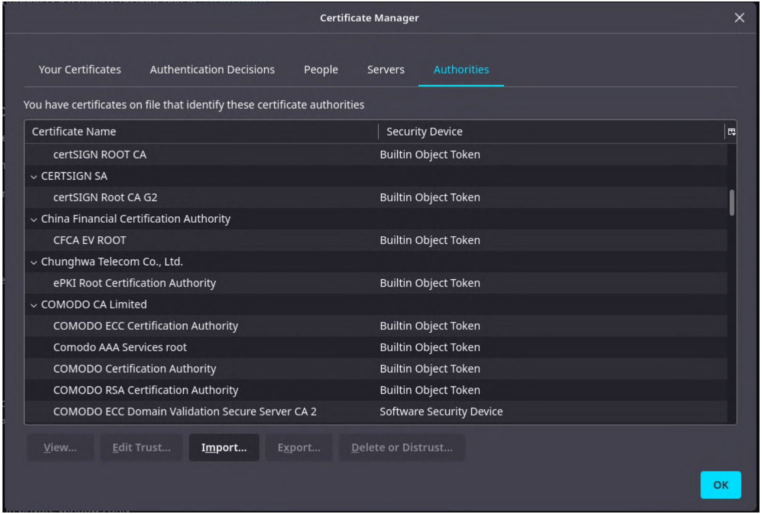
- the state
- a regulator
- the organization management

CA already (de facto standard)



Do you trust your operating system? Do you trust the list of root certificates that ship with it?

A recent browser certificate storage



How to distribute the trusted element?

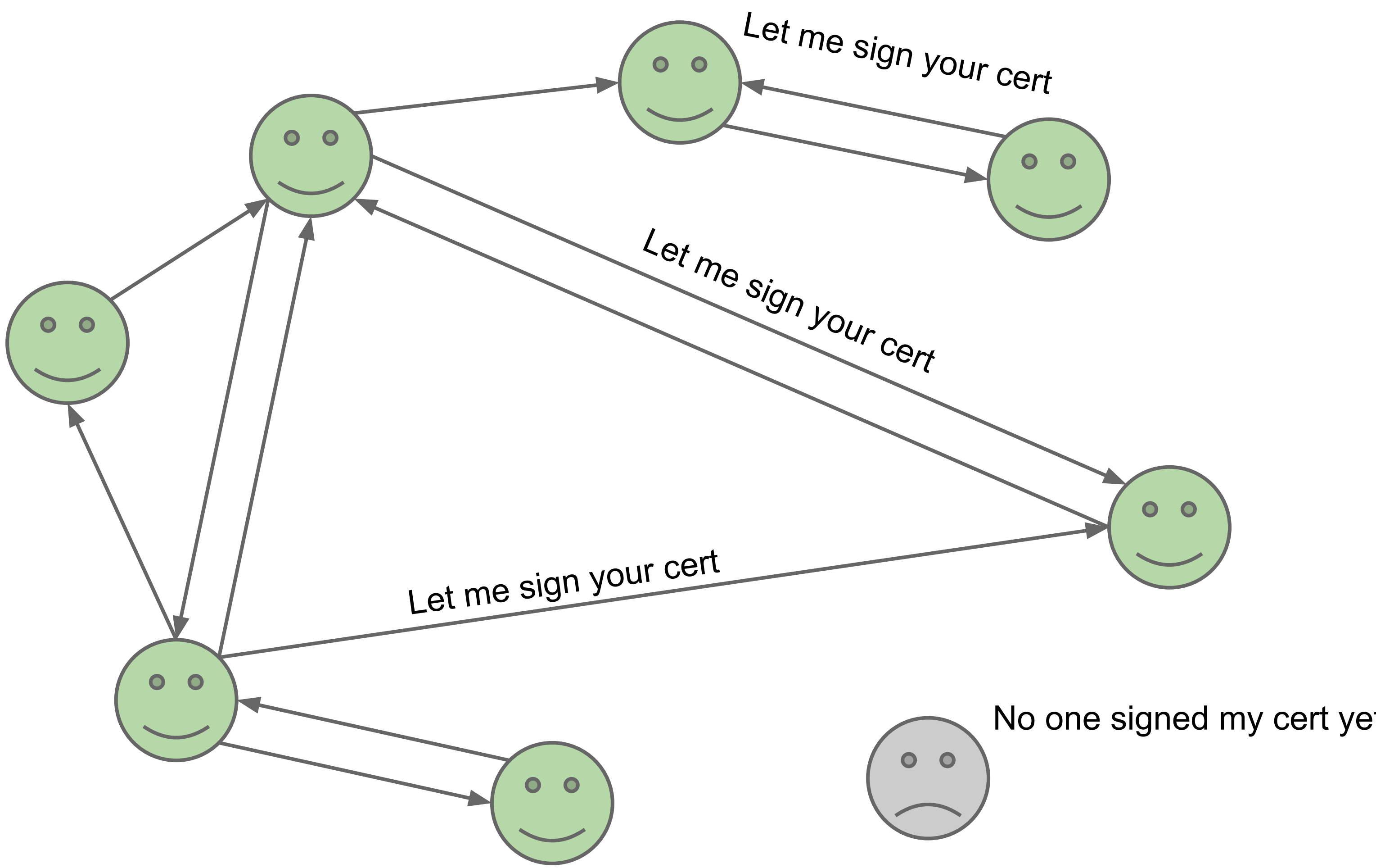
An **authority** releases it

- the state
- a regulator
- the organization management

CA already (de facto standard)

Decentralizing trust (e.g., PGP web-of-trust)

Decentralizing Trust: Web of Trust



Certificate Revocation Issues

- Signatures cannot be revoked (destroyed).
- **Certificates** need to be revoked at times.
- Certificate Revocation Lists (CRL)

Verification Sequence for Certificates

1. Does the **signature** validate the document?
    - Hash verification as we have seen
2. Is the **public key** the one on the certificate?
3. Is the certificate the one of the **subject**?
    - Problems with omonimous subjects, DN
4. Is the **certificate** validated by the CA?
    - Validate the entire certification chain, up to the root
5. Is the **root** certificate trusted?
    - Need to be already in possession of the root cert
6. Is the certificate in a **CRL**?
    - How do we get to the CRL if we are not **online**?

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   - Need to be already in possession of the root cert
6. Is the certificate in a **CRL**?
   - How do we get to the CRL if we are not **online**?

Any missing check = vulnerability!

Case study: Italian “legal” digital signatures framework

Introduced in Italy with D.P.R. 513/97

- many modifications, in particular when implementing EU regulations

Original Italian scheme: a list of “screened” CAs

Result: each CA created their own **digital signature application** (i.e., trusted element)

Attacking Digital Signature Applications

Digital signature stronger than handwritten signature

- **Written** documents can be modified, written signatures can be copied.
- **Digital** signature value **tied to content**, and cannot be forged unless the algorithm is broken
- However, a digital signature is **brittle**: if a fake is forged, it cannot be told from real one

Crypto: *OK* – Software Design: *KO*

Italian signature standards use **strong**, **unbroken cryptographic algorithms**!

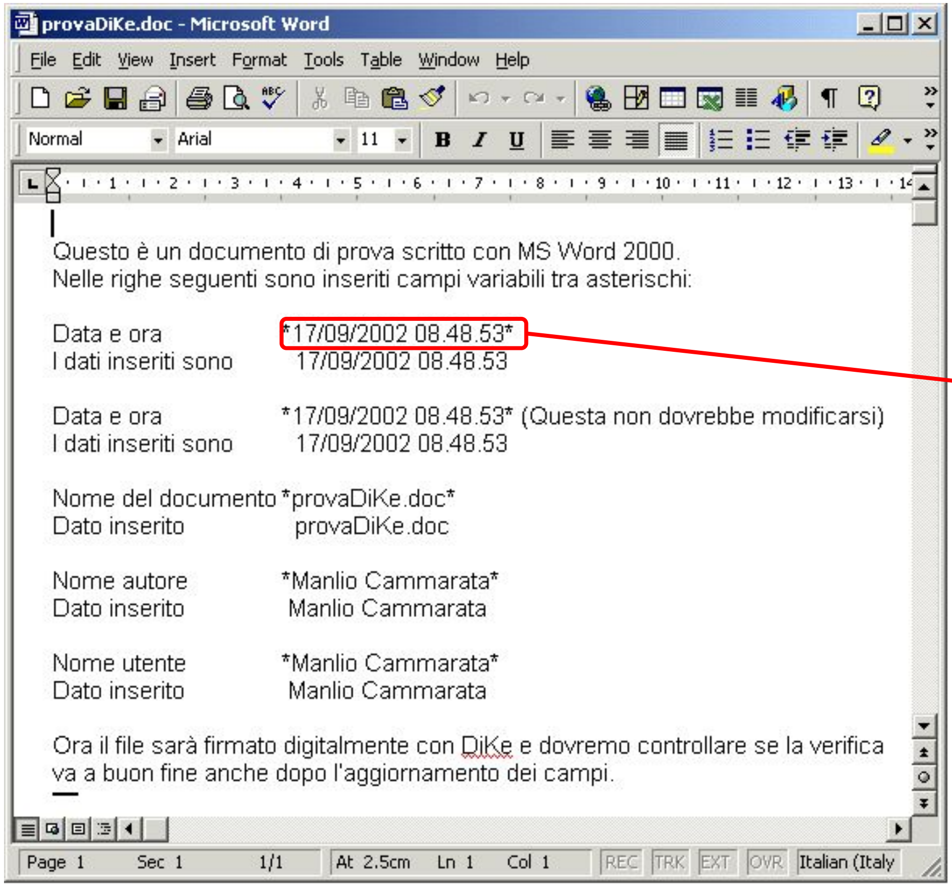
However, vulnerabilities did emerge

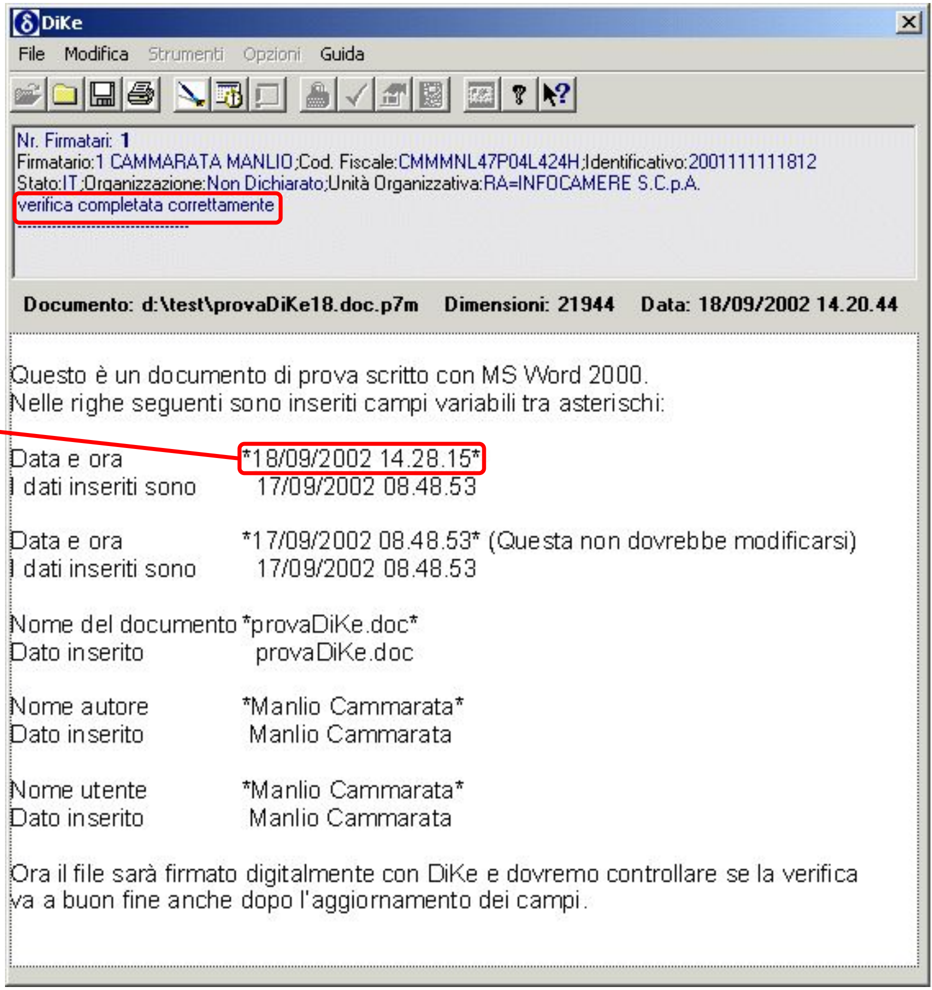
Do you remember the “bank vault door in a tent”?

Bug 1: Fields of pain

- Bug notified on 9/9/2002
- The software of several CAs (originally DiKe by Infocamere was the subject of scrutiny) allowed users to sign Word documents with **dynamic fields** or **macros** without notice
- A macro does not change the bit sequence of the document, so the **signature does not change** with the visualized content
- Examples and stuff on Prof. Zanero’s home: http://home.deib.polimi.it/zanero/bug-firma.html (Italian only, sorry for that!)

Example


provaDiKe.doc - Microsoft Word
File Edit View Insert Format Tools Table Window Help
Normal
Arial
11
Questo è un documento di prova scritto con MS Word 2000.
Nelle righe seguenti sono inseriti campi variabili tra asterischi:
Data e ora *17/09/2002 08.48.53*
I dati inseriti sono 17/09/2002 08.48.53
Data e ora *17/09/2002 08.48.53* (Questa non dovrebbe modificarsi)
I dati inseriti sono 17/09/2002 08.48.53
Nome del documento *provaDiKe.doc*
Dato inserito provaDiKe.doc
Nome autore *Manlio Cammarata*
Dato inserito Manlio Cammarata
Nome utente *Manlio Cammarata*
Dato inserito Manlio Cammarata
Ora il file sarà firmato digitalmente con DiKe e dovremo controllare se la verifica va a buon fine anche dopo l'aggiornamento dei campi.
Page 1 Sec 1 1/1 At 2.5cm Ln 1 Col 1 REC TRK EXT OVR Italian (Italy


DiKe
File Modifica Strumenti Opzioni Guida
Nr. Firmatari: 1
Firmatario:1 CAMMARATA MANLIO;Cod. Fiscale:CMMMNL47P04L424H;Identificativo:2001111111812
Stato:IT;Organizzazione:Non Dichiarato;Unità Organizzativa:RA=INFOCAMERE S.C.p.A.
verifica completata correttamente
Documento: d:\test\provaDiKe18.doc.p7m Dimensioni: 21944 Data: 18/09/2002 14.20.44
Questo è un documento di prova scritto con MS Word 2000.
Nelle righe seguenti sono inseriti campi variabili tra asterischi:
Data e ora *18/09/2002 14.28.15*
I dati inseriti sono 17/09/2002 08.48.53
Data e ora *17/09/2002 08.48.53* (Questa non dovrebbe modificarsi)
I dati inseriti sono 17/09/2002 08.48.53
Nome del documento *provaDiKe.doc*
Dato inserito provaDiKe.doc
Nome autore *Manlio Cammarata*
Dato inserito Manlio Cammarata
Nome utente *Manlio Cammarata*
Dato inserito Manlio Cammarata
Ora il file sarà firmato digitalmente con DiKe e dovremo controllare se la verifica va a buon fine anche dopo l'aggiornamento dei campi.

Reactions

- The CAs responded that this was *“intended behavior”* and that it did not violate the law
- However:
  - Microsoft, on 30/1/03, released an Office patch to allow **disabling macros** via API.
  - Nowadays, all software show a big alert when signing an Office document.
  - New legislation explicitly excludes modifiable and scriptable formats (but recommends PDF)
- The issue is actually much deeper
  - Decoders of complex formats should also be validated
  - Research field of “what you see is what you sign”

Horror story 1 overview

The importance of being static

- 2002-09-09: several pieces of software performing digital signatures with legal value in Italy allowed to sign MS Word documents *containing macros*
  - Macros allow to dynamically change the displayed text in a document according to, e.g., the current date
- Striking mismatch between what was thought to be signed (the visualized document) and the actual signed object (a program-document blend)
- Current standard for digital signatures on human-intended documents (PAdES,CAdES) target PDF and XML formats
  - n.b.: PDFs may embed Javascript, PDF/A do not

Bug 2: Firma&Cifra

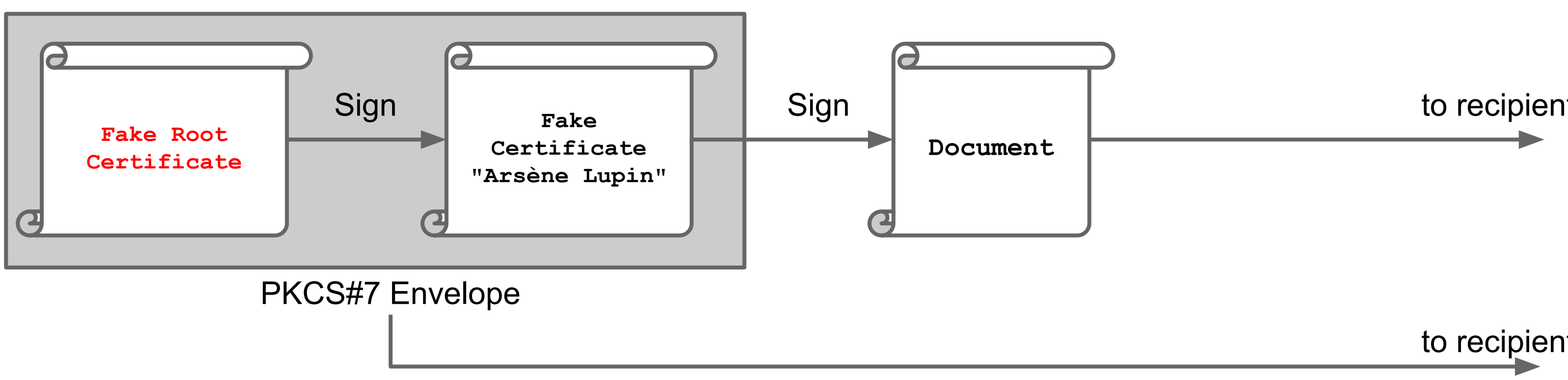
- Firma&Cifra was the digital signature application by PostECom
- Bug found by anonymous on 20/03/2003: http://www.interlex.it/docdigit/sikur159.htm
- **Result: creation** and **verification** of a **signature** with a **fake certificate**
- Also in this case: no cryptographic algorithm was broken to perform the show

Vulnerability Description

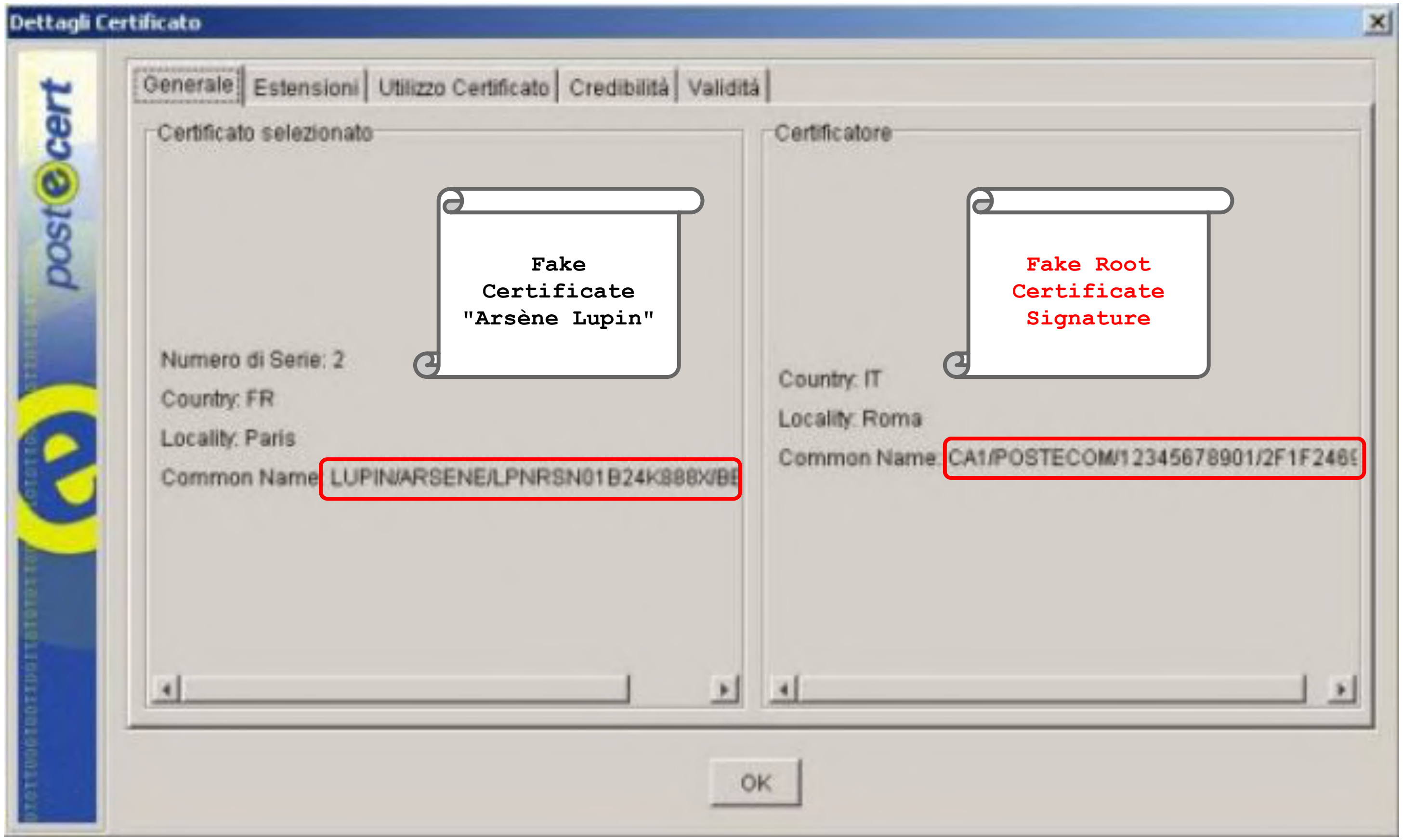
- In order to verify a signature, we need **author certificate** and the **certificate chain**
    - **Theoretically**, all available **online**
    - To allow **offline verification**, everything included with the document, in a PKCS#7 envelope
- Verification of root certificates must use preinstalled ones
    - Most **software** comes with them
    - The **root certificate storage** is a critical point!
- Firma&Cifra **trusts the root certificate in the PKCS#7 envelope**, and it even imports it in the secure storage area.

The Exploit: Arséne Lupin signature

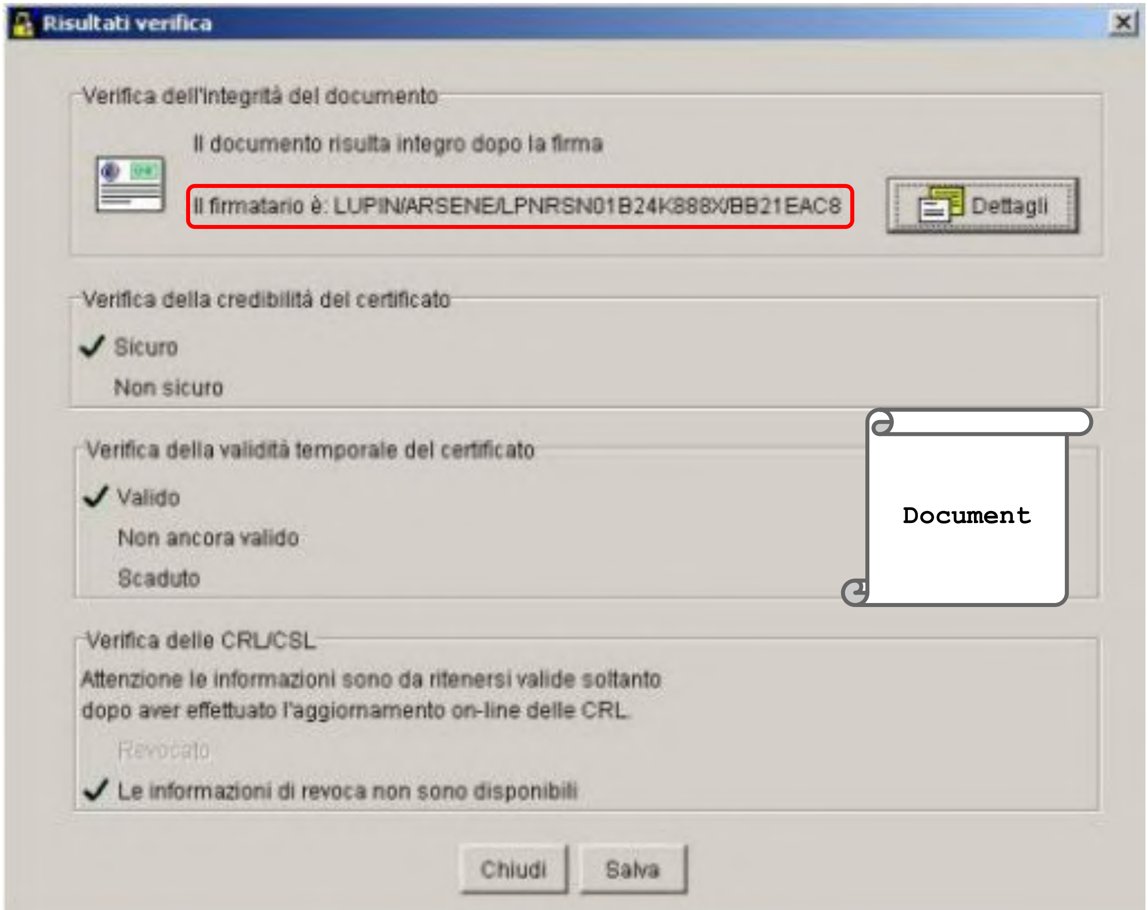
1. Generate a **fake *root* certificate** with the same name as a real one (e.g., PostECom itself)
2. Use this to generate a **fake *user* certificate** (in our example Arsène Lupin)
3. Use **Arsène Lupin's certificate** to sign theft and burglary confessions.
4. Include the fake root cert to the PKCS#7 envelope.



Les jeux sont faits: Lupin's Certficate



The Exploit: Arséne Lupin signature



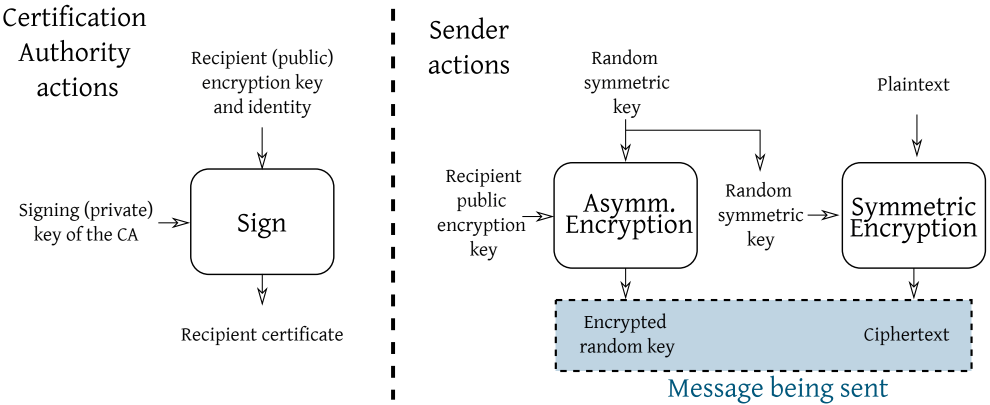
Best comment by Postecom: this is *"by design"*
(yep: wrong design, but still design!)

Horror story 2 overview

Trust with care

- 2003-02-20: Firma&Cifra was the digital signature application by PostECom
- When presented with any certificate bundled with a signed document, it considered the certificate authentic and added it to its trusted storage
- Signature forgery as easy as: 1) create your own CA certificate, 2) sign your target-user certificate, 3) sign the document
- Take away point: which certificates reside in your (applications') trusted storage determine who you trust

Putting it all together



This way of communicating is the mainstay of modern secure comm. protocols (TLS, OpenVPN, IPSec)

Directions in modern cryptography

Issues to solve, features to realize

- What if we have a quantum computer?
  - Some computationally hard problems are no longer hard
  - Move away from cryptosystems based on factoring/dlog
  - Alternatives available and being standardized (2022-04)
- What if we want to compute on encrypted data?
  - Yes, but it's moderately-to-horribly inefficient
- What if the attacker has physical access to the device computing the cipher (or some way of remotely measure it)
  - Take into account side channel information in the attacker model

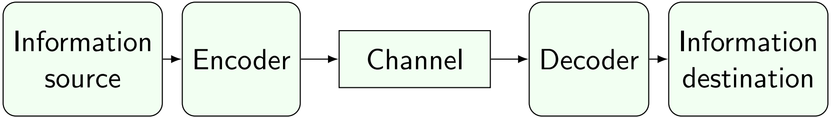
What is Shannon's information theory?

- Shannon's [3] way to mathematically frame communication
- A way to quantify information

What do we need this for (in this course)?

- Quantitatively frame "luck" and "guessing"

Basic Definitions



Basics

- A communication takes place between two endpoints
  - sender: made of an information source and an encoder
  - receiver: made of an information destination and a dencoder
- Information is carried by a channel in the form of a sequence of symbols of a finite alphabet

Losing uncertainty = Acquiring information

- The receiver gets information only through the channel
  - it will be uncertain on what the next symbol is, until the symbol arrives
  - thus we model the sender as a random variable
- Acquiring information is modeled as getting to know an outcome of a random variable $\mathcal{X}$
  - the amount of information depends on the distribution of $\mathcal{X}$
  - intuitively: the closer is $\mathcal{X}$ to a uniform distribution, the higher the amount of information I get from knowing an outcome
- Encoding maps each outcome as a finite sequence of symbols
  - More symbols should be needed when more information is sent

Measuring uncertainty: Entropy

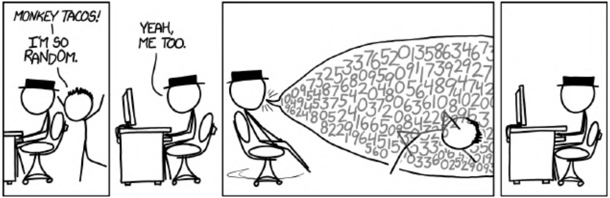
Desirable properties

- Non negative measure of uncertainty
- "combining uncertainties" should map to adding entropies

Definition

- Let $\mathcal{X}$ be a discrete r.v. with n outcomes in $\{x_0, \ldots, x_{n-1}\}$ with $\Pr(\mathcal{X} = x_i) = p_i$ for all $0 \leqslant i \leqslant n$
- The entropy of $\mathcal{X}$ is $H(\mathcal{X}) = \sum_{i=0}^{n-1} -p_i \log_b(p_i)$
- The measurement unit of entropy depends on the base b of the logarithm: typical case for $b = 2$ is bits

Examples



https://xkcd.com/1210/

$\mathcal{X}$: Uniformly random 6 letters word

- $\mathcal{X}$ is a sequence of 6 unif. random letters ($6^{26} \approx 3.1 \cdot 10^7$)
  - $H(\mathcal{X}) \approx \sum_{i=0}^{3.1 \cdot 10^7} -\frac{1}{3.1 \cdot 10^7} \log_b(\frac{1}{3.1 \cdot 10^7}) \approx 28.2b$
- $\mathcal{X}$ is a uniform pick from 6-letters English words
  - $H(\mathcal{X}) \approx \sum_{i=0}^{6300} -\frac{1}{6300} \log_b(\frac{1}{6300}) \approx 12.6b$

Shannon's noiseless coding theorem

Statement (informal)

It is possible to encode the outcomes n of i.i.d. random variables, each one with entropy $H(\mathcal{X})$, into no less than $nH(\mathcal{X})$ bits per outcome. If $< nH(\mathcal{X})$ bits are used, some information will be lost.

Consequences

- Arbitrarily compression of bitstrings is impossible without loss
  - Cryptographic hashes must discard some information
- Guessing a piece of information ($=$ one outcome of $\mathcal{X}$) is at least as hard as guessing a $H(\mathcal{X})$ bit long bitstring
  - overlooking for a moment the effort of decoding the guess

Min-Entropy

A practical mismatch

- It is possible to have distributions with the same entropies

Plucking low-hanging fruits

- We define the min-entropy of $\mathcal{X}$ as $H_\infty(\mathcal{X}) = -\log(\max_i p_i)$
- Intuitively: it's the entropy of a r.v. with uniform distribution, where the probability of each outcome is $(\max_i p_i)$
- Guessing the most common outcome of $\mathcal{X}$ is at least as hard as guessing a $H_\infty(\mathcal{X})$ bit long bitstring

Example

A very biased r.v.

$$\text{Consider } \mathcal{X} : \begin{cases} \mathcal{X} = 0^{128} & \text{with } \Pr \frac{1}{2} \\ \mathcal{X} = a, a \in 1\{0,1\}^{127} & \text{with } \Pr \frac{1}{2^{128}} \end{cases}$$

Intuition and quantification

- Predicting an outcome shouldn't be too hard: just say 0^{128}
- $H(\mathcal{X}) = \frac{1}{2}(-\log_2(\frac{1}{2})) + 2^{127}\frac{1}{2^{128}}(-\log_2(\frac{1}{2^{128}})) = 64.5\text{b}$
- $H_\infty(\mathcal{X}) = -\log_2(\frac{1}{2}) = 1\text{b}$
- Min-entropy tells us that guessing the most common output is as hard as guessing a single bit string

The Systems Perspective

“You have probably seen the door to a bank vault…10-inch thick, hardened steel, with large bolts…We often find the digital equivalent of such a vault door installed in a tent. The people standing around it are arguing over how thick the door should be, rather than spending their time looking at the tent.”

(Niels Ferguson & Bruce Schneier, Practical Cryptography)

Conclusions

Perfect ciphers *vs.* real world: brute-forcing

- Broken-unbroken ciphers: need for transparency
- Key lengths matters
- Symmetric, asymmetric algorithms and hash functions
- PKI and CAs and their complexity

We saw several case studies of attacks against crypto applications

- They had everything to do with systems security without even touching the algorithms themselves

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In Jonathan Katz and Hovav Shacham, editors, *Advances in Cryptology - CRYPTO 2017 - 37th Annual International Cryptology Conference, Santa Barbara, CA, USA, August 20-24, 2017, Proceedings, Part I*, volume 10401 of *Lecture Notes in Computer Science*, pages 570–596. Springer, 2017.

Marc Stevens, Alexander Sotirov, Jacob Appelbaum, Arjen K. Lenstra, David Molnar, Dag Arne Osvik, and Benne de Weger.
Short chosen-prefix collisions for MD5 and the creation of a rogue CA certificate.
In Shai Halevi, editor, *Advances in Cryptology - CRYPTO 2009, 29th Annual International Cryptology Conference, Santa Barbara, CA, USA, August 16-20, 2009. Proceedings*, volume 5677 of *Lecture Notes in Computer Science*, pages 55–69. Springer, 2009.

Further reading: a practical attack based on MD5 collisions

- It is known that MD5 allows a **chosen prefix collision** under certain constraints
  - Here the attack is used to create **two valid CA certificates with the same signature**: http://www.win.tue.nl/~bdeweger/CollidingCertificates/
  - Extended to threaten CAs in 2008: http://www.win.tue.nl/hashclash/rogue-ca/
- An evolution of the technique was used in **Flame**, a nasty malware used against several middle-Eastern targets
  - http://trailofbits.files.wordpress.com/2012/06/flame-md5.pdf