Online Learning Applications

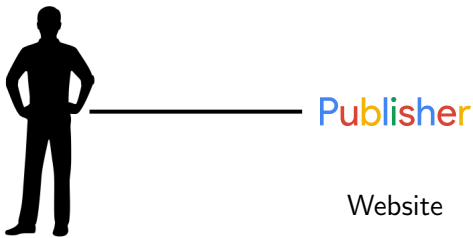
Part 6: Online advertising and auctions

Introduction to online advertisement

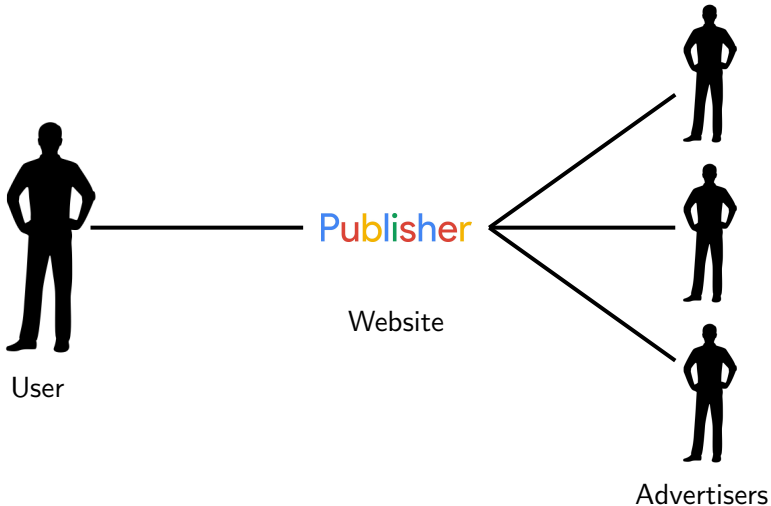
Online advertisement

User

Online advertisement



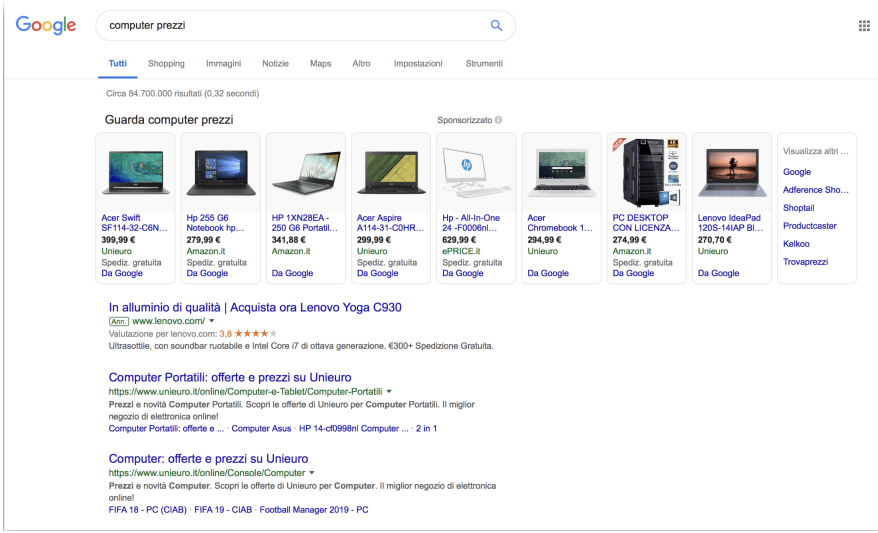
Online advertisement



Online advertisement

- **Advertisers** want to show **ads** to users visiting a web page
- An **user** visits the web page
- The **publisher** that handles the ad **slots** on the web page assigns each slot to an advertiser

Example


Google
computer prezzi
Tutti
Shopping
Immagini
Notizie
Maps
Altro
Impostazioni
Strumenti
Circa 84.700.000 risultati (0,32 secondi)
Guarda computer prezzi
Sponsorizzato
Acer Swift SF114-32-C6N…
399,99 €
Unieuro
Spediz. gratuita
Da Google
Hp 255 G6 Notebook hp…
279,99 €
Amazon.it
Spediz. gratuita
Da Google
HP 1XN28EA - 250 G6 Portatil…
341,88 €
Amazon.it
Da Google
Acer Aspire A114-31-C0HR…
299,99 €
Unieuro
Spediz. gratuita
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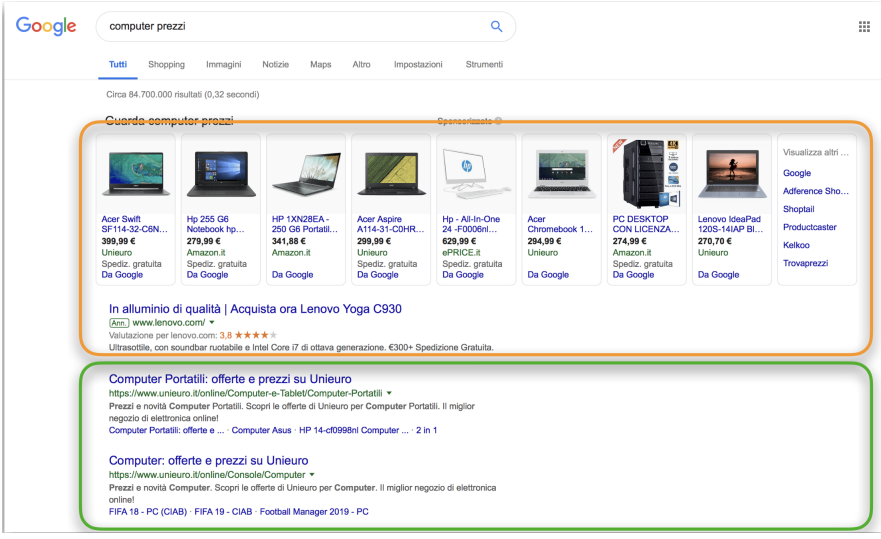
Hp - All-In-One 24 -f0006nl…
629,99 €
ePRICE.it
Spediz. gratuita
Da Google
Acer Chromebook 1…
284,99 €
Unieuro
Da Google
PC DESKTOP CON LICENZA…
274,99 €
Amazon.it
Spediz. gratuita
Da Google
Lenovo IdeaPad 120S-14IAP Bl…
270,70 €
Unieuro
Da Google
Visualizza altri …
Google
Adference Sho…
Shoptail
Productcaster
Kelkoo
Trovaprezzi
In alluminio di qualità | Acquista ora Lenovo Yoga C930
Ann. www.lenovo.com/
Valutazione per lenovo.com: 3,8
Ultrasottile, con soundbar ruotabile e Intel Core i7 di ottava generazione. €300+ Spedizione Gratuita.
Computer Portatili: offerte e prezzi su Unieuro
https://www.unieuro.it/online/Computer-e-Tablet/Computer-Portatili
Prezzi e novità Computer Portatili. Scopri le offerte di Unieuro per Computer Portatili. Il miglior negozio di elettronica online!
Computer Portatili: offerte e … · Computer Asus · HP 14-cf0998nl Computer … · 2 in 1
Computer: offerte e prezzi su Unieuro
https://www.unieuro.it/online/Console/Computer
Prezzi e novità Computer. Scopri le offerte di Unieuro per Computer. Il miglior negozio di elettronica online!
FIFA 18 - PC (CIAB) · FIFA 19 - CIAB · Football Manager 2019 - PC

Example



Ads

Results

Interaction

The interaction among the publisher and the advertisers goes as follows:

Interaction

1. An user arrives
2. Each advertiser **bids**
3. The publisher **assigns each slot** to an advertiser based on bids
4. The advertisers **pay** the publisher if their ad is clicked

Both advertisers and publisher have a problem to deal with.

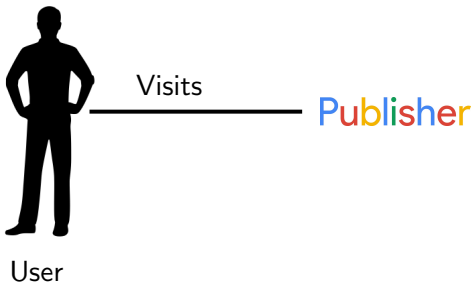
Advertiser's problem

Choose how to **bid**.

Publisher's problem

Choose how to **allocate slots** and **set payments**.

Example (single slot)



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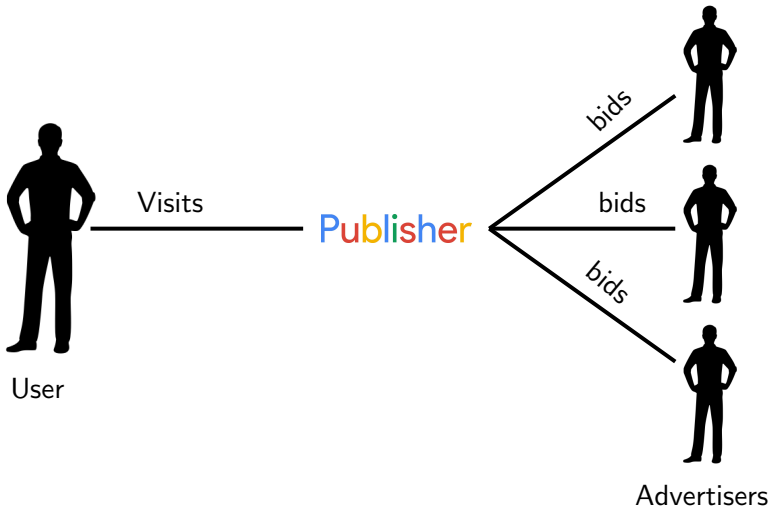

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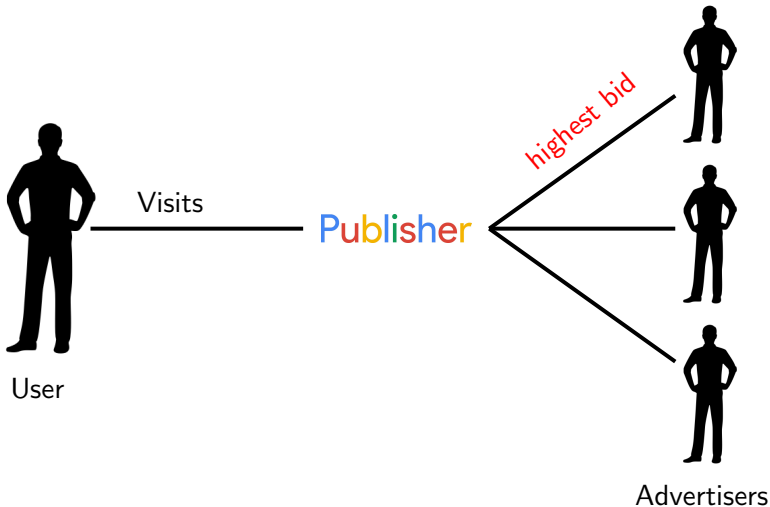

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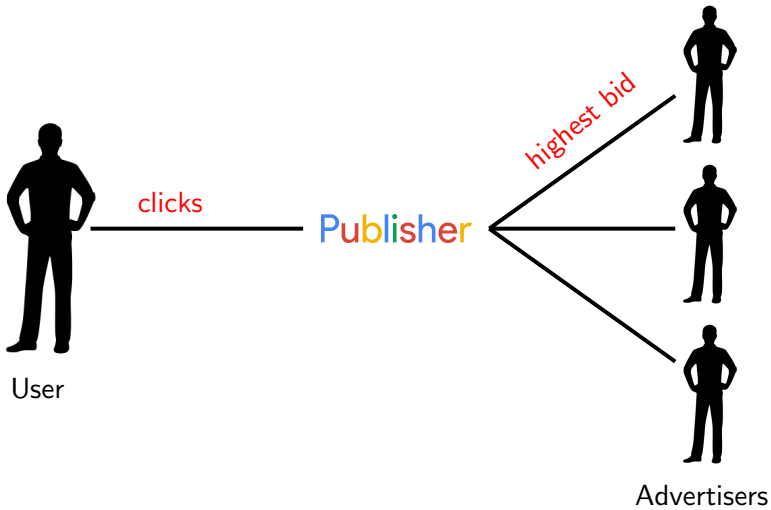

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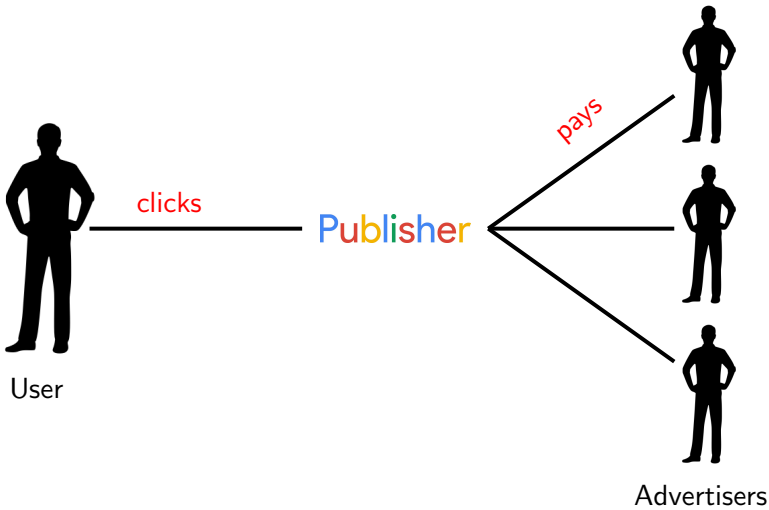
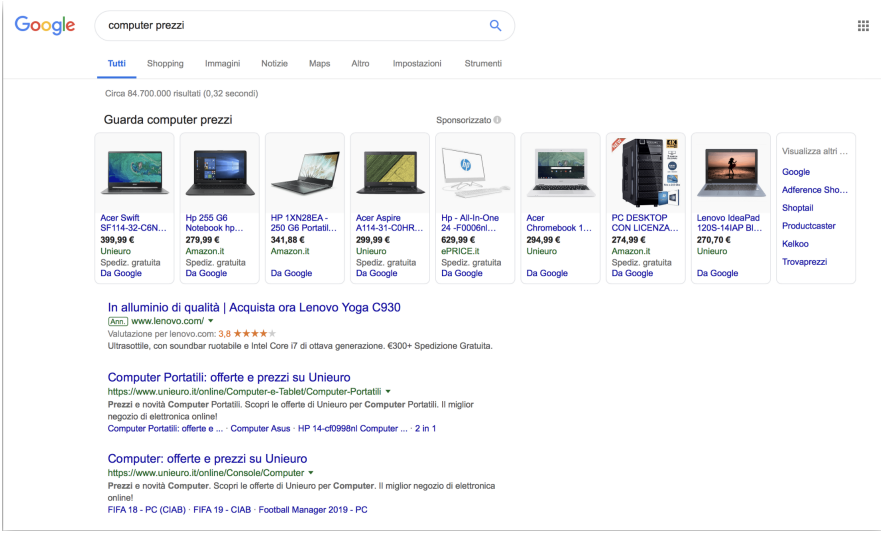


Slot prominence (some slots are better than others)



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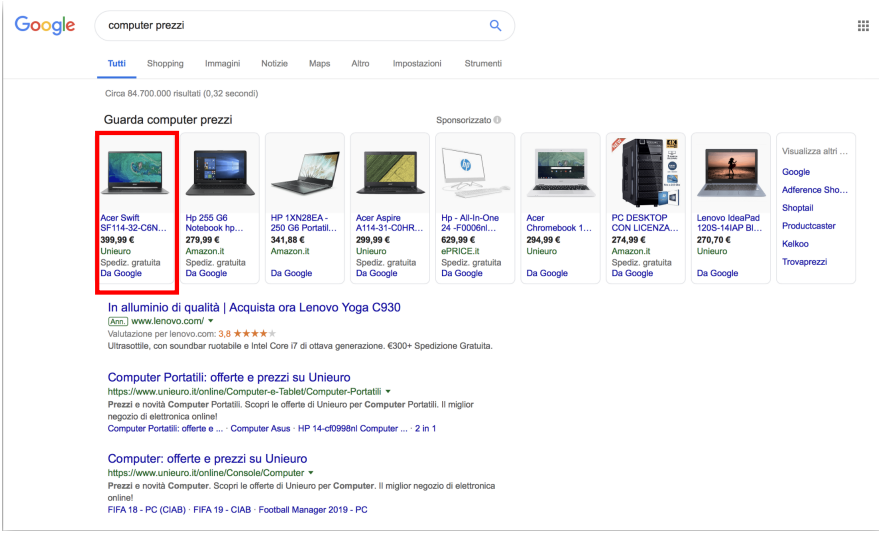
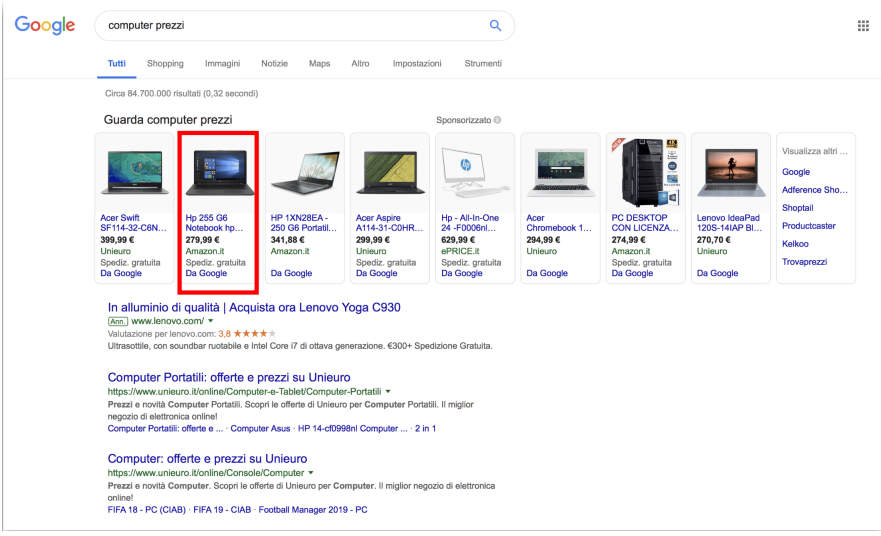


Best slot

Slot prominence (some slots are better than others)



Second-best slot

Publisher's perspective

How to set payments and allocation?

The utility of an **advertiser** a for visualizing an ad on a **slot** s is:

$$\lambda_s \; q_a \; v_a$$

- λ_s is the probability that the user observes the slot (**slot prominence**)
- q_a is the advertiser's click probability given the ad has been observed by the user (**ad quality**)
- v_a is the advertiser's value per click (**ad value**)
- $\lambda_s \; q_a$ is the advertiser's click probability (**click trough rate**)
- $q_a \; v_a$ is the **expected utility** of an advertiser if the slot is observed

How to allocate slots?

Suppose that the **publisher knows all the parameters**. The optimal allocation solves:

$$\max_{s(\cdot)} \sum_a \lambda_{s(a)} \; q_a \; v_a,$$

where:

- The function $s(\cdot)$ assigns to each ad at most one slot
- If an ad is not allocated, $\lambda_{s(a)} = 0$

The optimal $s()$:

- Assigns the best slot to the ad with largest $q_a v_a$
- assigns the second-best slot to the ad with second-largest $q_a v_a$
- ...

Known and unknown parameters

$$\lambda_s \quad q_a \quad v_a$$

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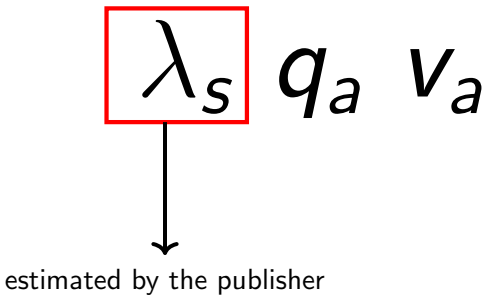
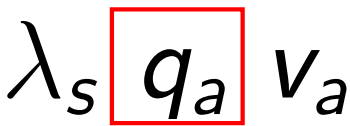
$\boxed{\lambda_s}\ q_a\ v_a$

Known and unknown parameters



Known and unknown parameters



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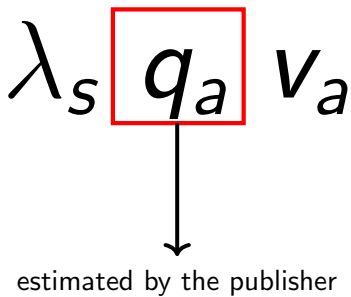

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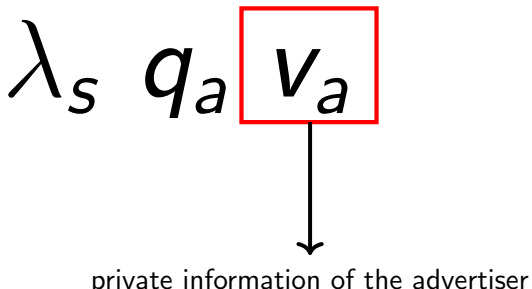

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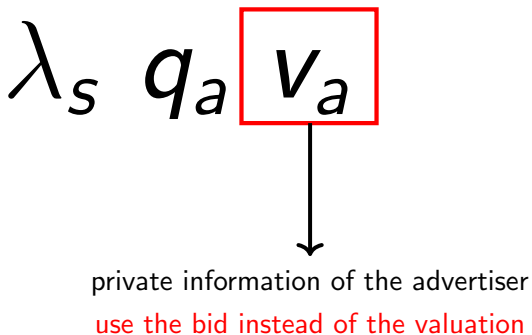

How to allocate slots?

The publisher chooses the allocation solving:

$$\max_{s(\cdot)} \sum_a \lambda_{s(a)}\ q_a\ b_a,$$

where:

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Truthful auctions

Problem

The publisher uses bids as an estimation of the valuations.

Truthful auctions

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Solution

Design an auction that incentivizes the advertisers to bid their valuations. This is called a **truthful auction**.

Second-price auctions

If there is only one slot, the publisher can use second-price auctions:

Second-price auction

- The advertiser a_1 with largest $q_a b_a$ wins the slot
- The advertiser pays $p_{a_1} = \frac{q_{a_2} b_{a_2}}{q_{a_1}}$ if the ad is clicked
- a_2 is the advertiser with the second largest $q_a b_a$

- The advertiser always pays $p_{a_1} \leq b_{a_1}$

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- **The auction is truthful $\rightarrow$ each advertiser is incentivized to report the true valuation**

VCG Auctions

If there are multiple slots, the publisher can use a VCG (Vickrey-Clarke-Groves) auction.

VCG auction

- Recall that $s()$ is the allocation function that assign slots to advertisers
- Recall that $s()$ assigns slots in decreasing order of expected value
- Each advertiser a pays $p_a = \frac{1}{\lambda_{s(a)} q_a}(Y_a - X_a)$ if the ad is clicked
- $X_a = \sum_{a' \neq a} \lambda_{s(a')} q_{a'} b_{a'}$ is the utility of the other advertisers in the optimal allocation
- $Y_a = \max_{s'(\cdot)} \sum_{a' \neq a} \lambda_{s'(a')} q_{a'} b_{a'}$ is the utility of the other advertisers if a does not take part to the auction

- Each advertiser "pays" his externalizes on the other advertisers

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- Each advertiser "pays" his externalizes on the other advertisers
- **The auction is truthful $\rightarrow$ each agent is incentivized to report the true valuation**
- If there is a single slot, VGC is equivalent to a second-price auction

First-price auctions

Truthful auctions have complex payments (see, e.g., VCG). Publishers also use **non-truthful auctions**:

- Non-truthful auctions are easier to understand
- With a single slot, the most common is the **first-price auction**

First-price auction

- The advertiser a_1 with largest $q_a b_a$ wins the slot
- The advertiser pays $p_{a_1} = b_{a_1}$ if the ad is clicked

- The winning advertiser pays exactly what they bid

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First-price auction

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- The advertiser pays $p_{a_1} = b_{a_1}$ if the ad is clicked

- The winning advertiser pays exactly what they bid
- ⚠ The auction is not-truthful $\rightarrow$ the advertisers are incentivized to reduce the bid in order to decrease the payment

Non-truthful auctions

First-price auctions can be generalized to multiple slots.

Generalized first-price auction

- Each advertiser a pays $p_a = b_a$ if the ad is clicked

Non-truthful auctions

First-price auctions can be generalized to multiple slots.

Generalized first-price auction

- Each advertiser a pays $p_a = b_a$ if the ad is clicked

Other non-truthful auctions commonly used are a generalization of second-price auctions.

Generalized second-price auction

- Recall that $s()$ is the allocation function that assign slots to advertisers
- Each advertiser a pays $p_a = \frac{q_{a+1}}{q_a} b_{a+1}$ if the ad is clicked
- $a + 1$ is the ad assign to the slot just after $s(a)$

Advertiser's perspective

Bidding strategies

Now, we consider the **advertiser's perspective**

- The advertiser wants to **maximize his utility**
- An advertiser takes part to a an auction every time a new user visit the page:
  - If the auction is **truthful**, they simply **bid their valuation**
  - If the auction is **not truthful**, they must **learn how to bid**

Learning to bid in non-truthful auctions

The problem of bidding is non-truthful auctions is an **online learning problem**:

- The advertiser chooses a bid at each auction $t \in [T]$
- Depending on the assumptions the setting can be adversarial or stochastic:
  - The setting is **adversarial** since the **other bidders** play unpredictably affecting the possibility of winning the auction and the payment (i.e., the reward)
  - the setting is **stochastic** if we assume that there are **lots of other bidders** and their joint behavior is stochastic

Bidding strategies

The goal of an advertiser is usually not unconstrained utility maximization.

Budget constraint

The advertiser assigns a **budget** to a marketing campaign.

Problem

The advertiser must choose how to spent the budget over time.

Example of campaign:

- The advertiser tries to display the ad whenever a specific keyword is searched on a search engine.
- The advertiser stops bidding when the budget is depleted

Example

user

Publisher

Advertisers

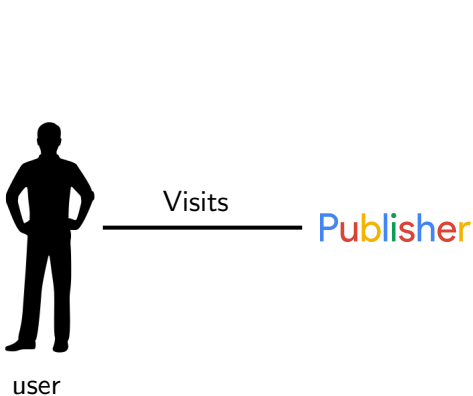
Example

user

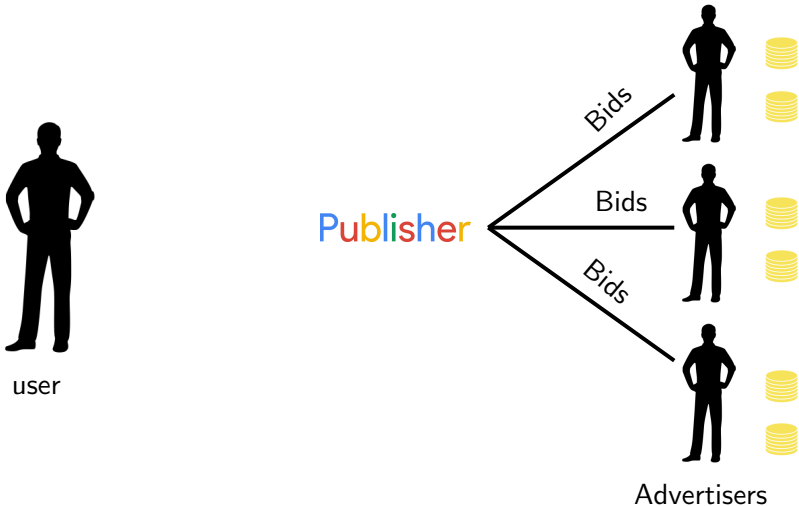
Publisher

Advertisers

Example



Example



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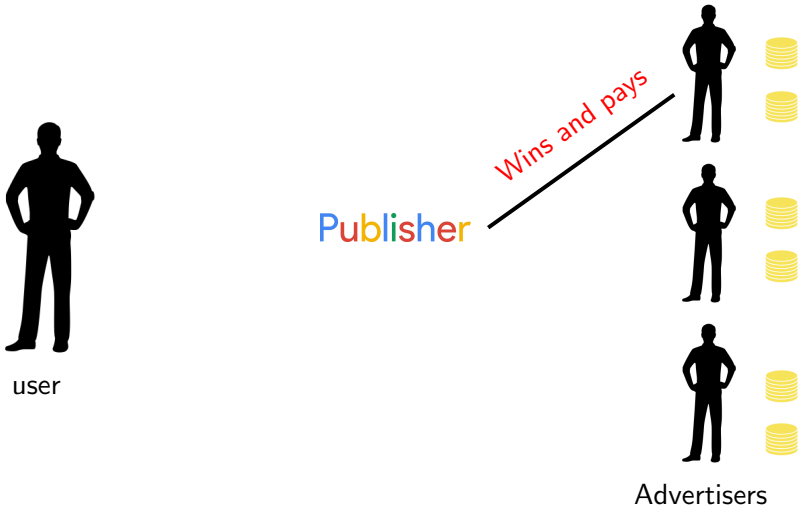


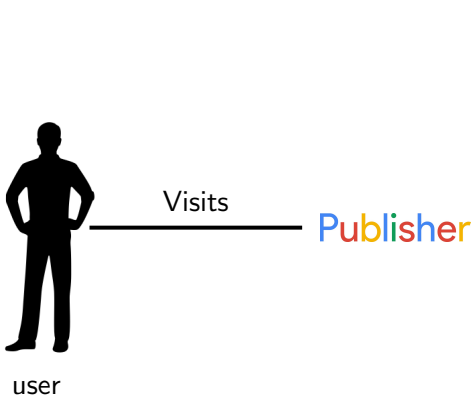
Example

user

Publisher

Advertisers

Example



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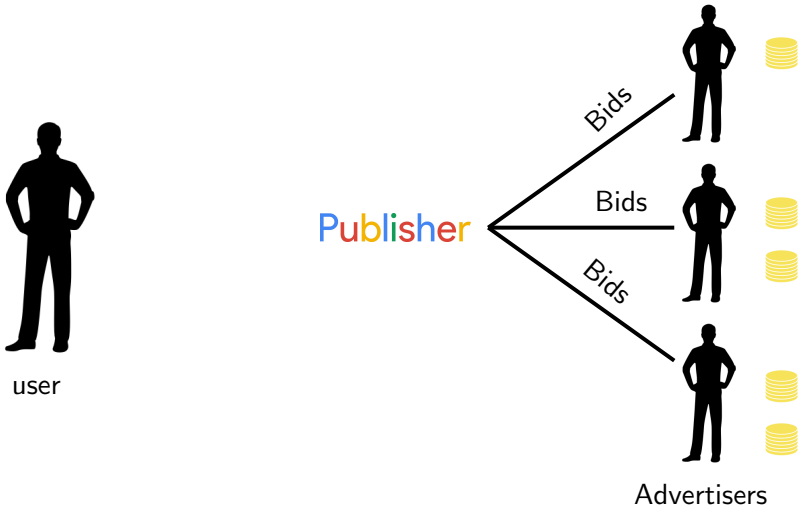


Example

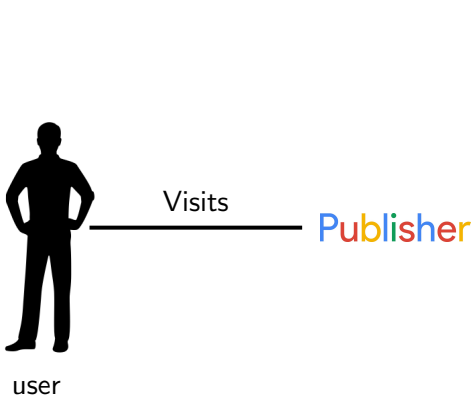
Example

user

Publisher

Advertisers

Example



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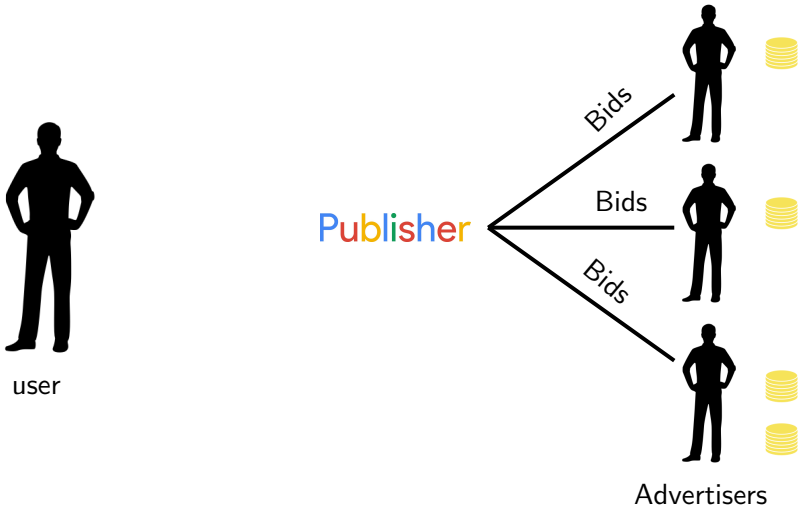

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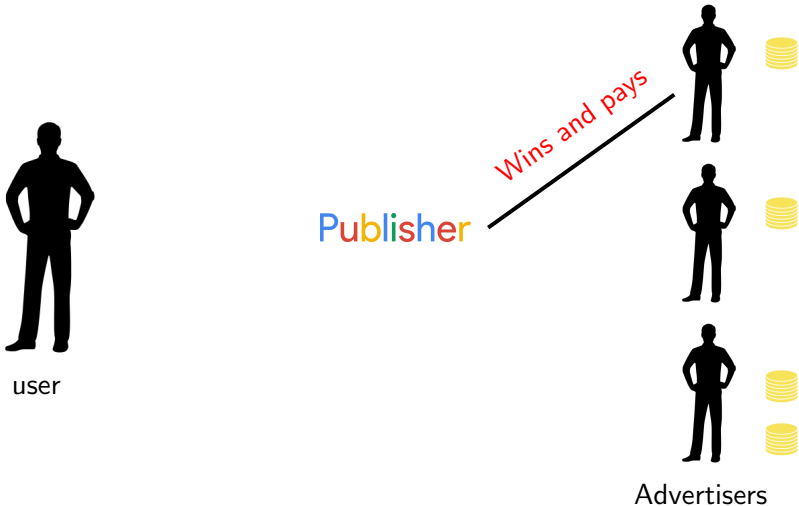


Example

user

Publisher

Advertisers

Example

The advertiser stops participating to auctions

user

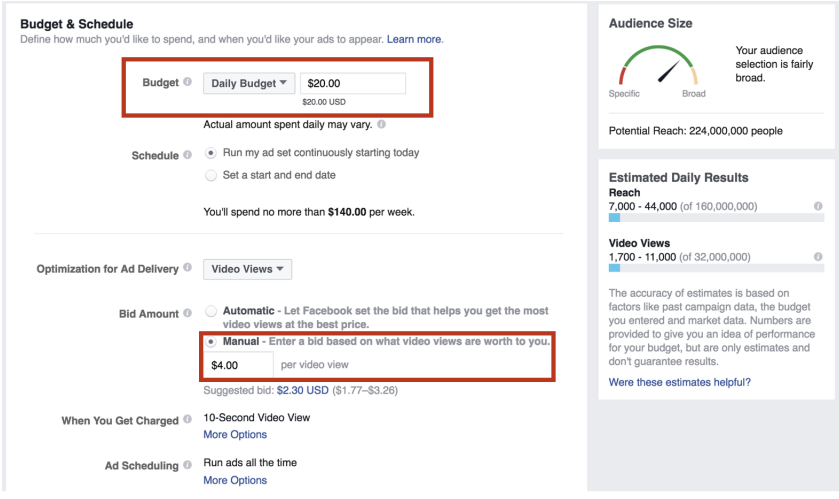
Publisher

Advertisers

Delegation of the campaign management

- Handling this repeated bidding problem is **complex**
- Advertisers usually **delegate** this task to:
  - Ad networks
  - Publishers
  - Ad exchange
- The advertiser sets a small number of parameters and a **proxy bidder** handles the bidding.

Delegation of the campaign management



Proxy Bidders

Proxy bidders take in input a small number of parameters, handle the auctions, and report only aggregated results to the advertiser.