Assignment- 4
ECS 322: Electromagnetic Theory

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Solutions

1. Here, I have attached all the images of the output and the theoretical calculations.

Lossy (AC Analysis)

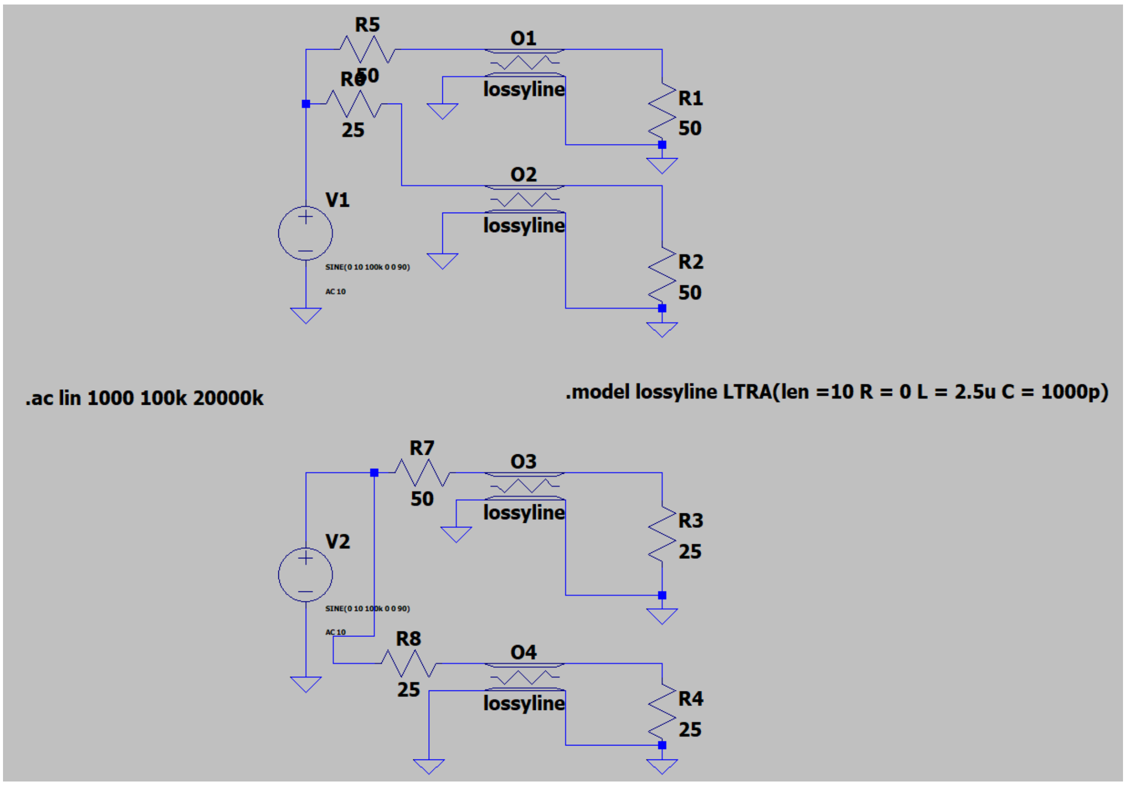


Fig.1. Circuit for Lossy AC analysis

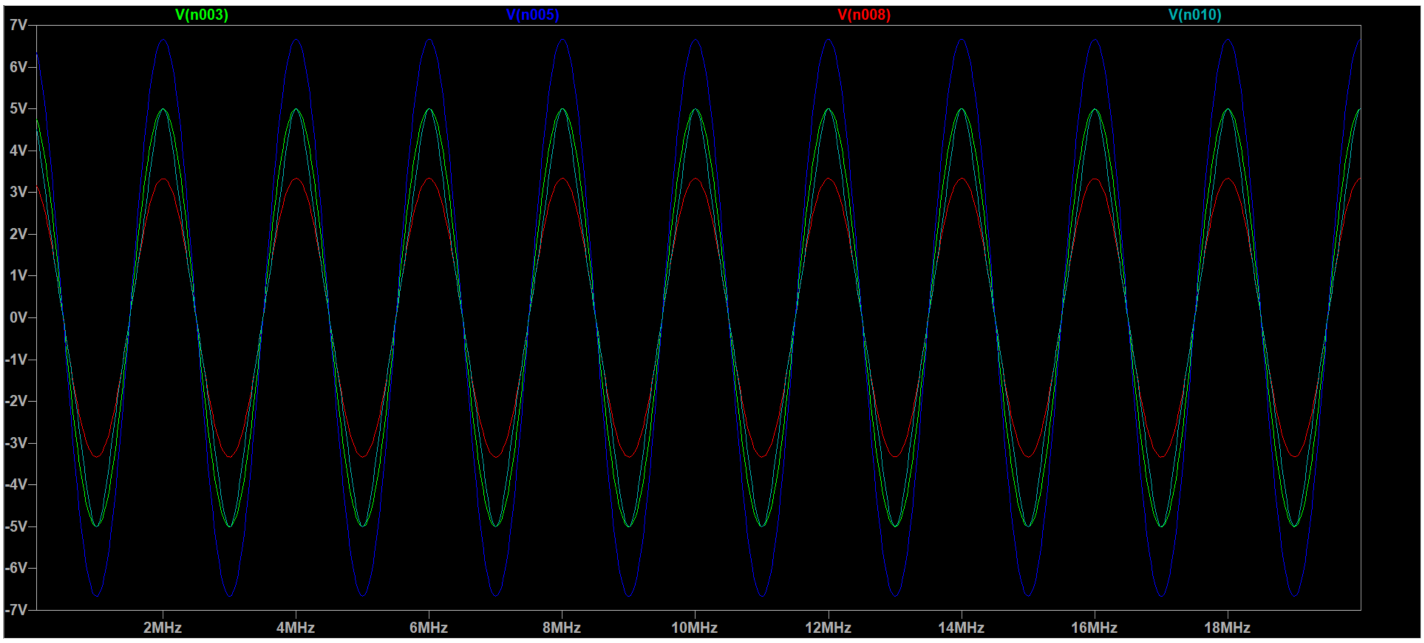


Fig.2. Lossy AC analysis

Lossy (Transient Analysis)

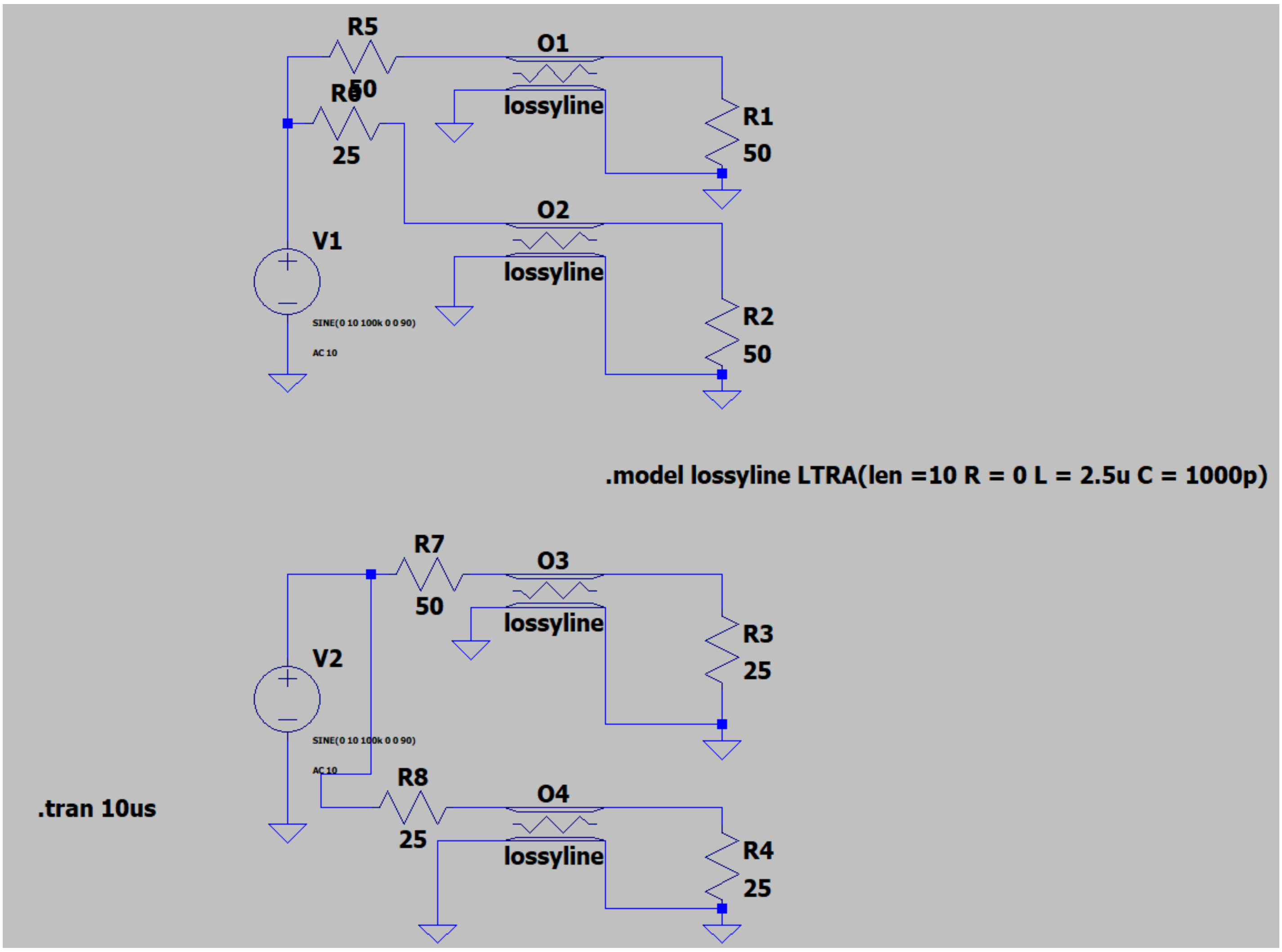


Fig.3. Circuit for Lossy Transient analysis

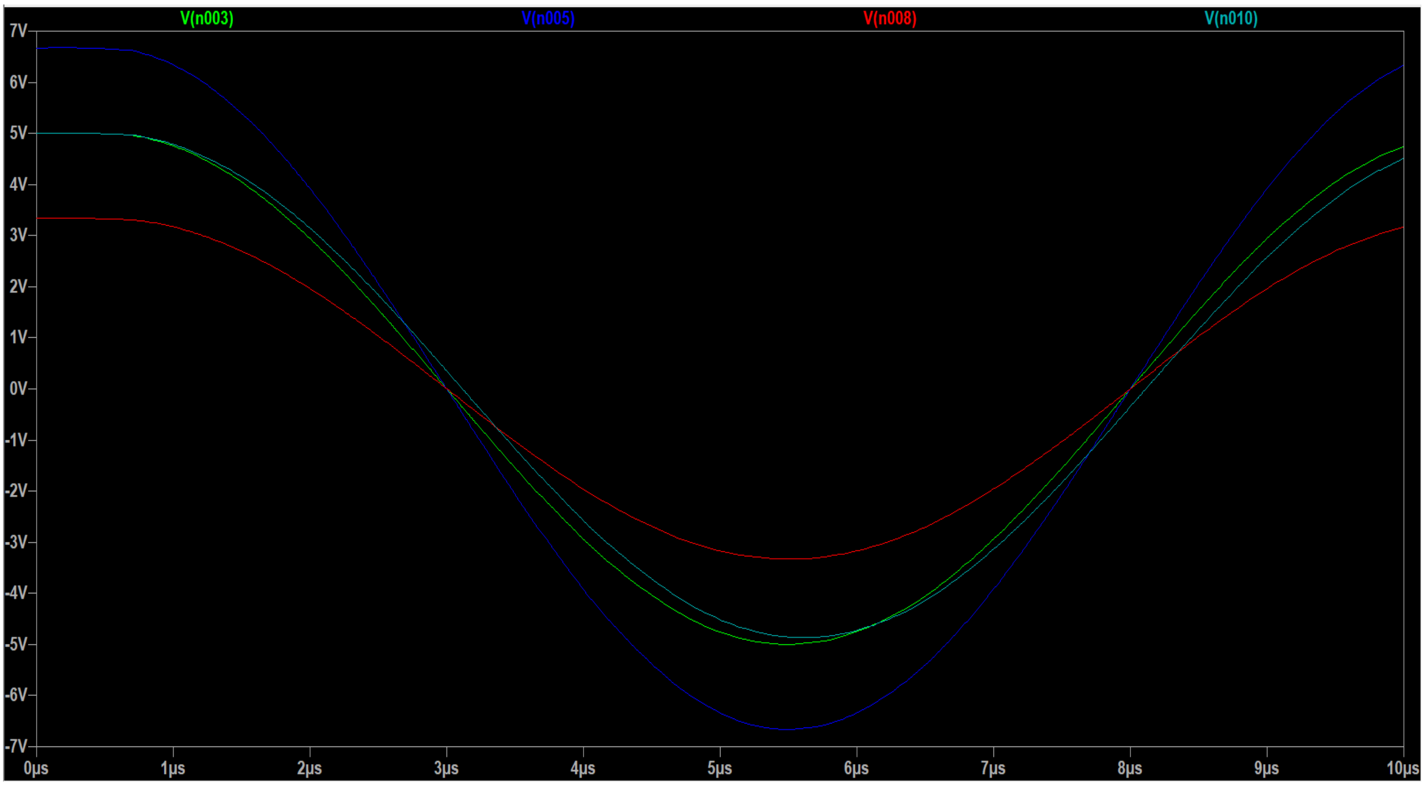


Fig.4. Lossy Transient analysis

Lossless (AC Analysis)

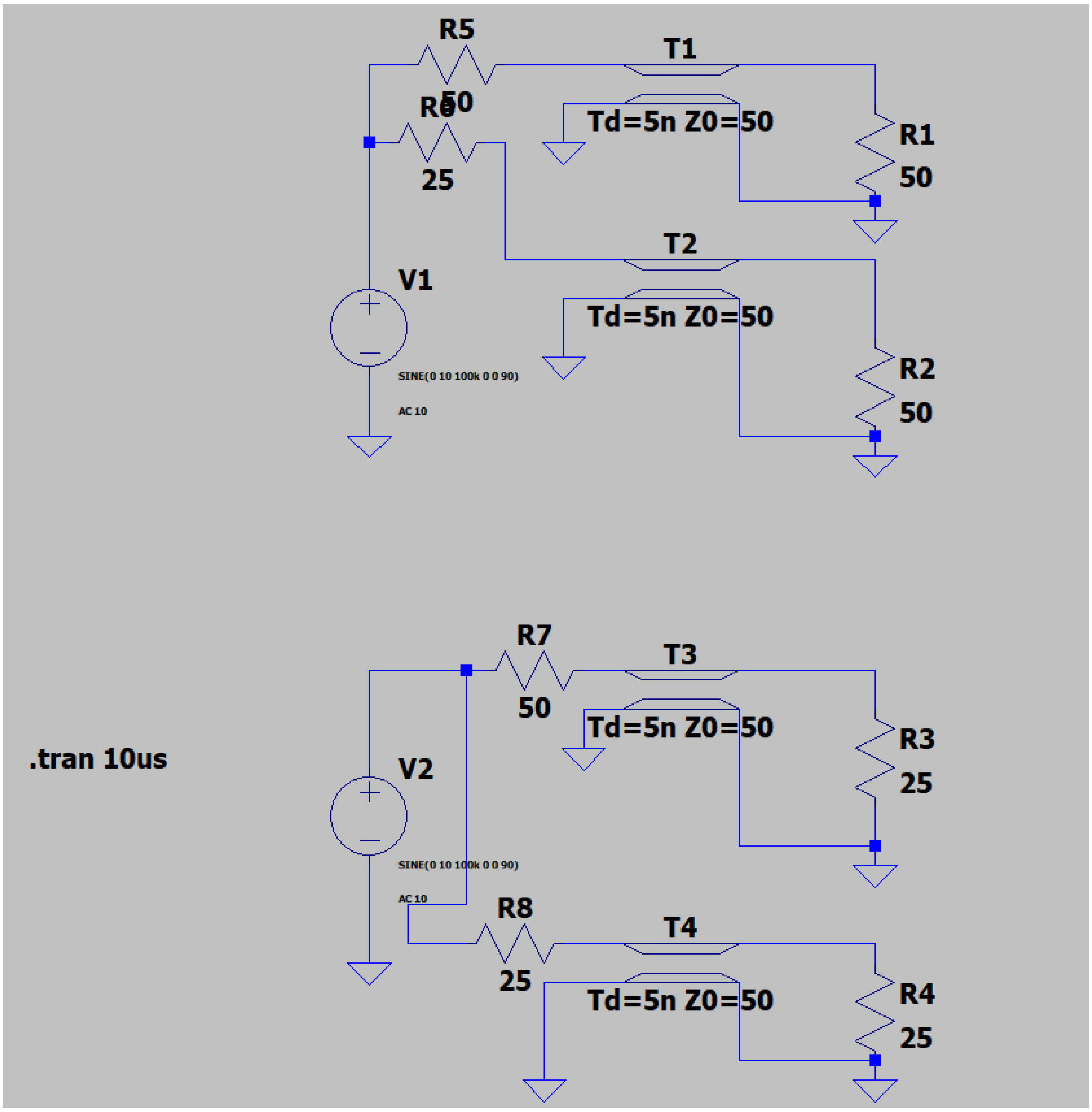


Fig.5. Circuit for Lossless AC analysis

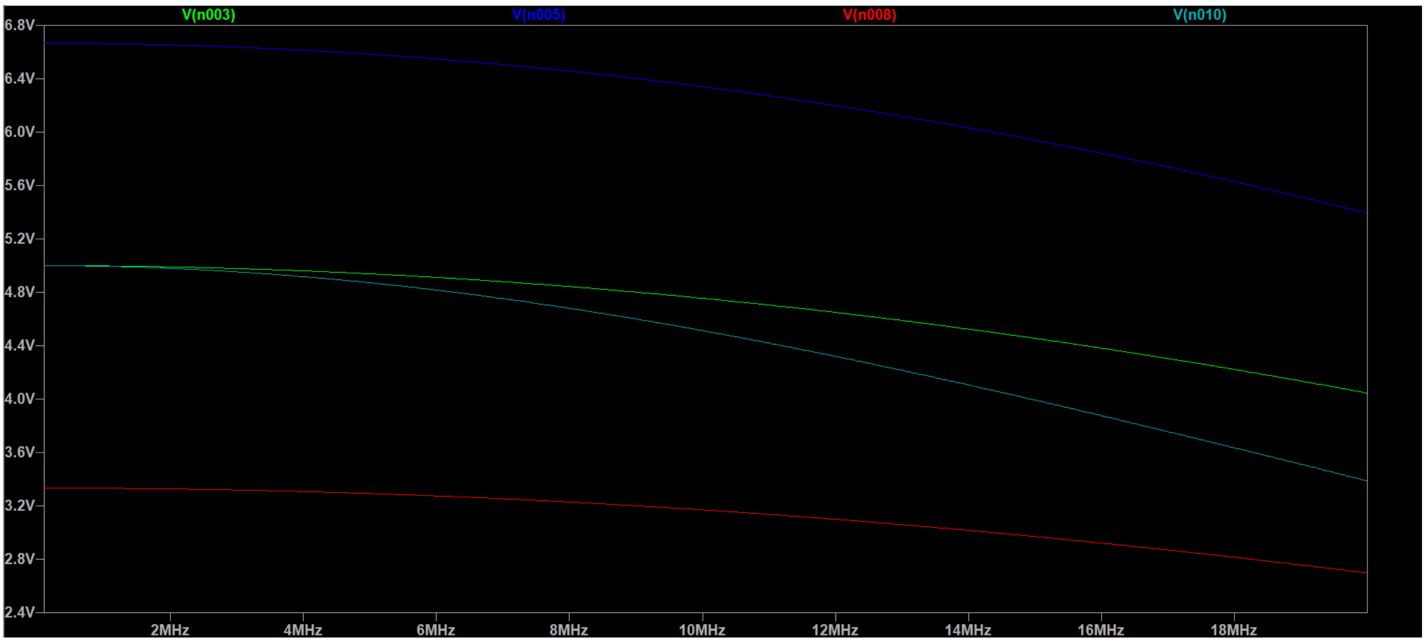


Fig.6. Lossless AC analysis

Lossless (Transient Analysis)

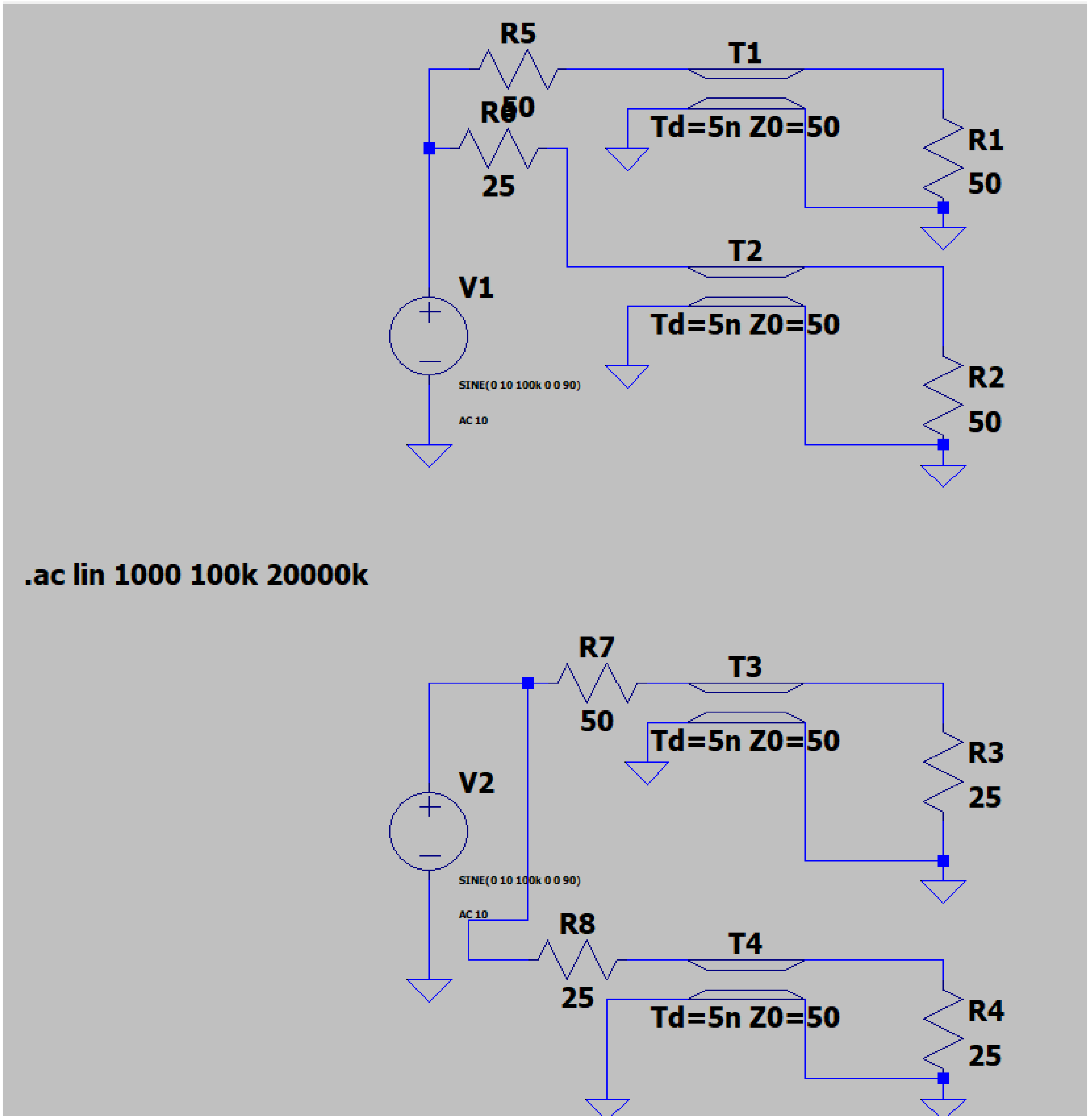


Fig.7. Circuit for Lossless Transient analysis

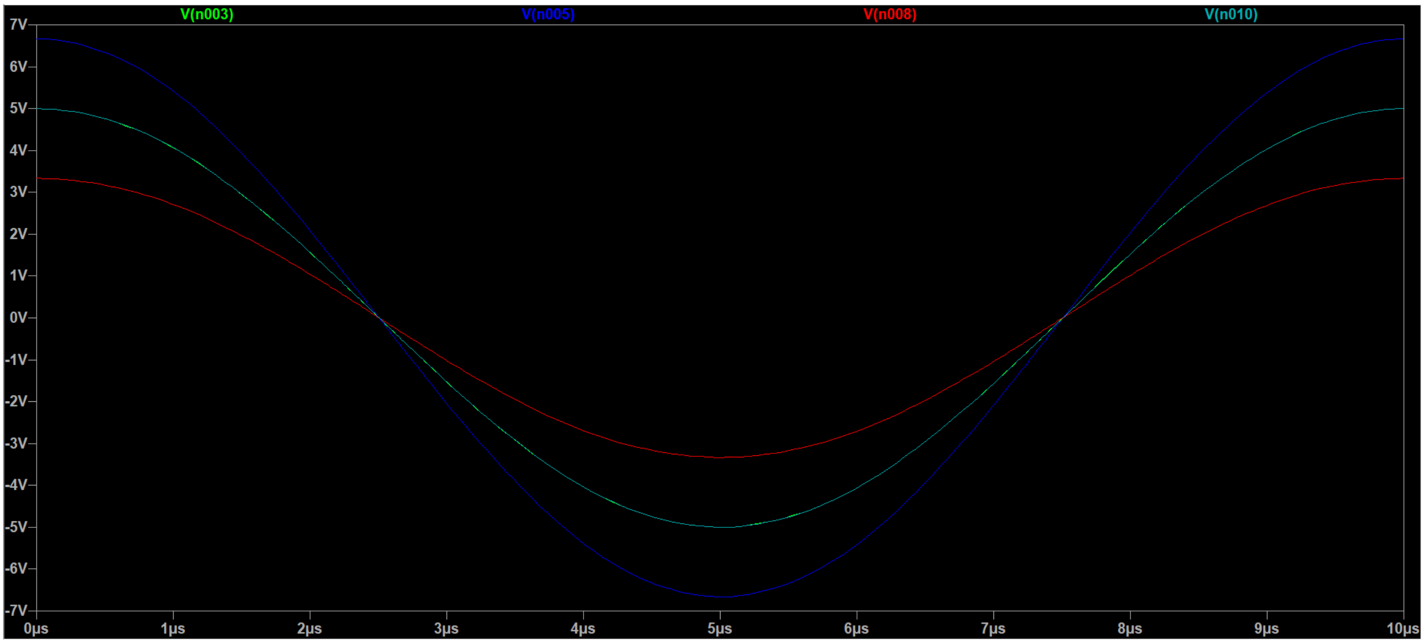


Fig.8. Lossless Transient analysis

Theoretical calculations:

1. Given, sinusoidal voltage, $V(t) = 10 \cos \omega t$

frequency $f = 100$ KHz

length $l = 10m$

$L' = 0.25$ MH/m

$C' = 100$ pF/m

$\varepsilon_r = 2.25$

(i) characteristic impedance $= \sqrt{\frac{L'}{C'}}$

$$= \sqrt{\frac{0.25}{100} \times 10^{-6+12}} \text{ ohm}$$

$$= \sqrt{\left(\frac{1}{4}\right) \times 10^{4}} \text{ ohm.}$$

$$= 0.5 \times 10^{2} \text{ ohm} = \boxed{50 \text{ ohm}}$$

(ii) Phase constant $(\beta) = 2\pi f \sqrt{L'C'}$

$$= 2\pi \times 100 \times 10^{3} \text{ Hz} \times \sqrt{L'C'}$$

Now, $v_p = \frac{\omega}{\beta} = \frac{2\pi f}{2\pi f \sqrt{L'C'}}$

or, $v_p = \frac{1}{\sqrt{L'C'}}$

or, $v_p = \frac{1}{\sqrt{0.25 \times 100 \times 10^{-18}}}$ m/s.

or, $v_p = \frac{1}{\sqrt{\left(\frac{1}{4}\right) \times 10^{-16}}}$ m/s $= \frac{1}{\frac{1}{2} \times 10^{-8}}$ m/s

$$\Rightarrow \boxed{v_p = 2 \times 10^8 \text{ m/s.}}$$

$$\text{Time delay} = \sqrt{L'C'}$$

$$= \sqrt{0.25 \times 100 \times 10^{-18}} \text{ s.}$$

$$= \sqrt{\left(\frac{1}{4}\right) \times 10^{-16}} \text{ s}$$

$$= (0.5 \times 10^{-8}) \text{ s.}$$

$$= (5 \times 10^{-9}) \text{ s}$$

$$= \boxed{5 \text{ ns.}}$$

Input impedance

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{v_p/f} = \frac{\pi f}{10^8} \text{ (rad/m).}$$

for scenario 1, $Z_L = 50\Omega$, $Z_0 = 50\Omega$

$$z_l = \frac{Z_L}{Z_0} = \frac{50}{50} = 1$$

$$Z_{in} = 50\left[\frac{1\cos\beta l + j\sin\beta l}{\cos\beta l + j1\sin\beta l}\right]$$

$$\boxed{Z_{in} = 50\,\Omega}$$

for scenario 2, $Z_L = 50\,\Omega$, $Z_0 = 50\,\Omega$, $z_l = 1$.

$$Z_{in} = 50\left[\frac{1\cos\beta l + j\sin\beta l}{\cos\beta l + j(1)\sin\beta l}\right]$$

$$\boxed{Z_{in} = 50\Omega}$$

for scenario 3, $Z_L = 25\Omega$, $Z_0 = 50\Omega$, $z_L = 1/2$

$$Z_{in} = 50\left[\frac{\frac{1}{2}\cos\beta l + j\sin\beta l}{\cos\beta l + j(\frac{1}{2})\sin\beta l}\right]$$

$$Z_{in} = 50\left[\frac{\cos(\pi f l \times 10^{-8}) + j\,2\sin(\pi f l \times 10^{-8})}{2\cos(\pi f l \times 10^{-8}) + j\sin(\pi f l \times 10^{-8})}\right]$$

for scenario 4, $Z_L = 25\Omega$, $Z_0 = 50\Omega$, $z_L = 1/2$.

$$Z_{in} = Z_0\left[\frac{z_L\cos\beta l + j\sin\beta l}{\cos\beta l + j z_L\sin\beta l}\right]$$

$$= 50\left[\frac{\frac{1}{2}\cos(\pi f l \times 10^{-8}) + j\sin(\pi f l \times 10^{-8})}{\cos(\pi f l \times 10^{-8}) + j(\frac{1}{2})\sin(\pi f l \times 10^{-8})}\right]$$

Time-averaged power

$$P_{av} = \frac{|V_0^+|^2}{2Z_0}\left(1 - |\Gamma|^2\right)$$

Here, $V_0^+ = 10$.

$$P_{av} = \frac{10^2}{2(50)}\left(1 - |\Gamma|^2\right)$$

$$P_{av} = 1\left(1 - |\Gamma|^2\right)$$

→ scenario 1 : $(z_L = 1)$

$$\Gamma = \frac{z_L - 1}{z_L + 1} = 0 \Rightarrow \boxed{P_{av} = 1W}$$

→ scenario 2 : → $z_L = 1$

$$\Gamma = \frac{z_L - 1}{z_L + 1} = 0 \Rightarrow \boxed{P_{av} = 1W}$$

→ scenario 3 : → $z_L = 1/2$.

$$\Gamma = \frac{Z_L - 1}{Z_L + 1} = -1/3.$$

$$P_{av} = 1\left(1 - \left(\frac{-1}{3}\right)^2\right)$$

$$\boxed{P_{av} = \frac{8}{9}\ W}$$

→ scenario 4, $z_L = 1/2$.

$$\Gamma = \frac{Z_L - 1}{Z_L + 1} = -\frac{1}{3}.$$

$$P_{av} = 1\left(1 - (-1/3)^2\right)$$

$$\boxed{P_{av} = \frac{8}{9}\ W}$$

2. We need to set up a project in COMSOL to determine the potential and electric field of a 3D point charge. Here are the plots.

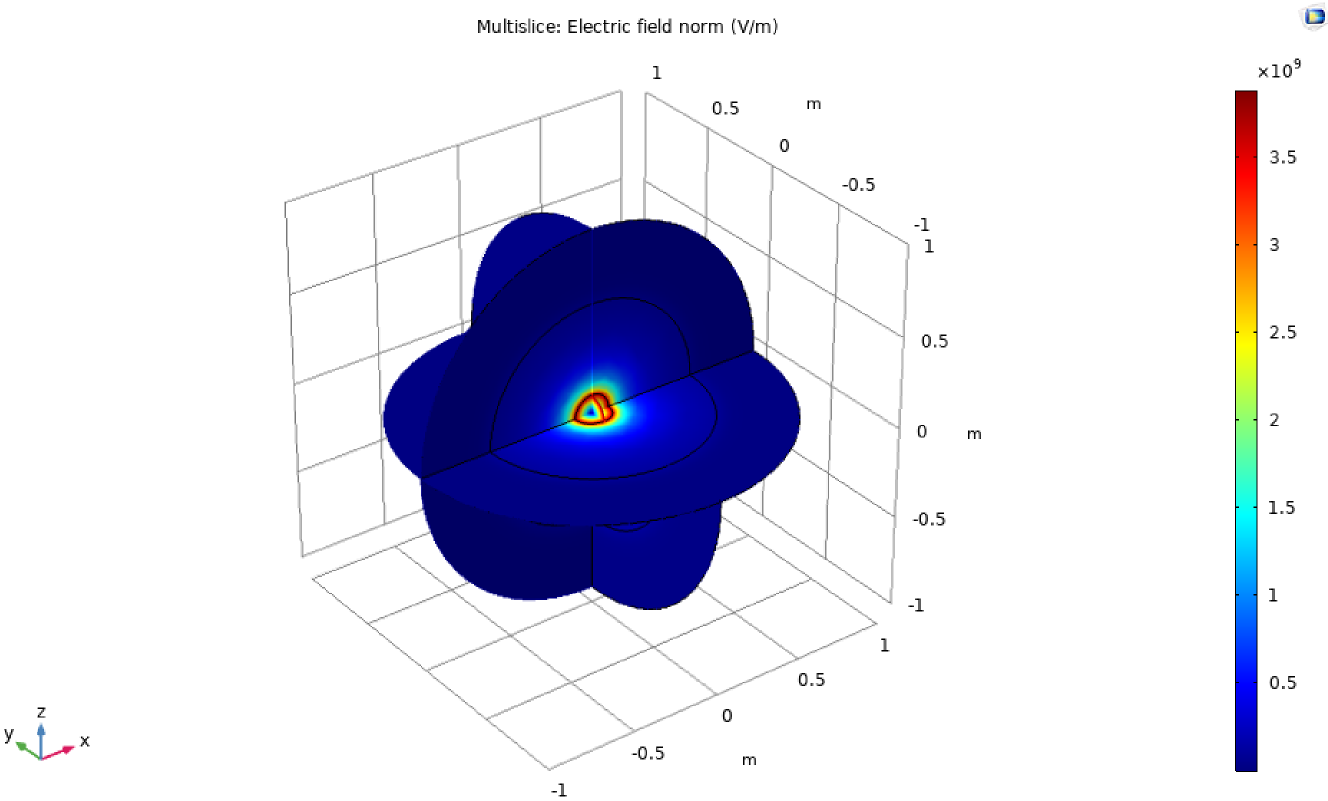


Fig.9. The electric field of a 3D point charge

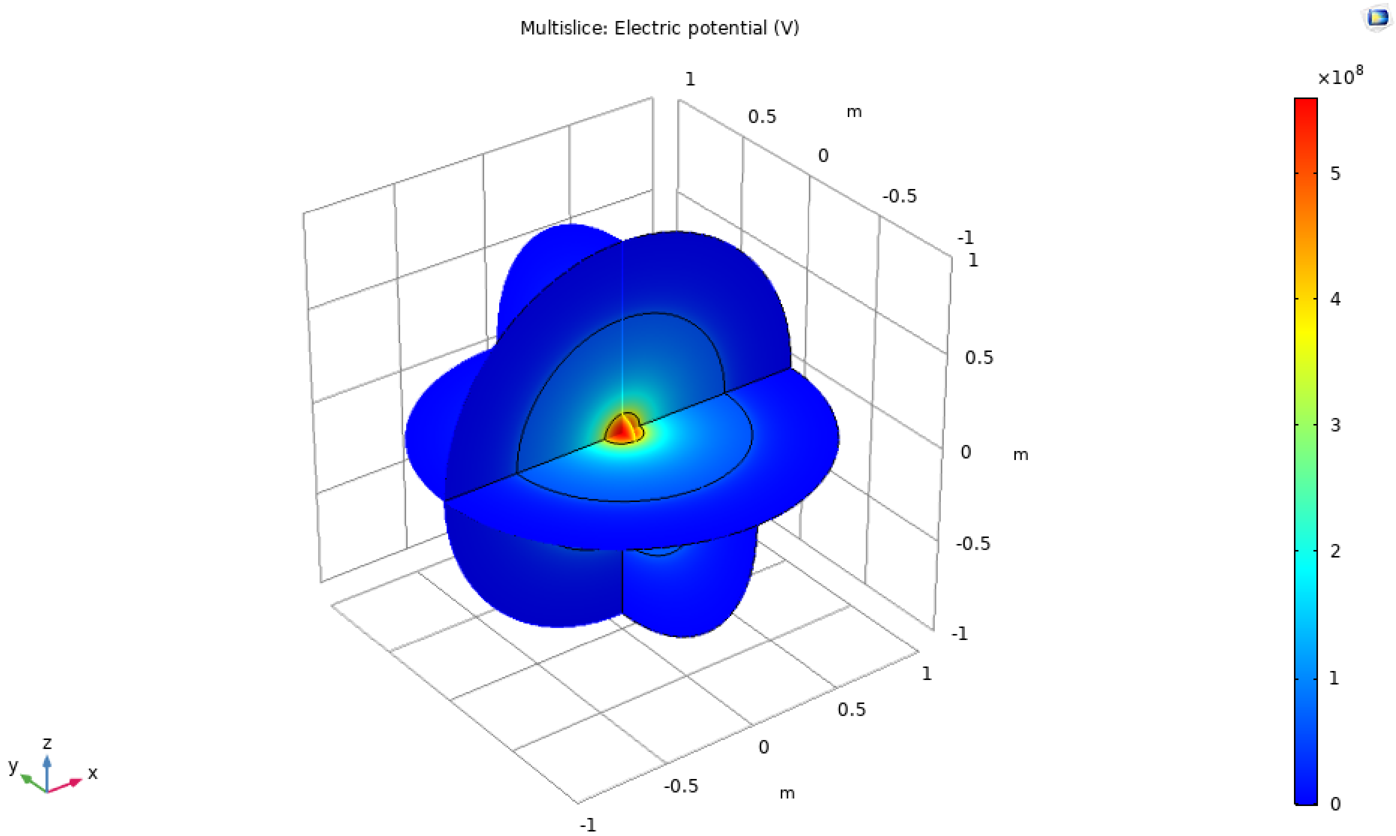


Fig.10. The electric potential of a 3D point charge

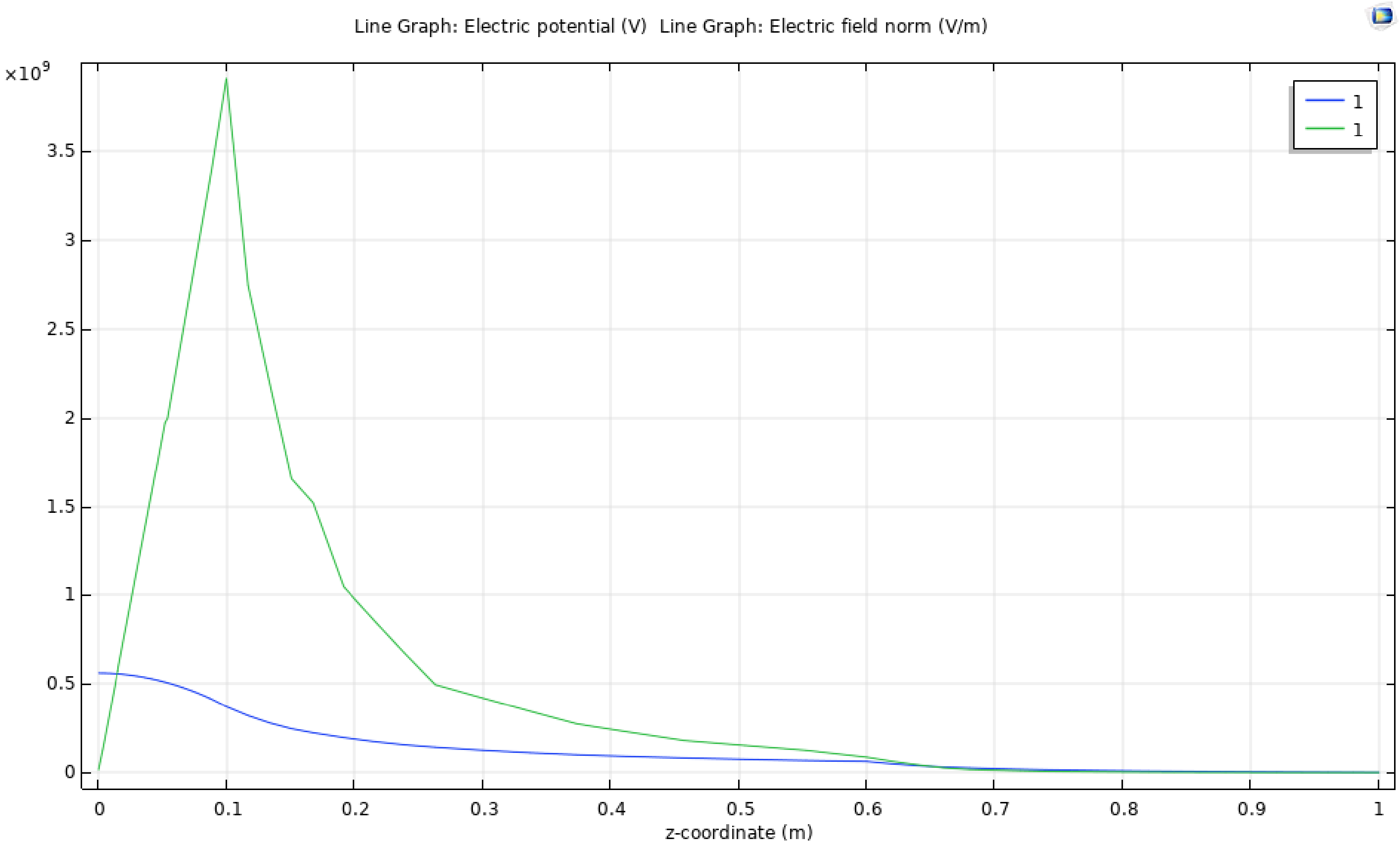


Fig.10. Line graph of the electric field and potential of a 3D point charge

Theoretical calculations:

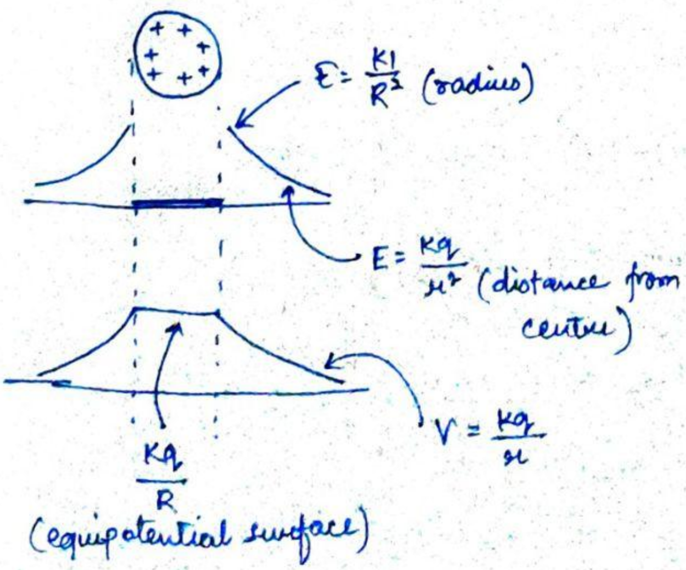
2. for a solid charged sphere

→ data from COMSOL:

Electric field at the surface of charged sphere = 3.76×10^9 V/m

Electric potential at the surface of charged sphere = 0.376×10^9

→ Theoretical data:



$E = \frac{Kq}{R^2}$ { where $K = 9 \times 10^9$, $R = 0.1$ m, $\rho_v = 1\ C/m^3$ }

$V = \frac{Kq}{R}$ { where $K = 9 \times 10^9$, $R = 0.1$m, $\rho_v = 1\ C/m^3$ }

given, $\rho_v = \frac{q}{v} = \frac{q}{\frac{4}{3} \times \pi \times (0.1)^3} = 1$

$\Rightarrow q = 4.2 \times 10^{-3}$ Columb

$E = \frac{9 \times 10^9 \times (4.2) \times 10^{-3}}{(0.1)^2} = 3.76 \times 10^9$ V/m ≈ Practical value

$V = \frac{9 \times 10^9 \times (4.2) \times 10^{-3}}{0.1} = 0.376 \times 10^9$ ≈ Practical value.

Both electric field and potential will gradually decrease to zero as we extend to infinity (∞).