Assignment - 3. - Ajay Choudhury - 18018.

Q.11. Here,

$$A = \begin{bmatrix} 0 & 1 \\ -k/m & -b/m \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 1/m \end{bmatrix}$$

$$C = [1 \quad 0]$$

$$D = 0.$$

So, $C_{A,B} = [B \;, AB] = \begin{bmatrix} 0 & 1/m \\ 1/m & -b/m^2 \end{bmatrix} \{ \det(C_{A,B}) \neq 0 \}$.

implies the system is controllable.

open loop characteristics polynomial = $\det(sI - A)$

$$= \det\left(\begin{bmatrix} s & -1 \\ k/m & s+\frac{b}{m} \end{bmatrix}\right)$$

$$= s\left(s + \frac{b}{m}\right) + \frac{k}{m}$$

$$= s^2 + \frac{b}{m}s + \frac{k}{m}.$$

$$\Rightarrow a_n = \left(\frac{b}{m} \quad \frac{k}{m}\right) \qquad A_n = \begin{pmatrix} 1 & b/m \\ 0 & 1 \end{pmatrix}$$

Now, desired closed loop polynomial $= s^2 + 2\mu_s \omega_n s + \omega_n^2$

$$= s^2 + 1.54s + 1.21$$

(when $\omega_n = 1.1$ & $\mu_s = 0.7$)

$\Rightarrow \alpha = \begin{pmatrix} 1.54 \\ 1.21 \end{pmatrix}^T.$

Thus $K = (\alpha - a_A)\left(\cancel{A_A}\right)\left(\frac{1}{C_{A,B}}\right)$

$$K_r = \frac{-1}{C(A-BK)^{-1}B}$$

D.12. From the augmented system.

$$A_1 = \begin{pmatrix} 0 & 1 & 0 \\ -k/m & -b/m & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} B \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1/m \\ 0 \end{pmatrix}$$

$t_r = 1.5 \quad \zeta = 0.7$

$$\therefore C_{A_1B_1} = [B_1, \; A_1B_1, \; A_1^2B_1] = \begin{pmatrix} 0 & 1/m & -b/m^2 \\ 1/m & -b/m^2 & \frac{b^2-k}{m^2} \\ 0 & 0 & -1/m \end{pmatrix}$$

here $\det(C_{A_1B_1}) \neq 0$

$\Rightarrow$ system is controllable.

open loop characteristics polynomial $= \det(sI - A_1)$.

$$= \det\begin{pmatrix} s & -1 & 0 \\ k/m & s+b/m & 0 \\ 1 & 0 & s \end{pmatrix} = s^3 + b/m\, s^2 + \frac{ks}{m}$$

$$\Rightarrow a_m = \begin{pmatrix} \frac{b}{m} & \frac{k}{m} & 0 \end{pmatrix}, \quad A_{A_1} = \begin{pmatrix} 1 & b/m & k/m \\ 0 & 1 & b/m \\ 0 & 0 & 1 \end{pmatrix}$$

desired close loop polynomial $= (s^2 + 2\xi_s \omega_n s + \omega_n^2)(s + P_1)$

$$= s^3 + (P_1 + 2\xi_s \omega_n)s^2 + (\omega_n^2 + 2\xi_s \omega_n P_1)s + \omega_n^2 P_1$$

$$\Rightarrow \alpha = \left((P_1 + 2\xi_s \omega_n),\ (\omega_n^2 + 2\xi_s \omega_n P_1),\ \omega_n^2 P_1 \right).$$

Hence,

$$K_1 = (\alpha - a_{A_1})\left(\frac{1}{A_{A_1}}\right)\left(C_{A_1B_1}^{-1}\right)$$

$$K_I = K_1(3)$$

$$K = K_1(1:2)$$