# Assignment - 1.

- Ajay Choudhury
-18018.

D.2. The kinetic energy of the mass spring damper system will be:

$$K = \frac{1}{2} m (\dot{z})^2$$

Since, the motion is only in one direction i.e z.

So, position of the system = $(z \; 0 \; 0)$

$$\therefore \boxed{K = \frac{1}{2} m(\dot{p}^T \dot{p}) = \frac{1}{2} m \dot{z}^2} \quad \text{——— (1)}$$

D.3.

Potential energy (PE) = $\frac{1}{2} K z^2$

Kinetic energy (KE) = $\frac{1}{2} m \dot{z}^2$. (from (1))

Now, we know Lagrangian = KE - PE.

$$= \frac{1}{2}(m\dot{z}^2 - Kz^2)$$

Now, we can get,

$$m\dot{z} = \frac{\partial L}{\partial \dot{z}} \quad \text{and} \quad -Kz = \frac{\partial L}{\partial z}$$

So, we know,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}}\right) - \frac{\partial L}{\partial z} = \dot{\tau} - B\dot{q}.$$

So replacing the above equation with found values,

$$\frac{d}{dt}(m\dot{z}) + Kz = f - b\dot{z}.$$

or, $\boxed{m\ddot{z} + Kz = f - b\dot{z}}$ ——— (2)

D4.

Here,

$$x = \begin{pmatrix} z \\ \dot{z} \end{pmatrix} \Rightarrow \dot{x} = \begin{pmatrix} \dot{z} \\ \ddot{z} \end{pmatrix}$$

from eqn (2), we get,

$$\ddot{z} = \frac{f - b\dot{z} - kz}{m}$$

$$\Rightarrow \dot{x} = \begin{pmatrix} \dot{z} \\ \frac{f - b\dot{z} - kz}{m} \end{pmatrix}$$

So, at equilibrium, $\dot{z} = 0$.

$\therefore \ddot{z} = 0$.

$\Rightarrow f - b\dot{z} - kz = 0$

or, $f - kz = 0$

or, $f = kz$ (at equilibrium).