Assignment - 2

-Ajay Choudhury
-18018.

D7.

(a) $$H(s) = \frac{1/m}{s^2 + \frac{b}{m}s + \frac{K}{m}}$$

$b_0 = 1/m$ $m = 5.7$ kg

$a_1 = b/m$ $K = 3.74$ N/m

$a_0 = K/m$ $b = 0.57$ N-s/m

$$P(s) = \frac{b_0}{s^2 + a_1 s + a_0}$$

$$\Delta P_L(s) = s^2 + \frac{b}{m}s + \frac{K}{m}$$

$$pol = -\frac{b}{2m} \pm \sqrt{\left(\frac{b}{2m}\right)^2 - \left(\frac{K}{m}\right)}$$

$$= -\frac{0.57}{11.4} \pm \sqrt{0.0025 - 6.56140}$$

$$= -0.05 \pm \sqrt{-6.5589}.$$

$$= -0.05 \pm 2.56103i$$

Hence the open loop poles of the system : $\boxed{-0.05 \pm 2.56103i}$

$$Z(s) = \left(\frac{1/m}{s^2 + \frac{b}{m}s + \frac{K}{m}}\right)\left[K_p\left(Z_r - Z\right) - K_D s Z(s)\right]$$

$$Z(s) = \frac{\frac{1}{m}K_p}{s^2 + \left(\frac{b}{m} + \frac{1}{m}K_D\right)s + \left(\frac{K}{m} + \frac{1}{m}K_p\right)} Z_r(s)$$

Hence

$K/m = 0.85614$

$b/m = 0.1$

$1/m = 0.17543.$

while tracking error when the input is a ramp or higher order polynomial is ∞.

$\dot{y}(t)$ & $r(t)$ diverge at $t \to \infty$.

adding integrator.

$$u(s) = \frac{1/m}{s\left(s^2 + \frac{b}{m}s + \frac{K}{m}\right)} \left(K_D s^2 + K_p s + K_I\right)$$

→ The system type for reference tracking for this system is type 1.

$$P(s)\ L(s) = \frac{(1/m)\left[K_D s^2 + K_p s + K_I\right]}{s\left[s^2 + \frac{b}{m}s + \frac{k}{m}\right]}$$

→ Has one for integrator

→ tracking error when the input step is zero and the tracking error when the input is a ramp.

$$\lim_{t\to\infty} e(t) = \frac{1}{K_v} = \frac{1}{\lim_{s\to 0} s\,P(s)\,L(s)} = \frac{1}{K_I/K} = \boxed{\frac{K}{K_I}}$$

→ when it is parabola $y(t)$ and t diverges as $t \to \infty$

D.8. (a) $\omega_n = \frac{2.2}{2} \Rightarrow \omega_n = 1.1$

$$\Delta_{a}^{d} = s^2 + 2\zeta\omega_n s + \omega_n^2.$$

$$\boxed{\Delta_{a}^{d} = s^2 + 1.54s + 1.21}$$

$$\Delta_a(s) = s^2 + (0.1 + 0.175 K_D)s + (0.656 + 0.175 K_P)$$

$$0.1 + 0.175 K_D = 1.545$$

$$\Rightarrow K_D = \frac{1.545 - 0.1}{0.175} = 8.257$$

and, $0.656 + 0.175 K_P = 1.21$

or, $K_P = \frac{1.21 - 0.656}{0.175} = 3.165$

D.9. $P(s)\,C(s) = \dfrac{1/m}{s\left(s + \frac{b}{m} + \frac{k}{ms}\right)} (K_D s + K_P)$ (open loop system)

do not have any free integrator thus the system is type 0.

The tracking error when the input is a step is

$$\lim_{t\to\infty} e(t) = \frac{1}{1 + N_P} = \frac{1}{1 + \lim_{s\to 0} P(s)C(s)} = \frac{1}{1 + \frac{K_P}{K}}$$

$$= \frac{k}{k+k_p}$$

$\frac{k}{m} = 0.65614$

$\frac{b}{m} = 0.1$

$1/m = 0.17543$

(b) $$P_{cl}(s) = s^2 + \left(\frac{b}{m} + \frac{1}{m}k_D\right)s + \left(\frac{k}{m} + \frac{1}{m}k_P\right)$$

$$P_{cl}(s) = -\frac{(b/m) + 1/m\, k_D}{2} \pm \sqrt{\left[\frac{(b/m + 1/m\, k_D)}{2}\right]^2 - \left(\frac{k}{m} + \frac{1}{m}k_P\right)}$$

$$P_{cl}(s) = -\frac{(0.1) + 0.175\,k_D}{2} \pm \sqrt{\left[\frac{0.1 + 0.175\,k_D}{2}\right]^2 - \left(0.656 + 0.175\,k_D\right)}$$

$$= -\frac{(0.1) + 0.175\,k_D}{2} \pm \sqrt{(0.05 + 0.0875\,k_D) - (0.656 + 0.175\,k_D)}$$

$$= -((0.05) + 0.0875\,k_D) \pm \sqrt{(0.05 + 0.0875\,k_D) - (0.656 + 0.175\,k_D)}$$

(c) $P_1 = -1$ & $P_2 = -1.5$.

$$\Delta_{cl}^{d} = (s+1)(s+1.5)$$

$$s^2 + (0.1 + 0.175\,k_D)s + (0.656 + 0.175\,k_P) = s^2 + 2.5s + 1.5$$

$$0.1 + 0.175\,k_D = 2.5 \Rightarrow k_D = 13.714$$

$$0.656 + 0.175\,k_P = 1.5 \Rightarrow k_P = 4.822$$

(b) Input disturbance (without integrator)

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0}\left[\frac{\left(\frac{1/m}{s^2+K/m\,s+K/m}\right)}{1+\left(\frac{1/m}{s^2+\frac{b}{m}s+\frac{K}{m}}\right)(K_D s+K_P)}\right] = \frac{1}{s^q}$$

$$\Rightarrow \lim_{t\to\infty} e(t) = \frac{A}{K+K_P} \qquad \boxed{q=0}$$

(with integrator)

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0}\left(\frac{\left(\frac{1/m}{s^2+b/m\,s+\frac{K}{m}}\right)}{1+\left(\frac{1/m}{s^2+\frac{b}{m}s+\frac{K}{m}}\right)\left(\frac{K_D s^2+K_P+K_I}{s}\right)}\right)\frac{A}{s^q}$$

$$\Rightarrow \left(\frac{\frac{1/m}{\cancel{\left(s^2+\frac{b}{m}s+\frac{K}{m}\right)}}}{\frac{s\left(s^2+\frac{b}{m}s+\frac{K}{m}\right)+\frac{1}{m}\left(K_D s^2+K_P s+K_I\right)}{\cancel{\left(s^2+\frac{b}{m}s+\frac{K}{m}\right)}}}\right)\frac{A}{s^q}$$

$$\frac{1/m}{1}\;\frac{s}{s\left(s^2+\frac{b}{m}s+\frac{K}{m}\right)+\frac{1}{m}\left(K_D s^2+K_P s+K_I\right)}\;\frac{A}{s^q}$$

$$\Rightarrow \frac{s/m}{K_I/m} = \boxed{\frac{A}{K_I}} \quad \text{if } q=1$$