Project 1

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PROBLEM P1

1.2

Given the linear system which can yield $d_1, ..., d_{N-1}$ in terms of $x_0, ..., x_N$,

$$\begin{bmatrix} \frac{7}{2} & 1 & & & \\ 1 & 4 & 1 & & 0 \\ & \ddots & \ddots & \ddots & \\ 0 & & 1 & 4 & 1 \\ & & & 1 & \frac{7}{2} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_{N-2} \\ d_{N-1} \end{bmatrix} = \begin{bmatrix} 6x_1 - \frac{3}{2}d_0 \\ 6x_2 \\ \vdots \\ 6x_{N-2} \\ 6x_{N-1} - \frac{3}{2}d_N \end{bmatrix}$$

When evaluated and simplified, yields the following equations,

$$\begin{aligned} \frac{1}{4}d_0 + \frac{7}{12}d_1 + \frac{1}{6}d_2 &= x_1 \\ \frac{1}{6}d_1 + \frac{4}{6}d_2 + \frac{1}{6}d_3 &= x_2 \\ &\vdots \\ \frac{1}{6}d_{N-3} + \frac{4}{6}d_{N-2} + \frac{1}{6}d_{N-1} &= x_{N-2} \\ \frac{1}{6}d_{N-2} + \frac{7}{12}d_{N-1} + \frac{1}{4}d_N &= x_{N-1}. \end{aligned}$$

These x's correspond to every 1^{st} and 4^{th} control point of all cubic bezier curve segments derived from the de Boor control points, e.g. x_1 and x_2 correspond to b_0^2 and b_3^2, x_2 and x_3 correspond to b_0^3 and b_3^3, and x_i and x_{i+1} correspond to b_0^{i+1} and b_3^{i+1}, etc.

Furthermore, these points $x_1, ..., x_n$ can be used to compute $d_1, ..., d_{N-1}$, because over the course of deCasteljau's algorithm, these points remain constant. All of the other control points in between become forgotten with each subdivision. As a result, these points are guaranteed to remain on the polygonal line approximating the curve regardless of the depth of recursion.