Project 2

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PROBLEM 2

Applying the `haar` function to the various strings w, yields a similar waveform across the different variations, whereby the lower index coefficients are identical and the higher index coefficients are the only ones to change. This is consistent with the notion that the coefficient vector produced by the haar transform captures coarse information at the lower index coefficients and fine information at the higher index coefficients. When the same vector is concatenated on itself multiple times, intuitively the coarser features of the signal would remain the same, whereas the higher frequency details may have changed.

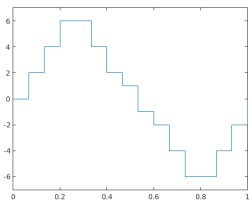

Figure 1: Original u.

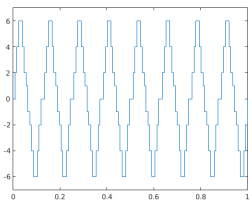

Figure 2: Concatenation of u with itself 8 times, w.

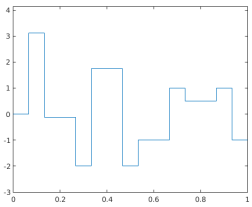

Figure 3: Haar Transform of original u.

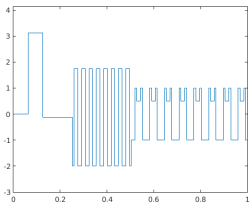

Figure 4: Haar Transform of w.

PROBLEM 3

Once $k = 4$, zeros begin to appear in the coefficient vector c. For $k = 4,5,6,7$, the output of the transform, c, no longer changes. This suggests that despite the number of rounds of averaging and differencing, each vector has an upper bound of compression. In other words, a point is reached where an input vector u can not be compressed further with more averaging and differencing.

The result of `haar(handel, 1)` is the original song becomes duplicated and each iteration is half as long as the original. Furthermore, the overall amplitude of the second iteration seems attenuated with respect to the first iteration. When $k = 2$, the same effect takes place on the first iteration of the song from the output of $k = 1$. Moreover, this yields a total of three iterations of the song whereby the first two iterations are of the same length and the third iteration being twice as long as both. The same attenuation occurs to the second iteration as compared to the first iteration. Finally, for each $k > 2$, the same thing occurs on the first instance of the song which lives in the output of the previous $k-1$ steps. For $k = 3$, there are five iterations of the song.

After zeroing out the detail coefficients and reconstructing the original signal, the song sounds much more grainy with static noise introduced to it.

PROBLEM 4

The third entry in the second row of the P of Ames' paper, is incorrect. This value should be 1152 instead of 1156.

After reconstructing `Xdurer` from its haar coefficients, it is indistinguishable from the original image.

PROBLEM 5

$$A = \begin{pmatrix} 100 & 103 & 99 & 97 & 93 & 94 & 78 & 73 \\ 102 & 97 & 100 & 111 & 113 & 104 & 96 & 82 \\ 99 & 109 & 104 & 95 & 93 & 92 & 88 & 76 \\ 114 & 104 & 99 & 102 & 93 & 82 & 74 & 74 \\ 96 & 91 & 91 & 87 & 79 & 78 & 77 & 76 \\ 90 & 88 & 83 & 78 & 77 & 74 & 76 & 76 \\ 92 & 81 & 73 & 72 & 69 & 65 & 66 & 62 \\ 75 & 70 & 69 & 65 & 60 & 55 & 61 & 65 \end{pmatrix} \quad C = \begin{pmatrix} 682 & 52 & 15 & 21 & 6 & 3 & 8 & 8 \\ 78 & 6 & -8 & 22 & -5 & -4 & 2 & 8 \\ 8 & -14 & -8 & 7 & 1 & -5 & -1 & 2 \\ 38 & -1 & -3 & 2 & -3 & 1 & -2 & 0 \\ -17 & 12 & 7 & -1 & -4 & 6 & -5 & -5 \\ 4 & -10 & -3 & -2 & -10 & 6 & -5 & 6 \\ 8 & 5 & -3 & 2 & 1 & -0 & -1 & 0 \\ 15 & 5 & 6 & 6 & 3 & -1 & -0 & 4 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 100 & 100 & 95 & 95 & 92 & 92 & 76 & 76 \\ 103 & 103 & 98 & 98 & 106 & 106 & 90 & 90 \\ 99 & 109 & 99 & 99 & 96 & 96 & 81 & 81 \\ 114 & 104 & 104 & 104 & 91 & 91 & 76 & 76 \\ 91 & 91 & 86 & 86 & 76 & 76 & 76 & 76 \\ 91 & 91 & 86 & 86 & 76 & 76 & 76 & 76 \\ 82 & 82 & 76 & 76 & 66 & 66 & 66 & 66 \\ 74 & 74 & 69 & 69 & 58 & 58 & 59 & 59 \end{pmatrix} \quad C_2 = \begin{pmatrix} 682 & 51 & 14 & 22 & 0 & -0 & -0 & -0 \\ 78 & 1 & -0 & 22 & 0 & -0 & -0 & -0 \\ -0 & -14 & -0 & 1 & -0 & -0 & -0 & -0 \\ 38 & -1 & -1 & 0 & -0 & -0 & -0 & -0 \\ -17 & 11 & -0 & 0 & -0 & -0 & -0 & -0 \\ -0 & -10 & -0 & -0 & -10 & -0 & -0 & -0 \\ -0 & -0 & -0 & -0 & -0 & -0 & -0 & -0 \\ 15 & -0 & 1 & 1 & -0 & -0 & -0 & -0 \end{pmatrix}$$