Distributed Calibration of Time Domain UWB Ranging Radios

PulsON® 400 Series

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This paper discusses a method of measurement of the intrinsic Time of Flight (TOF) delay between antenna and internal timing electronics of a UWB ranging radio. This technique uses many (at least 3) radios and isolates the bias in each through a system of equations.

Introduction

Time Domain radios estimate distance through measurement of the bidirectional Time of Flight (TOF) between requesting and responding devices. In order to adjust for the TOF delay between antenna and radio electronics each radio is pre-programmed with an internal delay estimate representing the delay between the radio's internal timer/sampler and the face of the antenna. The resulting range calculation is adjusted by the "Electrical Delays" of both requester and responder.

With only two radios the bias errors may be applied to one or averaged across both. However when a system of radios is used this technique allows estimation of individual errors in each radio.

This paper describes a method of estimating the unique electrical delay biases of each radio in a group of *n* radios by solving a system of *n(n-1)* equations, one equation for each measured link or edge between nodes. The error estimates are solved through linear least-squares regression.

Approach

Each range measurement in a system of radios is modeled as a combination of the true distance, with the addition of two error factors. Thus for each link between radio *i* to radio *j* in a system of *n* radios

$$r_{i,j} = d_{i,j} + \epsilon_i + \epsilon_j, \qquad \{1 \le i, j \le n, i \ne j\} \tag{1}$$

where:

$r_{i,j}$ is the UWB-estimated range between antennas of nodes *i* and *j*,

$d_{i,j}$ is the true range between antennas of nodes *i* and *j*, &

ϵ_i & ϵ_j are the internal electrical delay biases of nodes *i* and *j*.

The goal is determination of ϵ_i and ϵ_j given known (manually measured) $d_{i,j}$ and a collection of (radio-measured) range measurements $r_{i,j}$. This set of equations is converted to matrix form and solved for the error vector *E*

$$E = X^{-1}[R - D] \tag{2}$$

where:

$E = [\epsilon_1\ \epsilon_2\ \dots\ \epsilon_n]^T$ is the column vector of *n* bias errors in each radio 1 to *n*,

$R = \left[r_{1,2}\ r_{1,3}\ \dots\ \epsilon_{n,n-1}\right]^T$ is the column vector of *l*=*n*(*n*-*1*) radio link measurements,

$D = \left[d_{1,2} d_{1,3}\ \dots\ d_{n,n-1}\right]^T$ is the column vector of *l*=*n*(*n*-*1*) manual measurements, and

X^{-1} is the *l x n* (#links by #nodes) "design" matrix mapping the nodes to the links.

Note with $n > 3$ the number of links, l, may exceed the number of nodes, n. In general, the more radios and more links used will produce more accurate results, spreading any residual statistical error across the entire system.

Waveform Collection Rate

The simplest system contains of only three radios with three links. The basis equations for simultaneous solution are

$$\begin{aligned} r_{1,2} &= d_{1,2} + e_1 + e_2 \\ r_{1,3} &= d_{1,3} + e_1 + e_3 \\ r_{2,3} &= d_{2,3} + e_2 + e_3 \end{aligned} \tag{3}$$

Conversion to matrix form reveals the design matrix X

$$\begin{bmatrix} r_{1,2} \\ r_{1,3} \\ r_{2,3} \end{bmatrix} = \begin{bmatrix} d_{1,2} \\ d_{1,3} \\ d_{2,3} \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \tag{4}$$

In matrix notation

$$R = D + XE \tag{5}$$

Solving for the error vector E

$$E = X^{-1}(R - D) \tag{6}$$

Note in the 3 node case X is square and can be inverted directly as in equation 6. However if the number of links is greater than the number of radios (as is the case when n>3) one may use the left-pseudo inverse $X^{-1} = (X^T X)^{-1} X$. Thus, the general solution is

$$E = (X^T X)^{-1} X(R - D) \tag{6}$$

Computing X^{-1} and expanding the system of equations reveals the working system of equations

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} .5 & .5 & -.5 \\ .5 & -.5 & .5 \\ -.5 & .5 & .5 \end{bmatrix} \left(\begin{bmatrix} r_{1,2} \\ r_{1,3} \\ r_{2,3} \end{bmatrix} - \begin{bmatrix} d_{1,2} \\ d_{1,3} \\ d_{2,3} \end{bmatrix} \right) \tag{7}$$

Thus the biases for radios 1, 2, and 3 are solved as errors e_1, e_2, and e_3, respectively based on manually measured link distances $d_{1,2}$, $d_{1,3}$, and $d_{2,3}$, and radio range measurements $r_{1,2}$, $r_{1,3}$, and $r_{2,3}$. In practice, once the system is configured, D is measured once and R is measured many times. Solving equation 7 for many sets of R radio measurements can provide a statistical mean and standard deviation for the biases E.

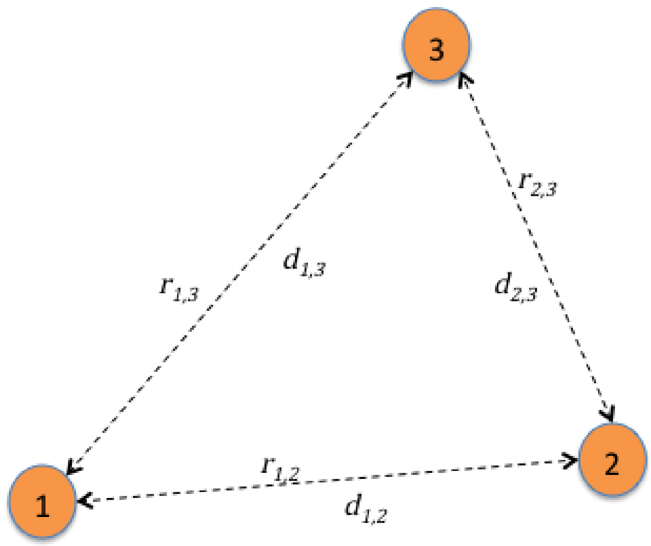

Fig. 1: A system of 3 nodes and 3 links. Manual link measurements are denoted $d_{i,j}$, and radio link measurements $r_{i,j}$