

Project 1

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Contribution of each group member:

Ajay Danda:

- Solved Q1 a
- Solved Q1 b] iv, v, vi
- Equal contribution in solving Q1 c
- Equal contribution in solving Q 2
- Equal contribution in Documenting the Report.

Satyam Bhikadiya:

- Solved Q1 b] i, ii, iii
- Equal contribution in solving Q1 c
- Equal contribution in solving Q 2
- Equal contribution in Documenting the Report.

Section 1:

(a) Use the above density function to analytically compute the probability that the lifetime of the satellite exceeds 15 years.

Answer:

Section 1:

(b) Use the following steps to take a Monte Carlo approach to compute $E(T)$ and $P(T > 15)$.

- i. Simulate one draw of the block lifetimes X_A and X_B . Use these draws to simulate one draw of the satellite lifetime T .
- ii. Repeat the previous step 10,000 times. This will give you 10,000 draws from the distribution of T . Try to avoid 'for' loop. Use 'replicate' function instead. Save these draws for reuse in later steps.
- iii. Make a histogram of the draws of T using 'hist' function. Superimpose the density function given above. Try using 'curve' function for drawing the density. Note what you see.
- iv. Use the saved draws to estimate $E(T)$. Compare your answer with the exact answer given above.
- v. Use the saved draws to estimate the probability that the satellite lasts more than 15 years. Compare with the exact answer estimated in part (a).
- vi. Repeat the above process of obtaining an estimate of $E(T)$ and an estimate of the probability four more times. Note what you see.

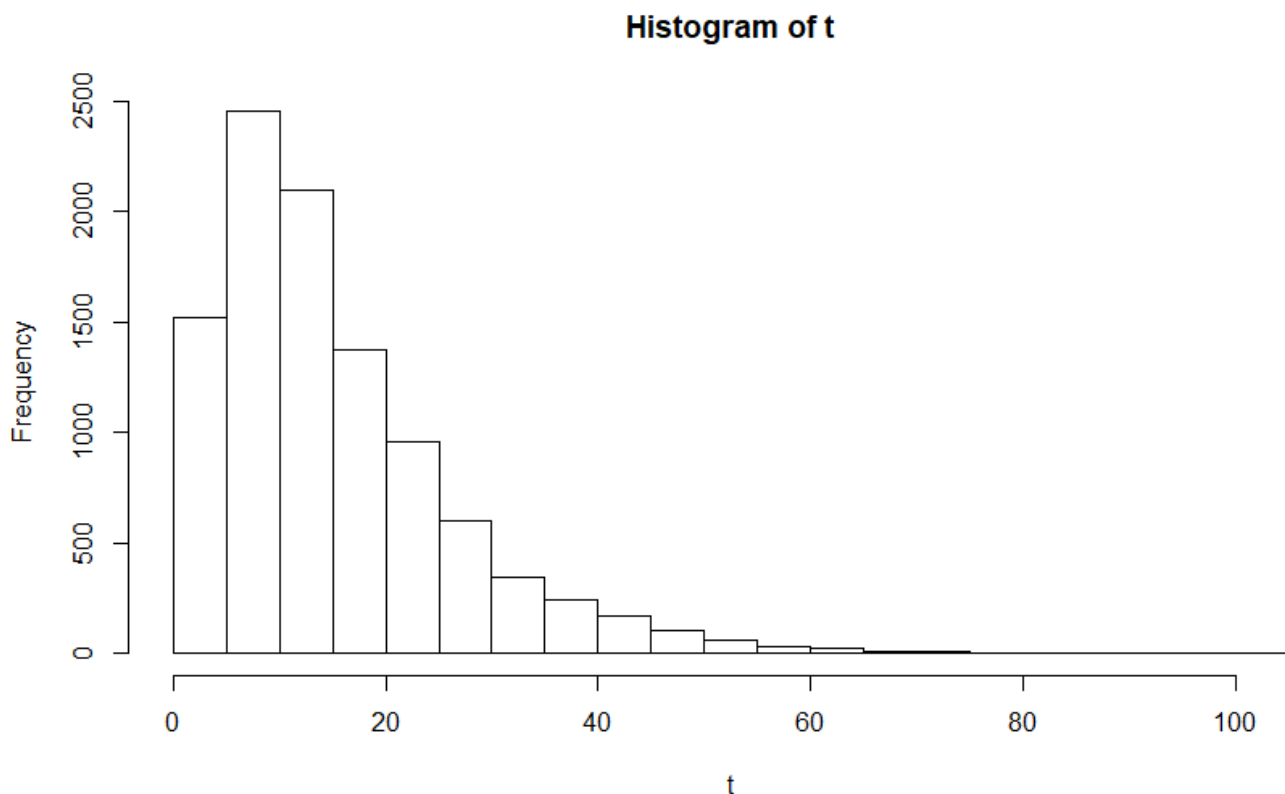
Answer:

Case 1: The R code for simulation at $n=10,000$ is as follows:

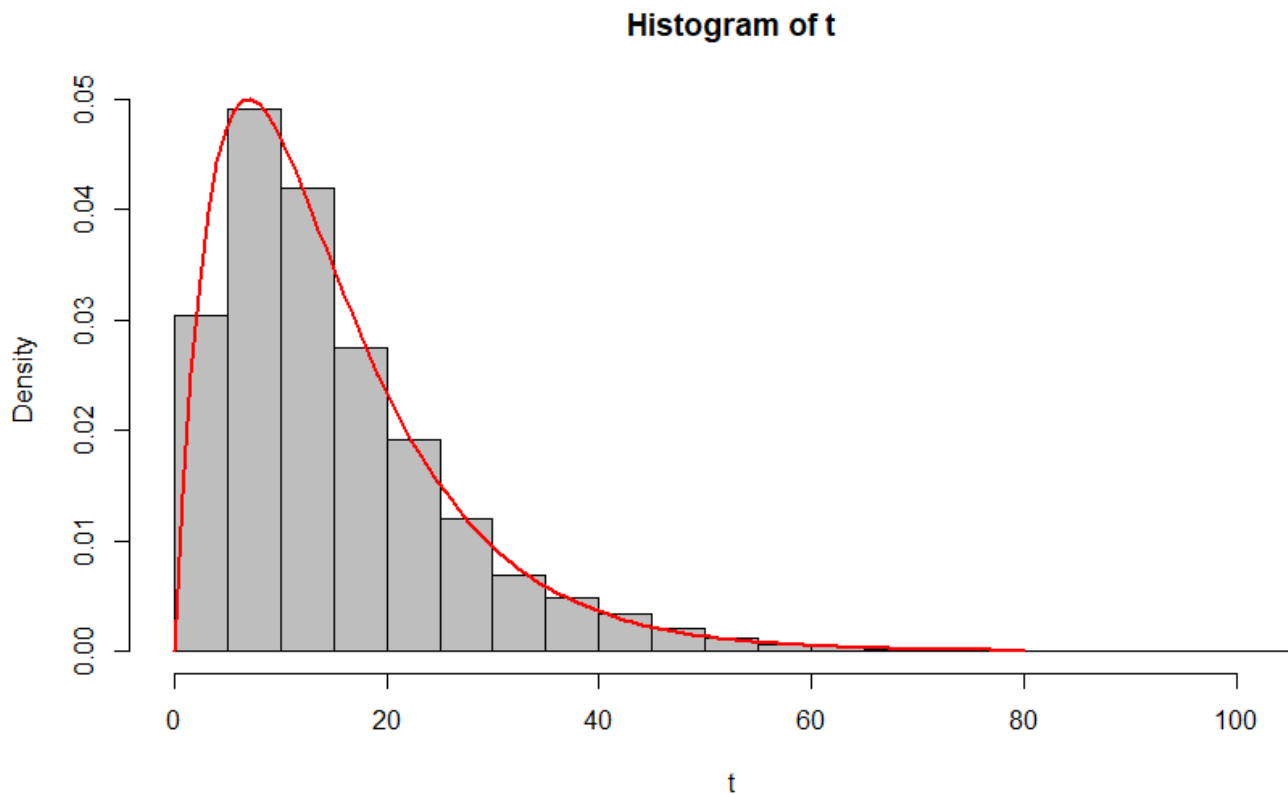
```
>t=replicate(10000,max(rexp(1,0.1),rexp(1,0.1)), simplify = "array" ) #replication of 2 random exponential 10000
> hist(t) # plotting the histogram of the same
> f=function(t){0.2*exp(-0.1*t)-0.2*exp(-0.2*t)} # Function of given probability density function of T
> I=integrate(f,15,Inf) # Integrating function to obtain probability that the satellite lasts more than 15 years.
> hist(t,prob=TRUE,col = "grey")
> hist(t,prob=TRUE,col = "grey", add=TRUE) # Plotting histogram to superimpose it with the curve later.
> curve(f,0,80,add=TRUE,col="red", lwd=2) # Plotting the curve of function f to superimpose it with the histogram.
> a=mean(t) # computing E(T) to compare it with the value of E(T) given.
```

1st Iteration outcomes are as follows:

Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



$E(T)$ and the probability that the satellite lasts more than 15 years:

EnvironmentHistoryConnections

Import Dataset

Global Environment

List

Data

I

List of 5

value : num 0.396

abs.error : num 5.13e-05

subdivisions: int 6

message : chr "OK"

call : language integrate(f = f, lower = 15, upper = Inf)

- attr(*, "class")= chr "integrate"

values

a

15.1190099747582

t

num [1:10000] 1.22 28.93 21.09 10 22.67 ...

Functions

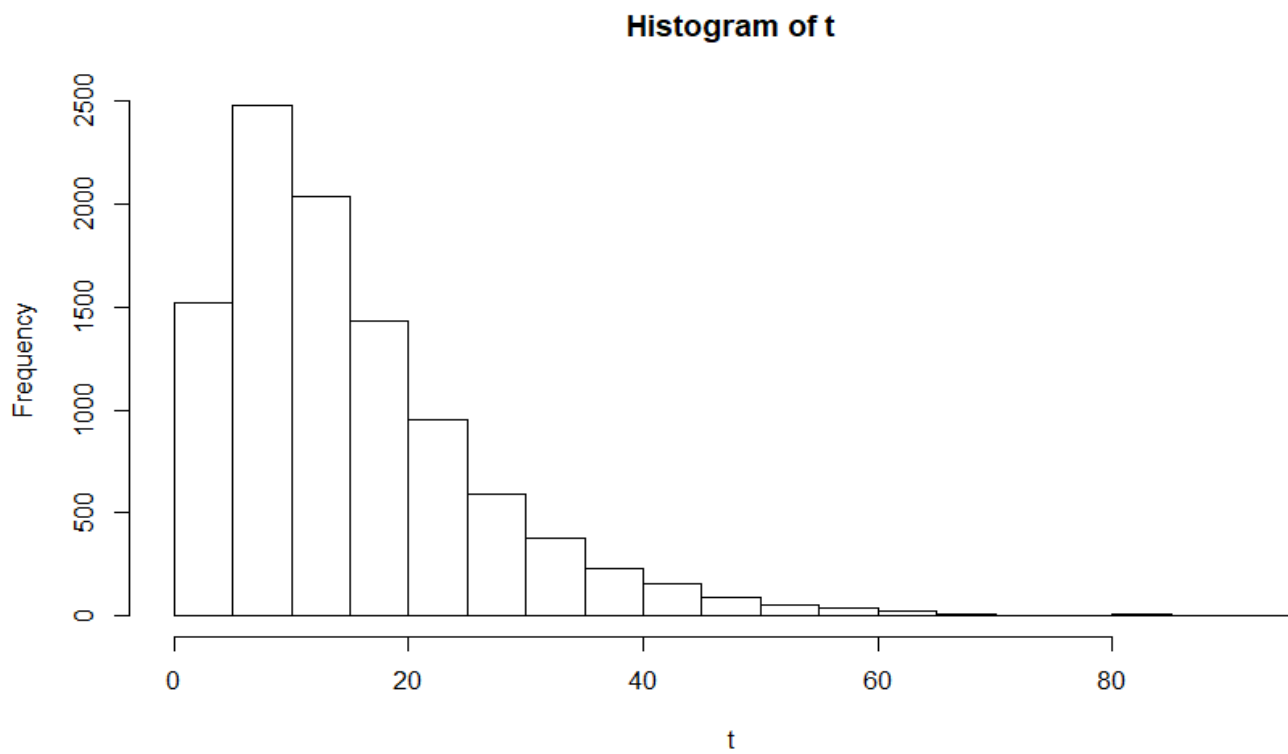
f

function (t)

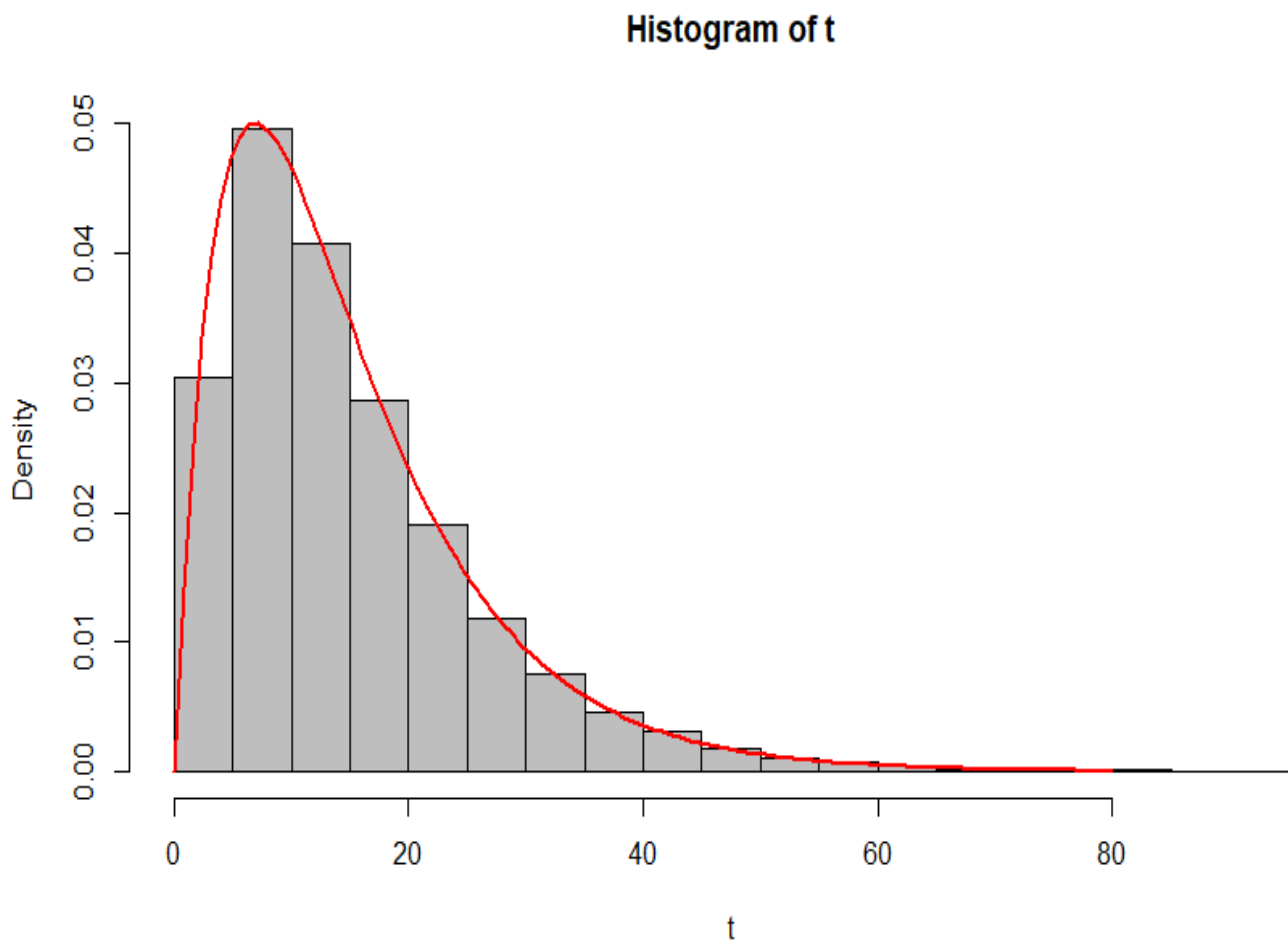
Observation: The actual mean value and the estimated mean value is very close to each other for the 1st iteration and the probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

2nd Iteration outcomes are as follows:

Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



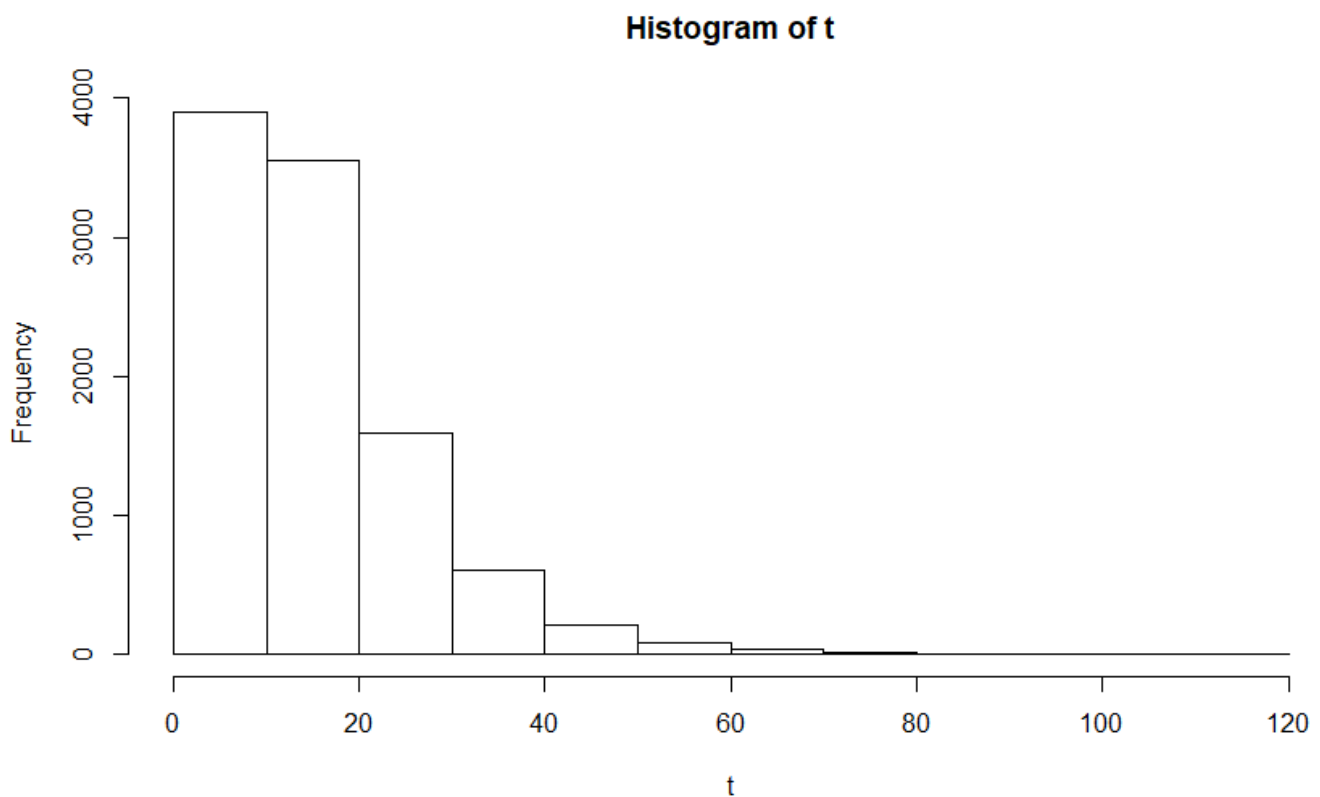
E(T) and the probability that the satellite lasts more than 15 years:

Project: (None)	
Environment	History
Connections	
Import Dataset	
Global Environment	
Data	
I	List of 5
value	num 0.396
abs.error	num 5.13e-05
subdivisions	int 6
message	chr "ok"
call	language integrate(f = f, lower = 15, upper = Inf)
	- attr(*, "class")= chr "integrate"
values	
a	15.0524337976861
t	num [1:10000] 48.16 18.09 19.14 3.69 12.17 ...
Functions	
f	function (t)

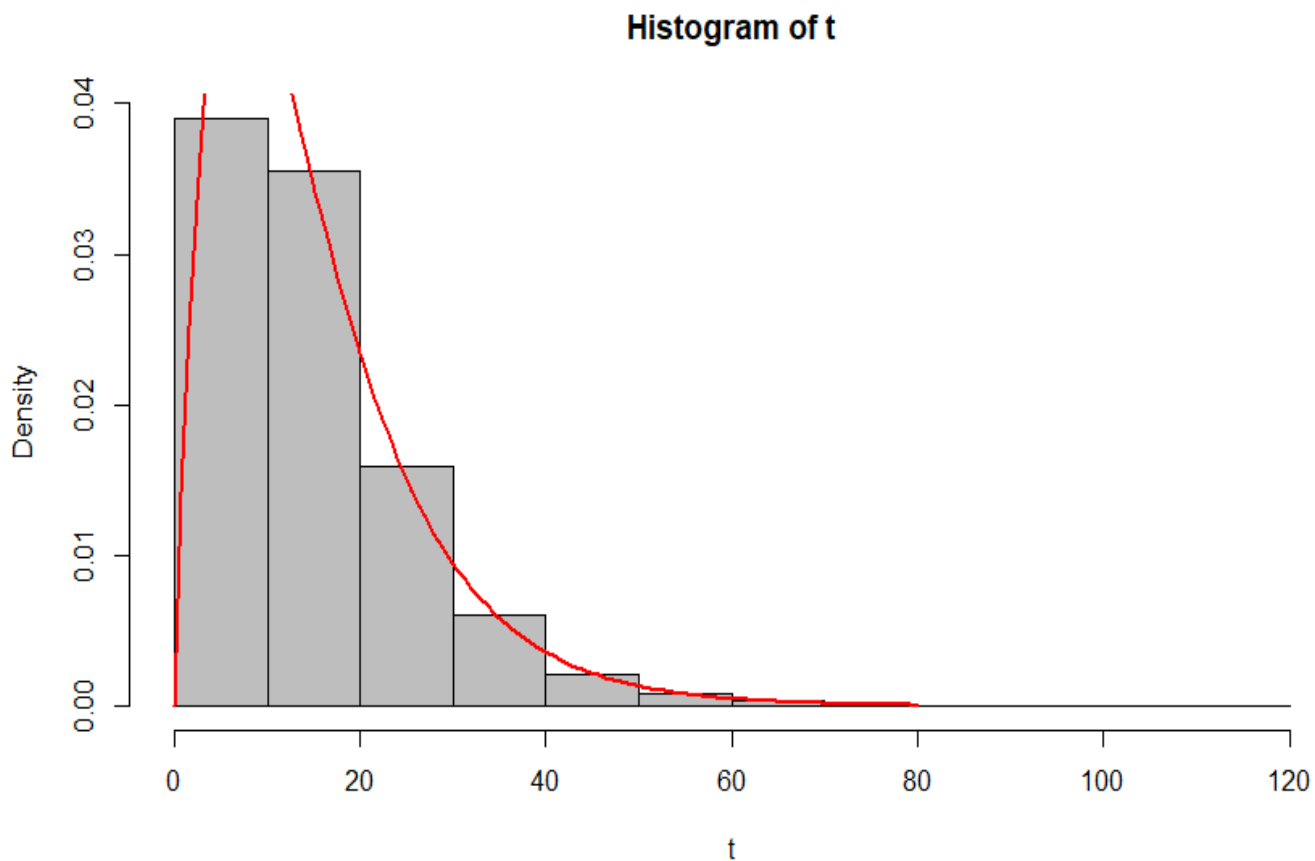
Observation: The actual mean value and the estimated mean value is very close to each other for the 2nd iteration and the probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

3rd Iteration outcomes are as follows:

Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



$E(T)$ and the probability that the satellite lasts more than 15 years:

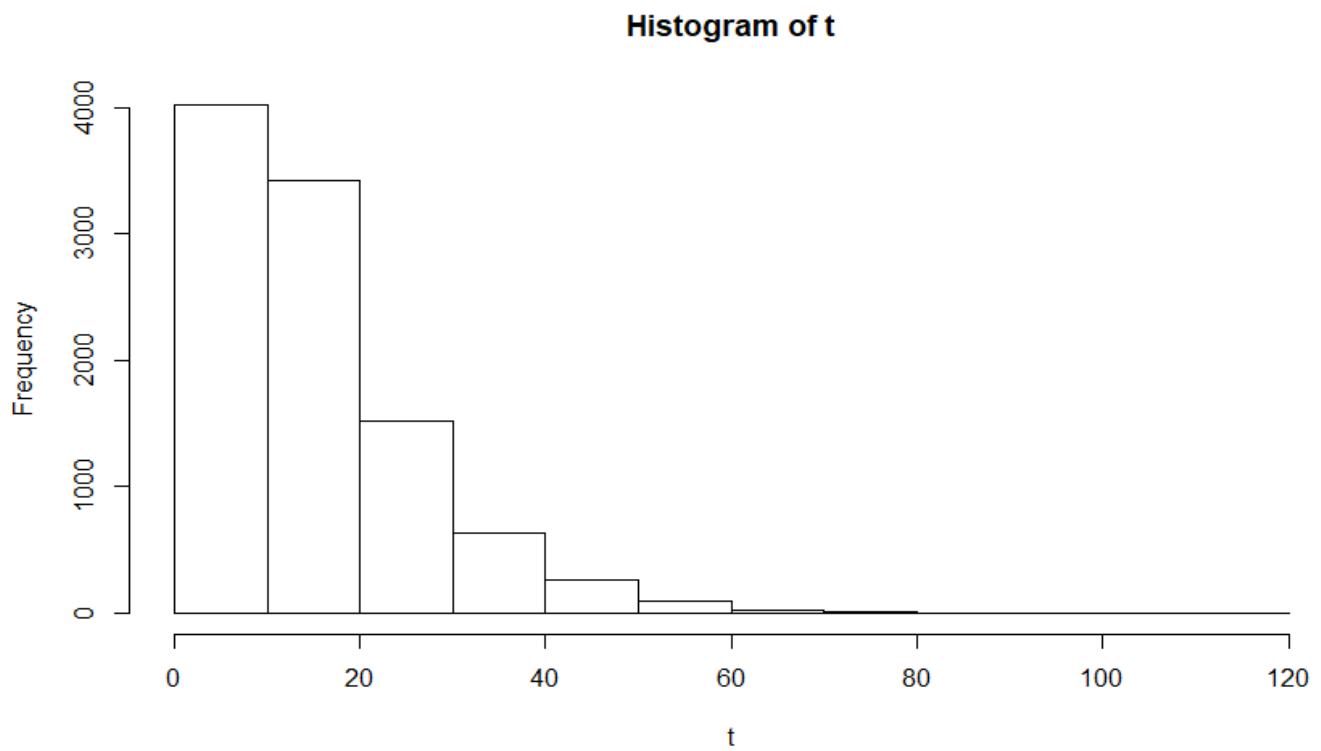
Project: (None) ▾

Environment		History	Connections
<div> <div> <div>📁</div> <div>📄</div> <div>📡 Import Dataset ▾</div> <div>🔧</div> </div> <div> <div>☰ List ▾</div> <div>🔄</div> </div> </div>			
Global Environment ▾ 🔍			
Data			
🔵 I	List of 5 🔍		
value :	num 0.396		
abs.error :	num 5.13e-05		
subdivisions:	int 6		
message :	chr "ok"		
call :	language integrate(f = f, lower = 15, upper = Inf)		
- attr(*, "class")=	chr "integrate"		
values			
a	15.064707754656		
t	num [1:10000] 8.2 2.36 3.6 1.84 19.25 ...		
Functions			
f	function (t) 📄		

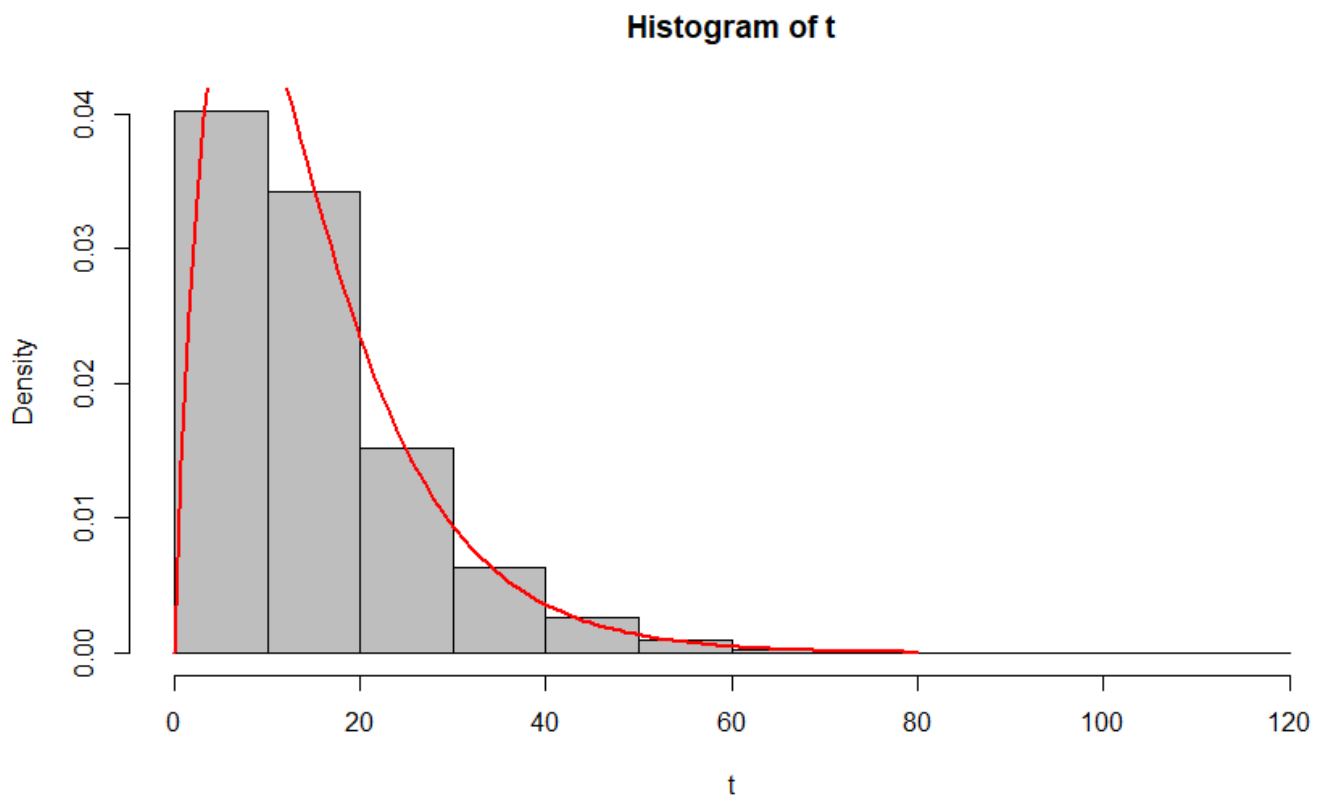
Observation: The actual mean value and the estimated mean value is very close to each other for the 3rd iteration and the probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

4th Iteration outcomes are as follows:

Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



E(T) and the probability that the satellite lasts more than 15 years:

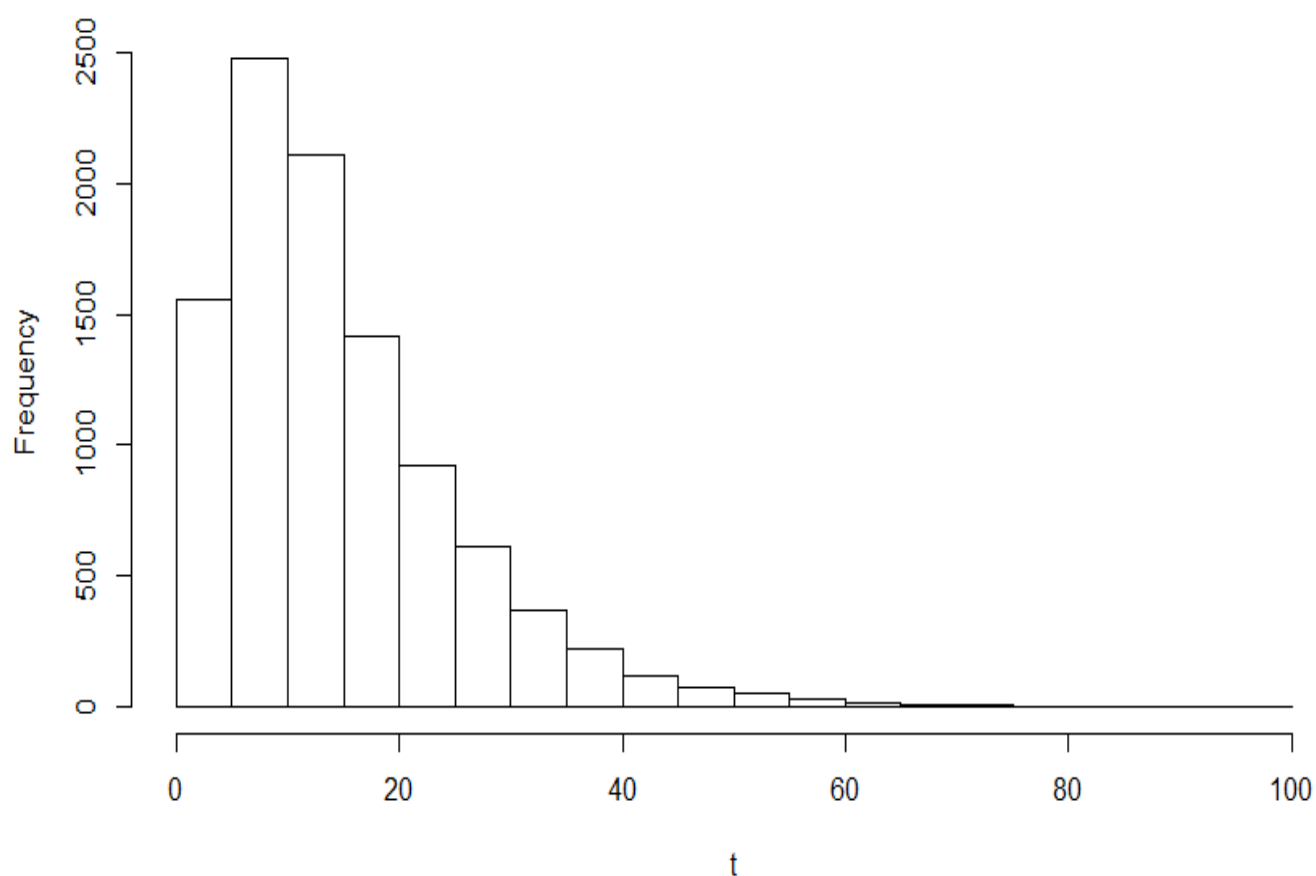
Project: (None)	
Environment	History
Connections	
Import Dataset	
Global Environment	
Data	
I	List of 5
value	num 0.396
abs.error	num 5.13e-05
subdivisions	int 6
message	chr "ok"
call	language integrate(f = f, lower = 15, upper = Inf)
	- attr(*, "class")= chr "integrate"
Values	
a	15.0875392425855
t	num [1:10000] 17.22 20.54 41.14 17.72 5.02 ...
Functions	
f	function (t)

Observation: The actual mean value and the estimated mean value is very close to each other for the 4th iteration and the probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

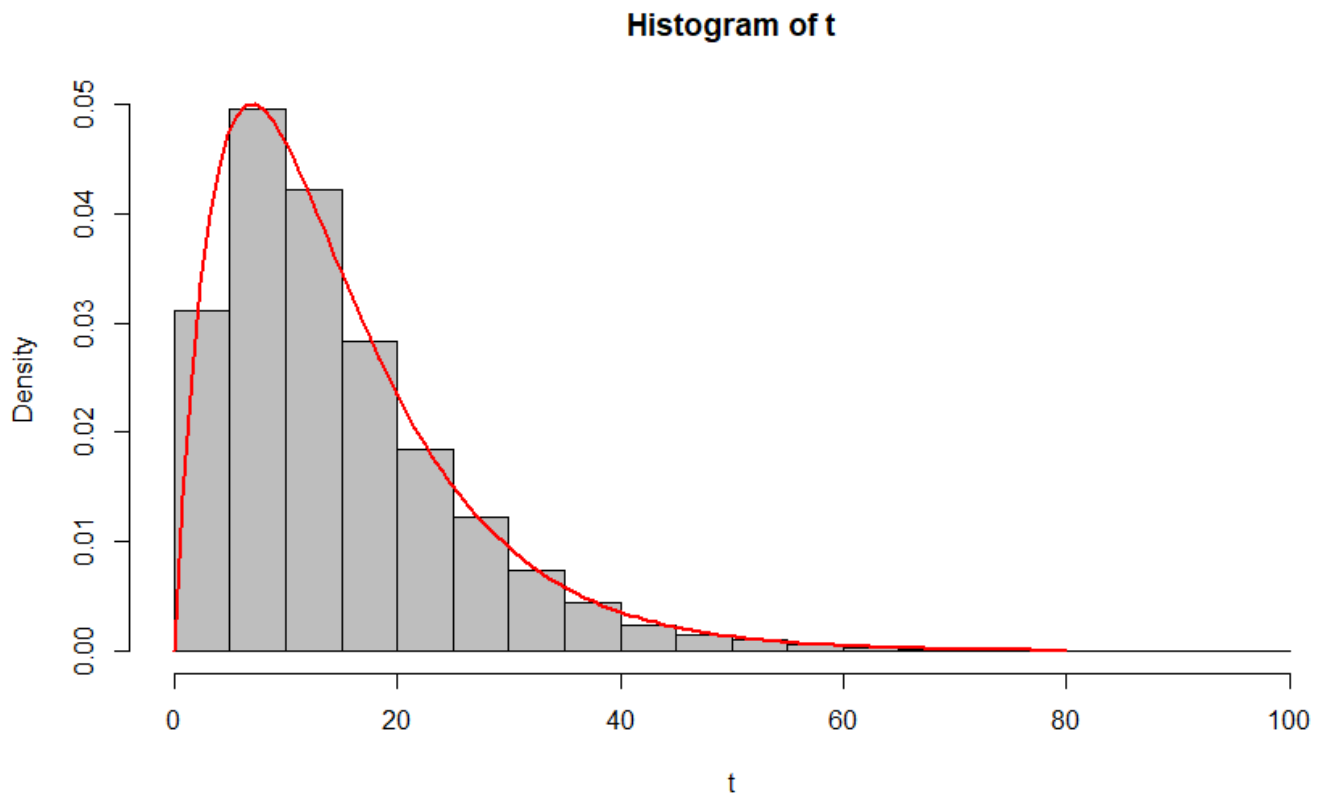
5th Iteration outcomes are as follows:

Histogram of the lifetime of a satellite (t):

Histogram of t



Density function and Histogram of the lifetime of a satellite (t):



$E(T)$ and the probability that the satellite lasts more than 15 years:

Project: (None) ▾

Environment History Connections

Import Dataset ▾

Global Environment ▾

Data

I List of 5

value : num 0.396

abs.error : num 5.13e-05

subdivisions: int 6

message : chr "OK"

call : language integrate(f = f, lower = 15, upper = Inf)

- attr(*, "class")= chr "integrate"

Values

a 14.7298275114027

t num [1:10000] 20.79 4.31 9.89 32.94 14.17 ...

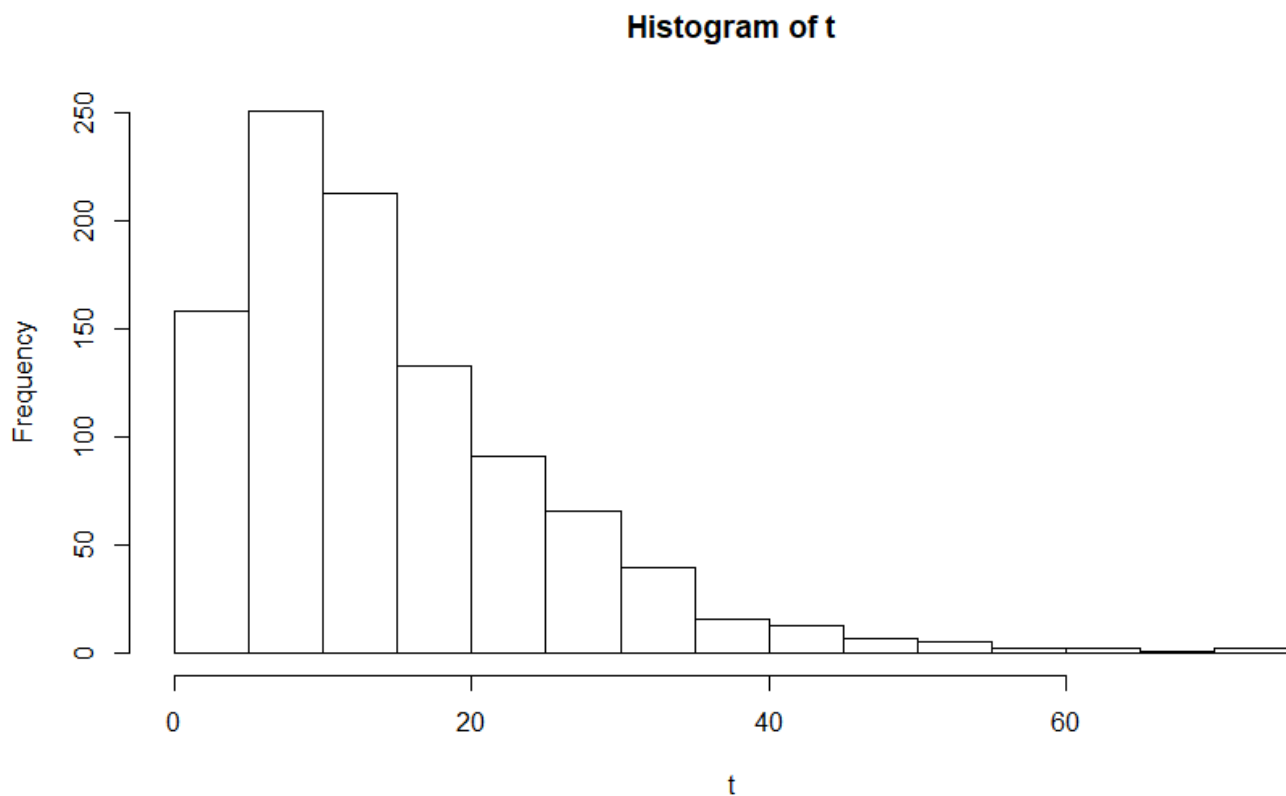
Functions

f function (t)

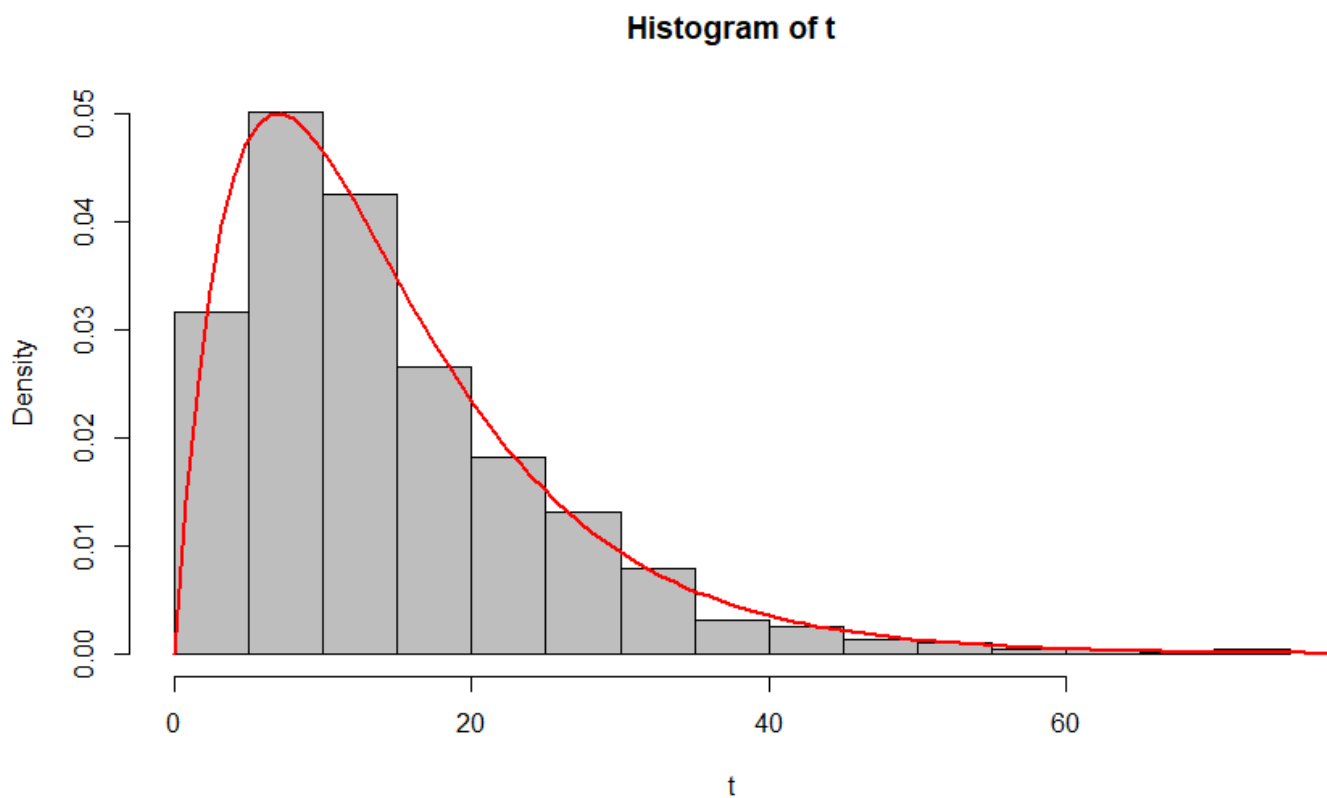
Case 2: The R code for simulation at $n=1,000$ is same as that of the above R code for 10,000. The only change is that replicate the values 1,000 times instead of 10,000 times.

1st Iteration outcomes are as follows:

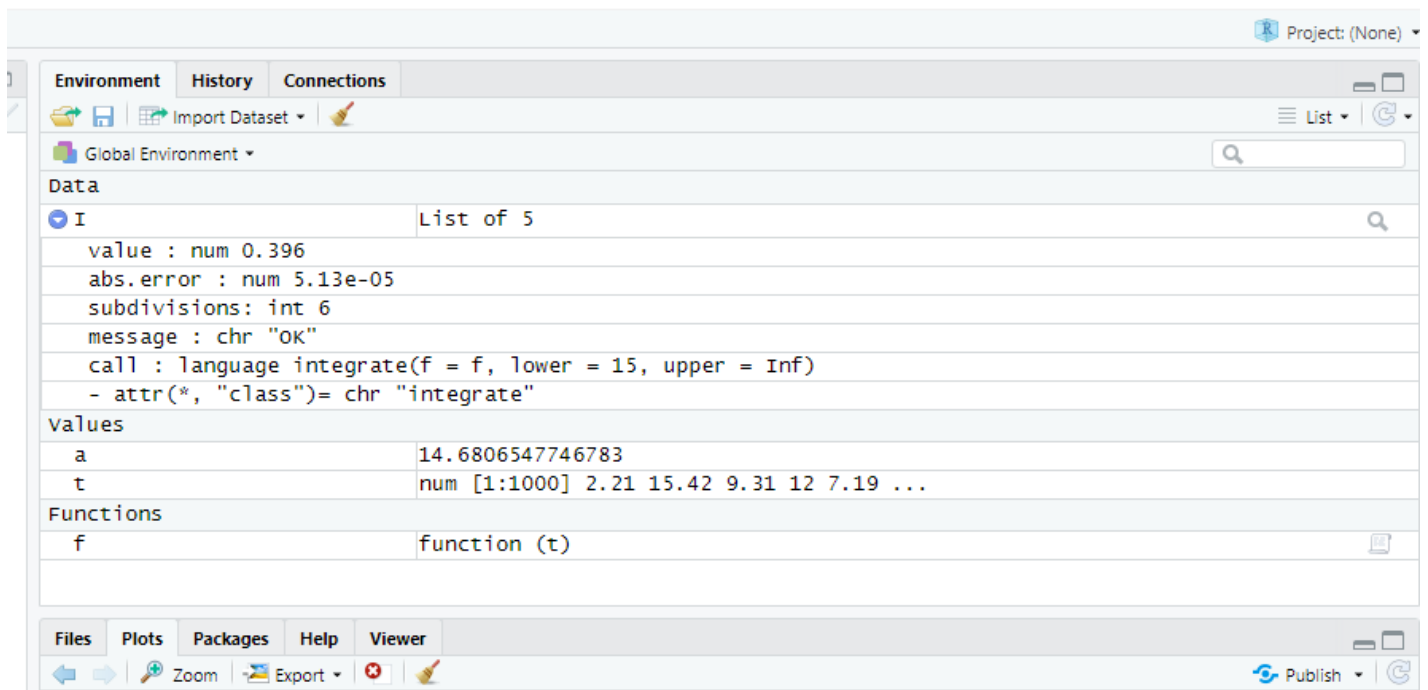
Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



$E(T)$ and the probability that the satellite lasts more than 15 years:



The RStudio Environment window displays the results of a function call. The variable 'I' is a list of 5 elements. The 'value' is 0.396, 'abs.error' is 5.13e-05, 'subdivisions' is 6, and 'message' is 'ok'. The 'call' shows the function 'language integrate(f = f, lower = 15, upper = Inf)' was executed. The 'values' section shows 'a' as 14.6806547746783 and 't' as a numeric vector of 1000 values starting with 2.21, 15.42, 9.31, 12, 7.19, etc. The 'Functions' section shows 'f' as a function of 't'.

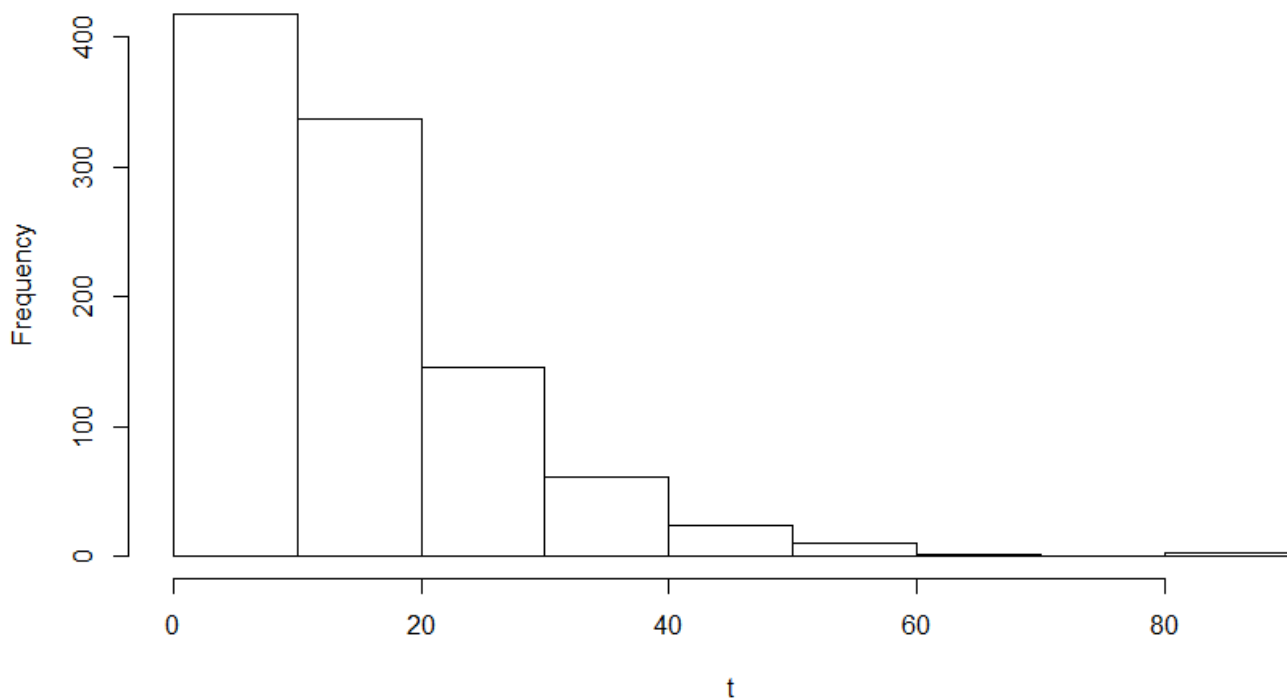
Environment	History	Connections
Global Environment		
Data		
I	List of 5	
value	num 0.396	
abs.error	num 5.13e-05	
subdivisions	int 6	
message	chr "ok"	
call	language integrate(f = f, lower = 15, upper = Inf)	
- attr(*, "class")	chr "integrate"	
Values		
a	14.6806547746783	
t	num [1:1000] 2.21 15.42 9.31 12 7.19 ...	
Functions		
f	function (t)	

Observation: The actual mean value and the estimated mean value are not that close to each other for the 1st iteration as compared to how close they were when $n=10,000$. The probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

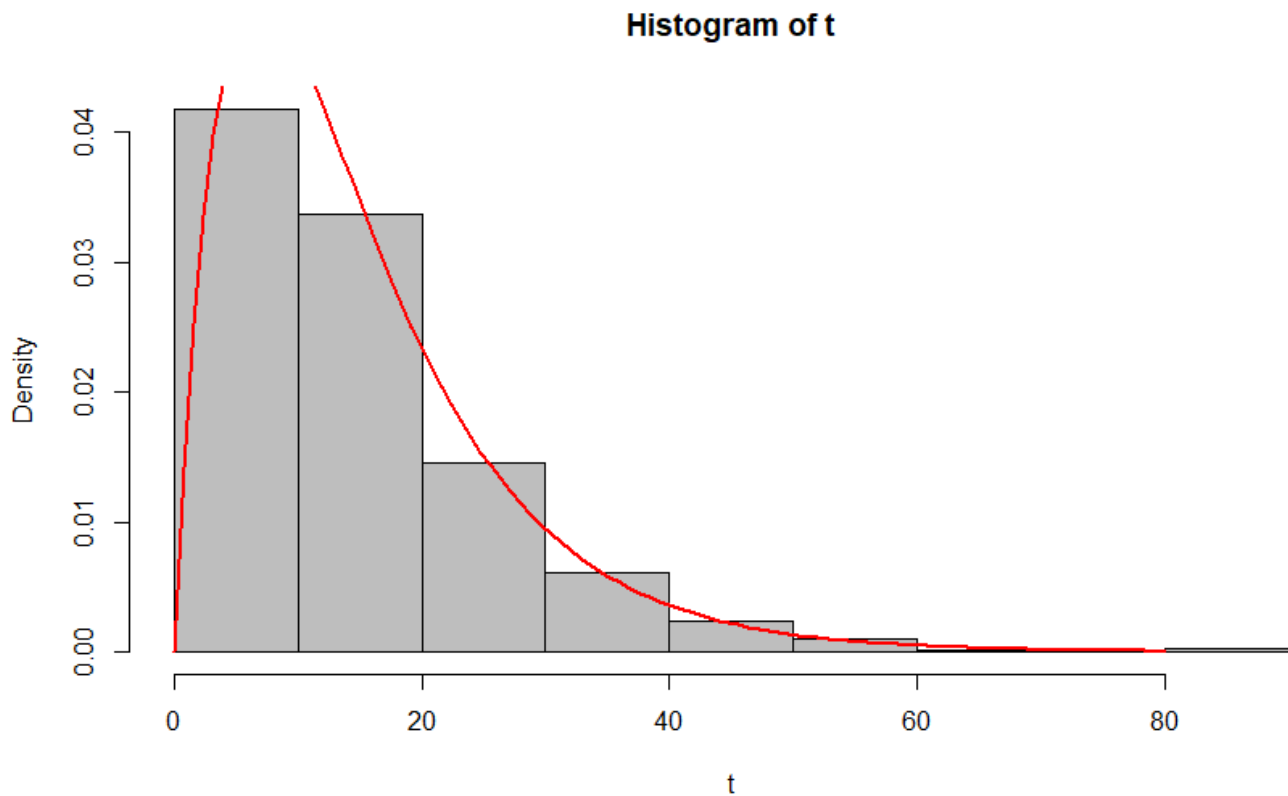
2nd Iteration outcomes are as follows:

Histogram of the lifetime of a satellite (t):

Histogram of t



Density function and Histogram of the lifetime of a satellite (t):



E(T) and the probability that the satellite lasts more than 15 years:

The screenshot shows the RStudio environment pane. At the top, there are tabs for "Environment", "History", and "Connections". Below the tabs is a toolbar with icons for file operations and a search bar. The main area displays the "Global Environment" with a search icon. Under the "Data" section, there is a variable "I" which is a "List of 5". The list contains the following elements:

- value : num 0.396
- abs.error : num 5.13e-05
- subdivisions: int 6
- message : chr "ok"
- call : language integrate(f = f, lower = 15, upper = Inf)
- attr(*, "class")= chr "integrate"

Below the "Data" section, there is a "values" section showing the values of the variables in the list:

- a : 14.823000353658
- t : num [1:1000] 13.3 17.2 16.1 10.5 17.1 ...

At the bottom, there is a "Functions" section showing the function "f":

- f : function (t)

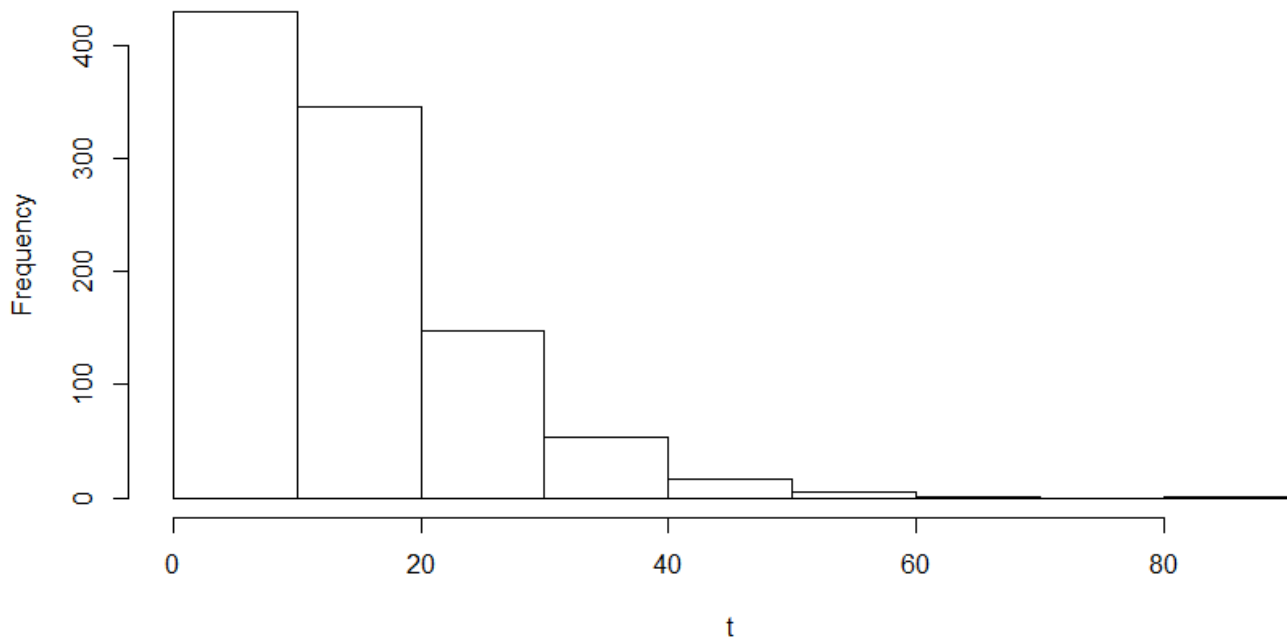
Observation: The actual mean value and the estimated mean value are gradually coming close to each other for the 2nd iteration. The probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

3rd Iteration outcomes are as follows:

Histogram of the lifetime of a satellite (t):

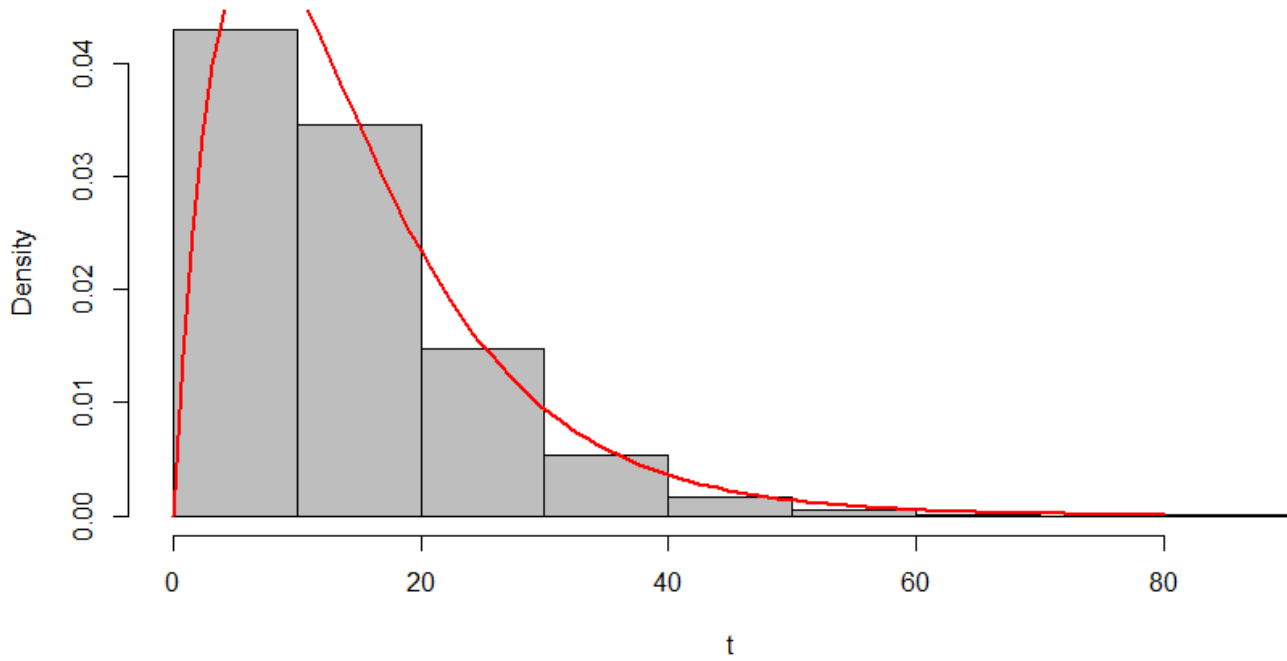
Density function and Histogram of the lifetime of a satellite (t):

Histogram of t



Density function and Histogram of the lifetime of a satellite (t):

Histogram of t



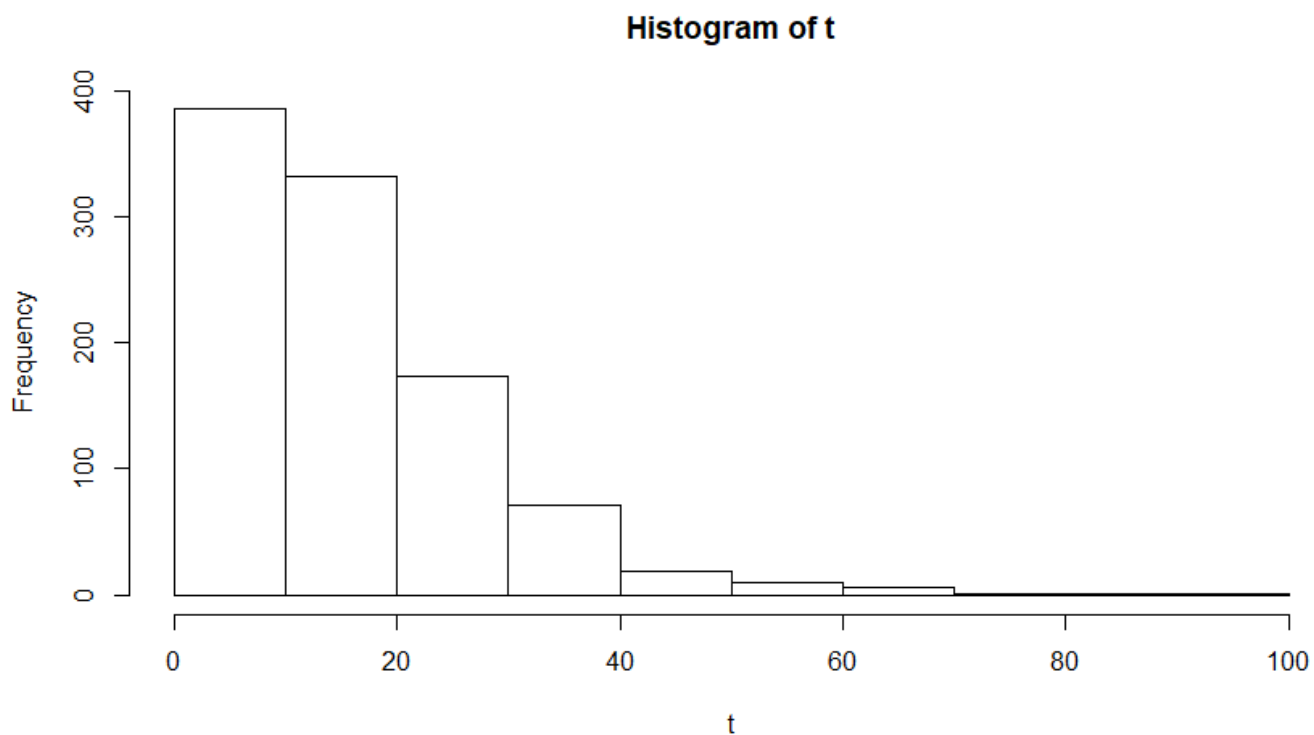
$E(T)$ and the probability that the satellite lasts more than 15 years:

Environment	History	Connections
Global Environment		
Data		
I	List of 5	
value : num 0.396		
abs.error : num 5.13e-05		
subdivisions: int 6		
message : chr "ok"		
call : language integrate(f = f, lower = 15, upper = Inf)		
- attr(*, "class")= chr "integrate"		
values		
a	13.9444842365028	
t	num [1:1000] 14.38 4.42 11 4.15 20.61 ...	
Functions		
f	function (t)	

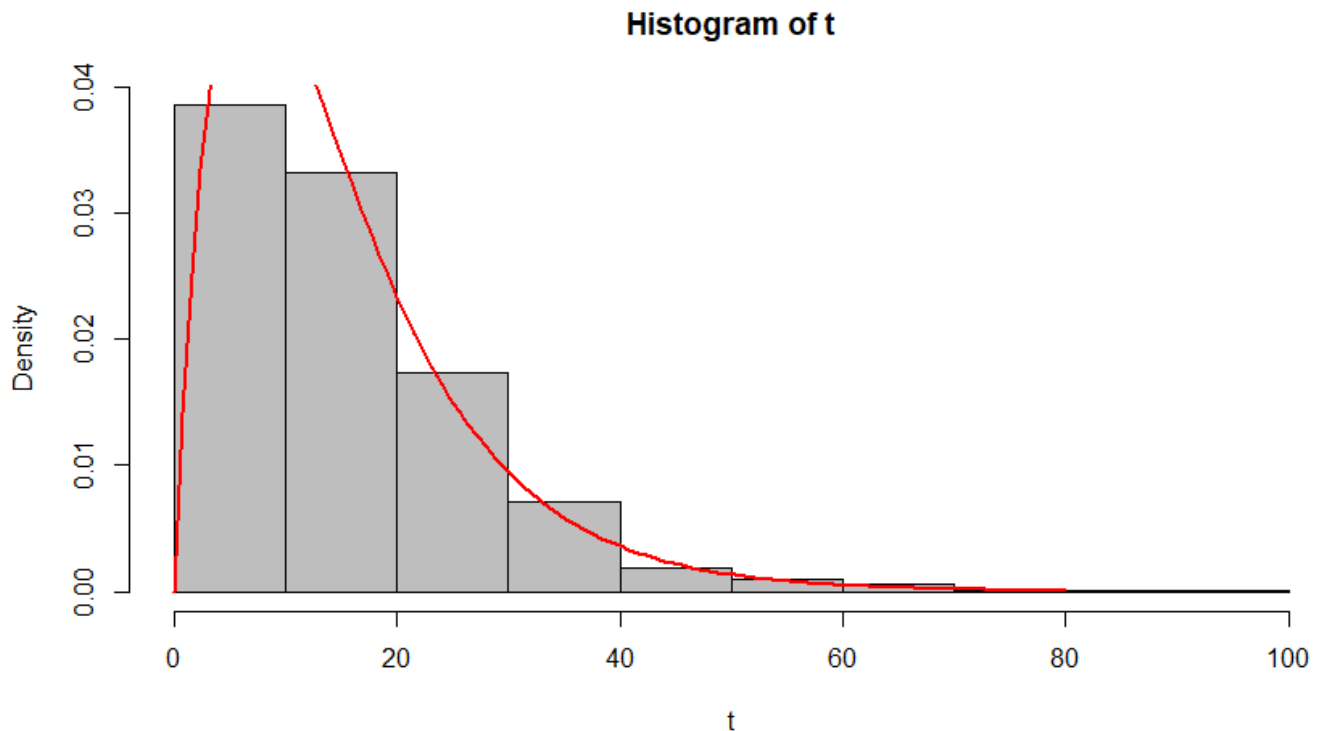
Observation: The actual mean value and the estimated mean value are very far from each other for the 3rd iteration. They were closer in the 2nd Iteration. The probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

4th Iteration outcomes are as follows:

Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



E(T) and the probability that the satellite lasts more than 15 years:

The screenshot shows the RStudio Environment pane. At the top, there are tabs for 'Environment', 'History', and 'Connections'. Below the tabs, there are icons for 'Import Dataset' and a search bar. The 'Global Environment' is selected. The 'Data' section shows a variable 'I' of type 'List of 5'. The list contains the following elements:

- value : num 0.396
- abs.error : num 5.13e-05
- subdivisions: int 6
- message : chr "ok"
- call : language integrate(f = f, lower = 15, upper = Inf)
- attr(*, "class")= chr "integrate"

Below the list, the 'values' section shows the attributes 'a' and 't':

- a: 15.4231027679278
- t: num [1:1000] 41.88 38.48 2.54 11.91 7.98 ...

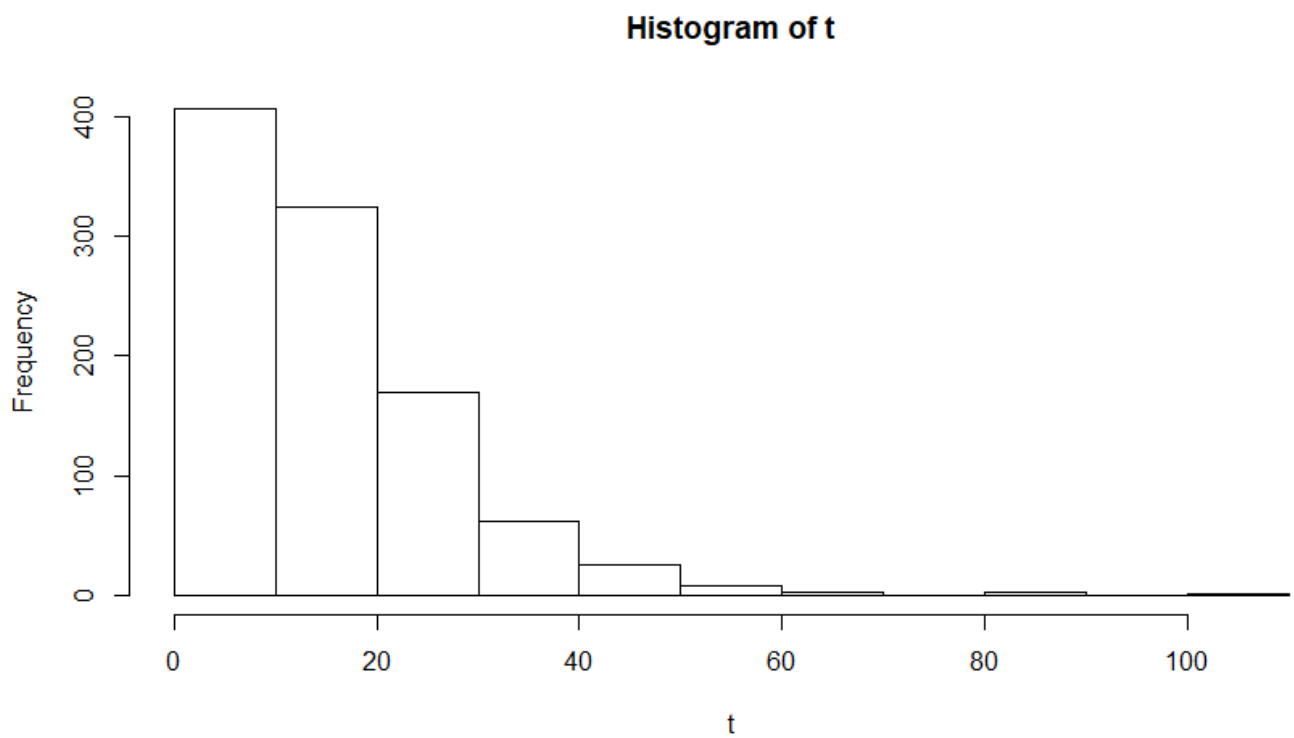
The 'Functions' section shows the function 'f':

- f: function (t)

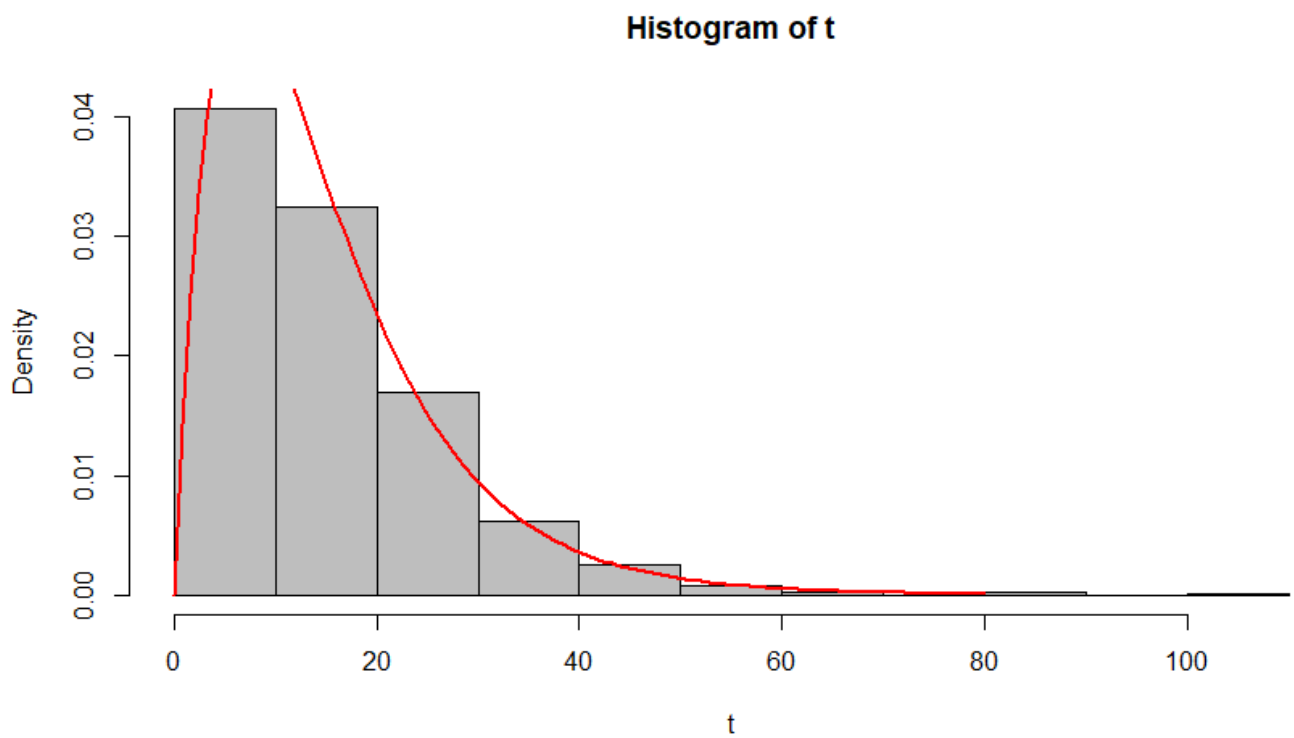
Observation: The actual mean value and the estimated mean value is close to each other for the 4th iteration. The value exceeds the actual mean value but is better than the 3rd Iteration. The probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

5th Iteration outcomes are as follows:

Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



$E(T)$ and the probability that the satellite lasts more than 15 years:

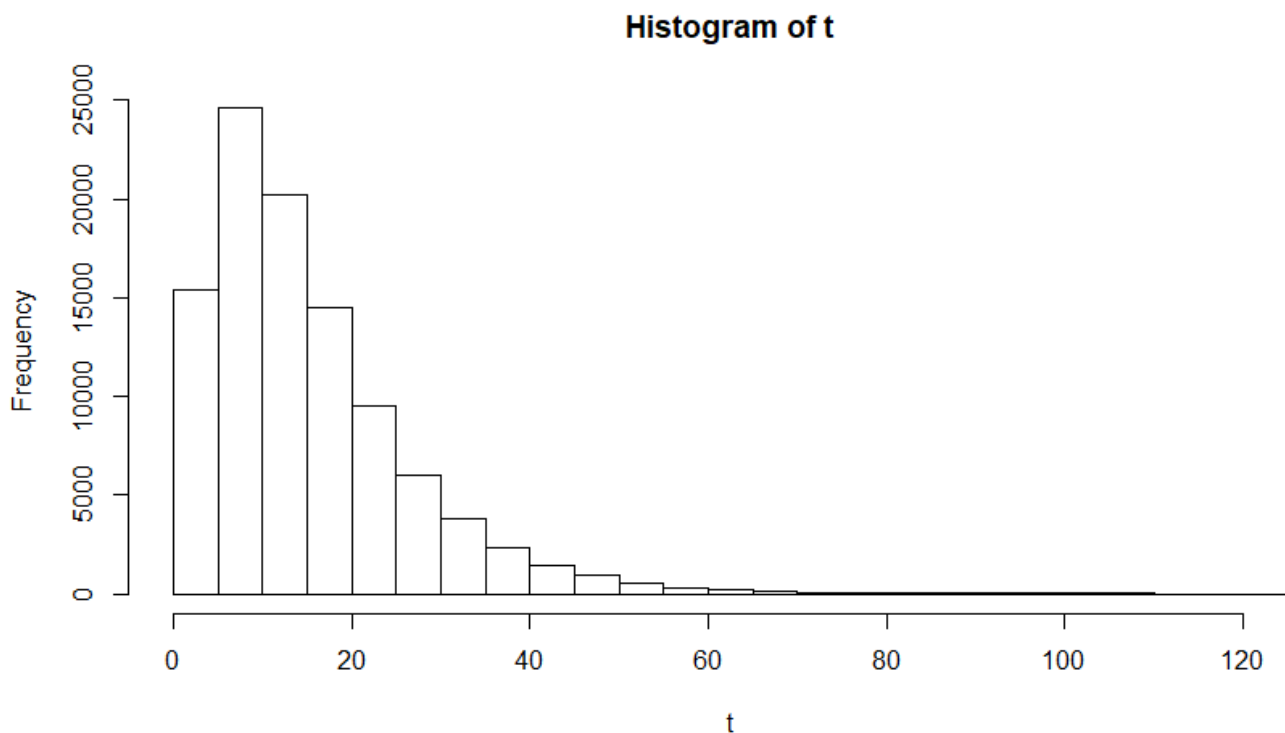
Environment		History	Connections
Global Environment		Import Dataset	
Data			
I	List of 5		
value	num 0.396		
abs.error	num 5.13e-05		
subdivisions	int 6		
message	chr "OK"		
call	language integrate(f = f, lower = 15, upper = Inf)		
	- attr(*, "class")= chr "integrate"		
Values			
a	15.0204292023693		
t	num [1:1000] 10.97 11.43 2.72 15.35 8.95 ...		
Functions			
f	function (t)		

Observation: The actual mean value and the estimated mean value is very close to each other for the 5th iteration. The probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

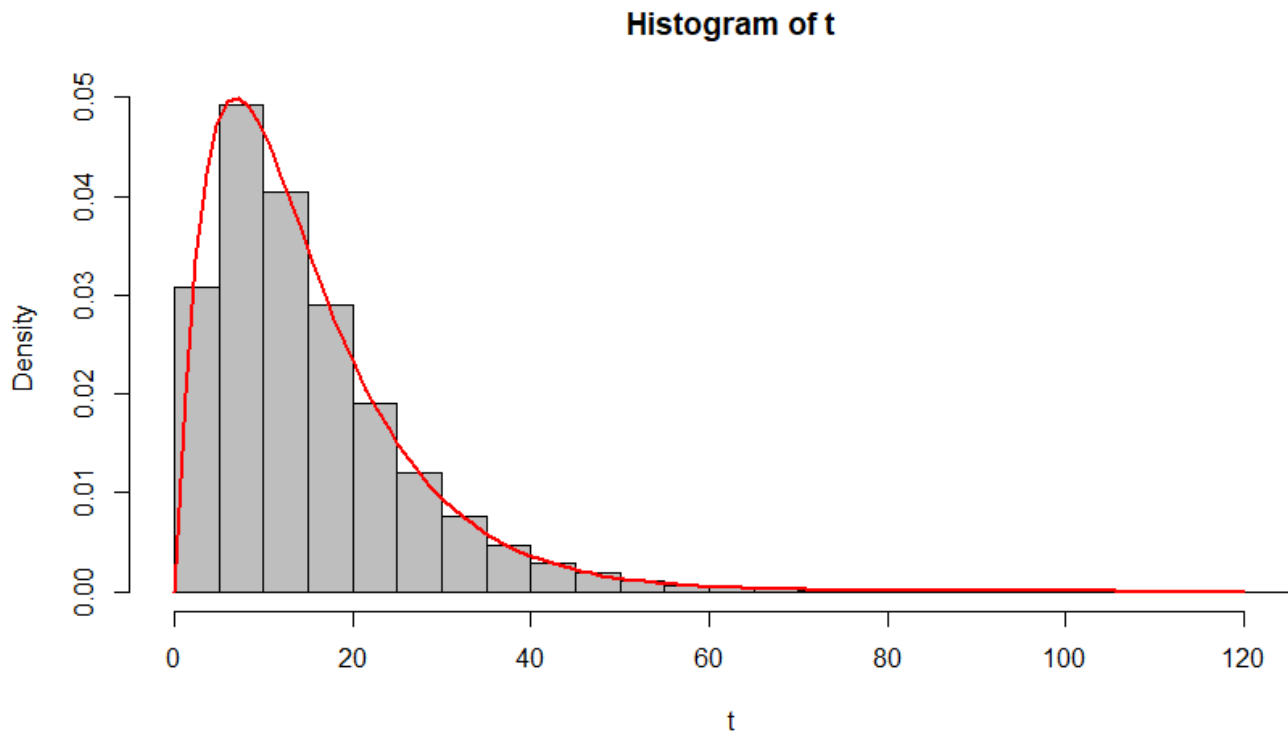
Case 3: The R code for simulation at $n=1,00,000$ is same as that of the above R code for 10,000. The only change is that we replicate the values 1,00,000 times instead of 10,000 times. There is also change in the range of values for which the histogram and curve is drawn. The range is (0,120).

1st Iteration outcomes are as follows:

Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



$E(T)$ and the probability that the satellite lasts more than 15 years:

Environment History Connections

Import Dataset

Global Environment

Data

I List of 5

value : num 0.396
abs.error : num 5.13e-05
subdivisions: int 6
message : chr "ok"
call : language integrate(f = f, lower = 15, upper = Inf)
- attr(*, "class")= chr "integrate"

values

a 15.0444500162022

t Large numeric (100000 elements, 781.3 kb)

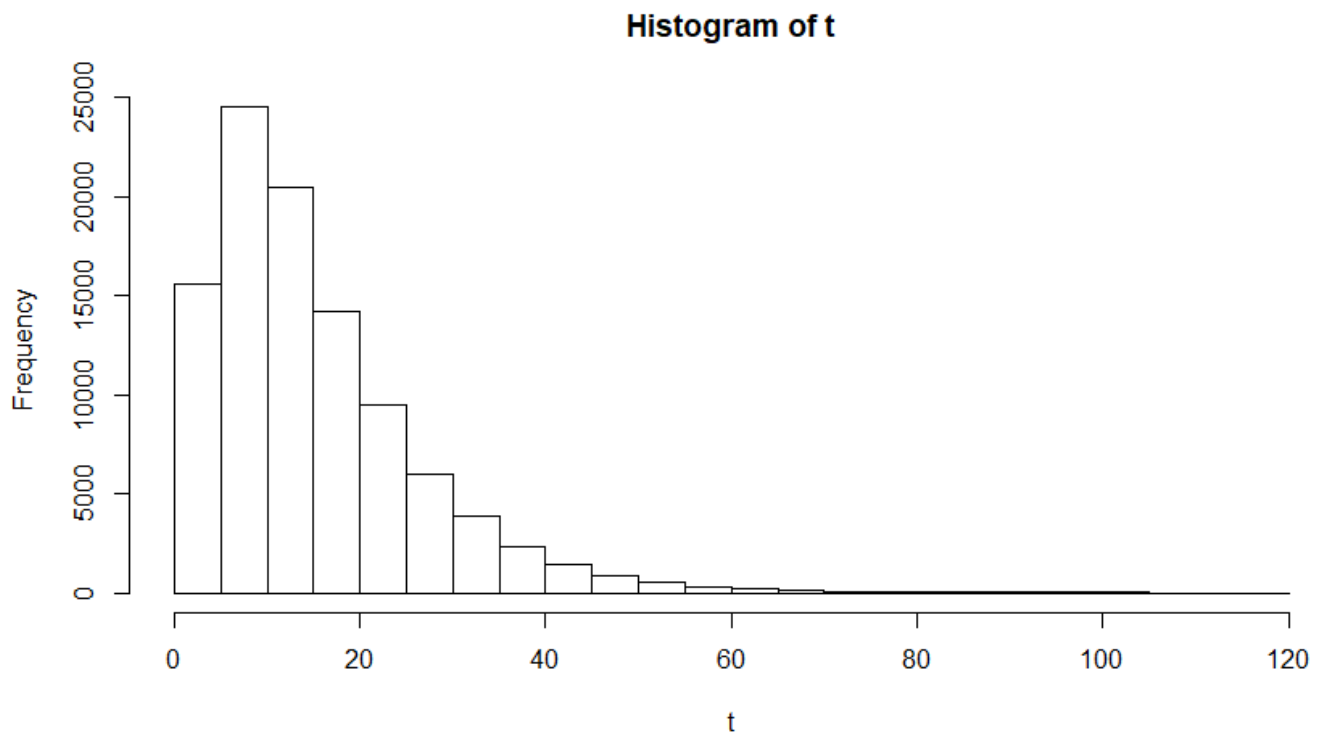
Functions

f function (t)

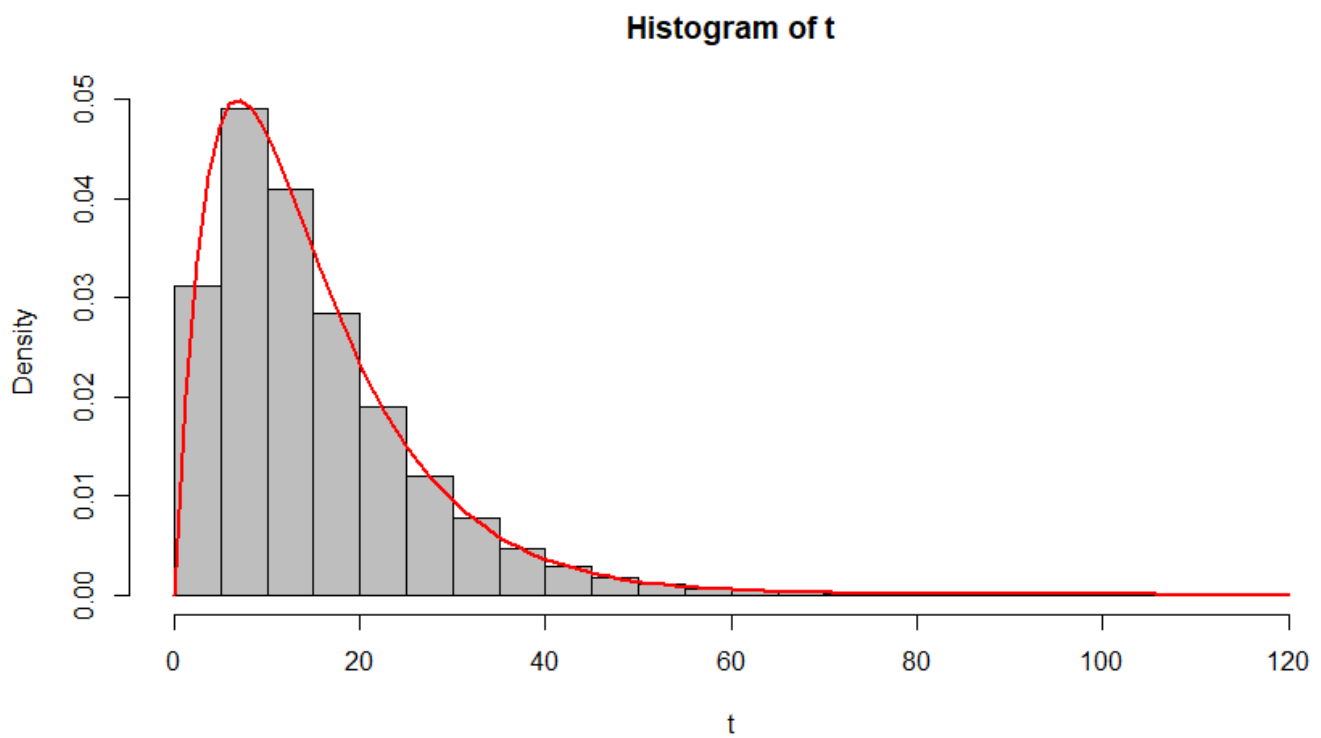
Observation: The actual mean value and the estimated mean value is very close to each other for the 1st iteration and the probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

2nd Iteration outcomes are as follows:

Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



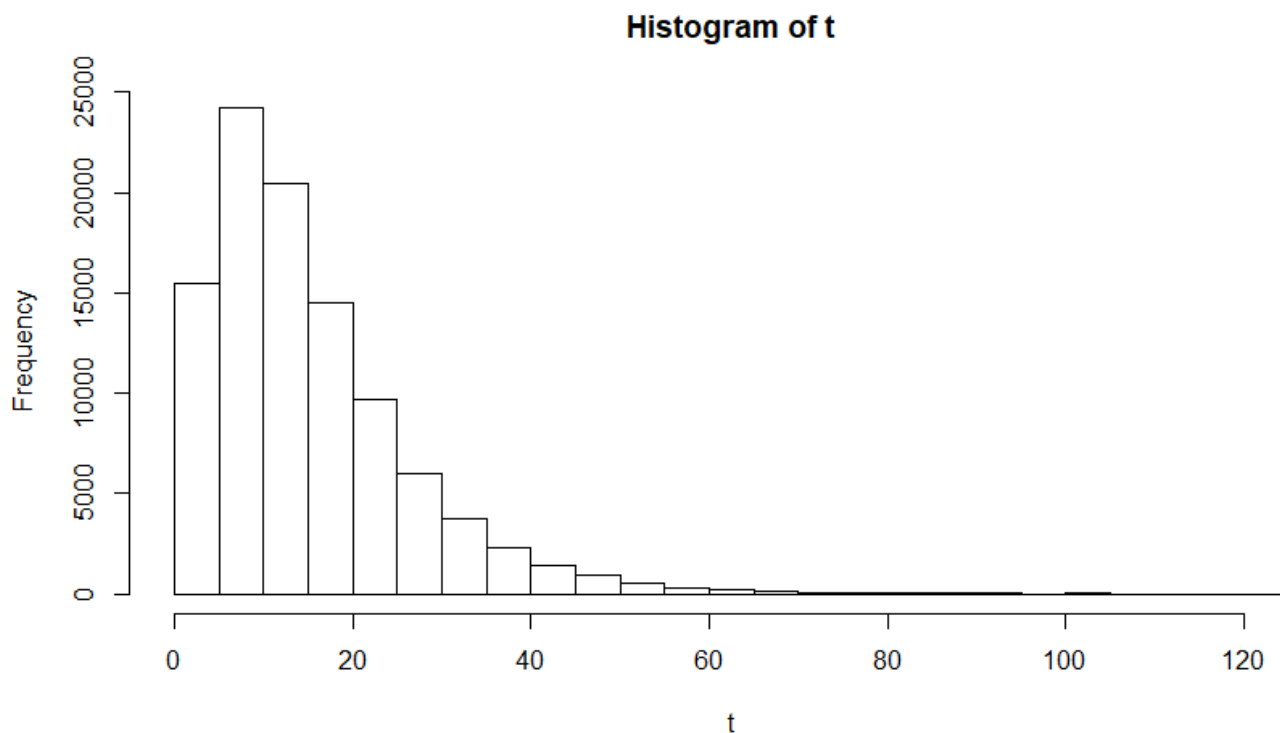
$E(T)$ and the probability that the satellite lasts more than 15 years:

Environment	History	Connections
Import Dataset		
Global Environment		
Data		
I	List of 5	
value : num 0.396		
abs.error : num 5.13e-05		
subdivisions: int 6		
message : chr "OK"		
call : language integrate(f = f, lower = 15, upper = Inf)		
- attr(*, "class")= chr "integrate"		
Values		
a	14.9799979608599	
t	Large numeric (100000 elements, 781.3 kb)	
Functions		
f	function (t)	

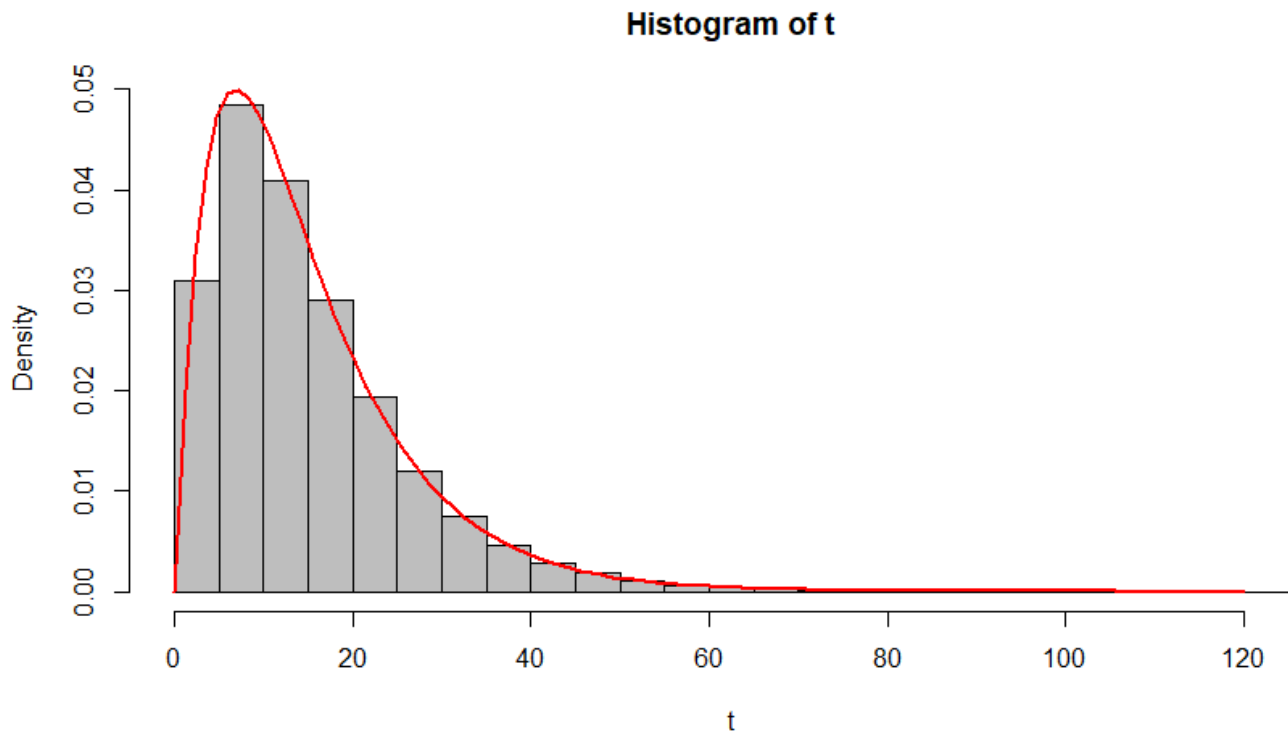
Observation: The actual mean value and the estimated mean value is very close to each other for the 2nd iteration. It is not as close as Iteration 1 but it is not bad. The probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

3rd Iteration outcomes are as follows:

Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



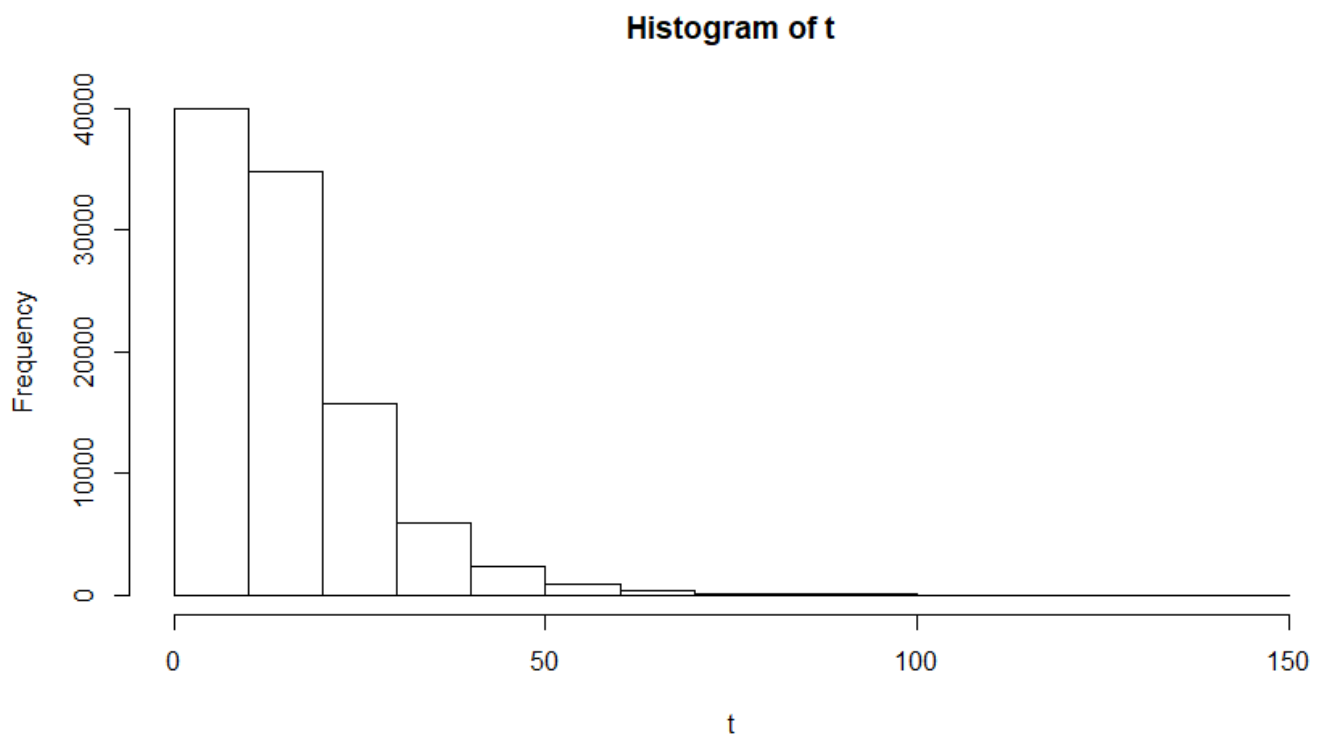
$E(T)$ and the probability that the satellite lasts more than 15 years:

Environment		History	Connections
Global Environment		Import Dataset	
Data			
I	List of 5		
value	num 0.396		
abs.error	num 5.13e-05		
subdivisions	int 6		
message	chr "OK"		
call	language integrate(f = f, lower = 15, upper = Inf)		
	- attr(*, "class")= chr "integrate"		
Values			
a	15.0465209659512		
t	Large numeric (100000 elements, 781.3 Kb)		
Functions			
f	function (t)		

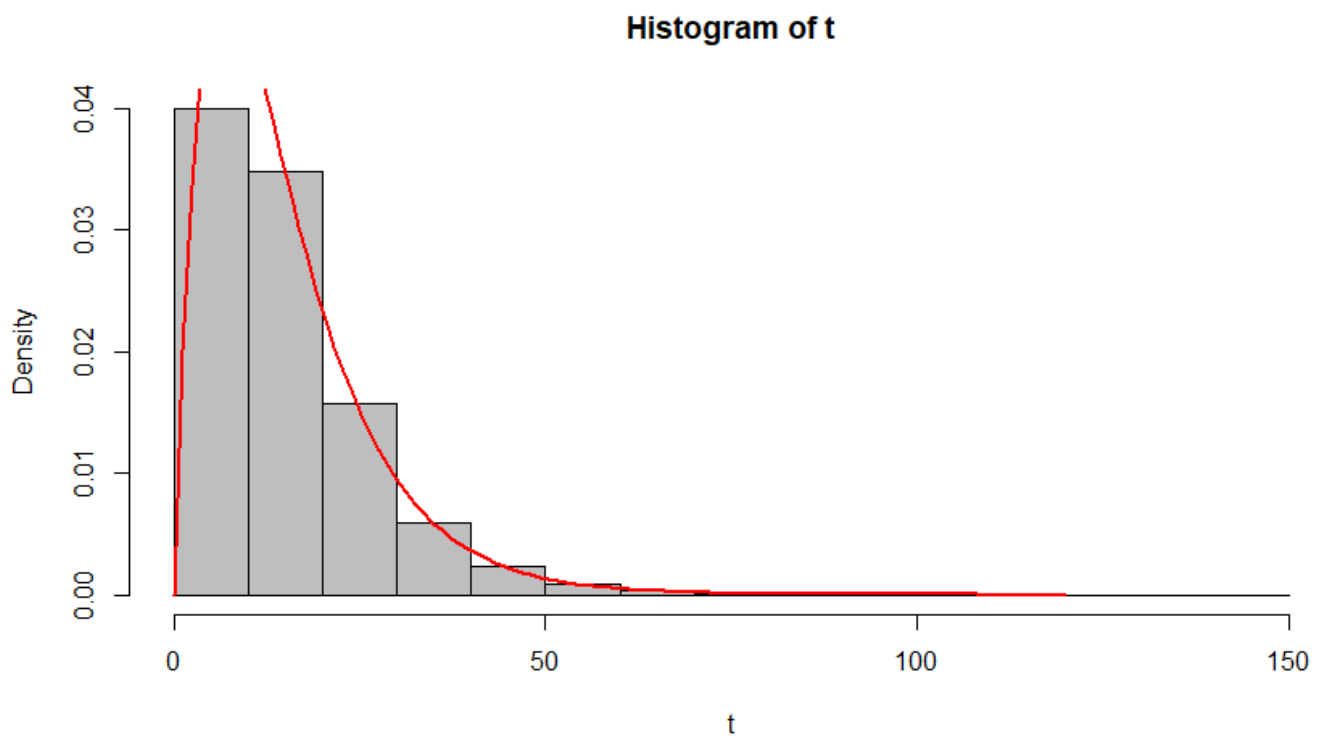
Observation: The actual mean value and the estimated mean value is very close to each other for the 3rd iteration. It is better than the 2nd Iteration and similar to 1st Iteration. The probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

4th Iteration outcomes are as follows:

Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



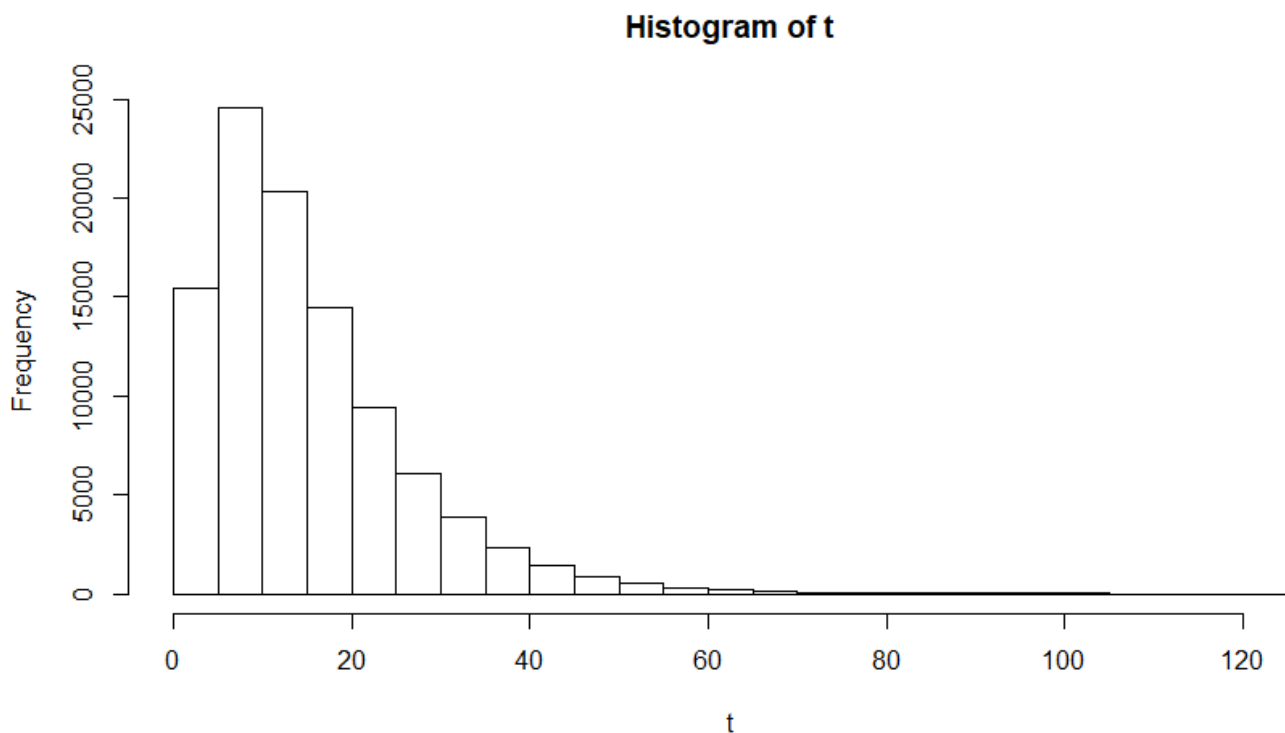
$E(T)$ and the probability that the satellite lasts more than 15 years:

Environment	History	Connections
Global Environment		
Data		
I	List of 5	
value : num 0.396		
abs.error : num 5.13e-05		
subdivisions: int 6		
message : chr "ok"		
call : language integrate(f = f, lower = 15, upper = Inf)		
- attr(*, "class")= chr "integrate"		
values		
a	14.985326134463	
t	Large numeric (100000 elements, 781.3 kb)	
Functions		
f	function (t)	

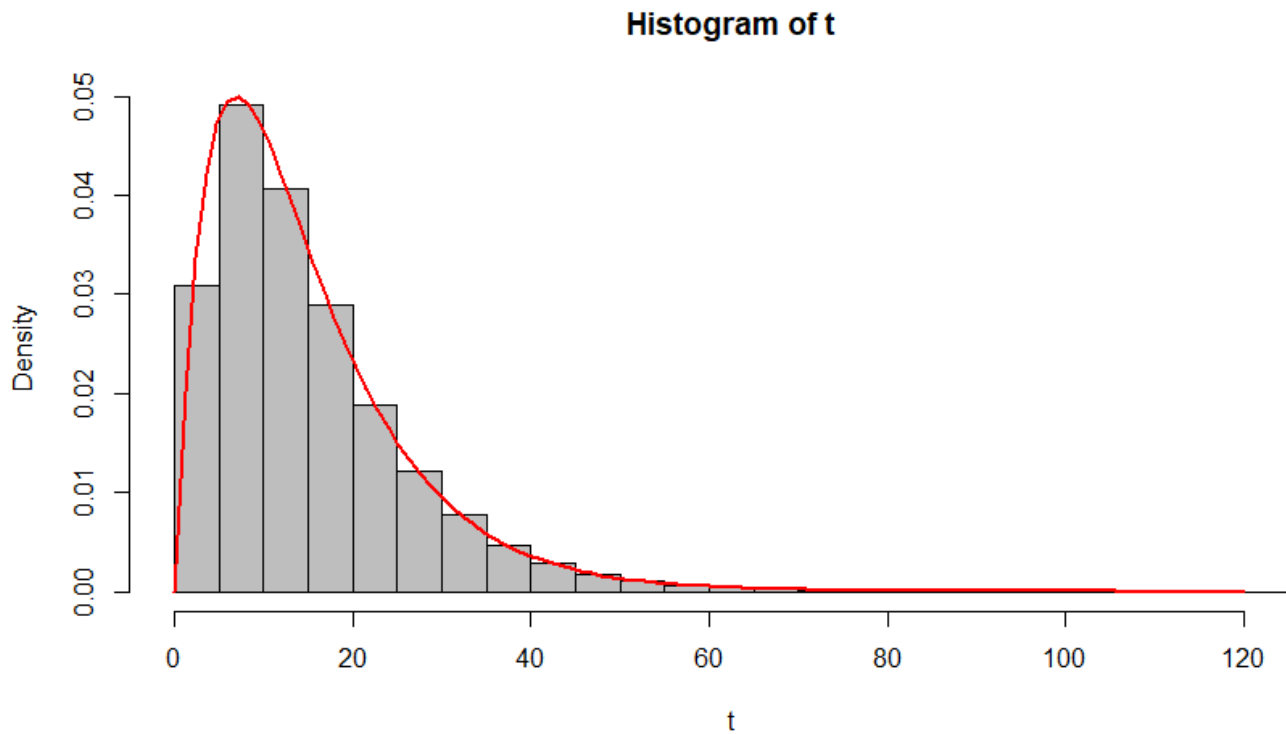
Observation: The actual mean value and the estimated mean value is close to each other for the 4th iteration and the probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

5th Iteration outcomes are as follows:

Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



$E(T)$ and the probability that the satellite lasts more than 15 years:

Environment		History	Connections
Global Environment		Import Dataset	
Data			
I	List of 5		
value :	num 0.396		
abs.error :	num 5.13e-05		
subdivisions:	int 6		
message :	chr "ok"		
call :	language integrate(f = f, lower = 15, upper = Inf)		
	- attr(*, "class")= chr "integrate"		
values			
a	14.9889821780794		
t	Large numeric (100000 elements, 781.3 kb)		
Functions			
f	function (t)		

Observation: The actual mean value and the estimated mean value is close to each other for the 5th iteration. The probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

Table of Observations:

VALUE OF N	ITERATION	E(T)	P(X>=15)
N= 10,000	Iteration 1	15.11	0.396
	Iteration 2	15.05	0.396
	Iteration 3	15.06	0.396
	Iteration 4	15.08	0.396
	Iteration 5	14.72	0.396
N= 1,000	Iteration 1	14.68	0.396
	Iteration 2	14.82	0.396
	Iteration 3	13.94	0.396
	Iteration 4	15.42	0.396
	Iteration 5	15.02	0.396
N= 1,00,000	Iteration 1	15.04	0.396
	Iteration 2	14.97	0.396
	Iteration 3	15.04	0.396
	Iteration 4	14.98	0.396
	Iteration 5	14.98	0.396

Final Conclusion:

With different sets of random value, as the number of replication (N) increases from 1,000 to 10,000 the expected mean value is approximately very close to the actual mean value. Hence, it proves the Law of Large Number. For a smaller number of replications, the variance between the actual mean value and the expected value is large. For larger number of replications, the variance between the actual mean value and the expected mean value is small. For larger number of replications, the range of t increases from (0,80) to (0,120). We also observe that the Density function is a normal distribution which skewed towards the right.

Q2. Use a Monte Carlo approach estimate the value of π based on 10,000 replications.

The R code for estimating value of π is as follows:

```
circle = function(long){  
+   m=rep(0,long)  
+   count=0  
+   for(i in 1:long){  
+     x=runif(2,0,1)  
+     if(sqrt(x[1]*x[1] + x[2]*x[2]) <= 1){  
+       count=count+1  
+     }  
+     proba=count/i  
+     piCap = proba*4  
+     m[i]=piCap  
+   }  
+   return(m)  
+ }  
> n=10000  
> sim=circle(n)  
> initial = 1  
> plot(sim[initial:n],type="l")  
> lines(rep(pi,n)[initial:n],col="red")
```

#Creating a function circle
Replicates long values having value 0
Initialising counter value to 0

Generate 2 uniform values between range (0,1)
Computing distance between two points.
Increment counter if condition is TRUE.

Probability of point lying within the circle.
Multiplying probability by 4 to obtain single value of π
Storing the value to plot it later.

Returning value of m to caller function.

#Initialising n to 10000 for replications
#Calling circle function

#Plotting the values obtained.

The plot of the same is as follows:

