Project 1
Name of group members: 1. Ajay Danda (AXD180068) 2. Satyam Bhikadiya (SXB180124)
Contribution of each group member:
Ajay Danda:

- Solved Q1 a
- Solved Q1 b] iv, v, vi
- Equal contribution in solving Q1 c
- Equal contribution in solving Q 2
- Equal contribution in Documenting the Report.

Satyam Bhikadiya:

- Solved Q1 b] i, ii, iii
- Equal contribution in solving Q1 c
- Equal contribution in solving Q 2
- Equal contribution in Documenting the Report.

Section 1:

(a) Use the above density function to analytically compute the probability that the lifetime of the satellite exceeds 15 years.

Answer:

Section 1:

(b) Use the following steps to take a Monte Carlo approach to compute E(T) and P(T > 15).

i. Simulate one draw of the block lifetimes XA and XB. Use these draws to simulate one draw of the satellite lifetime T.

ii. Repeat the previous step 10,000 times. This will give you 10,000 draws from the distribution of T. Try to avoid `for' loop. Use `replicate' function instead. Save these draws for reuse in later steps.

iii. Make a histogram of the draws of T using `hist' function. Superimpose the density function given above. Try using `curve' function for drawing the density. Note what you see.

iv. Use the saved draws to estimate E(T). Compare your answer with the exact answer given above.

v. Use the saved draws to estimate the probability that the satellite lasts more than 15 years. Compare with the exact answer estimated in part (a).

vi. Repeat the above process of obtaining an estimate of E(T) and an estimate of the probability four more times. Note what you see.

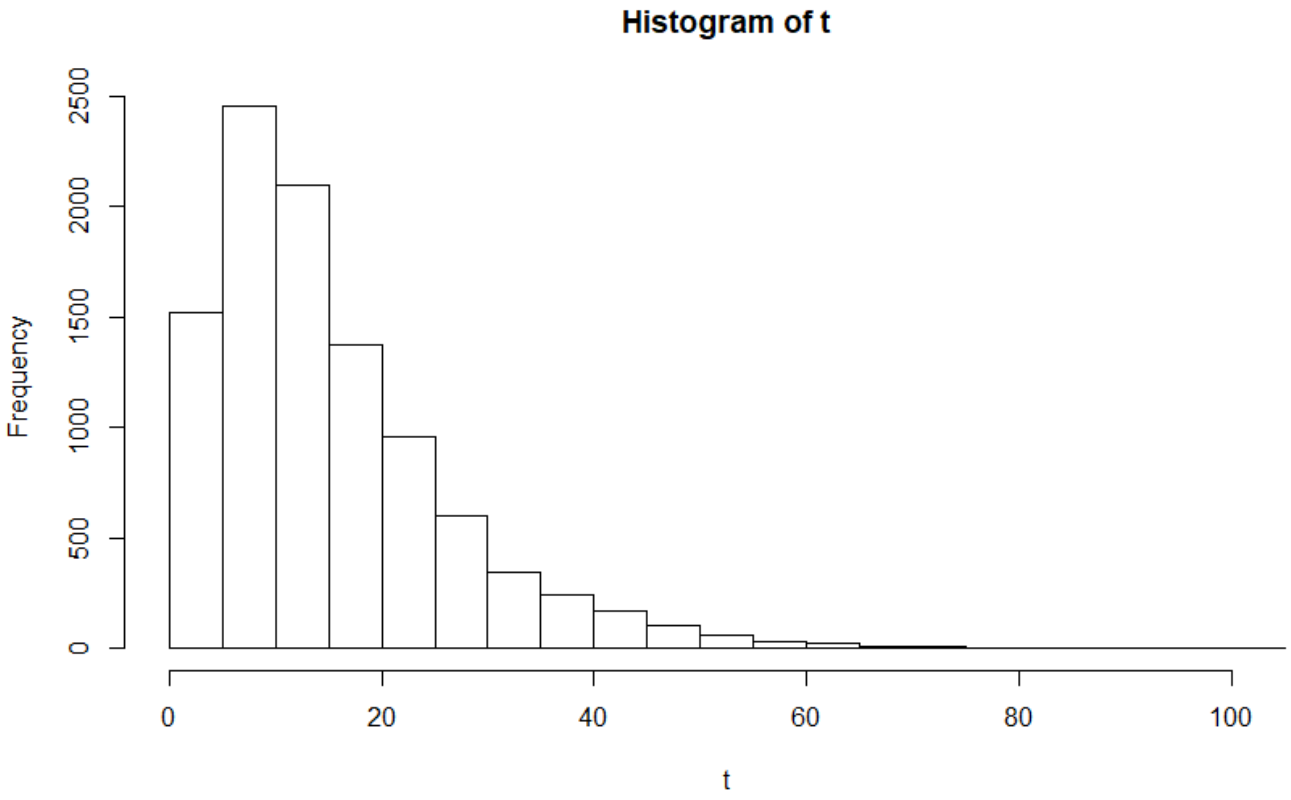
Answer:

Case 1: The R code for simulation at n=10,000 is as follows:

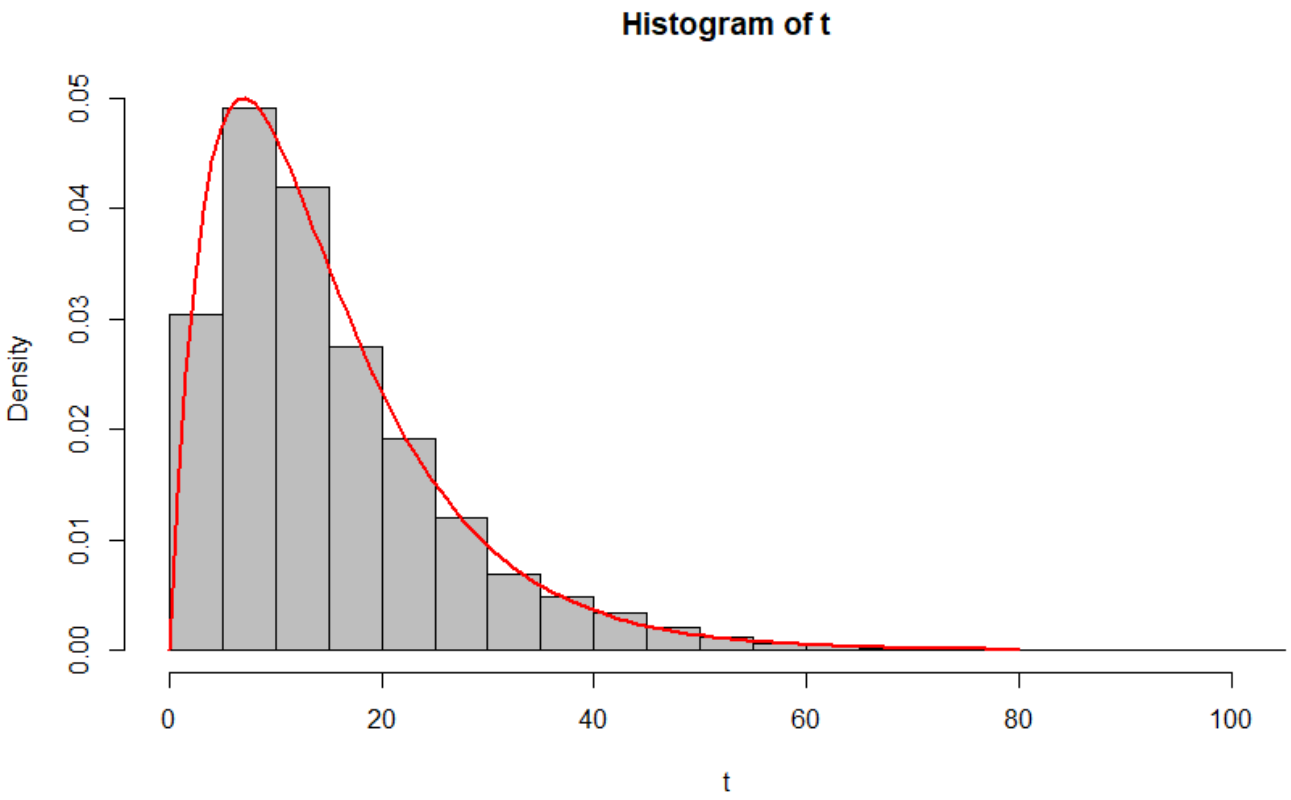
```
>t=replicate(10000,max(rexp(1,0.1),rexp(1,0.1)), simplify = "array" )  #replication of 2 random exponential 10000
> hist(t) # plotting the histogram of the same
> f=function(t){0.2*exp(-0.1*t)-0.2*exp(-0.2*t)} # Function of given probability density function of T
> I=integrate(f,15,Inf) # Integrating function to obtain probability that the satellite lasts more than 15 years.
> hist(t,prob=TRUE,col = "grey")
> hist(t,prob=TRUE,col = "grey", add=TRUE) # Plotting histogram to superimpose it with the curve later.
> curve(f,0,80,add=TRUE,col="red", lwd=2) # Plotting the curve of function f to superimpose it with the histogram.
> a=mean(t)  # computing E(T) to compare it with the value of E(T) given.
```

1st Iteration outcomes are as follows:

Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



E(T) and the probability that the satellite lasts more than 15 years:

Environment | History | Connections

Import Dataset | List

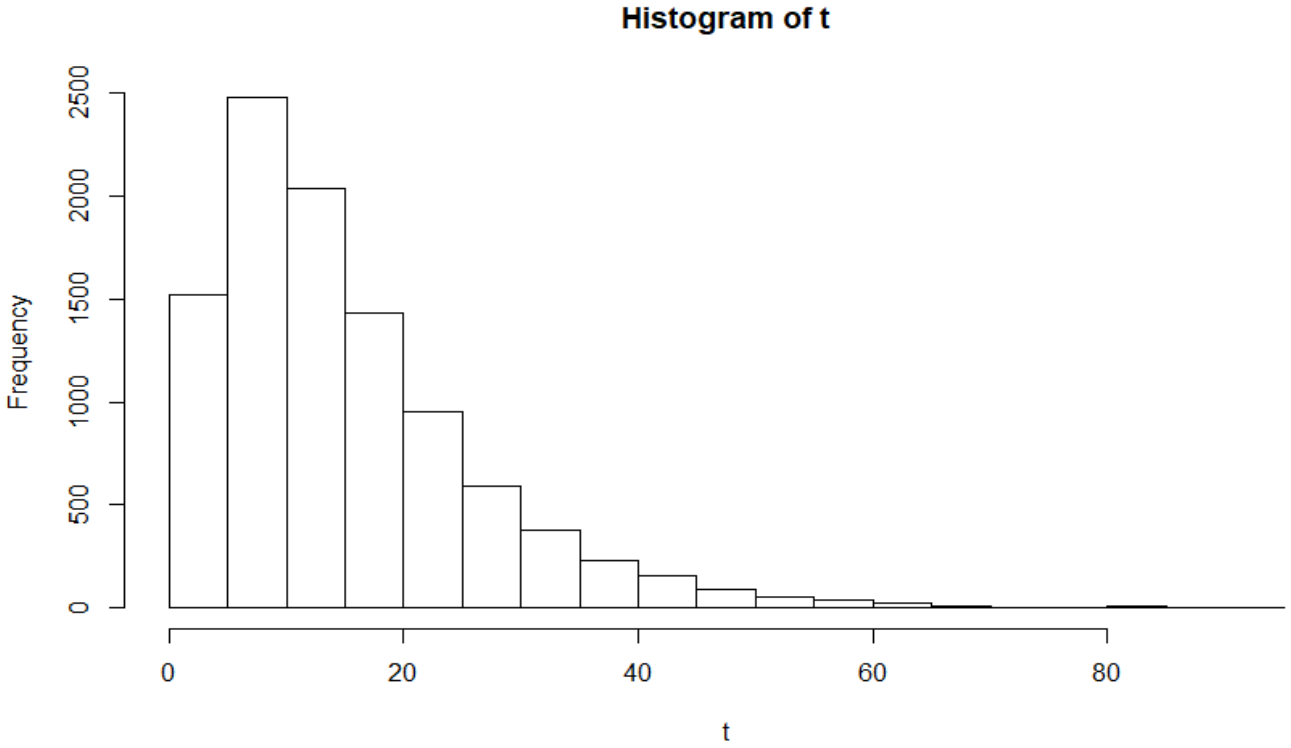
Global Environment

Data	
I	List of 5
value : num 0.396	
abs.error : num 5.13e-05	
subdivisions: int 6	
message : chr "OK"	
call : language integrate(f = f, lower = 15, upper = Inf)	
- attr(*, "class")= chr "integrate"	
Values	
a	15.1190099747582
t	num [1:10000] 1.22 28.93 21.09 10 22.67 ...
Functions	
f	function (t)

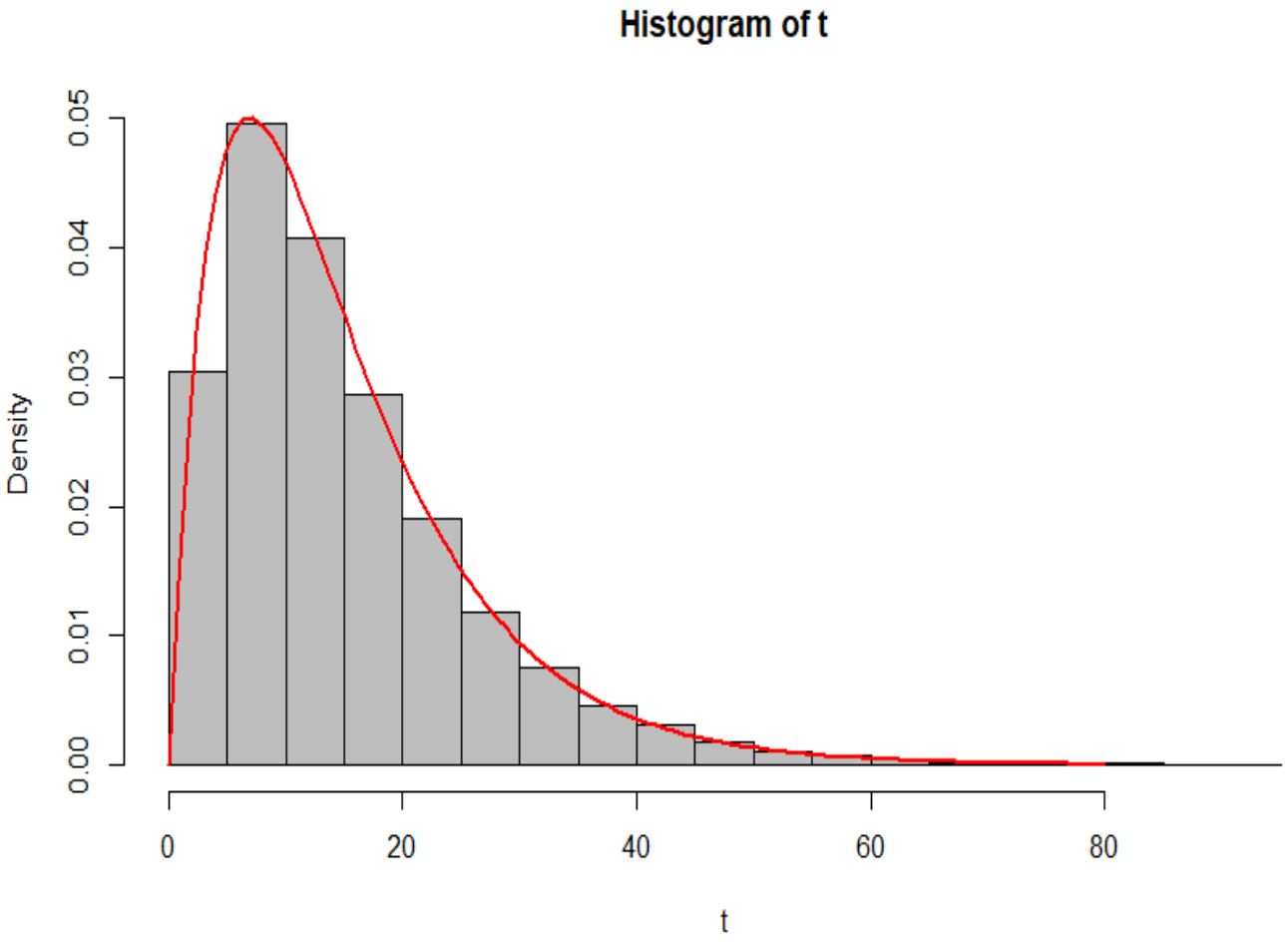
Observation: The actual mean value and the estimated mean value is very close to each other for the 1st iteration and the probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

2nd Iteration outcomes are as follows:

Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



E(T) and the probability that the satellite lasts more than 15 years:

Project: (None)

Environment | History | Connections

Import Dataset | List

Global Environment

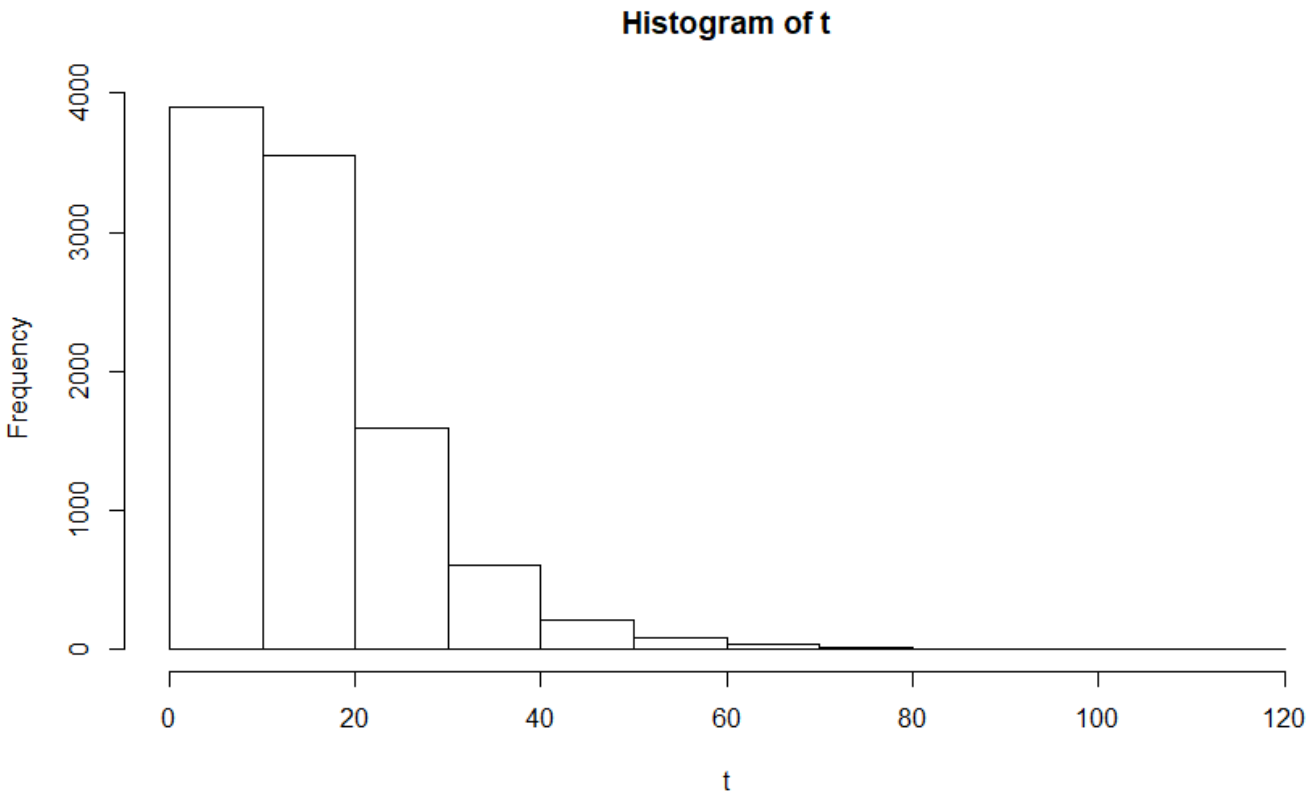
Data

I	List of 5
value : num 0.396	
abs.error : num 5.13e-05	
subdivisions: int 6	
message : chr "OK"	
call : language integrate(f = f, lower = 15, upper = Inf)	
- attr(*, "class")= chr "integrate"	
Values	
a	15.0524337976861
t	num [1:10000] 48.16 18.09 19.14 3.69 12.17 ...
Functions	
f	function (t)

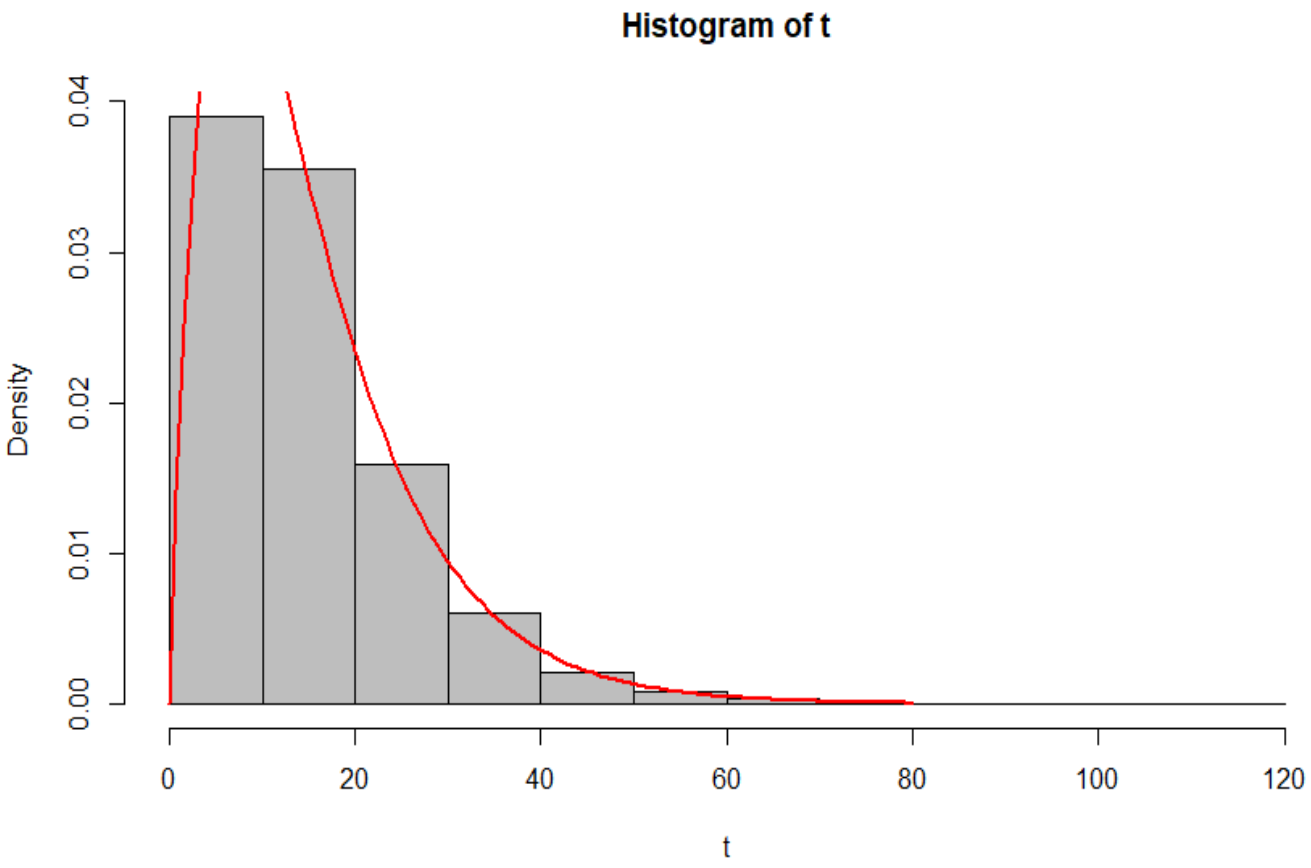
Observation: The actual mean value and the estimated mean value is very close to each other for the 2nd iteration and the probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

3rd Iteration outcomes are as follows:

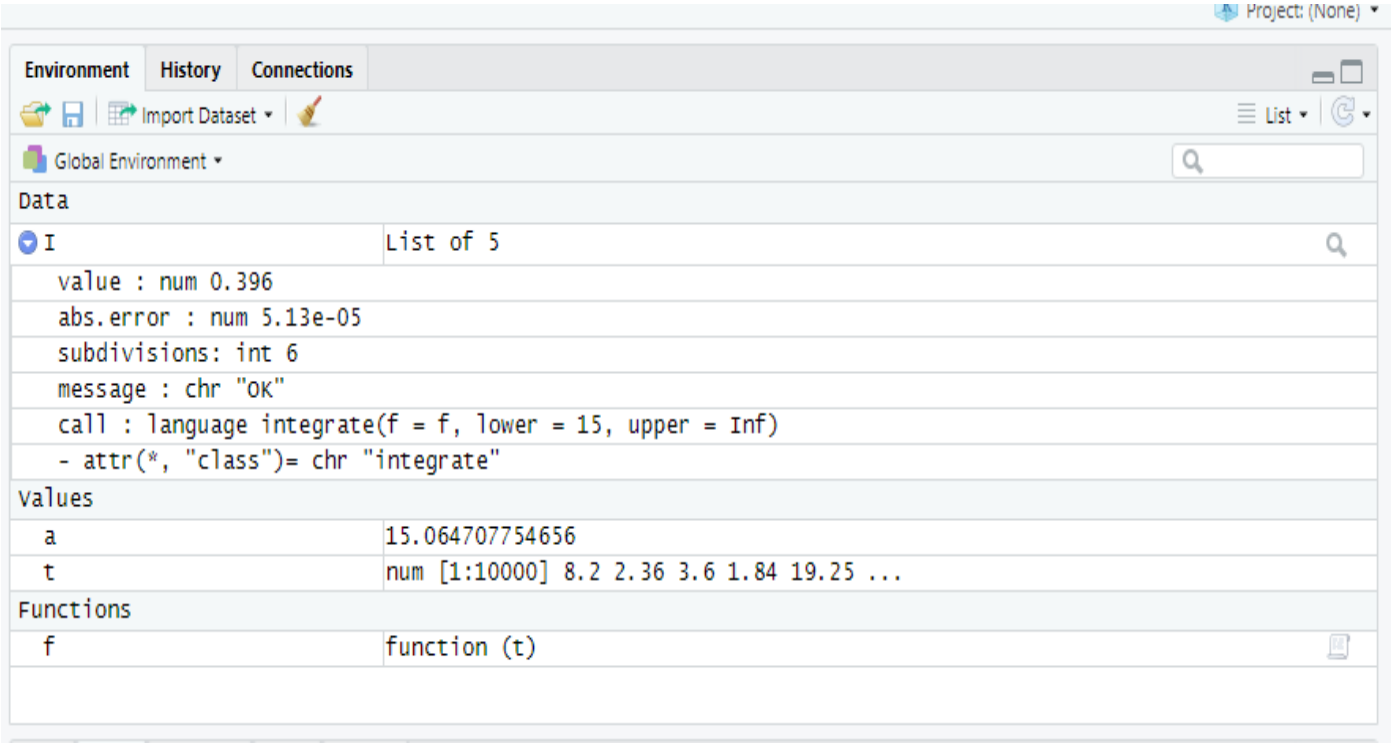
Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



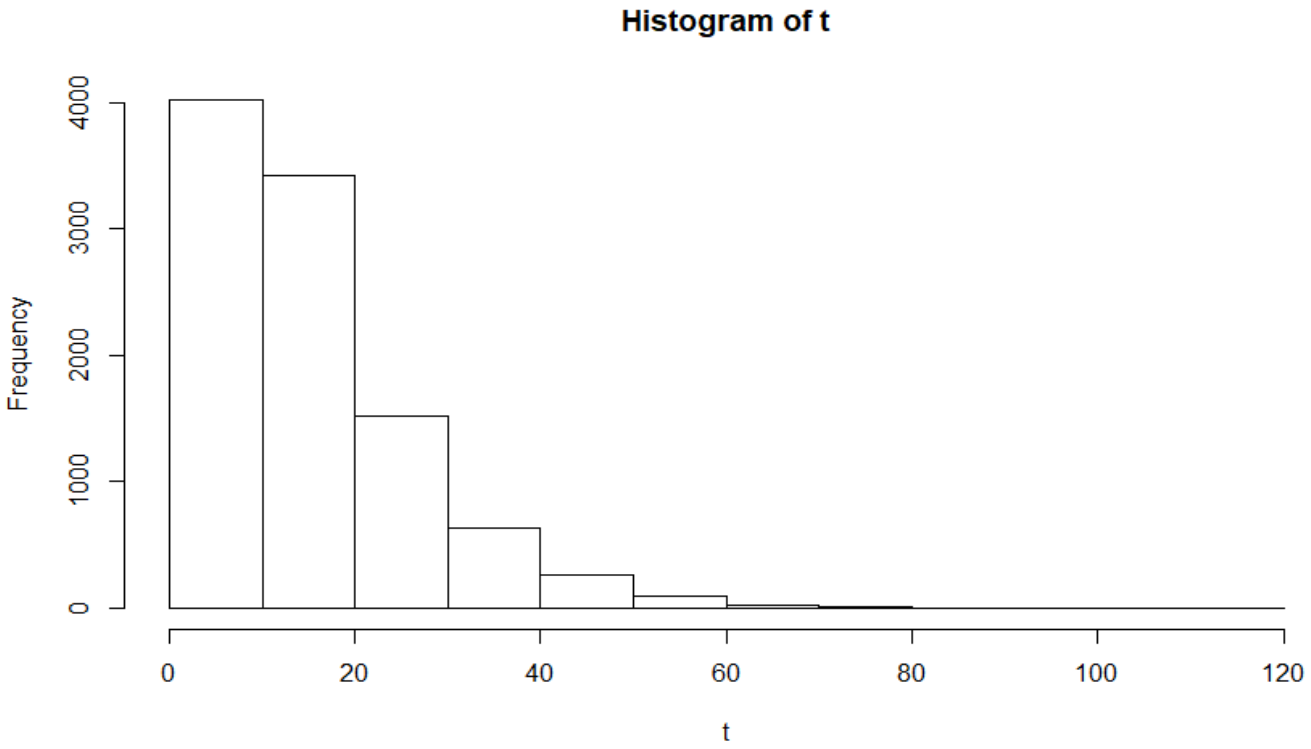
E(T) and the probability that the satellite lasts more than 15 years:



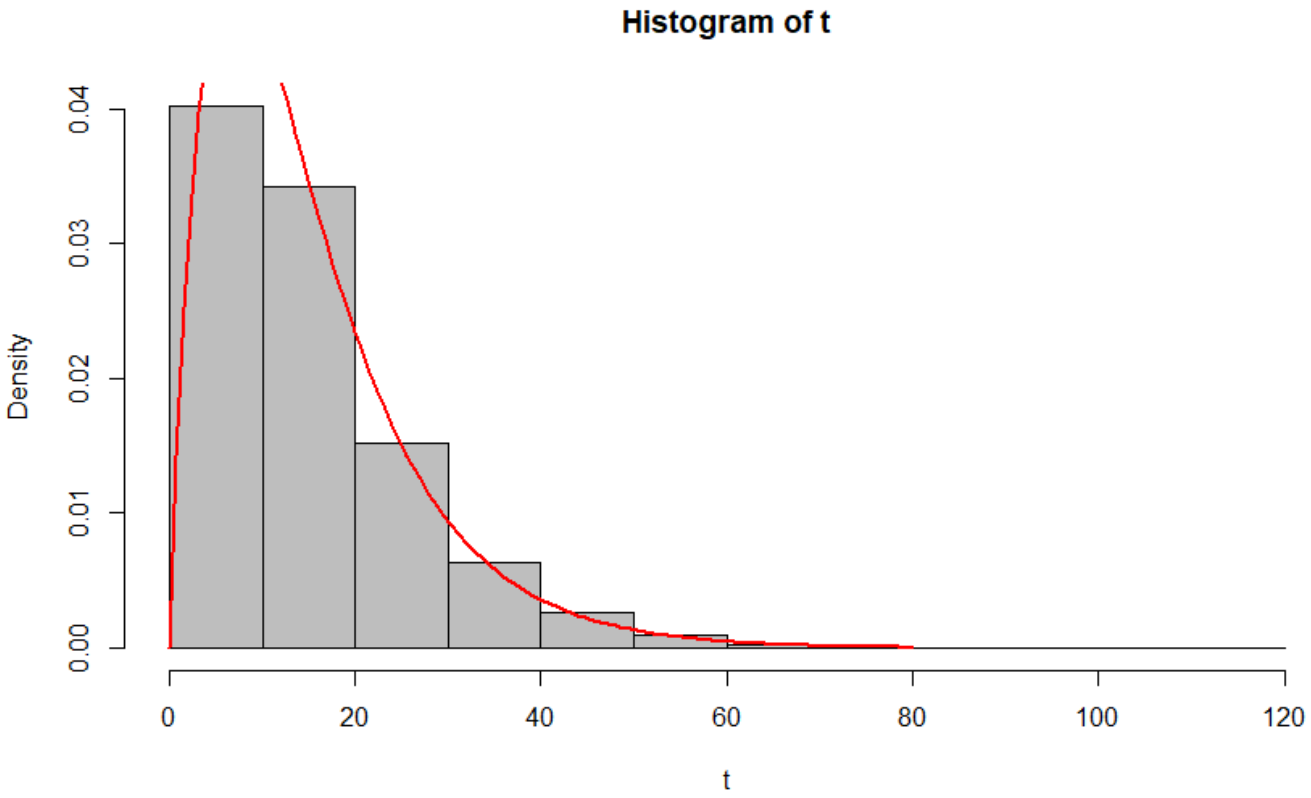
Observation: The actual mean value and the estimated mean value is very close to each other for the 3rd iteration and the probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

4th Iteration outcomes are as follows:

Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



E(T) and the probability that the satellite lasts more than 15 years:

Project: (None)

Environment | History | Connections

Import Dataset | List

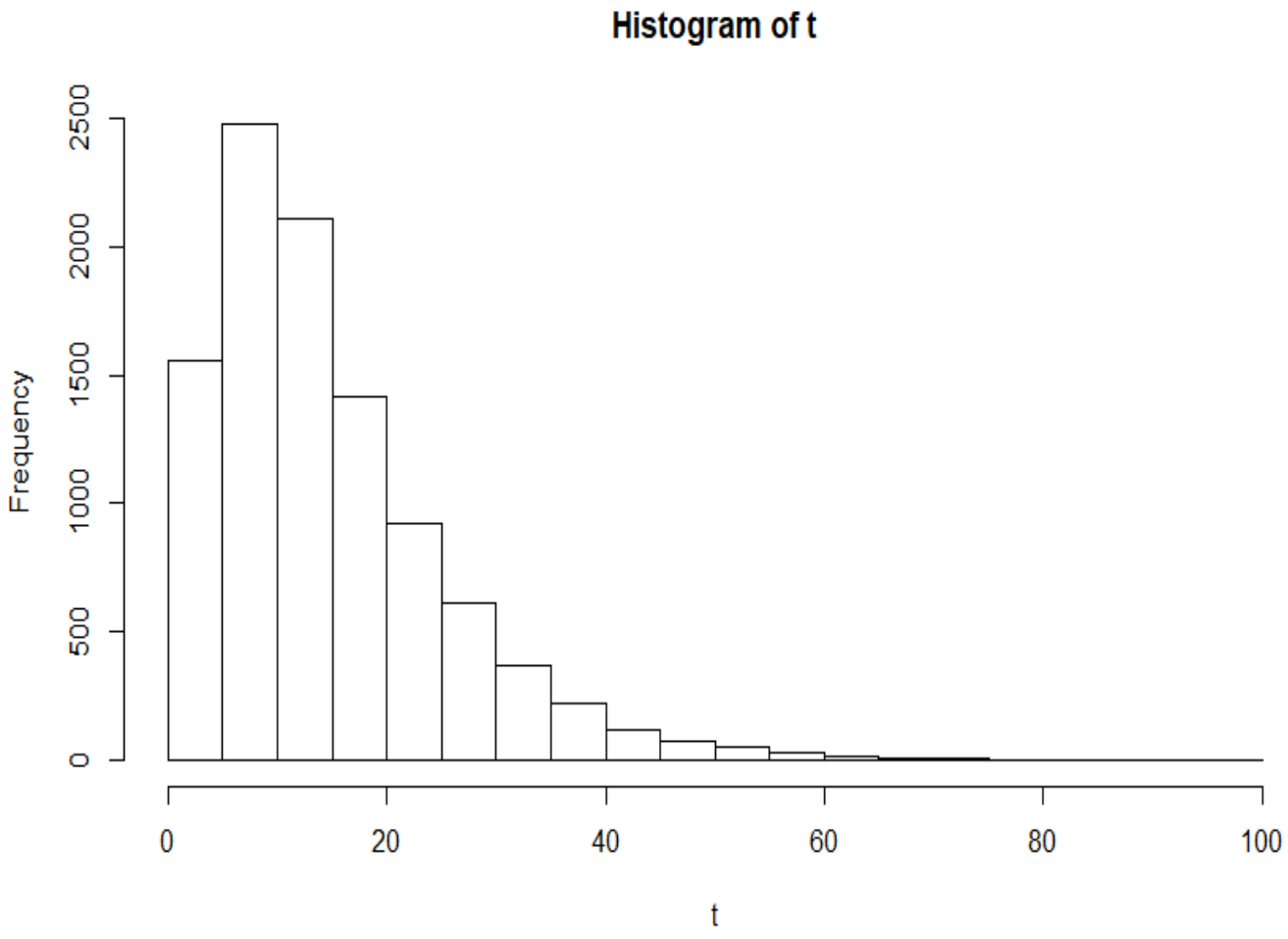
Global Environment

Data	
I	List of 5
value : num 0.396	
abs.error : num 5.13e-05	
subdivisions: int 6	
message : chr "OK"	
call : language integrate(f = f, lower = 15, upper = Inf)	
- attr(*, "class")= chr "integrate"	
Values	
a	15.0875392425855
t	num [1:10000] 17.22 20.54 41.14 17.72 5.02 ...
Functions	
f	function (t)

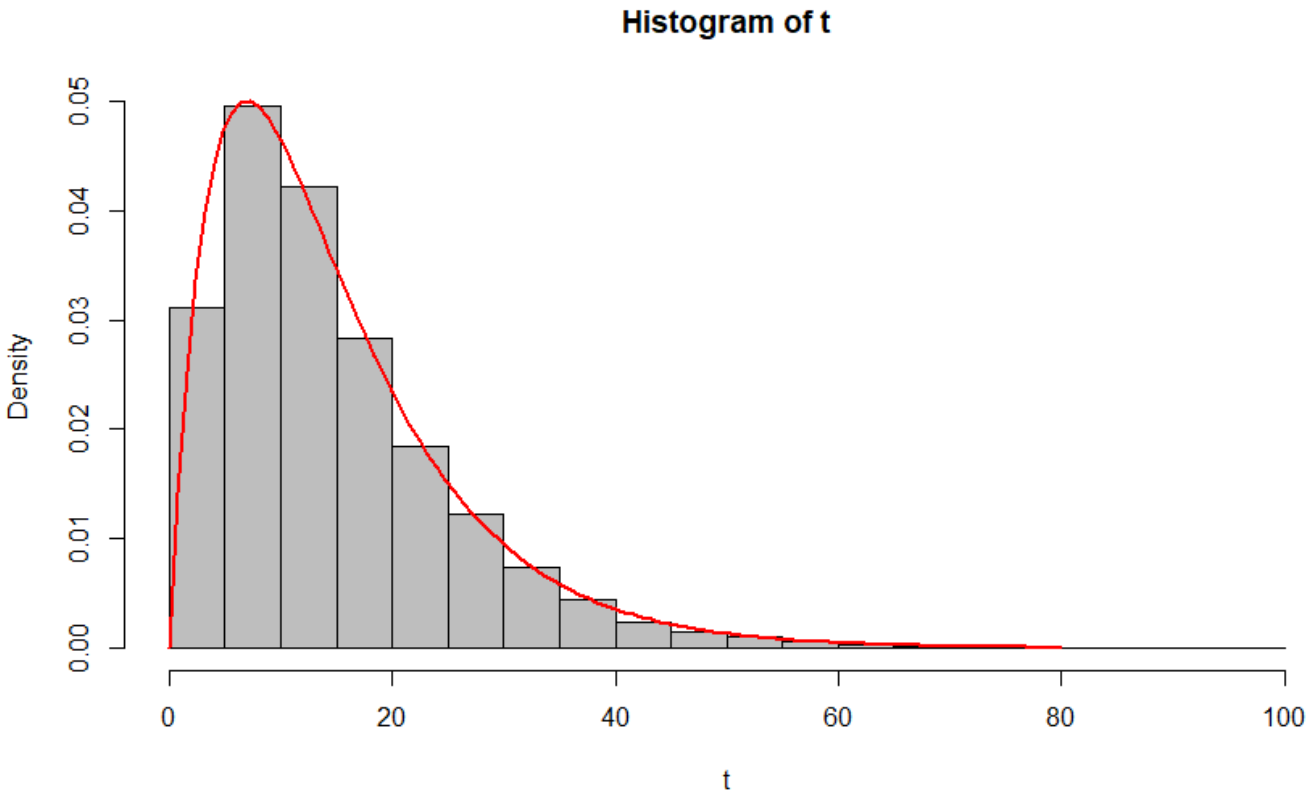
Observation: The actual mean value and the estimated mean value is very close to each other for the 4th iteration and the probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

5th Iteration outcomes are as follows:

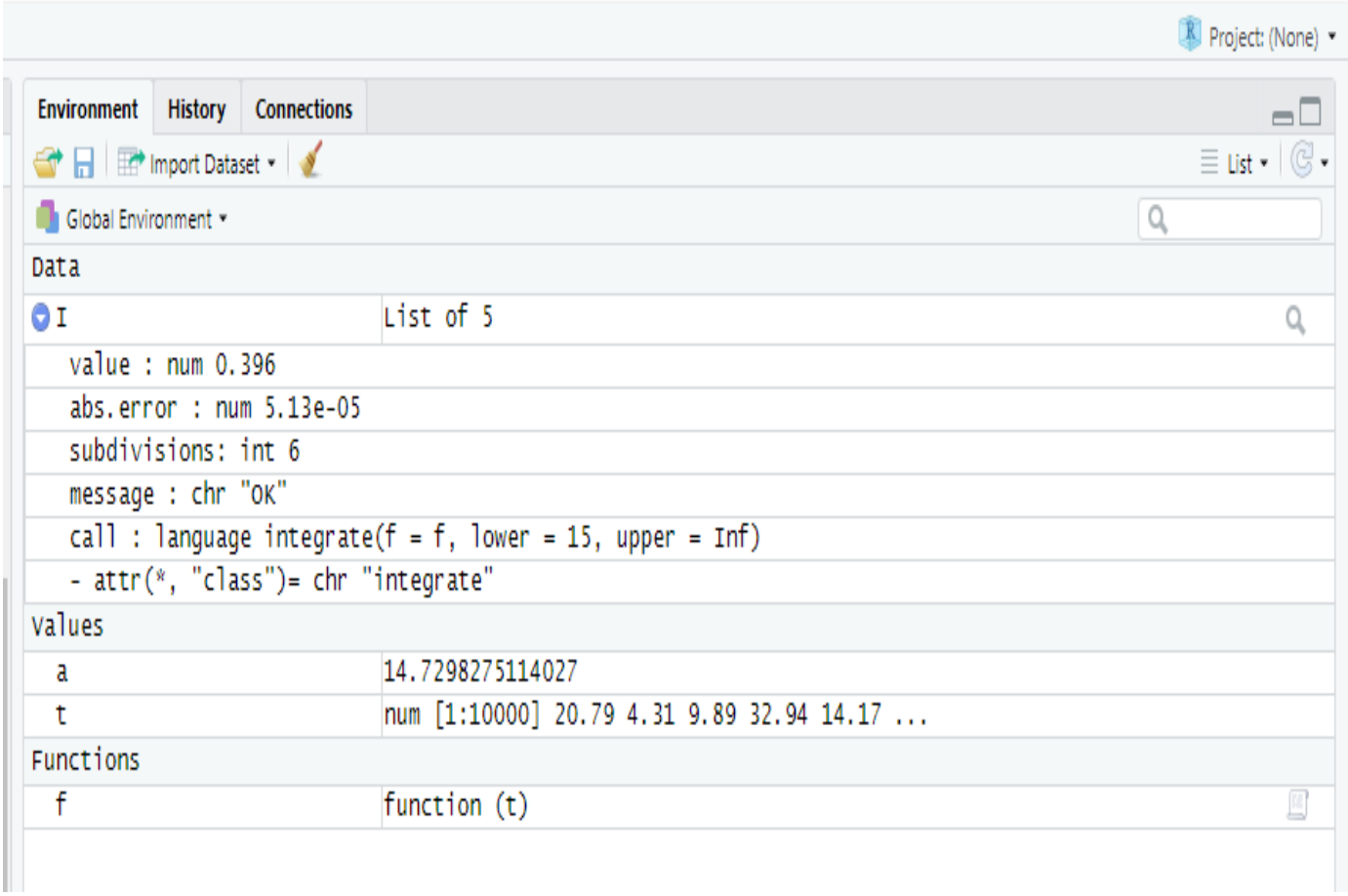
Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



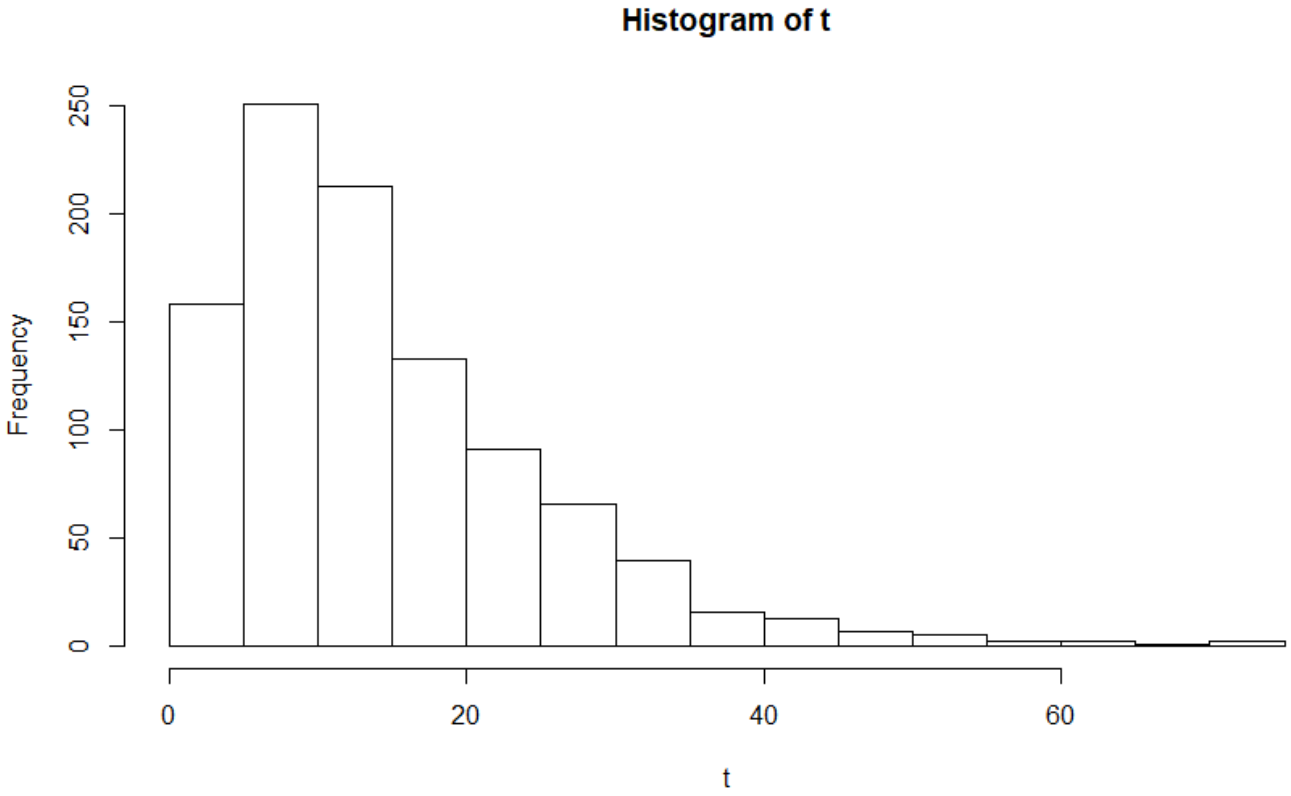
E(T) and the probability that the satellite lasts more than 15 years:



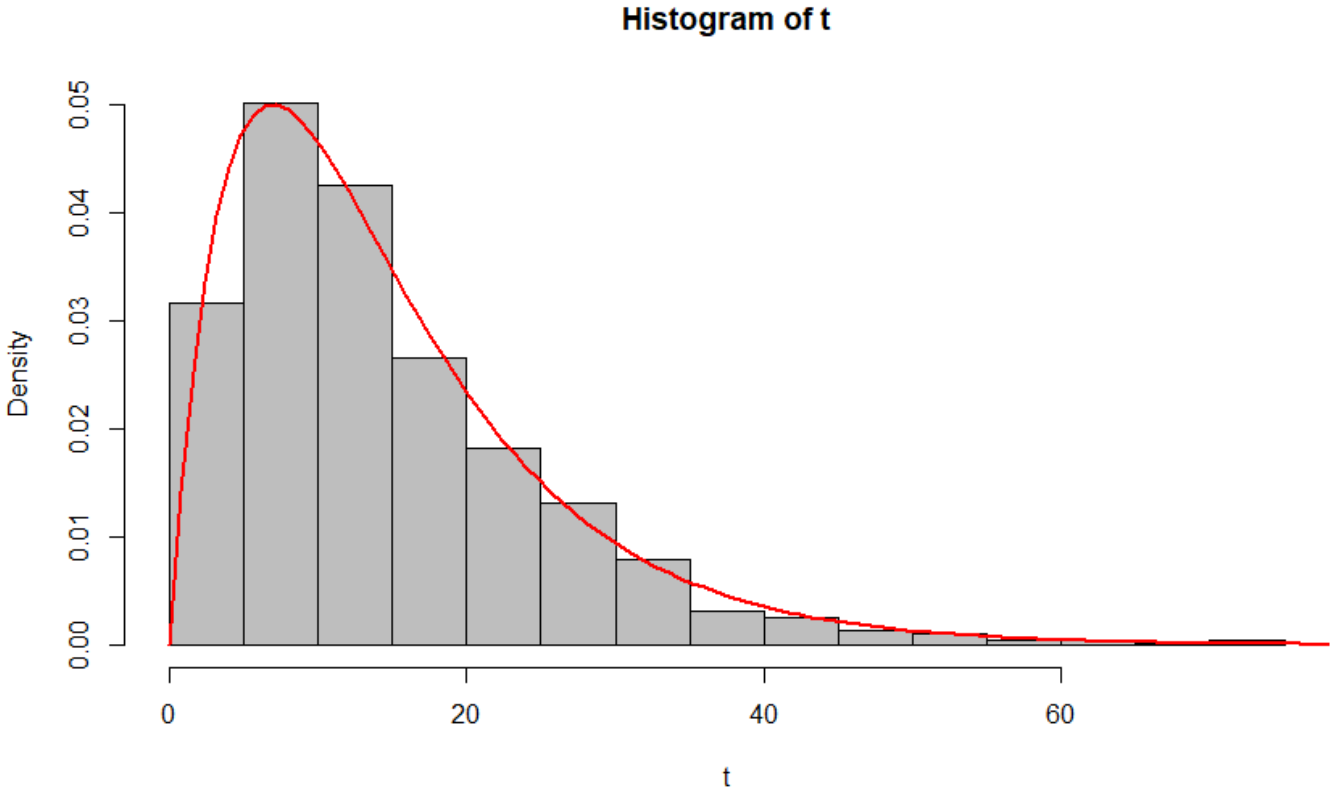
Case 2: The R code for simulation at n=1,000 is same as that of the above R code for 10,000. The only change is that replicate the values 1,000 times instead of 10,000 times.

1st Iteration outcomes are as follows:

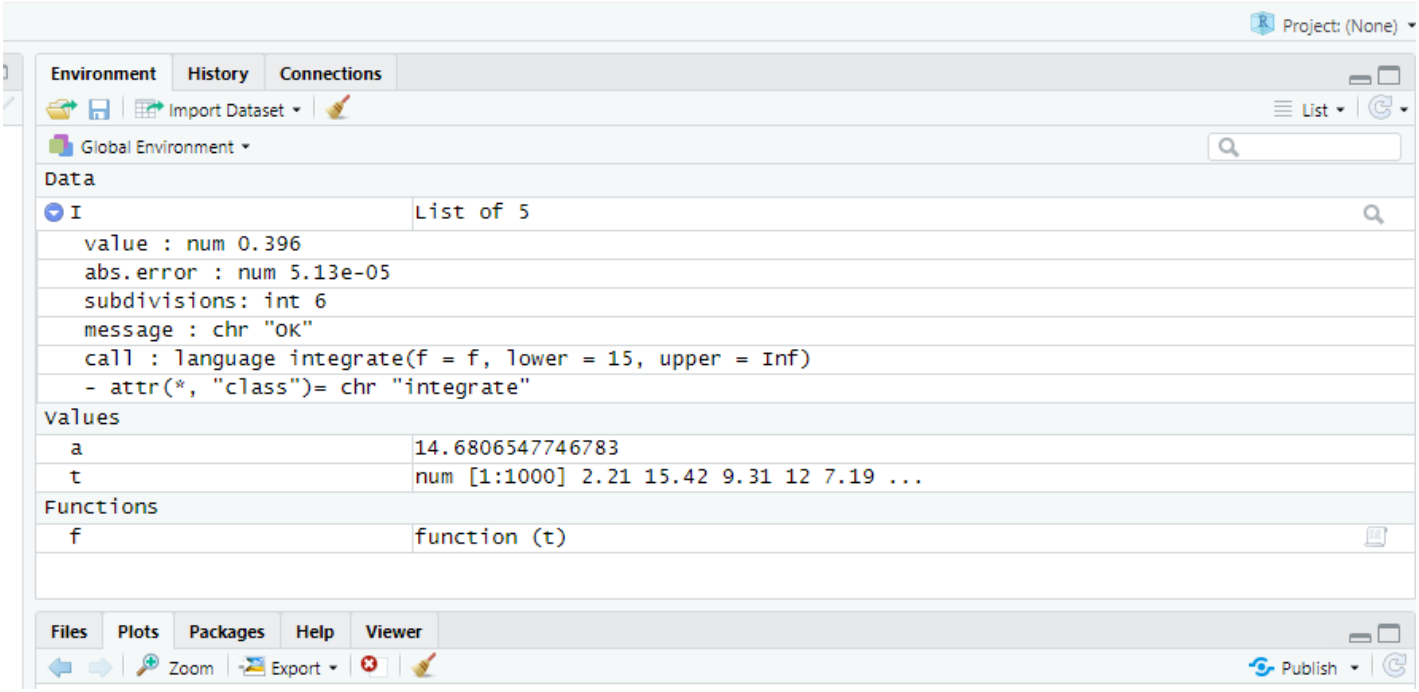
Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



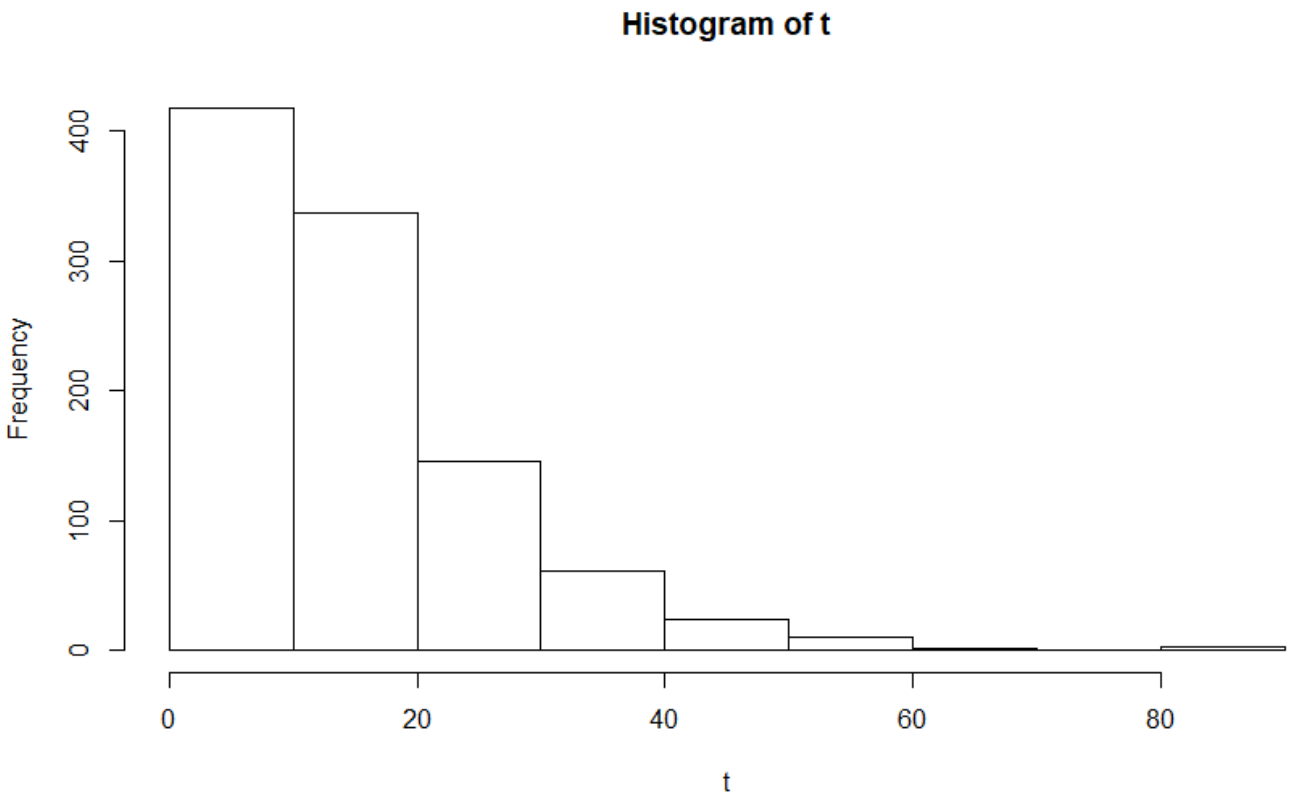
E(T) and the probability that the satellite lasts more than 15 years:



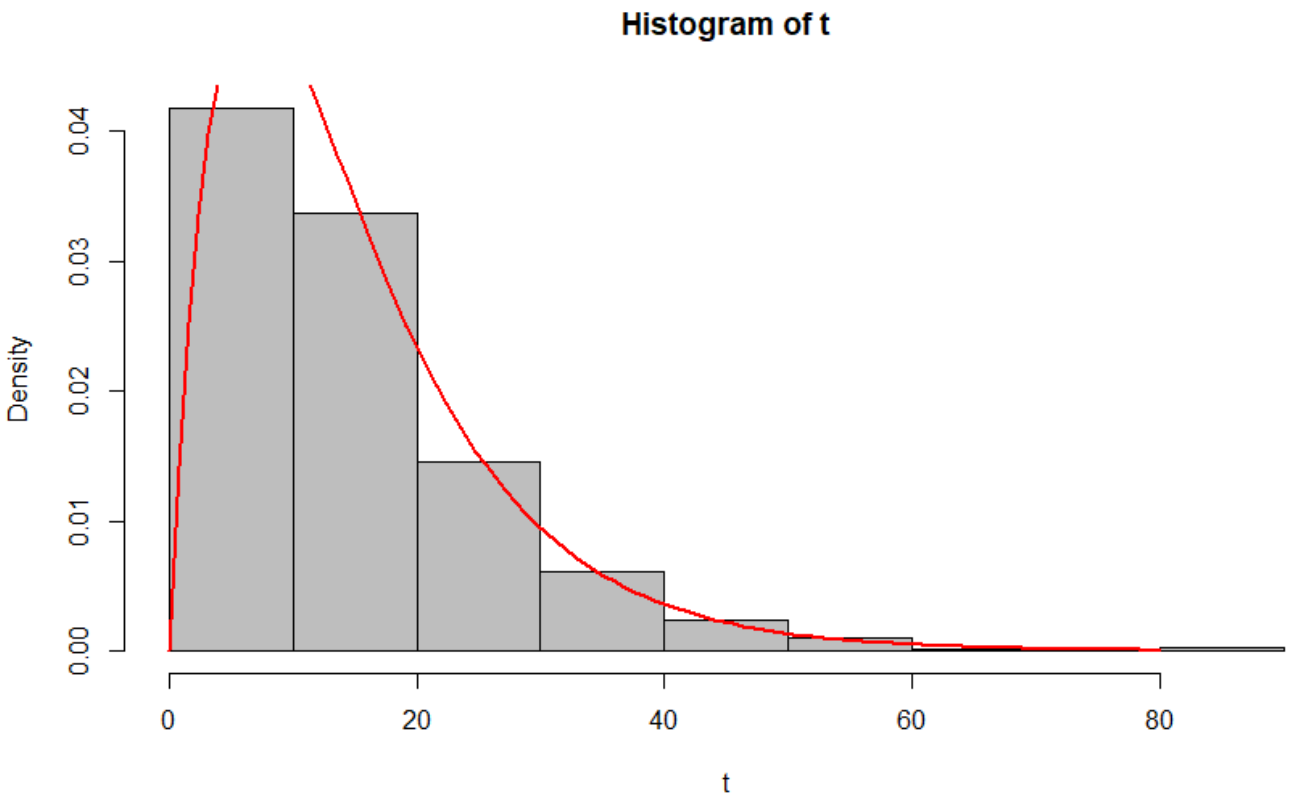
Observation: The actual mean value and the estimated mean value are not that close to each other for the 1st iteration as compared to how close they were when n=10,000. The probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

2nd Iteration outcomes are as follows:

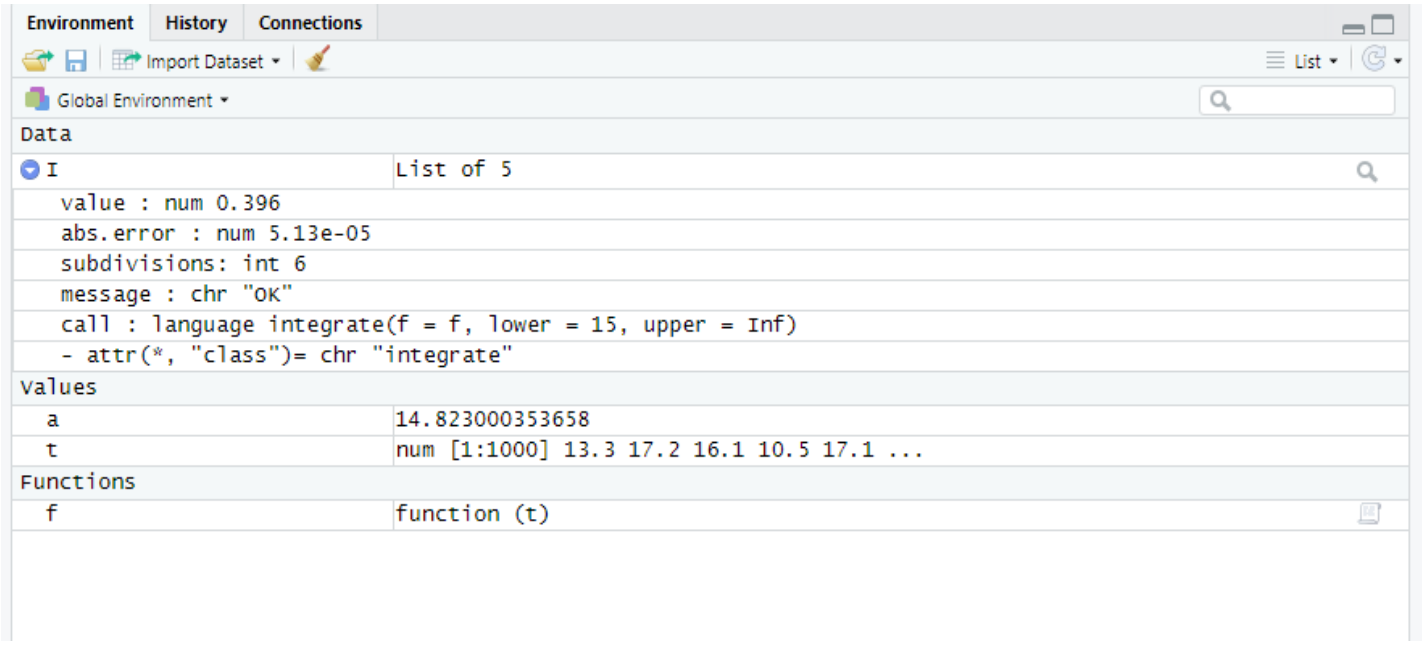
Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



E(T) and the probability that the satellite lasts more than 15 years:

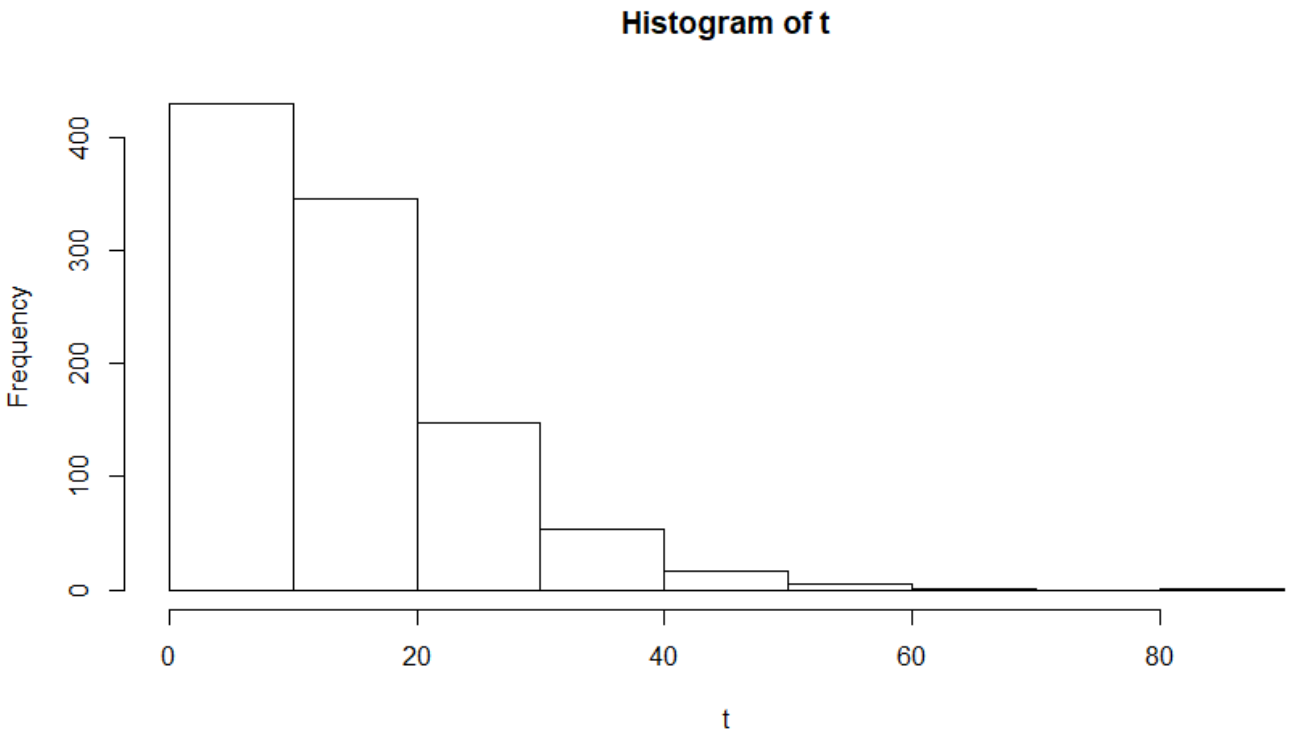


Observation: The actual mean value and the estimated mean value are gradually coming close to each other for the 2nd iteration. The probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

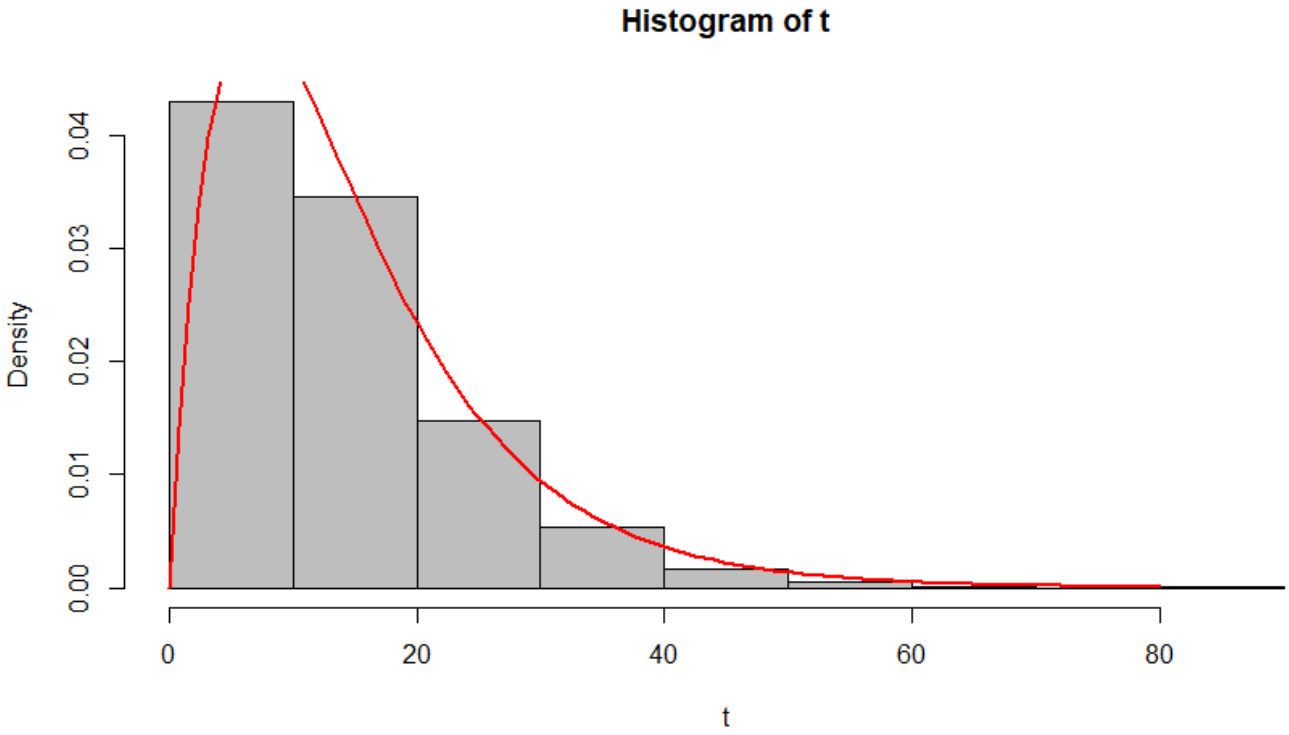
3rd Iteration outcomes are as follows:

Histogram of the lifetime of a satellite (t):

Density function and Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



E(T) and the probability that the satellite lasts more than 15 years:

Environment | History | Connections

Import Dataset | List

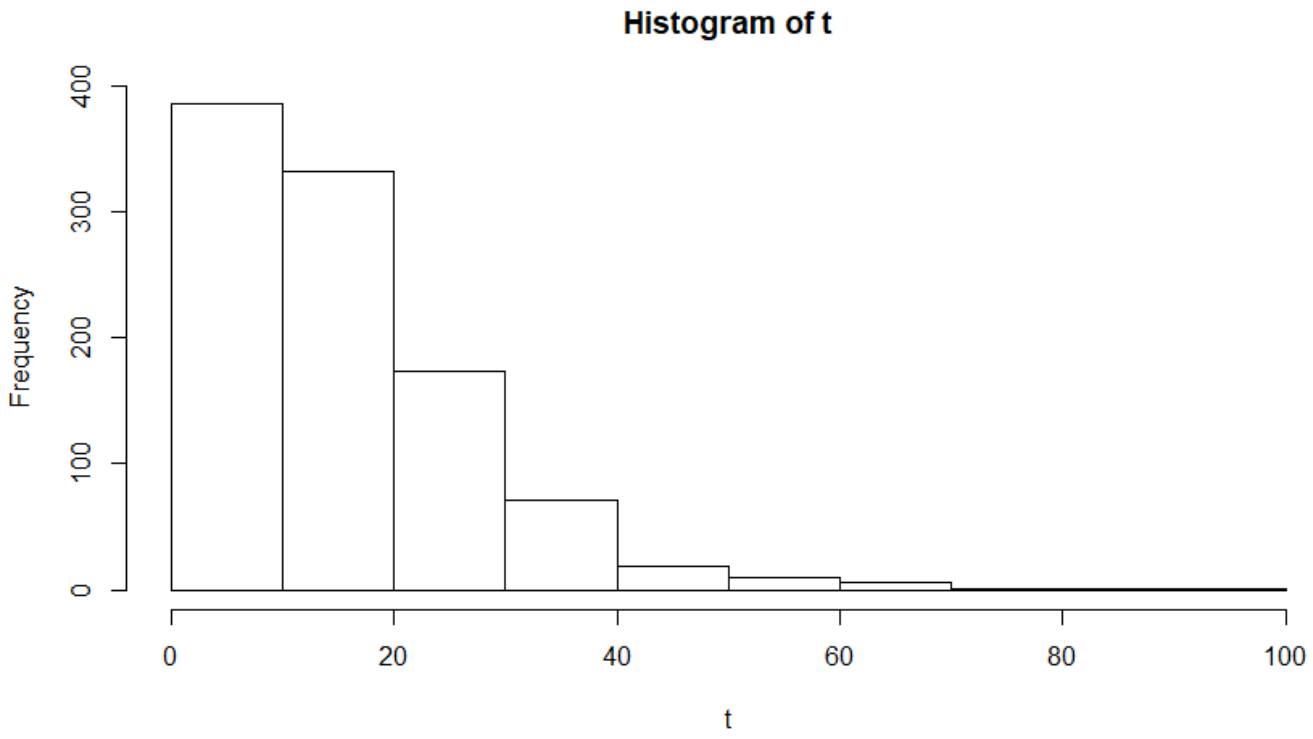
Global Environment

Data	
I	List of 5
value : num 0.396	
abs.error : num 5.13e-05	
subdivisions: int 6	
message : chr "OK"	
call : language integrate(f = f, lower = 15, upper = Inf)	
- attr(*, "class")= chr "integrate"	
Values	
a	13.9444842365028
t	num [1:1000] 14.38 4.42 11 4.15 20.61 ...
Functions	
f	function (t)

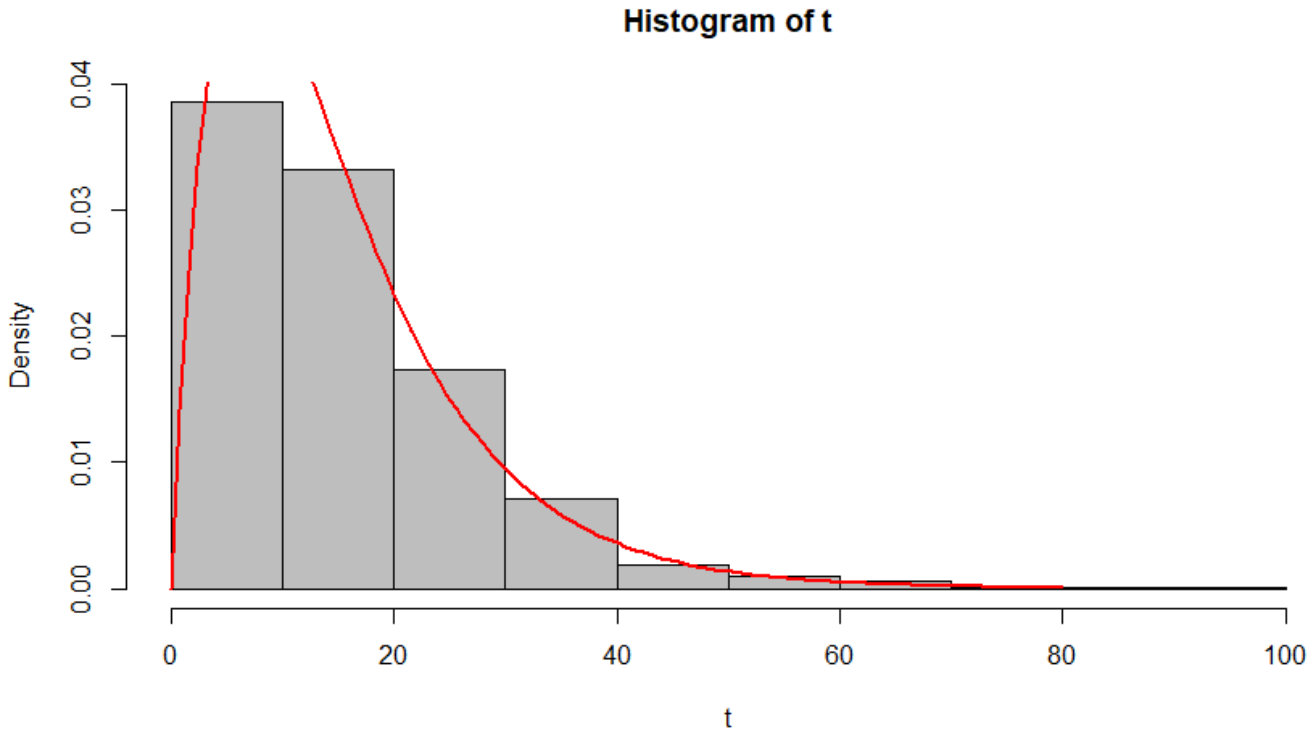
Observation: The actual mean value and the estimated mean value are very far from each other for the 3rd iteration. They were closer in the 2nd Iteration. The probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

4th Iteration outcomes are as follows:

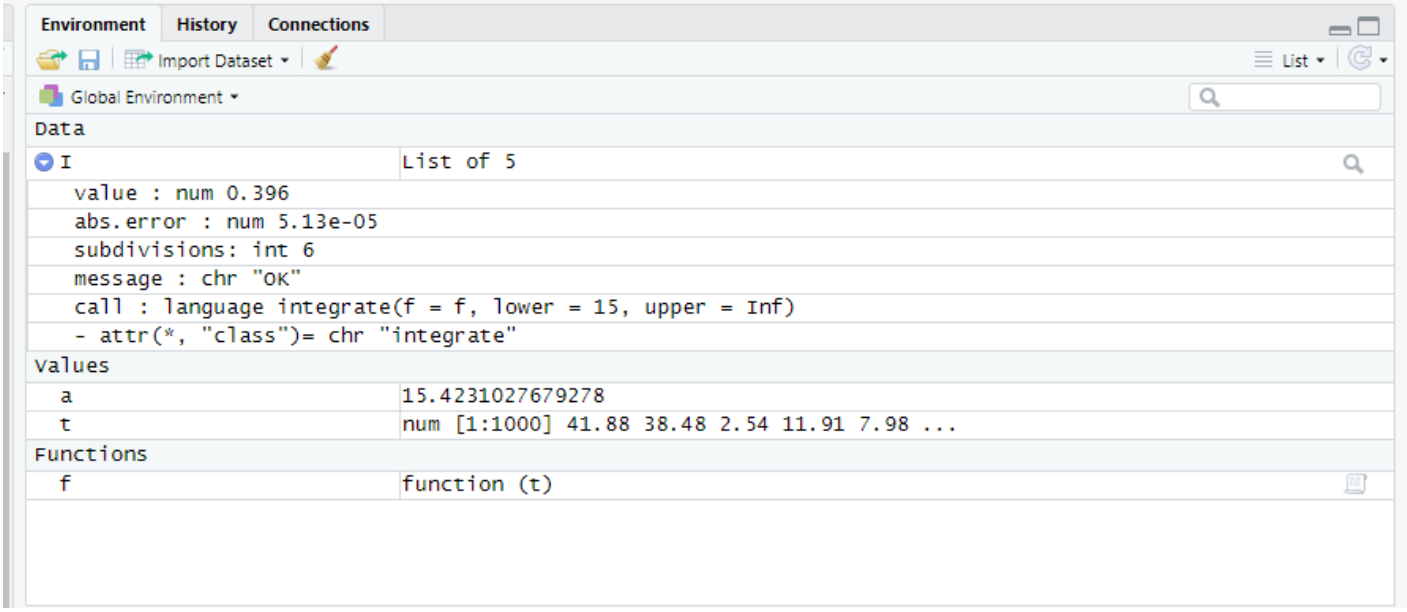
Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



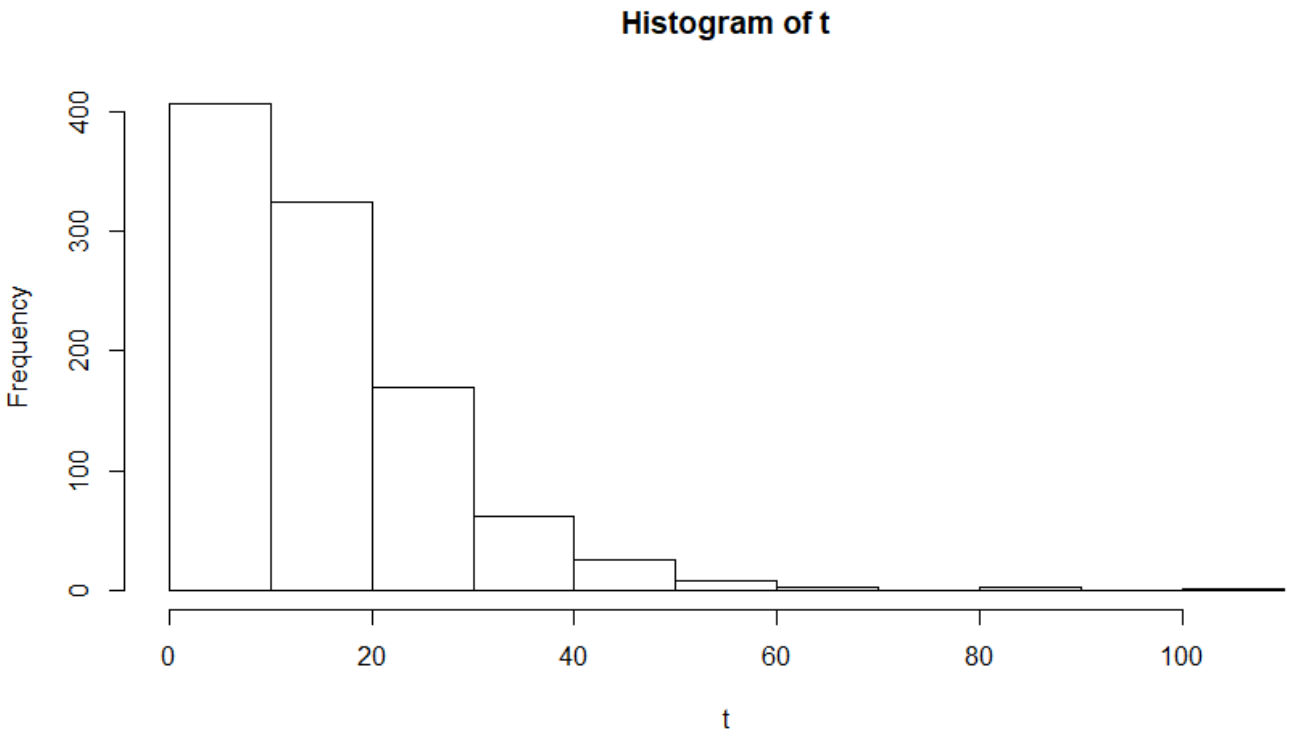
E(T) and the probability that the satellite lasts more than 15 years:



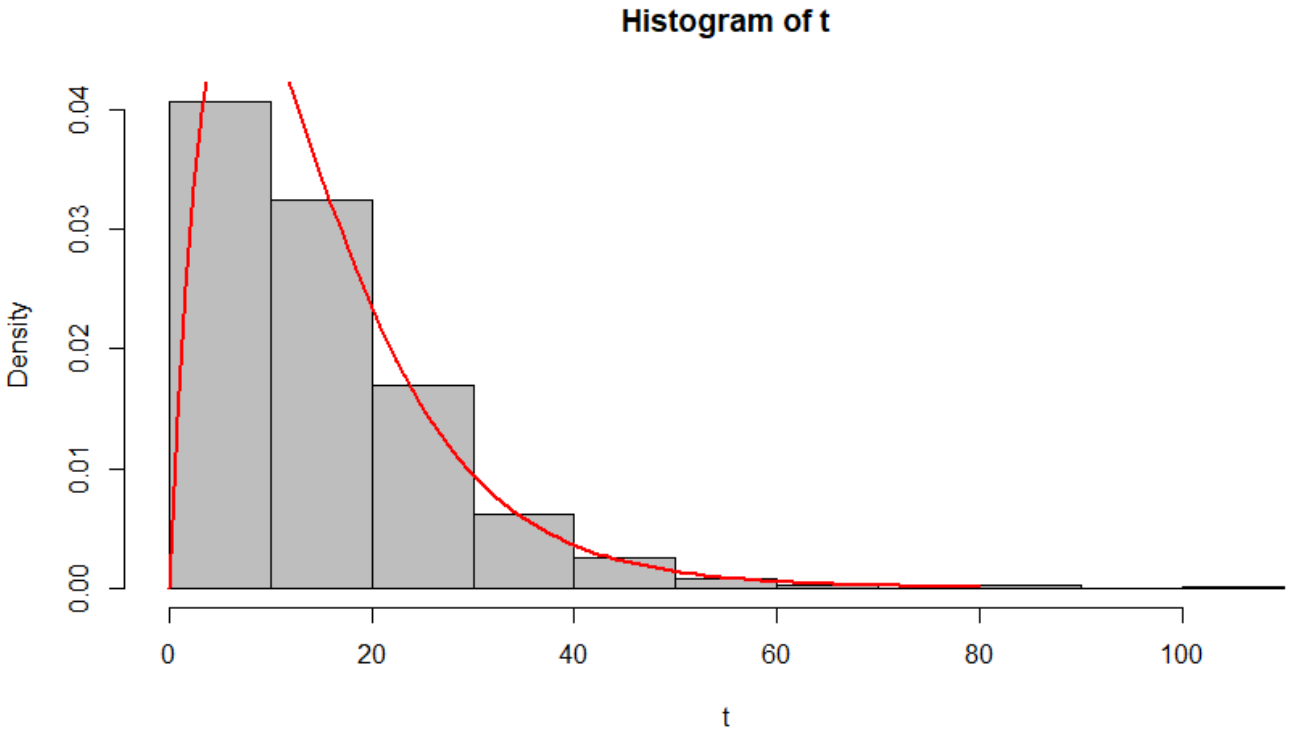
Observation: The actual mean value and the estimated mean value is close to each other for the 4th iteration. The value exceeds the actual mean value but is better than the 3rd Iteration. The probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

5th Iteration outcomes are as follows:

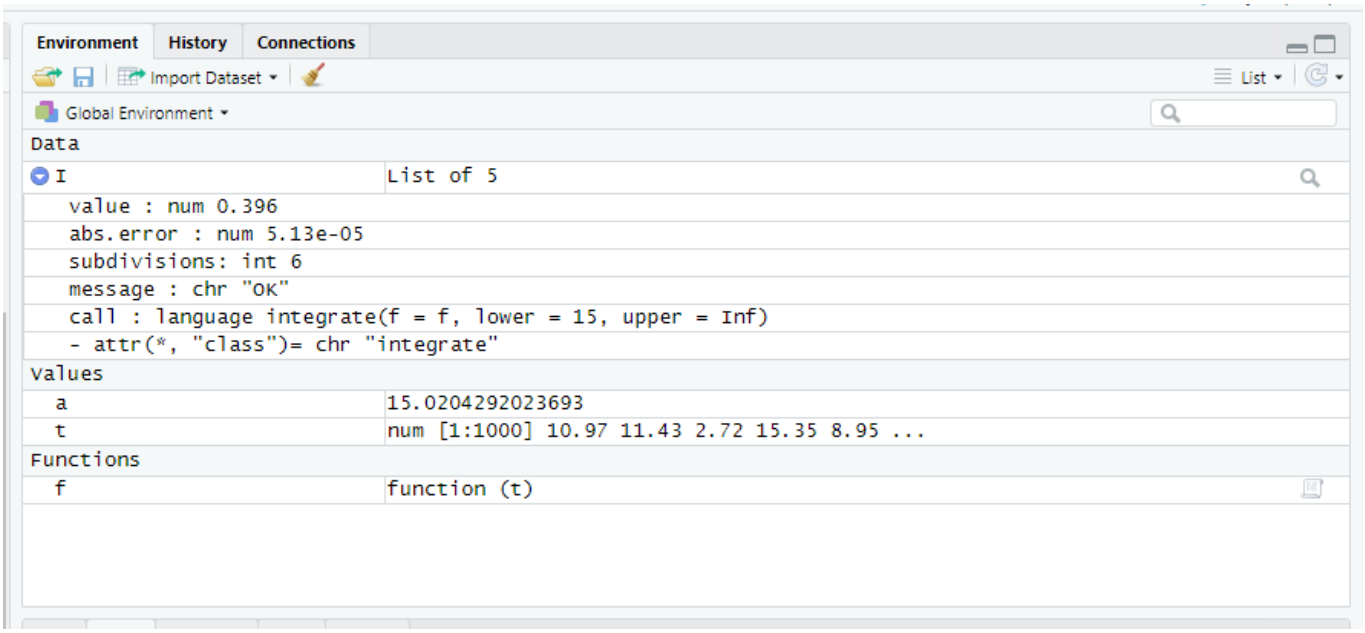
Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



E(T) and the probability that the satellite lasts more than 15 years:

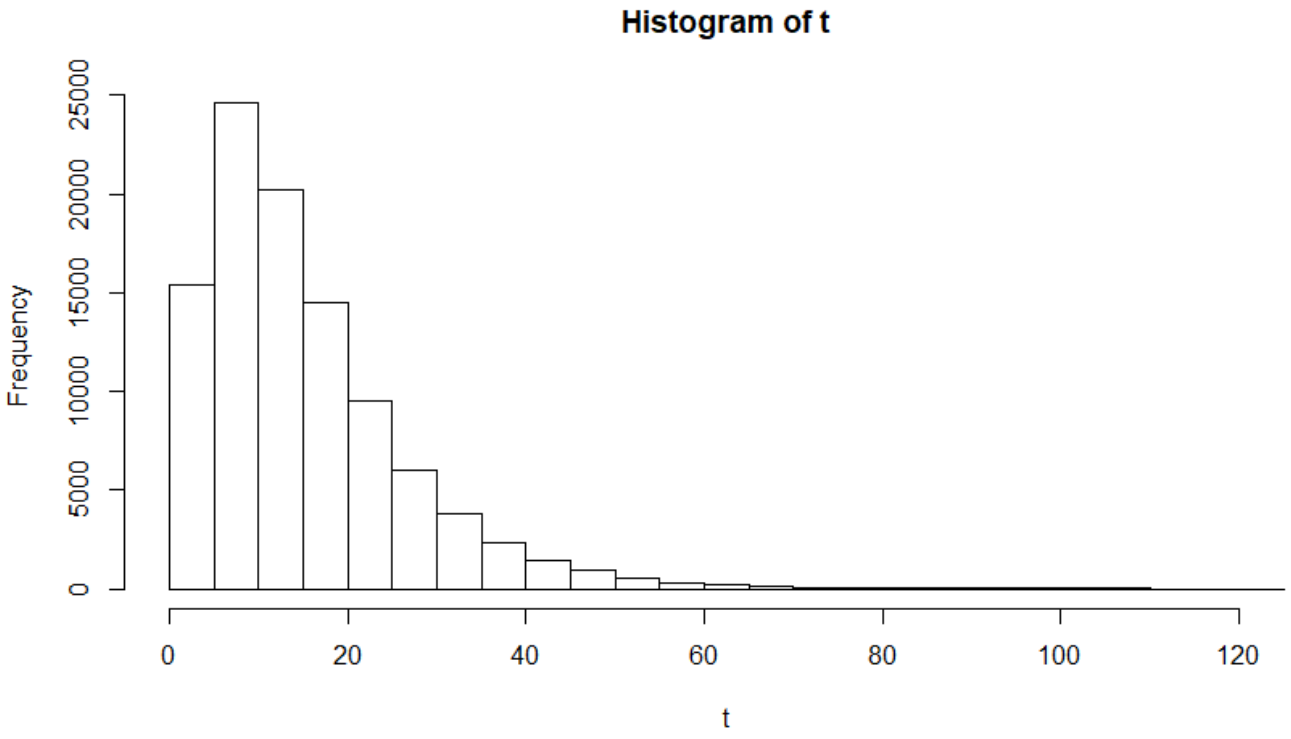


Observation: The actual mean value and the estimated mean value is very close to each other for the 5th iteration. The probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

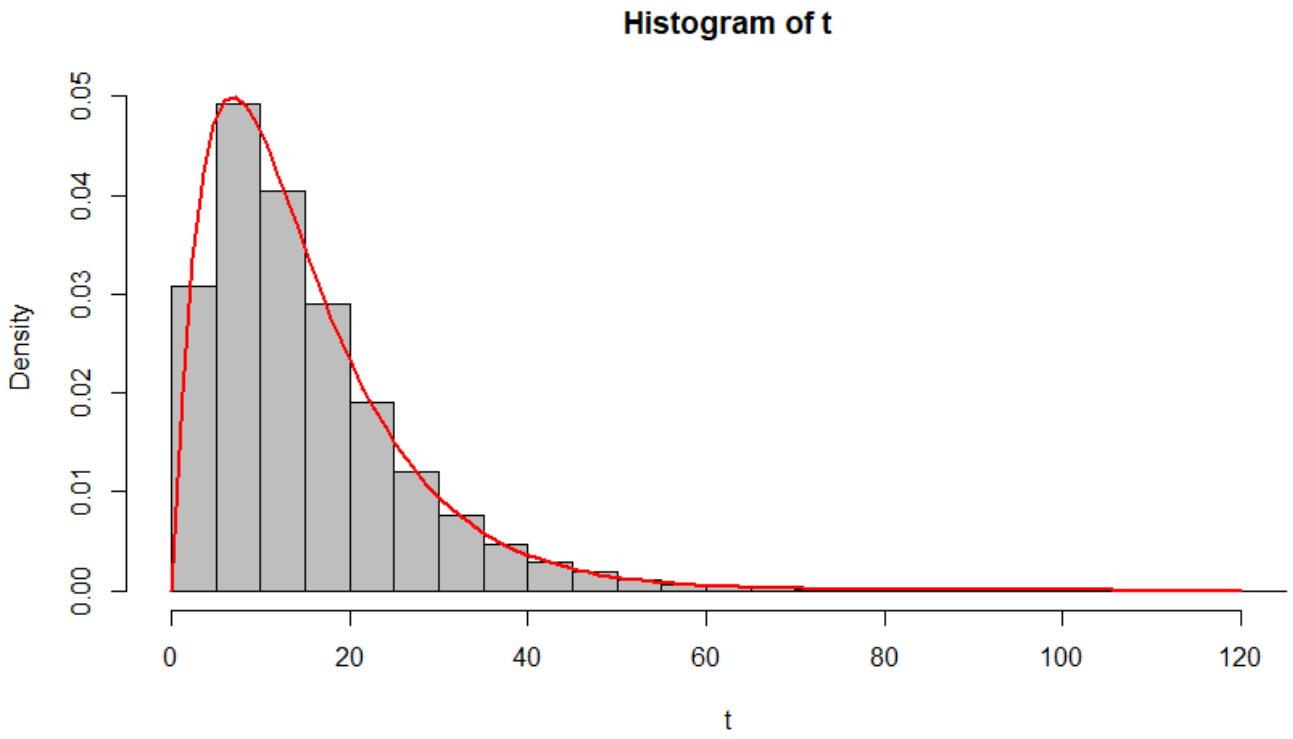
Case 3: The R code for simulation at n=1,00,000 is same as that of the above R code for 10,000. The only change is that we replicate the values 1,00,000 times instead of 10,000 times. There is also change in the range of values for which the histogram and curve is drawn. The range is (0,120).

1st Iteration outcomes are as follows:

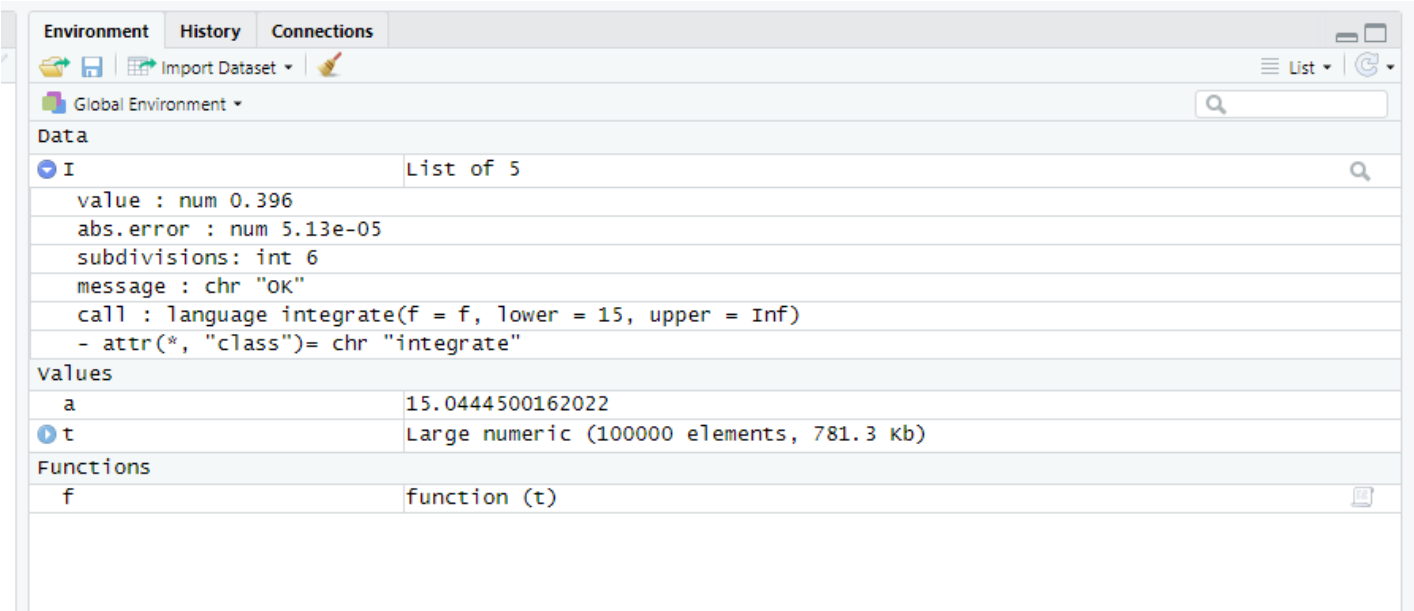
Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



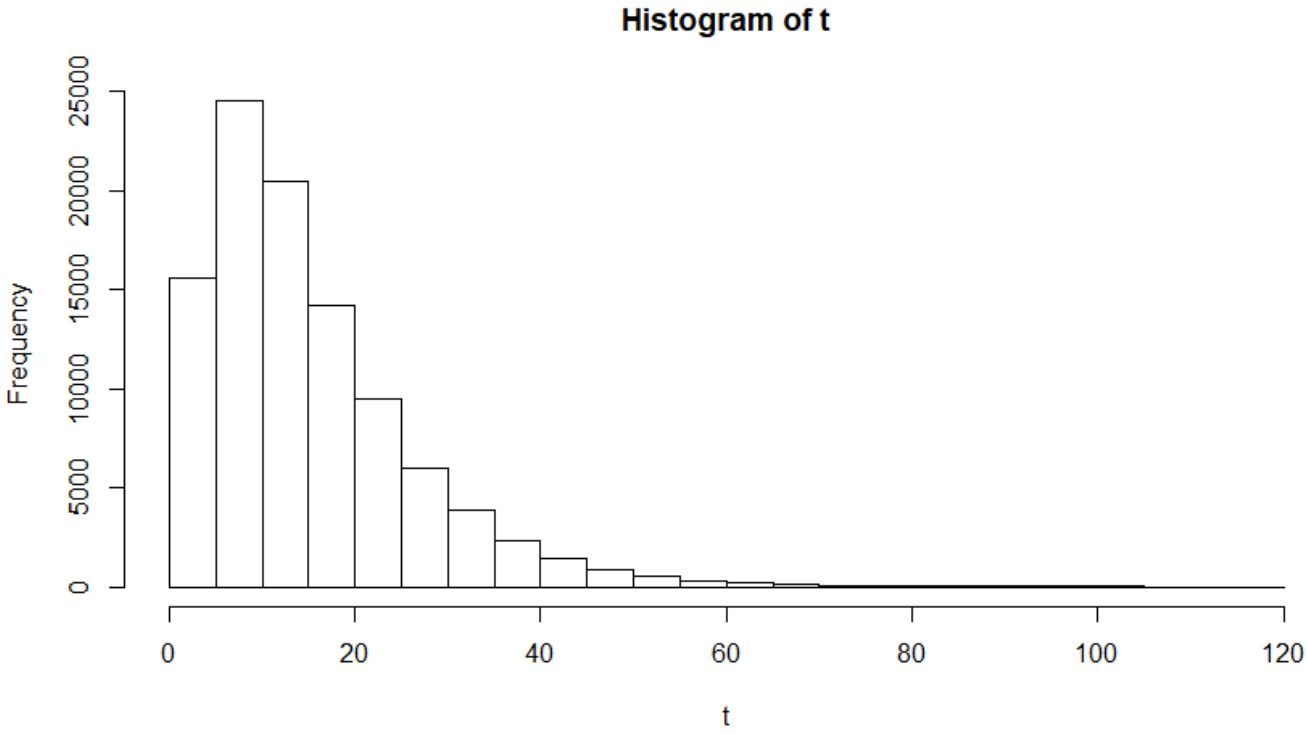
E(T) and the probability that the satellite lasts more than 15 years:



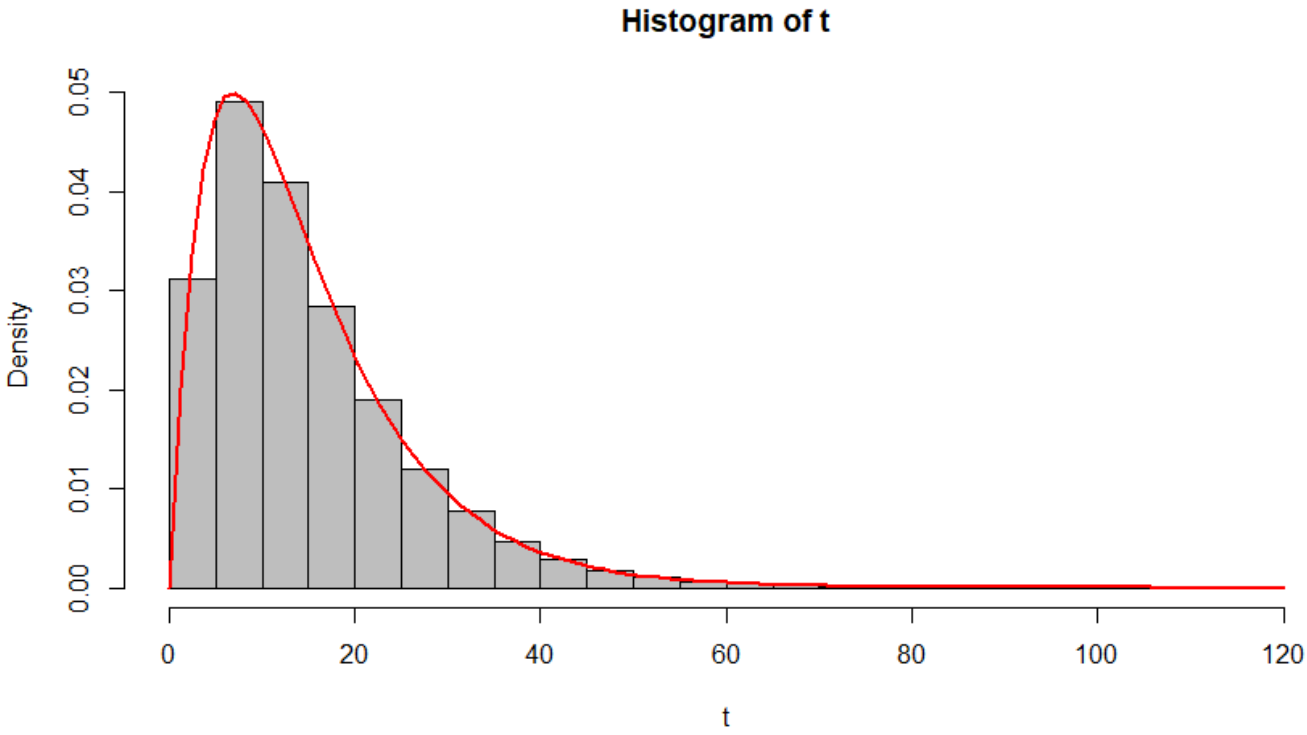
Observation: The actual mean value and the estimated mean value is very close to each other for the 1st iteration and the probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

2nd Iteration outcomes are as follows:

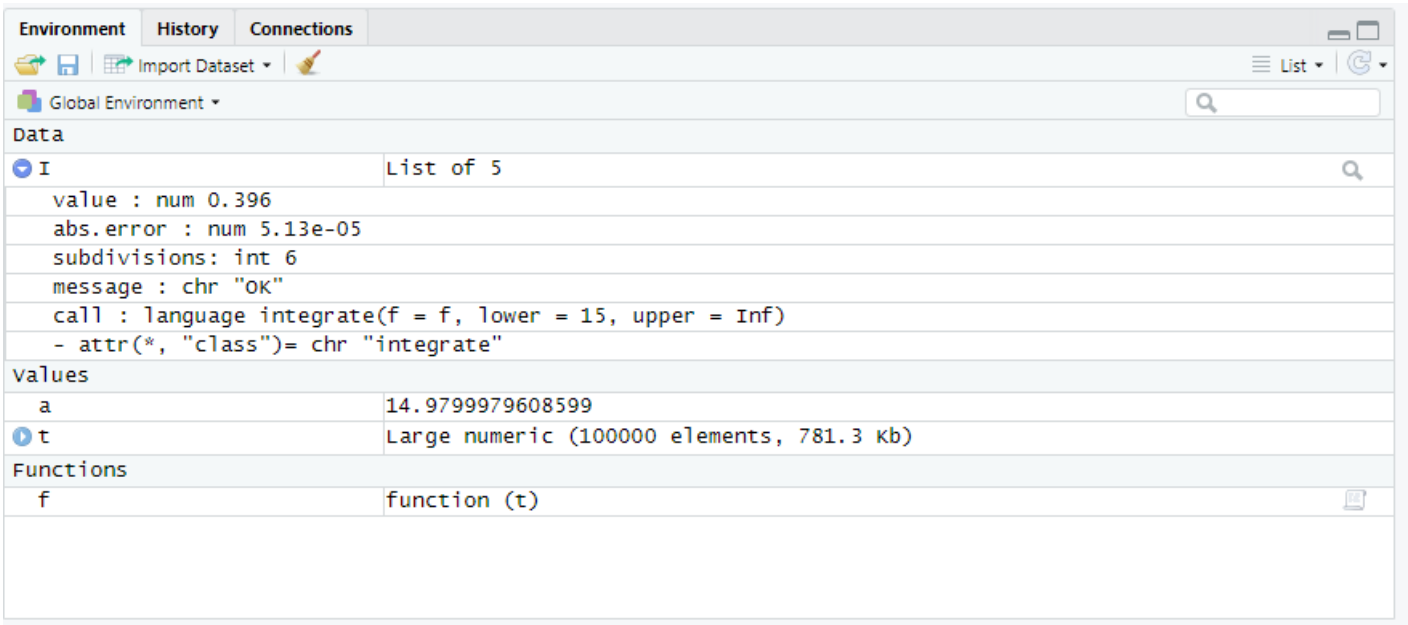
Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



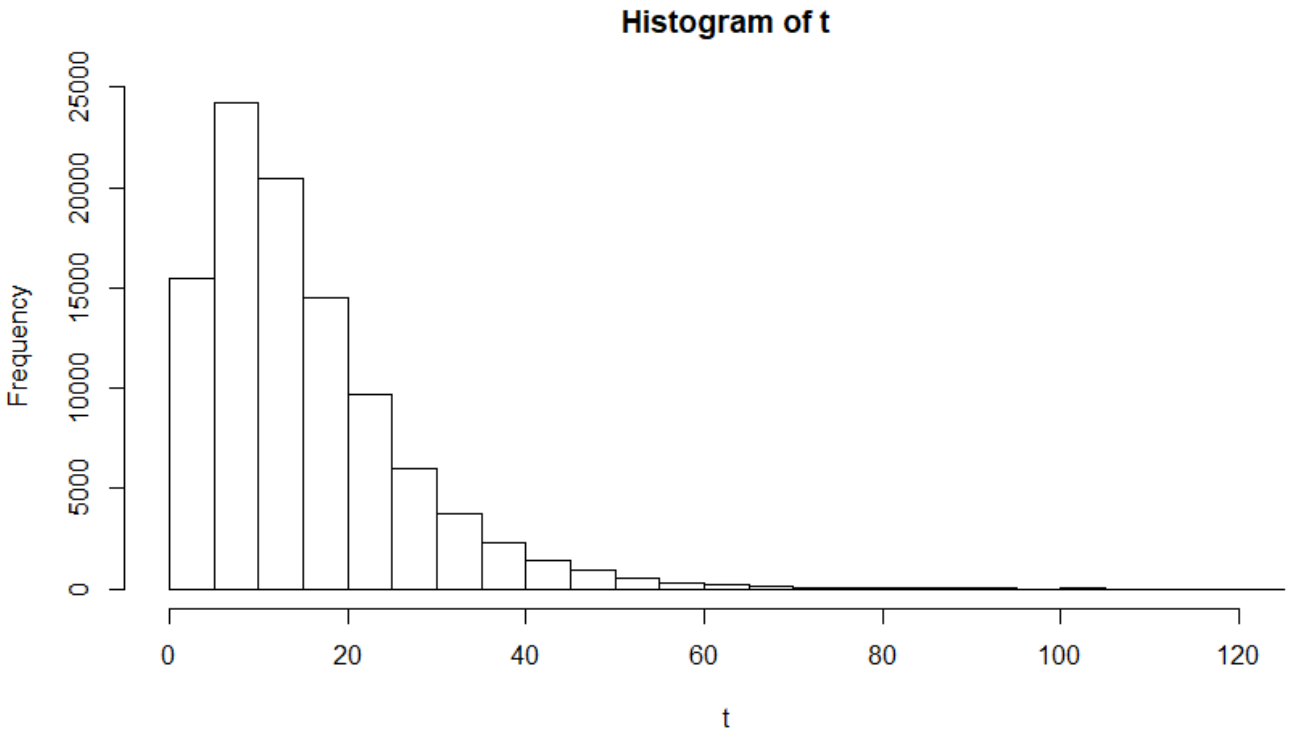
E(T) and the probability that the satellite lasts more than 15 years:



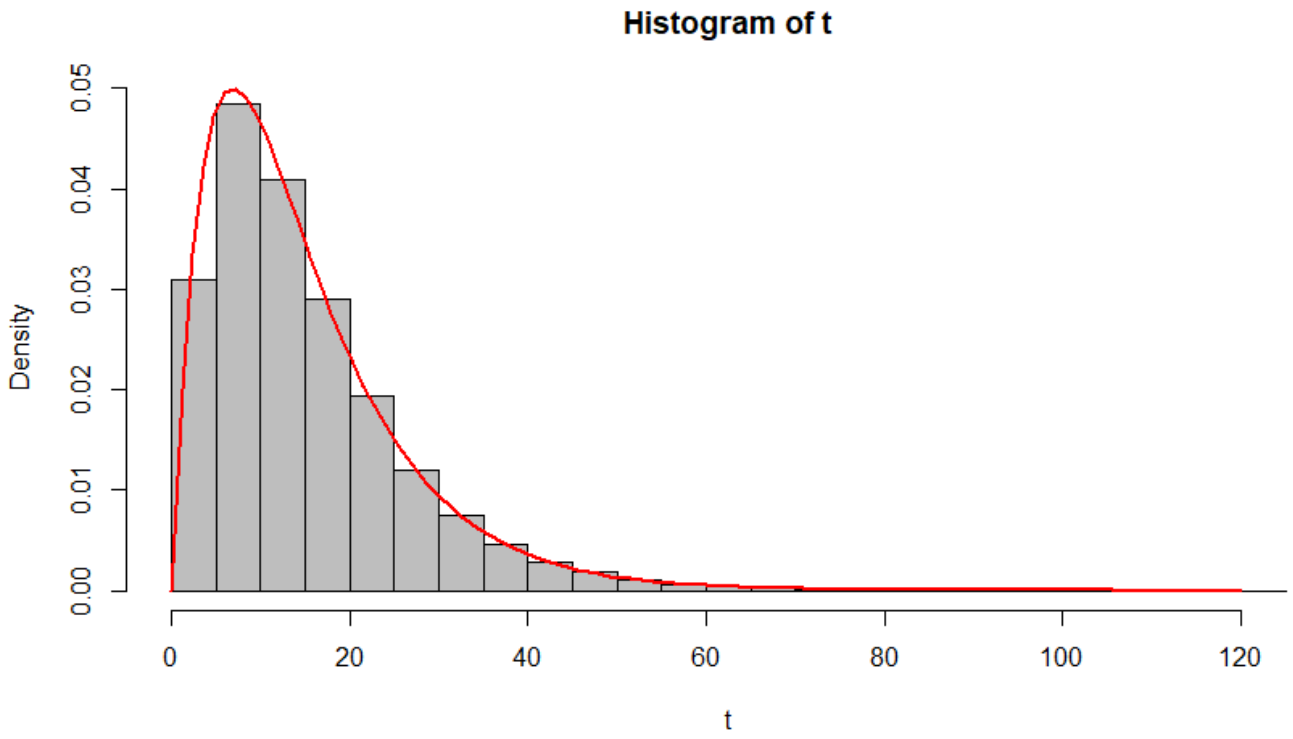
Observation: The actual mean value and the estimated mean value is very close to each other for the 2nd iteration. It is not as close as Iteration 1 but it is not bad. The probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

3rd Iteration outcomes are as follows:

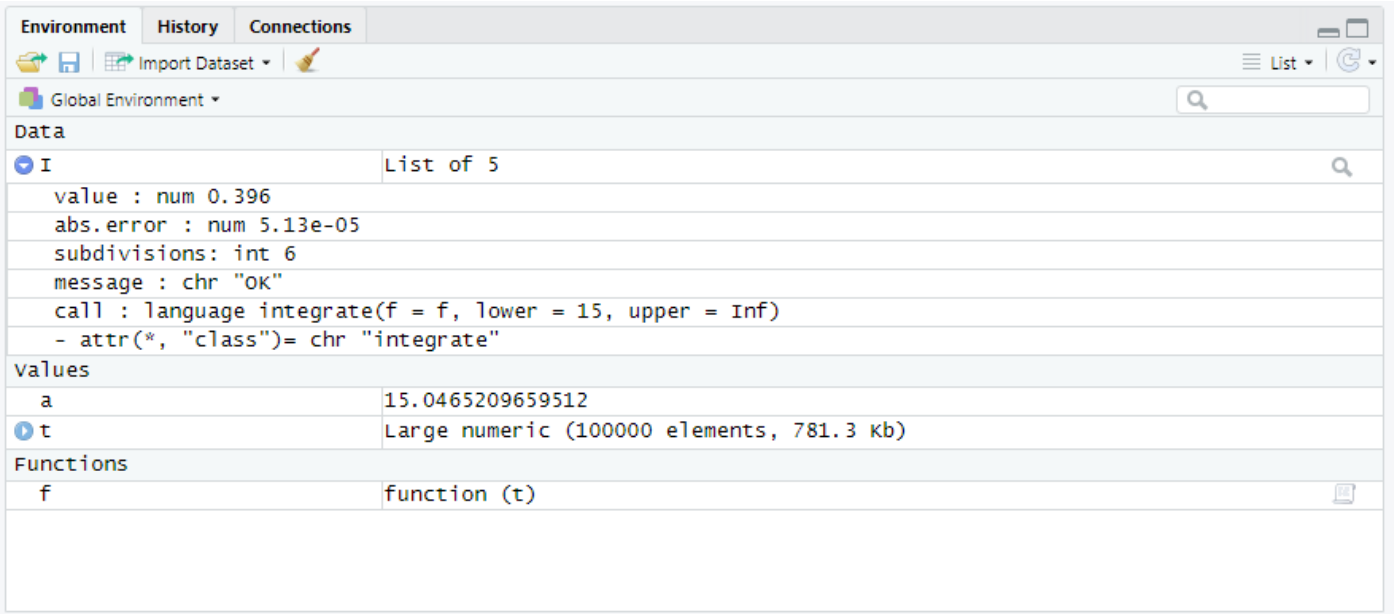
Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



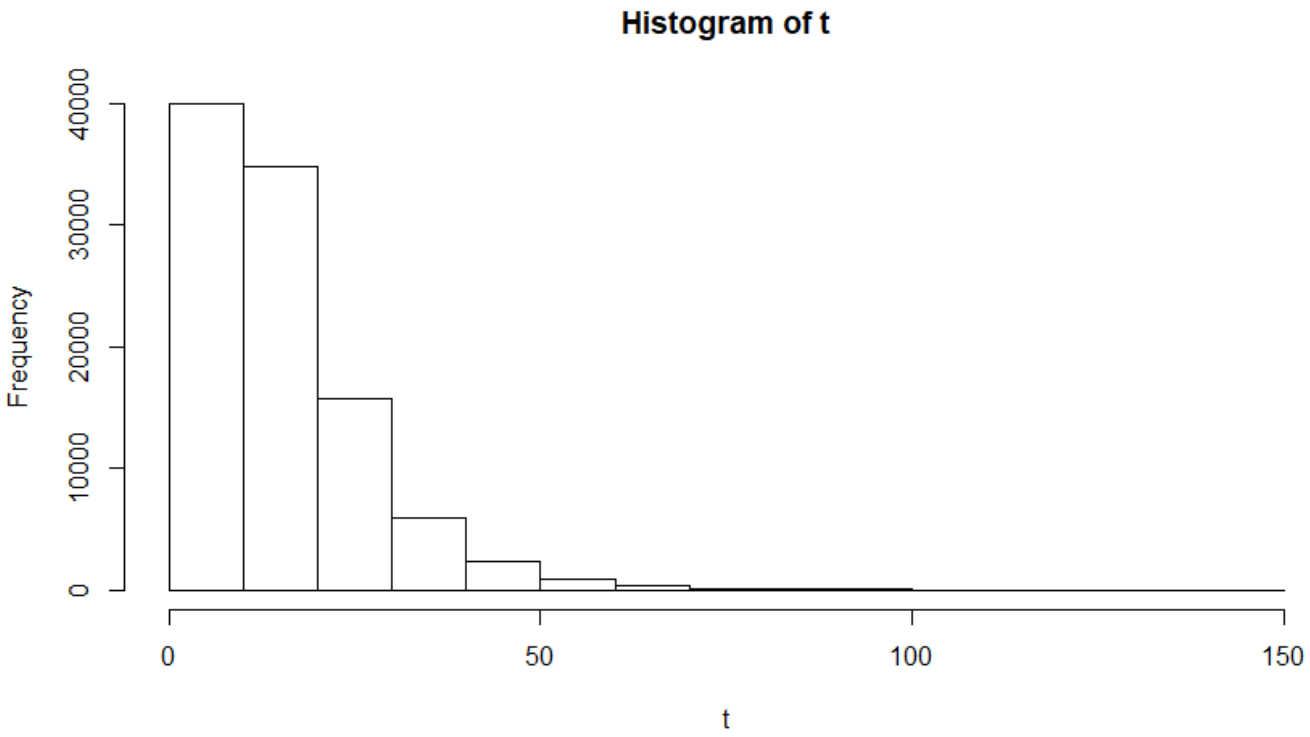
E(T) and the probability that the satellite lasts more than 15 years:



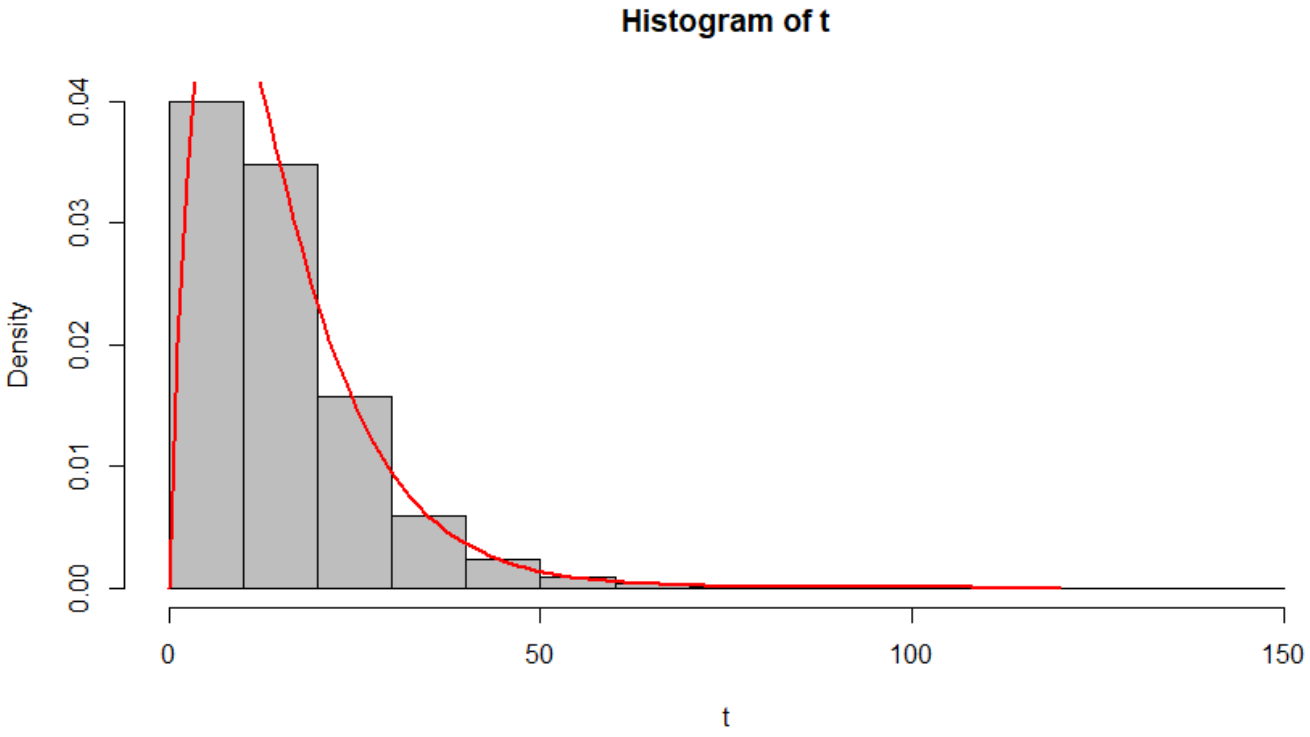
Observation: The actual mean value and the estimated mean value is very close to each other for the 3rd iteration. It is better than the 2nd Iteration and similar to 1st Iteration. The probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

4th Iteration outcomes are as follows:

Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



E(T) and the probability that the satellite lasts more than 15 years:

Environment | History | Connections

Import Dataset | List

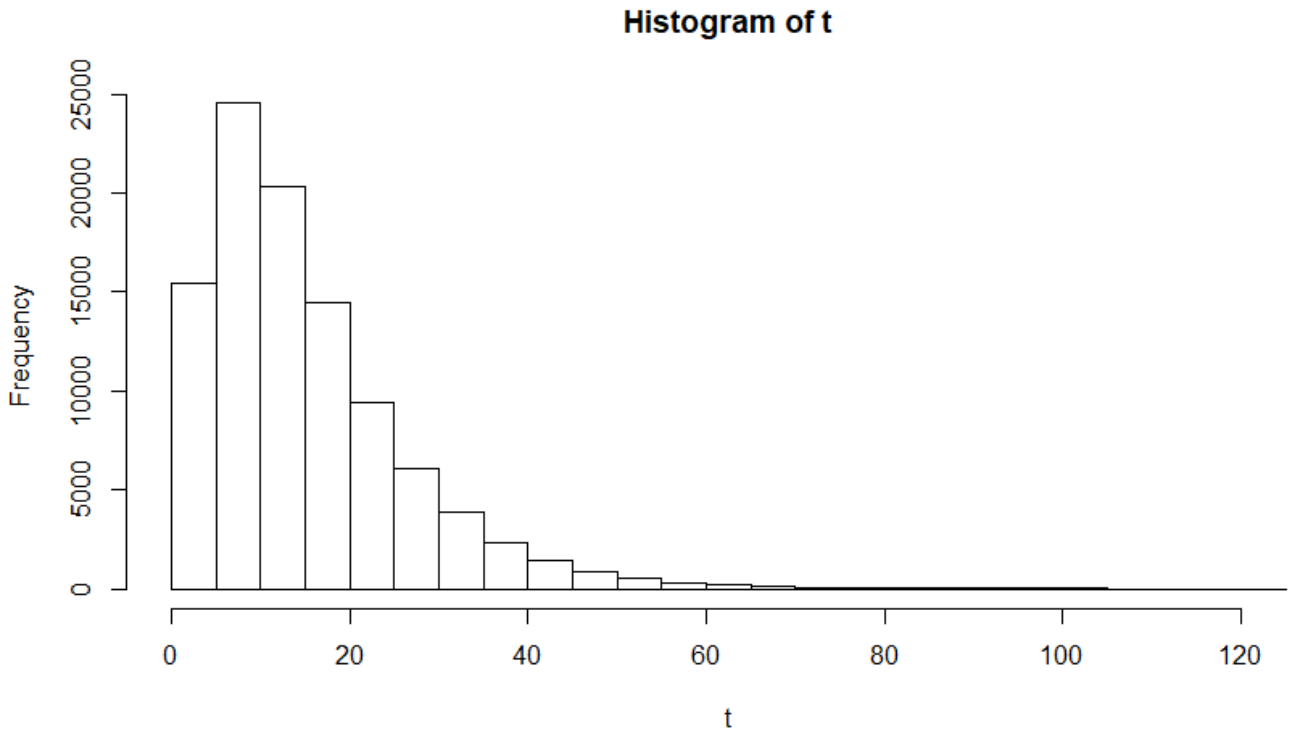
Global Environment

Data	
I	List of 5
value : num 0.396	
abs.error : num 5.13e-05	
subdivisions: int 6	
message : chr "OK"	
call : language integrate(f = f, lower = 15, upper = Inf)	
- attr(*, "class")= chr "integrate"	
Values	
a	14.985326134463
t	Large numeric (100000 elements, 781.3 Kb)
Functions	
f	function (t)

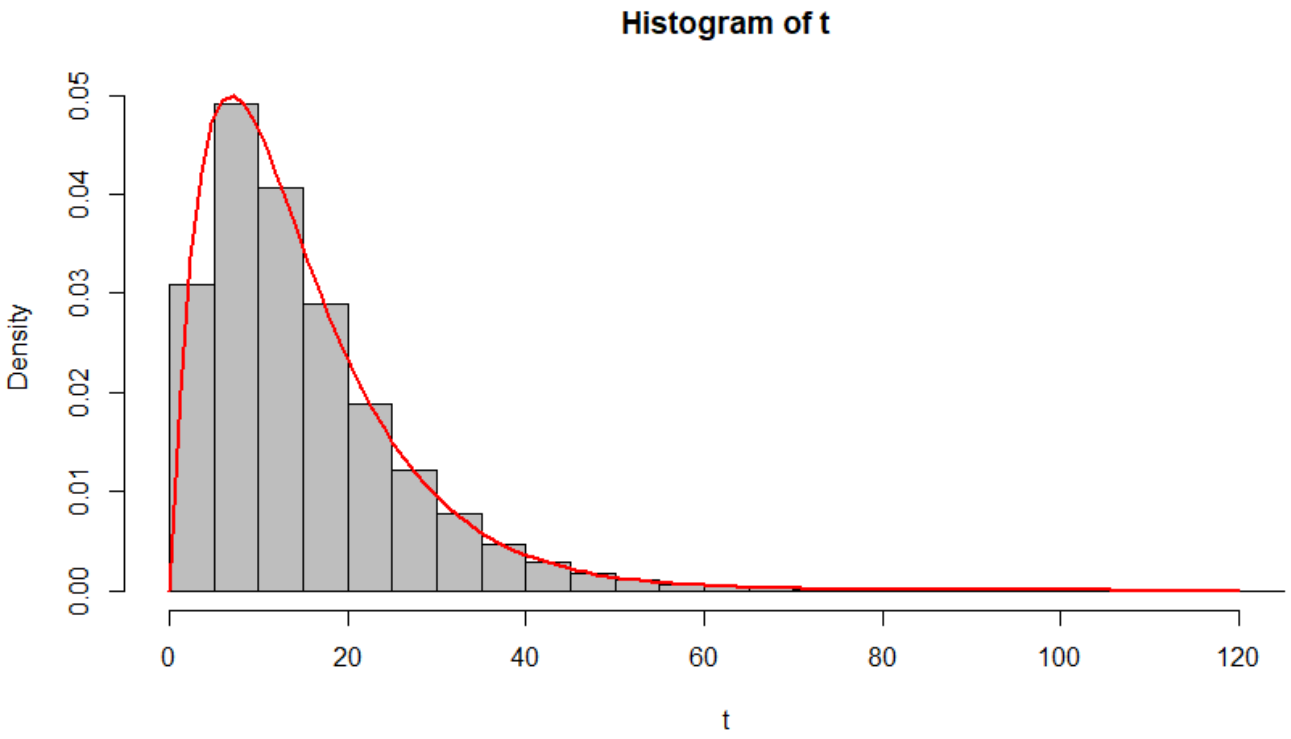
Observation: The actual mean value and the estimated mean value is close to each other for the 4th iteration and the probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

5th Iteration outcomes are as follows:

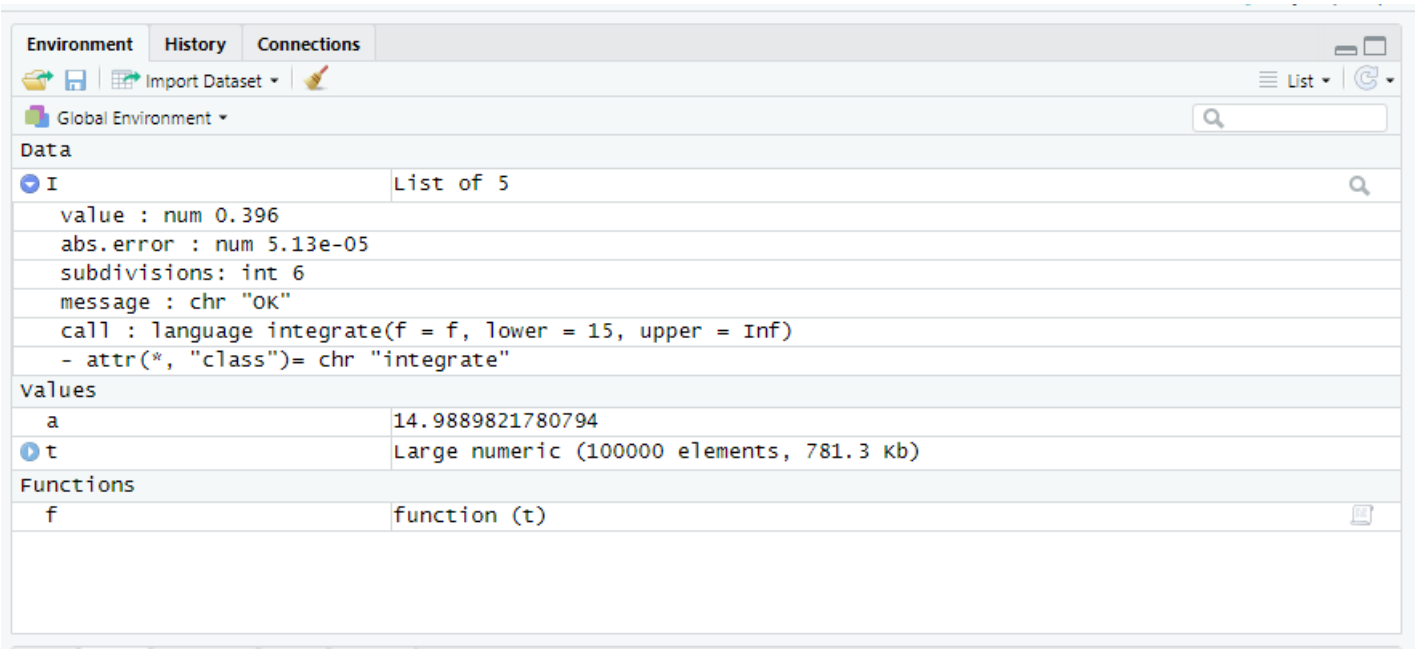
Histogram of the lifetime of a satellite (t):



Density function and Histogram of the lifetime of a satellite (t):



E(T) and the probability that the satellite lasts more than 15 years:



Observation: The actual mean value and the estimated mean value is close to each other for the 5th iteration. The probability that the satellite last for more than 15 years is 0.396 or 39.6 %.

Table of Observations:

VALUE OF N	ITERATION	E(T)	P(X>=15)
N= 10,000	Iteration 1	15.11	0.396
	Iteration 2	15.05	0.396
	Iteration 3	15.06	0.396
	Iteration 4	15.08	0.396
	Iteration 5	14.72	0.396
N= 1,000	Iteration 1	14.68	0.396
	Iteration 2	14.82	0.396
	Iteration 3	13.94	0.396
	Iteration 4	15.42	0.396
	Iteration 5	15.02	0.396
N= 1,00,000	Iteration 1	15.04	0.396
	Iteration 2	14.97	0.396
	Iteration 3	15.04	0.396
	Iteration 4	14.98	0.396
	Iteration 5	14.98	0.396

Final Conclusion:

With different sets of random value, as the number of replication (N) increases from 1,000 to 10,000 the expected mean value is approximately very close to the actual mean value. Hence, it proves the Law of Large Number. For a smaller number of replications, the variance between the actual mean value and the expected value is large. For larger number of replications, the variance between the actual mean value and the expected mean value is small. For larger number of replications, the range of t increases from (0,80) to (0,120). We also observe that the Density function is a normal distribution which skewed towards the right.

Q2. Use a Monte Carlo approach estimate the value of π based on 10,000 replications.

The R code for estimating value of π is as follows:

```
circle = function(long){                          #Creating a function circle
+   m=rep(0,long)                                 # Replicates long values having value 0
+   count=0                                       # Initialising counter value to 0
+   for(i in 1:long){
+     x=runif(2,0,1)                              # Generate 2 uniform values between range (0,1)
+     if(sqrt(x[1]*x[1] + x[2]*x[2]) <= 1){       # Computing distance between two points.
+       count=count+1                             # Increment counter if condition is TRUE.
+     }
+     proba=count/i                               # Probability of point lying within the circle.
+     piCap = proba*4                             # Multiplying probability by 4 to obtain single value of π
+     m[i]=piCap                                  # Storing the value to plot it later.
+   }
+   return(m)                                     # Returning value of m to caller function.
+ }
> n=10000                                         #Initialising n to 10000 for replications
> sim=circle(n)                                   #Calling circle function
> initial = 1
> plot(sim[initial:n],type="l")                   #Plotting the values obtained.
> lines(rep(pi,n)[initial:n],col="red")
```

The plot of the same is as follows: