

Fuzzy Logic:
Human-like decision making

Topics of this lecture

- Definition of fuzzy set
- Membership function
- Notation of fuzzy set
- Operations of fuzzy set
- Fuzzy number and operations
- Extension principle
- Fuzzy rules
- De-fuzzification
- Fuzzy control
- ファジィ集合の定義
- メンバシップ関数
- ファジィ集合の記述方法
- ファジィ集合の演算
- ファジィ数とその演算
- 拡張原理
- ファジィルール
- 非ファジィ化
- ファジィ制御

History of fuzzy logic

- The opposite word (antonym) of fuzzy is crisp. **Fuzzy** means un-clear or ambiguous, and **crisp** means clear, clean, and sharp.
- Fuzzy logic was proposed by Zadeh in 1965, and was applied in steam engine control by Mamdani in 1974.
- Fuzzy logic was made practically useful in Japan in the 1990s.



Conventional set vs. fuzzy set

- X : universe of discourse. Set A is defined by

$$A = \{ x \mid \mu(x) = T \wedge x \text{ in } X \}$$

where $\mu(x)$ is a **membership function**.

- For conventional set, the range of $\mu(x)$ is $\{T, F\}$
- For fuzzy set, the range of $\mu(x)$ is $[0,1]$. So, the above definition cannot be used for fuzzy set. We cannot say clearly if x is in A or not when $0 < \mu(x) < 1$.

Examples of fuzzy set

- ✓ young people, old people, kind people
- ✓ temperature is hot, just good, a little bit cold

Description of fuzzy set

- Membership function of a fuzzy set A : $\mu_A: X \rightarrow [0,1]$
- If the universe of discourse $X = \{x_1, x_2, \dots, x_N\}$, A is described as follows:

$$\begin{aligned} A &= \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \dots + \mu_A(x_N)/x_N \\ &= \sum \mu_A(x_i)/x_i \end{aligned}$$

where $/$ is a separator, and $+$ is the logic OR.

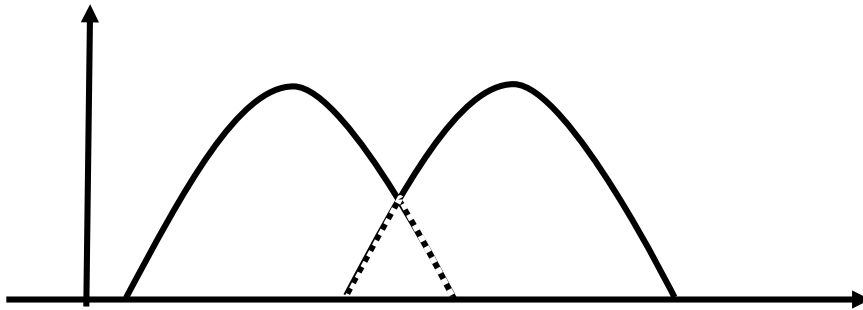
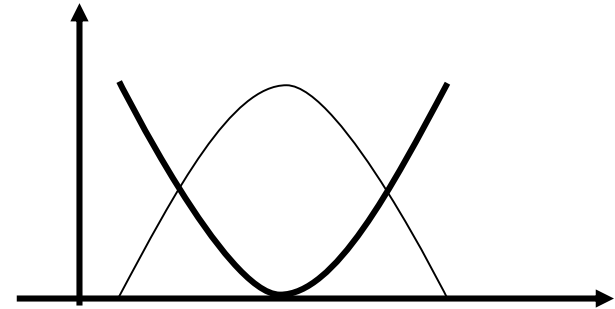
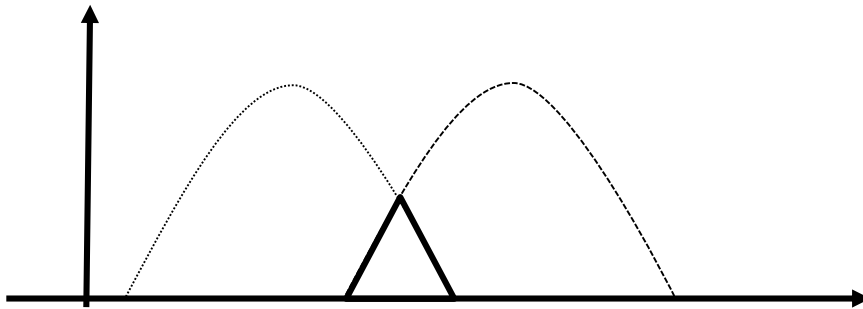
- If X is a continuous space, integral is used instead of the summation.



Operations of fuzzy sets

Meaning	Set notation	Logic notation	Membership function
Equivalence	$A = B$	$\mu_A(x) \Leftrightarrow \mu_B(x)$	$\mu_A(x) = \mu_B(x)$
Implication	$A \subseteq B$	$\mu_A(x) \Rightarrow \mu_B(x)$	$\mu_A(x) \leq \mu_B(x)$
Complement (negation)	\bar{A}	$\neg \mu_A(x)$	$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$
Union, OR (disjunction)	$A \cup B$	$\mu_A(x) \vee \mu_B(x)$	$\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$
Intersection, AND (conjunction)	$A \cap B$	$\mu_A(x) \wedge \mu_B(x)$	$\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}$

Examples



Upper-left: AND
Lower-left: OR
Upper-right: Negation

Example 5.1 p. 89

- The universe of discourse:

$X = \{\text{Masahiro, Tsuyoshi, Takuya, Saburo, Masami}\}$

- Fuzzy sets A = “young persons” and B = “Tall persons” are defined by
 - $A = 0.4/\text{Masahiro} + 0.6/\text{Tsuyoshi} + 0.8/\text{Takuya} + 1.0/\text{Saburo} + 0.9/\text{Masami}$
 - $B = 0.3/\text{Masahiro} + 0.5/\text{Tsuyoshi} + 0.9/\text{Takuya} + 0.6/\text{Saburo} + 0.9/\text{Masami}$
- The OR and AND of A and B are as follows:
 - $A \cup B = 0.4/\text{Masahiro} + 0.6/\text{Tsuyoshi} + 0.9/\text{Takuya} + 1.0/\text{Saburo} + 1.0/\text{Masami}$
 - $A \cap B = 0.3/\text{Masahiro} + 0.5/\text{Tsuyoshi} + 0.8/\text{Takuya} + 0.6/\text{Saburo} + 0.9/\text{Masami}$

Fuzzy number

- A fuzzy number is a fuzzy set of (real) numbers.
- The membership function μ_A of a fuzzy number A satisfies the following properties:
 - There is an x , such that $\mu_A(x)=1$;
 - The support $\{x \mid \mu_A(x)>0\}$ is bounded;
 - The α -cut $\{x \mid \mu_A(x)>\alpha\}$ is closed.
- Examples of fuzzy number:
 - The distance between Aizuwakamatsu city and Sendai is **about 200 km**.
 - If we go from Aizuwakamatsu city to Sendai by train, it takes **about 2** hours.

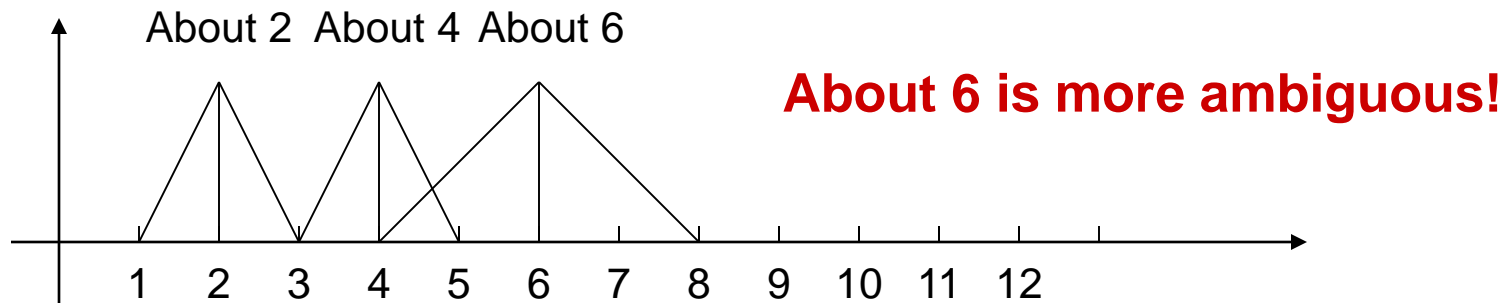
Computation with fuzzy numbers

- Based on the **extension principle**, a binary operation \circ between two fuzzy numbers can be calculated by

$$C = A \circ B : \mu_{A \circ B}(y) = \sup_{x_1 \circ x_2 = y} \{ \mu_A(x_1) \wedge \mu_B(x_2) \}$$

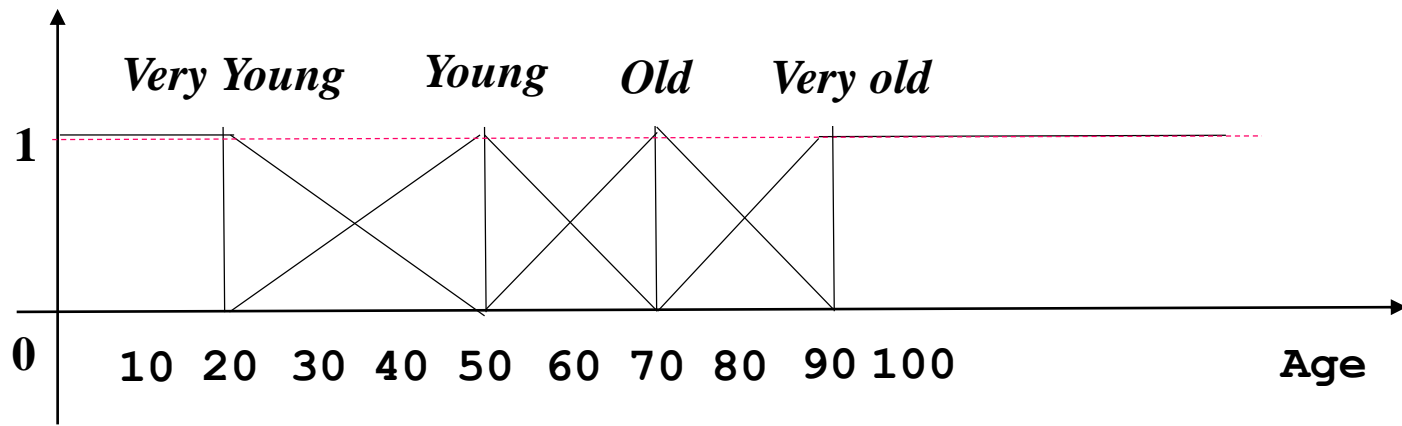
Superior (上限)

- The operation \circ can be $+$, $-$, \times , or \div .



Linguistic values of a fuzzy number

- To inference (reasoning) with fuzzy numbers, we may read a fuzzy number using “about”, “approximately”, or “roughly”, but these representations may not help us to understand the physical meaning.
- For example, to say a body temperature is about 39 degree, we may not understand that this number is relatively dangerous for an adult person. Rather, if we say the body temperature is “very high” or “very dangerous”, we can get the meaning more easily.



Fuzzy rules

Number of attributes : N

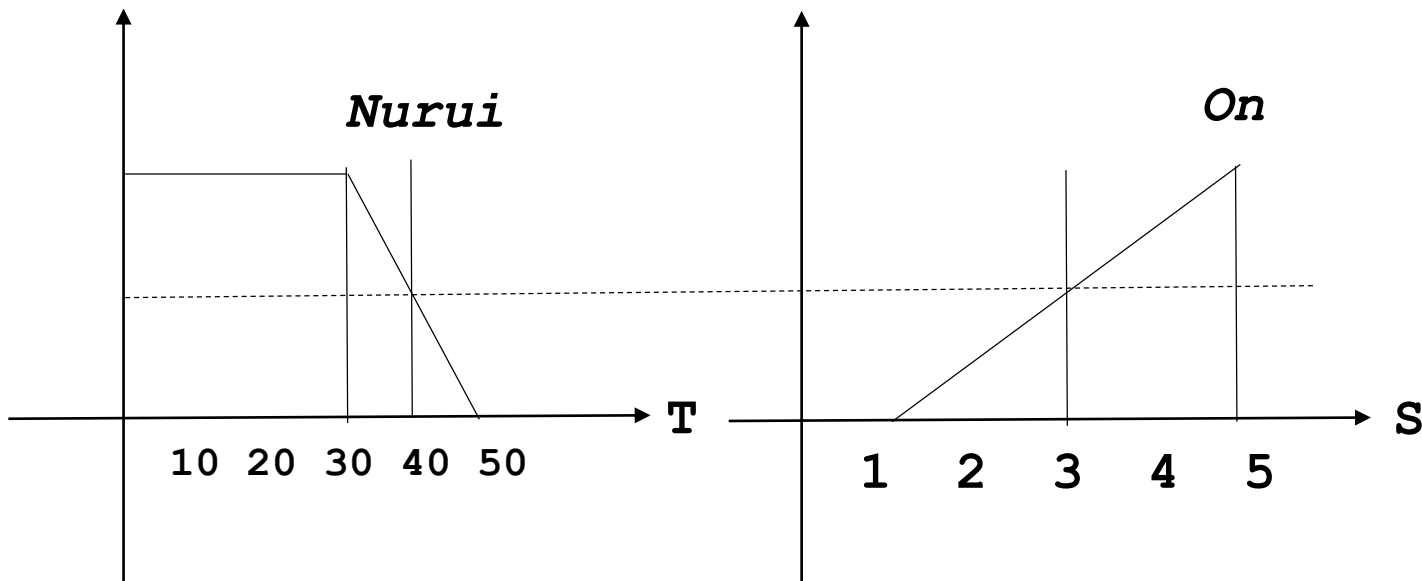
Number of rules : K

The k - th fuzzy rule :

$$R_k : \text{if } (x_1 = F_{k1} \wedge x_2 = F_{k2} \wedge \cdots \wedge x_N = F_{kN}) \text{ then } (y = B_k)$$

where $B_k, F_{kj}, k = 1, 2, \dots, K; j = 1, 2, \dots, N$, are
linguistic values.

Example of fuzzy rule



If (Temperature = Nurui) Then (Oidaki = On)



Pattern matching for fuzzy rules

For a given input datum $x = (x_1, x_2, \dots, x_N)$, the similarity between x and the condition of the k - th rule is

$$S(x, R_k) = \mu_{F_{k1}}(x_1) \wedge \mu_{F_{k2}}(x_2) \wedge \dots \wedge \mu_{F_{kN}}(x_N)$$

where \wedge is defined as the "min".

The similarity can also be considered the degree of matching.

Fuzzy inference

For a given input datum $x = (x_1, x_2, \dots, x_N)$, we can make a fuzzy decision as follows :

Step 1 : Find the membership function of the result B^* , which is also a fuzzy number using the following equation :

$$\mu_{B^*}(y) = \mu_{R_1}(x) \vee \mu_{R_2}(x) \vee \dots \vee \mu_{R_K}(x)$$

where \vee is "max", and $\mu_{R_k}(x) = S(R_k) \wedge \mu_{B_k}(y)$.

Fuzzy inference

Step 2: The final output is calculated as follows:

$$b^* = \frac{\int_Y y \times \mu_{B^*}(y) dy}{\int_Y \mu_{B^*}(y) dy}$$

- This is the gravity center of the fuzzy number B^* .
- We may also use median or the maximum value.
- The process for finding a concrete number from a fuzzy number is called **de-fuzzification**.

Example 5.3 pp. 93-96

- According to Japanese Road laws, road around the corner must be constructed based on the relation between the curvature and the speed limitation.
- For example, if the speed limit is 50 km/h, the radius of the curve must be 100m or more (see Table 5.4 in p. 94).

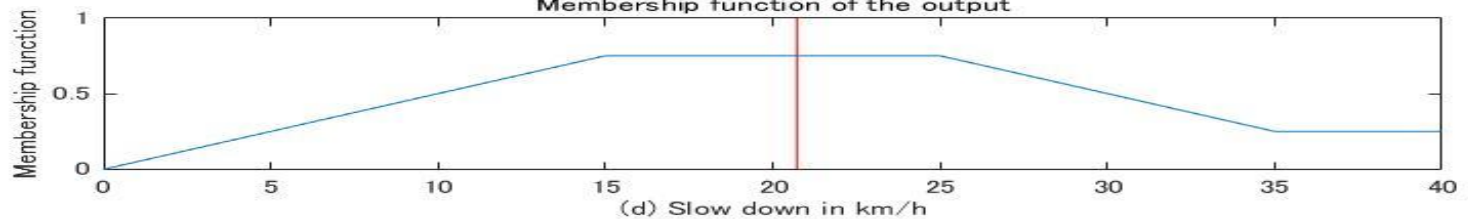
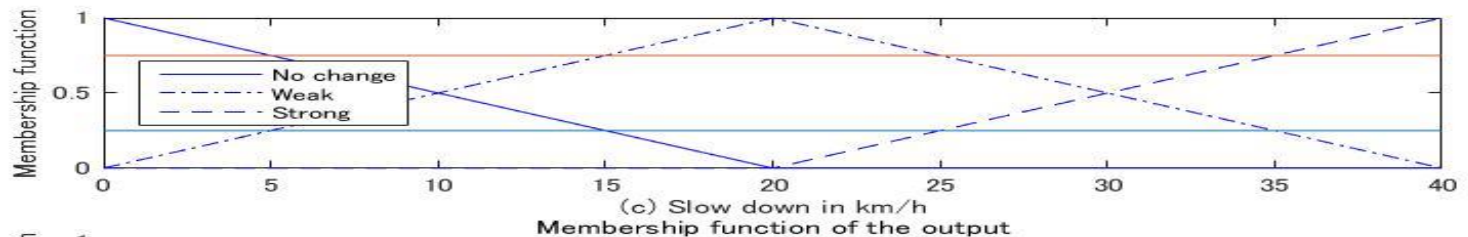
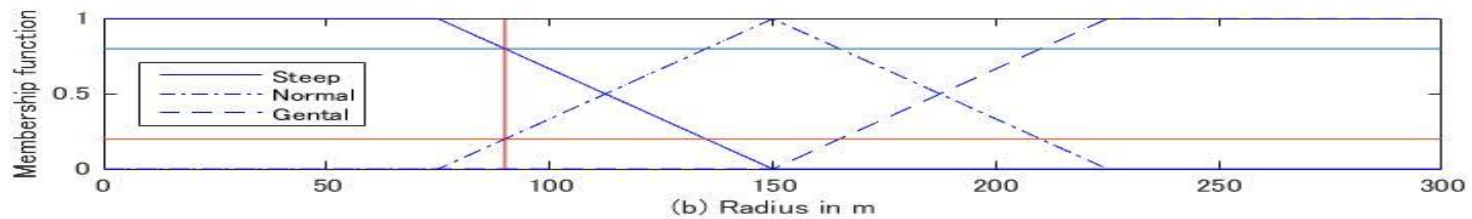
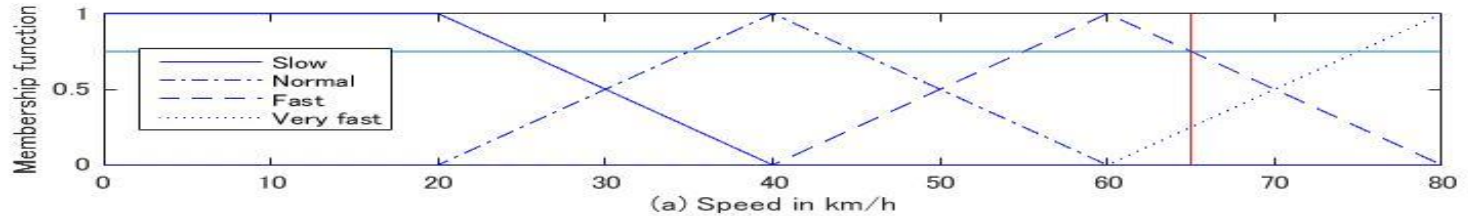


Fuzzy rules for speed control

- R1: If(v =Normal \wedge r =Sharp curve)
Then(B =Weak)
- R2: If(v =A little fast \wedge r =Sharp curve)
Then(B =Weak)
- R3: If(v =Fast \wedge r =Normal)
Then(B =Weak)
- R4: If(v =Fast \wedge r =Sharp curve)
Then(B =Strong)

Speed (v)	Curvature (radius r)	Break (b)
Normal	Slow curve	As is
Normal	Normal	As is
Normal	Sharp curve	Weak
A little fast	Slow curve	As is
A little fast	Normal	As is
A little fast	Sharp curve	Weak
Fast	Slow curve	As is
Fast	Normal	Weak
Fast	Sharp curve	Strong

Linguistic values of the fuzzy numbers



Proper speed for a given condition

Step 1: First, find the similarity (degree of matching) for each rule as follows:

- $S(R_1) = \min(m_{\text{normal}}(65), m_{\text{sharp}}(90)) = \min(0, 0.8) = 0$
- $S(R_2) = \min(m_{\text{a little fast}}(65), m_{\text{sharp}}(90)) = \min(0.75, 0.8) = 0.75$
- $S(R_3) = \min(m_{\text{fast}}(65), m_{\text{normal}}(90)) = \min(0.25, 0.2) = 0.20$
- $S(R_4) = \min(m_{\text{fast}}(65), m_{\text{sharp}}(90)) = \min(0.25, 0.8) = 0.25$

$$\mu_{B^*}(y) = \max[\min(0.75, \mu_{\text{weak}}(y)), \min(0.25, \mu_{\text{strong}}(y))]$$

Proper speed for a given condition

- Step 2: Defuzzification

$$b^* = \frac{\int_Y y \times \mu_{B^*}(y) dy}{\int_Y \mu_{B^*}(y) dy} = \mathbf{20.7 \text{ km/h}}$$

- That is, if the speed is reduced from 65 to 44.3 km/h, the car can pass the curve safely.

Problems in using fuzzy control

- Fuzzy control is a kind of soft control that uses human knowledge. The performance depends on the membership functions of the linguistic values.
- To find the optimal membership functions, we must use some other tools.
- For example, we may genetic algorithm to fine-tune the parameters of the membership functions (e.g. to determine the shape or type).
- We may also use neural networks to capture the best membership functions through learning.

Homework for lecture 9 (1)

- Solve problem 5.4 in p. 97 in the textbook.
- That is, for the membership functions of the linguistic values in Example 5.3, find their equations and draw the figures.
- Submit before end of the exercise class.

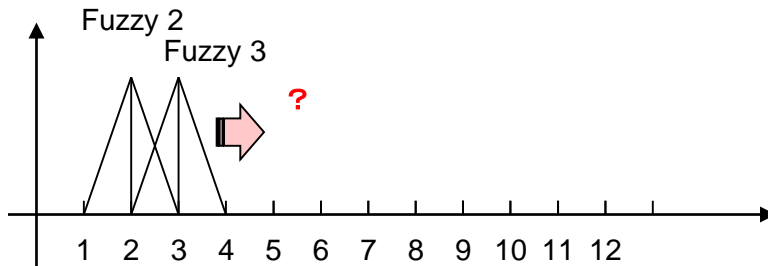
Homework for lecture 9 (2)

- Based on the skeleton program, write a program to implement a fuzzy system for speed control, and test your program using $v=70$ km/h and $r=90$ m.
- Plot the results using “gnuplot”, and save the 3-D figure.
- The name file should be “plot.png”, and the coordinates are as follows:
 - $(X, Y, Z) = (v, r, b)$
- From the figure, try to describe the relation between b and (v, r) , and add your description in “summary_09.txt”.

Quizzes for lecture 9

(1) Find the complement set of A given in Example 5.1.

(2) Using the extension principle, find the membership function of “fuzzy 5”, when “fuzzy 2” and “fuzzy 3” are defined below.



(3) Below is a fuzzy rule for controlling a robot based on light sensors.

$R_s : \text{if } (x_1 = \text{VeryBright} \wedge x_2 = \text{VeryBright} \wedge$
 $x_3 = \text{VeryDark} \wedge x_4 = \text{VeryDark})$
 $\text{then}(y = \text{GoForwardQuickly})$

Suppose that the minimum and maximum values of each sensor are 0 and 1024. Try to define the membership functions for the linguistic values VeryBright and VeryDark.

