

# Multiresolution compressive sensing for laser scanning microscope with content-aware sampling

## Introduction

Compressive sensing enables reconstructing of high-dimensional signal  $x$  from low-dimensional measurement  $y = Ax$ . In general, solving for  $\tilde{x}$  is an ill-posed problem i.e, no unique solution, and/or the solution is not robust to small data perturbations [@EstrelaTotal](#). Thus, to make it a well-posed problem ,we introduce the regularize term ( $\phi$ ) and solve as a optimization problem:

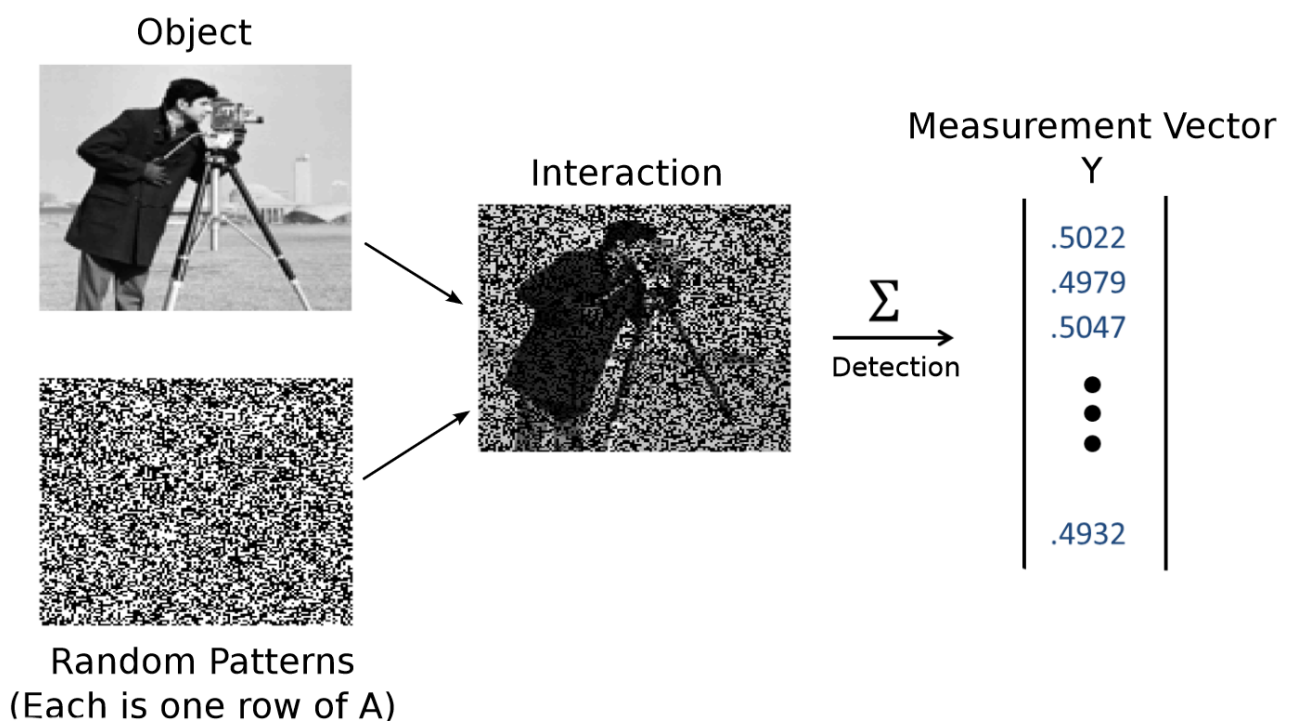
$$\arg \min_x \phi(x) \quad s. t. Ax = y$$

As explained in [@Farnell2019Total](#), Two of the most popular choices for  $\phi(x)$  are : (1)  $L_1$  norm where  $\phi(x) = \|x\|_{l_1}$  and (2) Total Variation (TV) norm where  $\phi(x) = \|x\|_{TV}$

$$\|x\|_{TV} = \|\nabla_x x\| + \|\nabla_y x\| = \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} |x_{i+1,j} - x_{i,j}| + |x_{i,j+1} - x_{i,j}|$$

A detailed review of various algorithms for solving  $L_1$  norm and TV norm is provided in ref [@Sher2019Review](#). Based on this review paper, We chose TVAL3 due to its fast reconstruction. TVAL3 (Total Variation Augmented Lagrangian ALternating-direction ALgorithm) [@LiEfficient](#), [@ZhangEfficienta](#). **Need to explain a bit about TVAL3** The basics intuition of it and why its fast.

Typically, Compressive sensing is implemented using a Digital Micromirror Device (DMD) or Coded Aperture, as shown in the figure. Each row of  $A$  corresponds to a unique binary mask.  $M$  unique binary masks sample the object of interest sequentially to obtain each element of the measurement matrix [@Marcia2011Compressed](#),[@Duarte2008Singlepixel](#). Further details on various optical architectures, like principal components and pseudo-random projections for compressive sensing, are detailed in ref [@NeifeldOptical](#).



From: [@HowlandCompressive](#)

This work focuses on applying compressive sensing to standard laser scanning. i.e., without any encoding device such as DMD or coded aperture. This will enable the deployment of the proposed algorithm without any change to the existing hardware. Such setup forms the basis of multiple imaging modalities such as confocal, two-photon, and optical coherence tomography.

## | Relevant work

Pavilion et al. in 2016 is the first work to implement on std. Laser scanning system.

[@Pavillon2016Compressed](#). They showed that by including the point spread function of the microscope in the optimization, it is possible to extend cs to std. Laser scanning system. (Need to find it cons!)

In 2018, Francis et al. [@Francis2018Multiresolutionbased](#) used a multi-resolution approach where a low-resolution image is used to estimate a compensation term in the regularize term leading to better reconstruction. However, this work was not implemented on the hardware.

In 2021, Hu et al. showed that we can vary the sampling density using bright field images to get better reconstruction [@Hu2021Fast](#). (Need to find it cons!)

## | Existing Problem

TVAL is nearly three times as fast as NESTA and ten times faster than L1 Magic, so it is the obvious choice for well-conditioned problems. TVAL's shortcoming lies in situations where the number of measurements is close to the information limit or when there is significant measurement noise, where the quality of reconstruction decreases rapidly [@Sher2019Review](#).

## | Our contribution

In this work , we try to address this combining PSF ([@Pavillon2016Compressed](#)) + Multi-resolution [@Francis2018Multiresolutionbased](#) + Intelligent sampling [@Hu2021Fast](#) with the goal of obtaining better and faster reconstruction, enabling faster data acquisition for confocal laser scanning microscope.

## | Key Bottleneck

- As the resolution of the image increase, the computation gets harder and the reconstruction suffers. So, we need methodologies way to solve it.
- 

## | Methodologies

- Better framing of the optimization problem like PSF
- multi-resolution
- context aware
- scanning strategy

## | Existing Work

## | Prior Data Available

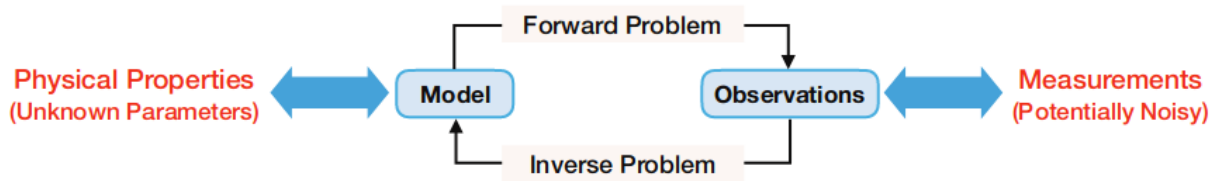


Image from: [@Bhandari2022Computational](#)

## | To Do

[@Sher2019Review](#)

[@Kravets2022Progressive](#)

[@Wang2022Total](#)

[@Stern2017Optical](#)

## | Literature Review

Point scanning vs Switching-mask-based single-pixel cameras.

1. [@Zhang2021Deep](#), [@Zhang2020Compressed](#) UC Davis  
They project a pattern on a small block and then sequentially scan and recover using neural network. Showed demo on classical images.
2. [@Yuan2021Compressive](#) - Not Clear  
it excites a sample with an array of foci, randomly interleaves the scattering projection positions at the entrance of an imaging spectrometer, and then compresses multiple coded spectra into single spectral acquisition channels. SIRI randomly displaces the Raman spectra of adjacent foci that would otherwise be smeared on a detector because of their spectral similarities, which is crucial for an unambiguous and high-fidelity hyperspectral reconstruction.
3. [@Hu2021Fast](#) uses BF Images as a mask. Excellent Idea. Yeast Cells
4. [@Soldevila2019Fast](#) Mouse cerebellum brain, Under sample and then use matrix completion algos.
5. [@Zhang2018Dynamic](#) supervised approach for chemical API
6. [@Lin2017Spectroscopic](#) fungal cells + uniform pseudo-random scanning scheme by designing a three-dimensional (3D) triangular Lissajous trajectory with a high least common multiplier (LCM) for the axis frequencies.  
<https://github.com/jianzhongcs/ISTA-Net-PyTorch>
7. [@Woringer2017Faster](#) for 3D in z-axis and have sample code.
8. Mouse embryonic fibroblast (MEF) cells and used PSF [@Pavillon2016Compressed](#)

9. [@Chauffert2014Variable](#) proposes random walking and travelling salesman problem.

## | Issues and Solutions

1. Can we use Image Pyramids
2. Fastest way to matrix multiplane stressgen algo and space librararay?
3. Eigenspace

## |

1. <https://github.com/Adrian-Markelov/IHT-tutorial>
2. <https://github.com/vegarant/cilib>
3. <https://github.com/DIPlib/diplib>

## | Compressive Sensing

The mathematical framework is as follows:

$$y = Cx$$

(1)

Where  $y$  is the measured/observed data,  $C$  is the sampling matrix and  $x$  is the source signal (image). Now,  $x$  can be expressed in a universal basis  $\Psi$  such as fourier transform, wavelet, shearlets, and curvelets.

$$x = \Psi s$$

(2)

Where  $s$  is the sparse coefficient. Rewriting equation (1) in terms of (2):

$$y = C\Psi s$$

(3)

In compressive sensing  $y, C, \Psi$  are known and sparse coefficients  $s$  is the unknown which is computationally solved.

## | Tailored Sensing

Step 1: Build custom eigen basis or principal components ( $\Psi_r$ ) from our training data using techniques such as SVD.

$$y = C_{opt}\Psi_r a$$

(4)

Step 2: Solve for the tailored sampling matrix  $C_{opt}$  by pivoted QR on  $\Psi_r$ . This tailored sampling matrix ( $C_{opt}$ ) correspond to optimal reconstruction for the signal.

Step 3: Measure the signal  $x$  using sampling matrix  $C_{opt}$  to obtain measure data  $y$ :

$$y = C_{opt}x$$

(5)

Step 4: Solve for  $a$  by inverting  $\Theta$ , where  $\Theta = C_{opt}\Psi_r$

$$y = C_{opt}\Psi_r a$$

(6)

$$y = \Theta a$$

(7)

$$a = \Theta^{-1}y$$

(8)

Step 5: Reconstruct  $x$  by:

$$x = \Psi_r a$$

(9)

## **| Matlab Simulation**

[GitHub Link](#)

**2.1 Discrete Mask**



**2.2 Reconstructed Image: Discrete Compressive Sensing**



**3.1 Continuous Spiral Mask**



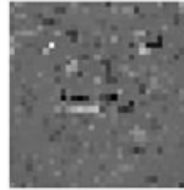
**3.2 Reconstructed Image: Continuous Compressive Sensing**



**4.1 Discrete Random Mask**



**4.2 Reconstructed Image: Tailored Sensing**



**5.1 QR Optimal Mask**



**5.2 Reconstructed Image: Tailored Sensing (Optimal Mask)**



**6.1 QR Optimal Mask Continuous**



**6.2 Reconstructed Image: Tailored Sensing (Continuous)**

