Module 1:
The Gaussian beam q parameter

Course 2 of *Optical Engineering*: Optical efficiency and resolution

with **Dr. Robert R. McLeod and Dr. Amy C. Sullivan**

Gaussian beam q parameter

The D (=1,2) dimensional Gaussian beam expression with normalization:

$$E(\vec{r}) = A_0 \left(\frac{1}{\sqrt{\pi} w_0} \right)^{D/2} \frac{w_0}{w(z)} e^{-\frac{\rho^2}{w^2(z)} - jkz - jk\frac{\rho^2}{2R(z)} + j\zeta(z)}$$

Define the complex radius of curvature

$$\frac{1}{q(z)} = \frac{1}{R(z)} - j\frac{\lambda}{\pi w^2(z)}$$

What is q(z)?

$$q(z) = \frac{1}{\dfrac{1}{z\left[1+\left(\frac{z_0}{z}\right)^2\right]} - j\dfrac{1}{z_0\left[1+\left(\frac{z}{z_0}\right)^2\right]}}$$

$$= \left[\frac{z - j\,z_0}{z^2 + z_0^2}\right]^{-1}$$

$$= z + j\,z_0$$

Some other useful relations:

$$\arg(q) = \tan^{-1}\frac{z_0}{z}$$

The phase of j/q(z) is ζ.

$$\frac{|q|}{z_0} = \sqrt{1+\left(z/z_0 \right)^2} = \frac{w(z)}{w_0}$$

Can now write the Gaussian beam above as

$$\boxed{E(\vec{r}) = j\,A_0 \left(\frac{1}{\sqrt{\pi} w_0} \right)^{D/2} \frac{z_0}{q(z)} e^{-jk\frac{\rho^2}{2q(z)} - jkz}}$$