Module 4: Optical Path Length of a Paraxial Lens

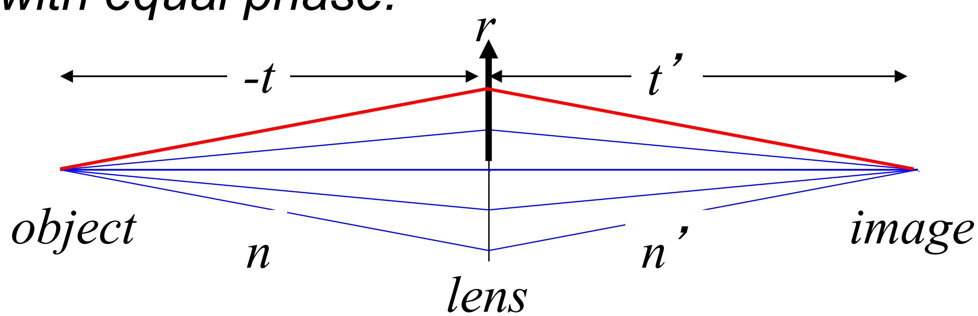
Course 1 of *Optical Engineering*: First Order Optical System Design

with **Dr. Robert R. McLeod and Dr. Amy C. Sullivan**

University of Colorado **Boulder**

What is a lens? Paraxial thin lens via Fermat

Define a lens as a thin phase function that connects all the rays from object to image *with equal phase.*



OPL on axis = OPL off axis

$$-nt + n't' = n\sqrt{t^2 + r^2} + n'\sqrt{t'^2 + r^2} + S_{lens}(r)$$

Fermat's principle

$$\approx -nt - \frac{r^2}{2}\frac{n}{t} + n't' + \frac{r^2}{2}\frac{n'}{t'} + S_{lens}(r)$$

Binomial approx.

Solve for OPL of lens

$$\boxed{S_{lens}(r) = -\frac{r^2}{2}\left(-\frac{n}{t} + \frac{n'}{t'}\right) = -\frac{r^2}{2f}}$$

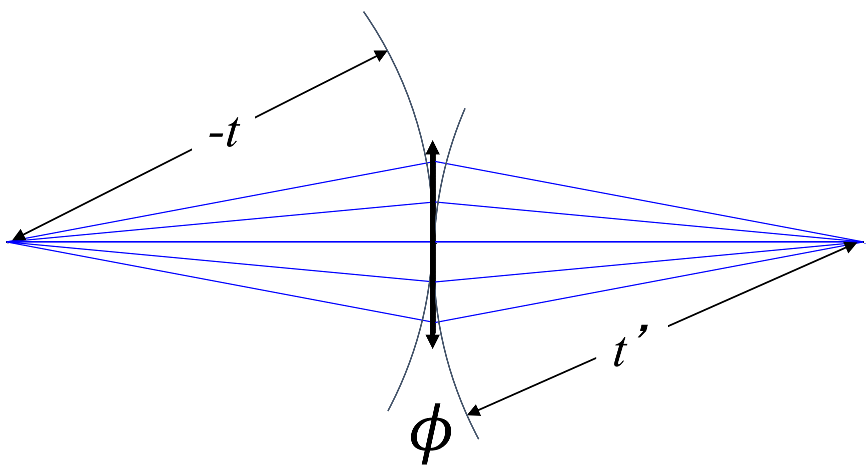
…using the Gaussian thin lens equation

$$\frac{1}{f} \equiv \phi = -\frac{n}{t} + \frac{n'}{t'}$$

Variables

f	Focal length of lens	[m]
ϕ=1/f	Power of lens	[diopters]

What is a lens? Transforms wavefront curvature



The power of a lens is the algebraic increment in curvature added to the incident wavefront.

$$\frac{1}{t'} = \frac{1}{t} + \phi \qquad \text{In air}$$

Module 4: Power of a Single Curved Surface

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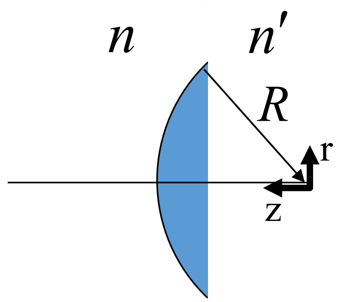
What is a lens? Paraxial power via Fermat

Consider a curved surface between to refractive indices



z coordinate of surface is

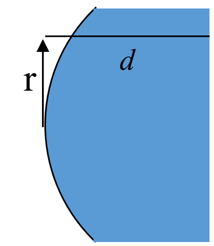
$$r^2 + z^2 = R^2$$

Sphere

$$z = \sqrt{R^2 - r^2} \approx R\left(1 - \frac{1}{2}\frac{r^2}{R^2}\right) = R - \frac{1}{2}\frac{r^2}{R}$$

Paraxial = parabola

Calculate optical path length along z at some height r



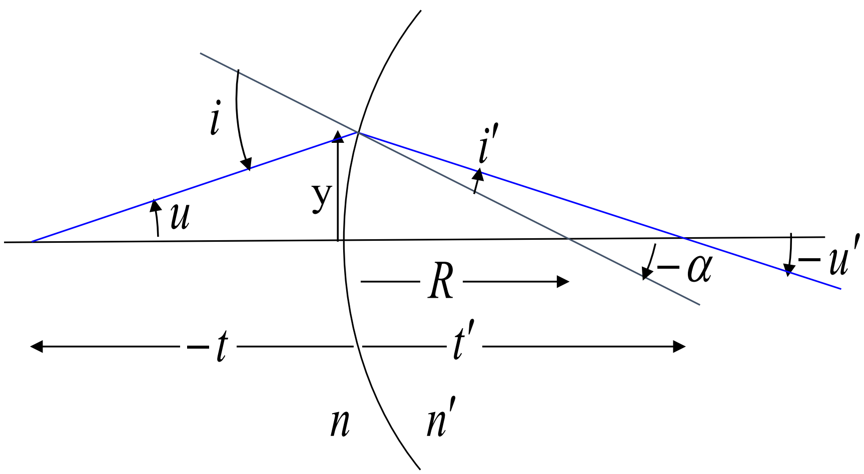
$$\begin{aligned} S &= nL + n'L' \\ &= n\left(\frac{1}{2}\frac{r^2}{R}\right) + n'\left(d - \frac{1}{2}\frac{r^2}{R}\right) \\ &= -(n'-n)\left(\frac{1}{2}\frac{r^2}{R}\right) + n'd \\ &\equiv -\frac{r^2}{2f} \end{aligned}$$

Set equal to our definition of OPL for thin lens

$$\boxed{\phi = \frac{1}{R}(n'-n) = c(n'-n)}$$

Paraxial power of curved surface

What is a lens? Paraxial power via Snell



$$ni = n'i'$$ Paraxial Snell's Law

$$n(u-\alpha) = n'(u'-\alpha)$$ Replace refraction with ray angles

$$n\left(\frac{y}{-t} - \frac{-y}{R}\right) = n'\left(\frac{-y}{t'} - \frac{-y}{R}\right)$$ Paraxial approximations to angle, obeying sign conv.

$$\frac{n}{-t} + \frac{n'}{t'} = \frac{1}{R}(n'-n) = \phi$$ Cancel *y*, rearrange.

which agrees with the previous derivation

Module 4:
Lens Maker's Equation

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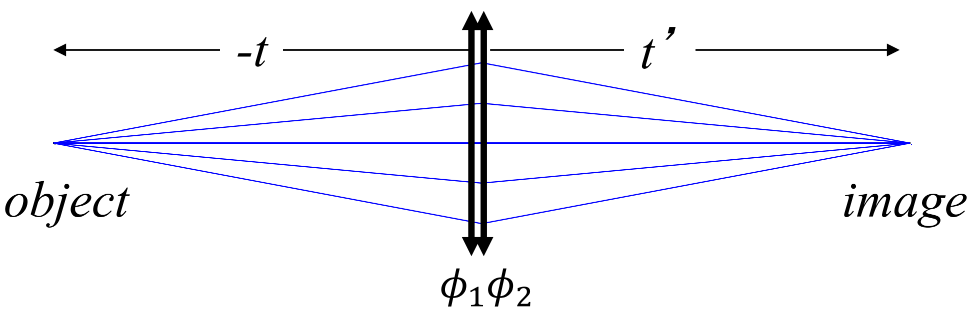
Two thin lenses in contact: First 2 lens formula

What is the equivalent focal length F of two thin lenses in contact?



Remember that the optical path length is

$$S \equiv \int_A^B n(\vec{r})\mathrm{d}s$$

So optical path lengths of contacted lenses just add

$$S_{tot} = S_1 + S_2$$
$$= -\frac{r^2}{2f_1} - \frac{r^2}{2f_2}$$
$$= -\frac{r^2}{2}\left(\frac{1}{f_1} + \frac{1}{f_2}\right)$$

Thus

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

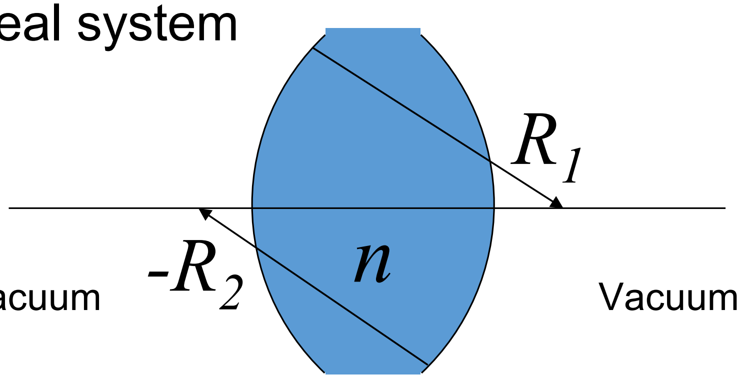
or

$$\boxed{\Phi = \phi_1 + \phi_2}$$

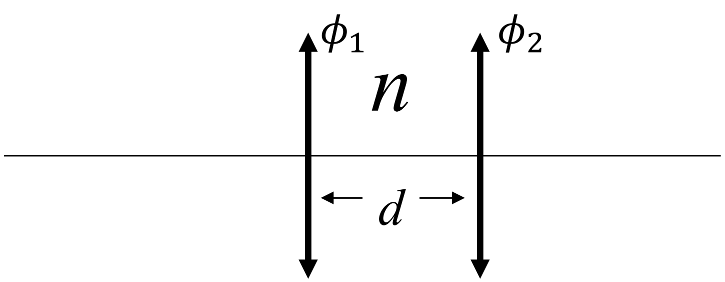
Powers add

Lens maker's equation: How to design singlet lenses

Real system



Equivalent thin lens system



Two thin surfaces separated by d

$$\Phi = \phi_1 + \phi_2 - \frac{d}{n}\phi_1\phi_2$$

We don't yet have the tools to derive this, so just accept it for now

$$= \frac{1}{R_1}(n-1) + \frac{1}{R_2}(1-n) + \frac{d}{n}\frac{1}{R_1R_2}(n-1)^2$$

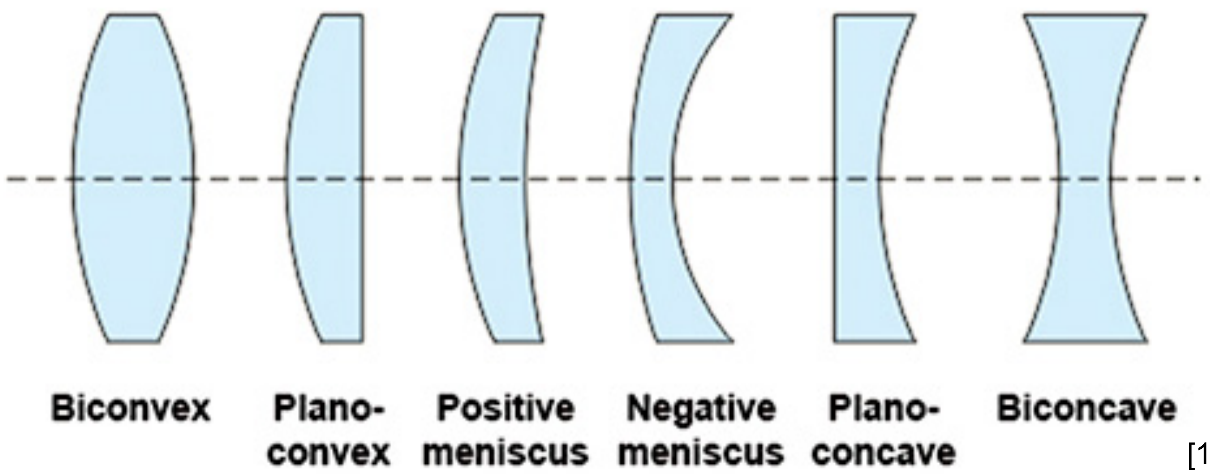
$$\boxed{= c_1(n-1) + c_2(1-n) + \frac{d}{n}c_1c_2(n-1)^2}$$

If d is << both R_1 and R_2

$$\Phi \approx (c_1 - c_2)(n-1)$$

Field guide to singlet lenses

Singlets are sufficiently common that it's worth giving all the possible combinations of surfaces and powers names



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Module 4:
Power of Curved Mirrors

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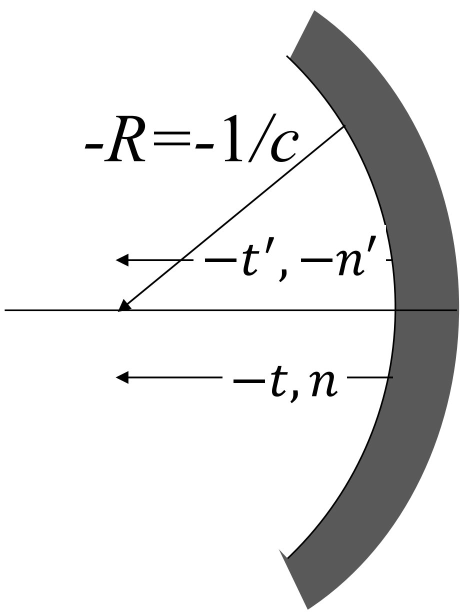
Mirrors: The "lens maker's equation" for mirrors



Place object at center of curvature. Since ray strikes surface normally, it must return to the same point.

Sign convention

1. t' is negative since it is to the left of the mirror vertex
2. We use a negative index n'=-n upon reflection in any lens equation
3. All quantities (as always) labeled are +

$$\frac{n'}{t'} - \frac{n}{t} = \frac{-n}{R} - \frac{n}{R} = \frac{1}{f}$$

Thus

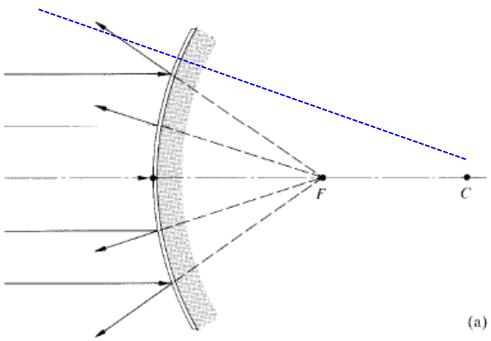
$$\boxed{\frac{1}{f} = \phi = -\frac{2n}{R} = -2nc}$$

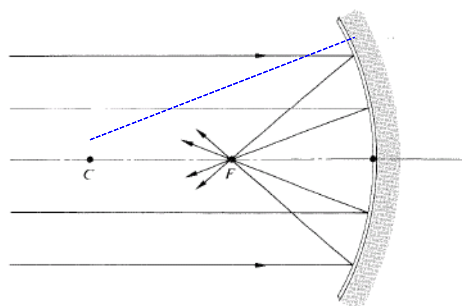
Remember that by our sign convention, R<0 in the geometry shown, yielding a lens power which is positive.

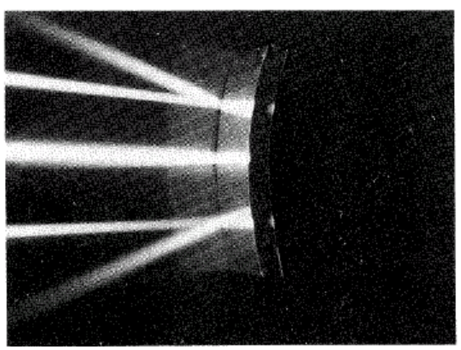
Positive and negative mirrors

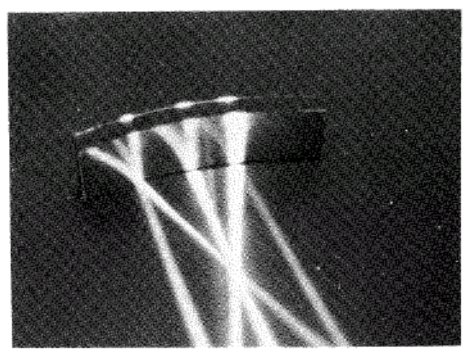
$f < 0$

$f > 0$







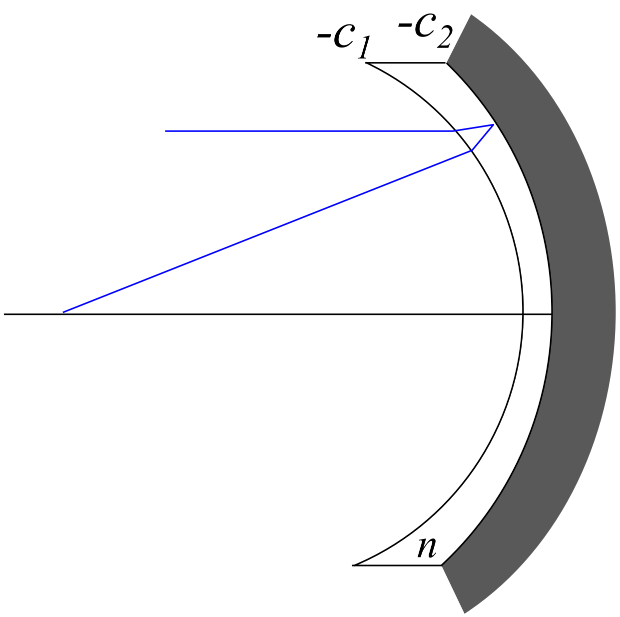


In vacuum

$$f = -\frac{R}{2}$$

Lenses and mirrors in contact: The Mangin mirror

Negative meniscus with reflective outer surface.
Cancels spherical aberration.



$$\Phi \approx \Phi_1 + \Phi_2 + \Phi_3$$

Three surfaces in contact

$$= c_1(n-1) - 2nc_2 + c_1(-1+n)$$

Lens equation, $c_1 < 0$ as drawn

Derived on previous page, $c_2 < 0$ as drawn

Lens equation but using negative indices per sign convention when light travels R → L

$$= 2c_1(n-1) - 2nc_2$$

= a symmetric concave lens with curvatures c_1 + a convex mirror of curvature c_2

Module 4: Design with Thick Optics

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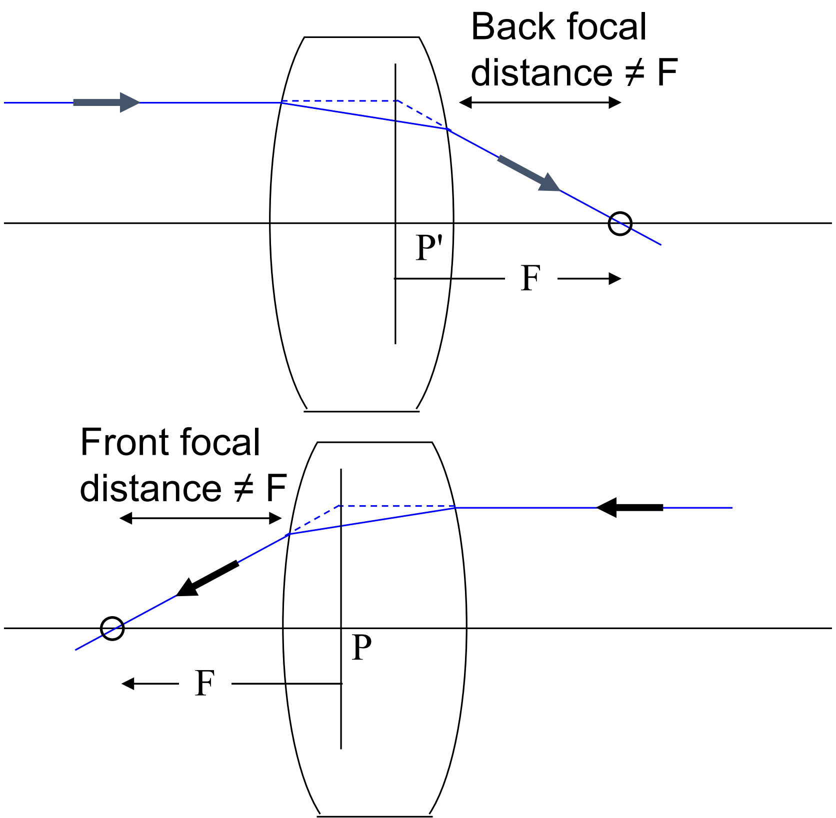
Design with thick optics: aka Gaussian design



Rear principle plane P’ ≡ Where a forwards axial ray appears to refract

Front principle plane P ≡ Where a backwards axial ray appears to refract

Effective focal length F ≡ The distance between the principle plane and the ray intersection with the axis

- In the paraxial limit, all forwards rays exiting a compound lens will appear to have encountered a single plane P’ with power $\Phi = 1/F$.
- The same is true for backwards rays but at plane P.
- Effective focal length is the same in both directions.

Module 4:
Applying Thick Optics Concepts

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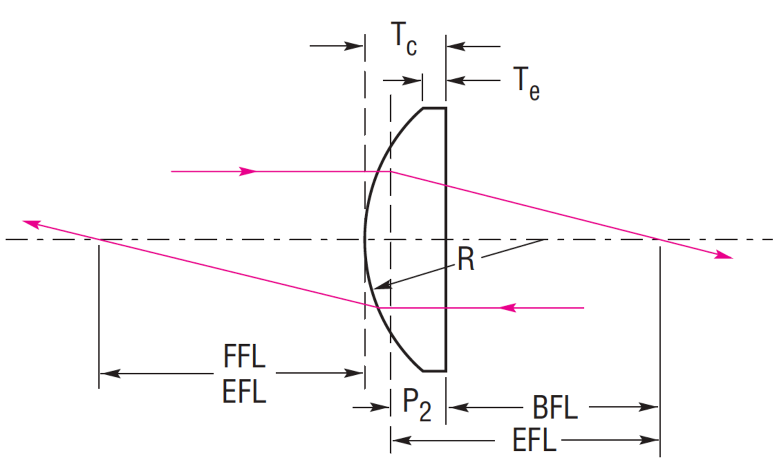
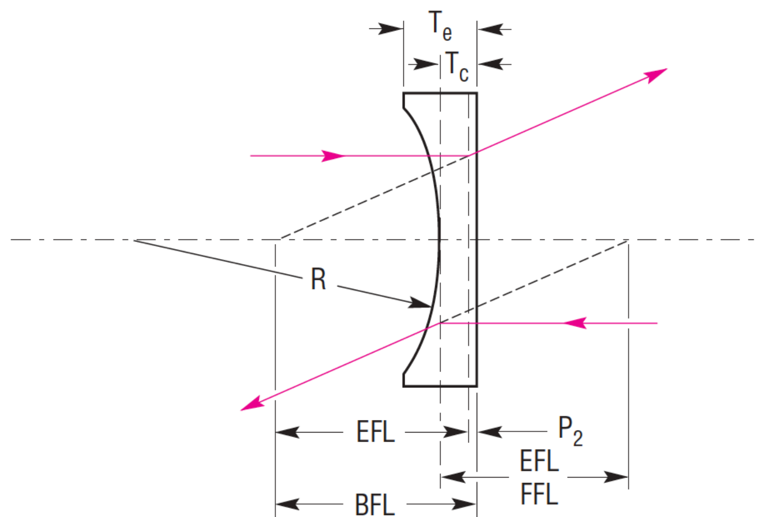
Example of principle planes: Plano singlets





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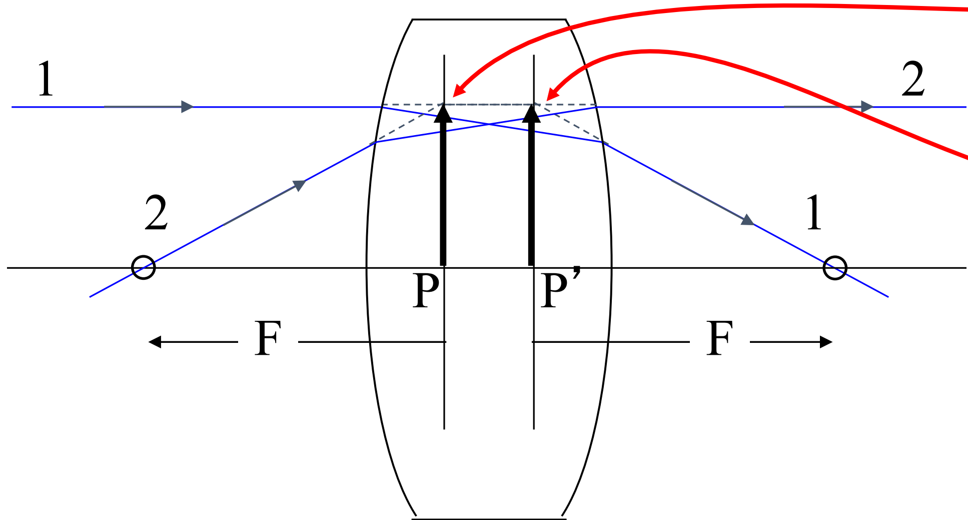
- Rays only bend at frond surface so Effective Focal Length (F) and Front Focal Distance are the same.
- Back Focal Length is < Effective Focal Length because rear principle plane P' (here P_2) is inside the lens

- Negative lens has back focal point to the left of the lens.
- Effective and Front focal lengths are again the same

How to use principle planes: Teleport rays



1 and 2 converge towards a virtual object at P

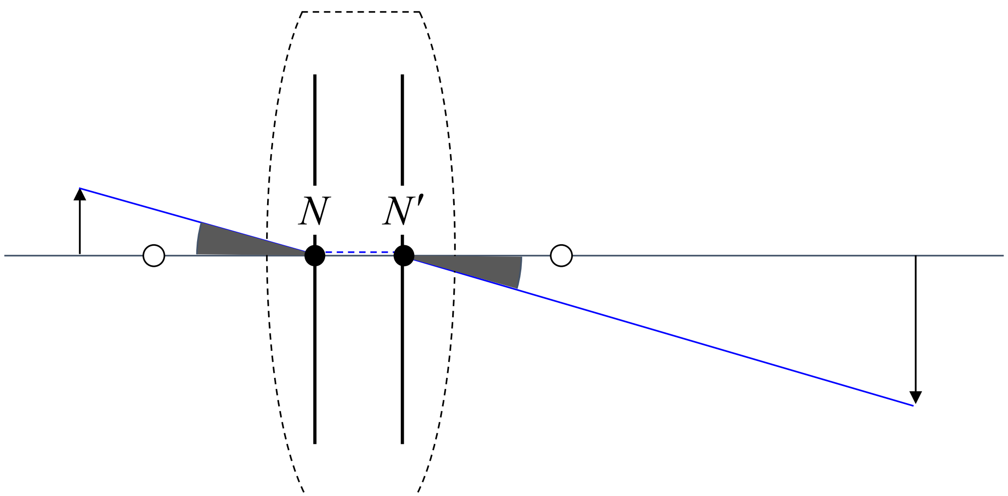
1 and 2 emerge from a virtual image at P'

Both rays behave like a thin lens IF they "teleport" from P to P', then apply thin lens at P'

1. Forward ray used to find P'
2. Reverse ray used to find P, now propagating forward

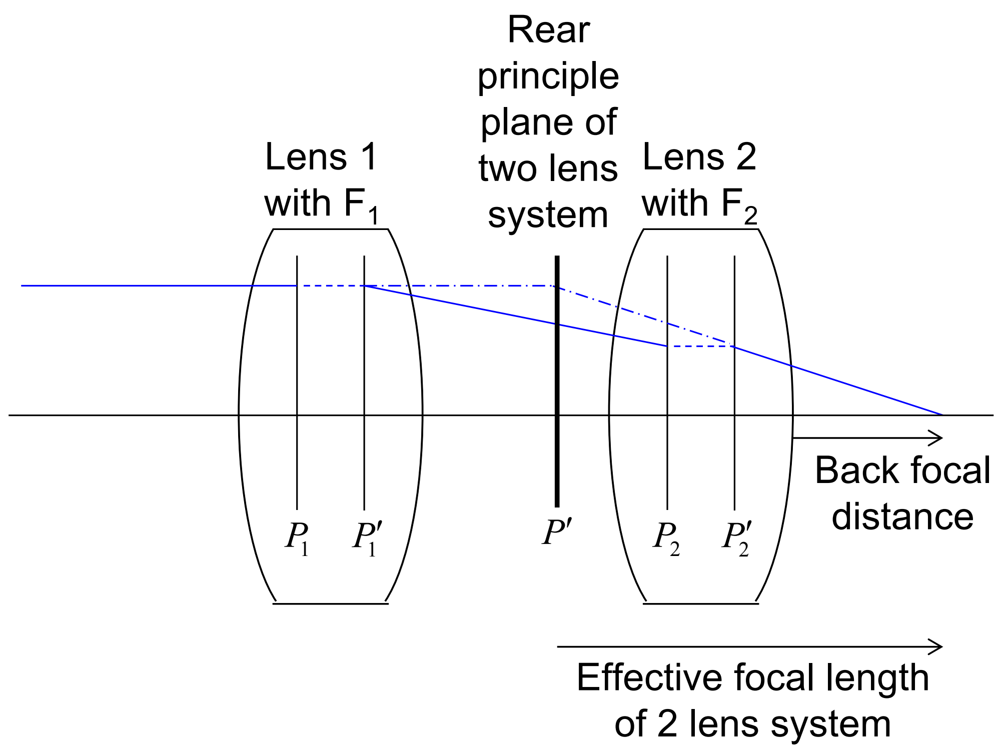
Formally, P and P' are the "conjugates of unit magnification"

Nodal points



- A ray crossing the axis at the first nodal point emerges from the second nodal points parallel to itself.
- If n = n' , the nodal points are at the intersection of the principal planes and the axis.
- This is not true in general (e.g. the eye).

Applying thick lens concept to a 2 lens system



1. Teleport incident ray from P_1 to P_1' and apply thin lens of focal length F_1
2. Propagate ray to lens 2
3. Teleport incident ray from P_2 to P_2' and apply thin lens of focal length F_2
4. Intersect ray with axis to find back focal distance of combination
5. Project back to intersection with incident ray to find P' rear principle plane for combination