Module 5: First-order ray tracing with ABCD matrices

Course 1 of *Optical Engineering*: First Order Optical System Design

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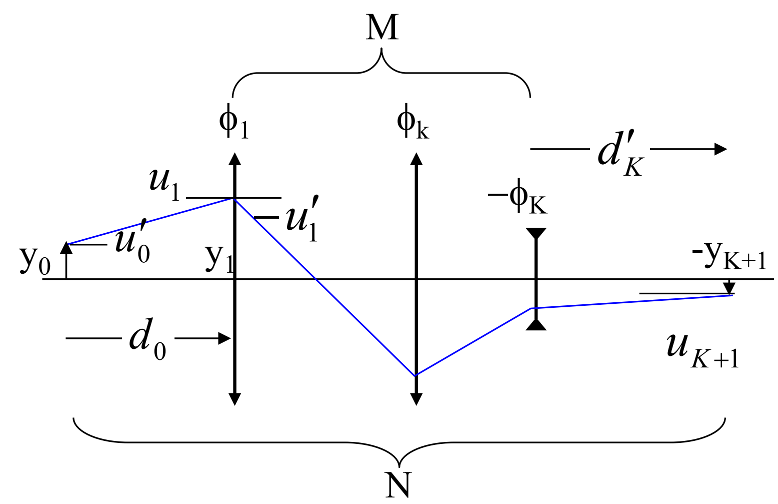
y-u tracing with matrices

Refraction equation

$$n'_k u'_k = n_k u_k - y_k \phi_k \Rightarrow \begin{bmatrix} y_k \\ u'_k \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\dfrac{\phi_k}{n'_k} & \dfrac{n_k}{n'_k} \end{bmatrix} \begin{bmatrix} y_k \\ u_k \end{bmatrix} \equiv \mathbf{R}_k \begin{bmatrix} y_k \\ u_k \end{bmatrix}$$

Transfer equation

$$y_{k+1} = y_k + u'_k d'_k \Rightarrow \begin{bmatrix} y_{k+1} \\ u_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & d'_k \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_k \\ u'_k \end{bmatrix} \equiv \mathbf{T}_k \begin{bmatrix} y_k \\ u'_k \end{bmatrix}$$



For K optical elements: 0 (object) < k < K+1 (image)

We can define two useful matrices:

M The *system matrix* describes the optical system minus the object and image distance transfers.

$$\Rightarrow \begin{bmatrix} y_K \\ u'_K \end{bmatrix} = \mathbf{R}_K \mathbf{T}_{K-1} \ldots \mathbf{T}_1 \mathbf{R}_1 \begin{bmatrix} y_1 \\ u_1 \end{bmatrix} \equiv \mathbf{M} \begin{bmatrix} y_1 \\ u_1 \end{bmatrix}$$

N The *conjugate matrix* includes the object and image distances.

$$\Rightarrow \begin{bmatrix} y_{K+1} \\ u_{K+1} \end{bmatrix} = \mathbf{T}_K \mathbf{R}_K \mathbf{T}_{K-1} \ldots \mathbf{T}_1 \mathbf{R}_1 \mathbf{T}_0 \begin{bmatrix} y_0 \\ u'_0 \end{bmatrix} \equiv \mathbf{N} \begin{bmatrix} y_0 \\ u'_0 \end{bmatrix} = \mathbf{T}_K \mathbf{M} \mathbf{T}_0 \begin{bmatrix} y_0 \\ u'_0 \end{bmatrix}$$