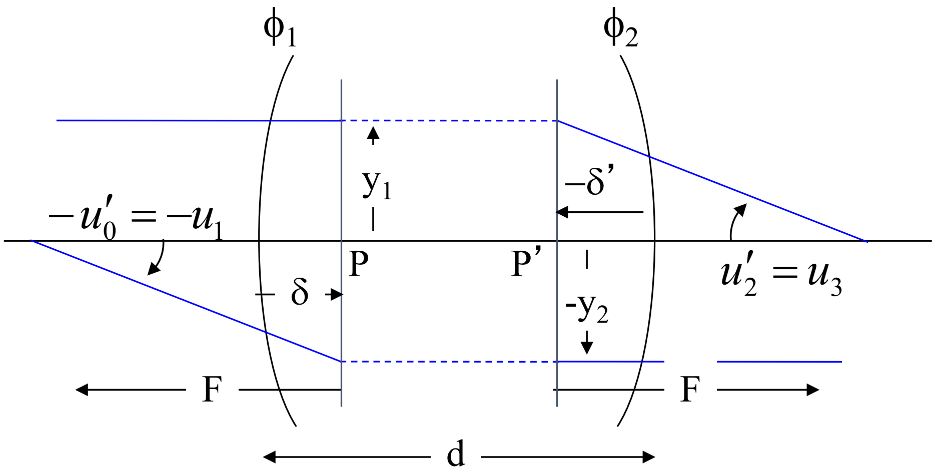
# Module 5:
# Properties of the 2 Lens System

**Course 1 of *Optical Engineering*: First Order Optical System Design**

with **Dr. Robert R. McLeod and Dr. Amy C. Sullivan**

# Two lens system: Effective power




$$\mathrm{N} = \begin{bmatrix} M & 0 \\ -\Phi & 1/M \end{bmatrix}$$

$$\mathrm{N}_{21} = -\Phi \Rightarrow \boxed{\Phi = 1/F = \phi_1 + \phi_2 - d\phi_1\phi_2}$$

$$\mathrm{N} = T(t_2')R(\phi_2)T(d)R(\phi_1)T(-t_1)$$

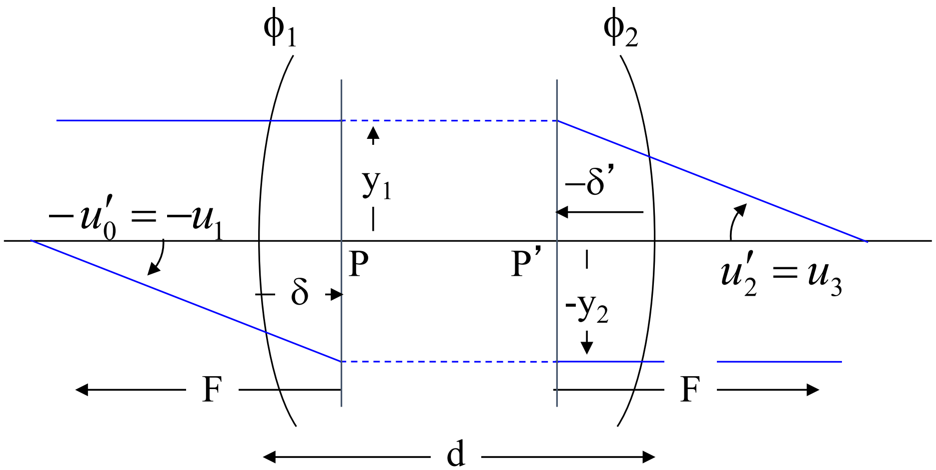
$$= \begin{bmatrix} 1 - t_2'(\phi_1 + \phi_2) + d\phi_1(t_2'\phi_2 - 1) & t_2' - d(t_1\phi_1 + 1)(t_2'\phi_2 - 1) + t_1(t_2'(\phi_1 + \phi_2) - 1) \\ d\phi_1\phi_2 - \phi_1 - \phi_2 & 1 - d\phi_2 - t_1(d\phi_1\phi_2 - \phi_1 - \phi_2) \end{bmatrix}$$

Compare to thin lens formula for $d$ = 0:

$$\Phi = \phi_1 + \phi_2$$

✓

# Two lens system: Magnification




$$\mathrm{N} = \begin{bmatrix} M & 0 \\ -\Phi & 1/M \end{bmatrix}$$

$$\mathrm{N}_{11} = M \quad \Rightarrow \boxed{M = \frac{1}{1 + t_1\Phi - d\phi_2}}$$

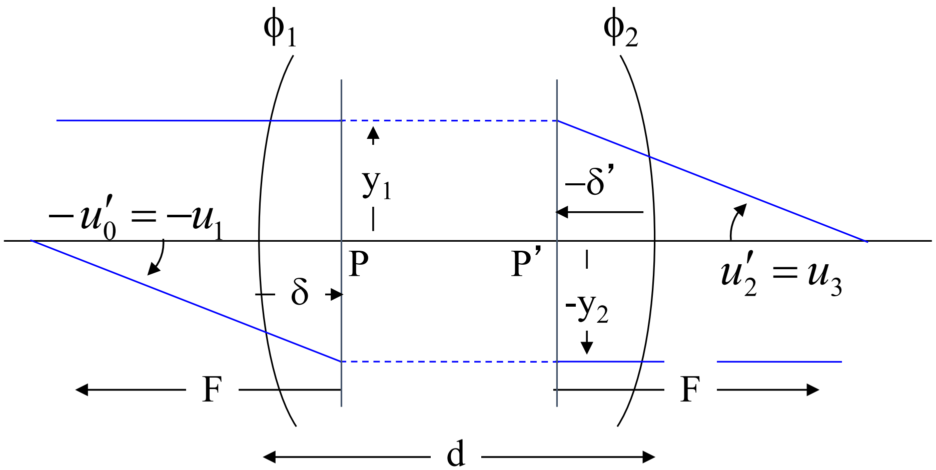
$$\mathrm{N} = T(t_2')R(\phi_2)T(d)R(\phi_1)T(-t_1)$$

$$= \begin{bmatrix} 1 - t_2'(\phi_1 + \phi_2) + d\phi_1(t_2'\phi_2 - 1) & t_2' - d(t_1\phi_1 + 1)(t_2'\phi_2 - 1) + t_1(t_2'(\phi_1 + \phi_2) - 1) \\ d\phi_1\phi_2 - \phi_1 - \phi_2 & 1 - d\phi_2 - t_1(d\phi_1\phi_2 - \phi_1 - \phi_2) \end{bmatrix}$$

Compare to Newton thin lens formula for $d$ = 0:

$$M = \frac{F}{z} = \frac{1}{(F + t_1)\Phi} = \frac{1}{1 + t_1\Phi} \quad ✓$$

# Two lens system: Imaging condition




$$N = \begin{bmatrix} M & 0 \\ -\Phi & 1/M \end{bmatrix}$$

$$N_{12} = 0 \;\Rightarrow\; \boxed{t_2' = M\left[t_1 - d\left(1 + \phi_1 t_1\right)\right]}$$

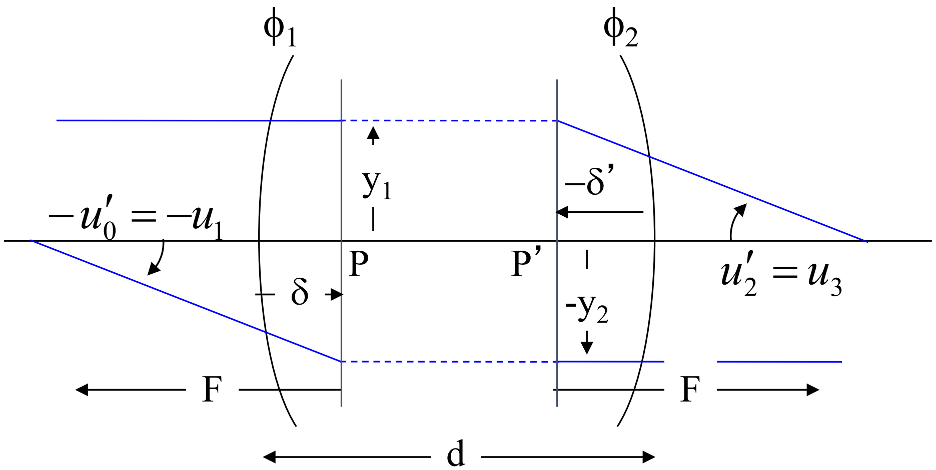
$$N = T(t_2')R(\phi_2)T(d)R(\phi_1)T(-t_1)$$

$$= \begin{bmatrix} 1 - t_2'(\phi_1+\phi_2) + d\phi_1(t_2'\phi_2 - 1) & t_2' - d(t_1\phi_1 + 1)(t_2'\phi_2 - 1) + t_1(t_2'(\phi_1+\phi_2) - 1) \\ d\phi_1\phi_2 - \phi_1 - \phi_2 & 1 - d\phi_2 - t_1(d\phi_1\phi_2 - \phi_1 - \phi_2) \end{bmatrix}$$

Compare to Gaussian thin lens formula for $d$ = 0:

$$t_2' = \frac{t_1}{1 + t_1\Phi} = \frac{1}{\frac{1}{t_1} + \frac{1}{F}} \qquad ✓$$

# Two lens system: Front Focal Plane




$$N_{22} = 0 \;\Rightarrow\; \boxed{t_1^{ffp} = F\left(d\phi_2 - 1\right)}$$

$$N = T(t_2')R(\phi_2)T(d)R(\phi_1)T(-t_1)$$

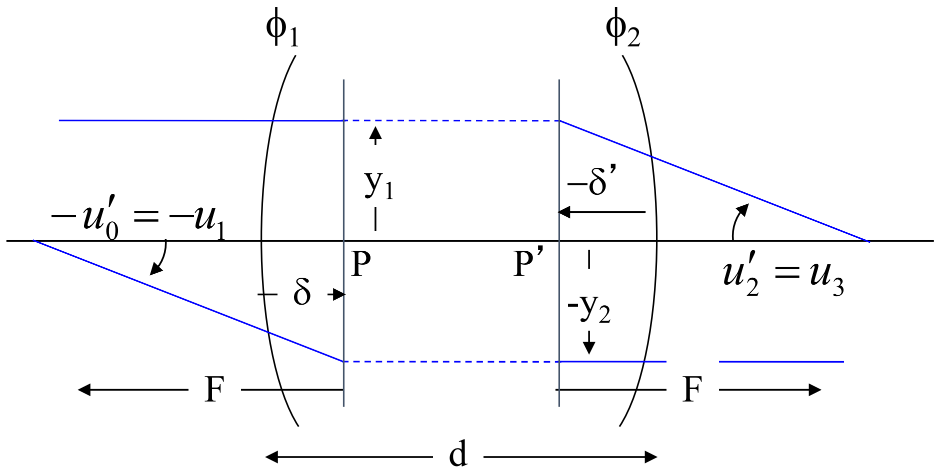
$$= \begin{bmatrix} 1 - t_2'(\phi_1+\phi_2) + d\phi_1(t_2'\phi_2 - 1) & t_2' - d(t_1\phi_1+1)(t_2'\phi_2-1) + t_1(t_2'(\phi_1+\phi_2)-1) \\ d\phi_1\phi_2 - \phi_1 - \phi_2 & 1 - d\phi_2 - t_1(d\phi_1\phi_2 - \phi_1 - \phi_2) \end{bmatrix}$$

Set $d$ or $\phi_2 = 0$

$$t_1^{ffp} = -F$$

✓

# Two lens system: Back Focal Plane




$$N_{11} = 0 \quad \Rightarrow \boxed{t_2'^{bfp} = F\left(1 - d\phi_1\right)}$$

$$N = T(t_2')R(\phi_2)T(d)R(\phi_1)T(-t_1)$$

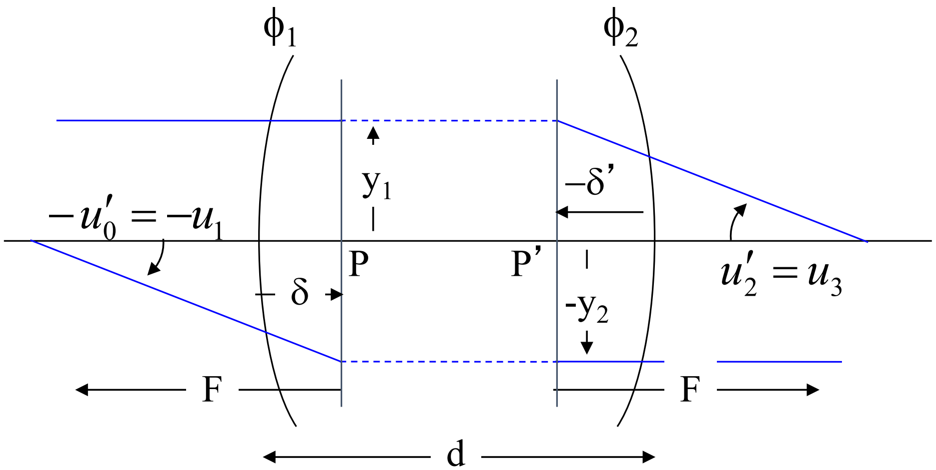
$$= \begin{bmatrix} 1 - t_2'(\phi_1 + \phi_2) + d\phi_1(t_2'\phi_2 - 1) & t_2' - d(t_1\phi_1 + 1)(t_2'\phi_2 - 1) + t_1(t_2'(\phi_1 + \phi_2) - 1) \\ d\phi_1\phi_2 - \phi_1 - \phi_2 & 1 - d\phi_2 - t_1(d\phi_1\phi_2 - \phi_1 - \phi_2) \end{bmatrix}$$

Set $d$ or $\phi_1 = 0$

$$t_2'^{bfp} = F$$

✓

# Two lens system: Front principle plane




The front principle plane is one effective focal length to the right of the front focal plane:

$$t_1^{ffp} + F \Rightarrow \boxed{\delta = \frac{d\phi_2}{\Phi}}$$

$$\text{N} = T(t_2')R(\phi_2)T(d)R(\phi_1)T(-t_1)$$

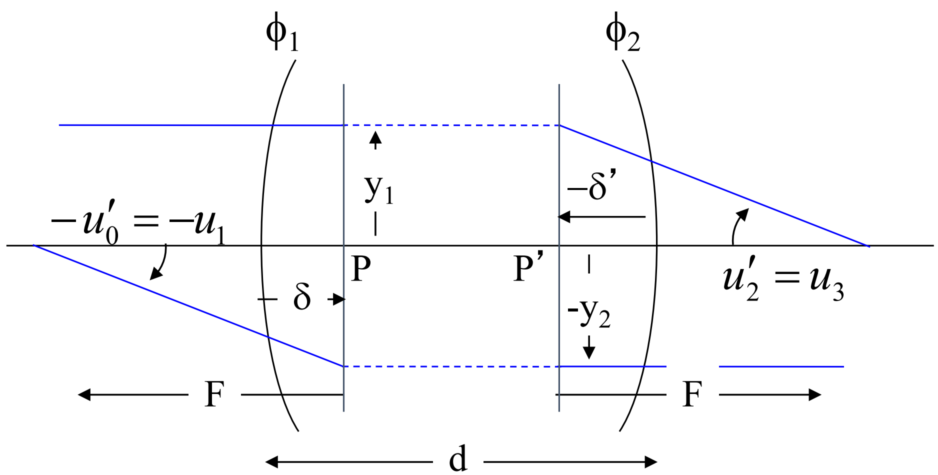
$$= \begin{bmatrix} 1 - t_2'(\phi_1+\phi_2) + d\phi_1(t_2'\phi_2 - 1) & t_2' - d(t_1\phi_1+1)(t_2'\phi_2-1) + t_1(t_2'(\phi_1+\phi_2)-1) \\ d\phi_1\phi_2 - \phi_1 - \phi_2 & 1 - d\phi_2 - t_1(d\phi_1\phi_2 - \phi_1 - \phi_2) \end{bmatrix}$$

Set $d$ or $\phi_2 = 0$

$$\delta = 0$$

✓

# Two lens system: Back principle plane




The back principle plane is one effective focal length to the left of the back focal plane:

$$t_2'^{bfp} - F \Rightarrow \boxed{\delta' = -\frac{d\phi_1}{\Phi}}$$

$$\mathrm{N} = T(t_2')R(\phi_2)T(d)R(\phi_1)T(-t_1)$$

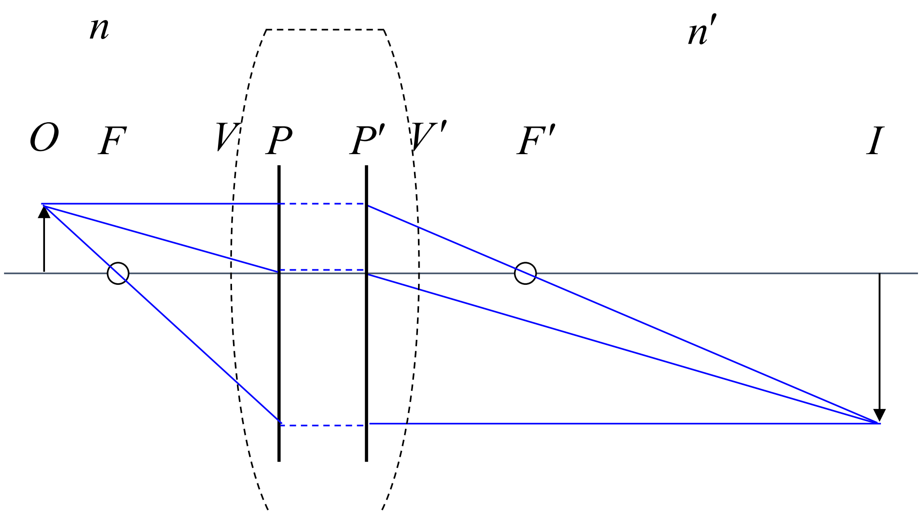
$$= \begin{bmatrix} 1 - t_2'(\phi_1+\phi_2) + d\phi_1(t_2'\phi_2 - 1) & t_2' - d(t_1\phi_1+1)(t_2'\phi_2-1) + t_1(t_2'(\phi_1+\phi_2)-1) \\ d\phi_1\phi_2 - \phi_1 - \phi_2 & 1 - d\phi_2 - t_1(d\phi_1\phi_2 - \phi_1 - \phi_2) \end{bmatrix}$$

Set $d$ or $\phi_1 = 0$

$$\delta' = 0$$

✓

# Other useful matrices




OI: Conjugate matrix

$$\begin{bmatrix} M & 0 \\ -\Phi & 1/M \end{bmatrix}$$

FF´: Focal matrix

$$\begin{bmatrix} 0 & 1/\Phi \\ -\Phi & 0 \end{bmatrix}$$

PP´: Nodal matrix

$$\begin{bmatrix} n/n' & 0 \\ -\Phi & n'/n \end{bmatrix}$$

VV´: System matrix for two surfaces,
$\tau$ = reduced thickness = VV´ / index

$$\begin{bmatrix} 1-\phi_1\tau & \tau \\ -\Phi & 1-\phi_2\tau \end{bmatrix}$$

# Sources

1. https://www.westechoptical.com/products/singlets
2. https://www.newport.com/c/spherical-lenses