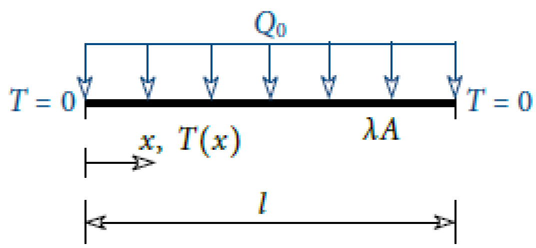
Project Simulation & Optimization 2025

Given is a rod with cross-section A and thermal conductivity λ with a constant heat supply Q_0 along the rod.



The temperature profile T in the rod is described by the differential equation

$$T'' + \frac{Q_0}{\lambda\, A} = 0$$

.

We are looking for:

1. Approximation of the differential equation using a finite difference method. The second derivative of a function f can be approximated with the function values of f at the discrete points x_i with
$$f'' \approx \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2}$$
with the distance between two points $h = x_{i+1} - x_i$.

2. Set up the system of equations that is necessary to determine the temperature at discrete points x_i. Note: Also take the boundary conditions into account!

3. Solve the system of equations using the solvers discussed in the lecture and plot the result graphically. Which solver is best suited for this task?

4. Adapt a suitable solver to the specific structure of the matrix to increase efficiency (reduction of memory requirements and effort).

System parameters: $l = 1\,\mathrm{m}$, $\lambda = 5\,\mathrm{J}/^\circ\mathrm{Cms}$, $A = 1\mathrm{cm}^2$, $Q_0 = 15 J/m^2 s$. The entire calculation and documentation can be done in a Python Jupyter Notebook.

<u>Due date:</u> February 22th, 2026