Multiple Integral

Date: 30th October 2020

Evaluation of double integral with limits

Type-1 (both independent limits)

$$\int_4^3 \int_0^1 xy^2 \, dydx$$

https://www.youtube.com/watch?v=mY_Je8GX7pM&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h&index=4

$$\int_0^2 \int_0^1 xy \; dy \, dx$$

https://www.youtube.com/watch?v=pQOyykCa1hQ&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h&index=5

Type-2 (one independent limit another one dependent limit)

Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy \; dy \, dx$

https://www.youtube.com/watch?v=_37G6weR9Wo&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h&index=10

Evaluate $\int_0^{\sqrt{2}} \int_{-\sqrt{4-2y^2}}^{\sqrt{4-2y^2}} y \; dy \, dx$

https://www.youtube.com/watch?v=d5iPrK0oJog

Evaluate $\int_0^{\pi} \int_0^{x} \sin y \; dy \, dx$

https://www.youtube.com/watch?v=T_KEmHi3uuI

Evaluate $\int_1^{\log 8} \int_0^{\log y} e^{x+y} \; dy \, dx$

https://www.youtube.com/watch?v=-Cm44vdzZyE

Evaluation of triple integral with limits

Type-1 (all independent limits)

1. A) Evaluate $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z} \; dx \, dydz$ B) Evaluate $\int_0^a \int_0^a \int_0^a (yz + zx + xy) \, dx \, dydz$

https://www.youtube.com/watch?v=EPJAnWEElRs

Type-2 (one independent limit remaining dependent limits)

2. Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\, dz\, dx dy$

https://www.youtube.com/watch?v=jzI3tsZAX2U

3. A) Evaluate $\int_1^3 \int_{1/x}^1 \int_0^{\sqrt{xy}} xyz\; dz\, dy dx$ B) Evaluate $\int_0^1 \int_1^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\; dz\, dy dx$ C) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\, dz\, dy dx$

https://www.youtube.com/watch?v=7SNQ56QkRm0

4. Evaluate $\int_0^{\log 2} \int_0^x \int_0^{x+\log y} e^{x+y+z} dz dy dx$

https://www.youtube.com/watch?v=3cwCG6tm5Yg&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h&index=8

5. *Evaluate* $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{1-x^2-y^2-z^2}} dz dy dx$

https://www.youtube.com/watch?v=XlNqvLtsBoY&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h&index=13

Multiple Integrals in polar coordinates (when limits are given)

1. A) Evaluate $\int_0^{\pi} \int_0^{a\sin\theta} r\, d\, r d\theta$

B) Evaluate $\int_0^{\pi/2} \int_{a(1+\cos\theta)}^{a} r\, dr d\theta$

C) Evaluate $\int_b^{\pi/2} \int_0^{2a\cos\theta} r^2 \sin\theta \;\; dr d\theta$

https://www.youtube.com/watch?v=FwjpyZ1nyE0&t=447s

$\int_0^{\pi} \int_0^{a(1+\cos\theta)} r^2 \cos\theta dr d\theta$

https://www.youtube.com/watch?v=_V9FEfzTSxw&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h&index=3

Solve $\int_0^{\pi} \int_0^{a\cos\theta} dr d\theta$

https://www.youtube.com/watch?v=lUzFBROiTFc&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h

4. Evaluate $\int_0^{\pi/2} \int_0^{a\sin\theta} \int_0^{\frac{a^2-r^2}{a}} r\, dz dr d\theta$

https://www.youtube.com/watch?v=yruU9YNV4Es

5. Evaluate $\int_b^{\pi/2} \int_0^{a\cos\theta} \int_0^{\sqrt{a^2-r^2}} r\;\, dz\, dr d\theta$

https://www.youtube.com/watch?v=QQLGMS4hlOg

Find the limits after changing the order of integration for $\int_0^b \int_0^{\frac{a}{b}\sqrt{b^2-y^2}} f(x,y)\,dydx$

https://www.youtube.com/watch?v=BGDogvhOcM4&t=93s

https://www.youtube.com/watch?v=BGDogvhOcM4&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h&index=7

Evaluation of double integral over the region (limits they wont give)

Evaluate (finding limits) $\iint_R f(x,y)dxdy$ where R is region

i) Bounded in the +ve quadrant of circle $x^2+y^2=a^2$

ii) Bounded in the +ve quadrant of circle $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

https://www.youtube.com/watch?v=PFyakrPmIkc

- Evaluate $\iint \sin\pi\left(x^2+y^2\right)dxdy$ over the region bounded by the circles $x^2+y^2=1$ by changing into polar coordinates.
  https://www.youtube.com/watch?v=Qz9ZK88cg48&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h&index=19
- Evaluate $\iint\left(a^2-x^2-y^2\right)dxdy$, over the semi-circle $x^2+y^2=ax$ in the +ve quadrant by changing to polar coordinates.
  https://www.youtube.com/watch?v=FUVn9RtebAs&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h&index=20

Evaluate $\iint_R (x^2+y^2)dxdy$ where R is a region bounded by positive quadrant for which

$x+y\le 1$ by changing into polar coordinates

https://www.youtube.com/watch?v=DtR-ibVlfkQ

Evaluate $\iint\sqrt{a^2-x^2-y^2}\,dxdy$ over the semi-circle

$x^2+y^2=ax$ in the positive quadrant

https://www.youtube.com/watch?v=7AZ0d4CuuC0&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h&index=14

- Evaluate $\iint_R xy(x+y)dydx$ where R is a region over the area between $y=x^2 and\ y=x$

https://www.youtube.com/watch?v=J5wg0qJ5I00

Evaluate $\iint_R x^2y^2dxdy$ over the circle $x^2+y^2=1$

https://www.youtube.com/watch?v=UAJRT6wK4so

- Evaluate $\iint r^3 dr d\theta$ over the area included between the circles $r=2\sin\theta\ and\ r=4\sin\theta$

https://www.youtube.com/watch?v=eGqQS3fPUfw&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h&index=24

Show that $\iint_R r^2 \sin\theta dr d\theta = \frac{2a^3}{3}$ where R is the region. Bounded by the semi circle $r = 2a\cos\theta$ above the initial line.

https://www.youtube.com/watch?v=BvyvCDAlWyA&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h&index=25

- Find, by double integration, the area lying inside the cardioid $r = a(1+\cos\theta)$ and outside the circle $r = a$

https://www.youtube.com/watch?v=HxwzMx9NJjs&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h&index=28

Evaluate $\int\int_R r\sin\theta \; dr d\theta$ over the cardioids $r = a(1-\cos\theta)$ above the initial line

https://www.youtube.com/watch?v=gPV18ElvYn8&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h&index=9

Evaluate $\iint \frac{rd\; dr}{\sqrt{a^2+r^2}}$ over one loop of the lemniscates $r^2 = a^2\cos 2\theta$

https://www.youtube.com/watch?v=FM-LBiyu5cc&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h&index=15

By changing the order of integration

Evaluation of double integral by changing the order of integration $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 dydx$

https://www.youtube.com/watch?v=n0c-tK0bGhw&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h&index=11

Evaluation of double integral by changing the order of integration $\int_0^1 \int_{4y}^4 e^{x^2} dxdy$

https://www.youtube.com/watch?v=6R9wErh5EXY

Evaluation of double integral by changing the order of integration $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}} dydx$

https://www.youtube.com/watch?v=mvN-rMr2qVg

Evaluation of double integral by changing the order of integration $\int_0^\pi \int_x^\pi \frac{\sin y}{y} dydx$

https://www.youtube.com/watch?v=R5XNSM6GSXY&t=34s (in telugu)

https://www.youtube.com/watch?v=fSjexcMEjno

Evaluation of double integral by changing the order of integration $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dxdy$

https://www.youtube.com/watch?v=1kEvIpJJl4U (in telugu)

https://www.youtube.com/watch?v=tRYlYmeWV4s

Evaluation of double integrals by Change of variables

- Evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2} dxdy$ by changing to polar coordinates

https://www.youtube.com/watch?v=6_HaCy0e3xc&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h&index=16

- Evaluate $\int_0^a \int_y^a \frac{x^2}{\sqrt{x^2+y^2}} dxdy$ by changing to polar coordinates.

https://www.youtube.com/watch?v=gCEFsdVIIME&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h&index=18

Evaluation of double integration by change of variable method $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$

https://www.youtube.com/watch?v=MuMCpF1SNno&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h&index=2

- Evaluate $\iint_R (x^2+y^2)^{\frac{7}{2}} dxdy$, over the circle $x^2+y^2=1$ by changing to polar coordinates.

https://www.youtube.com/watch?v=QoirsCInvTk&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h&index=21

- Evaluate $\int_{-a}^{a} \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dydx$ by changing to polar coordinates

https://www.youtube.com/watch?v=fbXOJmac1ZU&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h&index=23

- Evaluate $\iint_R xy(x^2+y^2)^{\frac{3}{2}} dxdy$ over the quadrant of the circle $x^2+y^2=1$, by changing into polar coordinates.

https://www.youtube.com/watch?v=LwmmbLI7-9k&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h&index=22

Miscellaneous problems

Show that the area between the parabolas $y^2 = 4ax \text{ and } x^2 = 4ay \text{ is } \frac{16}{3}a^2$

https://www.youtube.com/watch?v=P_Xpu8NgeVE&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h&index=6

Find the volume of the region bounded by $z = x^2+y^2, z=0, x=-a, x=a \text{ and } y=a, y=a$

https://www.youtube.com/watch?v=UzK_9IK_RR4&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h&index=12

- If $\overline{F} = \left(3x^2-2z\right)i - 4xyj - 5xk$, evaluate $\int_v curl\ \overline{F} dv$ where v is a volume bounded by planes.

$x=0 y=0, z=0 \ ad\ 3x+2y-3z=6$

https://www.youtube.com/watch?v=STl4mRQYJuk&list=PL3b73JEqFpKbsEkzC7fYYgMxY7z2Gn28h&index=17

Dear students, for remaining topics I need time, happy to help you