Codeforces

Combinatorics

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Probability

Probability -> Denotes the likelihood of an event

$0 <= P(e) <= 1$

$P(e)$ = no of favourable outcomes / total no of possible outcomes

$P(\neg e) = 1 - P(e)$

$P(a \cup b) = P(a) + P(b) - P(a \text{ and } b)$

If a and b are disjoint events

$P(a \text{ and } b) = 0$

Conditional Probability

P(a/b) = Probability of happening of event "a" given that b has already occurred

$P(a/b) = P(a \cap b)/P(b)$

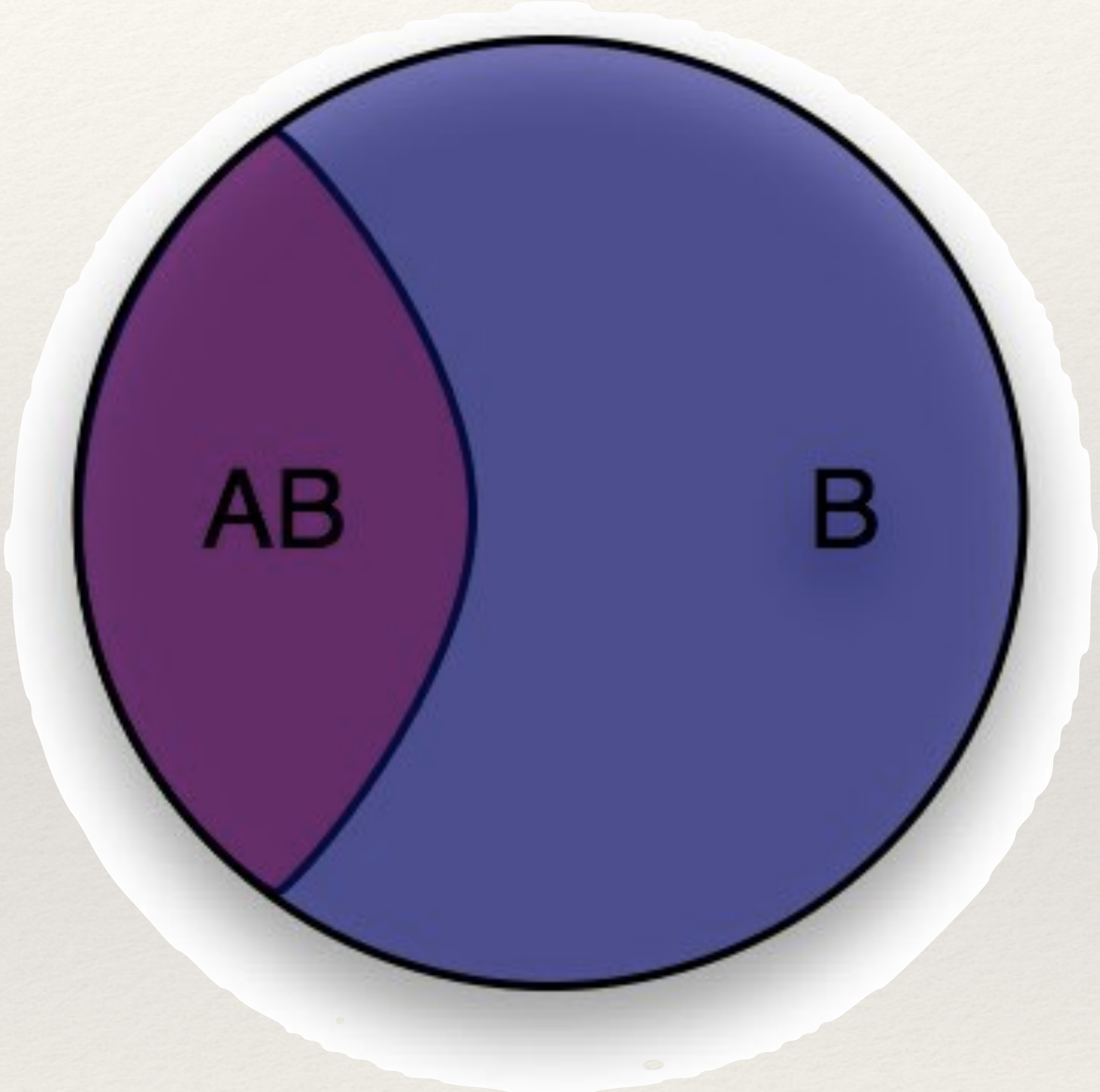
$P(b/a) = P(b \cap a)/P(a)$

alternatively

$P(a \cap b) = P(a/b) * P(b) = P(b/a) * P(a)$

Bayes Theoram

$$P(a/b) = \frac{P(b/a) \times P(a)}{P(b)}$$



Independent Events

Two event a and b are independent iff

P(a/b) = P(a) or P(b/a) = P(b)

P(a and b) = P(a)*P(b)

Random variable

RV is a value which is generated by some random process.

X = sum of outcomes

X = sum of odd outcomes

X = (sum of those outcomes)%5

Problem

There are N chits, out of which, A chits have 1 written on them and B $(A + B = N)$ chits have 0 written on them.

Chef and Chefina are taking alternate turns. They both have a personal score which is initially set to 0. Chef starts first. On their turn, a player will:

- Pick a random chit among the available chits;
- Add the number written on the chit to their personal score;
- Remove the chit that was picked.

Determine the expected score of Chef after all chits have been picked.

The expected score can be written in the form of $\frac{P}{Q}$ such that Q is not divisible by 998244353 and its guaranteed that for testcases that Q is not a multiple of 998244353. Output the value of $P \cdot Q^{-1}$ mod 998244353.

Input Format

- The first line of input will contain a single integer T, denoting the number of test cases.
- Each test case consists of two integers A and B, the number of chits having 1 and 0 respectively.

Output Format

For each test case, output the expected score of Chef.
The expected score can be written in the form of $\frac{P}{Q}$. Output the value of $P \cdot Q^{-1}$ mod 998244353.

I have seenn the comment of this question is if you have basic knowledge of binomial distribution you can solve this can easily

Observation

Line 5

Suppose we arrange a number of 1s and b numbers of 0s.
Chef will get the odd places and Chefina will get the even places.

Thus, the expected total for Chef will be the sum over all t; the probability of getting a sum equal to t times t.

Suppose k is the number of odd places.
We need to evaluate the below expression

$$\sum_t t \cdot \frac{{}^kC_t \cdot {}^{n-k}C_{a-t}}{{}^nC_a}$$

Approach

Line 8

Consider the Binomial expansions

$$(1+x)^k = {}^kC_0 + {}^kC_1 x + \ldots + {}^kC_k x^k = \sum_{i=0}^{k} {}^kC_i x^i$$

Differentiating the above polynomial on both sides,

$$k \cdot (1+x)^{k-1} = \sum_{i=0}^{k} i \cdot {}^kC_i x^{i-1}$$

$$(1+x)^{n-k} = \sum_{i=0}^{n-k} {}^{n-k}C_i x^i$$

Multiplying the above polynomials, we get

$$k \cdot (1+x)^{n-k+k-1} = \sum_{i=0}^{n-1} \sum_t t \cdot {}^kC_t x^{t-1} \cdot {}^{n-k}C_{i+1-t} x^{i+1-t}$$

The above equation simplifies to

$$k \cdot (1+x)^{n-1} = \sum_{i=0}^{n-1} \sum_{t} t \cdot {}^{k}C_{t} \cdot {}^{n-k}C_{i+1-t} x^{i}$$

Putting $a = i + 1$ for the required expression and computing the co-efficient of x^{a-1} on LHS, we get the below expression.

$$k \cdot {}^{n-1}C_{a-1} = \sum_{t} t \cdot {}^{k}C_{t} \cdot {}^{n-k}C_{a-t}$$

Thus, the required expression is

$$\frac{k \cdot {}^{n-1}C_{a-1}}{{}^{n}C_{a}} = \frac{k \cdot a}{n}$$

We need to compute $P * Q^{-1} mod\ 998244353$.
This is just computing *Modular multiplicative inverse* of Q modulo 998244353.
As 998244353 is a prime, from Little Fermat Theorem we have:
$Q^{-1} \, mod \, p \equiv Q^{p-2} \, mod \, p$
So we just need to calculate $P * Q^{998244353-2} \, mod \, 998244353$.
This can be done in logarithmic time using Modular Exponentiation.

PROBLEM:

There are A ones and B zeros.
Alice and Bob alternate turns; with Alice moving first.
On their turn, a player chooses (uniformly at random) one of the remaining elements, adds it to their score, and then discards that value.

What is Alice's expected final score?

EXPLANATION:

If you're familiar with linearity of expectation, this is a rather straightforward task.
Each 1 contributes independently to the answer, so let's find the probability that Alice chooses a specific 1 and then multiply this by A: this will be the final answer.

Note that Alice will choose exactly $k = \lceil \frac{N}{2} \rceil$ of the $N = A + B$ elements available. Since each choice is fully random, each set of size k is equally likely to be chosen.
So, there are $\binom{N}{k}$ choices of what the final set can be.

Of these, we'd like to count the number of sets that include a specific 1.
This is not hard: if a 1 is fixed, the other $k - 1$ elements of Alice's set must be chosen from the remaining $N - 1$ elements, giving us $\binom{N-1}{k-1}$ possible choices; again, each one is equally likely.

So, the required probability is $\frac{\binom{N-1}{k-1}}{\binom{N}{k}}$.
Expanding this in terms of factorials and cancelling out will reduce this to just $\frac{k}{N}$.

The final answer is thus simply $A \cdot \frac{k}{N}$, where $k = \lceil \frac{N}{2} \rceil$.
This can be computed in $\mathcal{O}(\log MOD)$, since all that needs to be done is to invert N with respect to MOD.
If you don't know how to compute inverses, a tutorial is linked in the prerequisites above.

- Discuss the problem 1641 leetcode contest

Problem 1 codeism

Homework:

https://codeforces.com/problemset/problem/1543/C

https://codeforces.com/problemset/problem/1525/E