DSP LAB - Experiment 6

Discrete Time Fourier Transform (DTFT)

Ajay Krishnan K
EE22BTECH11003

February 28, 2024

Aim

To compute the Discrete Time Fourier Transform (DTFT) of a given sequence.

Theory

The Discrete Time Fourier Transform (DTFT) of a sequence $x[n]$ is given by

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \tag{1}$$

where $X(\omega)$ is a continuous function of ω.

The DTFT of a sequence is a continuous function of frequency and is periodic with period 2π. It provides the frequency content of the given sequence $x[n]$. The magnitude of $X(w)$ gives the strength of the frequency components and the phase of $X(w)$ gives the phase shift of the frequency components.

Procedure

The DTFT of a given sequence can be computed using the following steps:

1. Generate the sequence $x[n]$.
2. Compute the DTFT $X(w)$ using the formula (1).
3. Plot the $X(w)$.

Implementation

The following MATLAB code computes the DTFT of a given sequence $x[n]$ and plots $X(w)$.

```
fs = 4000;
duration = 1;
t = 0:1/fs:duration;

% Create the input signal
f1 = 500;
f2 = 1000;
f3 = 700;

x1 = sin(2*pi*f1*t);
x2 = sin(2*pi*f2*t);
x3 = sin(2*pi*f3*t);

x = x1 + 2*x2 + 1.5*x3;

% take the fft of the input signal
w = linspace(-pi,pi,length(x));
X = dtft(x, w);

% reduce the amplitude of the output
X = X / max(abs(X));

% create the frequency axis
f = linspace(-fs/2,fs/2,length(X));

plot the input signal
figure(1)
subplot(2,1,1)
plot(t,x)
xlabel('Time (s)')
ylabel('Amplitude')
title('Input Signal')
subplot(2,1,2)
plot(f,abs(X))
xlabel('Frequency (Hz)')
ylabel('Magnitude')
title('Frequency Spectrum of Input Signal')

% define the dtft
function X = dtft(x, w)
    X = zeros(1,length(w));
    for i = 1:length(w)
        for n = 1:length(x)
            X(i) = X(i) + x(n)*exp(-1j*w(i)*(n-1));
        end
    end
end
```

Results

The DTFT of the given sequence $x[n]$ is computed and plotted. The magnitude plot of $X(w)$ is shown in the figure below.

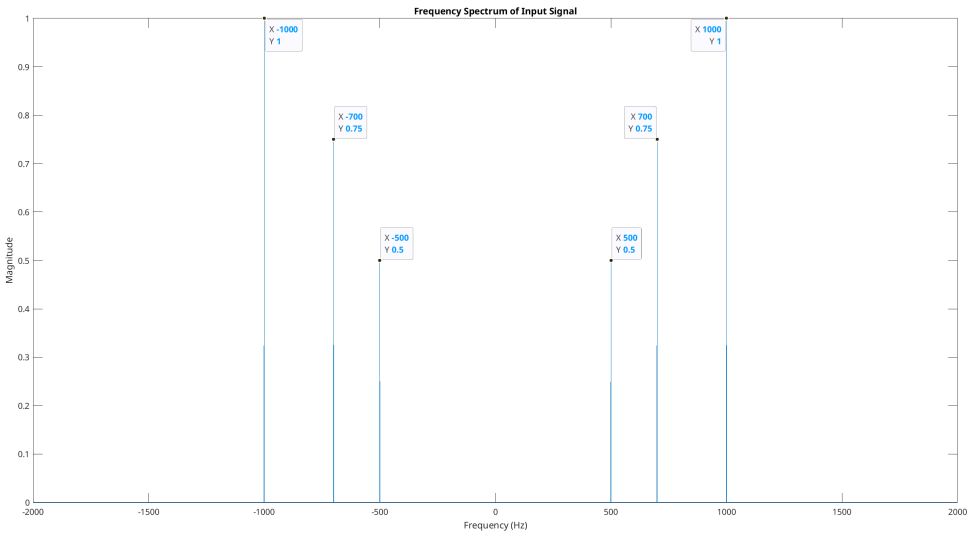

Figure 1: Magnitude of $X(w)$

Observations

The magnitude of $X(w)$ is plotted in Fig. 1. The figure shows three peaks at $f = 500Hz$, $f = 1000Hz$ and $f = 700Hz$ (same at the negative frequencies), with magnitudes $|X(w)| = 0.5$, $|X(w)| = 1$ and $|X(w)| = 0.75$ respectively. This indicates the presence of frequency components at these frequencies in the input sequence $x[n]$. This magnitudes are also consistent with the input sequence $x[n]$.

Conclusion

The Discrete Time Fourier Transform (DTFT) of a given sequence has been computed and plotted. This method can be used to analyze the frequency content of a given sequence. From the plot, we can observe the frequency components present in the input sequence and the magnitude of $X(w)$ is consistent with the input sequence $x[n]$.