

# Assignment 1-b

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## Solution for Problem 1.1.6

### Problem Statement

The area of  $\triangle ABC$  is defined as

$$\frac{1}{2} \|(\mathbf{A} - \mathbf{B}) \times \mathbf{A} - \mathbf{C}\| \quad (1)$$

where

$$\mathbf{A} \times \mathbf{B} \triangleq \begin{vmatrix} 1 & -4 \\ -1 & 6 \end{vmatrix} \quad (2)$$

Find the area of  $\triangle ABC$ .

### Solution

Given triangle with vertices

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (3)$$

Now,

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad (4)$$

$$= \begin{pmatrix} 5 \\ -7 \end{pmatrix} \quad (5)$$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (6)$$

$$= \begin{pmatrix} 4 \\ 4 \end{pmatrix} \quad (7)$$

Since

$$\mathbf{A} \times \mathbf{B} \triangleq \begin{vmatrix} 1 & -4 \\ -1 & 6 \end{vmatrix} \quad (8)$$

We get,

$$(\mathbf{A} - \mathbf{B}) \times (\mathbf{A} - \mathbf{C}) = \begin{pmatrix} 5 \\ -7 \end{pmatrix} \times \begin{pmatrix} 4 \\ 4 \end{pmatrix} \quad (9)$$

$$= \begin{vmatrix} 5 & 4 \\ -7 & 4 \end{vmatrix} \quad (10)$$

$$= (48) \quad (11)$$

Since

$$\|\mathbf{B} - \mathbf{A}\| \triangleq \sqrt{(\mathbf{B} - \mathbf{A})^\top \mathbf{B} - \mathbf{A}} \quad (12)$$

We get,

$$\|(\mathbf{A} - \mathbf{B}) \times (\mathbf{A} - \mathbf{C})\| = \sqrt{(48)^\top (48)} \quad (13)$$

$$= \sqrt{(48)^2} \quad (14)$$

$$= 48 \quad (15)$$

Therefore, the area of  $\triangle ABC$

$$= \frac{1}{2} \|(\mathbf{A} - \mathbf{B}) \times \mathbf{A} - \mathbf{C}\| \quad (16)$$

$$= \frac{1}{2} \times 48 \quad (17)$$

$$= 24 \text{ sq.units} \quad (18)$$