

Assignment 2

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Question 12.13.6.16

- 1) Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

Solution:

Random Variable	Value of Random Variable	Event
X	0	Ball is Red
	1	Ball is black
Y	0	Ball drawn from bag I
	1	Ball drawn from bag II

TABLE I: Random Variable Distribution

Let X_1 be the state before ball is transferred and

X_2 be the state after ball is transferred.

When ball is not transferred,

$$\Pr(X_1 = 0, Y = 0) = \frac{3}{7} \quad (1)$$

$$\Pr(X_1 = 1, Y = 0) = \frac{4}{7} \quad (2)$$

When the ball being transferred is red,

$$\Pr(X_2 = 0, Y = 1|X_1 = 0) = \frac{5}{10} \quad (3)$$

$$= \frac{1}{2} \quad (4)$$

When the ball being transferred is black,

$$\Pr(X_2 = 0, Y = 1|X_1 = 1) = \frac{4}{10} \quad (5)$$

$$= \frac{2}{5} \quad (6)$$

Now the probability of the transferred ball is black given drawn ball being red is

(According to Bayes' theorem)

$$\Pr(X_1 = 1|X_2 = 0, Y = 1) = \frac{\Pr(X_1 = 1) \Pr(X_2 = 0, Y = 1|X_1 = 1)}{\Pr(X_1 = 0) \Pr(X_2 = 0, Y = 1|X_1 = 0) + \Pr(X_1 = 1) \Pr(X_2 = 0, Y = 1|X_1 = 1)} \quad (7)$$

$$= \frac{\frac{4}{7} \times \frac{2}{5}}{\frac{3}{7} \times \frac{1}{2} + \frac{4}{7} \times \frac{2}{5}} \quad (8)$$

$$= \frac{16}{31} \quad (9)$$