# Gate ST-37.2023

Ajay Krishnan K
EE22BTECH11003

**Question**

1) Let $X$ be a random variable with probability density function

$$f(x) = \begin{cases} \frac{1}{x^2} & if x \geq 1 \\ 0 & otherwise. \end{cases} \tag{1}$$

If $Y = \log_e X$, then $\Pr(Y < 1 | Y < 2)$ equals

**Solution:** Given, the probability density function of $X$ is

$$f(x) = \begin{cases} \frac{1}{x^2} & if x \geq 1 \\ 0 & otherwise. \end{cases} \tag{2}$$

Also, $Y = \log_e X$.

Consider the cumulative distribution function(CDF) of $X$,

$$F_X(x) = \Pr(X \leq x) \tag{3}$$
$$= \int_1^x \frac{1}{x^2} dx \tag{4}$$
$$= 1 - \frac{1}{x}, x \geq 1 \tag{5}$$

Now, we need to find the CDF of $Y$.

$$F_Y(y) = \Pr(Y \leq y) \tag{6}$$
$$= \Pr(\log_e X \leq y) \tag{7}$$
$$= \Pr(X \leq e^y) \tag{8}$$
$$= F_X(e^y) \tag{9}$$
$$= 1 - \frac{1}{e^y}, y \geq 0 \tag{10}$$

Now, we need to find $\Pr(Y < 1 | Y < 2)$. For that, we need to find $F_Y(1)$ and $F_Y(2)$.

Using the equation for CDF,

$$F_Y(1) = 1 - \frac{1}{e} \tag{11}$$

and

$$F_Y(2) = 1 - \frac{1}{e^2} \tag{12}$$

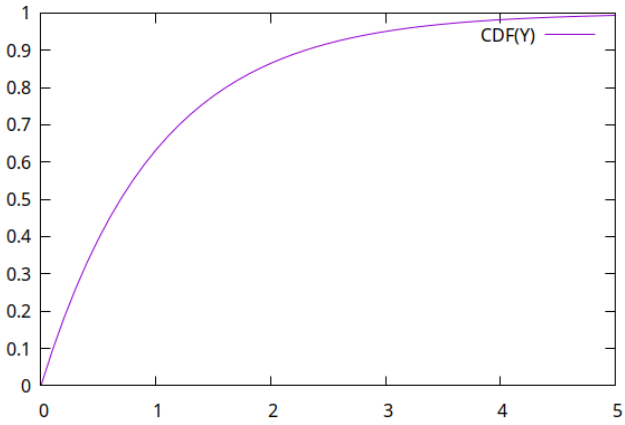


Fig. 1: CDF of $Y$

Now, we can find $\Pr(Y < 1 | Y < 2)$ as follows,

$$\Pr(Y < 1 | Y < 2) = \frac{\Pr(Y < 1, Y < 2)}{\Pr(Y < 2)} \tag{13}$$
$$= \frac{\Pr(Y < 1)}{\Pr(Y < 2)} \tag{14}$$
$$= \frac{F_Y(1)}{F_Y(2)} \tag{15}$$
$$= \frac{1 - \frac{1}{e}}{1 - \frac{1}{e^2}} \tag{16}$$
$$= \frac{e(e-1)}{e^2 - 1} \tag{17}$$
$$= \frac{e}{e+1} \tag{18}$$

2) Steps to plot the cdf of $Y$.
   a) Generate a uniform random variable between 0 and 1.
   b) Given the PDF of $X$ as $\frac{1}{x^2}$, use the inverse transform method to generate $X$.
      The inverse transform method is given by,
      $$X = F^{-1}(U) \tag{19}$$
      where $U$ is a uniform random variable between 0 and 1 and $F^{-1}$ is the inverse of CDF of $X$.
   c) The CDF of $X$ is given by,
      $$F(x) = \int_1^x \frac{1}{x^2} dx = 1 - \frac{1}{x} \tag{20}$$

The inverse of CDF of $X$ is given by,

$$F^{-1}(x) = \frac{1}{1-x} \tag{21}$$

d) Use $X$ to generate $Y = \log_e X$.
e) Calculate the CDF of $Y$ using the equation $1 - \frac{1}{e^y}$.
f) Store the values of CDF in data file.
g) Plot the CDF using GNUPlot.