Given a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} \tag{1}$$

## 1 Section 1

| Parameters | Values | Description |
|---|---|---|
| **m1** | $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ | **AB** |
| **m2** | $\begin{pmatrix} 6 \\ -1 \end{pmatrix}$ | **BC** |
| **m3** | $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$ | **AC** |
| $\|\mathbf{B}-\mathbf{A}\|$ | 6.08 | Length of $BC$ |
| Rank | 3 | Non-Collinear |
| $\mathbf{n_1}$ | $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ | Normal to **AB** |
| $c_1$ | -3 | |
| $\mathbf{n_2}$ | $\begin{pmatrix} -1 \\ -6 \end{pmatrix}$ | Normal to **BC** |
| $c_2$ | 7 | |
| **n3** | $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$ | Normal to **AC** |
| $c_3$ | 1 | |
| Area | 1.5 | Area of $\triangle ABC$ |
| $\angle A$ | 161.565 | Angle |
| $\angle B$ | 9.46 | |
| $\angle C$ | 8.97 | |

TABLE 1: Section 1

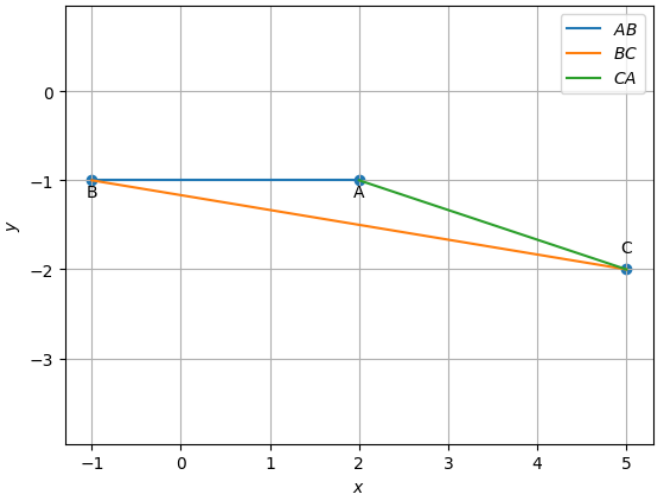


Fig. 1: Triangle using python

## 2 Section 2

| Parameters | Values | Description |
|---|---|---|
| $D$ | $\begin{pmatrix} 2 \\ -1.5 \end{pmatrix}$ | Midpoint of $BC$ |
| $E$ | $\begin{pmatrix} 3.5 \\ -1.5 \end{pmatrix}$ | Midpoint of $CA$ |
| $F$ | $\begin{pmatrix} 1.5 \\ 1.0 \end{pmatrix}$ | Midpoint of $AB$ |
| $\mathbf{n_1}$ | $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$ | $AD$ |
| $c_1$ | -2 | |
| **n2** | $\begin{pmatrix} -1 \\ -9 \end{pmatrix}$ | $BE$ |
| $c_2$ | 10 | |
| **n3** | $\begin{pmatrix} 2 \\ 9 \end{pmatrix}$ | $CF$ |
| $c_3$ | -8 | |
| $G$ | $\begin{pmatrix} 2.0 \\ -1.33 \end{pmatrix}$ | Centroid |
| Rank | 2 | Collinear |

TABLE 2: Section 2

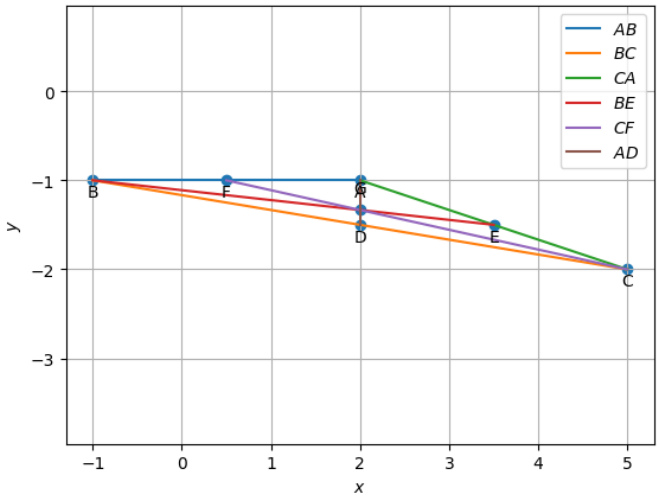


Fig. 2: Centroid using python

## 3 Section 3

| Parameters | Values | Description |
|---|---|---|
| **D** | $\begin{pmatrix}1.92\\-1.49\end{pmatrix}$ | Foot on $BC$ |
| **E** | $\begin{pmatrix}-0.7\\-0.1\end{pmatrix}$ | Foot on $CA$ |
| **F** | $\begin{pmatrix}5\\-1\end{pmatrix}$ | Foot on $AB$ |
| $\mathbf{n_1}$ | $\begin{pmatrix}-6\\1\end{pmatrix}$ | $AD$ |
| $\mathbf{m_1}$ | $\begin{pmatrix}0.49\\-0.08\end{pmatrix}$ | |
| $c_1$ | -7 | |
| $\mathbf{n_2}$ | $\begin{pmatrix}3\\-1\end{pmatrix}$ | $BE$ |
| $\mathbf{m_2}$ | $\begin{pmatrix}-0.9\\0.3\end{pmatrix}$ | |
| $c_2$ | -2 | |
| $\mathbf{n_3}$ | $\begin{pmatrix}3\\0\end{pmatrix}$ | $CF$ |
| $\mathbf{m_3}$ | $\begin{pmatrix}-1\\0\end{pmatrix}$ | |
| $c_3$ | 15 | |
| **H** | $\begin{pmatrix}5\\17\end{pmatrix}$ | Orthocentre |

TABLE 3: Section 3

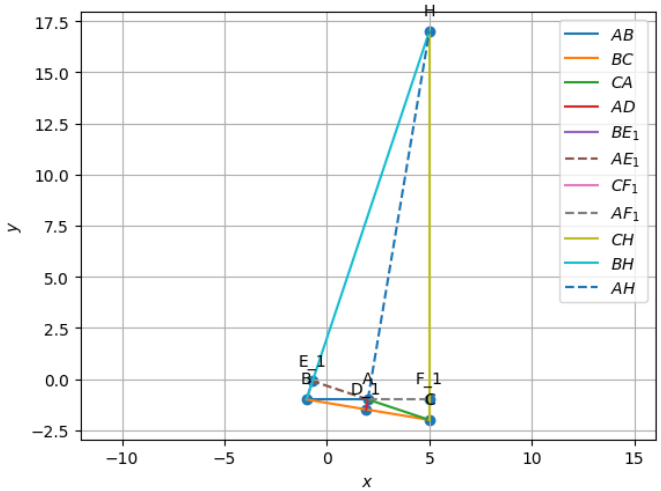


Fig. 3: Altitudes using python

## 4 Section 4

| Parameters | Values | Description |
|---|---|---|
| $\mathbf{m_1}$ | $\begin{pmatrix}3\\0\end{pmatrix}$ | Perpendicular Bisector of $AB$ |
| $c_1$ | 1.5 | |
| $\mathbf{m_2}$ | $\begin{pmatrix}-6\\1\end{pmatrix}$ | Perpendicular Bisector of $BC$ |
| $c_2$ | $-13.5$ | |
| $\mathbf{m_3}$ | $\begin{pmatrix}3\\-1\end{pmatrix}$ | Perpendicular Bisector of $CA$ |
| $c_3$ | 12 | |
| **O** | $\begin{pmatrix}0.5\\-10.5\end{pmatrix}$ | Circumcentre |
| **R** | 9.62 | Circumradius |
| $\angle BOC$ | 323.13° | $\angle BOC = 2\angle BAC$ |
| $\angle BAC$ | 161.56° | |

TABLE 4: Section 4

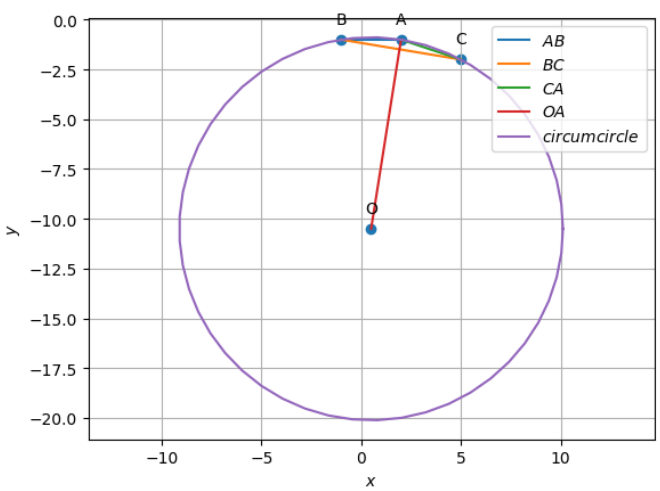


Fig. 4: Circumcircle using python

## 5 Section 5

| Parameters | Values | Description |
|---|---|---|
| $\mathbf{m_1}$ | $\begin{pmatrix}0.05\\0.32\end{pmatrix}$ | Angle bisector of $\angle A$ |
| $c_1$ | $-0.68$ | |
| $\mathbf{m_2}$ | $\begin{pmatrix}1.99\\-0.16\end{pmatrix}$ | Angle bisector of $\angle B$ |
| $c_2$ | 2.15 | |
| $\mathbf{m_3}$ | $\begin{pmatrix}1.93\\-0.48\end{pmatrix}$ | Angle bisector of $\angle C$ |
| $c_3$ | $-1.47$ | |
| $\mathbf{I}$ | $\begin{pmatrix}1.96\\-1.24\end{pmatrix}$ | Incentre |
| $\mathbf{D_3}$ | $\begin{pmatrix}1.92\\-1.49\end{pmatrix}$ | Point of contact with $BC$ |
| $\mathbf{E_3}$ | $\begin{pmatrix}1.96\\-1\end{pmatrix}$ | Point of contact with $CA$ |
| $\mathbf{F_3}$ | $\begin{pmatrix}2.04\\-1.01\end{pmatrix}$ | Point of contact with $AB$ |
| $\|\mathbf{I}-\mathbf{D_3}\|$ | 0.24 | $ID_3 = IF_3 = IE_3 = r$ |
| $\|\mathbf{I}-\mathbf{E_3}\|$ | | |
| $\|\mathbf{I}-\mathbf{F_3}\|$ | | |
| $r$ | | |
| $\angle BAI$ | 80.78° | $\angle BAI = \angle CAI$ |
| $\angle CAI$ | 80.78° | |

TABLE 5: Section 5

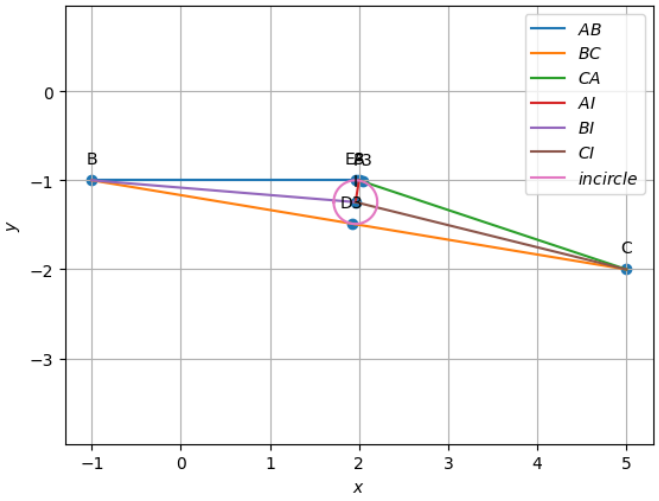


Fig. 5: Incircle using python