Assignment 1-b

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Solution for Problem 1.1.6

Problem Statement
The area of $\triangle ABC$ is defined as

$$\frac{1}{2}\|(A - B) \times A - C\| \tag{1}$$

where

$$A \times B \triangleq \begin{vmatrix} 1 & -4 \\ -1 & 6 \end{vmatrix} \tag{2}$$

Find the area of $\triangle ABC$.
Solution
Given triangle with vertices

$$A = \begin{pmatrix} 1 \\ -1 \end{pmatrix},\ B = \begin{pmatrix} -4 \\ 6 \end{pmatrix},\ C = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{3}$$

Now,

$$A - B = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{4}$$

$$= \begin{pmatrix} 5 \\ -7 \end{pmatrix} \tag{5}$$

$$A - C = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{6}$$

$$= \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{7}$$

Since

$$A \times B \triangleq \begin{vmatrix} 1 & -4 \\ -1 & 6 \end{vmatrix} \tag{8}$$

We get,

$$(A - B) \times (A - C) = \begin{pmatrix} 5 \\ -7 \end{pmatrix} \times \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{9}$$

$$= \begin{vmatrix} 5 & 4 \\ -7 & 4 \end{vmatrix} \tag{10}$$

$$= 48 \tag{11}$$

Therefore, the area of $\triangle ABC$

$$= \frac{1}{2}\|(A - B) \times A - C\| \tag{12}$$

$$= \frac{1}{2} \times \|48\| \tag{13}$$

$$= \frac{1}{2} \times 48 \tag{14}$$

$$= 24 \text{ sq.units} \tag{15}$$