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Question Let X be a random variable with probability density function Now, we can find $\Pr(Y < 1|Y < 2)$ as follows,

$$f(x) = \begin{cases} \frac{1}{x^2} & \text{if } x \geq 1 \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

If $Y = \log_e X$, then $\Pr(Y < 1|Y < 2)$ equals

$$\Pr(Y < 1|Y < 2) = \frac{\Pr(Y < 1, Y < 2)}{\Pr(Y < 2)} \quad (10)$$

$$= \frac{\Pr(Y < 1)}{\Pr(Y < 2)} \quad (11)$$

$$= \frac{F_Y(1)}{F_Y(2)} \quad (12)$$

$$= \frac{1 - \frac{1}{e}}{1 - \frac{1}{e^2}} \quad (13)$$

$$= \frac{e(e-1)}{e^2-1} \quad (14)$$

$$= \frac{e}{e+1} \quad (15)$$

Solution: Given, the probability density function of X is

$$f(x) = \begin{cases} \frac{1}{x^2} & \text{if } x \geq 1 \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

Also, $Y = \log_e X$.

Consider the cumulative distribution function(CDF) of Y ,

$$F_Y(y) = \Pr(Y \leq y) \quad (3)$$

$$= \Pr(\log_e X \leq y) \quad (4)$$

$$= \Pr(X \leq e^y) \quad (5)$$

$$= \int_1^{e^y} \frac{1}{x^2} dx \quad (6)$$

$$= 1 - \frac{1}{e^y}, y \geq 0 \quad (7)$$

Now, we need to find $\Pr(Y < 1|Y < 2)$. For that, we need to find $F_Y(1)$ and $F_Y(2)$.

Using the equation for CDF,

$$F_Y(1) = 1 - \frac{1}{e} \quad (8)$$

and

$$F_Y(2) = 1 - \frac{1}{e^2} \quad (9)$$