JEE EXPERT

ANSWER KEY

RANK ELEVATOR TEST SERIES

(RETS/PT-01)

12TH (Zenith X01 & X02)

Date 18.08.2019

PHYSICS

1	(B)	2	(A)	3	(D)	4	(B)	5	(C)
6	(C)	7	(D)	8	(C)	9	(D)	10	(B)
11	(C)	12	(B)	13	(B)	14	(D)	15	(A)
16	(A)	17	(C)	18	(C)	19	(D)	20	(C)
21	(A)	22	(C)	23	(C)	24	(A)	25	(B)
26	(D)	27	(D)	28	(C)	29	(C)	30	(B)

CHEMISTRY

31	(C)	32	(C)	33	(C)	34	(C)	35	(D)
36	(C)	37	(B)	38	(C)	39	(C)	40	(A)
41	(B)	42	(B)	43	(C)	44	(D)	45	(C)
46	(C)	47	(C)	48	(D)	49	(D)	50	(C)
51	(B)	52	(A)	53	(B)	54	(C)	55	(B)
56	(D)	57	(A)	58	(D)	59	(B)	60	(B)

MATHEMATICS

61	(C)	62	(B)	63	(C)	64	(B)	65	(C)
66	(B)	67	(A)	68	(C)	69	(C)	70	(B)
71	(C)	72	(A)	73	(B)	74	(B)	75	(A)
76	(A)	77	(D)	78	(B)	79	(A)	80	(B)
81	(C)	82	(B)	83	(B)	84	(C)	85	(B)
86	(A)	87	(B)	88	(C)	89	(D)	90	(A)

JEE EXPERT

SOLUTIONS

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MATHEMATICS

61. **(C)** Given equation can be written as $l(x+2y+1)+m(3x+2y-1)=0$.

Dividing by l, it becomes $(x+2y+1)+\frac{m}{l}(3x+2y-1)=0$ …(i)

The family of lines (i) passes through the point of intersection of $x+2y+1=0$ and $3x+2y-1=07$ $\therefore$ $(x,\ y)\equiv(1,\ -1)$

62. **(B)** Let y = m_1x and y = m_2x

$\therefore$ $m_1+m_2=\frac{-2h}{b},\quad m_1m_2=\frac{a}{b}$

in new position $m_1m_1'=-1 \Rightarrow m_1'=-\frac{1}{m_1}$

Similarly, $m_2'=-\frac{1}{m_2}$

New lines are $y=\left(-\frac{1}{m_1}\right)x$ and $y=\left(-\frac{1}{m_2}\right)x$

$\therefore$ $(m_1y+x)(m_2y+x)=0 \quad \Rightarrow \quad bx^2-2hxy+ay^2=0$

63. **(C)** Any line through (1, 2) can be written as $\frac{x-1}{\cos\theta}=\frac{y-2}{\sin\theta}=r$

where θ is the angle which this line makes with positive direction of x-axis. Any point on this line is (r cosθ + 1, r cosθ + 2) when $|r|=\frac{1}{3}\sqrt{6}$, this point lies on the line x + y = 4.

i.e. r cosθ + 1 + r sinθ + 2 = 4,

$|r|=\frac{1}{3}\sqrt{6}$

$\Rightarrow \quad r(\cos\theta + \sin\theta) = 1, \; |r| = \frac{1}{3}\sqrt{6} \;\Rightarrow\; r^2(1 + 2\sin\theta\cos\theta) = 1, \; r^2 = \frac{6}{9}$

$\Rightarrow \quad 1 + \sin 2\theta = \frac{1}{r^2} = \frac{9}{6} \quad \Rightarrow \quad \sin 2\theta = \frac{1}{2} \Rightarrow 2\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6} \quad \Rightarrow$

$\theta = \frac{\pi}{12}$ or $\frac{5\pi}{12}$

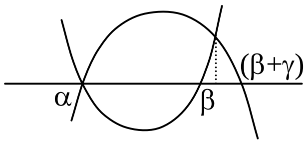
64. **(B)** As $\frac{PB}{PA} = 3 \quad \Rightarrow \quad AB = (PB - PA) = 2\sqrt{2} \quad \{\because \; (PA)(PB) = \text{PT}^2\}$

As point P is (2, 2) and the given circle $x^2 + y^2 = 2$ has diameter $2\sqrt{2}$.

$\Rightarrow$ AB is diameter of the circle.

$\Rightarrow$ Line must pass through the centre (0, 0) also.

65. **(C)**



66. **(B)**

67. **(A)** The circle is

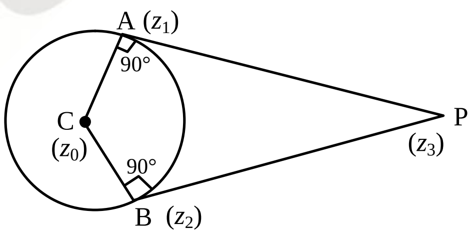
$\left(x^2 + y^2 - a^2\right) + \lambda(2x - a) = 0$

$\therefore$ it passes through (2a, 0), then we have, $\lambda = -a$

68. **(C)** Centre $\equiv (-g, -f)$

The normal to given circle passes through centre.

69. **(C)** Let z_1, z_2 are represented by A, B whereas z_0 is represented by C.



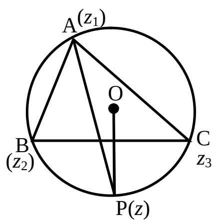
Let P represent z_3

$$\frac{z_3 - z_1}{z_0 - z_1} = \frac{PA}{AC} e^{\frac{i\pi}{2}}; \quad \frac{z_0 - z_2}{z_3 - z_2} = \frac{BC}{PB} e^{\frac{i\pi}{2}}$$

$$\frac{z_3 - z_1}{z_0 - z_1} \cdot \frac{z_0 - z_2}{z_3 - z_2} = \frac{PA}{\text{radius}} \cdot \frac{\text{radius}}{PB} e^{i\pi} = -1 \quad (\because \; PA = PB)$$

70. **(B)** Since $OP \perp BC$

$\Rightarrow \quad \arg\left(\frac{z}{z_3 - z_2}\right) = -\frac{\pi}{2}$



71. **(C)** We have,

$$x^n - 1 = (x-1)(x-\alpha)(x-\alpha^2)......(x-\alpha^{n-1})$$

$$\Rightarrow \quad \frac{x^n-1}{x-1} = (x-\alpha)(x-\alpha^2)......(x-\alpha^{n-1})$$

$$\Rightarrow \quad 1 + x + x^2 + ... + x^{n-1} = (x-\alpha)(x-\alpha)^2......(x-\alpha^{n-1})$$

Putting x = 3, we get

$$1 + 3 + 3^2 + ... + 3^{n-1} = (3-\alpha)(3-\alpha^2)....(3-\alpha^{n-1})$$

$$\Rightarrow \quad (3-\alpha)(3-\alpha^2)......(3-\alpha^{n-1}) = \frac{3^n-1}{3-1} = \frac{3^n-1}{2}$$

72. (A)

73. **(B)** We have, $(z + \alpha\beta)^3 = \alpha^3$

$\Rightarrow$ $z = \alpha - \alpha\beta$, $z = \alpha w - \alpha\beta$, $z = \alpha w^2 - \alpha\beta$.

Thus, the vertices A, B and C of ΔABC are respectively, $\alpha - \alpha\beta$, $\alpha w - \alpha\beta$ and $\alpha w^2 - \alpha\beta$.

Clearly, $AB = BC = AC = |\alpha|\ |1 - w| = \sqrt{3}|\alpha|$.

74. **(B)** As the sum of the roots = 3

$\Rightarrow$ both the roots cannot lie in the interval (0, 1).

75. **(A)** The roots of the equation $ax^2 + bx + c = 0$ are of opposite signs if they are real and their product is negative.

i.e. if $b^2 - 4ac \ge 0$ and $\frac{c}{a} < 0$

i.e. if $b^2 - 4ac \ge 0$ and $ac < 0$

i.e. if $ac < 0$

($\because$ when $ac < 0$, $b^2 - 4ac$ is automatically ≥ 0)

Hence, the roots of the given equation are of opposite signs if $2(a^2 - 2a) < 0$

i.e. if $a^2 - 2a < 0$

i.e. $0 < a < 2$.

76. **(A)** Since $2 + i\sqrt{3}$ is a root of $x^2 + px + q = 0$, therefore $2 - i\sqrt{3}$ is also a root of the equation.

Sum of the roots = –p

$\Rightarrow$ $(2 + i\sqrt{3}) + (2 - i\sqrt{3}) = -p$

$\Rightarrow$ $p = -4$

Now, product of the roots = q

$\Rightarrow$ $7 = q$.

77. **(D)** Clearly $\alpha + \beta = p$ and $\alpha\beta = -(p + q)$

Now $(\alpha + 1)(\beta + 1) = \alpha + \beta + \alpha\beta + 1 = p - p - q + 1 = 1 - q$ …(i)

The given expression = $\dfrac{(\alpha+1)^2}{(\alpha+1)^2+(q-1)} + \dfrac{(\beta+1)^2}{(\beta+1)^2+q-1}$

$$= \frac{2(\alpha+1)^2(\beta+1)^2+(q-1)[(\alpha+1)^2+(\beta+1)^2]}{(\alpha+1)^2(\beta+1)^2+(q-1)^2+(q-1)[(\alpha+1)^2+(\beta+1)^2]}$$

$$= \frac{2(1-q)^2+(q-1)[(\alpha+1)^2+(\beta+1)^2]}{2(1-q)^2+(q-1)[(\alpha+1)^2+(\beta+1)^2]} = 1; \text{ [By (i)]}$$

78. **(B)** $2+|e^x-1| = (e^x)^2 - 2e^x + 1 = |e^x-1|^2$

y $|e^x-1|^2 - |e^x-1| - 2 = 0$

or $|e^x-1| = 2, -1 \Rightarrow |e^x-1| = 2$

$\Rightarrow e^x - 1 = 2, -2 \Rightarrow e^x = 3, -1 \Rightarrow e^x = 3$

79. **(A)** We have $\tan\alpha + \tan\beta + \tan\gamma = 0$

$\Rightarrow$ $\tan\gamma = -h$, and since $\tan\gamma$ satisfies the given equation,

$a\tan^3\gamma + (2a - x)\tan\gamma + y = 0$

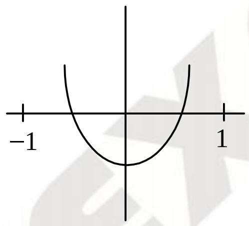
or $a(-h)^3 + (2a - x)(-h) + y = 0$

or $ah^3 + (2a - x)h = y$

80. **(B)** $(2x - |\sin\theta|)^2 - 1 \geq -1$

81. **(C)** $f(x) = ax^2 + bx + c$ represents a parabola as shown in figure

$\therefore$ Roots of $ax^2 + bx + c = 0$ lies in the interval $(-1, 1)$.



82. **(B)** α, β, γ are roots of $x^3 + qx + r = 0$

$\therefore$ $\alpha+1, \beta+1, \gamma+1$ are roots of $(x-1)^3 + q(x-1) + r = 0$

or $x^3 - 3x^2 + x(3+q) + (r-q-1) = 0$

Product of roots = $(-1)^3\dfrac{(r-q-1)}{1}$ or $(\alpha+1)(\beta+1)(\gamma+1) = q+1-r$

83. **(B)** Given $\alpha < \beta$ …(i)

$c < 0 < b$ …(ii)

This $\Rightarrow$ c is negative and b is positive …(iii)

$x^2 + bx + c = 0 \Rightarrow \alpha + \beta = -b$ ...(iv)

$\alpha\beta = c$ ...(v)

$\Rightarrow$ $\alpha\beta$ is negative so α and β are of opposite signs such that their sum is negative.

$\Rightarrow |\alpha| > \beta \Rightarrow \alpha < 0 < \beta < |\alpha|$

84. **(C)** $y^2 + p^2x^2 - 2pxy = a^2 + a^2p^2 \Rightarrow p^2(x^2 - a^2) - p(2xy) + y^2 - a^2 = 0$

Roots are equal, so D = 0

$\Rightarrow 4x^2y^2 = 4(x^2 - a^2)(y^2 - a^2) \Rightarrow x^2 + y^2 = a^2$

85. **(B)** $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

Now, $(1 + \alpha + \alpha^2)(1 + \beta + \beta^2)$

$$= 1 + (\alpha + \beta) + (\alpha + \beta)^2 + \alpha\beta(\alpha + \beta) + \alpha^2\beta^2 - \alpha\beta$$

$$= 1 - \frac{b}{a} + \frac{b^2}{a^2} - \frac{bc}{a^2} + \frac{c^2}{a^2} - \frac{c}{a}$$

$$= \frac{a^2 - ab + b^2 - bc + c^2 - ac}{a^2} = \frac{a^2 + b^2 + c^2 - ab - bc - ac}{a^2}$$

$$= \frac{(a-b)^2 + (b-c)^2 + (c-a)^2}{2a^2} \geq 0$$

Hence given expression is always positive.

86. **(A)** If AB subtends γ at the centre of the circle, then $\gamma = 2C$. Similarly $\beta = 2B, \alpha = 2A$

$\Rightarrow$ Arithmetic mean of three given quantities

$$= \frac{1}{3}\left[\cos\left(2A + \frac{\pi}{2}\right) + \cos\left(2B + \frac{\pi}{2}\right) + \cos\left(2C + \frac{\pi}{2}\right)\right] = \frac{1}{3}[-\sin 2A - \sin 2B - \sin 2C]$$

$$= \frac{-1}{3}[\sin 2A + \sin 2B + \sin 2C]$$

This will be minimum if $\sin 2A + 2B + \sin 2C$ is maximum, which is there at $A = B = C = \frac{\pi}{3}$

and minimum value $= -\frac{1}{3}\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{3}}{2}$

87. **(B)** We have, $a_n = S_n - S_{n-1}$ ...(i)

and $a_{n-1} = S_{n-1} - S_{n-2}$ ...(ii)

$\therefore d = a_n - a_{n-1} = (S_n - S_{n-1}) - (S_{n-1} - S_{n-2})$ [From (i) and (ii)]

$= S_n - 2S_{n-1} + S_{n-2}$

88. **(C)** $t_n = \frac{2n+3}{n(n+1)}.\frac{1}{3^n} = \left(\frac{3}{n} - \frac{1}{n+1}\right)\frac{1}{3^n} = \frac{1}{n.3^{n-1}} - \frac{1}{(n+1)3^n}$

$$\sum_{n=1}^{n} t_n = 1 - \frac{1}{n+1}.\frac{1}{3^n}$$

89. **(D)** tan x tan 4x = 1 $\Rightarrow$ $\cos 4x \cos x - \sin 4x \sin x = 0$

$\Rightarrow \quad \cos 3x = 0 \Rightarrow 3x = (2n+1)\frac{\pi}{2} \Rightarrow x = \frac{(2n+1)\pi}{6},\ n \in I \Rightarrow x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

90. **(A)** From the given relations we have

sin2B = (3/2) sin2A

and $3\sin^2 A = 1 - 2\sin^2 B = \cos 2B$

so that $\tan 2B = \frac{(3/2)\,.\sin 2A}{3\sin^2 A} = \cot A$

or $1 - \tan 2B \tan A = 0$

$\Rightarrow \quad A + 2B = \frac{\pi}{2} \quad \Rightarrow B = \frac{\pi}{4} - \frac{A}{2}$