

# OPTICS

# SOLUTIONS

1.  $\beta = \frac{\lambda D}{d}$  &  $\beta' = \frac{\lambda' D}{d}$

$$\frac{\beta'}{\beta} = \frac{\lambda'}{\lambda} = \frac{1}{\mu}$$

$$\beta' = \beta / \mu = 0.3 \text{ mm}$$

2. Given  $I_1 = I$ , &  $I_2 = 4I$

$$I_A = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \pi/2 = 5I$$

$$I_B = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \pi = I$$

$$I_A - I_B = 4I$$

3. By relation,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

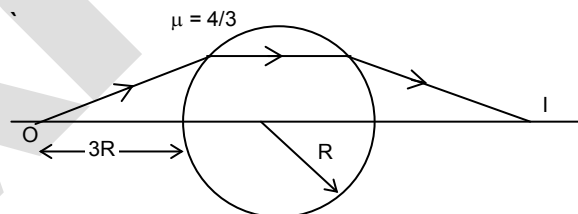
$$\frac{4}{3v} - \frac{1}{-3R} = \frac{4/3 - 1}{R}$$

$$\text{or } \frac{4}{3v} = 0 \Rightarrow v = \infty$$

So image formed is at infinity so refracted ray is parallel to principal axis hence

By symmetry the final image is also formed at  $3R$  from the surface of sphere.

So distance of image from the centre =  $3R + R = 4R$



4. Since the ray coming out of the convex lens is parallel so combined focal length of these lenses is  $\infty$ .

$$\text{or } \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\text{i. e. } \frac{1}{\infty} = -\frac{1}{10} + \frac{1}{f_2} - \frac{10}{-10 \times f_2}$$

$$\Rightarrow f_2 = 20 \text{ cm.}$$

5.  $A = r + r = 60^\circ$

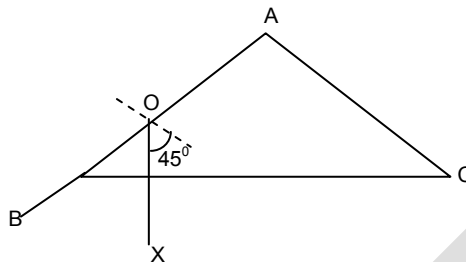
$$r = 30^\circ$$

$$\mu = \sin 90 / \sin 30 = 2$$

6. Using Snell's law at face AB

$$\frac{\sin 90}{\sin r} = \sqrt{2}$$

$$\Rightarrow \sin r = \frac{1}{\sqrt{2}} \Rightarrow r = 45^\circ$$



so ray OX is  $\perp$  to face BC  
 $\Rightarrow$  Angle of incidence is  $= 0$

7. 9 I and I

8. According to Newton's formula  $xy = f^2$

Note that  $m = \frac{f}{f-u} = \frac{f-v}{f}$

or  $\frac{f}{x} = \frac{y}{x}$

$\Rightarrow xy = f^2$

9. Let us take the lens to be stationary and screen is moving with velocity  $V$  away from the lens.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$-\frac{1}{v^2} \cdot \frac{dv}{dt} + \frac{1}{u^2} \frac{du}{dt} = 0$$

$$\frac{du}{dt} = \frac{u^2}{v^2} \frac{dv}{dt}$$

$$\dot{u} = \frac{1}{m^2} \cdot \dot{v}$$

Thus the object is moving with velocity  $(1/m^2)V$  with respect to the lens and towards it (i.e., towards the screen). Velocity of the object with respect to the screen,

$$v_{OS} = v - v/m^2$$

As,  $m = 1/2$ , Hence  $v_{OS} = \left[ 1 - \frac{1}{(1/2)^2} \right] v = -3v$

$= 3\text{ms}^{-1}$  towards the screen.

10.  $\mu_1 \times \sin 90 = \sin \alpha \times \mu_g$  ( $\therefore \mu_1 = 1$ )

$$\delta \alpha = \frac{1}{\mu_g}$$

if  $\sin \alpha \leq \frac{1}{\mu_g}$  then all points can be observed in

area which is having radius CD.

Again  $\mu_g \times \sin \alpha = \mu_w \delta \beta$

$$\sin \beta = \frac{\mu_g}{\mu_w} \sin \alpha = \frac{1.5}{1.4} \sin \alpha$$

if  $\sin \alpha < \frac{1}{\mu_g}$

$$\sin \beta = \frac{1}{\mu_w}$$

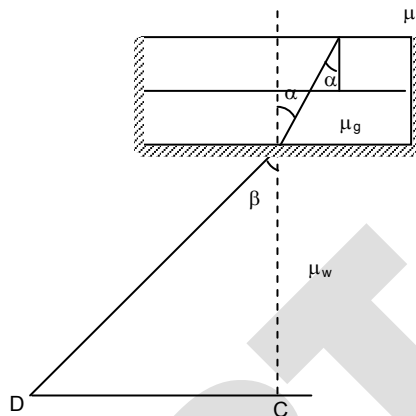
$$\beta = \sin^{-1} (1/1.4)$$

$$\beta = 45.58$$

$$\therefore CD = 5 \tan \beta = 5.10$$

$$\therefore \text{Area} = \pi (CD)^2$$

$$= 81.78 \text{ m}^2$$



11. Refraction of flat surface:  
Apparent depth = (Real depth) / (R.I.)

$$\Rightarrow t_1 = \frac{t}{n} \quad \dots (1)$$

Refraction of curved surface

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

Here  $n_1 = n$ ,  $n_2 = 1$ ,  $u \cong -t$ ,  $R$  is  $-R$

$$\Rightarrow \frac{1}{v} - \frac{n}{-t} = \frac{1-n}{-R}$$

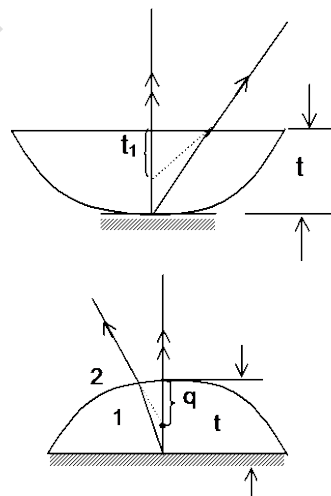
$$\Rightarrow \frac{1}{v} = \frac{n-1}{R} - \frac{n}{t}$$

Putting  $v = t_2$  we obtain,  $-\frac{1}{t_2} = \frac{n-1}{R} - \frac{n}{t}$   $\dots (2)$

$$\frac{1}{f} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right), \text{ Putting } R_1 = R \text{ and } R_2 = \infty \text{ we obtain}$$

$$f = \frac{R}{n-1} \quad \dots (3)$$

Putting  $\frac{n-1}{R} = \frac{1}{f}$  from (3) in (2) we obtain,



$$-\frac{1}{t_2} = \frac{1}{f} - \frac{n}{t} \Rightarrow \frac{1}{f} = \frac{n}{t} - \frac{1}{t_2}$$

$$\Rightarrow f = \frac{tt_2}{nt_2 - t}$$

Putting the value of n from equation (1)

$$\Rightarrow f = \frac{t_1 t_2}{\frac{t}{t_1} t_2 - t} = \frac{tt_1 t_2}{tt_2 - tt_1} = 75 \text{ cm.}$$

12.  $1/f = (n-1) \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$

$$\Rightarrow 1/f = \left( \frac{1.5}{1} - 1 \right) \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \text{ when the lens is placed in air and}$$

$$1/xf = \left( \frac{1.5}{y} - 1 \right) \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \text{ when the lens is placed in the liquid.}$$

Where y = R.I. of the liquid

$$\text{Solving we get, } y = \frac{3}{2 + 1/x}.$$

13. According to Bohr theory

$$\frac{1}{\lambda} = RZ^2 \left[ \frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

$$\text{Thus } \lambda \propto \frac{1}{Z^2}$$

More the atomic number, smaller is the wavelength obtained in the transition of electron (for identical transition).

14.  $\frac{\mu - \mu_0}{\mu_0} = \frac{1}{f(\mu - 1)}$

15. As  $n_1 \lambda_1 = n_2 \lambda_2$   
 $\frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2} = \frac{7.8 \times 10^{-8}}{5.2 \times 10^{-8}} = \frac{3}{2}$

$$\text{also } n_2 - n_1 = 1 \quad \text{i.e. } n_1 = 2$$

16. Let the position of the image formed due to the refraction at the first surface be  $v_1$  and of the final image be  $v$ . Using the coordinate sign convention

$$\frac{\mu_1}{-\infty} + \frac{\mu_2}{v_1} = \frac{\mu_2 - \mu_1}{R} \text{ and } \frac{\mu_2}{-v_1} + \frac{\mu_3}{v} = \frac{\mu_3 - \mu_2}{R}$$

Adding these equations,  $\frac{\mu_3}{v} = \frac{\mu_3 - \mu_1}{R}$ , giving the focal length of the lens

$$f = v = \frac{\mu_3 R}{\mu_3 - \mu_1}$$

17. If a transparent plate of thickness  $t$  and refractive index  $\mu$  is introduced in the path of one of the interfering waves, the entire fringe pattern shifts by

$$y_0 = \frac{D}{d} (\mu - 1) t$$

$$\text{Now } y_0 = 30 \beta = 30 \frac{D\lambda}{d}$$

$$\therefore 30 \frac{D\lambda}{d} = \frac{D}{d} (\mu - 1) t$$

$$\therefore t = 3.6 \times 10^{-3} \text{ cm.}$$

18. Phase difference  $\phi = \frac{2\pi}{\lambda} \times \lambda/3 = \frac{2\pi}{3}$

$$\Rightarrow I_P = I_{\max} \cos^2 (\pi/3)$$

$$\Rightarrow I_P / I_{\max} = 1/4$$

19. (i) Let at the time  $t$  the particle moves parallel to mirror.

$$V' \sin \theta = u \sin \alpha - gt \quad \dots (1)$$

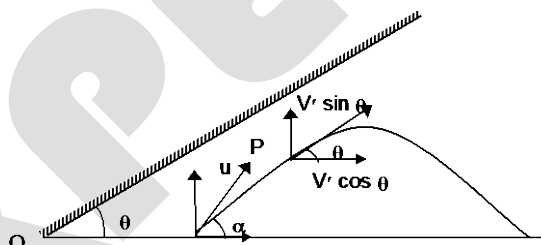
$$\text{And } v' \cos \theta = u \cos \alpha$$

$$v' = \frac{u \cos \alpha}{\cos \theta} \quad \dots (ii)$$

From (i) and (ii)

$$\frac{u \cos \alpha}{\cos \theta} \cdot \sin \theta = u \sin \alpha - gt$$

$$t = \frac{u \cos \alpha (\tan \alpha - \tan \theta)}{g}$$



20. 40/3 cm

21. For the convex lens L,  $\mu = 3/2$ ,  $r_1 = +30$  cm,  $r_2 = -40$  cm.

$$\frac{1}{f_g} = \left( \frac{3}{2} - 1 \right) \left( \frac{1}{30} + \frac{1}{40} \right) \therefore f_g = \frac{240}{7} \text{ cm}$$

For water lens,

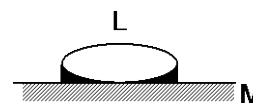
$$\mu = \frac{4}{3}, \quad r_1 = -40 \text{ cm}, \quad r_2 = \infty$$

$$\frac{1}{f_w} = \left( \frac{4}{3} - 1 \right) \left( -\frac{1}{40} \right) \therefore f_w = -120 \text{ cm}$$

$\therefore$  Focal length of combination,  $f$  is given by

$$\frac{1}{f} = 2 \left( \frac{1}{f_w} + \frac{1}{f_g} \right) \Rightarrow f = +24 \text{ cm}$$

+ve sign indicates that it is a converging system.



22. Path difference =  $(\mu - 1)t = n\lambda$ ; For minimum  $t$ ,  $n = 1$ ;  $\therefore t = 2\lambda$

23. For an object placed at infinity the image after first refraction will be formed at  $v_1$

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{-\infty} = \frac{\mu_2 - \mu_1}{+R} \quad \dots (i)$$

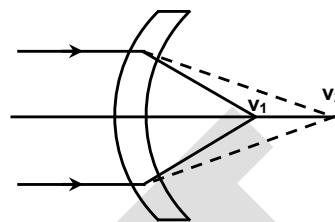
The image after second refraction will be found at  $v_2$

$$\frac{\mu_3}{v_2} - \frac{\mu_2}{v_1} = \frac{\mu_3 - \mu_2}{+R} \quad \dots (ii)$$

adding (i) and (ii)

$$\frac{\mu_3}{v_2} = \frac{\mu_3 - \mu_1}{R} \Rightarrow v_2 = \frac{\mu_3 R}{\mu_3 - \mu_1}$$

Therefore focal length will be  $\frac{\mu_3 R}{\mu_3 - \mu_1}$



24.  $D = 1.5 \text{ m}$   
 $d = 0.025 \text{ mm}$   
 $\lambda_1 = 4300 \text{ \AA}, \lambda_2 = 5100 \text{ \AA}$   
 $n = 3$

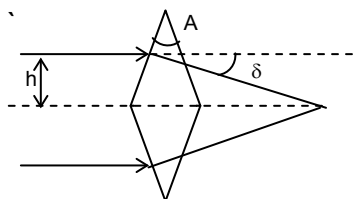
$$x_1 = \frac{n\lambda_1 D}{d} \quad \&$$

$$x_2 = \frac{n\lambda_2 D}{d}$$

$$x_2 - x_1 = \frac{nD}{d}(\lambda_2 - \lambda_1)$$

$$= 1.44 \text{ cm.}$$

25.  $\tan \delta = h / f$   
 $f = \frac{h}{\tan \delta}$  for small  $\delta$ ,  $\tan \delta = \delta$   
 further  $\delta = (\mu - 1) A$   
 $f = \frac{h}{(\mu - 1)A}$



26. (a) The fringe pattern shifts upward

$$(b) \text{FS} = (\mu - 1)t \frac{\omega}{\lambda}$$

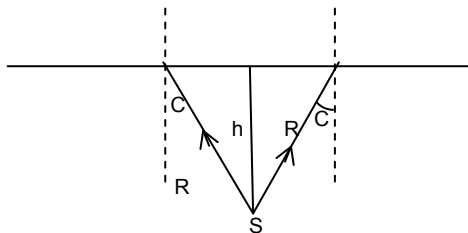
$$\text{FS} = 20 \omega; t = \ell,$$

$$\therefore (\mu - 1)\ell = 20 \lambda \quad \text{or } \mu = 1 + \frac{20\lambda}{\ell}$$

27. Fraction =  $\frac{2\pi R^2(1 - \cos \theta)}{4\pi R^2}$

$$= \frac{1 - \cos \theta}{2} = \frac{1}{2} \left\{ 1 - \sqrt{1 - \frac{1}{17^2}} \right\} = 0.17$$

$\therefore$  Percentage = 17 %



28. The distance of mth bright fringe from the central fringe =  $y_m = m\lambda \frac{D}{d} = m\beta$

where  $\beta$  = fringe width.

$Y_9 = 9\beta$  . . . (i)

Distance of the mth dark fringe from the central fringe =  $y'_m = \left(m - \frac{1}{2}\right)\beta$

$y'_2 = \frac{3}{2}\beta$  . . . . (ii)

From (1) and (2), we get

$y_9 - y'_2 = 9\beta - \frac{3}{2}\beta = \frac{15}{2}\beta$  .  $\therefore \frac{15\beta}{2} = 8.835 \text{ mm}$

or  $\beta = 1.178 \times 10^{-3} \text{ m}$

Now,  $\lambda = \frac{\beta d}{D} = 5.89 \times 10^{-7} \text{ m} = 5890 \text{ \AA}$

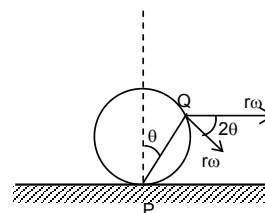
29. Velocity of point Q w.r.t mirror is

$$= (r\omega + r\omega \cos 2\theta)\hat{i} + (r\omega \sin 2\theta)(-\hat{j})$$

Horizontal velocity of image is same but vertical velocity is in opposite direction

so velocity of image w.r.t. Q in vertical direction is =

$2r\omega \sin 2\theta = 4 r\omega \sin \theta \cos \theta$



30. i at hypotenuse =  $45^\circ$   
Hence ray is totally reflected, if  $45^\circ \geq C$   
 $\therefore \sin 45^\circ \geq \sin C \Rightarrow \mu \geq \sqrt{2}$   
 $\therefore \mu_{\min} = 1.414$

31.  $\delta$

32.  $d/\lambda D$

33.  $n \times 7.8 \times 10^{-7} = (n + 1)5.2 \times 10^{-7}$   
 $\Rightarrow n = 2$

34. The speed of light in any medium is given as

$$v = \frac{C}{n} \text{ where } C = 3 \times 10^8 \text{ m/sec.}$$

The ratio of speeds of light in water and glass is given as

$$\frac{v_w}{v_g} = \frac{C/n_w}{C/n_g} = \frac{n_g}{n_w}$$

Since frequency  $f$  of light does not change,  $v_w = \lambda_w f$  and  $v_g = \lambda_g f$  we obtain,

$$\frac{v_w}{v_g} = \frac{\lambda_w}{\lambda_g} = \frac{n_g}{n_w}$$

$$\Rightarrow \frac{\lambda_w}{\lambda_g} = \frac{3/2}{4/3} = \frac{9}{8}$$

$$\Rightarrow \frac{\lambda_w - \lambda_g}{\lambda_g} = \frac{9-8}{8} = \frac{1}{8}$$

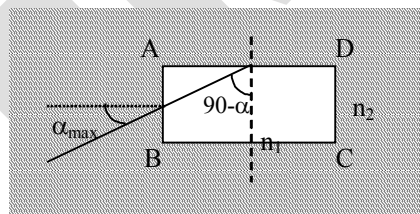
$$\Rightarrow \frac{\Delta\lambda}{\lambda_g} = \frac{1}{8}$$

$$35. \quad \frac{\sin \alpha_{\max}}{\sin r} = \frac{n_1}{n_2}$$

$$\frac{\sin(90 - \alpha)}{\sin 90} = \frac{n_2}{n_1}$$

$$\cos \alpha = \frac{n_2}{n_1}$$

$$\alpha_{\max} = \sin^{-1} \left[ \frac{n_1}{n_2} \sin \left( \cos^{-1} \frac{n_2}{n_1} \right) \right]$$



36. For A:–  
The velocity of approach of the first image (without reflection) =  $v_1$

$$= v + \frac{\mu_1 v}{\mu_2} = v \left( 1 + \frac{\mu_1}{\mu_2} \right)$$

and the velocity of approach of the second image (after reflection) =  $v_2$

$$= v - \frac{\mu_1 v}{\mu_2} = v \left( 1 - \frac{\mu_1}{\mu_2} \right)$$

$$\therefore \text{Velocity of separation of the two images} = v_1 - v_2 = v \left( \frac{2\mu_1}{\mu_2} \right) = 2 \left( \frac{\mu_1}{\mu_2} \right) v$$

$$37. \quad \frac{nD\lambda_1}{d} = \frac{mD\lambda_2}{d}$$

$$\frac{n}{m} = \frac{5200}{6500} = \frac{4}{5}$$

$$y = \frac{4 \times 120 \times 6500 \times 10^{-6}}{2/1000} = 1.56 \text{ mm}$$



38. As incident ray retraces its path the ray is incident normally on the silvered face.

From Snell's law at surface AC

$$\frac{\sin 45}{\sin r} = \sqrt{2}$$

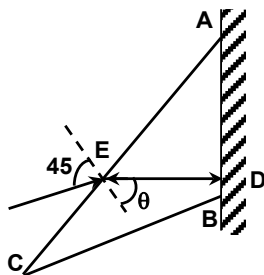
$$\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \sin r$$

$$r = 30^\circ$$

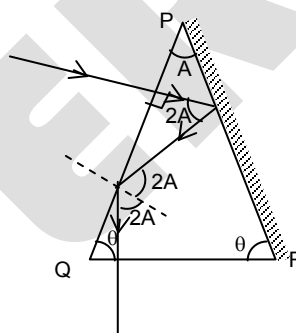
in  $\triangle AED$

$$\angle A + \angle(90 - 30) = 90$$

$$\angle A = 30$$



39.  $A + 2\theta = 180^\circ$  ... (i)  
 Also,  $\theta = 2A$  ... (ii)  
 From (i) and (ii)  
 $A = 36^\circ$ .



40. Path difference at angle  $\theta$  is  
 $\Delta x = d \sin(90 - \theta) = d \cos \theta$   
 corresponding phase difference between 1 and 2 is

$$\phi' = \left( \frac{2\pi}{\lambda} \right) \Delta x$$

$$\phi' = \left( \frac{2\pi}{\lambda} \right) d \cos \theta$$

$\therefore$  net phase difference will be

$$\phi_{\text{net}} = \phi' + \phi = \frac{2\pi}{\lambda} d \cos \theta + \phi$$

for maximum intensity

$$\phi_{\text{net}} = 2n\pi \quad [n = 0, \pm 1, \pm 2, \dots]$$

$$\Rightarrow \theta = \cos^{-1} \left[ \frac{\lambda}{2\pi d} (2n\pi - \phi) \right] \Rightarrow \theta = \cos^{-1} \left[ \frac{\lambda}{d} \left( n - \frac{\phi}{2\pi} \right) \right]$$

41. Fringe width  $\beta = \frac{14.73 - 12.34}{10} = 0.239 \text{ mm}$ .

$$\text{Or } \frac{D\lambda}{d} = 0.239 \times 10^{-3} \text{ m}$$

$$\text{With } \lambda = 6000 \text{ \AA}, \frac{D}{d} = \frac{0.239 \times 10^{-3}}{6000 \times 10^{-10}} = 398.3$$

$$\text{Again with } \lambda' = 5000 \text{ \AA} \quad \beta' = \left( \frac{D}{d} \right) \lambda' = 398.3 \times 5000 \times 10^{-10}$$

$$= 0.199 \text{ mm}$$

The position of zero order maxima remains unchanged at 12.34 mm from the reference point while the 10<sup>th</sup> order maxima occurs at  $12.34 + 10 \beta' = 14.33 \text{ mm}$

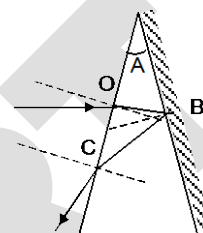
42. Path of the ray is shown in the figure refraction occurs at O and C while reflection takes place at B.

$$\text{Deviation at O} = i - r = (\mu - 1) r = (\mu - 1) \frac{A}{2} \text{ (Clockwise)}$$

$$\text{Deviation of B} = \pi - 2r = \pi - A \text{ (clockwise)}$$

$$\text{Deviation at C} = (\mu - 1) \frac{3A}{2} \text{ (anticlockwise)}$$

$$\text{Net deviation } \delta = \delta_1 + \delta_2 - \delta_3 = \pi - \mu A$$



43. Using Snell's law

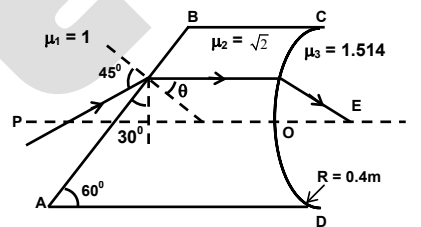
$$\mu_1 \sin 45^\circ = \mu_2 \sin \theta$$

$$\theta = 30^\circ$$

i.e. ray moves parallel to axis

$$\frac{\mu_3}{OE} - \frac{\mu_2}{\infty} = \frac{(\mu_3 - \mu_2)}{R}$$

$$OE = 6.056 \text{ m} \approx 6.06 \text{ m}$$



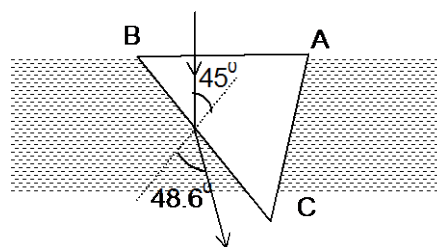
44.  $i$  at hypotenuse  $= 45^\circ$   
Hence ray is totally reflected, if  $45^\circ \geq C$

$$\therefore \sin 45^\circ \geq \sin C \Rightarrow \mu \geq \sqrt{2}$$

$$\therefore \mu_{\min} = 1.414$$

Let  $r$  be the angle of refraction in water. Then

$$\sqrt{2} \sin 45^\circ = \frac{4}{3} \sin r \Rightarrow r = 48.6^\circ$$



45. When the curved surface is in contact  
 $\mu = (\text{real depth})/(\text{apparent depth}) = 4/3$   
When the plane surface is in contact,

$$\text{Using } \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \text{ we get } R = -25 \text{ cm}$$

$$\text{Again using for the plano-convex lens } \frac{1}{f} = (\mu - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \text{ we get } f = 75 \text{ cm}$$

46. If a transparent plate of thickness  $t$  and refractive index  $\mu$  is introduced in the path of one of the interfering waves, the entire fringe pattern shifts by

$$y_0 = \frac{D}{d} (\mu - 1) t$$

$$\text{Now } y_0 = 30 \beta = 30 \frac{D\lambda}{d}$$

$$\therefore 30 \frac{D\lambda}{d} = \frac{D}{d}(\mu - 1)t$$

$$\therefore t = 3.6 \times 10^{-3} \text{ cm.}$$

47. The distance of  $m$ th bright fringe from the central fringe  $= y_m = m\lambda \frac{D}{d} = m\beta$

where  $\beta$  = fringe width.

$$Y_9 = 9\beta \quad \dots (i)$$

$$\text{Distance of the } m\text{th dark fringe from the central fringe} = y'_m = \left(m - \frac{1}{2}\right)\beta$$

$$y'_2 = \frac{3}{2}\beta \quad \dots (ii)$$

From (1) and (2), we get

$$y_9 - y'_2 = 9\beta - \frac{3}{2}\beta = \frac{15}{2}\beta.$$

$$\therefore \frac{15\beta}{2} = 8.835 \text{ mm}$$

$$\text{or } \beta = 1.178 \times 10^{-3} \text{ m}$$

$$\text{Now, } \lambda = \frac{\beta d}{D} = 5.89 \times 10^{-7} \text{ m} = 5890 \text{ \AA}$$

48. For the convex lens L,  $\mu = 3/2$ ,  $r_1 = +30 \text{ cm}$ ,  $r_2 = -40 \text{ cm}$ .

$$\frac{1}{f_g} = \left(\frac{3}{2} - 1\right) \left(\frac{1}{30} + \frac{1}{40}\right)$$

$$\therefore f_g = \frac{240}{7} \text{ cm}$$

For water lens,

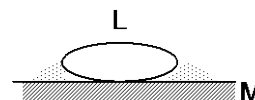
$$\mu = \frac{4}{3}, \quad r_1 = -40 \text{ cm}, \quad r_2 = \infty$$

$$\frac{1}{f_w} = \left(\frac{4}{3} - 1\right) \left(-\frac{1}{40}\right) \quad \therefore f_w = -120 \text{ cm}$$

$\therefore$  Focal length of combination,  $f$  is given by

$$\frac{1}{f} = \frac{1}{f_w} + \frac{1}{f_g} \Rightarrow f = +48 \text{ cm}$$

+ve sign indicates that it is a converging system.



49. Also  $R = 1\text{ m}$   
In triangle PQO

$$\frac{PQ}{PO} = \sin 2r$$

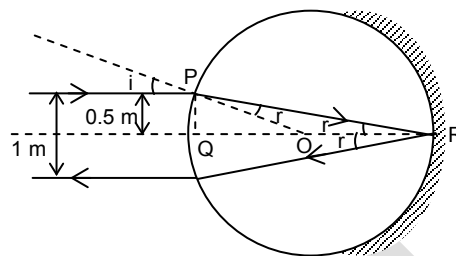
$$\frac{0.5}{1} = \sin 2r$$

$$\Rightarrow 2r = 30^\circ \text{ or } r = 15^\circ$$

It is clear from figure that

$$i = 2r = 30^\circ$$

$$\text{so } \mu = \frac{\sin i}{\sin r} = \frac{\sin 30}{\sin 15} = \frac{1}{2 \sin 15^\circ}$$



50. For refraction at P

$$n \sin r_2 = n_1$$

$$\sin r_2 = \frac{n_1}{n} \quad \dots (i)$$

also  $r_1 + r_2 = 45^\circ = \text{angle of prism}$   
at point Q

$$i \sin i = n \sin r_1 \quad \dots (ii)$$

$$i \sin i = n \sin r_1 \quad \dots (iii)$$

from (i), (ii) and (iii)

$$\sin i = n \sin (45^\circ - r_2)$$

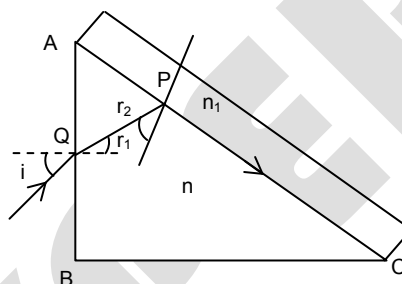
$$= n [\sin 45^\circ \cos r_2 - \sin r_2 \cos 45^\circ]$$

$$= \frac{n}{\sqrt{2}} [\sqrt{1 - \sin^2 r_2} - \sin r_2]$$

$$= \frac{n}{\sqrt{2}} \left[ \sqrt{1 - \frac{n_1^2}{n^2}} - \frac{n_1}{n} \right]$$

$$\Rightarrow i = \sin^{-1} \left\{ \frac{n}{\sqrt{2}} \left( \sqrt{1 - \frac{n_1^2}{n^2}} - \frac{n_1}{n} \right) \right\}$$

$$= \sin^{-1} \left[ \frac{1}{\sqrt{2}} (\sqrt{n^2 - n_1^2} - n_1) \right]$$



51. Equivalent focal length is given by

$$\frac{1}{f} = \frac{2}{f_l} + \frac{1}{f_m}$$

$$f_l = +20\text{ cm}; \quad f_m = \frac{25}{2}$$

$$\therefore \frac{1}{f} = \frac{2}{20} + \frac{2}{25} = \frac{45}{250} = \frac{9}{50}$$

the lens act as a concave mirror,

$$u = -10\text{ cm}; \quad f = \frac{-50}{9}\text{ cm}$$

Mirror formula,  $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$$\frac{1}{v} = \frac{-9}{50} + \frac{1}{10} = \frac{-9+5}{50} = \frac{-4}{50}$$

$$v = -12.5\text{cm}$$

52. For an object placed at infinite image will be formed at focus.

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{20}$$

$$\Rightarrow \frac{1.5}{v} - \frac{1}{\infty} = \frac{1.5 - 1}{20}$$

$$\Rightarrow v = 60\text{ cm}$$

Now this will act as an object for second surface, we have

$$\frac{\mu_3}{v} - \frac{\mu_2}{60} = \frac{\mu_3 - \mu_2}{-20}$$

$$\Rightarrow \frac{1.3}{v} = \frac{3}{2 \times 60} + \frac{-0.2}{-20} = \frac{1}{40} + \frac{2}{200} = \frac{5+2}{200}$$

$$\Rightarrow \frac{1.3}{v} = \frac{7}{200} \Rightarrow 7v = 260$$

$$v = 37.14\text{ cm}$$

53. (a) As there is symmetry about the line SP, fringes will be circular.

$$(b) \frac{I_{\min}}{I_{\max}} = \left( \frac{\sqrt{I} - \sqrt{0.36I}}{\sqrt{I} + \sqrt{0.36I}} \right)^2 = \left( \frac{0.4}{1.6} \right)^2 = \frac{1}{16}$$

- (c) For maximum at P, path difference =  $n\lambda$ .

If AB is shifted by a distance x, it will cause an additional path difference of  $2x$ .

$$2x = \lambda \quad (\text{for minimum value of } x)$$

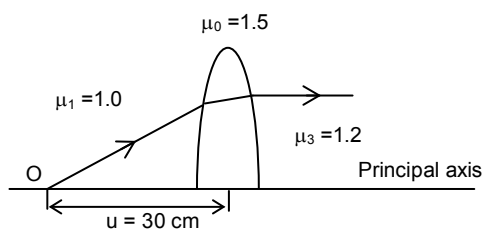
$$\Rightarrow x = \lambda/2 = 300\text{ nm}$$

54. The equivalent focal length of the upper half of the lens is

$$\frac{1}{f_1} = \frac{1}{f_0} (2\mu_0 - \mu_1 - \mu_3)$$

$$\frac{1}{f_1} = \frac{1}{24} [2(1.5) - 1 - 1.2] = \frac{1}{30}$$

$$\text{or } f_1 = 30\text{ cm}$$



Using lens formula

$$\frac{\mu_3}{v_1} - \frac{\mu_1}{u} = \frac{1}{f_1}$$

$$\text{or } v_1 = \frac{\mu_3 u f_1}{u + \mu_1 f_1}$$

here  $u = -30$  cm;  $f_1 = +30$  cm

$$\therefore v_1 = \frac{1.2(-30)(+30)}{-30 + 30} = \infty$$

The equivalent focal length for the lower half is

$$\frac{1}{f_2} = \frac{1}{f_0} (2\mu_0 - \mu_2 - \mu_4)$$

$$\frac{1}{f_2} = \frac{1}{24} (2(1.5) - 1.1 - 1.6)$$

$$\text{or } f_2 = 80 \text{ cm}$$

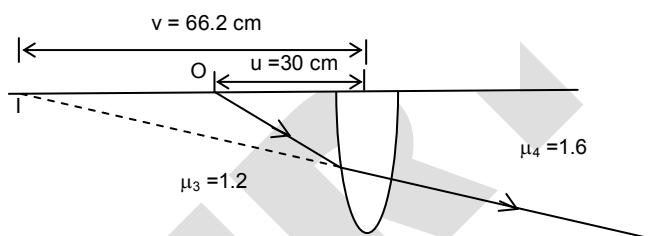
Using lens formula

$$\frac{\mu_4}{v_2} - \frac{\mu_2}{u} = \frac{1}{f_2}$$

$$\text{or } v_2 = \frac{\mu_4 u f_2}{u + \mu_2 f_2}$$

Here  $u = -30$  cm,  $f_2 = 80$  cm

$$v_2 = \frac{1.6(-30)(80)}{-30 + (1.1)(80)} = -66.2 \text{ cm}$$



55. The object distance corresponding to this ray can be found from its intersection with the x-axis.

$$x = 1$$

$$\therefore u = -(40 - 1) = -39$$

$$\frac{1}{v} - \frac{1}{-39} = \frac{1}{30}; v = 130 \text{ cm}$$

The equation of the refracted ray is

$$y = \frac{1}{130} (x - 170)$$

If the space  $x > 40$  cm, on the side of the lens is filled with a liquid of refractive index,  $\mu = 4/3$ , then we get,

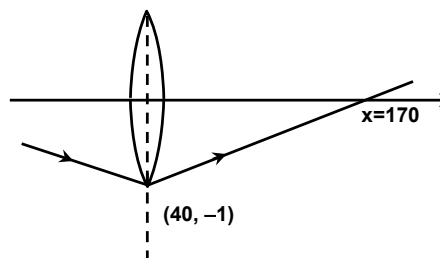
$$\frac{1.5}{v_1} - \frac{1.0}{-39} = \frac{0.5}{30}$$

$$\frac{4}{3} - \frac{1.5}{v_1} = \frac{4}{3} - 1.5$$

$$\text{or, } v = -390 \text{ cm}$$

The equation of the refracted ray is

$$y = -\frac{1}{390} (x + 350)$$



56. Optical path difference  $\Delta = 2 \int_0^{L/2} \mu dx - L = 2\mu_0 \int_0^{L/2} \left(1 + \frac{2x}{L}\right) dx - L$

$$= \frac{3}{2} \mu_0 L - L = (1.5\mu_0 - 1)L$$

Thus  $\Delta = N\lambda$

or  $N = \frac{\Delta}{\lambda} = (1.5\mu_0 - 1) \frac{L}{\lambda}$

57. Image formed by upper lens

$$\frac{1}{v_1} - \frac{1}{(-90)} = \frac{1}{30} \Rightarrow v_1 = 45\text{cm}$$

This will act as an object for mirror

$$\therefore \frac{1}{v_2} + \frac{1}{15} = \frac{-1}{30}$$

$$v_2 = -10\text{cm}$$

coordinates of image, (110, 0)

Image formed directly by mirror

$$\frac{1}{v} - \frac{1}{120} = \frac{-1}{30} ; \frac{1}{v} = \frac{-1}{30} + \frac{1}{120} = \frac{-3}{120} = -\frac{1}{40}$$

$$v = -40\text{cm}$$

$\therefore$  coordinates (80, 0)

Image formed by lower lens.

$$\frac{1}{v_c} - \left(\frac{1}{-90}\right) = \frac{1}{30} \Rightarrow v_c = 45\text{cm}.$$

This acts as an objects for mirror

$$\therefore \frac{1}{v_m} + \frac{1}{15} = \frac{-1}{30} \Rightarrow v_m = -10\text{cm}.$$

$$\text{Magnification due to lens} = -\frac{45}{90} = -\frac{1}{2};$$

coordinates of image formed by lower lens = (135, -0.75)

$$\text{Magnification due to mirror} = \frac{+10}{15} = \frac{+2}{3}$$

$\therefore$  Coordinates of final image = (110, -0.5)

58. Phase difference  $\phi = \frac{2\pi}{\lambda} \times \lambda/3 = \frac{2\pi}{3}$

$$\Rightarrow I_P = I_{\max} \cos^2(\pi/3)$$

$$\Rightarrow I_P / I_{\max} = 1/4$$

59. (a)  $\frac{(\mu_s - 1)tD}{d} = 2 \times 10^{-3}$

$$\Rightarrow \mu_s - 1 = \frac{2 \times 10^{-3} \times 10^{-3}}{4 \times 10^{-6} \times 1}$$

$$\Rightarrow \mu_s = 1.5 \quad \text{and} \quad \left( \frac{\mu_s}{\mu_m} - 1 \right) \frac{tD}{d} = 0.5 \times 10^{-3}$$

$$\Rightarrow \left( \frac{\mu_s}{\mu_m} - 1 \right) = \frac{1}{8}$$

$$\mu_m = \frac{\mu_s}{\left( \frac{1}{8} + 1 \right)} = \frac{3}{2} \times \frac{8}{9}$$

$$\Rightarrow \mu_m = \frac{4}{3}$$

$$(b) \text{ Fringe width } \beta = \frac{\lambda D}{\mu_m d}$$

$$\Rightarrow \beta = \frac{1.5}{4} \times 10^{-3} = 0.375 \text{ mm}$$

$\therefore$  The first maximum is the nearest maximum from the point O and its distance is 0.125 mm,

60. Initial path difference =  $D_2 - D_1 = \frac{xd}{D}$

After introduction of slits the path difference becomes

$$= D_2 + (\mu_2 - 1)2t - D_1 - (\mu_1 - 1)t$$

$$= \frac{xd}{D} + 2(\mu_2 - 1)t - (\mu_1 - 1)t$$

For zero order fringe occurs at a distance given by

$$\frac{xd}{D} + 2(\mu_2 - 1)t - (\mu_1 - 1)t = 0$$

$$\text{i.e. } x = [(\mu_1 - 1)t - 2(\mu_2 - 1)t] D/d \quad \dots (i)$$

$$\text{but } x = 5\beta = \frac{5\lambda D}{d} \quad \dots (ii)$$

so from (i) and (ii)

$$\text{we get } t_1 = 2.4 \times 10^{-6} \text{ m}$$

$$t_2 = 2t_1 = 4.8 \times 10^{-6} \text{ m}$$

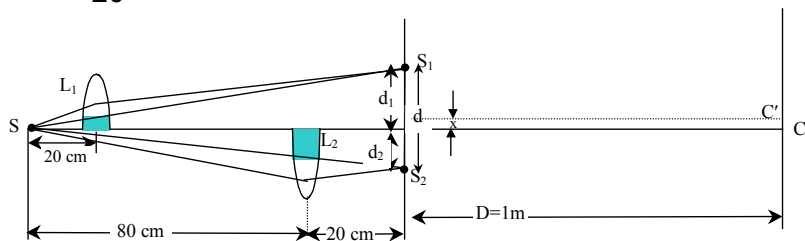
61.  $L_1, \frac{1}{v} = \frac{1}{16} - \frac{1}{20}, v_1 = 80 \text{ cm}$

$$L_2 = \frac{1}{v} = \frac{1}{16} - \frac{1}{80}, v_2 = 20 \text{ cm}$$

$$\text{Magnification } m = \frac{v}{u}$$



$$m_1 = \frac{80}{-20} = -4$$



$$\text{Hence } d_1 = 5\Delta = 5 \times 0.40 = 2 \text{ mm}$$

$$\text{Similarly } d_2 = \frac{3\Delta}{4} + 3\Delta = \frac{15}{4}\Delta = 1.5 \text{ mm.}$$

$$d = d_1 + d_2 = 3.5 \text{ mm.}$$

$$D = 2 - 1 \text{ metre.} \quad \Rightarrow x = d_1 - \frac{d}{2} = 0.25 \text{ mm}$$

$$\text{As shifting } x = \frac{(\mu - 1)tD}{d}; \quad 0.25 \times 10^{-3} = \frac{(\mu - 1) \times 2 \times 10^{-6} \times 1}{3.5 \times 10^{-3}}$$

$$\text{which gives } \mu = 1.437.$$

62. Using new-Cartesian sign convention separately

$$\text{for lens and mirror, we get } \frac{1}{v_\ell} - \frac{1}{u_\ell} = \frac{1}{f_\ell}$$

$$\text{putting } u_\ell = -20 \text{ cm, } f_\ell = 15 \text{ cm; } v_\ell = 60 \text{ cm}$$

$\Rightarrow$  Image after refraction forms 30 cm behind the mirror hence,  $u_m = 30 \text{ cm}$

$$\text{Now, } \frac{1}{v_m} + \frac{1}{u_m} = \frac{1}{f_m} \quad \text{Where } f_m = -30 \text{ cm}$$

$$\text{Solving we get, } v_m = -15 \text{ cm.}$$

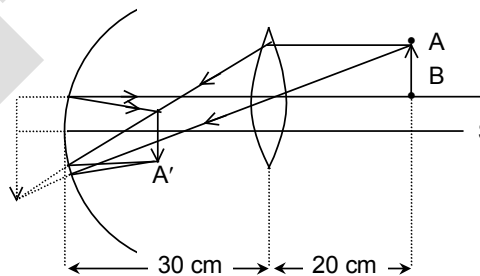
Using the values obtained we get,

Magnifications by lens and mirror are thus  $-3$  &  $\frac{1}{2}$  respectively.

$$\text{Overall magnification} = -3 \times \frac{1}{2} = -1.5 \quad \Rightarrow \quad A'B' = 1.8 \text{ cm.}$$

$B'$  would be at height  $0.6 (1/2) \text{ cm} = 0.3 \text{ cm}$  above RS.

$A'$  would be  $\{ (1.2 \times 3) - 0.6 \} (1/2) = 1.5 \text{ cm}$  below RS.



$$63. \quad 4\mu t = (2n - 1)\lambda$$

$$t_{\min} = \lambda/4\mu = 90 \text{ nm.}$$

64.  $\therefore \mu \cdot \sin \theta_x = \text{constant}$ ,  $\theta_x = \text{angle made with X-axis at any point.}$

$$\therefore \mu \sin \theta_x = \mu_0 \sin (\pi/2)$$

$$\therefore \sin \theta_x = \frac{\mu_0}{\mu} = \frac{\mu_0}{\mu_0 \{1 - x/r\}^{-1}}$$

$$\therefore \frac{dy}{dx} = \tan \theta_x = \frac{1 - x/r}{\sqrt{1 - (1 - x/r)^2}}$$

$$\therefore \int dy = \int \frac{(1 - x/r)}{\sqrt{1 - (1 - x/r)^2}} dx$$

$$\text{Putting } z = 1 - (1 - x/r)^2 \quad \therefore \quad x^2 + y^2 - 2xr = 0$$

$$\text{Putting } y = d \text{ and } x = x_A, \quad x_A = r\{1 \pm \sqrt{1 - (d/r)^2}\}$$

$$\text{Putting the condition, as } r \rightarrow \infty, \quad x_A \rightarrow 0, \quad x_A = r\{1 - \sqrt{1 - (d/r)^2}\}$$

65. (a) Central maximas of both the waves will be at the centre of the screen

$$(b) \quad n \frac{\lambda_1 D}{2d} = \frac{m \lambda_2 D}{2d}$$

$$\Rightarrow \quad \frac{n}{m} = \frac{\lambda_2}{\lambda_1} = \frac{26}{14}$$

minimum values of  $n$  and  $m$  will be 13 and 7

$$\text{so location will be } 13 \times \frac{\lambda_1 D}{2d} = \frac{13 \times 14000 \times 10^{-10} \times 1}{2 \times 10^{-2}}$$

$$= 9.1 \times 10^{-4} \text{ m. (from centre on either side)}$$

$$(c) \quad \frac{(2n+1)\lambda_1 D}{4d} = \frac{(2m+1)\lambda_2 D}{4d}$$

$$\frac{2n+1}{2m+1} = \frac{\lambda_2}{\lambda_1} = \frac{26}{14} = \frac{13}{7}$$

minimum values of  $n$  and  $m$  will be 6 and 3.

$$\text{So location will be } \frac{13 \times 14000 \times 10^{-10} \times 1}{2 \times 2 \times 10^{-2}} = 4.55 \times 10^{-4} \text{ m}$$

$$66. \quad d_{\max} = \frac{\lambda}{Q_{\min}} = \frac{6 \times 10^{-7} \times 60 \times 180}{\pi} = 2.06 \text{ mm}$$

$$y_3 = \frac{D}{d}(3n) = \frac{1 \times 3 \times 6 \times 10^{-7}}{2.06 \times 10^{-3}} \text{ m}$$

$$= 0.874 \text{ mm}$$

$$y_5 = \frac{D}{d}(2n-1)\frac{\lambda}{2} = \frac{1 \times (2 \times 5 - 1) \times 6 \times 10^{-7}}{(2.06 \times 10^{-3}) \times 2} \text{ m} = 1.31 \text{ mm}$$

67. If  $I_m$  be the intensity on the screen, intensity at a point where phase difference between the incident disturbances equals  $\phi$  is given by

$$I = 2I_0 + 2I_0 \cos(\phi)$$

$$\text{as } I = \frac{3}{4} I_m \quad \text{and} \quad I_m = 4I_0$$

$$\text{We have } 3I_0 = 2I_0 + 2I_0 \cos(\phi) \Rightarrow \cos(\phi) = 1/2$$

If optical path difference corresponding to phase difference  $\Delta\phi$  be  $p$  then

$$\cos\left[\frac{p(2\pi)}{\lambda}\right] = \frac{1}{2} \Rightarrow \left[\frac{p(2\pi)}{\lambda}\right] = 2n\pi + \frac{\pi}{3}$$

$$\Rightarrow \quad p = \lambda[n \pm (1/6)]$$

As  $4\lambda < p < 4.5\lambda$ ,  $p$  can be

$$\therefore p = \lambda \left[ 4 + \frac{1}{6} \right] \quad (1)$$

If  $t$  be the thickness of each glass and  $\mu_1, \mu_2$  be the refractive indices,

$$p = (\mu_2 - 1)t - (\mu_1 - 1)t$$

$$\Rightarrow p = 0.3t \quad [\mu_2 = 1.7, \mu_1 = 1.4]$$

$$\Rightarrow t = \frac{p}{0.3} = \frac{25}{6} \left( \frac{\lambda}{0.3} \right) = 7.5 \mu\text{m}$$

68. (a) The first image is formed at the focus of the first lens. This is at 20 cm from the first lens and hence at  $u = -40$  cm from the second. Using the lens formula for the second lens,

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f} = -\frac{1}{40} + \frac{1}{20} \quad \text{or } v = 40 \text{ cm}$$

The final image is formed 40 cm to the right of the second lens.

- (b) The equivalent focal length is

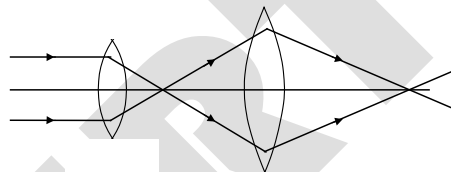
$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} = \frac{1}{20} + \frac{1}{20} - \frac{60}{(20)^2} \quad \text{or } F = -20 \text{ cm}$$

It is a divergent lens. It should be kept at a distance

$$D = \frac{dF}{f_1} \quad \text{behind the second lens.}$$

$$\text{here } D = \frac{60 \times (-20)}{20} = -60 \text{ cm}$$

Thus the equivalent divergent lens should be placed at a distance of 60 cm to the right of the second lens. The final image is formed at the focus of this divergent lens i.e. 20 cm to the left of it. It is, therefore, 40 cm to the right of the second lens.



69. Consideration refraction at glass – water interface

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\Rightarrow \frac{4}{3v} - \frac{3}{-2r} = \frac{(4/3) - (3/2)}{r}$$

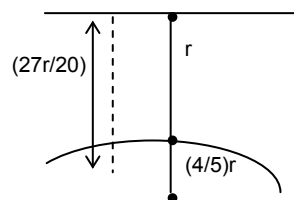
$$\Rightarrow \frac{4}{3v} + \frac{3}{2r} = -\frac{1}{6r}$$

$$\therefore v = -\frac{4}{6}$$

Now refraction at water air surface

$$u = -\left(r + \frac{4}{5}r\right) = -\frac{9r}{5}$$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{\infty} \Rightarrow \frac{1}{v} + \frac{4 \times 5}{3 \times 9r} = \frac{1 - (4/3)}{\infty}$$



$$\frac{1}{v} = -\frac{20}{27r}$$

$$v = -\frac{27r}{20}$$

$$\text{so height above centre} = 2r - \frac{27r}{20}$$

$$= \frac{40r - 27r}{20} = \frac{13}{20}r.$$

70. According to Moseley's equation for  $k_\alpha$  radiation ;

$$\frac{1}{\lambda} = R(z-1)^2 \left( \frac{1}{1^2} - \frac{1}{2^2} \right)$$

$$\frac{\lambda_1}{\lambda} = \frac{(z-1)^2}{(z_1-1)^2} = \frac{0.7092}{1.5405}$$

$$z = 29 \text{ for cu, hence } z_1 - 1 = 28 \sqrt{\frac{1.5405}{1.6578}} = 27$$

$$\text{or } z_1 = 4z$$

$\therefore$  Impurity is molybdenum

$$\text{similarly ; } \frac{\lambda_2}{\lambda_1} = \frac{(z-1)^2}{(z_2-1)^2} = \frac{1.65768}{1.5405}$$

$$\text{or } z_2 - 1 = 28 \sqrt{\frac{1.5405}{1.6578}} = 27$$

$$z_2 = 28$$

It is atomic number of Nickel.

Hence the other impurity is Nickel.

71. Path difference  $= D_2 - D_1 = \frac{xd}{D}$

$$\text{New path difference} = D_2 + (\mu_2 - 1)2t - D_1 - (\mu_1 - 1)t$$

$$= \frac{xd}{D} + 2(\mu_2 - 1)t - (\mu_1 - 1)t$$

For, zero order fringe occurs at a distance given by

$$\frac{xd}{D} + 2(\mu_2 - 1)t - (\mu_1 - 1)t = 0$$

$$\therefore x = [(\mu_1 - 1)t - 2(\mu_2 - 1)t] \frac{D}{d}$$

$$= [(\mu_1 - 1)t - 2(\mu_2 - 1)t] \frac{\beta}{\lambda} \quad \left[ \because \beta = \frac{\lambda D}{d} \right]$$

$$\text{But } x = 5\beta$$

$$\therefore t = -2.4 \times 10^{-6} \text{ m}$$

-ve sign indicates that shift would take place towards 2<sup>nd</sup> slit and hence x should have been

-ve because t cannot be negative.

$$\therefore t = 2.4 \mu\text{m} \text{ and } 2t = 4.8 \mu\text{m}.$$

72. Let us take the lens to be stationary and screen is moving with velocity  $V$  away from the lens.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$-\frac{1}{v^2} \cdot \frac{dv}{dt} + \frac{1}{u^2} \frac{du}{dt} = 0$$

$$\frac{du}{dt} = \frac{u^2}{v^2} \frac{dv}{dt}$$

$$\dot{u} = \frac{1}{m^2} \cdot \dot{v}$$

Thus the object is moving with velocity  $(1/m^2)V$  with respect to the lens and towards it (i.e., towards the screen). Velocity of the object with respect to the screen,

$$v_{OS} = v - v/m^2$$

As,  $m = 1/2$ , Hence  $v_{OS} = \left[1 - \frac{1}{(1/2)^2}\right]v = -3v$

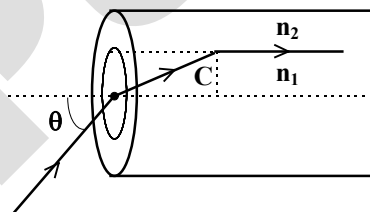
$= 3\text{ms}^{-1}$  towards the screen.

73. be the maximum angle of incidence at the interface of  $n_1$  &  $n_2$  should be minimum. i.e. critical angle  $C$

$$\sin C = \frac{n_2}{n_1}$$

At the interface of air &  $n_1$

$$N_1 = \frac{\sin(90 - C)}{\sin \theta} \Rightarrow \sin \theta = \sqrt{n_1^2 - n_2^2}$$



74. (a)  $\frac{dy}{dx} = \tan \theta = \frac{\sin \theta}{\sqrt{1 - \sin^2 \theta}}$
- According to the Snell's law
- $$\mu \sin \theta = \mu_0 \sin \theta_0 = \text{constant}$$
- or  $\sin \theta = \frac{(1) \sin 30^\circ}{\mu} = \frac{1}{2\mu}$

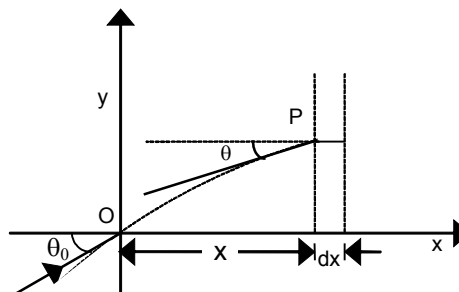
$$\frac{dy}{dx} = \frac{1}{\sqrt{4\mu^2 - 1}} \quad \text{since } \mu = \mu_0 \sqrt{1 + 0.25x}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{4(1 + 0.25x) - 1}} = \frac{1}{\sqrt{3 + x}}$$

or  $dy = \frac{dx}{\sqrt{3 + x}}$

Integrating,  $y = 2\sqrt{3 + x} + C$  At  $x = 0$ ,  $y = 0$

$$\therefore C = -2\sqrt{3}$$



$$\text{Thus, } y = 2[\sqrt{3+x} - \sqrt{3}]$$

(b) The ray comes out of the medium at

$$y = 2[\sqrt{3+1} - \sqrt{3}] = 0.54\text{m}$$

75. In order that all the rays entering at A emerge from B, the angle of incident  $i$  for a ray, should be equal to or greater than critical angle  $\theta_c$  for air glass interface. So,

$$i \geq \theta_c = \sin^{-1}\left(\frac{1}{\mu}\right) \Rightarrow \sin i \geq \left(\frac{1}{\mu}\right)$$

$$\sin i = \frac{PQ}{OQ} = \frac{R}{R+d} \Rightarrow \frac{R}{R+d} \geq \frac{1}{1.5} \quad \text{or} \quad 1.5 \geq R+d$$

$$0.5 \geq d \quad \text{or} \quad d/R \leq 0.5$$

$$\left(\frac{d}{R}\right)_{\max} = 0.5$$

76.  $\frac{1}{v} + \frac{1}{u} = \frac{2}{R} = \frac{1}{f}$   
 $v = -0.2 \quad u = -\infty$

$$-\frac{1}{20} + \frac{1}{\infty} = \frac{2}{R}$$

$$R = -40\text{cm}$$

Focal length of lens formed by water

$$\frac{1}{f_\ell} = \left(\frac{3}{2} - 1\right) \left(\frac{1}{40} + \frac{1}{40}\right)$$

$$\frac{1}{f_0} = \frac{1}{40}$$

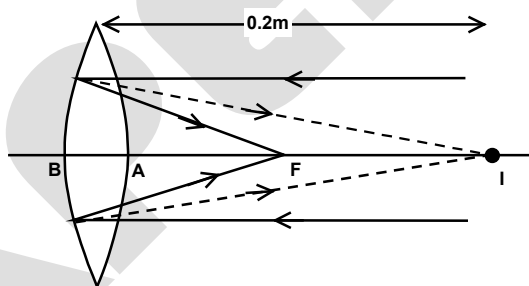
Power of effective mirror

$$P'_M = 2P_\ell + P_M$$

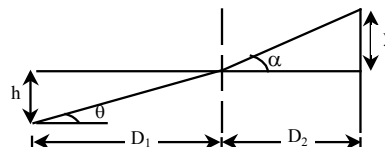
$$P'_M = \left(\frac{2}{40} + \frac{1}{20}\right) \times 100$$

$$P'_M = \frac{100}{10} \Rightarrow f'_M = -10\text{cm}$$

Hence image will be formed at  $-10\text{cm}$ .



77. (a) For central maxima, optical path difference between the two rays must be zero. Also, since  $(D_1 \text{ and } D_2) \gg d$   
 $\therefore d \sin \theta = d \sin \alpha$   
 $\Rightarrow \tan \theta = \tan \alpha \Rightarrow h/D_1 = y/D_2 \Rightarrow y = 2\text{ cm}$



- (b)  $(\mu - 1)D_2 = d \sin \theta$

solving we get,  $\mu = 1.0016$ .

78. (a)  $\beta(6000 \text{ \AA}) = \frac{D\lambda}{d} = \frac{1 \times 6 \times 10^{-7}}{2 \times 10^{-4}} = 3 \times 10^{-3} \text{ m.}$

$$\beta(8000 \text{ \AA}) = \frac{1 \times 8 \times 10^{-7}}{2 \times 10^{-4}} = 4 \times 10^{-3} \text{ m.}$$

(b) Path difference  $= \Delta = S_2A - S_1A = \frac{dx}{D} = \frac{2 \times 10^{-4} \times 10^{-3}}{1} \text{ m}$

phase difference,  $\phi_1(6000 \text{ \AA}) = \frac{2\pi}{\lambda} \Delta = \frac{2\pi \times 2 \times 10^{-7}}{6 \times 10^{-7}} = \frac{2\pi}{3}$

phase difference,  $\phi_2(8000 \text{ \AA}) = \frac{2\pi}{\lambda'} \Delta = \frac{\pi}{2}$

If the individual intensities at P, then

(c)  $I_1 + I_2 = 9I_0$

$$I_1 \cos^2(\phi_1/2) + I_2 \cos^2(\phi_2/2) = 4I_0$$

$$\Rightarrow I_2 = 7I_0, I_1 = 2I_0$$

$$I_1 : I_2 = 2 : 7$$

79. (a)  $\frac{0.5\text{m}}{\mu_1} = \frac{h_2}{\mu_1}$ ;  $h_2$  = distance of image from the surface of water.

$$h_2 = \frac{\mu_2}{\mu_1} \cdot 0.5\text{m} = \frac{2}{3} \text{ m}$$

Magnification = 1,

Lateral component of velocity (along x)

= velocity of image along x-axis

velocity of image along y =  $\frac{4}{3} \times$  velocity along y of object.

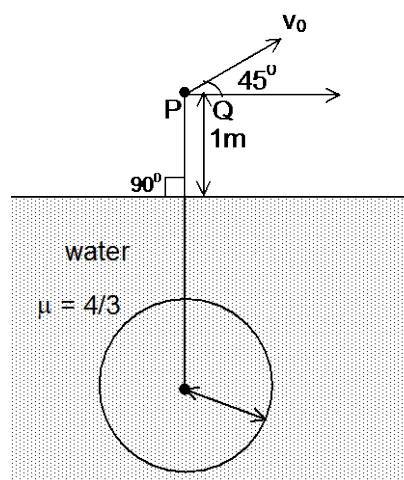
$$= \frac{4}{3} \frac{v_0}{\sqrt{2}}$$

$$\therefore \vec{v}_{\text{image}} = \left( \frac{v_0}{\sqrt{2}} \hat{i} + \frac{4}{3} \frac{v_0}{\sqrt{2}} \hat{j} \right) = \frac{v_0}{\sqrt{2}} \left( \hat{i} + \frac{4}{3} \hat{j} \right)$$

$$= 2\hat{i} + (8/3)\hat{j}.$$

(b) After reflection on the glass surface, we can apply

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{2}{R}, \quad R = 0.1 \text{ m}$$



$$u = -\left(\frac{4}{3} + 0.5 + 0.5\right) = -\frac{14}{3}$$

$$\frac{1}{v} + \frac{1}{(-14/3)} = \frac{2}{0.1} = 20$$

$$\frac{1}{v} + \frac{3}{14} = 20$$

$$v = 14/277.$$

$$\text{size of image} = \left| \left( \frac{14/277}{-14/3} \right) \right| \times 3 \text{ cm} = 0.32 \text{ mm}.$$

(c) Position and size of image formed by single refraction

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}; \quad u = -14/3 \text{ m}, v = 14/277 \mu_1 = 4/3, \mu_2 = 3/2$$

$$\Rightarrow v = 1.31 \text{ m}$$

$$\text{The size can be calculated using the magnification: } \frac{\mu_1 v}{\mu_2 u} = \frac{(4/3)(1.31)}{(3/2)(-14/3)} = -0.249$$

$$\text{size} = 0.249 \times 3 \text{ cm}$$

80. Radius of curvature of the curved surfaces of the lens = R, say.

$$\frac{1}{f} = (\mu_g - 1) \left( \frac{1}{R} - \frac{1}{-R} \right)$$

$$\Rightarrow \frac{1}{30} = \left( \frac{3}{2} - 1 \right) \left( \frac{2}{R} \right) \Rightarrow R = 30 \text{ cm}$$

Refraction at first lens,

$$\text{1st refraction: } \frac{\mu_g}{v_1} - \frac{1}{-90} = \frac{\mu_g - 1}{30} \Rightarrow v_1 = 270 \text{ cm}$$

$$\text{2nd refraction} = \frac{\mu_g}{v_2} - \frac{3/2}{270} = \frac{3}{30} - \frac{3}{270}$$

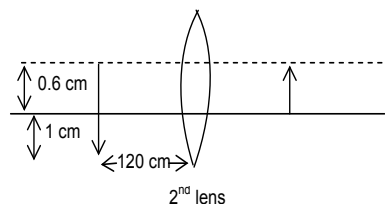
$$\Rightarrow v_2 = 120 \text{ cm}$$

$$\text{and magnification} = \frac{\mu_g(270)}{\mu_a(-90)} \times \frac{\mu_o(120)}{(\mu_g)(270)} = -\frac{16}{9}$$

The image due to first lens is magnified, inverted and midway between the two lens.

$$\text{Image size} = \frac{16}{9} \times 0.9 = 1.6 \text{ cm}$$

Clearly, the second lens will account for the final image at 90 cm from it & outside the tank.





And the portion of the image above the axis of the second lens

$$= (1) \left( \frac{16}{9} \right) = \frac{9}{16} \text{ cm} = 0.5625 \text{ cm}$$

and that below it

$$= (0.6) \left( \frac{9}{16} \right) = 0.3375 \text{ cm}$$

81.  $L_1$  and  $L_2$  form two coherent light sources, say  $S_1$  and  $S_2$ .  
Let their distance be  $v$  from the lens.

$$\text{Then, } \frac{1}{v} - \frac{1}{(-15)} = \frac{1}{10} \Rightarrow v = 30 \text{ cm}$$

$$\text{Now, we can write } S_1 S_2 = d + d \left( \frac{v}{u} \right)$$

$$\Rightarrow \frac{S_1 S_2}{(0.05)} = \frac{15 + 30}{15}$$

$$\Rightarrow S_1 S_2 = 0.15 \text{ cm}$$

$$D = 180 - v = (180 - 30) \text{ cm} = 150 \text{ cm}$$

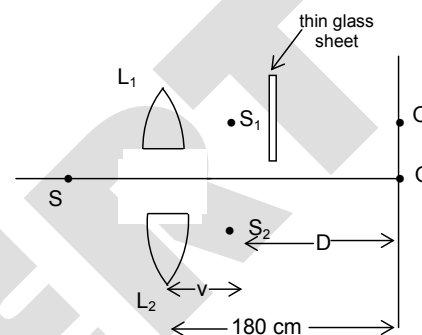
$$\text{fringe width} = \frac{\lambda D}{(S_1 S_2)} = \frac{(600 \times 10^{-7} \text{ cm})(150)}{0.15} = 0.6 \text{ mm}$$

The shift in the central bright fringe

$$= OO' = \frac{D}{(S_1 S_2)} (\mu - 1)t, \text{ where } t = \text{thickness of glass sheet}$$

$$\Rightarrow \frac{5}{10} \text{ cm} = \frac{(150 \text{ cm})}{(0.15 \text{ cm})} (1.5 - 1)t$$

$$\Rightarrow t = 10^{-3} \text{ cm}$$



82. When the curved surface is in contact  
 $\mu = (\text{real depth})/(\text{apparent depth}) = 4/3$

When the plane surface is in contact,

$$\text{Using } \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \text{ we get } R = -25 \text{ cm}$$

$$\text{Again using for the plano-convex lens } \frac{1}{f} = (\mu - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \text{ we get } f = 75 \text{ cm}$$

83. Path difference  $= D_2 - D_1 = \frac{xd}{D}$

$$\text{New path difference} = D_2 + (\mu_2 - 1)2t - D_1 - (\mu_1 - 1)t$$

$$= \frac{xd}{D} + 2(\mu_2 - 1)t - (\mu_1 - 1)t$$

For, zero order fringe occurs at a distance given by  $\frac{xd}{D} + 2(\mu_2 - 1)t - (\mu_1 - 1)t = 0$

$$\begin{aligned}\therefore x &= [(\mu_1 - 1)t - 2(\mu_2 - 1)t] \frac{D}{d} \\ &= [(\mu_1 - 1)t - 2(\mu_2 - 1)t] \frac{\beta}{\lambda} \quad [\because \beta = \frac{\lambda D}{d}] \end{aligned}$$

But  $x = 5\beta$

$$\therefore t = -2.4 \times 10^{-6} \text{ m}$$

-ve sign indicates that shift would take place towards 2<sup>nd</sup> slit and hence x should have been -ve because t can not be negative.

$$\therefore t = 2.4 \mu\text{m} \text{ and } 2t = 4.8 \mu\text{m}.$$

84.  $4\mu t = (2n - 1)\lambda$   
 $t_{\min} = \lambda/4\mu = 90 \text{ nm}.$

85. Image formed by upper lens

$$\frac{1}{v_1} - \frac{1}{(-90)} = \frac{1}{30} \Rightarrow v_1 = 45\text{cm}$$

This will act as an object for mirror

$$\therefore \frac{1}{v_2} + \frac{1}{15} = \frac{-1}{30}$$

$$v_2 = -10\text{cm}$$

coordinates of image, (110, 0)

Image formed directly by mirror

$$\frac{1}{v} - \frac{1}{120} = \frac{-1}{30}$$

$$\frac{1}{v} = \frac{-1}{30} + \frac{1}{120} = \frac{-3}{120} = -\frac{1}{40}$$

$$v = -40\text{cm}$$

$\therefore$  coordinates (80, 0)

Image formed by lower lens.

$$\frac{1}{v_c} - \left( \frac{1}{-90} \right) = \frac{1}{30} \Rightarrow v_c = 45\text{cm}.$$

This acts as an objects for mirror

$$\therefore \frac{1}{v_m} + \frac{1}{15} = \frac{-1}{30} \Rightarrow v_m = -10\text{cm}.$$

$$\text{Magnification due to lens} = -\frac{45}{90} = -\frac{1}{2};$$

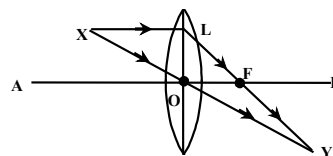
coordinates of image formed by lower lens = (135, -0.75)

$$\text{Magnification due to mirror} = \frac{+10}{15} = \frac{+2}{3}$$

$\therefore$  Coordinates of final image = (110, -0.5)

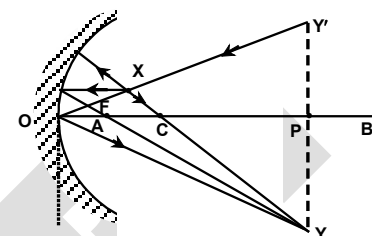
86. - O is the optical centre of the lens, where the line XY cuts the optic axis AB.

- Draw a line parallel to AB. It cuts the lens at L.
- F is the point where line LY cuts the optic axis AB.
- Image is inverted  $\Rightarrow$  convex lens



- Locate a point Y' such that YY' is perpendicular and PY = PY'.

- O is the pole of the mirror where line XY' cuts the line AB.
- C is the centre of curvature where the line XY cuts the optic axis AB.
- Centre point of C is the focus (F) of the mirror.



87. (a) For central maxima, path difference = 0

$$SS_2 + S_2C - (SS_1 + S_1C) = 0$$

$$\Rightarrow (SS_2 - SS_1) + (S_2C - S_1C) = 0$$

$$\Rightarrow \frac{dy}{b} + \frac{dy_1}{D} = 0$$

$$\Rightarrow y_1 = -\frac{yD}{b}$$

$$= -2(0.5 \sin \pi t) \text{ mm}$$

$$y_1 = -(\sin \pi t) \text{ mm}$$

(b) Path difference at point P =  $SS_2 - SS_1 + (S_2P - S_1P)$

$$\Delta p = \frac{dy}{b} + \frac{d(d/2)}{D}$$

For intensity to be maximum

$$\Delta p = n\lambda \quad \Rightarrow \quad \frac{dy}{b} + \frac{d^2}{2D} = n\lambda$$

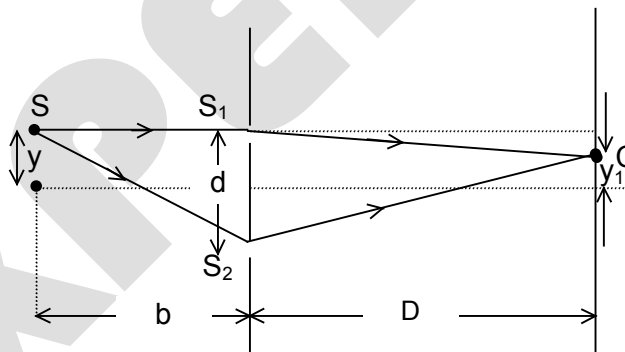
Putting values, we get

$$0.5 \sin \pi t + 0.25 = 0.5 n$$

$$\Rightarrow \sin \pi t = \frac{0.5n - 0.25}{0.5}$$

For minimum value of t,  $n = 1$

$$\Rightarrow t = \frac{1}{6} = 0.167 \text{ s.}$$



88. (a)  $(S_1P)_{OP} = \mu_m(S_1P)$

$$(S_2P)_{OP} = \mu_m(S_2P) + (\mu_g - \mu_m)t_0$$

$$\therefore \text{Optical path difference : } \Delta_{OP} = (S_2P)_{OP} - (S_1P)_{OP}$$

$$\Delta_{OP} = \mu_m(S_2P - S_1P) + (\mu_g - \mu_m)t_0$$

$$\Delta_{OP} = \frac{\mu_m y d}{D} + (\mu_g - \mu_m) t_0$$

For Central max.,  $\Delta_{OP} = 0$

$$\therefore y = -\frac{(\mu_g - \mu_m) t_0 D}{\mu_m d}$$

$$y = \frac{(4 - t) t_0 D}{(10 - t) d}$$

When at 'O',  $y = 0$

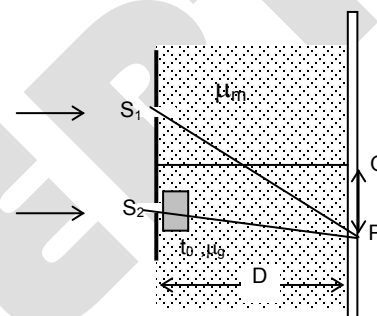
$$\therefore t = 4 \text{ sec.}$$

(b) Speed of central max.

$$v = \left| \frac{dy}{dt} \right| = \frac{6Dt_0}{(10 - t)^2 d}$$

When it is at O,  $t = 4 \text{ sec.}$

$$v = \frac{6Dt_0}{36d} = \frac{1 \times 36 \times 1 \times 10^{-6}}{6 \times 2 \times 10^{-3}} = 3 \times 10^{-3} \text{ m/sec} = 3 \text{ mm/sec.}$$



89. The object distance  $u = \infty$

$$\frac{3/2}{v_1} - \frac{1}{u} = \frac{3/2 - 1}{30} \quad \dots \dots (i)$$

$$\frac{4/3}{v} - \frac{3/2}{v_1} = \frac{-1/6}{-30} \quad \dots \dots (ii)$$

from (i) and (ii)

$$v \approx 60 \text{ cm}$$

after reflection from the mirror, the light rays appear to converge at a point 40 cm to the right of the convex lens now refraction from the two surfaces of the lens

$$\frac{3/2}{v_1} - \frac{4/3}{40} = \frac{3/2 - 4/3}{30} = \frac{1/6}{30}$$

$$\text{and } \frac{1}{v} - \frac{3/2}{v_1} = \frac{-1/2}{-30}$$

$$\Rightarrow v = 18 \text{ cm to the right of the convex lens.}$$

90. (a) Total phase difference at C,  $\Delta\phi = kd \sin \phi - kt(\mu - 1)$   
for centre maxima at C,

$$\Delta\phi = 0$$

$$t = \frac{d \sin \phi}{(\mu - 1)} \Rightarrow \frac{2 \times 10^{-3} \times \sin 30^\circ}{(\mu - 1)} = 5 \times 10^{-3}.$$

$$\mu' = 1.2 \Rightarrow \mu = 1.2 \times (4/3) = 1.6$$

Hence refractive index of mica slab = 1.6.

(b) A black line is formed at the position where dark fringe are formed for both the wavelength.

The distance of first black line from centre bright line

$$y = \frac{(2n-1)\lambda D}{2d} \quad \dots (I)$$

For black line ;  $\frac{(2n_1-1)\lambda'_1 D}{2d} = \frac{(2n_2-1)\lambda'_2 D}{2d}$

$$\frac{(2n_1-1)}{(2n_2-1)} = \frac{\lambda'_2}{\lambda'_1}, \quad \text{where } \lambda'_1 = \frac{\lambda_1}{\mu_w} \text{ and } \lambda'_2 = \frac{\lambda_2}{\mu_w}$$

$$\frac{(2n_1-1)}{(2n_2-1)} = \frac{7}{5}$$

for minimum value,  $n_1 = 4$  and  $n_2 = 3$ .

Hence distance of first black line

$$y = \frac{(2 \times 4 - 1)4000 \times 10^{-10} \times 40 \times 10^{-2} \times 3}{2 \times 2 \times 10^{-3} \times 4}$$

$$= 2.1 \times 10^{-4} \text{ m} = 210 \mu\text{m}.$$

91. (i) Lensmaker's formula  $\frac{1}{f} = (\mu - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$

For concavo-convex lens  $R_1 = -60 \text{ cm}$ ;  $R_2 = -20 \text{ cm}$ ;  $\mu = 1.5$

$$\frac{1}{f_1} = (1.5 - 1) \left( \frac{1}{-60} - \frac{1}{-20} \right) = \frac{1}{60}$$

Focal length of Lens-mirror combination

$$\frac{1}{F_1} = \frac{2}{f_1} + \frac{2}{R_2} = \frac{2}{60} + \frac{2}{20} = \frac{4}{30} = \frac{1}{7.5}$$

The object should be placed at  $2F_1$  above the lens, i.e.

$$u_1 = 2(7.5) = 15 \text{ cm}$$

(ii) Focal length of the plano-convex lens of water

$$\mu = 4/3, \quad R = -\infty; \quad R_2 = -60 \text{ cm}$$

$$\frac{1}{f_2} = \left( \frac{4}{3} - 1 \right) \left( \frac{1}{-\infty} - \frac{1}{-60} \right) = \frac{1}{180}$$

Focal length of the new-combination,

$$\frac{1}{F_2} = \frac{2}{f_1} + \frac{2}{f_2} + \frac{2}{20} = \frac{2}{180} + \frac{2}{60} + \frac{2}{20} = \frac{13}{90}$$

The object should be placed at  $u_2 = 2F_2 = 2 \left( \frac{90}{13} \right) = 13.85 \text{ cm}$

Downward displacement of the pin =  $15 - 13.85 = 1.15 \text{ cm}$

92. When the curved surface is in contact

$$\mu = (\text{real depth})/(\text{apparent depth}) = 4/3$$

When the plane surface is in contact,

Using  $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$  we get  $R = -25 \text{ cm}$

Again using for the Plano-convex lens  $\frac{1}{f} = (\mu - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$  we get  $f = 75 \text{ cm}$

93. (a)  $\mu_{\text{air}} \sin 60^\circ = \mu_p \sin r$

$$\Rightarrow \frac{\sqrt{3}}{2} = \sqrt{3} \sin r$$

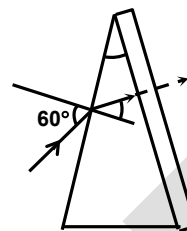
$$\Rightarrow r = 30^\circ$$

The refracted ray inside the prism hits the other face at  $90^\circ$ ; hence deviation produced by this face is zero and hence angle of emergence is zero.

(b) Multiple reflection occurs between the surfaces of the film for minimum thickness

$$\Delta x = 2\mu t = \lambda, \text{ where } t = \text{thickness}$$

$$\Rightarrow t = \frac{\lambda}{2\mu} = 125 \text{ nm}$$



94. (a) At  $t = 0$   $\sin r = \frac{3}{5} \Rightarrow \mu = \frac{5}{3\sqrt{2}}$

(b) Let at time  $t$ , insect is at a distance  $x$  from centre of the tank.

$$\frac{x_1}{h} = \tan r = \frac{3}{4}$$

$$x_1 = \frac{3}{4}h \Rightarrow x + (H - h) + x_1 = 3$$

$$x = 4 - h + \frac{3}{4}h - 3 \quad \text{or} \quad x = 1 - \frac{h}{4}$$

$$\left( \frac{dx}{dt} \right) = \frac{1}{4} \left( - \frac{dh}{dt} \right)$$

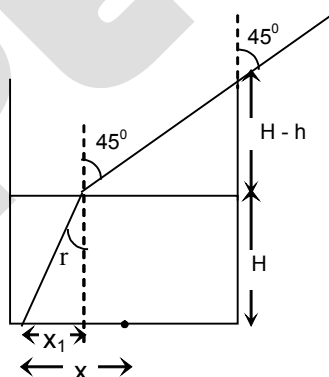
from equation of continuity,  $-A_1 \left( \frac{dh}{dt} \right) = A_2 \sqrt{2gh}$

$$- \int_H^h \frac{dh}{\sqrt{h}} = \frac{\pi(3 \times 10^{-2})^2}{\pi(3)^2} \sqrt{2 \times 9.8} \int_0^t dt$$

$$h = (2 - 2.21 \times 10^{-4} t)^2$$

$$- \frac{dh}{dt} = 4.42 \times 10^{-4} (2 - 2.21 \times 10^{-4} t) \quad \text{putting } H = 4 \text{ m}$$

$$\therefore \text{speed of insect } v = \frac{dx}{dt} = \frac{1}{4} \left( - \frac{dh}{dt} \right) = 1.1 \times 10^{-4} (2 - 2.21 \times 10^{-4} t) \text{ m/s.}$$



95.  $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow -\frac{1}{v^2} \frac{dv}{dt} + \frac{1}{u^2} \frac{du}{dt} = 0$

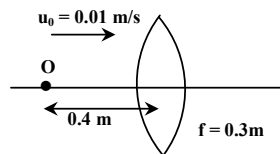
$$\Rightarrow \frac{dv}{dt} = \frac{v^2}{u^2} \frac{du}{dt} \quad \dots (i)$$

$$\frac{1}{30} = \frac{1}{v} - \frac{1}{-40} \Rightarrow v = 120 \text{ cm}$$

$$\Rightarrow \frac{dv}{dt} = 0.09 \text{ m/sec}$$

$$\Rightarrow m = \frac{dv}{du} = \frac{v^2}{u^2} = \left(1 - \frac{v}{f}\right)^2$$

$$\frac{dm}{dt} = -\frac{2}{f} \left(1 - \frac{v}{f}\right) \frac{dv}{dt} = \frac{-2}{0.3} \left(1 - \frac{120}{30}\right) \times 0.09 = 1.8 \text{ s}^{-1}$$



96. RI of lens material =  $\mu_2$ ; RI of first liquid =  $\mu_3$ ; RI of air =  $\mu_1$ ; RI of second liquid =  $\mu'_3$

In general  $(\mu_3/v) - (\mu_1/u) = \{(\mu_2 - \mu_1)/R\} + \{(\mu_3 - \mu_2)/R\}$

Here  $u = -15 \text{ cm}$ ,  $v = \infty$

Now,  $1/(+15) = [(3/2) - 1]/R - [(\mu'_3 - (3/2))/R]$

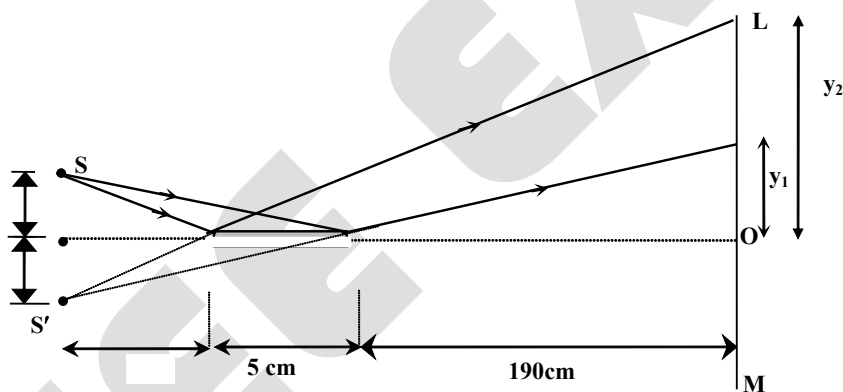
$\Rightarrow R = 10 \text{ cm}$

For the second liquid, similarly,

$(1/25) = (0.5/R) - [(\mu'_3 - (3/2))/R]$

$\Rightarrow \mu'_3 = 1.6$

97.



Using geometry,

$$y_1 = (190) \frac{0.1}{10} = 1.9 \text{ cm}$$

$$y_2 = (195) \frac{(0.1)}{5} = 3.9 \text{ cm}$$

The region in which interference occurs is

$$y_2 - y_1 = 3.9 - 1.9 = 2 \text{ cm}$$

The fringe width is  $\omega = \frac{\lambda D}{2d}$

Where  $D = 190 + 5 + 5 = 200 \text{ cm}$ ;  $2d = 2 \text{ mm}$

$$\lambda = \frac{c}{v} = \frac{3 \times 10^8}{6 \times 10^{14}} = 0.5 \mu\text{m}$$

$$\therefore \omega = (0.5 \times 10^{-6}) \left( \frac{2000}{2} \right) = 0.5 \text{ mm}$$

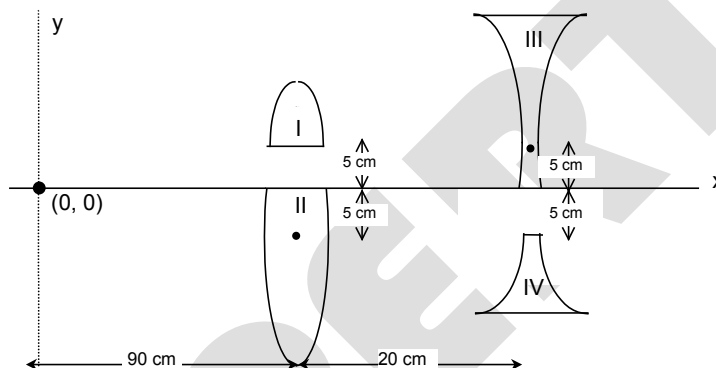
$$\text{no. of fringes} = \frac{(y_2 - y_1)}{\omega} = \frac{(20)}{0.5} = 40.$$

98. Location of the image due to convex lens.

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{30} = \frac{1}{v} - \frac{1}{-90}$$

$$\Rightarrow v = +45 \text{ cm from convex lens.}$$



Location of the image formed by the concave lens after convex lens.

$$\frac{1}{-50} = \frac{1}{v} - \frac{1}{+25} \Rightarrow v = +50 \text{ from concave lens.}$$

For lens numbered (I)

Optical centre at point (90, 0)

so image will be on point (135, 0)

This will behave as an object for lens numbered (III)

$$m = \frac{v}{u} = \frac{I}{O} \Rightarrow \frac{50}{25} = \frac{I}{-5} \Rightarrow I = -10 \text{ cm.}$$

so location of final image due to this combination will be at (160, -5) cm

For lens numbered (II)

Optical centre at point (90, -5) cm

$$m = \frac{v}{u} = \frac{I}{O} \Rightarrow \frac{45}{-90} = \frac{I}{+5} \Rightarrow I = -2.5 \text{ cm.}$$

so location of image due to this lens will be

at [135, -7.5] cm

and this will also be a location of real image for the gap between concave lens.

This will also behave as an object for lens numbered (IV)

whose optical centre at (110, 0) cm.

so for final image for the combination (II) and (IV) will be

$$m = \frac{v}{u} = \frac{I}{O} \Rightarrow \frac{50}{25} = \frac{I}{7.5} \Rightarrow I = -15 \text{ cm.}$$

So location will be (160, -15) cm

Total three real images.

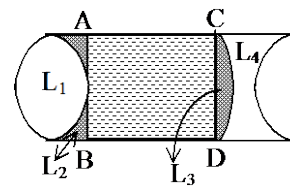


99. The situation is shown in fig.

The whole system may be considered to consist of

(i) Double convex lens (Glass)  $L_1$ ,

(ii) Plano concave lens (water)  $L_2$



The above two lenses form one group of lens, say focal length  $F_1$ .

(iii) Plano-convex lens (water)  $L_3$

(iv) Double concave lens (Glass)  $L_4$

The above two lenses form one group of lens, say focal length  $F_2$ . The above two groups are separated by parallel plane water slab of thickness 4 cm.

$$\text{Apparent thickness of water slab} = \frac{4}{4/3} = 3 \text{ cm},$$

Thus the above two groups can be regarded to be separated by air of thickness 3 cm.

$$\text{For } L_1; \frac{1}{f_1} = (1.5 - 1) \left( \frac{1}{4} + \frac{1}{4} \right) \text{ or } f_1 = 4 \text{ cm.}$$

$$\text{For } L_2; \frac{1}{f_2} = \left( \frac{4}{3} - 1 \right) \left( -\frac{1}{4} + \frac{1}{\infty} \right) \text{ or } f_2 = -12 \text{ cm.}$$

$$\text{Combined focal length } \frac{1}{F_1} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{4} - \frac{1}{12} = \frac{1}{6}$$

$$\text{or } F_1 = 6 \text{ cm.}$$

$$\text{For } L_3; \frac{1}{f_3} = \left( \frac{4}{3} - 1 \right) \left( \frac{1}{8} + \frac{1}{\infty} \right) = \frac{1}{24}$$

$$\text{or } f_3 = 24 \text{ cm.}$$

$$\text{For } L_4; \frac{1}{f_4} = (1.6 - 1) \left( -\frac{1}{8} - \frac{1}{8} \right) = -\frac{1.2}{8}$$

$$\text{or } f_4 = -6.67 \text{ cm.}$$

$$\frac{1}{F_2} = \frac{1}{f_3} + \frac{1}{f_4} = \frac{1}{24} + \left( -\frac{1.2}{8} \right) = -\frac{13}{120}$$

Hence, combined focal length.

$$\frac{1}{F} = \frac{1}{F_1} + \frac{1}{F_2} - \frac{d}{F_1 F_2}$$

$$= \frac{1}{6} - \frac{13}{120} + \frac{3}{6 \times (120/13)}$$

$$F = 8.9 \text{ cm.}$$

100. Since the medium inside the glass pieces is air, its optical power will be zero. When its rear surface is polished it works as a concave mirror of focal length  $f_m = -r/2$ .

Since magnification  $m = 2$

$$\therefore \text{Distance of object from the mirror} = \frac{a}{m} = 30 \text{ cm.}$$

For the mirror

$$v = -a = -60 \text{ cm, } u = -30 \text{ cm, } f = f_m = -r/2.$$

$$\text{Using mirror formula } \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$f_m = -20 \text{ cm or } r = 40 \text{ cm}$$

When water is filled in space between glass pieces, an equi-convex lens of water is formed whose one surface is silvered.

Effective focal length of the combination

$$\frac{1}{F} = \frac{1}{f_m} + \frac{2}{f_\ell}$$

For equiconvex lens

$$\frac{1}{f} = (\mu - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$R_1 = +r, R_2 = -r, \mu = 4/3$$

$$F = f_\ell = 60 \text{ cm.}$$

$$\text{Hence } \frac{1}{F} = \frac{2}{60} + \frac{1}{20} = \frac{1}{12}$$

$$F = 12 \text{ cm.}$$

Since, a sharp image is again formed on the screen, therefore, for this effective mirror

$$F = -12 \text{ cm, } v = -a = -60 \text{ cm, } u = ?$$

$$\frac{1}{F} = \frac{1}{v} + \frac{1}{u}$$

which gives  $u = -15 \text{ cm.}$

Hence the object must be moved through  $30 - 15 = 15 \text{ cm}$  towards the mirror.

101. If  $I_m$  be the intensity on the screen, intensity at a point where phase difference between the incident disturbances equals  $\phi$  is given by

$$I = 2I_0 + 2I_0 \cos(\phi)$$

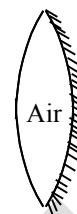
$$\text{as } I = \frac{3}{4} I_m \quad \text{and} \quad I_m = 4I_0$$

$$\text{We have } 3I_0 = 2I_0 + 2I_0 \cos(\phi) \Rightarrow \cos(\phi) = 1/2$$

If optical path difference corresponding to phase difference  $\Delta\phi$  be  $p$  then

$$\cos \left[ \frac{p(2\pi)}{\lambda} \right] = \frac{1}{2} \Rightarrow \left[ \frac{p(2\pi)}{\lambda} \right] = 2n\pi + \frac{\pi}{3}$$

$$\Rightarrow p = \lambda[n \pm (1/6)]$$



As  $4\lambda < p < 4.5\lambda$ ,  $p$  can be

$$\therefore p = \lambda \left[ 4 + \frac{1}{6} \right] \quad (1)$$

If  $t$  be the thickness of each glass and  $\mu_1, \mu_2$  be the refractive indices,

$$p = (\mu_2 - 1)t - (\mu_1 - 1)t$$

$$\Rightarrow p = 0.3t \quad [\mu_2 = 1.7, \mu_1 = 1.4]$$

$$\Rightarrow t = \frac{p}{0.3} = \frac{25}{6} \left( \frac{\lambda}{0.3} \right) = 7.5 \mu\text{m}$$

102. If  $f_0$  be the focal length of the lens in air, then modified focal length of the lens in this case is

$$\frac{1}{f} = \frac{1}{f_0} (2\mu_2 - \mu_1 - \mu_3)$$

$$\frac{1}{f} = \frac{1}{20} (2(1.5) - 1.2 - 2.0)$$

$$\text{or } f = -100 \text{ cm}$$

The convex lens acts a diverging lens.

Let  $I'$  be the image formed by the convex lens

$$\therefore \frac{\mu_3}{v} - \frac{\mu_1}{u} = \frac{1}{f}$$

here  $u = -40 \text{ cm}$ ;  $v = -x$ ;  $f = -100 \text{ cm}$

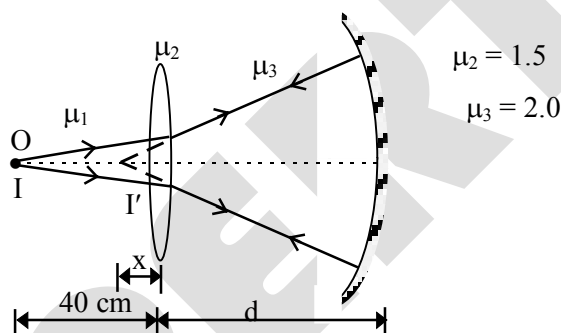
$$\therefore \frac{2}{-x} - \frac{1.2}{-40} = \frac{1}{-100}$$

$$\text{or } x = - \frac{(2)(40)(-100)}{40 - (1.2)(-100)} = 50 \text{ cm}$$

If the rays have to retrace their paths, then  $I'$  must lie on the centre of curvature of the mirror.

$$\therefore R = d + x$$

$$\text{or } d = R - x = 80 - 50 = 30 \text{ cm.}$$



103. If  $I_m$  be the intensity on the screen, intensity at a point where phase difference between the incident disturbances equals  $\phi$  is given by

$$I = 2I_0 + 2I_0 \cos(\phi)$$

$$\text{as } I = \frac{3}{4} I_m \quad \text{and} \quad I_m = 4I_0$$

$$\text{We have } 3I_0 = 2I_0 + 2I_0 \cos(\phi) \Rightarrow \cos(\phi) = 1/2$$

If optical path difference corresponding to phase difference  $\Delta\phi$  be  $p$  then

$$\cos \left[ \frac{p(2\pi)}{\lambda} \right] = \frac{1}{2} \Rightarrow \left[ \frac{p(2\pi)}{\lambda} \right] = 2n\pi + \frac{\pi}{3}$$

$$\Rightarrow p = \lambda [n \pm (1/6)]$$

As  $4\lambda < p < 4.5\lambda$ ,  $p$  can be

$$\therefore p = \lambda \left[ 4 + \frac{1}{6} \right] \quad (1)$$

If  $t$  be the thickness of each glass and  $\mu_1, \mu_2$  be the refractive indices,

$$p = (\mu_2 - 1)t - (\mu_1 - 1)t$$

$$\Rightarrow p = 0.3t \quad [\mu_2 = 1.7, \mu_1 = 1.4]$$

$$\Rightarrow t = \frac{p}{0.3} = \frac{25}{6} \left( \frac{\lambda}{0.3} \right) = 7.5 \mu\text{m}$$

104. (a)  $\Delta x = (\ell_2 - \ell_1) + d \sin \theta$

$$\Delta x = n\lambda, n = 0, 1, 2, 3, \dots$$

$$\Rightarrow \sin \theta = \frac{1}{10 \times 10^{-6}} [n \times 500 \times 10^{-9} - 20 \times 10^{-6}]$$

$$\theta = \sin^{-1} \left[ 2 \left( \frac{n}{40} - 1 \right) \right]$$

(b)  $|\sin \theta| \leq 1$

$$20 \leq n \leq 60$$

Hence number of maxima = 41.

(c) Maxima will appear on C.

for minima at C,  $(\mu - 1)t = \lambda/2$

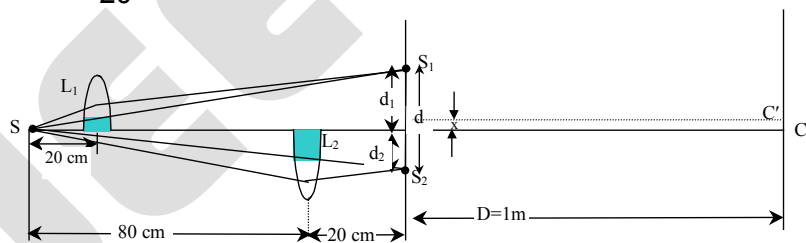
$$\text{Hence } t = \frac{\lambda}{2(\mu - 1)} = \frac{500 \times 10^{-9}}{2 \times 0.5} = 500 \text{ nm.}$$

105.  $L_1, \frac{1}{v} = \frac{1}{16} - \frac{1}{20}, v_1 = 80 \text{ cm}$

$$L_2 = \frac{1}{v} = \frac{1}{16} - \frac{1}{80}, v_2 = 20 \text{ cm}$$

$$\text{Magnification } m = \frac{v}{u}$$

$$m_1 = \frac{80}{-20} = -4$$



$$\text{Hence } d_1 = 5\Delta = 5 \times 0.40 = 2 \text{ mm}$$

$$\text{Similarly } d_2 = \frac{3\Delta}{4} + 3\Delta = \frac{15}{4}\Delta = 1.5 \text{ mm.}$$

$$d = d_1 + d_2 = 3.5 \text{ mm.}$$

$$D = 2 - 1 \text{ metre.} \quad \Rightarrow x = d_1 - \frac{d}{2} = 0.25 \text{ mm}$$

As shifting  $x = \frac{(\mu - 1)tD}{d}$  ;  $0.25 \times 10^{-3} = \frac{(\mu - 1) \times 2 \times 10^{-6} \times 1}{3.5 \times 10^{-3}}$   
 which gives  $\mu = 1.437$ .

106. (a)  $\frac{(\mu_s - 1)tD}{d} = 2 \times 10^{-3}$

$$\Rightarrow \mu_s - 1 = \frac{2 \times 10^{-3} \times 10^{-3}}{4 \times 10^{-6} \times 1}$$

$$\Rightarrow \mu_s = 1.5 \quad \text{and} \quad \left( \frac{\mu_s}{\mu_m} - 1 \right) \frac{tD}{d} = 0.5 \times 10^{-3}$$

$$\Rightarrow \left( \frac{\mu_s}{\mu_m} - 1 \right) = \frac{1}{8}$$

$$\mu_m = \frac{\mu_s}{\left( \frac{1}{8} + 1 \right)} = \frac{3}{2} \times \frac{8}{9}$$

$$\Rightarrow \mu_m = \frac{4}{3}$$

(b) Fringe width  $\beta = \frac{\lambda D}{\mu_m d}$

$$\Rightarrow \beta = \frac{1.5}{4} \times 10^{-3} = 0.375 \text{ mm}$$

$\therefore$  The first maximum is the nearest maximum from the point O and its distance is 0.125 mm,

107. If  $I_m$  be the intensity on the screen, intensity at a point where phase difference between the incident disturbances equals  $\phi$  is given by

$$I = 2I_0 + 2I_0 \cos(\phi)$$

as  $I = \frac{3}{4} I_m$  and  $I_m = 4I_0$

$$\text{We have } 3I_0 = 2I_0 + 2I_0 \cos(\phi) \Rightarrow \cos(\phi) = 1/2$$

If optical path difference corresponding to phase difference  $\Delta\phi$  be  $p$  then

$$\cos \left[ \frac{p(2\pi)}{\lambda} \right] = \frac{1}{2} \Rightarrow \left[ \frac{p(2\pi)}{\lambda} \right] = 2n\pi + \frac{\pi}{3}$$

$$\Rightarrow p = \lambda \left[ n \pm \left( \frac{1}{6} \right) \right]$$

As  $4\lambda < p < 4.5\lambda$ ,  $p$  can be

$$\therefore p = \lambda \left[ 4 + \frac{1}{6} \right] \quad (1)$$

If  $t$  be the thickness of each glass and  $\mu_1, \mu_2$  be the refractive indices,

$$p = (\mu_2 - 1)t - (\mu_1 - 1)t$$

$$\Rightarrow p = 0.3t \quad [\mu_2 = 1.7, \mu_1 = 1.4]$$

$$\Rightarrow t = \frac{p}{0.3} = \frac{25}{6} \left( \frac{\lambda}{0.3} \right) = 7.5 \mu\text{m}$$

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