ELECTROSTATICS

1. ELECTRIC CHARGE

Charge is the property associated with matter due to which it produces and experiences electrical and magnetic effects. The study of electrical effects of charge at rest is called electrostatics.

A body may be positively or negatively charged depending upon deficiency or excess of electrons in the body. If a body is neutral then it has equal amount of +ve and – ve charge.

Units of charge

The SI unit of charge is 'coulomb'.
[1 coulomb = 1 amp-sec.]

In cgs electrostatic system, the unit of charge is state coulomb or esu of charge or frankline.
[1 statcoulomb = 3.336×10^{-10} coulomb]

Methods of Charging

A body can be charged by these three methods
(i) friction
(ii) conduction
(iii) induction

Properties of Charge

Characteristics of charge

1. Charge is a scalar quantity.

2. Charge is always associated with mass i.e. charge can not exist without mass, though mass can exist without charge. Particles such as neutrino or photon have no rest mass, so they can have no charge.

3. Charge is transferable from one charged body to another body which may be charged or uncharged, if they are put in contact. The process of charge transfer is called conduction. Whole of the charge can not be transferred by conduction from one body to another except in case when a charged body is enclosed by a conducting body and connected to it.

4. An accelerated charge always radiates energy in the form of electromagnetic waves.

5. Charge can be detected and measured with the help of gold leaf electroscope, voltmeter or ballistic galvanometer.

6. Charge is invariant i.e. charge on a body does not change, whatever be its speed, specific charge depends on speed as mass depends on speed.

specific charge = q/m

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

where m is the dynamic mass and m_0 is the rest mass.
The dimension of the body also changes with the speed as

$$l = l_0\left(\sqrt{1-\frac{v^2}{c^2}}\right)$$

where l_0 is the length when observer is stationary with respect to the object and l is the length parallel to the direction of motion. When there is a relative velocity between the object and the observer. Due to change in dimension and mass, charge density (q/l) and the specific charge (q/m) also change.

7. A body can be charged by friction. (rubbing together). Induction or conduction. In case of induction, maximum induced charge is given by

$q' = -q\,[1-(1/k)]$ (to be discussed later)

where q' = induced charge

q = free charge which is responsible for induction

k = dielectric constant.

8. Quantization of charge :

Charge exists in discrete packets rather than in continuous amount. One such packet is named as one quanta, this is referred as charge quantization. One quanta is the smallest discrete value of charge that can exist in nature and it is equal to the charge of an electron. Charge on electron is -1.6×10^{-19}C.

B In some theoretical specialation, the elementary particle such as proton and neutron are supposed to be compose of quarks having charge $\left(\pm\frac{1}{3}e\right)$ and $\left(\pm\frac{2}{3}e\right)$. However quarks do not exist in free state.

9. Conservation of charge :

The total charge (the difference between the amount of positive and negative charge) within an isolated system is always constant or conserved.

Illustration : 1

A copper sphere contains about 2 x 10^{22} atoms. The charge on the nucleus of each atom is 29e. what fraction of the electrons must be removed from the sphere to give it a charge of $+2\mu$ C ?

Solution:

The total number of electrons is 29 (2×10^{22}) = 5.8×10^{23}.

Electrons removed = (2×10^{-6}C) / (1.6×10^{-19}C) = 1.25×10^{13},

so the fraction removed = electrons removed / total number electrons = 2.16×10^{-11}.

Since loss or gain of electron is responsible for creating charge on a body and electron is a particle with mass, every charged body will have mass also.

Illustration : 2

Which of the following charge can not be possible.

(i) 1.6×10^{-19} C (ii) 1.6×10^{-18} C (iii) 1.6×10^{-20} C (iv) 1 C

Solution:

1.6×10^{-20} C is not possible because it does not obeys quantization of charge

2. COULOMB'S LAW

Force between two point charges (interaction force) is directly proportional to the product of magnitude of charges (q_1 and q_2) and is inversely proportional to the square of the distance between them i.e., ($1/r^2$). This force is conservative in nature. This is also called inverse square law. The direction of force is always along the line joining the point charges.

$$F \propto \frac{q_1 q_2}{r^2}$$

$$F = K\frac{q_1 q_2}{r^2} \quad \text{where K is a constant}$$

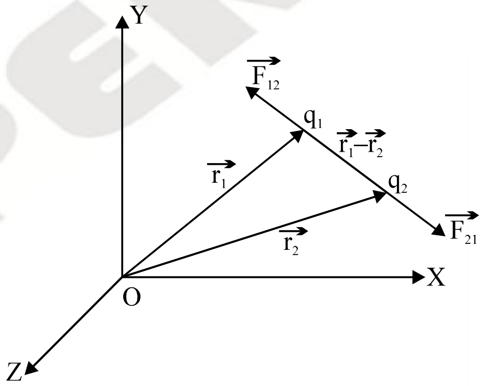
$$K = \frac{1}{4\pi\varepsilon_0}$$

$K = 9 \times 10^9$ $C^2/N\text{-}m^2$

ε_0 = permittivity of free space $= 8.85 \times 10^{-12}$ $N\text{-}m^2/C^2$

Coulomb's Law in Vector Form

Suppose the position vectors of two charges q_1 and q_2 are $\vec{r}_1$ and $\vec{r}_2$, then, electric force on charge q_1 due to charge q_2 is,



$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)$$

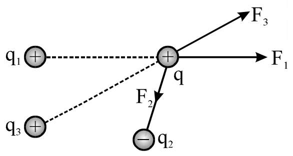
Similarly, electric force on q_2 due to charge q_1 is

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1)$$

Here q_1 and q_2 are to be substituted with sign. Position vector of charges q_1 and q_2 are $\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ and $\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$ respectively. Where (x_1, y_1, z_1) and (x_2, y_2, z_2) are the co-ordinates of charges q_1 and q_2.

Superposition Theorem

The interaction between any two charges is independent of the presence of all other charges. Electrical force is a vector quantity therefore, the net force on any one charge is the vector sum of the all the forces exerted on it due to each of the other charges interacting with it independently i.e.

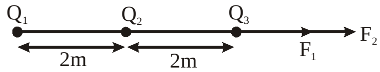


Net force on charge q,

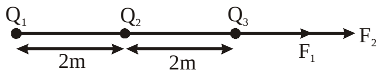
$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots\ldots\ldots\ldots$$

Illustration : 3

Three charges each of $20\mu C$ are placed along a straight line, successive charges being 2 m apart as shown in Figure. Calculate the force on the charge on the right end.



Solution:



$$F = F_1 + F_2 \qquad F_1 = \frac{kQ_1Q_2}{r^2} = \frac{(9x10^9)(20x10^{-6})^2}{4^2} = 0.225N$$

$$F_2 = \frac{kQ_2Q_3}{r^2} = \frac{(9x10^9)(20x10^{-6})^2}{2^2} = 0.9\,N$$

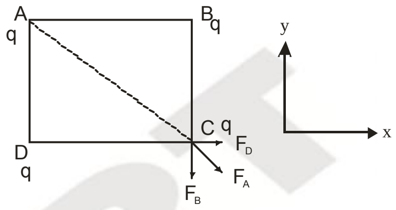
$F = 0.225+0.9 = 1.125$ N to the right

Illustration : 4

Four identical point charges each of magnitude q are placed at the corners of a square of side a. Find the net electrostatic force on any of the charge.

Solution :

Let the concerned charge be at C then charge at C will experience the force due to charges at A, B and D. Let these forces respectively be $\vec{F}_A$, $\vec{F}_B$ and $\vec{F}_D$ thus forces are given as



$$\vec{F}_A = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{AC^2} \text{ along AC} = \frac{q^2}{4\pi\varepsilon_0 2a^2}\left(\frac{\hat{i}}{\sqrt{2}} - \frac{\hat{j}}{\sqrt{2}}\right)$$

$$\vec{F}_B = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{BC^2} \text{ along BC} = \frac{q^2}{4\pi\varepsilon_0 a^2}(-\hat{j})$$

$$\vec{F}_D = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{DC^2} \text{ along DC} = \frac{q^2}{4\pi\varepsilon_0 a^2}(\hat{i})$$

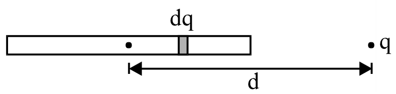
$$\vec{F}_{net} = \vec{F}_A + \vec{F}_B + \vec{F}_D$$

$$= \frac{q^2}{4\pi\varepsilon_0 a^2}\left[\hat{i}\left(\frac{1}{2\sqrt{2}}+1\right) - \hat{j}\left(\frac{1}{2\sqrt{2}}+1\right)\right]$$

$$\left|\vec{F}_{net}\right| = \sqrt{2}\left(\frac{1}{2\sqrt{2}}+1\right)\frac{q^2}{4\pi\varepsilon_0 a^2} = \left(\frac{1}{2}+\sqrt{2}\right)\frac{q^2}{4\pi\varepsilon_0 a^2}$$

Illustration : 5

Find the force between a uniformlly charged rod of length ℓ and charge Q and a point charge q placed at a distance d from the centre of the rod at its axiel position.



Solution :

$$F = \frac{Kqdq}{r^2}$$

$$dq = \lambda dx$$

$$F = \int_{d-\ell/2}^{d+\ell/2} \frac{Kq\lambda dx}{x^2}$$

$$F = \frac{Kq\lambda\ell}{\left(d^2 - \frac{\ell^2}{4}\right)} \quad \Rightarrow \quad F = \frac{KqQ}{\left(d^2 - \frac{\ell^2}{4}\right)}$$

ELECTRIC FIELD

Every charge sets up around it a space in which any other charge will experience electrostatic force

. This space is known as the electric field. Electric field at a point is given as the force experienced per unit charge at that position.

The charge over which field is checked is purposely taken small so that it does not affect set up of other charges. Filed is denoted by $\vec{E}$. Mathematically.

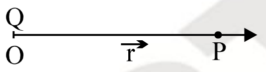
$$\vec{E} = \underset{q \to 0}{Lt}\left(\frac{\vec{F}}{q}\right).$$

SI Unit of Electric field is N/C

Electric Field Due to A point Charge :

A point charge Q is lying at origin (O), force on a charge q at P

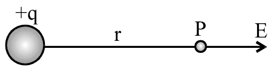
$$\vec{F} = \frac{1}{4\pi \in_0} \frac{Qq}{r^2}$$

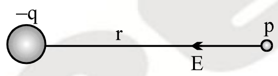


Force on unit charge at P

$$\vec{E} = \frac{\vec{F}}{q} = \frac{1}{4\pi \in_0} \frac{Q}{r^2}$$

In Vector Form,





$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q\vec{r}}{|\vec{r}|^3}$$

Where $\vec{r}$ is the position vector of the point P with respect to the charge. If the position vectors of point P and charge q are $\vec{r}_p$ and $\vec{r}_q$ respectively.

then $\quad \vec{r} = \vec{r}_p - \vec{r}_q$

if $\quad \vec{r}_p = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\quad \vec{r}_q = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

then $\quad \vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q\left[(x_1 - x_2)\hat{i} + (y_1 - y_2)\hat{j} + (z_1 - z_2)k\right]}{[(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2]^{3/2}}$

hence $\quad E_x = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q(x_1 - x_2)}{\{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2\}^{3/2}}$

Similarly expression for E_y and E_z can be written.

SUPERPOSITION THEOREM

In case of many point charges field at a point is given as

$$\vec{E} = \sum \vec{E}_i$$

{Where $\sum \vec{E}_i$ represents vector sum of the field at P due to individual point charges}

- For a given setup of charges field is the property of a point.
- Field is a vector quantity hence it can be added vectorially only, not algebraically.

Illustration : 6

A charge $q = 1\ \mu C$ is placed at point (1m, 2m, 4m). Find the electric field at point P(0m, –4m, 3m).

Solution :

Here,

and

or

Now,

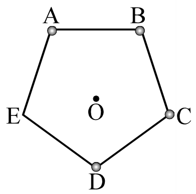
Substituting the values, we have

Illustration : 7

Four particles, each having a charge q, are placed on the four vertices of a regular pentagon. The distance of each corner from the centre is a. Find the electric field at the centre of the pentagon.

Solution :

Let the charges be placed at the vertices A, B, C and D of the pentagon ABCDE. If we put a charge q at the corner E also, the field at O will be zero by symmetry. Thus, the field at the centre due to the charges at A, B, C and D is equal and opposite to the field due to the charge q at E alone.



The field at O due to the charge q at E is

along EO.

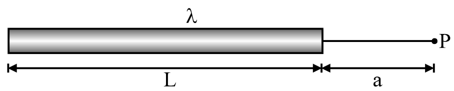
Thus, the field at O due to the given system of charges is along OE.

ELECTRIC FIELD DUE TO VARIOUS CHARGE DISTRIBUTION

1. Electric field due to point charge

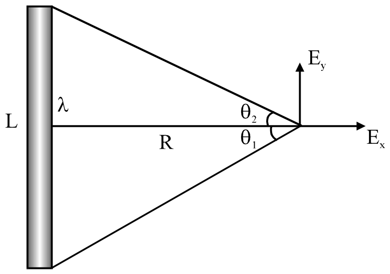
or

2. Electric field due to rod



(i)

(ii)

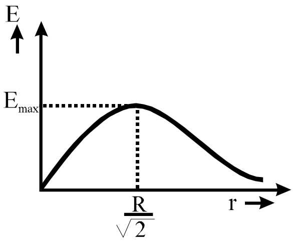


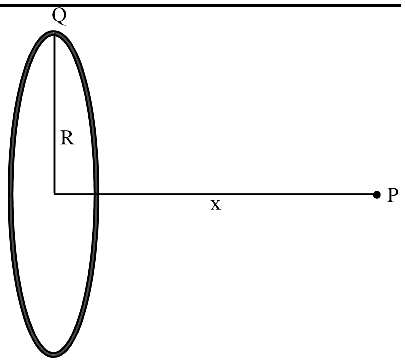
$$E_y = \frac{K\lambda}{R}[\cos\theta_2 - \cos\theta_1]$$

3. Electric field due to a charged ring

at a point on the axis

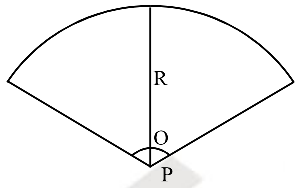
$$E = \frac{KQx}{(x^2 + R^2)^{3/2}}$$





4. Due to arc at the centre of acr

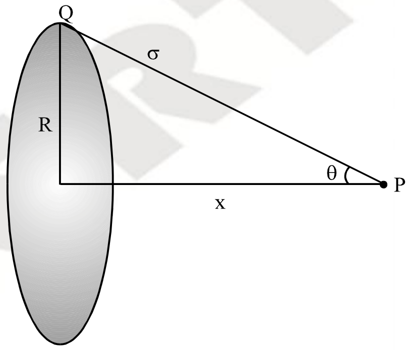
$$E = \frac{2K\lambda}{R}\sin\frac{\theta}{2}$$



5. Electric field due to disc

$$E = \frac{\sigma}{2\varepsilon_0}(1 - \cos\theta)$$

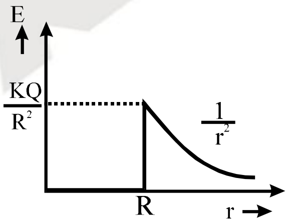
$$= \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$$

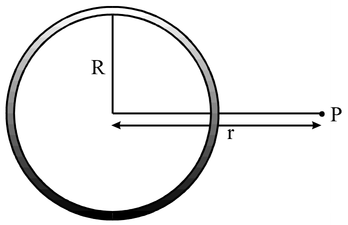


6. Electric field due to spherical shell
(Conductor and non-conductor)

$$E_0 = \frac{kQ}{r^2} \qquad r \geq R$$

$$E_I = \text{zero} \qquad r < R$$

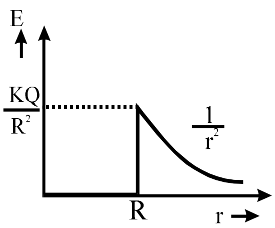


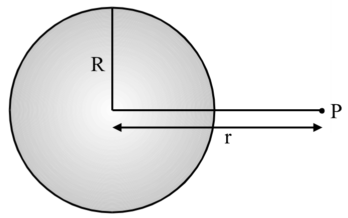


7. Electric field due to solid sphere
(i) conducting $\equiv$ spherical shell

$$E_{out} = \frac{kQ}{r^2} \qquad r \geq R$$

$$E_{In} = \text{zero} \qquad r < R$$

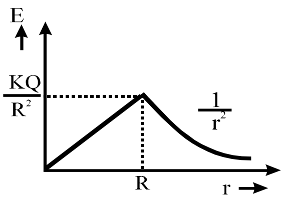




(ii) non-conducting, uniformly charged solid sphere

$$E_O = \frac{kQ}{r^2} \qquad r \geq R$$

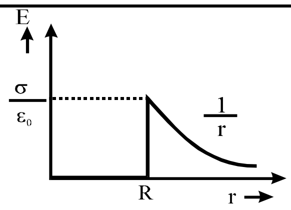
$$E_{In} = \frac{kQr}{R^3} = \frac{\rho r}{3\varepsilon_0} \qquad r < R$$

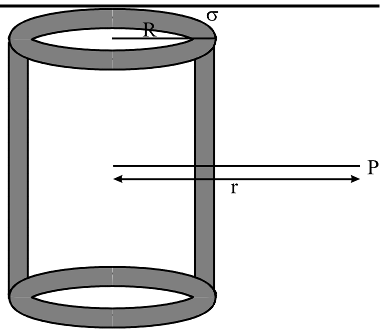


8. Electric field due to infinite cylendrical shell

$$E = \frac{\sigma R}{\varepsilon_0 r} \qquad r \geq R$$

$$E_0 = \text{zero} \qquad r < R$$

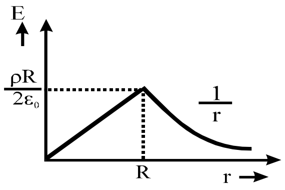




9. Electric field due to infinite unformly charged cylinder (non-conducting)

$$E_0 = \frac{\rho R^2}{2\varepsilon_0 r} \qquad r \geq R$$

$$E_I = \frac{\rho r}{2\varepsilon_0} \qquad r < R$$



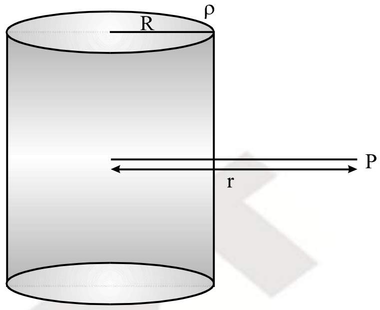
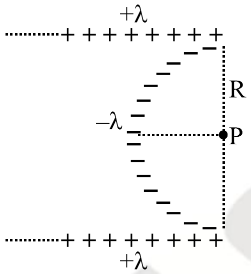


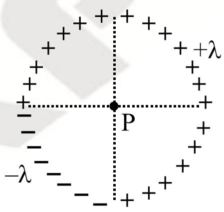
Illustration : 8

In the following diagrams, electric charges are distributed uniformly with l Cm^{-1}. Determine the field intensity $\vec{E}$ at the point P in each case.

(i)



(ii)



Solution :

(i) λ/R and due to semi circular ring field of magnitude 2Kλ/R is towards left hence

$E_{net} = E_{wire} + E_{ring} = 0$

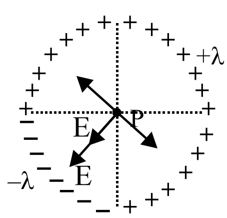
(ii) Field due to arc in second quadrant and forth quadrant cancel out. The net electric field is due to ring in first and third quadrant

$$E_{net} = 2E$$

$$E = \frac{2K\lambda}{R}\sin\frac{\theta}{2}$$

$$\theta = \pi/2$$

$$E_{net} = \frac{2\sqrt{2}\ K\lambda}{R}$$



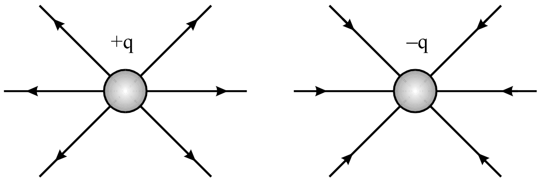
ELECTRIC LINES OF FORCE

Definition

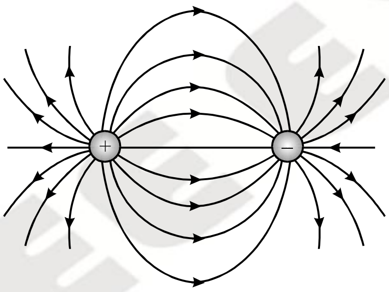
Electric lines of the forces are the imaginary path on which a unit positive test charge moves or a electric line of force is an imaginary curve, a tangent to which at a point gives the direction of intensity of the electric field at that point. The idea of lines of force was introduced by Michael Faraday.

Properties of Electric lines of force

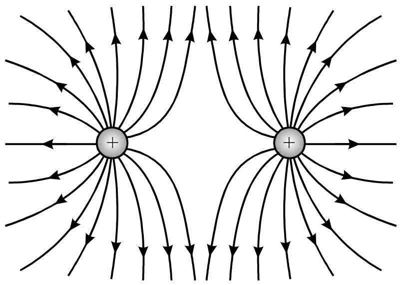
1. They usually originate from a positive charge and terminate to a negative charge



2. The number of lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge. This means, for example that if 100 lines are drawn leaving a + 4 μC charge then 75 lines would have to end on a –3 μC.
3. Lines of force never cross each other because if they do so, then at the point of intersection the net intensity will have two different directions which is not possible.
4. They never form a closed loop when produced by a static charge because if they do so, then the work done round a closed path will not be zero which contradicts its conservative nature.
5. They contract longitudinally (lengthwise) producing attraction between opposite charges and expand laterally producing repulsion between similar charges.

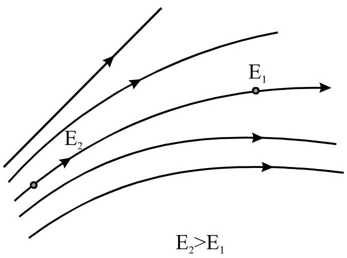


Attraction

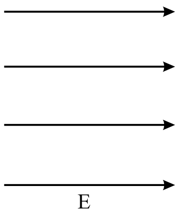


Repulsion

6. The number of lines per unit area crossing a surface at right angle to the field direction at every point is proportional to the electric intensity and hence the lines of force are closely spaced where the intensity is large and are widely separated where the intensity is small.
If the lines of force are parallel and equally spaced then they represent a uniform electric field.



$E_2 > E_1$
[Non-uniform electric field]



[uniform electric field]

7. Electric field lines also give us an indication of the equipotential surface (surface which has the same potential)

8. Electric field lines are always directed from higher potential to lower potential.

9. In a region where there is no electric field, lines are absent. This is why inside a conductor (where electric field is zero) there, cannot be any electric field line.

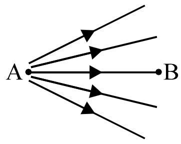
10. Electric lines of force ends or starts normally from the surface of a conductor.

Illustration : 9

Electric field lines are shown in figure. Then which is correct

(A) $E_A > E_B$ (B) $E_B > E_A$

(C) $E_A = E_B$ (D) none of these



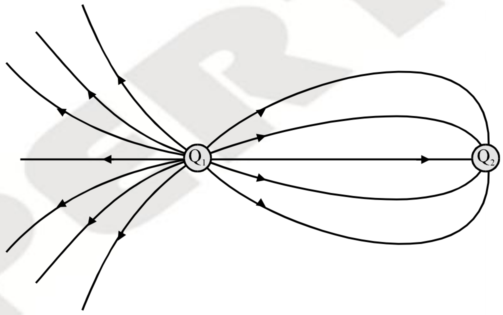
Solution :

(A)

The density of the electric lines of forces at A is more then density at point B so $E_A > E_B$.

Illustration : 10

If $Q_1 = 12\ \mu C$ find the value of Q_2 in the given figure.



Solution :

$Q_1/Q_2 = -12/5$

Therefore $Q_2 = -5\ \mu C$

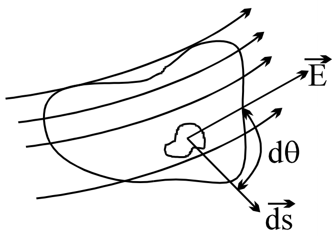
5. GAUSS'S LAW

Electrical Flux (scalar quantity)

If the lines of force pass through a surface then the surface is said to have flux linked with it. Mathematically it can be formulated as follows :

The flux linked with small area element on the surface of the body :

$$d\phi = \vec{E}.d\vec{s}$$



Where $d\vec{s}$ is the area vector of the small area element. The area vector of a closed surface is always in the direction of outward drawn normal.

The total flux linked with whole of the body,

$$\phi = \int \vec{E}.d\vec{s} \quad \text{total electrical flux}$$

$$\phi = \oint \vec{E}.d\vec{s} \quad \text{total flux linked with closed surface}$$

where $\oint$ is referred to closed integral done for a closed surface.

Gauss's Theorem

This law gives a relation between the electric flux through any closed surface (called Gaussian surface) and the charge enclosed by the surface. It states that the total electrical flux ¤ through any closed surface is equal to 1/£$_0$ times the net charge enclosed by the surface i.e. total flux–

$$¤:= \oint \vec{E}.d\vec{s} = \frac{q_{in}}{\varepsilon_0}$$

[q_{in} Ĝ:Charge enclosed within the Gaussian surface]

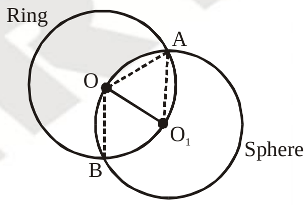
Note : *Gauss theorem is a fundamental theorem and it is applicable only for the closed surfaces, the closed surfaces may be real or hypothetical and of any shape or size.*
The above formula is always applicable whether the electric field be uniform or variable.

Applications of Gauss's Law for the Calculation of Electric field

Gauss's law is useful when there is symmetry in the charge distribution, as in the case of uniformly charged sphere, long cylinders, and flat sheets. In such cases, it is possible to find a simple Gausssian surface over which the surface integral can be easily evaluated.
These are steps to apply the Gauss's law for finding electric field.
(i) Use the symmetry of the charge distribution to determine the pattern of the lines
(ii) Choose a Gaussian surface for which $\vec{E}$ is either parallel to $d\vec{S}$ or perpendicular to $d\vec{S}$
(iii) If $\vec{E}$ is parallel to $d\vec{S}$, then the magnitude of $\vec{E}$ should be constant over this part of the surface. The integral then reduces to a sum over area elements.
(iv) Find enclosed charged in the taken Gaussian surface
(v) Apply gauss law to find electric field.

Illustration : 11

A charge Q is distributed uniformly on a ring of radius r. A sphere of equal radius r is constructed. With its centre at the periphery of the ring (fig.) Find the flux of the electric field through the surface of the sphere.

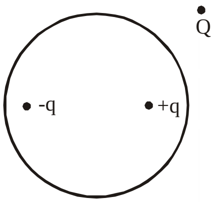


Solution:

From the geometry of the fig. $OA = OO_1$ and $O_1A = O_1O$. Thus, OAO_1 is an equiliateral triangle. Hence $\angle AOO_1 = 60^0$ OR $\angle AOB = 120^0$
The arc $AO_1B$of the ring subtends an angle 120^0 at the centre O. Thus, one third of the ring is inside the sphere. The charge enclosed by the sphere $= \frac{Q}{3}$. Thus flux our sphere $\frac{Q}{3\in_0}$

Illustration : 12

Three point charges –q, +q and Q are placed in a region as shown. P is a point on imaginary Gaussian sphere enclosing –q and +q and Q is outside.



Solution :

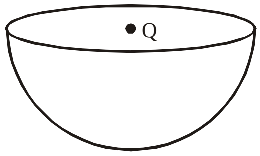
(a) What is the flux over sphere

$$\phi = \frac{\Sigma q_{in}}{\in_0} = \frac{+q-q}{\in_0} = \frac{0}{\in_0} = 0$$

(b) Which of the charges contribute to the intensity of the field at P?
field at point P is due to all the three charges.

Illustration : 13

A charge Q is placed at the centre of an imaginary hemispherical surface. Using symmetry arguments and the Gauss's law, find the flux of the electric field due to this charge through the surface of the hemisphere (figure).



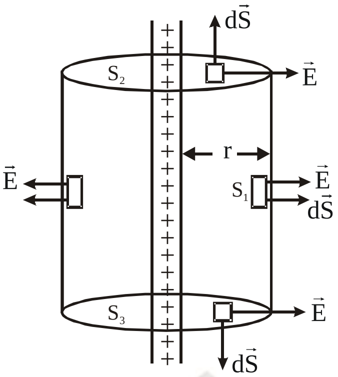
Solution :

Let us imagine another identical hemispherical surface over given one.
Both being symmetric with respect to Q, hence flux will be same through both the hemisphere $(\phi_1 = \phi_2)$.

$$\phi_1 + \phi_2 = \frac{Q}{\varepsilon_0}$$

$$\therefore \qquad \phi_1 = \phi_2 = \frac{Q}{2\varepsilon_0}$$

ELECTRIC FIELD DUE TO LONG LINE OF CHARGE

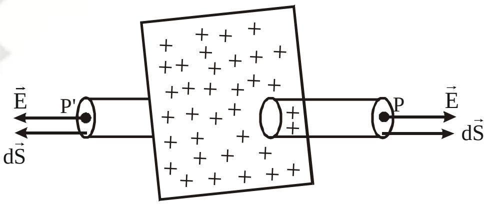


Consider an infinite line which has a linear charge density λ. Using Gauss's law, let us find the electric field at a distance 'r' from the line charge.
The cylindrical symmetry tells us that the field strength will be the same at all points at a fixed distance r from the line. Thus, the field lines are directed radially outwards, perpendicular to the line charge.
The appropriate choice of Gaussian surface is a cylinder of radius r and length L. On the flat end faces, S_2 and S_3, $\vec{E}$ is perpendicular $d\vec{S}$, which means flux is zero on them. On the curved surface S_1, $\vec{E}$ is parallel $d\vec{S}$, so that $\vec{E},\ d\vec{S} = EdS$.
The charge enclosed by the cylinder is $Q = \lambda L$.
Applying Gauss's law to the curved surface, we have

$$E\oint dS = E(2\pi rL) = \frac{\lambda L}{\varepsilon_0}$$

or $$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Field Due to an Infinite Charged nonconducting Plane Sheet



Let us consider a thin non-conducting charged plane sheet, infinite in extent, and having a surface charge density (charge per unit area) $\sigma\ C/m^2$. Let P be a point, distance r from the sheet, at which the electric intensity is required.
Let us choose a point P′ symmetrical with P, on the other side of the sheet. Let us now draw a Gaussian cylinder cutting through the sheet, with its plane ends parallel to the sheet and passing through P and P′. Let A be the area of each end.
By symmetry, the electric intensity at all points on either side near the sheet will be perpendicular to the sheet, directed outward (if the sheet is positively charged). Thus $\vec{E}$ is perpendicular to the plane ends of the cylinder and parallel to the curved surface. Also its magnitude will be the same at P and P'. Therefore, the flux through the two plane ends is

$$\phi_E = \int \vec{E}.d\vec{S} + \int \vec{E}.d\vec{S} = \int E\,dS + \int E\,dS$$

$$= EA + EA = 2EA$$

The flux through the curved surface of the Gaussian cylinder is zero because $\vec{E}$ and $d\vec{S}$ are at right angles everywhere on the curved surfaces.
Hence, the total flux through the Gaussian cylinder is

$$\phi_E = 2EA$$

The charge enclosed by the Gaussian surface $q = \sigma A$
Applying Gauss's law, we have

$$2EA = \frac{\sigma A}{\varepsilon_0}$$

$$\Rightarrow \qquad E = \frac{\sigma}{2\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0} \qquad \text{(as there is no field inside the conductor)}$$

Field Due to an Infinite Charged Conducting Plane Sheet

In this case charge on sheet is uniformly distributed on its both sides.

$\therefore \quad Q_{enc} = s \times 2A$

$\therefore \quad \phi_E = 2EA = \dfrac{\sigma \times 2A}{\varepsilon_0}$

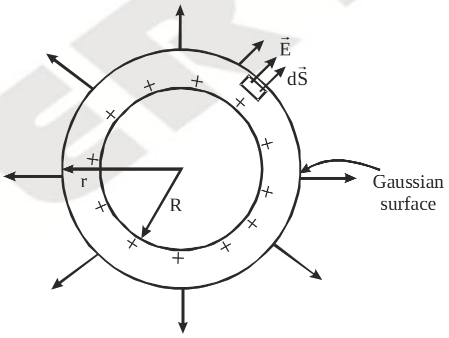
$\therefore \quad E = \dfrac{\sigma}{\varepsilon_0}$

Electric Field Due to an Uniformly Charged Spherical Shell

Using Gauss's law, let us find the intensity of the electric field due to a uniformly charged spherical shell or a solid conducting sphere

Case I: At an external point.

In case of an isolated charged spherical conductor any excess charge on it is distributed uniformly over its outer surface same as that of charged spherical shell or hollow sphere. Hence field lines must point radially outward. Also, the field strength will have the same value at all points on any imaginary spherical surface concentric with the imaginary spherical surface concentric with the charged conducting sphere or the shell. This symmetry leads us to choose the Gaussian surface to be a sphere of radius $r > R$. Any arbitrary element of area $d\vec{S}$ is parallel to the local $\vec{E}$, so $\vec{E}.d\vec{S} = EdS$ at all points on the surface.



According to Gauss's law

$$\oint \vec{E}.d\vec{S} = \oint EdS = E\oint dS = E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

Therefore,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

For points outside the charged conducting sphere or the charged spherical shell, the field is same as that of a point charge at the centre.

Case II : At an Internal Point (r < R)

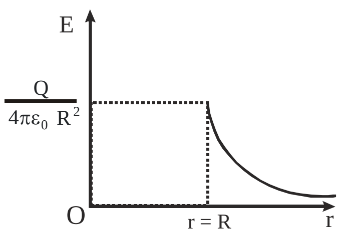
The field still has the same symmetry and so we again pick a spherical Gaussian surface, but now with radius r less than R. Since the charge enclosed is zero, from Gauss's law we have

$$E(4\pi r^2) = 0$$

$\therefore \quad E = 0$

Thus, we conclude that E = 0 at all points inside a uniformly charged conducting sphere or the charged spherical shell.

Variation of E with the distance from the centre (r)

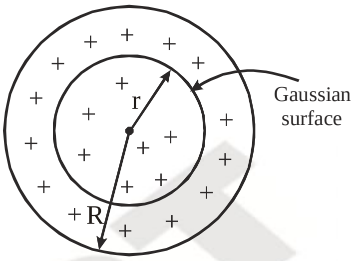


Electric Field Due to an Uniformly Charged Sphere

A Non-conducting uniformly charged sphere of radius R has a total charge Q uniformly distributed throughout its volume. Using the Gauss's Law, Let us find the field

Case I: at an internal point (r < R)

Positive charge Q is uniformly distributed throughout the volume of sphere of radius R. For finding the electric field at a distance (r < R) from the centre, we choose a spherical Gaussian surface of radius r, concentric with the charge distribution. From symmetry the magnitude E of the electric field has the same value at every point on the Gaussian surface, and the direction of $\vec{E}$ is radial at every point on the surface.



So, applying Gauss's law

$$\oint \vec{E}.d\vec{s} = E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

Here, $Q' = \left(\frac{4}{3}\pi r^3\right)\rho = \left(\frac{4}{3}\pi r^3\right) \times \frac{Q}{4\pi R^3} = \frac{Qr^3}{R^3}$

Where ρ is volume charge density

Therefore

$$E(4\pi r^2) = \frac{Qr^3}{R^3\varepsilon_0}$$

or $$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}r$$

The field increases linearly with distance form the centre

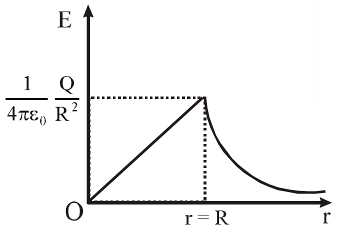
Case II : At an external point $(r > R)$

To find the electric field outside the charged sphere, we use a spherical Gaussian surface of radius (r >R). This surface encloses the entire charged sphere. So from Gauss's law, we have

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

or, $$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

The field at points outside the sphere is a same as that of a point charge at the centre. Variation of E with the distance from the centre (r)



CONDUCTOR

Conductors (such as metals) possess free electrons. If a resultant electric field exists in the conductor these free charges will experience a force which will set a current flow. When no current flows, the resultant force and the electric field must be zero. Thus, under electrostatic conditions the value of $\vec{E}$ at all points within a conductor is zero. This idea, together with the Gauss' law can be used to prove several interesting facts regarding a conductor.

Properties of conductor

(i) Excess charge on a conductor resides on its outer surface.

(ii) Electric field at any point close to the charged conductor $\dfrac{\sigma}{\epsilon_0}$.

(iii) Electric field and field lines are normal to the surface of a conductor.

(iv) The potential of a charged conductor throughout its volume is same

6. ELECTRIC POTENTIAL

Electric potential at a point in electric field is defined to be equal to the minimum work done by an external agent in moving a unit positive charge from infinity or a referance point to that point against the electrical force of the field.

Electric potential at a point in electric field is numerically equal but opposite in sign to the work performed by electrical force to bring unit positive charge from infinity to that point.

If W is the work done by external agent in bringing a positive test charge q_0 from infinity to a point then the potential V at that point.

$$V = \frac{W_{ext}}{q_0} \qquad V = -\frac{W_E\text{(work done by electric field)}}{q_0}$$

Unit of potential is joule/coulomb or volt. (S.I. unit)

Electric potential difference

Potential deference between two points is equal to the minimum work one in moving a unit positive test charge from one point to the other.

$$V_B - V_A = \frac{W_{AB}}{q_0} \Rightarrow W_{AB} = q_0\,(V_B - V_A)$$

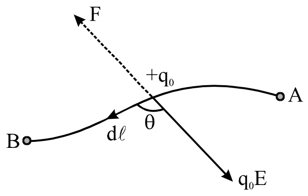
W_E is the work done by the electric field then ;

$$-(V_B - V_A) = \frac{W_E}{q_0}$$
$$W_E = -q_0\,(V_B - V_A)$$
$$W_{ext.(min.)} = -W_E$$

Relation between electric field and electric potential

Consider a charge q_0 which is moved in an electric field.

The external agent would have applied an equal and opposite force on the charge to move it without acceleration.



$$\vec{F} = -q_0\vec{E}$$

If due to the force $\vec{F}$, charge moves a small distance $d\vec{\ell}$ along the path, then minimum work done by the external agent is :

$$dW = \vec{F}\cdot d\vec{\ell} = -q_0\vec{E}\cdot d\vec{\ell}$$

Therefore total work done in moving a charge from A to B ;

$$W_{AB} = -q_0 \int_A^B \vec{E}.d\vec{\ell}$$

$$\frac{W_{AB}}{q_0} = -\int_A^B \vec{E}.d\vec{\ell}$$

$$V_B - V_A = -\int_A^B \vec{E}.d\vec{\ell}$$

Electric Potentail Due to point charge

To calculate the potential at point P, due to the point charge, calculate the potential difference between the point P and infinity which automatically will be the potential of the point, let any point lies between P and infinity at a distance x from the given point charge.

Potential difference across dx

$$dV = -\vec{E}\,.\,d\vec{x}$$

$$dV = -Edx\cos 0°$$

$$dV = -Edx$$

$$dV = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{x^2}\,dx$$

$$\int_{V_p}^{0} dV = -\frac{1}{4\pi\varepsilon_0}\cdot q\int_r^{\infty}\frac{dx}{x^2}$$

$$V_P = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}$$

Superposition Theorem

The potential at any point due to a group of point charges is the algebraic sum of the potentials contributed at the same point by all the individual point charges.

$$V = V_1 + V_2 + V_3 + ...$$

The electric potential due to a continuous charge distribution is the sum of potentials of all the infinitesimal charge elements in which the distribution may be divided

$$V = \int dV$$

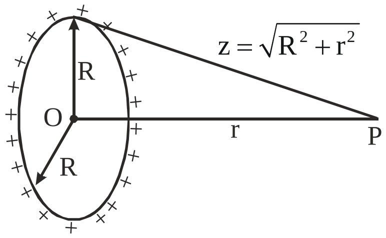
$$V = \int \frac{dq}{4\pi\varepsilon_0 r}$$

ELECTRIC POTENTIAL DUE TO VARIOUS CHARGE DISTRIBUTION

Electric Potential due to a Charged Ring

A charge Q is uniformly distributed over the circumference of a ring. Let us calculate the electric potential at an axial point at a distance r from the centre of the ring.

The electric potential at P due to the charge element dq of the ring is given by



$$dV=\frac{1}{4\pi\varepsilon_0}\frac{dq}{Z}=\frac{1}{4\pi\varepsilon_0}\frac{dq}{(R^2+r^2)^{1/2}}$$

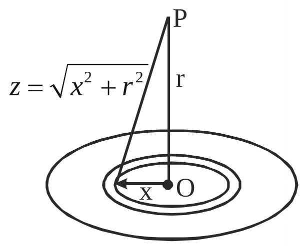
Hence, the electric potential at P due to the uniformly charged ring is given by

$$V=\int\frac{1}{4\pi\varepsilon_0}\frac{dq}{(R^2+r^2)^{1/2}}=\frac{1}{4\pi\varepsilon_0}\frac{1}{(R^2+r^2)^{1/2}}\int dq$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{(R^2+r^2)}}.$$

Electric Potential Due to a Charged Disc at a Point on the Axis

A non-conducting disc of radius 'R' has a uniform surface charge density $\sigma\ C/m^2$. Let us calcuate the potential at a point on the axis of the disc at a distance 'r' from its centre. The symemtry of the disc tells us that the appropriate choice of element is a ring of radius x and thickness dx. All points on this ring are at the same distance $Z=\sqrt{x^2+r^2}$, from the point P. The charge on the ring is $dq=\sigma dA=\sigma(2\pi x\ dx)$ and so the potential due to the ring is



$$dV=\frac{1}{4\pi\varepsilon_0}\frac{dq}{Z}=\frac{1}{4\pi\varepsilon_0}\frac{\sigma(2\pi\ x\ dx)}{\sqrt{x^2+r^2}}$$

Since potential is scalar

The potential due to the whole disc is given by

$$V=\frac{\sigma}{2\varepsilon_0}\int_0^R\frac{x\ dx}{\sqrt{x^2+r^2}}$$

$$=\frac{\sigma}{2\varepsilon_0}\left[(x^2+r^2)^{1/2}\right]_0^R$$

$$=\frac{\sigma}{2\varepsilon_0}\left[(R^2+r^2)^{1/2}-r\right]$$

Let us see this expression at large distance when $r>>R$.

$$V=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}\text{, where } Q=\pi r^2\sigma \text{ is the total charge on the disc.}$$

Thus, we conclude that at large distance, the potential due to the disc is the same as that of a point charge Q.

Electric Potential Due to a Shell

A shell of radius R has a charge Q uniformly distributed over its surface. Let us calcuate the potential at a point

(a) outside the shell; $(r > R)$

(b) inside the shell $(r < R)$.

Solution:

(a) At points outside a uniform spherical distribution, the electric field is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

since $\vec{E}$ is radial, $\vec{E}.d\vec{r} = Edr$

since $V(\infty) = 0$, we have

$$V(\alpha) - V(r) = -\int \vec{E}.d\vec{r}$$

$$0 - V = -\int_r^{\alpha} \frac{Q}{4\pi\varepsilon_0 r^2}dr$$

$$\Rightarrow \qquad V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

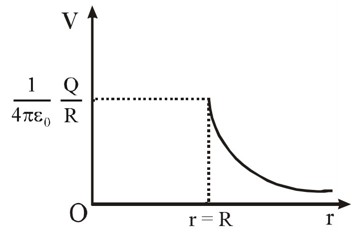
We see that the potential due to a uniformly charged shell is the same as that due to a point charge Q at the centre of the shell.

(b) At an internal Point

At points inside the shell, $E = 0$. So, the work done in bringing a unit positive charge from a point on the surface to any point inside the shell is zero. Thus, the potential has a fixed value at all points within the spherical shell and is equal to the potential at the surface.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

Variation of electric potential with the distance from the centre (r)



All the above results hold for a "conducting sphere also whose charge lies entirely on the outer surface.

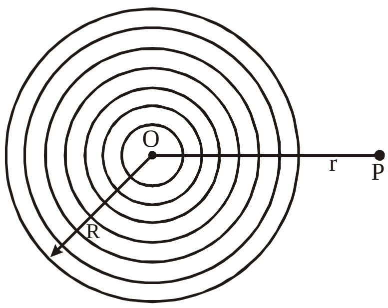
Electric Potential due to a Non-conducting Charged Sphere

A charge Q is uniformly distributed throughout a non-conducting spherical volume of radius R. Let us find expressions for the potential at an (a) external point (r > R); (b) internal point (r < R) where r is the distance of the point from the centre of the sphere.

(a) At an external point

Let O be the centre of a non-conducting sphere of radius R, have a charge Q distributed uniformly over its entire volume.

Let us divide the sphere into a large number of thin concentric shells carrying charges q_1, q_2, q_3, ... etc. The potential at the point P due to the shell of charge q_1 is $\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$.



Now, potential is a scalar quantity. Therefore the potentials V due to the whole sphere is equal to the sum of the potentials due to all the shells.

$$\therefore \qquad V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r} + ...$$

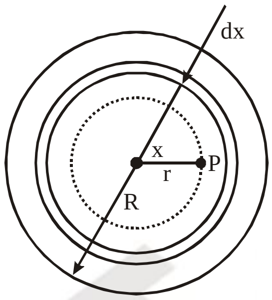
$$= \frac{1}{4\pi\varepsilon_0 r}[q_1 + q_2 + q_3 + ...]$$

But $q_1 + q_2 + q_3 + ... = Q$, the charge on the sphere

$$\therefore \qquad V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

(b) Potential at an internal point

Suppose the point P lies inside the sphere at a distance r from the centre O, if we draw a concentric sphere through the point P, the point P will be external for the solid sphere of radius r, and internal for the outer spherical shell of internal radius r and external radius R.



The charge on the inner solid spheres $\frac{4}{3}\pi r^3\rho$. Therefore the potential V_1 at P due to this sphere is given by

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{4/3\pi r^3\rho}{r} = \frac{r^2\rho}{3\varepsilon_0}$$

Let us now find the potential at P due to the outer spherical shell. Let us divide this shell into a number of thin concentric shells and consider one such shell of radius x and infinitesimally small thickness dx.

The volume of this shell = surface area × thickness = $4\pi x^2\ dx$. The charge on this shell, dq = $4\pi x^2\ dx\rho$. The potential at P due to this shell

$$dV_2 = \frac{1}{4\pi\varepsilon_0}\frac{dq}{x} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi x^2(dx)\rho}{x}$$

$$= \frac{\rho x\ dx}{\varepsilon_0}$$

The potential V_2 at P due to the whole shell of internal radius r and axternal radius R is given by

$$V_2 = \int_r^R \frac{\rho}{\varepsilon_0} x\,dx = \frac{\rho}{\varepsilon_0}\left|\frac{x^2}{2}\right|_r^R$$

$$= \frac{\rho(R^2 - r^2)}{2\varepsilon_0}$$

Since the potential is a scalar quantity, the total potential V at P is given by

$$V = V_1 + V_2$$

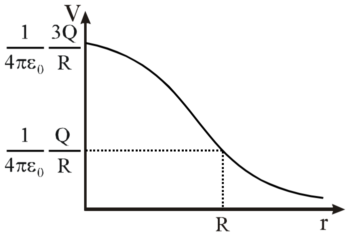
$$= \frac{r^2\rho}{3\varepsilon_0} + \frac{\rho(R^2 - r^2)}{2\varepsilon_0}$$

$$= \frac{\rho(3R^2 - r^2)}{6\varepsilon_0}$$

But $$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$\therefore \qquad V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R^3}\left[3R^2 - r^2\right]$$

Variation of potential with distance from centre.



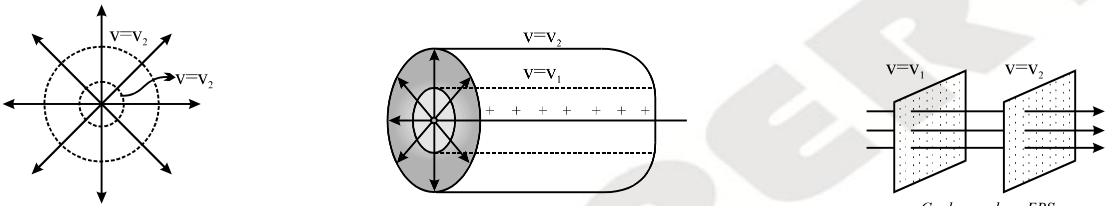
Equipotential surface

If all the points of the surface are at the same potential, the surface is called an equipotential surface. Work done in the moving a charge between any two points on an equipotential surface zero.

Equipotential surfaces can never cross each other because there will be two normals at the point of intersection giving two different directions of electric field which is absurd.

Equipotential surface are always perpendicular to lines of force.

For a point charge the equipotential surface is spherical. For a line charge equipotential surface is cylindrical and for uniform field the equipotential surface is planar.



Concentric spherical EPS for a point charge

Co-axial cylindrical EPS for a line charge

Coplanar plane EPS for uniform field

Equipotential surfaces are closely spaced where electric field intensity is large and widely spaced where electric field intensity is small.

Properties of Equipotential surface

(i) Potential difference between two points in an equipotential surface is zero.

(ii) It a test charge q_0 is moved from one point to the other on such a surface, the electric potential energy q_0V remains constant.

(iii) No work is done by the electric force when the test charge is moved along this surface.

(iv) Two equipotential surfaces can never intersect each other because otherwise the point of intersection will have two potentials which is of course not acceptable.

(v) Field lines and equipotential surfaces are always mutually perpendicular.

Relation between field and potential

In rectangular components, the electric field is

$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}\,;$$

and an infinitesimal displacement is $d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$

Thus,

$$\int dV = -\int \vec{E}.d\vec{r}$$

$$= -[E_x\ dx + E_y\ dy + E_z\ dz]$$

for a displacement in the x-direction,

$$dy = dz = 0 \text{ and so}$$

$$dV = -E_x\ dx. \text{ Therefore,}$$

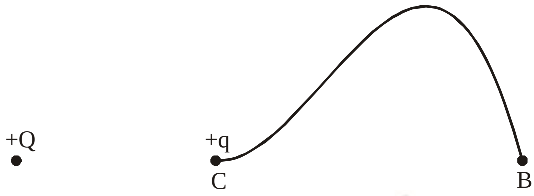
$$E_x = -\left(\frac{dV}{dx}\right)_{y,\ z\,constant}$$

A derivative in which all variables except one are held constant is called partial derivative and is written with ∂ instead of d. The electric field is, therefore,

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

ELECTROSTATICS POTENTIAL ENERGY

In the figure, if a charge +q is moved from B to C in the electric field of charge +Q, the work will have to be done by some outside agent in pushing the charge +q against the force of field of +Q.



This situation is very similar to that of a mass moved in gravitational field of earth away from it. Work done against the gravitational pull of earth is stored in Gravitational potential energy and can be recovered back. Similarly in electric field, work done against an electric field is stored in the form of electric potential energy & can be recovered back. If the charge +q is taken back from C to B, the electric force will try to accelerate the charge and hence to recover the potential energy stored in the form of kinetic energy.

As the work done against an electric field can be recovered back, electrostatic forces and fields fall under the category of conservative forces and fields. Another property of these fields is that the work done is independent of path taken from the one point to the another.

Potential Energy of a System of Two Point Charges

The potential energy possessed by a system of two-point charges q_1 and q_2 separated by a distance r is the work required to bring them to this arrangement from infinity. This electrostatic potential energy is given by

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

Note : While writting potential or potential energy charges must be multiplied with their signs.

Electric Potential Energy of a System of Point Charges

The electric potential energy of such a system is the work done in assembling this system starting from infinite separation between any two-point charges.

For a system of point charges $q_1, q_2, ...q_n$, the potential energy is

$$U = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{q_i q_j}{4\pi\varepsilon_0 r_{ij}}(i \neq j)$$

It simply means that we have to consider all the pairs that are possible.

Important points regarding Electrostatic potential energy

(i) Work done required by an external agency to move a charge q from A to B in an electric field with constant speed

$$W_{A\to B} = q[V_B - V_A]$$

(ii) When a charge q is let free in an electric field, it loses potential energy and gains kinetic energy, if it goes from A to B, then loss in ponetial energy = gain in kinetic energy

or $$q(V_B - V_A) = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

Illustration : 14

The electric field in a region is given by $\vec{E} = (A/x)^3 \hat{i}$. Write a suitable SI unit for A. Write an expression for the potential in the region assuming the potential at infinity to be zero.

Solution:

The SI unit of electric field is N/C or V/m. Thus,

The unit of A is $\dfrac{N - m^3}{C}$ or V-m^2.

$$V(x, y, z) = -\int_{\infty}^{(x,y,z)} \vec{E}.d\vec{r}$$

$$= -\int_{\infty}^{(x, y, z)} \frac{A\,dx}{x^3} = \frac{A}{2x^2}.$$

Illustration : 15

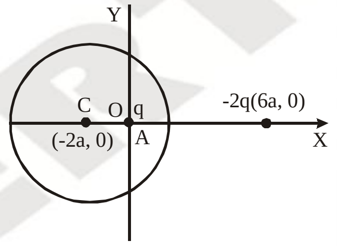
Two points charge q and –2q are placed at a distance 6a apart. Find the locus of the point in the plane of charges where the field potential is zero.

Solution:

Let us take the charge on X-axis;

q at A (0, 0) and –2q at B(6a, 0)

Potential at a point P(x, y) is



$$V = \frac{q}{4\pi\varepsilon_0\sqrt{x^2 + y^2}} + \frac{-2q}{4\pi\varepsilon_0\sqrt{(x-6a)^2 + y^2}}$$

$$V = 0$$

$$\Rightarrow \qquad \frac{q^2}{x^2 + y^2} = \frac{4q^2}{(x-6a)^2 + y^2}$$

$\Rightarrow$ the locus is $(x - 6a)^2 = 4x^2 + 3y^2$.

$$3x^2 + 3y^2 + 2(6a)x = 36a^2$$

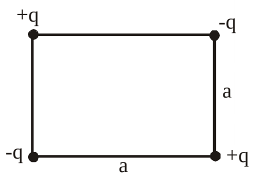
$$\Rightarrow \qquad x^2 + y^2 + 4ax = 12a^2$$

$$(x + 2a)^2 + y^2 = 16a^2$$

$\therefore$ Locus is a circle with centre (-2a, 0) and radius 4a.

Illustration : 16

What is work done by the electrostatic field when we put the four charges together, as shown in the figure. Each side of the square has a length a. Initially charges were at infinity.



Solution:

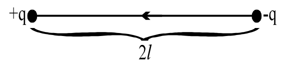
$U_i = 0$ [Where charges are separated by infinite distance]

$$U_f = \frac{1}{4\pi\varepsilon_0}\left(\frac{-4q^2}{a} + \frac{q^2}{\sqrt{2}a} + \frac{(-q)^2}{\sqrt{2}a}\right) \qquad \text{[for 6 pairs of charges]}$$

Work done by field $= -\Delta U = U_i - U_f = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{a}\left(4 - \dfrac{\sqrt{2}}{a}\right)$

ELECTRIC DIPOLE

Two equal and opposite point charges placed a short distance apart, form an electric dipole.



The product of the magnitude of either charge and the distance between the charges is called the electric dipole moment. It is a vector quantity whose direction is along the axis of dipole pointing from negative towards the positive charge.
dipole moment (p) = q. (2*l*)
The S.I. unit of dipole moment is Coulomb-Metre
The practical unit of dipole moment is Debye
I debye (D) = 3.3×10^{-30} Coulomb-meter

Dipole Moment of a System of Discrete Charges

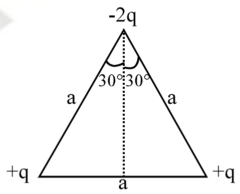
Since dipole moment is a vector quantity, hence in case of two or more than two dipoles, the resultant dipole moment will be the vector sum of the dipole moments of individual dipoles.

$$\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots\ldots\ldots\ldots$$

. . . . $_2$O and CO_2.H_2O has some dipole moment but CO_2 being linear has zero dipole moment

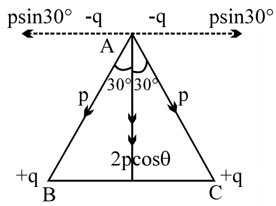
Illustration : 17

Three points charges +q, –2q, +q are arranged on the vertices of an equilateral triangle as shown in the figure. Find the dipole moment of the system.



Solution:

Arrangement of the charges is equivalent to two dipoles having dipole moment p each as shown above.

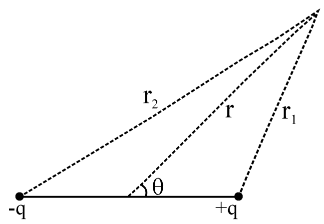


Net dipole moment

$$P_{net} = p\cos 30° + p\cos 30°$$
$$P_{net} = 2p\cos 30°$$
$$= p\sqrt{3} = qa\sqrt{3}$$

Electric Potential Due to Dipole

$$V_{(+)} = \frac{Kq}{r_1} \approx \frac{Kq}{r - d\cos\theta}, \quad [r_1 \approx r - d\cos\theta]$$



$$V_{(-)} = \frac{Kq}{r_2} \approx \frac{-Kq}{r + d\cos\theta}, \quad [r_2 \approx r + d\cos\theta]$$

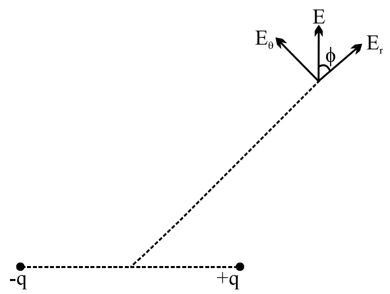
$$V = V_1 + V_2 = Kq\left[\frac{1}{r - d\cos\theta} - \frac{1}{r + d\cos\theta}\right]$$

$$= Kq\left[\frac{r + d\cos\theta - r + d\cos\theta}{r^2 - d^2\cos^2\theta}\right]$$

$$V = \frac{Kp\cos\theta}{r^2\left[1 - \frac{d^2\cos^2\theta}{r^2}\right]} \approx \frac{Kp\cos\theta}{r^2}$$

Electric Field due to Dipole

There are two component of field one in radial and one in perpendicular direction



$$E_r = -\frac{\partial V}{\partial r}$$

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta}$$

$$E_r = \frac{2Kp\cos\theta}{r^3}$$

$$E_\theta = -\frac{1}{r}\left[\frac{-Kp\sin\theta}{r^2}\right] = \frac{Kp\sin\theta}{r^3}$$

$$E = \sqrt{(E_r)^2 + (E_\theta)^2}$$

$$E = \frac{Kp}{r^3}\sqrt{1+3\cos^2\theta}$$

Angle of Field from Radial Direction

$$\tan\phi = \frac{E_\theta}{E_r} = \frac{1}{2}\tan\theta$$

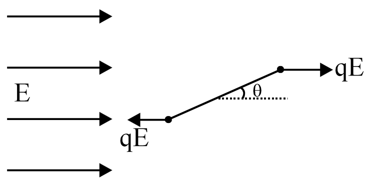
$$\tan\phi = \frac{1}{2}\tan\theta$$

Dipole in electric field

If a dipole is placed in a uniform electric field E,
Force on the dipole is zero.
Torque on the dipole is given as

$$\tau = pE\sin\theta \text{ or } \vec{\tau} = \vec{p} \times \vec{E}$$

where θ is the angle between $\vec{p}$ and $\vec{E}$.



The potential energy of the dipole is

$$U = -pE\cos\theta \text{ or } U = -\vec{p}.\vec{E} \text{ (taking } U = 0 \text{ at } \theta = 90^0)$$

When $\theta = 0^0$, the dipole moment $\vec{p}$ is in the direction of the field $\vec{E}$ and the dipole is in *stable equilibrium.* If it is slightly displaced, it performs oscillations.

When $\theta = 180^0$, the dipole moment $\vec{p}$ is opposite to the direction of the field $\vec{E}$ and the dipole is in *unstable equilibrium.*

Illustration : 18

For a given dipole at a point (away from the center of dipole) intensity of the electric field is E. Charges of the dipole are brought closer such that distance between point charges is half, and magnitude of charges are also halved. Find the intensity of the field now at the same point

Solution:

$$P_i = 2q\ell$$

$$P_f = 2\frac{q}{2}\frac{\ell}{2} = \frac{P_i}{4}$$

$r_f = r_i \quad \theta_f = \theta_i$

$$E = \frac{p}{4\pi \in_0 r^3}\sqrt{1+3\cos^2\theta}$$

$$\Rightarrow \qquad E_f = \frac{E_i}{4}.$$

$\therefore$ Final intensity of electric field is E/4.

Illustration : 19

A dipole of dipole moment P lies in a uniform electric field E such that dipole direction is along field. If dipole is rotated through 180º such that dipole direction becomes opposite to the field direction. Find the work done by the electrostatic field.

Solution:

$U_i = -\vec{P}.\vec{E} = -PE\cos O = -PE$

$U_f = -P.E.\cos(180^\circ) = PE$

work done by the field $= -\Delta U = U_i - U_f = -2PE$

CAPACITORS

Basics

Capacitor is an arrangement of two conductors carrying charges of equal magnitudes and opposite sign and separated by an insulating medium.

1. The net charge on the capacitor as a whole is zero. When we say that a capacitor has a charge Q, we mean that the positively charged conductor has charge +Q and negatively charged conductor has a charge –Q.
2. The positively charged conductor is at a higher potential than the negatively charged conductor. The potential difference V between the conductor is proportional to the charge magnitude Q and the ratio Q/V is known as *capacitance* C of the capacitor.

$$C = \frac{Q}{V}$$

Units of capacitance is farad (F). The capacitance is usually measured in microfarad (μF).

1 «F = 10^{-6} F

3. In a circuit, a capacitor is represented by the symbol : 
4. Capacitors work as a charge – storing or energy – storing devices. A capacitor stores energy in the form of electric field.

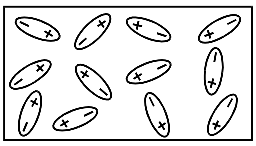
DIELECTRICS

In dielectric materials, effectively there are no free electrons.

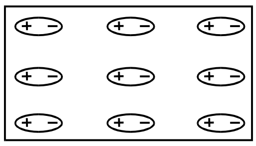
In monatomic materials the centre of the negative charge coincides with the centre of the positive charge whereas in polyatomic materials, on the other hand, the center of the negative charge may or may not coincide with the centre of the positive charge distribution. If it does not coincide, each molecule behaves as a dipole with dipole moment $\vec{p}$. Such materials are known as polar materials.

If such a material is placed in an electric field, the individual dipoles experience torque due to the field and they try to align along the field.

The charge appearing on the surface of a dielectric when placed in an electric field is called ***induced charge***. As the induced charge appears due to a shift in the electrons bound to the nuclei, this charge is also called bound charge.



Dielectric in absence of electric field



Dielectric in presence of electric field

Because of the induced charges, an extra electric field is produced inside the material. If $\vec{E}_0$ be the applied field due to external sources and $\vec{E}_P$ be the field due to polarization. The resultant field is $\vec{E} = \vec{E}_0 + \vec{E}_P$. For homogeneous and isotropic dielectrics, the direction of $\vec{E}_P$ is opposite to the direction of $\vec{E}_0$. The resultant field $\vec{E}$ is in the same direction as the applied field $\vec{E}_0$ but its magnitude is reduced. We can write

$$\vec{E} = \frac{\vec{E}_0}{K}$$

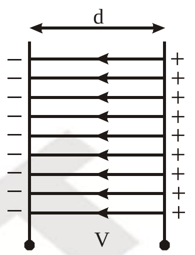
where K is a constant for given dielectric which has a value greater than one. This constant K is called the dielectric constant or relative permittivity of the dielectric.

Types of Capacitor

(a) Parallel plate capacitor
(b) Spherical capacitor
(c) Cylindrical capacitor

Parallel Plate Capacitor

The parallel plate capacitor consist of two metal plates placed parallel to each other and separated by a distance that is very small as compared to the dimension of the plates.



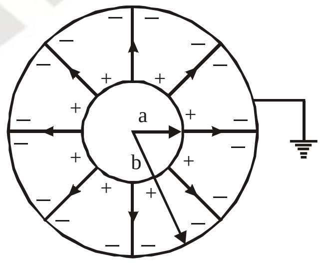
The capacitance is given by

$$C = \frac{K \in_0 A}{d}$$

Where K the dielectric constant of the medium between the plates, d is separation between the plates and A is the area of each plate.

Spherical Capacitor

A spherical capacitor consist of two concentric spheres of radii a and b as shown. The inner sphere is positively charged to potential V and outer sphere is at zero potential. The inner surface of the outer sphere has an equal negative charge.



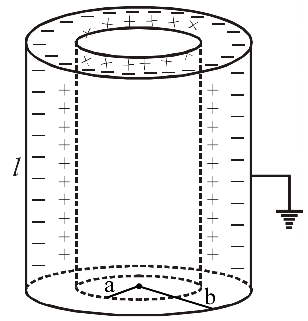
The capacitance is given by

$$C = \frac{4\pi K \in_0 ab}{b - a}$$

Where K is the dielectric constant and a and b are inner and outer radius of the sphere.

Cylindrical Capacitor

Cylindrical capacitor consist of two co-axial cylinders of radii a and b and length *l*. The electric fields exists in the region between the cylinders. Let K be the dielectric constant of the material between the cylinders. The capacitance is given by :



$$C = \frac{2\pi K \in_0 \ell}{\ln \frac{b}{a}}$$

Where K is the dielectric constant, a and b are inner and outer radius of the sphere and ℓ is the length of the cylinder.

CAPACITORS IN SERIES AND PARALLEL COMBINATION

Series Combinations

When two or more than two capacitors are connected in such a way that plates of capacitors are conneted with each other the combination is known as series. [Only first plate of first capactiors and second plate of last capacitor is connected to source.

When capacitors are connected in series, the magnitude of charge Q on each capacitor is same. The potential difference across C_1 and C_2 is different i.e., V_1 and V_2.

$$Q = C_1 V_1 = C_2 V_2$$

The total potential difference across combination is :

$$V = V_1 + V_2$$

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} \qquad \frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

The ratio Q/V is called as *the equivalent capacitance* C between point a and b.

The equivalent capacitance C is given by : $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

The potential difference across C_1 and C_2 is V_1 and V_2 respectively, given as follows :

$$V_1 = \frac{C_2 V}{C_1 + C_2} \;\&\; V_2 = \frac{C_1}{C_1 + C_2} V$$

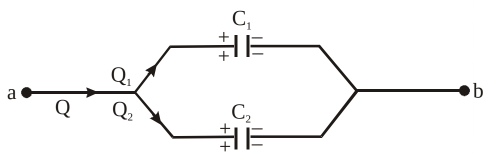
In case of more than two capacitors, the relation is :

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + ...$$

$$\Rightarrow \qquad \frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}.$$

Parallel Combinations

When two or more than two capacitors are connected in such a way that one plate of all capacitors are connected to one point and other plate of all capacitors are connected to other single point such a combination arragement of capacitors is known as perallel combination.



When capacitors are connected in parallel, the potential difference V across each is same and the charge on C_1, C_2 is different i.e., Q_1 and Q_2.

The total charge is Q given as :

$$Q = Q_1 + Q_2$$

$$Q = C_1 V + C_2 V$$

$$\frac{Q}{V} = C_1 + C_2$$

Equivalent capacitance between a and b is :

$$C = C_1 + C_2$$

The charges on capacitors is given as :

$$Q_1 = \frac{C_1}{C_1 + C_2} Q$$

$$Q_2 = \frac{C_2}{C_1 + C_2} Q$$

In case of more than two capacitors,

$$C = C_1 + C_2 + C_3 + C_4 + C_5 + ...$$

Illustration : 20

Two capacitors of capacitance $C_1 = 6\ \mu F$ and $C_2 = 3\ \mu F$ are connected in series across a cell of emf 18 V.

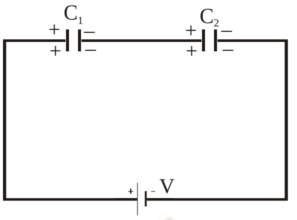
Calculate :

(a) the equivalent capacitance

(b) the potential difference across each capacitor

(c) the charge on each capacitor.

Solution :



(a) $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$

$\Rightarrow\ C = \dfrac{C_1C_2}{C_1 + C_2} = \dfrac{6\times3}{6+3} = 2\mu F.$

(b) $V_1 = \dfrac{C_2}{C_1 + C_2} V = \dfrac{3}{6+3}\times 18 = 6\,\text{volts}$

$V_2 = \dfrac{C_1}{C_1 + C_2} V = \dfrac{6}{6+3}\times 18 = 12\,\text{volts}$

Note that the smaller capacitor C_2 has a larger potential difference across it.

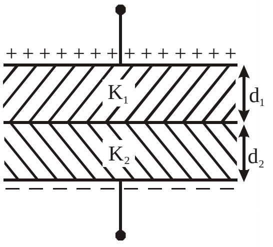
(c) $Q_1 = Q_2 = C_1V_1 = C_2V_2 = CV$

charge on each capacitor $= C_{eq}\,V$

$= 2\mu F \times 18\,\text{volts} = 36\mu C$

Illuatration : 21

Find the capacitance of the system in which dielectric is filled as shown in the figure. Each plates are of area A.



Solution:

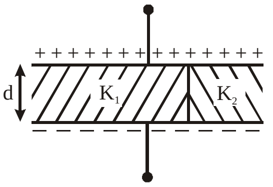
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C} = \frac{d_1}{k_1 \in_0 A} + \frac{d_2}{k_2 \in_0 A}$$

$$C = \frac{\in_0 A}{\frac{d_1}{k_1} + \frac{d_2}{k_2}}$$

Illuatration : 22

Find the capacitance of the system in which dielectric is filled as shown in the figure.



Solution:

$C = C_1 + C_2$

$$C = \frac{k_1 \in_0 A_1}{d} + \frac{k_2 \in_0 A_2}{d}$$

$$C = \frac{\in_0}{d}[k_1A_1 + k_2A_2].$$

Energy Stored by Capacitor

Consider a capacitor of capacity C connected to a source of which maintains a constant potential difference V across it. Let at any instant charge on capacitor is q which increased to q + dq in next instant.

Potential difference between the plates at the instant charge an capacitor is q, is

$$\Rightarrow \qquad V = \frac{q}{C}$$

When extra charge dq is transferred, the increment of work required,

$$dW = Vdq$$

$$\therefore \qquad \int dW = \int_O^Q Vdq \qquad \Rightarrow \qquad \int_O^Q \frac{q}{C}dq = \frac{Q^2}{2C}$$

$$\therefore \qquad \text{energy stored} = \frac{Q^2}{2C} = \frac{(CV)^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Work done by battery

W = Charge that flow through battery × battery EMF

Whenever there is a charging of capacitor.Work done by battery, in part is stored as electrostatic energy in between capacitor plate and remaining is dissipated as heat due to charge flow through connecting wires.

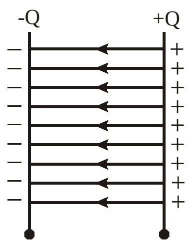
- In general, Heat produced is given by

  $\Rightarrow$ H = (Work done by battery - Energy stored in capacitor)

Force on any plate of parallel plate capacitor due to other

Intensity of the field at surface of any plate due to other is half of the field between plates

$$= \frac{E}{2} = \frac{\sigma}{2\epsilon_0}$$



Force for area dS on any plate dF $= \sigma ds \dfrac{E}{2} = \dfrac{\sigma^2 ds}{2\epsilon_0}$

Net force on any plate $F = \int dF = \dfrac{\sigma^2 A}{2\epsilon_0}$

Force per unit area $= \dfrac{\sigma^2}{2\epsilon_0}$

Illuatration : 23

An uncharged capacitor is connected to a battery. Show that half the energy supplied by the battery is lost as heat while charging the capacitor.

Solution:

Suppose the capacitance of the capacitor is C and the emf of the battery is V. The charge given to the capacitor is Q = CV. The work done by the battery is

$$W = QV$$

The battery supplies this energy. The energy stored in the capacitor is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

The remaining energy $QV - \dfrac{1}{2}QV = \dfrac{1}{2}QV$ is lost as heat.

Thus, half the energy supplied by the battery is lost as heat.

Illuatration : 24

A capacitor of capacitance C is charged by connecting it to a battery of emf ε. The capacitor is now disconnected and reconnected to the battery with the polarity reversed. Calculate the heat developed in the connecting wires.

Solution:

When the capacitor is connected to the battery, a charge $Q = C\varepsilon$ appears on one plate and $-Q$ on the other. When the polarity is reversed, a charge $-Q$ appears on the first plate and $+Q$ on the second. A charge 2Q, therefore, passes through the battery from the negative to the positive terminal. The battery does a work

$$W = (2Q)\varepsilon = 2C\varepsilon^2$$

in the process. The energy stored in the capacitor is the same in the two cases. Thus, the work done by the battery appears as heat in the connecting wires. The heat produced is, therefore, $2C\varepsilon^2$.

Illustration : 25

A parallel plate air capacitor is made using two plates 0.2 m square, spaced 1 cm apart. It is connected to a 50 V battery.

(a) what is the capacitance ?

(b) what is the charge on each plate ?

(c) what is the energy stored in the capacitor ?

(d) what is the electric field between the plates ?

(e) if the battery is disconnected and then the plates are pulled apart to a separation of 2 cm, what are the answers to the above parts ?

Solution :

(a) $$C_0 = \frac{\varepsilon_0 A}{d_0} = \frac{8.85\times10^{-12}\times0.2\times0.2}{0.01}$$

$$C_0 = 3.54\times10^{-5}\ \mu F$$

(b) $$Q_0 = C_0V_0 = \left(3.54\times10^{-5}\times50\right)\mu C = 1.77\times10^{-3}\mu C$$

(c) $$U_0 = 1/2C_0V_0^2 = 1/2\left(3.54\times10^{-11}\right)(50)^2$$

$$U_0 = 4.42\times10^{-8}\,J.$$

(d) $$E_0 = \frac{V_0}{d_0} = \frac{50}{0.01} = 5000\,V/m.$$

(e) If the battery is disconnected, the charge on the capacitor plates remains constant while the potential difference between plates can change.

$$C = \frac{A\epsilon_0}{2d} = 1.77\times10^{-5}\ \mu F$$

$$Q = Q_0 = 1.77\times10^{-3}\ \mu C$$

$$V = \frac{Q}{C} = \frac{Q_0}{C_0/2} = 2V_0 = 100\ \text{volts}.$$

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q_0^2}{(C_0/2)} = 2U_0 = 8.84\times10^{-8}\,J$$

$$E = \frac{V}{c}\frac{2V_0}{2d_0} = E_0 = 5000\,V/m.$$

work has to be done against the attraction of plates when they are separated. This gets stored in the energy of the capacitor.

1. Two point charges +q and –q are held fixed at (-d, 0) and (d, 0) respectively of a (X, Y) coordinate system. Then

(a) The electric field $\vec{E}$ at all points on the X-axis has the same direction.

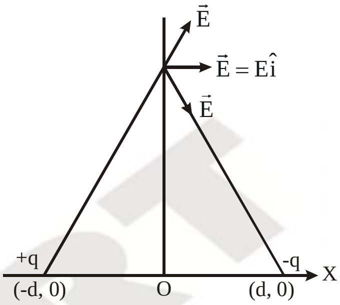
(b) $\vec{E}$ at all points on the Y-axis is along $\hat{i}$.

(c) Work has to be done in bringing a test charge from infinity to the origin.

(d) The dipole moment is 2qd directed along $\hat{i}$.

Ans. ***(b)***

Solution :



The diagrammatic representation of the given problem is shown in fig.

The electrical field $\vec{E}$ at all points on the X-axis will not have the same direction.

The electrical field $\vec{E}$ at all points on the Y-axis will be parallel to the X-axis (i.e. $\hat{i}$ direction).

The electric potential at the origin due to both the charge is zero, hence, no work is done in bringing a test charge from infinity to the origin.

Dipole moment is directed from the –q charge to the +q charge (i.e. –x direction).

2. A charge +q is fixed at each of the points $x = x_0$, $x = 3x_0$, $x = 5x_0$, … inf. on the x-axis and a charge –q is fixed at each of the points $x = 2x_0$, $x = 4x_0$, $x = 6x_0$, …. inf. Here x_0 is a positive constant. Take the electric potential at a point due to a charge Q at a distance r from it to be $Q/(4\pi\varepsilon_0 r)$. Then, the potential at the origin due to the above system of charges is :

(a) 0 (b) $\dfrac{q}{8\pi\varepsilon_0 x_0 \ln 2}$ (c) ∞ (d) $\dfrac{q \ln 2}{4\pi\varepsilon_0 x_0}$.

Ans. ***(d)***

Solution:

Potential at origin

$$V = \frac{q}{4\pi\varepsilon_0}\frac{1}{x_0}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - ....\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{\ln 2}{x_0}$$

3. There are points on a straight line joining two fixed opposite charges of same magnitude. There is :

(a) no point where potential is zero

(b) only one point where potential is zero

(c) no point where electric field is zero

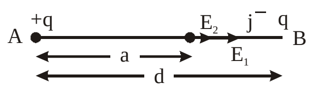
(d) only one point where electric field is zero.

Ans. ***(c)***

Solution:

Let two opposite charges +q and –q be situated at points A and B respectively.

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}$$



$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(d-a)^2}$$

$E = E_1 + E_2$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{a^2} + \frac{1}{(d-a)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{\left[d^2 - 2ad + a^2 + a^2\right]}{a^2(d-a)^2}$$

Hence, there can be no point where electric field is zero.

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{a}; V_2 = \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{(d-a)}$$

$$V = V_1 + V_2$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{d-a}\right] = \frac{q(d-2a)}{4\pi\varepsilon_0 a(d-a)}$$

$\therefore$ Potential is zero only at d=2a or a=d/2.

4. A certain charge Q is divided into two parts q and (Q-q). For the maximum coulomb force between them, the ratio (q/Q) is :

(a) 1/16 (b) 1/8 (c) 1/4 (d) 1/2.

Ans. ***(d)***

Solution:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q(Q-q)}{r^2}$$

for F to be maximum, $\frac{dF}{dq} = 0$

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{1}{r^2}[Q - q + q(-1)] = 0$$

$$Q - 2q = 0$$

$$\frac{q}{Q} = \frac{1}{2}.$$

5. A charge is situated at a certain distance from an electric dipole in the end-on position experiences a force F. If the distance of the charge is doubled, the force acting on the charge will be :

(a) F/4 (b) F/8 (c) 2F (d) F/2.

Ans. ***(b)***

Solution:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

$$E \propto \frac{1}{r^3} \quad \Rightarrow \quad F \propto \frac{1}{r^3}$$

Hence, the force will become F/8.

6. Two identical metal plates are given positive charges Q_1 and Q_2 ($<Q_1$) respectively. If they are now brought close together to form a parallel plate capacitor with capacitance C, the potential difference between them is :

(a) $\frac{Q_1 + Q_2}{2C}$ (b) $\frac{Q_1 + Q_2}{C}$ (c) $\frac{Q_1 - Q_2}{C}$ (d) $\frac{Q_1 - Q_2}{2C}$.

Ans. ***(d)***

Solution :

Within the capacitor,

$$E_1 = \frac{Q_1}{2\varepsilon_0 A}; E_2 = \frac{Q_2}{2\varepsilon_0 A}$$

$$E = E_1 - E_2 = \frac{1}{2\varepsilon_0 A}(Q_1 - Q_2)$$

Hence, $V = Ed = \frac{1}{2}\frac{d}{\varepsilon_0 A}(Q_1 - Q_2) = \frac{Q_1 - Q_2}{2C}$.

7. A 6×10^{-9} F parallel plate capacitor is connected to a 500 V battery. When air is replaced by another dielectric material. 7.5×10^{-6} C charge flows into the capacitor. The dielectric constant of the material is :

(a) 1.0 (b) 3.5 (c) 2.0 (d) 4.5.

Ans. ***(b)***

Solution:

$Q = CV$

$Q_1 = 6 \text{ x } 10^{-9} \times 500$

$= 3 \text{ x } 10^{-6}$ C

After insertion of dielectric

$Q'_1 = (3+7.5) \times 10^{-6}$ C

$= 10.5 \times 10^{-6}$ C

$Q'_1 = CVK$

$10.5 \times 10^{-6} = 6 \times 10^{-9} \times 500\,K$

$K = 3.5$

8. A conducting liquid drop has charge uniformly distributed over the surface. Electrostatic energy of drop E_0. Now this drop is broken in 8 small liquid drops such that mass and charge get equally distributed. What is the change in electrostatic energy of the system is the process. Assume drops to be widely separated after break up.

(a) 0 (b) $-\frac{3E_0}{4}$ (c) $\frac{E_0}{2}$ (d) $\frac{3E_0}{4}$.

Ans. ***(b)***

Solution:

Let initial charge on drop is Q and final charge on each drop q

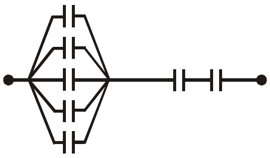
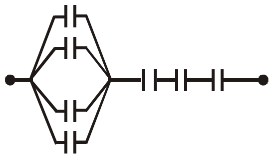
$$U_i = \frac{1}{4\pi \in_0}\frac{Q^2}{2R} = E_0$$

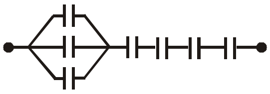
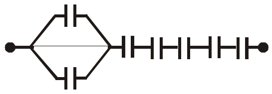
$$U_f = 8\frac{1}{4\pi \in_0}\frac{(Q/8)^2}{R/2} \qquad \left(\text{where } q = \frac{Q}{8}, r = R/2\right)$$

$$= \frac{E_0}{4}$$

$$\Delta U = U_f - U_i = -\frac{3E_0}{4}.$$

9. Seven capacitors each of capacitance $2\mu F$ are to be connected in a configuration to obtain an effective capacitance of $(10/11)\mu F$. Which of the combination(s), shown in figure below, will achieve the desired result?

(a)  (b) 

(c)  (d) .

Ans. (a)
Solution:

(a) $\frac{1}{C}=\frac{1}{5\times 2}+\frac{2}{2}=\frac{11}{10}$ or $C=\frac{10}{11}\mu F$

(b) $\frac{1}{C}=\frac{1}{4\times 2}+\frac{3}{2}=\frac{13}{8}$ or $C=\frac{8}{13}\mu F$

(c) $\frac{1}{C}=\frac{1}{3\times 2}+\frac{4}{2}=\frac{13}{6}$ or $C=\frac{6}{13}\mu F$

(d) $\frac{1}{C}=\frac{1}{2\times 2}+\frac{5}{2}=\frac{11}{4}$ or $C=\frac{4}{11}\mu F$.

10. A particle of charge q and mass m moves rectilinearly under the action of electric field E = A – Bx, where B is positive constant and x is distance from the point where particle was initially at rest then the distance traveled by the particle before coming to rest and acceleration of particle at that moment are respectively :

(a) $\frac{2A}{B},0$ (b) $0,\frac{-qA}{m}$ (c) $\frac{2A}{B},\frac{-qA}{m}$ (d) $\frac{-2A}{B},\frac{-qA}{m}$.

Ans. (c)
Solution :

F=qE = q(A – Bx)
ma = q(A-Bx)

$a=\frac{q}{m}(A-Bx)$ ...(1)

$\frac{vdv}{dx}=\frac{q}{m}(A-Bx)$

$vdv=\frac{q}{m}(A-Bx)dx$

$\int_0^0 vdv=\frac{q}{m}\int_0^x (A-Bx)dx$

$Ax-\frac{Bx^2}{2}=0$

$x=0,\ x=\frac{2A}{B}$ ...(2)

From eq. (1) and (2)

$\frac{q}{m}(A-Bx)=\frac{q}{m}\left(A-B\times\frac{2A}{B}\right)$

$=\frac{q}{m}(A-2A)=\frac{-qA}{m}$.

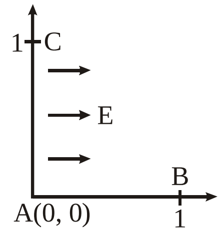
11. A uniform electric field pointing in positive x-direction exists in a region. Let A be the origin, B be the point on the x-axis at x = +1 cm and C be point on the y-axis at y = + 1 cm. Then the potentials at the points A, B and C satisfy :

(a) $V_A < V_B$ (b) $V_A > V_B$ (c) $V_A < V_C$ (d) $V_A > V_C$.

Ans. (b)

Solution:

Direction of electric field is in the direction of potential drop



$\Rightarrow$ $V_A > V_B$

$V_A = V_C$

12. An electron of mass m_e initially at rest moves through a certain distance in a uniform electric field in time t_1. A proton of mass m_p also initially at rest takes time t_2 to move through an equal distance in this uniform electric field. Neglecting the effect of gravity the ratio of $\frac{t_2}{t_1}$ is nearly equal to :

(a) 1 (b) $\left(\frac{m_p}{m_e}\right)^{\frac{1}{2}}$ (c) $\left(\frac{m_e}{m_p}\right)^{\frac{1}{2}}$ (d) 1836.

Ans. (b)

Solution :

Force on a charge particle in a uniform electric field

$$F = qE$$

The acceleration imparted to the particle is

$$a = \frac{qE}{m}$$

The distance traveled by the particle in time t is

$$d = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{qE}{m}\right)t^2$$

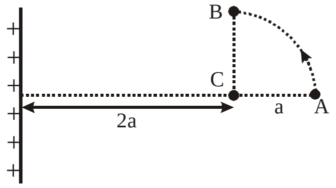
For the given problem

$$\frac{t_p^2}{m_p} = \frac{t_e^2}{m_e}$$

$$\frac{t_p^2}{t_e^2} = \frac{m_p}{m_e}$$

$$\Rightarrow \quad \frac{t_p}{t_e} = \sqrt{\frac{m_p}{m_e}}$$

13. The arc AB with the centre C and the infinitely long wire having linear charge density λ are lying in the same plane. The minimum amount of work to be expended to move a point charge q_0 from point A to B through a circular path AB of radius a is equal to :



(a) $\frac{q_0}{2\pi\varepsilon_0}\ln\frac{2}{3}$ (b) $\frac{q_0\lambda}{2\pi\varepsilon_0}\ln\frac{3}{2}$

(c) $\frac{q_0\lambda}{2\pi\varepsilon_0}\ln\frac{2}{3}$ (d) $\frac{q_0\lambda}{\sqrt{2\pi\varepsilon_0}}$.

Ans. (b)

Solution:

$$E = \frac{\lambda}{2\pi \epsilon_0 x}$$

$$\int_{V_A}^{V_B} dV = -E\,dx = -\frac{\lambda}{2\pi\varepsilon_0}\int_{3a}^{2a}\frac{dx}{x}$$

$$\Rightarrow \qquad V_B - V_A = \frac{\lambda}{2\pi \epsilon_0}\ln\left(\frac{3}{2}\right)$$

work done by agent $= \dfrac{q_0\lambda}{2\pi\varepsilon_0}\text{In}\dfrac{3}{2}$.

14. Two identical thin rings, each of radius R, are coaxially placed a distance R apart. If Q_1 and Q_2 are respectively the charges uniformly spread on the two rings, the work done in moving a charge q from the centre of one ring that of the other is :

(a) zero

(b) $q\,(Q_1 - Q_2)\,\dfrac{\sqrt{2}-1}{\sqrt{2}\,4\pi\varepsilon_0 R}$

(c) $q\,\sqrt{2}\,\dfrac{Q_1+Q_2}{4\pi\varepsilon_0 R}$

(d) $q\,(Q_1/Q_2)\,\dfrac{\sqrt{2}+1}{\sqrt{2}\,4\pi\varepsilon_0 R}$.

Ans. (b)

Solution:

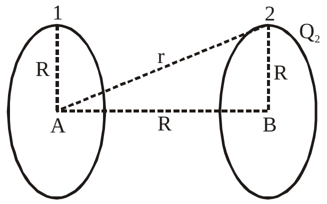
The potential at A due to the charge on the ring 1 is given as :

$$V_{A1} = \frac{Q_1}{4\pi\varepsilon_0}\cdot\frac{1}{R}$$

The potential at A due to the charge Q_2 on the ring 2 is given as :

$$V_{A2} = \frac{Q_2}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{R+R^2}}$$

$$= \frac{Q_2}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{R^2+R^2}} = \frac{1}{\sqrt{2}}\cdot\frac{Q_2}{4\pi\varepsilon_0}\cdot\frac{1}{R}$$



Total potential at A is

$$V_A = V_{A1} + V_{A2} = \frac{1}{4\pi\varepsilon_0 R}\left(Q_1 + \frac{Q_2}{\sqrt{2}}\right)$$

The potential energy of charge q at A is

$$U_A = V_{Aq} = \frac{q}{4\pi\varepsilon_0 R}\left(Q_1 + \frac{Q_2}{\sqrt{2}}\right)$$

Similarly, the potential energy of charge q at B is

$$U_B = \frac{q}{4\pi\varepsilon_0 R}\left(\frac{Q_1}{\sqrt{2}} + Q_2\right)$$

The work done in moving a charge q from point A to B is :

$$W = \Delta U = -U_A + U_B$$

$$= \frac{q}{4\pi\varepsilon_0 R}\cdot\left(\frac{Q_1}{\sqrt{2}} + Q_2 - Q_1 - \frac{Q_2}{\sqrt{2}}\right)$$

$$\therefore \qquad = \frac{q}{4\pi\varepsilon_0 R}(Q_2 - Q_1)\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)$$

SOLVED PROBLEMS (SUBJECTIVE)

1. Three point charges q, 2q and 8q are to be placed on a 9 cm long straight line. Find the positions where the charges should be placed such that the potential energy of this system is minimum. In this situation, what is the electric field at the charge q due to the other two charges ?

Solution:

The maximum contribution may come from the charge 8q forming pairs with others. To reduce its effect, it should be placed at a corner and the smallest charge q in the middle. This arrangement shown in figure ensures that the charges in the strongest pair 2q, 8q are at the largest separation.

The potential energy is

$$U = \frac{q^2}{4\pi\varepsilon_0}\left[\frac{2}{x} + \frac{16}{9cm} + \frac{8}{9cm - x}\right].$$

This will be minimum if

$$A = \frac{2}{x} + \frac{8}{9cm - x} \text{ is minimum.}$$

For this, $\dfrac{dA}{dx} = -\dfrac{2}{x^2} + \dfrac{8}{(9cm - x)^2} = 0$ .... (i)

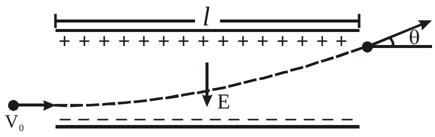
or, 9 cm – x = 2x or, x = 3 cm

The electric field at the position of charge q is

$$\frac{q}{4\pi\varepsilon_0}\left(\frac{2}{x^2} - \frac{8}{(9cm - x)^2}\right)$$

= 0.

2. A uniform electric field E is created between two parallel, charged plates as shown in figure. An electron enters the field symetrically between the plates with a speed u_0. The length of each plate is ℓ. Find the angle of deviation of the path of the electron as it comes out of the field.



Solution:

The acceleration of the electron is $a = \dfrac{eE}{m}$ in the upward direction. The horizontal velocity remains u_0 as there is no acceleration in this direction. Thus, the time taken in crossing the field is :

$$t = \frac{l}{u_0}$$

The upward component of the velocity of the electron as it emerges from the field region is

$$u_y = at = \frac{eEl}{mu_0}$$

The horizontal component of the velocity remains

$$u_x = u_0.$$

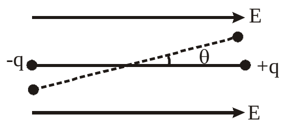
The angle θ made by the resultant velocity with the original direction is given by

$$\tan\theta = \frac{u_y}{u_x} = \frac{eEl}{mu_0^2}$$

Thus, the electron deviates by an angle

$$\theta = \tan^{-1}\frac{u_y}{u_x} = \frac{eEl}{mu_0^2}.$$

3. Figure shows an electric dipole formed by two particles fixed at the ends of a light rod of length l. The mass of each particle is m and the chargers are –q and +q. The system is placed in such a way that the dipole axis is parallel to a uniform electric field E that exists in the region. The dipole is slightly rotated about its centre and released. Show that for small angular displacement, the motion is angular simple harmonic and find its time period.



Solution:

Suppose, the dipole axis makes an angle θ with the electric field at an instant. The magnitude of the torque on it is

$$|\vec{\tau}| = |\vec{P} \times \vec{E}|$$

$$= q\,l\,E \sin\theta$$

This torque will be restoring & tend to rotate the elipole back towards the electric field. Also, for small angular displacement $\sin\theta = \theta$ so that

$$\tau = -q\,l\,E\theta$$

It the moment of inertia of the body about OA is I, the angular acceleration becomes.

$$\alpha = \frac{\tau}{I} = -\frac{qlE}{I}\theta \qquad \alpha = -\omega^2\theta$$

where $\omega^2 = \dfrac{qlE}{I}$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{qlE}}$$

Now, moment of inertia of the system about the axis of rotation is

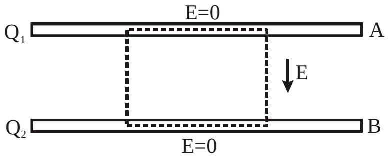
$$I = m\left(\frac{\ell}{2}\right)^2 + M\left(\frac{\ell}{2}\right)^2 = 2m\left(\frac{\ell}{2}\right)^2 = \frac{m\ell^2}{2}$$

So, $$T = 2\pi\sqrt{\frac{ml}{2qE}}.$$

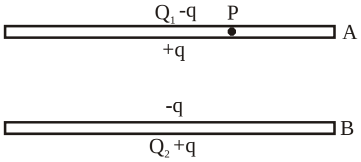
4. Two conducting plates A and B are placed parallel to each other. A is given a charge Q_1 and B a charge Q_2. Find the distribution of charges on the four surfaces.

Solution:

Consider a Gaussian surface as shown in figure. Two faces of this closed surface lie completely inside the conductor where the electric field is zero. The flux through these faces is, therefore, zero. The other parts of the closed surface which are outside the conductor are parallel to the electric field and hence the flux on these parts is also zero. The total flux of the electric field through the closed surface is, therefore, zero. From Gauss's law, the total charge inside this closed surface should be zero. The charge on the inner surface of A should be equal and opposite to the inner surface of B.



(a)



(b)

The distribution should be like the one shown in figure. To find the value of q, consider the field at a point P inside the plate A. Suppose, the surface area of each plate in A. Now since $E = \dfrac{\sigma}{2\varepsilon_0}$ the electric field at P

Due to the charge $Q_1 - q = \dfrac{Q-q}{2A\varepsilon_0}$ (downward),

Due to the charge $+q = \dfrac{q}{2A\varepsilon_0}$ (upward),

Due to the charge $-q = \dfrac{q}{2A\varepsilon_0}$ (downward),

And due to the charge $Q_2 + q = \dfrac{Q_2+q}{2A\varepsilon_0}$ (upward).

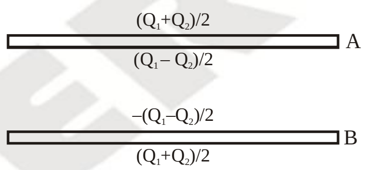
The net electric field at P due to all the four charged surfaces is (in the downward direction)

$$\frac{Q_1-q}{2A\varepsilon_0} - \frac{q}{2A\varepsilon_0} + \frac{q}{A\varepsilon_0} - \frac{Q_2+q}{2A\varepsilon_0}.$$

As the point P is inside the conductor, this field should be zero. Hence,

$$Q_1 - q - Q_2 - q = 0$$

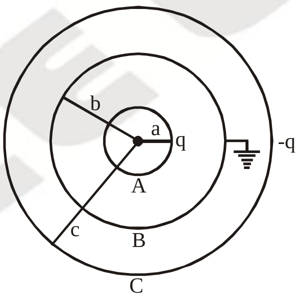
or, $q = \dfrac{Q_1 - Q_2}{2}$ ...(i)



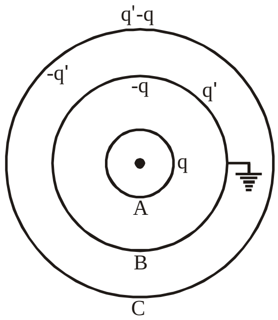
Thus, $Q_1 - q = \dfrac{Q_1 + Q_2}{2}$ ...(ii)

And $Q_2 + q = \dfrac{Q_1 + Q_2}{2}$ ...(iii)

5. Figure shows three concentric thin spherical shells A, B and C of radii a, b and c respectively. The shells A and C are given charges q and – q respectively and the shell B is earthed. Find the charges appearing on the surfaces of B and C.



(a)



(b)

Solution:

As shown in the previous worked out example, the inner surface of B must have a charge – q from the Gauss's law. Suppose, the outer a surface of B has a charge q'. The inner surface of C must have a charge $-q'$ from the Gauss's law. As the net charge on C must be – q, its outer surface should have a charge $q' - q$. The charge distribution is shown in figure. Potential of shall B due to the charge q on the surface of A.

$$= \frac{q}{4\pi\varepsilon_0 b},$$

due to the charge – q on the inner surface of B

$$= \frac{-q}{4\pi\varepsilon_0 b},$$

due to the charge q' on the outer surface of B

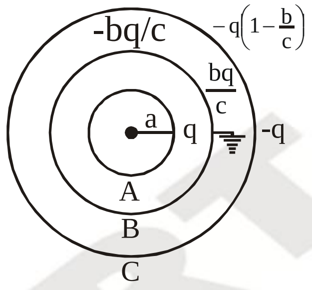
$$= \frac{q'}{4\pi\varepsilon_0 b},$$

due to the charge $-q'$, on the inner surface of C

$$= \frac{-q'}{4\pi\varepsilon_0 c}$$

and due to the charge $q'-q$ on the outer surface of C

$$= \frac{q'-q}{4\pi\varepsilon_0 c}.$$



The net potential is

$$V_B = \frac{q'}{4\pi\varepsilon_0 b} - \frac{q}{4\pi\varepsilon_0 c}.$$

This should be zero as the shall B is earthed. Thus,

$$q' = \frac{b}{c}q.$$

The charges on various surfaces are as shown in figure

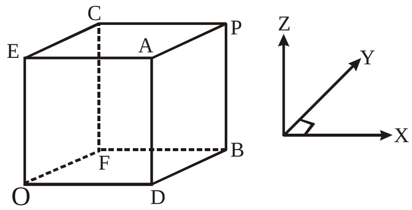
6. A cube of edge a metres carries a point charge q at each corner. Calculate the resultant force on any one of the charges.

Solution:

Let us take one corner of cube as origin O(0, 0, 0) and the opposite corner as P(a, a, a). We will calculate the electric field at P due to the other seven charges at corners.

Expressing the field of a point charge in vector form

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^3}\bar{r}$$



(i) Field at P due to A, B, C

$$\bar{E}_1 = \frac{q}{4\pi\varepsilon_0 a^3}\left[\overline{AP} + \overline{BP} + \overline{CP}\right]$$

$$= \frac{q}{4\pi\varepsilon_0 a^3}\left[a\hat{j} + a\hat{k} + a\hat{i}\right]$$

(ii) Field at P due to D, E, F

Now that DP = EP = FP = $a\sqrt{2}$

$$\bar{E}_2 = \frac{q}{4\pi\varepsilon_0 \left(a\sqrt{2}\right)^3}\left[\overline{DP} + \overline{EP} + \overline{FP}\right]$$

$$= \frac{q}{4\pi\varepsilon_0 (2\sqrt{2}a^3)}\left[\left(a\hat{j} + a\hat{k}\right) + \left(a\hat{i} + a\hat{j}\right) + \left(a\hat{i} + a\hat{k}\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0 \sqrt{2}a^2}\left[\hat{i} + \hat{j} + \hat{k}\right]$$

(iii) Field at P due to O

$$OP = a\sqrt{3}$$

$$\bar{E}_3 = \frac{q}{4\pi\varepsilon_0\left(a\sqrt{3}\right)^3}\overline{OP}$$

$$\bar{E}_3 = \frac{q}{4\pi\varepsilon_0(3\sqrt{3}a^2)}(a\hat{i}+a\hat{j}+a\hat{k})$$

$$\bar{E}_3 = \frac{q}{4\pi\varepsilon_0(3\sqrt{3}a^2)}\left(\hat{i}+\hat{j}+\hat{k}\right)$$

Resultant Field at P

$$\bar{E} = \bar{E}_1 + \bar{E}_2 + \bar{E}_3$$

$$\bar{E} = \frac{q(\hat{i}+\hat{j}+\hat{k})}{4\pi\varepsilon_0 a^2}\left[1+\frac{1}{\sqrt{2}}+\frac{1}{3\sqrt{3}}\right] \qquad \text{outward along OP}$$

Force on charge at P is F = q E

$$\Rightarrow \qquad F = \frac{q^2\sqrt{3}}{4\pi\varepsilon_0 a^2}\left[1+\frac{1}{\sqrt{2}}+\frac{1}{3\sqrt{3}}\right]$$

Outwards along diagonal OP

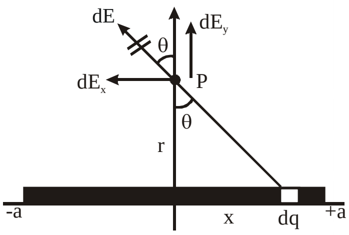
Note: In this problem, we have non-coplanar point charges and hence it is best to use vector approach in general form.

7. A uniform line charge λ (in coulombs per meter) exists along the X-axis from $x=-a$ to $x=+a$. Find the electric field E at point P a distance r along the perpendicular bisector.

Solution:

In all the problems which involve distributions of charge, we choose an element of charge dq to find the element of field dE' produced at the given location. The we sum all such dE's to find the total field E at that location.

You must note the symmetry of the situation. For each element dq located at positive X, there is a similar dq(see mirror-image in origin) located at the same negative value of x. The dE_x in the opposite direction due to the other dq. Hence, as we sum all the dq's along the line, all the dE_x components add to zero. So we need to sum only the dE_y components, a scalar sum since they all point in the same direction. The element of charge is $dq = \lambda\ dx$.



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2+x^2} = \frac{\lambda\ dx}{4\pi\varepsilon_0(r^2+x^2)}$$

$$E_x = \int dE_x = \int dE\sin\theta = 0 \qquad \text{(by symmetry)}$$

$$E_y = \int dE_y = \int dE\cos\theta$$

$$\int\frac{\lambda dx}{4\pi\varepsilon_0(r^2+x^2)}\frac{r}{\sqrt{r^2+x^2}} = \frac{\lambda r}{4\pi\varepsilon_0}\int\frac{dx}{(r^2+x^2)^{3/2}}$$

The integral on the right hand side can be evaluated by substituting $x = r\ \tan\alpha$ and $dx = r\sec^2\alpha\ d\alpha$

$$\int\frac{dx}{(r^2+x^2)^{3/2}} = \int\frac{r\sec^2\alpha\ d\alpha}{r^3\sec^3\alpha} = \int\frac{\cos\alpha}{r^2}d\alpha = \frac{\sin\alpha}{r^2}$$

$$\Rightarrow \qquad \int\frac{dx}{(r^2+x^2)^{3/2}} = \frac{1}{r^2}\frac{x}{\sqrt{x^2+r^2}}$$

$$E_y = \frac{\lambda r}{4\pi\varepsilon_0}\left|\frac{x}{r^2\sqrt{x^2+r^2}}\right|_{-a}^{a} = \frac{\lambda r}{4\pi\varepsilon_0}\frac{2a}{r^2\sqrt{a^2+r^2}}$$

The net field at P is E = E_y

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}\frac{a}{\sqrt{a^2+r^2}}$$

Note: In case of an infinite lien charge, the field is everywhere perpendicular to the line of charge. The field at a distance r from the line is calculated by taking $a \to \infty$ in the above result.
E (infinite line charge)

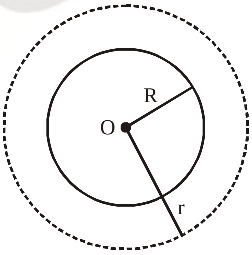
$$= \lim_{a\to\infty}\frac{\lambda}{2\pi\varepsilon_0 r}\frac{a}{\sqrt{a^2+r^2}} \qquad \Rightarrow \qquad E = \frac{\lambda}{2\pi\varepsilon_0 r}.$$

8. A system consists of a ball of radius R carrying a spherically symmetric charge Q and the surrounding space filled with a charge of volume density $\rho = a/r$ where a is a constant, r is the distance from the centre of ball. Find the ball's charge at which the magnitude of the electric field is independent of r outside the ball . How high is this strength?

Solution:

Let us consider a spherical surface of radius $r(r > R)$ concentric with the ball and apply Gauss's Law.

$$\oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$$



Let Q= total charge of the ball

$$\varepsilon_0 E(4\pi r^2) = Q + \int_R^r \rho 4\pi x^2\,dx$$

$$\varepsilon_0 E(4\pi r^2) = Q + 4\pi\int_R^r \frac{a}{x}x^2\,dx$$

$$\varepsilon_0 E(4\pi r^2) = Q + 2\pi a(r^2 - R^2)$$

$$\Rightarrow \quad E = \left(\frac{Q - 2\pi a R^2}{4\pi\varepsilon_0}\right)\frac{1}{r^2} + \frac{2\pi a}{4\pi\varepsilon_0}$$

For E to be independent of r,

$$Q = 2\pi a R^2$$

and the value of E is

$$E = \frac{a}{2\varepsilon_0}.$$

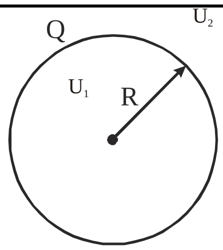
9. A charge Q is uniformly distributed over a spherical volume of radius R. Obtain an expression for the energy of the system.

Solution:

In this case, the electric field exists from centre of the sphere to infinity. Potential energy is stored in electric field with energy density.

$$u = \frac{1}{2}\varepsilon_0 E^2 (\text{energy / volume})$$

(i) Energy stored within the sphere (V_1)
Energy field at a distance r is ($r \leq R$)



$$E = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R^3}.r$$

$\therefore$ $$u = \frac{1}{2}\varepsilon_0 E^2$$

$$u = \frac{\varepsilon_0}{2}\left\{\frac{1}{4\pi\varepsilon_0}.\frac{Q}{R^3}r\right\}^2$$

Volume of element, $dV = (4\pi r^2)dr$

$\therefore$ Energy stored in this volume, $dU = u(dV)$

$$dU = \left(4\pi r^2 dr\right)\frac{\varepsilon_0}{2}\left\{\frac{1}{4\pi\varepsilon_0}.\frac{Q}{R^3}r\right\}^2$$

$$dU = \frac{1}{8\pi\varepsilon_0}.\frac{Q^2}{R^6}.r^4 dr$$

$$U_1 = \int_0^R dU = \frac{1}{8\pi\varepsilon_0}\frac{Q^2}{R^6}\int_0^R f^4 dr$$

$$= \frac{Q^2}{40\pi\varepsilon_0 R^6}\left[r^5\right]_0^R$$

$$U_1 = \frac{1}{40\pi\varepsilon_0}.\frac{Q^2}{R} \qquad \text{.... (1)}$$

(ii) Energy stored outside the sphere (U_2)

Electric field at a distance r is ($r \geq R$)

$$E = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r^2}$$

$$u = \frac{1}{2}\varepsilon_0 E^2$$

$$u = \frac{\varepsilon_0}{2}\left\{\frac{1}{4\pi\varepsilon_0}.\frac{Q}{r^2}\right\}^2$$

$dV = (4\pi r^2 dr)$

$$dU = udV = (4\pi r^2 dr)\left[\frac{\varepsilon_0}{2}\left(\frac{1}{4\pi\varepsilon_0}.\frac{Q}{r^2}\right)^2\right]$$

$$dU = \frac{Q^2}{8\pi\varepsilon_0}.\frac{dr}{r^2}$$

$$U_2 = \int_R^\infty dV = \frac{Q^2}{8\pi\varepsilon_0}\int_R^\infty \frac{dr}{r^2}$$

$$U_2 = \frac{Q^2}{8\pi\varepsilon_0 R} \qquad \text{... (2)}$$

Therefore, total energy of the system is

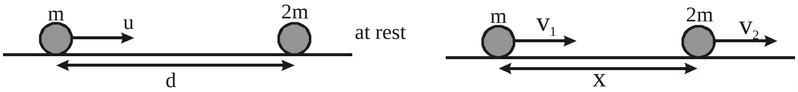
$$U = U_1 + U_2 = \frac{Q^2}{40\pi\varepsilon_0 R} + \frac{Q^2}{8\pi\varepsilon_0 R}$$

$$U = \frac{3}{20}\frac{Q^2}{\pi\varepsilon_0 R}.$$

10. Two particles of mass m and 2m carry a charge q each. Initially the heavier particle is at rest on a smooth horizontal plane and the other is projected along the plane directly towards the first from a distance d with speed u. Find the closest distance of approach.

Solution:

As the mass 2m is not fixed, it will also move away from m due to repulsion. The distance between the particles is minimum when their relative velocity is zero i.e., when they have equal velocities.



Hence at closest approach, $v_1 = v_2$

By conservation of momentum

$$mu = mv_1 + 2mv_2$$

$$v_2 = v_1 = u/3$$

By conservation of energy

Loss in KE = gain in PE

$$\frac{1}{2}mu^2 - \left(\frac{1}{2}mv_1^2 + \frac{1}{2}2mv_2^2\right) = \frac{q^2}{4\pi\varepsilon_0}\left(\frac{1}{x} - \frac{1}{d}\right)$$

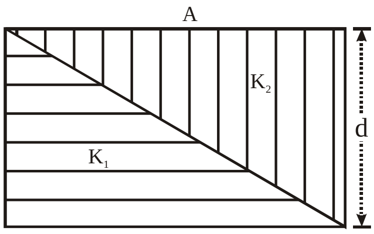
$$\frac{1}{2}mu^2 - \frac{1}{2}m\frac{u^2}{9}(1+2) = \frac{q^2}{4\pi\varepsilon_0}\left(\frac{1}{x} - \frac{1}{d}\right)$$

$$\frac{1}{3}mu^2 = \frac{q^2}{4\pi\varepsilon_0}\left(\frac{1}{x} - \frac{1}{d}\right)$$

$$\frac{1}{x} = \frac{1}{d} + \frac{4\pi\varepsilon_0 mu^2}{3q^2}$$

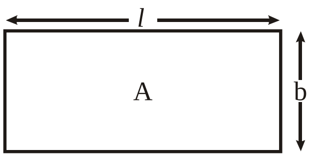
$$x = \frac{3q^2 d}{3q^2 + 4\pi\varepsilon_0 mu^2 d}.$$

11. The capacitance of a parallel plate capacitor with plate area A and separation d is C. The space between the plates is filled with two wedges of dielectric constants K_1 and K_2 respectively (Fig.). Find the capacitance of the resulting capacitor.



Solution:

Let length and breadth of the capacitor be *l* and b respectively and d be the distance between the plates as shown in fig. Then consider a strip at a distance x of width dx.



Now QR = x tan θ

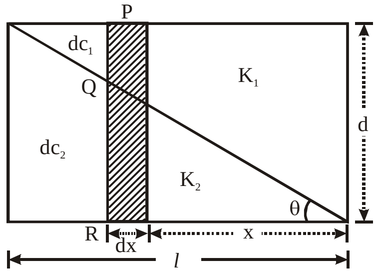
and PQ = d – x tan θ

Where tan θ = d/*l*,

Capacitance of PQ

$$dC_1 = \frac{k_1\varepsilon_0(b\,dx)}{d - x\tan\theta} = \frac{k_1\varepsilon_0(b\,dx)}{d - \dfrac{xd}{l}}$$

$$dC_1 = \frac{k_1\varepsilon_0 b l dx}{d(l-x)} = \frac{k_1\varepsilon_0 A(dx)}{d(l-x)}$$



and dC_2 = capacitance of QR

$$dC_2 = \frac{k_2\varepsilon_0 b(dx)}{d\tan\theta}$$

$$dC_2 = \frac{k_2\varepsilon_0 A(dx)}{x\,d} \quad ...... \quad \left\{\because \tan\theta = \frac{d}{l}\right\}$$

Now dC_1 and dC_2 are in series. Therefore, their resultant capacity dC will be given by

$$\frac{1}{dC} = \frac{1}{dC_1} + \frac{1}{dC_2}$$

then $$\frac{1}{dC} = \frac{1}{dC_1} + \frac{1}{dC_2}$$

$$= \frac{d(l-x)}{K_1\varepsilon_0 A(dx)} + \frac{x.d}{K_2\varepsilon_0 A(dx)}$$

$$\frac{1}{dC} = \frac{d}{\varepsilon_0 A(dx)}\left(\frac{l-x}{K_1} + \frac{x}{K_2}\right) \quad = \quad \frac{d[K_2(l-x) + K_1 x]}{\varepsilon_0 AK_1K_2(dx)}$$

$$dC = \frac{\varepsilon_0 AK_1K_2}{d[K_2(l-x)+K_1x]}dx \quad , \quad dC = \frac{\varepsilon_0 AK_1K_2}{d[K_2 l + (K_1 - K_1)x]}dx$$

All such elemental capacitor representing DC are connected in parallel.

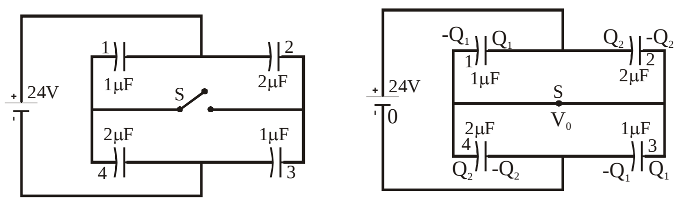
Now the capacitance of the given parallel plate capacitor is obtained by adding such infinitesimal capacitors parallel from x = 0 to x = l.

i.e. $$C = \int_{x=0}^{x=l} dC$$

$$= \int_0^l \frac{\varepsilon_0 AK_1K_2}{d[K_2 l + (K_1 - K_2)x]}dx$$

$$C = \frac{K_1K_2\varepsilon_0 A}{(K_1 - K_2)d}\,\text{In}\,\frac{K_2}{K_1}.$$

12. The connections shown in figure are established with the switch S open. How much charge will flow through the switch if it is closed?



(a) (b)

Solution:

Sample output to test PDF Combine only

When the switch is open, capacitors (2) and (3) are in series. Their equivalent capacitance is $\frac{2}{3}\mu F$.

The charge appearing on each of these capacitors is, therefore, $24V \times \frac{2}{3}\mu F = 16\mu C$.

The equivalent capacitance of (1) and (4), which are also connected in series, is also $\frac{2}{3}\mu F$ and the charge on each of these capacitors is also $16\mu C$. The total charge on the two plates of (1) and (4) connected to the switch is, therefore, zero.

The situation when the switch S is closed is shown in figure. Let the charges be distributed as shown in the figure. Q_1 and Q_2 are arbitrarily chosen for the positive plates of (1) and (2). The same magnitude of charges will appear at the negative plates (3) and (4).

Take the potential at the negative terminal to the zero and at the switch to be V_0.
Writing equations for the capacitors (1), (2), (3) and (4).

$$Q_1 = (24V - V_0) \times 1\ \mu F \qquad \text{...(i)}$$

$$Q_2 = (24V - V_0) \times 2\mu F \qquad \text{...(ii)}$$

$$Q_1 = V_0 \times 1\ \mu F \qquad \text{...(iii)}$$

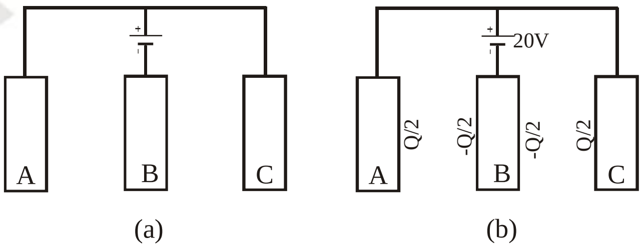
$$Q_2 = V_0 \times 2\mu F \qquad \text{...(iv)}$$

From (i) and (iii), $V_0 = 12V$.
Thus, from (iii) and (iv),

$$Q_1 = 12\mu C \text{ and } Q_2 = 24\ \mu C.$$

The charge on the two plates of (1) and (4) which are connected to the switch is, therefore $Q_2 - Q_1 = 12\mu C$. When the switch was open, this charge was zero. Thus, $12\mu C$ of charge has passed through the switch after it was closed.

13. Each of the three plates shown in figure has an area of 200 cm^2 on one side and the gap between the adjacent plates is 0.2 mm. The emf of the battery is 20 V. Find the distribution of charge on various surfaces of the plates. What is the equivalent capacitance of the system between the terminal points?



Solution:

Suppose the negative terminal of the battery gives a charge – Q to the plate B. As the situation is symmetric on the two sides of B, the two faces of the plate B will share equal charge –Q/2 each. From Gauss's law, the facing surfaces will have charge Q/2 each. As the positive terminal of the battery has supplied just this much charge (+Q) to A and C, the outer surfaces of A and C will have no charge. The distribution will be as shown in figure.
The capacitance between the plates A and B is

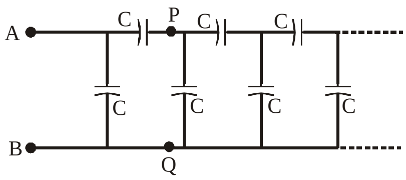
$$C = \frac{A\varepsilon_0}{d} = 8.85 \times 10^{-12}\ F/m \times \frac{200 \times 10^{-4}\ m^{-2}}{2 \times 10^{-4}\ m}$$

$$= 8.85 \times 10^{-10}\,F = 0.885 \text{ nF}.$$

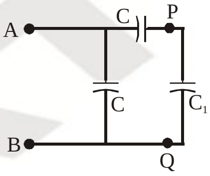
Thus, Q= 0885 nF × 20 V = 17.7 nC.
The distribution of charge on various surfaces may be written from figure
The equivalent capacitance is

$$\frac{Q}{20\,V} = 1.77 \text{ nF}.$$

14. Find the capacitance of the infinite ladder shown in figure.



Solution:



As the ladder is infinitely long, the capacitance of the ladder to the right of the points P, Q is the same as that of the ladder to the right of the points A, B. If the equivalent capacitance of the ladder is C_1, the given ladder may be replaced by the connections shown in figure.

The equivalent capacitance between A and B is easily found to be $C + \frac{CC_1}{C + C_1}$.

But being equivalent to the original ladder, the equivalent capacitance is also C_1.

Thus, $$C_1 = C + \frac{CC_1}{C + C_1}$$

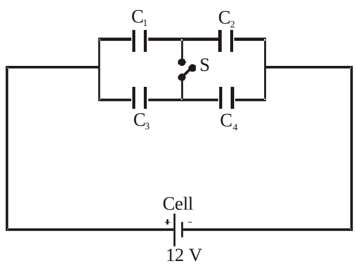
Or, $$C_1C + C_1^2 = C^2 + 2CC_1$$

Or, $$C_1^2 - CC_1 - C^2 = 0$$

Giving $$C_1 = \frac{C + \sqrt{C^2 + 4C^2}}{2} = \frac{1 + \sqrt{5}}{2} C.$$

Negative value of C_1 is rejected.

15. The emf of the cell in the circuit is 12 volts and the capacitors are : $C_1 = 1\ \mu f, C_2 = 3\ \mu f, C_3 = 2\ \mu f, C_4 = 4\ \mu f$. Calculate the charge on each capacitor and the total charge drawn from the cell when
(a) the switch s is closed
(b) the switch s is open.

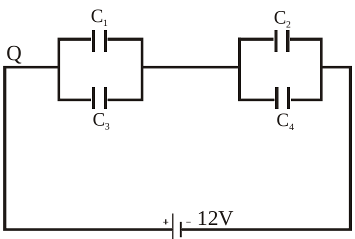


Solution:

(a) Switch S is closed :

$$C = \frac{(C_1 + C_3)(C_2 + C_4)}{(C_1 + C_3) + (C_2 + C_4)}$$

$$C = \frac{3 \times 7}{3 + 7} = 2.1 \mu F$$



total charge drawn from the cell is :

Q = C V = 2.1 μF × 12 volts = 25.2 μC

C_1, C_3 are in parallel and C_2, C_4 are in parallel.

Charge on C_1

$$Q_1 = \frac{C_1}{C_1 + C_3} Q = \frac{1}{1+2} \times 25.2\mu C = 8.4\mu C.$$

Charge on C_3

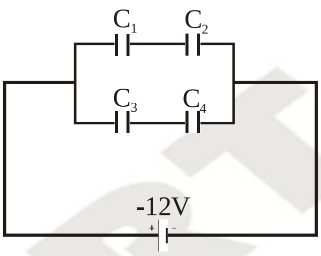
$$Q_3 = \frac{C_3}{C_1 + C_3} Q = \frac{2}{1+2} \times 25.2\mu C = 16.8\mu C.$$

Charge on C_2

$$Q_2 = \frac{C_2}{C_2 + C_4} Q = \frac{3}{3+4} \times 25.2\mu C = 10.8\mu C.$$

Charge on C_4

$$Q_4 = \frac{C_4}{C_2 + C_4} Q = \frac{4}{3+4} \times 25.2\mu C = 14.4\mu C.$$



(b) Switch S is open :

$$C = \frac{C_1 C_2}{C_1 + C_2} + \frac{C_3 C_4}{C_3 + C_4}$$

$$C = \frac{1 \times 3}{1+3} + \frac{2 \times 4}{2+4} = \frac{25}{12} \mu F$$

total charge drawn from battery is :

$$Q = CV = \frac{25}{12} \times 12 = 25\mu C$$

C_1 & C_2 are in series and the potential difference across combination is 12 volts.
charge on C_1 = charge on C_2

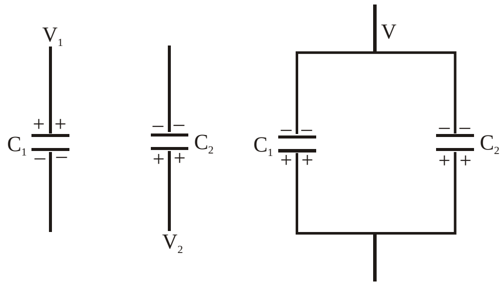
$$= \left(\frac{C_1 C_2}{C_1 + C_2} \right) V = \frac{3}{4} \times 12 = 9\mu C.$$

C_3 & C_4 are in series and the potential difference across combination is 12 volts.
charge on C_3 = charge on C_4

$$= \left(\frac{C_3 C_4}{C_3 + C_4} \right) V = \frac{8}{6} \times 12 = 16\mu C.$$

16. Two capacitors $C_1 = 1\mu F$ and $C_2 = 4\mu F$ are charged to a potential difference of 100 volts and 200 volts respectively. The charged capacitors are now connected to each other with terminals of opposite sign connected together. What is the
(a) final charge on each capacitor in steady state ?
(b) decrease in the energy of the system ?

Solution:



Initial charge on $C_1 = C_1 V_1 = 100\mu C$

Initial charge on $C_2 = C_2 V_2 = 800\mu C$

$$C_1 V_1 < C_2 V_2$$

when the terminals of opposite polarity are connected together, the magnitude of net charge finally is equal to the difference of magnitude of charges before connection.

(charge on $C_2)_i$ - (charge on $C_1)_i$

= (charge on $C_2)_f$ - (charge on $C_1)_f$

Let V be the final common potential difference across each.

The charges will be redistributed and the system attains a steady state when potential difference across each capacitor becomes same.

$$C_2V_2 - C_1V_1 = C_2V + C_1V$$

$$V = \frac{C_1V_2 - C_1V_1}{C_2 + C_1} = \frac{800-100}{5} = 140 \text{ volts}$$

Note that because $C_1V_1 < C_2V_2$, the final charge polarities are same as that of C_2 before connection.

Final charge on $C_1 = C_1 V = 140\ \mu C$

Final charge on $C_2 = C_2 V = 560\ \mu C$

Loss of energy $= U_i - U_f$,

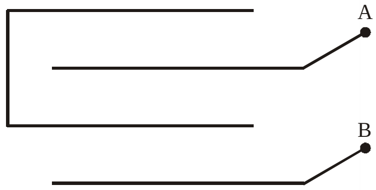
$$\text{Loss of energy} = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}C_1V^2 - \frac{1}{2}C_2V^2$$

$$= \frac{1}{2}1(100)^2 + \frac{1}{2}4(200)^2 - \frac{1}{2}(1+4)(140)^2$$

$$= 36000\mu J = 0.036 J$$

Note: The energy is lost as heat in the connected wires due to the temporary currents that flow while the charge is being redistributed.

17. Four identical metal plates are located in air at equal separations d as shown. The area of each plate is A. Calculate the effective capacitance of the arrangement across A and B.

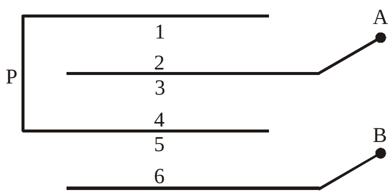


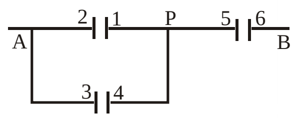
Solution :

Let us call the isolated plate as P. A capacitor is formed by a pair of parallel plates facing each other. Hence we have three capacitor formed by the pairs (1, 2), (3, 4) and (5, 6). The surface 2 and 3 are at same potential as that of A. The arrangement can be redrawn as a network of three capacitors.

$$C_{AB} = \frac{2C.C}{2C+C} = \frac{2C}{3}$$

$$= \frac{2}{3}\frac{\varepsilon_0 A}{d}$$

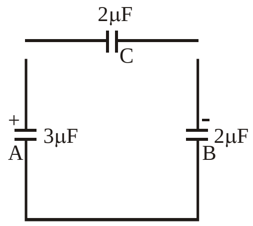




18. Two capacitors A and B with capacities $3\,\mu$F and 2μF are charged to a potential difference of 100V and 180V respectively. The plates of the capacitors are connected as shown in the figure with one wire of each capacitor free. The upper plate of A is positive and that of B is negative. An uncharged $2\,\mu$F capacitor C with lead wires falls on the free ends to complete the circuit. Calculate :

(i) the final charge on the three capacitors, and

(ii) the amount of electrostatic energy stored in the system before and afterthe completion of the circuit.



Solution

(i) Charge on capacitor A, before joining with an uncharged capacitor,

$$q_A = CV = (100) \times 3\ \mu c = 300\ \mu c$$

similarly charge on capacitor B,

$q_B = 180 \times 2\ \mu c$

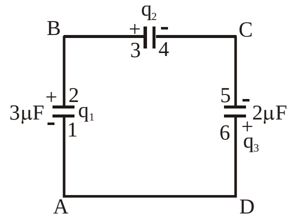
$= 360\ \mu c$

Let q_1, q_2 and q_3 be the charges on the three capacitors after joining them as shown in fig.

From conservation of charge,

Net charge on plates 2 and 3 before joining

= Net charge after joining



$\therefore$ $300 = q_1 + q_2$ .... (1)

Similarly, net charge on plates 4 and 5 before joining

= Net charge after joining

$-360 = -q_2 - q_3$

$360 = q_2 + q_3$ .... (2)

applying Kirchoff's 2nd law in loop ABCDA,

$$\frac{q_1}{3} - \frac{q_2}{2} + \frac{q_3}{2} = 0$$

$2q_1 - 3q_2 + 3q_3 = 0$ .... (3)

From equations (1), (2) and (3),

$q_1 = 90\ \mu c$, $q_2 = 90\ \mu c$ and $q_3 = 150\ \mu c$

(ii) (a) Electrostatic energy stored before completing the circuit,

$$U_i = \frac{1}{2}(3 \times 10^{-6})(100)^2 + \frac{1}{2}(2 \times 10^{-6})(180)^2 \qquad \left(U = \frac{1}{2}CV^2\right)$$

$= 4.74 \times 10^{-2}$ J

$= 47.4$ *m*J.

(b) Electrostatic energy stored after completing the circuit,

$$U_f = \frac{1}{2}(90 \times 10^{-6})^2 . \frac{1}{3 \times 10^{-6}} + \frac{1}{2}(90 \times 10^{-6})^2 . \frac{1}{2 \times 10^{-6}} + \frac{1}{2}(150 + 10^{-6})^2 . \frac{1}{2 \times 10^{-6}}$$

$$\left(U = \frac{1}{2}\frac{q}{c^2}\right)$$

$= 90 \times 10^{-4}$ J

$= 9$ mJ.

EXERCISE I

OBJECTIVE (Only one option is correct)

1. Three point charges q_1, q_2 and q_3 are taken such that when q_1 and q_2 are placed close together to form a single point charge, the force on q_3 at distance L from this combination is a repulsion of 2 units in magnitude. When q_2 and q_3 are so combined the force on q_1 at distance L is an attractive force of magnitude 4 units. Also q_3 and q_1 when combined exert an attractive force on q_2 of magnitude 18 unit at same distance L. The algebraic ratio of charges q_1, q_2 and q_3 is

(A) 1: 2: 3 (B) 2: –3 : 4 (C) 4 : –3 : 1 (D) 4: –3 : 2

2. A ring of radius R is made out of a thin metallic wire of area of cross section A. The ring has a uniform charge Q distributed on it. A charge q_0 is placed at the centre of the ring. If Y is the young's modulus for the material of the ring and ΔR is the change in the radius of the ring then,

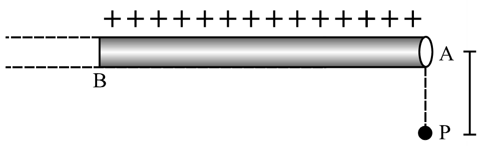
(A) $\Delta R = \frac{q_0 Q}{4\pi\varepsilon_0 RAY}$ (B) $\Delta R = \frac{q_0 Q}{4\pi^2\varepsilon_0 RAY}$

(C) $\Delta R = \frac{q_0 Q}{8\pi^2\varepsilon_0^2 RAY}$ (D) $\Delta R = \frac{q_0 Q}{8\pi^2\varepsilon_0 RAY}$

3. An electron of mass m_e, initially at rest, moves through a certain distance in a uniform electric field in time t_1. A proton of mass m_p, also initially at rest, takes time t_2 tom move through an equal distance in this uniform electric field. Neglecting the effect of gravity, the ratio $\frac{t_2}{t_1}$ is nearly equal to

(A) 1 (B) $\sqrt{\frac{m_P}{m_e}}$ (C) $\sqrt{\frac{m_e}{m_P}}$ (D) 1836

4. A semi-infinite insulating rod has linear charge density λ. The electric field at the point P shown in figure is



(A) $\frac{2\lambda^2}{(4\pi\varepsilon_0 r)}$ at 45° with AB

(B) $\frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 r^2}$ at 45° with AB

(C) $\frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 r}$ at 45° with AB

(D) $\frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 r}$ at 135° with AB

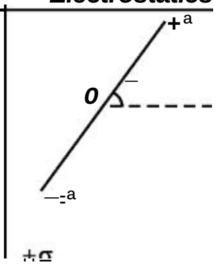
5. A circle of radius r has a linear charge density $\lambda = \lambda_0 \cos^2\theta$ along its circumference. Total charge on the circle is

(A) $\lambda_0(2\pi r)$ (B) $\lambda_0(\pi r)$ (C) $\lambda_0\left(\frac{\pi r}{2}\right)$ (D) $\lambda_0\left(\frac{\pi r}{4}\right)$

6. The electric field at the centre of a hemispherical surface having uniform surface charge density σ is

(A) $\frac{\sigma}{\varepsilon_0}$ (B) $\frac{\sigma}{2\varepsilon_0}$ (C) $\frac{\sigma}{4\varepsilon_0}$ (D) $\frac{\sigma}{8\varepsilon_0}$

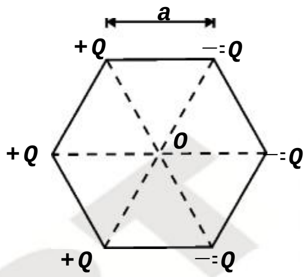
7. A largest sheet carries uniform surface charge density σ. A rod of length 2l has a linear charge density λ on one half and $-\lambda$ on the other half. The rod is hinged at mid point O and makes angle θ with the normal to the sheet. The torque experienced by the rod is



(a) $\frac{\sigma\lambda l^2}{2\epsilon_0}\cos\theta$ (b) $\frac{\sigma\lambda l}{\epsilon_0}\cos^2\theta$

(c) $\frac{\sigma\lambda l^2 \sin\theta}{2\epsilon_0}$ (d) $\frac{\sigma\lambda l \sin^2\theta}{\epsilon_0}$

8. Six charges are placed at the vertices of a regular hexagon as shown in the figure. The electric field on the line passing through point O and perpendicular to the plane of the figure at a distance of x (>>a) from O is



(a) $\frac{Qa}{\pi\epsilon_0 x^3}$ (b) $\frac{2Qa}{\pi\epsilon_0 x^3}$ (c) $\frac{\sqrt{3}Qa}{\pi\epsilon_0 x^3}$ (d) zero

9. The magnitude of electric field E in the annular region of a charged cylindrical capacitor
(A) is same throughout
(B) is higher near the outer cylinder than near the inner cylinder
(C) varies as 1/r, where r is the distance from the axis
(D) varies as $1/r^2$, where r is the distance from the axis.

10. A positively charged thin metal ring of radius R is fixed in the X - Y plane with its centre at the origin 0. A negatively charged particle P is released from rest at the point $(0,0,Z_0)$ where $(Z_0 > 0)$. Then the motion of P is
(A) periodic for all values of Z_0 satisfying $0<Z_0<-\infty$
(B) simple harmonic for all values of Z_0 satisfying $0<Z_0<R$
(C) approximately simple harmonic provided $Z_0 << R$
(D) such that P crosses O and continues to move along the negative Z-axis towards $Z = -\infty$

11. The charge density of a spherical charge distribution is given by

$$\rho(r) = \begin{cases} \rho_0 \frac{r}{a} & r \le a \\ 0 & r > a \end{cases}$$

The total charge on the distribution is

(A) $\frac{2}{3}\pi a^3\rho_0$ (B) $\frac{1}{3}\pi a^3\rho_0$ (C) $2\pi a^3\rho_0$ (D) $\pi a^3\rho_0$

12. The force between two point charge + Q and – Q placed 'r' distance apart is f_1 and force between two spherical conductors, each of radius R placed with their centers 'r' distance apart charged with charge +Q and –Q is f_2. If the separation 'r' is not much larger than R, then

(A) $f_1 > f_2$ (B) $f_1 = f_2$ (C) $f_1 < f_2$ (D) $f_2 = \frac{r}{R} f_1$

13. Three charge +2q, –q, –q are placed at the corners of an equilateral triangle. If V is electric potential and E and electric field at the centre of the triangle, then

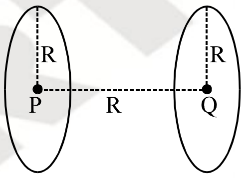
(A) $E = 0,\ V \neq 0$ (B) $E \neq 0,\ V = 0$ (C) $E \neq 0,\ V \neq 0$ (D) $E = 0,\ V = 0$

14. A hollow metallic sphere P is charged to a potential V volt. Another sphere Q is charged to a potential V/2. If Q is placed inside P and they are connected by a wire, then
(A) charge will flow from P to Q until the potential becomes 3V/4
(B) whole charge will flow from P to Q
(C) the whole charge will flow from Q to P
(D) no charge flows

15. A condenser of capacitance 2 μF is connected to a battery through a resistor 3 ohm. The battery spends energy of 4 m J to charge the condenser fully. The heat produced across the resistor during charging is equal to (in mJ)
(A) 1 (B) 3/2 (C) 2 (D) 2/3

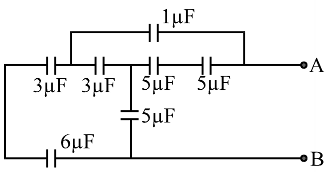
16. n identical capacitors are connected in parallel to a potential difference V. These capacitors are reconnected in series, their charges being left undisturbed. The potential difference obtained is
(A) V/n (B) (n – 1)V (C) n^2V (D) nV

17. Two rings each of radius R are kept perpendicular to the plane of the paper so that their centres P and Q are at a distance R. They carry uniform charge q each. The potential due to one ring at its centre is V. Then the potential due to both rings at the centre P will be



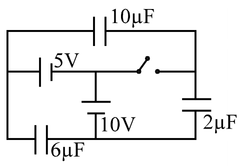
(A) 2V (B) $\sqrt{2}\,V$
(C) 0 (D) $(\sqrt{2}+1)V/\sqrt{2}$

18. The equivalent capacitance between points A and B is -



(A) $\frac{7}{3}$ μF (B) $\frac{7}{5}$ μF
(C) $\frac{7}{6}$ μF (D) $\frac{11}{3}$ μF

19. Find the charge flown through the switch when it is closed



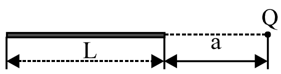
(A) 20 C (B) 30 μC
(C) 42 μC (D) 60 μC

20. Three capacitors 1 μF , 2 μF and 3 μF having withstanding voltage 100 volts are joined in series across a 300 V supply. Break down will take place first in -
(A) 1μF (B) 2 μF (C) 2 μF (D) in all three

21. A capacitor of capacitance C_0 is charged to a potential V_0 and them isolated. A small capacitor C is then charge from C_0, discharged and charged again and the process is repeated n times. Due to this potential of the larger capacitor is decreases to V. Value of C is -

(A) $C_0\left[\frac{V_0}{V}\right]^{1/n}$ (B) $C_0\left[\left(\frac{V_0}{V}\right)^{1/n}-1\right]$ (C) $C_0\left[\left(\frac{V}{V_0}\right)-1\right]^n$ (D) $C_0\left[\left(\frac{V}{V_0}\right)^n+1\right]$

22. It is given that the potential energy of a line charge is equal to U. If the same line charge is build in the presence of a point charge Q as shown in the diagram then what will be the work done in the building the line cahrge?



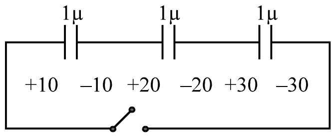
(A) $U + Q\lambda[\ln(a+L)/a]/(4\pi\varepsilon_0)$ (B) $U + Q\lambda[\ln(a+L)/a]/(2\pi\varepsilon_0)$
(C) $U - Q\lambda[\ln(a+L)/a]/(4\pi\varepsilon_0)$ (D) $-U + Q\lambda[\ln(a+L)/a]/(4\pi\varepsilon_0)$

23. Two point charges of +Q each have been placed at the positions (–a/2, 0, 0) and (a/2, 0, 0). The locus of the points where –Q charge can be placed such the that total electrostatic potential energy of the system call become equal to zero, is represented by which of the following equations?

(A) $X^2 + (Y - a)^2 = 2a^2$ (B) $Z^2 + (Y - a)^2 = 27a^2/4$

(C) $Z^2 + Y^2 = 15a^2/4$ (D) None

24. Three capacitors each of 1 µF are initially charged by 10V, 20V and 30v respectively. Now these are connected as shown in figure. If switch is closed, find the amount of heat produced on closing the switch -



(A) 600 µF (B) 350 µF

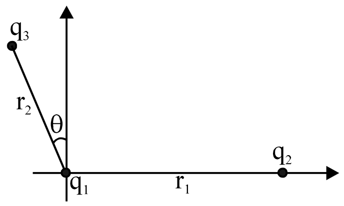
(C) 300 µF (D) 0

25. A parallel plate capacitor has an electric field of 5×10^6 V/m between the plates. If the charge on the capacitor plate is 0.5 µC, then the force on each capacitor plate is -

(A) 1.25 Nt (B) 2.5 Nt (C) 0.125 Nt (D) 0.25 Nt

SUBJECTIVE PROBLEM

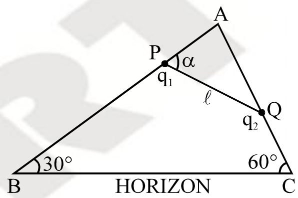
1. Figure shown charges q_1, q_2 & q_3. What force acts on q_1? Take $q_1 = -1\ \mu C$, $q_2 = +3\ \mu C$ and $q_3 = -2\mu C$, $r_1 = 15$ cm. $r_2 = 10$ cm & $\theta = 30^0$. Find only the X & Y components of the force.



2. The distance between two fixed positive charges 4e & e is l. How should a third charge 'q' be arranged for it to be in equilibrium? Under what condition will the equilibrium of the charge 'q' be stable (for displacement on the line joining 4e & e) and will it be unstable?

3. A particle having charge +q is fixed at a point O and a second particle of mass m and having charge $-q_0$ moves with constant speed in a circle of radius r about the charge +q. Find the energy required to be supplied to the moving charge to increase the radius of the path to 2r.

4. A rigid insulated wire frame in the form of a right angled triangle ABC, is set in a vertical plane as shown. Two bead of equal masses m each and carrying charges q_1 and q_2 are connected by a cord of length ℓ and slide without friction on the wires. Considering the case when the beads are stationary, determine.



(a) the angle α. (b) The tension in the cord &

(c) The normal reaction on the beads. If the cord is now cut, what are the values of the charges for which the beads continue to remain stationary.

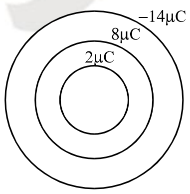
5. A particle symmetric charge distribution is given by $\rho = \rho_0(1 - ra)$ $r \le a$

$\rho = 0$ $r > a$

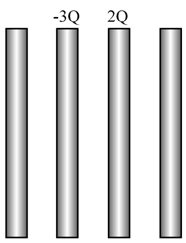
Calculate the electric field inside the outside the charge distribution and for what values of r is the field maximum and what is the value of the maximum field?

6. A clock face has negative charges –q, –2q, –3q, ..., –12q fixed at the position of the corresponding numerals on the dial. The clock hands do not disturb the net field due to point charges. At what time does the hour hand point in the same direction is electric field at the centre of the dial.

7. Distribute the charges



Conducting spherical shells



Conducting plates

8. Fig - 2 shows two square arrays of charged particle. The squares which are centered on point P, are misaligned. What is the magnitude and direction of electric field at point P?

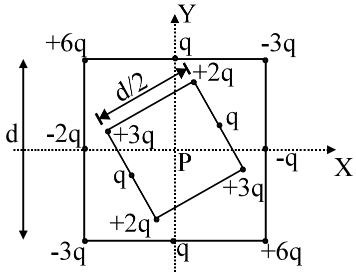


Fig. - 2

9. An electric dipole with dipole moment

$P = (3\hat{i} + 4\hat{j})(1.25 \times 10^{-30}\ C - m)$

is in an electric field $\vec{E} = (4000 N/C)\hat{j}$.

(a) What is the torque acting on the dipole?

(b) What is the potential energy of the dipole, supposing potential energy to the zero when the dipole moment makes zero angle with electric field.

(c) If an external agent turns the dipole until its electric dipole moment is $P = (-4\hat{i} + 3\hat{j})(1.25 \times 10^{-30}\ C - m)$, how much work is done by the agent?

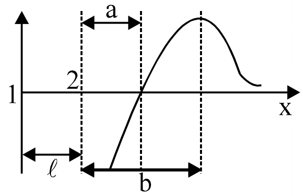
10. A line charge of infinite length with charge density λ_1 C/m is oriented along the z – axis. Another line charge of infinite length with charge density λ_2 C/m is placed parallel to the x – axis and passes through (0, a, 0). Find the force of electrostatic interaction between them.

11. Determine the electric field strength vector if the potential of the field depends on x, y coordinates as

(a) $V = a(x^2 - y^2)$ (b) $V = axy$

where a is constant.

12. Two point-like charges are positioned at point 1 and 2. The field intensity to the right of the charge Q_2 on the line that passes through the two charges varies according to a law that is represented schematically in the figure. The field intensity is assumed to be positive if its direction coincides with the positive direction on the x-axis. The distance between the charges is ℓ.



(a) Find the sign of each charge

(b) Find the ratio of the absolute value of the charges $\left|\dfrac{Q_1}{Q_2}\right|$

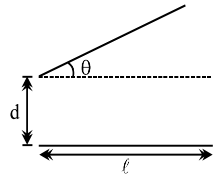
(c) Find the value of b, where the field intensity is maximum.

13. A 3 μF capacitor is charged to a potential of 300 volt & 2 μF is charged to 200 volt. the capacitor are connected in parallel, plates of same polarity being connected together. What is the final potential difference between the plates of the capacitor after they are connected. If the plates of opposite polarity are joined together after the capacitors are charged, what amount of charge will flow & from which capacitor does it come ?

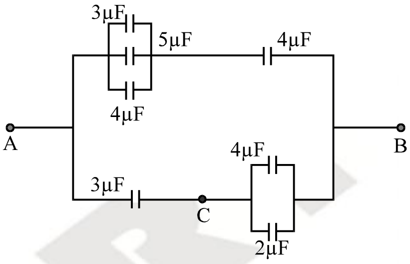
14. A parallel plate capacitor is filled with a dielectric and a certain p.d. is applied to its plates. The energy of the capacitor is 4×10^{-5} J. After the capacitor is disconnected from the power source, the dielectric is extracted from the capacitor. The work performed against the forces of the electric field in extracting the dielectric is 8×10^{-5} J. Find the dielectric constant of the dielectric.

15. A charge 200 μC is imparted to each of the two identical parallel plate capacitors connected in parallel. At t = 0, the plates of both the capacitors are 0.1 m apart. The plates of first capacitor move towards each other with velocity 0.001 m/s and plates of second capacitor move apart with the same velocity. Find the current in the circuit.

16. A capacitor has square plates, each of side a making an angle of θ with each other as shown in figure. Calculate the capacitance of the arrangement for small θ.



17. The capacitance of all the capacitors shown in figure are in micro farad. What is the equivalent capacitance between A and B ? If the charges on the 5μF capacitors is 120 μC, what is the potential difference between A and C ?



EXERCISE II

OBJECTIVE (More than one option is correct)

1. Four idenical charges are placed at the points (1, 0, 0), (0, 1, 0), (-1, 0, 0) and (0, -1, 0).
(A) The potential at the origin is zero
(B) The field at the origin is zero
(C) The potential at all points on the z-axis, other than the origin, is zero
(D) The field at all points on the z-axis, other than tthe origin, acts along the z-axis.

2. Four charges, all of the same magnitude, are placed a the four corners of a square. At the centre of the square, the potential is V and the field is E. By suitable choices of the signs of the four charges, which of the following can be obtained?
(A) $V = 0,\ E = 0$ (B) $V = 0,\ E \neq 0$ (C) $V \neq 0,\ E = 0$ (D) $V \neq 0,\ E \neq 0$

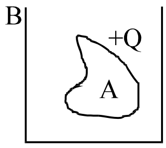
3. A particle A of mass m and charge Q moves directly towards a fixed particle B, which has charge Q. The speed of A is v when it is far away from B. The minimum separation between the particles is proportional to
(A) Q^2 (B) $\frac{1}{v^2}$ (C) $\frac{1}{v}$ (D) $\frac{1}{m}$

4. Two large, identical and parallel conducting plates have surfaces X and Y, facing each other. The charge per unit area on X is σ_1, and on Y it is σ_2.
(A) $\sigma_1 = -\sigma_2$ in all case
(B) $\sigma_1 = -\sigma_2$ only if a charge is given to one plate only
(C) $\sigma_1 = \sigma_2 = 0$ if equal charges are given to both the plates.
(D) $\sigma_1 > \sigma_2$ if X is given more charge than Y.

5. S_1 and S_2 are two equipotential surfaces on which the potentials are not equal.
(A) S_1 and S_2 cannot intersect.
(B) S_1 and S_2 cannot both be plane surfaces.
(C) in the region between S_1 and S_2, the field is maximum where they are closest to each other.
(D) A line of force from S_1 to S_2 must be perpendicular to both.

6. Two large, parallel conducting plates are placed close to each other. The inner surface of the two plates have surface charge densities $+\sigma$ and $-\sigma$. The outer surfaces are without charge. The electric field has a magnitude of
(A) $2\sigma/\varepsilon_0$ in the region between the plates (B) σ/ε_0 in the region between the plates
(C) σ/ε_0 in the region outside the plates (D) zero in the region outside the plates

7. A conducor A is given a charge of amount +Q and then placed inside a deep metal can B, without touching it.



(A) The potenial of A does not change when i is placed inside B.
(B) If B is earthed, +Q amount of charge flows from it into the earth.
(C) If B is earthed, the potential of A is reduced.
(D) Either (B) or (C) are true, or both are true only if the outer surface of B is connected o earh and not its inner surface.

8. A conducting sphere of radius R, carrying charge Q, lies inside an uncharged conducting shell of radius 2R. If they are joined by a metal wire,

(A) Q/3 amount of charge will flow from the sphere o the shell

(B) 2Q/3 amount of charge will flow from the sphere o the shell

(C) Q amount of charge will flow from the sphere o the shell

(D) $k\frac{Q^2}{4R}$ amoun of heat will be produced

9. The capacitance of a parallel-plate capacior is C0 when the region between he plates has air. This region is now filled with a dielectric slab of dielectric constant K. The capacitor is connected to a cell of emf ε, and the slab is taken out.

(A) Charge $\varepsilon C_0(K - 1)$ flows through the cell

(B) Energy $\varepsilon^2 C_0(K - 1)$ is absorbed by the cell.

(C) The energy sotred in the capacitor is reduced by $\varepsilon^2 C_0(K - 1)$.

(D) The external agent has to do $\frac{1}{2}\varepsilon^2 C_0(K-1)$ amount of work to take the slab out.

10. A parallel-plate air capacitor of capacitance C_0 is connected to a cell of emf E and then disconnected from it. A dielectric slab of dielectric constant K, which can just fill the air gap of the capacitor, is now inserted in it.

(A) The potential difference between the plates decreases K times.

(B) The energy stored in the capacitor decreases K times.

(C) The charge in energy is $\frac{1}{2}C^2 C_0(K-1)$

(D) The charge in energy is $\frac{1}{2}C^2 C_0\left(1-\frac{1}{K}\right)$

In each of the following questions, a statement of Assertion (A) is given followed by a corresponding statement of Reason (R) just below it. Of the statements, mark the correct answer -

(A) If both assertion and reason are true and reason is the correct explanation of assertion

(B) If both assertion and reason are true but reason is not the correct explanation of assertion

(C) If assertion is true but reason is false

(D) If both assertion and reason are false

(E) If assertion is false but reason is true.

11. Assertion : On going away from a point charge or small electric dipoel, electric field decreases at the same rate in both the cases.

Reason : Electric field is inversely proportional to square of distance from the charge.

12. Assertion : An applied electric field will polarize the polarize the polar dielectric material.

Reason : In polar dielectrics, each molecule has a permanent dipole moment but these are randomly oriented in the absence of an externally applied electric field.

13. Assertion : The whole charge of a conductor cannot be transferred to another isolated conducor

Reason : The total transfer of charge from one to another is not possible.

14. Assertion : A point charge is brought in an electric field. The field at a nearby point is increase, whatever be the nature of the charge.

Reason : The electric field is independent of the nature of charge.

15. **Assertion :** When charges are shared between two bodies, there occurs no loss of charge, but there does occur a loss of energy.

Reason : In case of sharing of charges energy of conservation fails.

16. Two large conducting spheres carrying charges Q_1 and Q_2 are brought close to each other. Is the magnitude of electrostatic force between them exactly given by $\frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}$ where r is the distance between their centres?

17. If Coulomb's law involved $1/r^3$ dependence (instead of $1/r^2$), would Gauss' law be still true?

18. A small test charge is released at rest at a point in an electrostatic field configuration. Will it travel along the line of force passing through that point?

19. What is the work done by the field of a nucleus in a complete circular orbit of the electron? What if the orbit is elliptical?

20. We know that electric field is discontinuous across the surface of charged conductor. Is electric potential also discontinuous there?

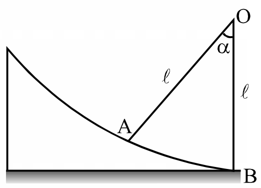
21. Guess a possible reason why water has a much greater di-electric constant (=80) than say mica(=6)

SUBJECTIVE QUESTION

1. Find the electric flux crossing the wire from ABCD of length ℓ and width b and whose centre is at a distance OP = d from an infinite line of change with linear charge density λ. Consider that the plane of frame $\perp$ to the line OP.

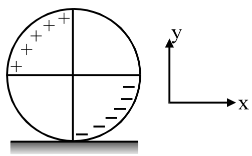
2. The inner and outer of three concentric spherical conducting shells of radii a, b and c (a < b < c) are earthed. The intermediate shell consists of two hemispherical shells in contact & carries a charge Q. Show that the hemispherical shell will separate unless $a > \frac{bc}{2c-b}$.

3. An electrometer consists of vertical metal bar at the top of which is attached a thin rod which gets deflected from the bar under the action of an electric charge (figure). The length of the rod is la and its mass is m. What will be the charge when the rod of such an electrometer is deflected through an angle a. Make the following assumptions :



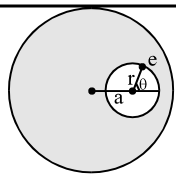
(a) the charge on the electrometer is equally distributed between the bar and the rod
(b) the charges are concentrated at point A on the rod and at point B on the bar.

4. A nonconducting ring of mass m and radius R is charged as shown. The charged density i.e. charge per unit length is λ. It is then placed on a rough nonconducting horizontal surface plane. At time t = 0, a uniform electric field $\vec{F} = E_0\hat{i}$ is switched on and the ring start rolling without sliding. Determine the friction force (magnitude and direction) acting on the ring, when it starts moving.



5. When an electrically neutral metal sphere of radius a is immersed in a uniform electric field of magnitude E, the surface charge density on the sphere is found to be, $\sigma = 3\varepsilon_0 E\cos\theta$, where θ is the angle between the direction of E and the radius to the point where σ exist. Show that total electric flux originating and terminating on the sphere is zero by integrating σ over that surface.

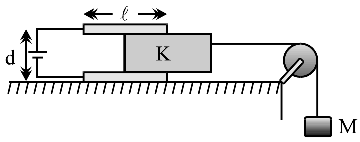
6. A cavity of radius r is present inside a solid dielectric sphere of radius R, having a volume charge density of ρ. The distance between the centres of the sphere and the cavity is a. An electron e is kept inside the cavity at an angle $\theta = 45°$ as shown. How long will it take to touch the sphere again?



7. Find potential difference V_{AB} between A(0, 0, 0) and B(1m, 1m, 1m) in an electric field.

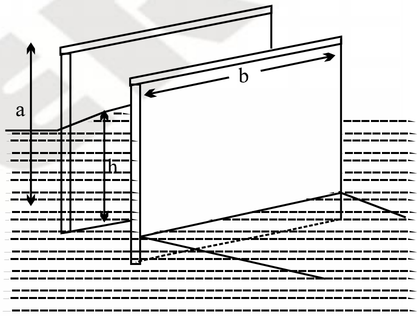
(a) $\vec{E} = y\hat{i} + x\hat{j}$ (b) $\vec{E} = 3x^2y\hat{i} + x^3\hat{j}$

8. Consider the situation shown in figure. The width of each plate is b. The capacitor plates are rigidly clamped in the laboratory and connected to a battery of emf ε. All the surfaces are frictionless. Calculate the value of M for which the dielectric slab will stay in equilibrium.



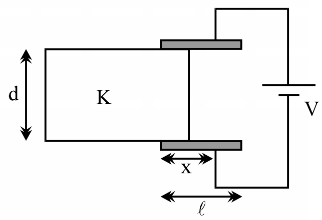
9. A parallel plate capacitor has plates with area A & separation d. A battery charges the plates to a potential difference of V_0. The battery is then disconnected & a dielectric slab of constant K & thickness d is introduced. Calculate the positive work done by the system (capacitor + slab) on the man who introduces the slab.

10. A large vessels is filled with a liquid of density ρ and dielectric constant K. Two vertical plates touch the surface of the liquid (figure). The dimensions of the plates are a and b, the distance between them is d. The plates have been charged by applying a voltage V_0 and then disconnected from the voltage source. To what height will the liquid rise > Ignore capillary effects.



11. In the above question, solve the previous problem assuming that the plates remain connected to the voltage source.

12. Figure shows a parallel plate capacitor with plate slab width b & length ℓ. The separation between the plates is d. The plates are rigidly clamped & connected to a battery of e.m.f. V. A dielectric slab of thickness d and di-electric constant K is slowly inserted between the plates.



(i) Calculate the energy of the system when a length x of the slab is introduced into the capacitor

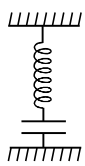
(ii) What force should be applied to the slab to ensure that it goes slowly into the capacitor ? Neglect any effect of friction or gravity ?

13. A parallel plate capacitor with air as a dielectric is arranged horizontally. The lower plate is fixed and the other connected with perpendicular spring. The area of each plate is A. In the steady position, the distance between the plates is d_0. When the capacitor is connected with an electric source with the voltage V, a new equilibrium appears, with the distance between the plates as d_1. Mass of the upper plates is m.

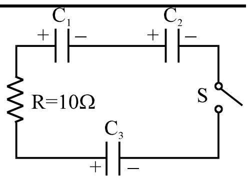
(i) Find the spring constant K

(ii) What is the maximum voltage for a given K in which an equilibrium is possible ?

(iii) what is the angular frequency of the oscillating system around the equilibrium value d_1. (take amplitude of oscillation $<< d_1$)



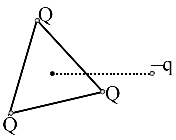
14. Three charged capacitors C_1, C_2 and C_3 of capacitances 3 μF, 6μF and 6μF respectively and having charge 30μ C each are connected with a resistor of resistance R = 10 Ω through a switch S as shown.



The switch S is closed at time t = 0. Find :
(a) the initial current in the circuit
(b) the final charge deposited on the capacitors
(c) heat dissipated in the resistor.

Passage # 1

Three charged particles each of +Q are fixed at the corners of an equilateral triangle of side ‘a’. A fourth particle of charge –q and mass m is placed at a point on the line passing through centroid of triangle and perpendicular to the plane of triangle at a distance x from the centre of triangle



39. Force on the fourth particle is

(*A) $\frac{1}{4\pi \in_0}\frac{9\sqrt{3}\,Qx}{(3x^2+a^2)^{3/2}}$ (B) $\frac{1}{4\pi \in_0}\frac{3\sqrt{3}\,Qx}{(3x^2+a^2)^{3/2}}$

(C) $\frac{1}{4\pi \in_0}\frac{2\sqrt{2}\,Qx}{(2x^2+a^2)^{3/2}}$ (D) $\frac{1}{4\pi \in_0}\frac{4\sqrt{2}\,Qx}{(2x^2+a^2)^{3/2}}$

40. Value of x for which the force is maximum is

(A) $\frac{a}{\sqrt{3}}$ (B) $\frac{a}{\sqrt{2}}$ (*C) $\frac{a}{\sqrt{6}}$ (D) $\frac{a}{\sqrt{5}}$

41. If particle is released from a distance ‘x’ its speed when it reaches the centre of triangle is

(A) $\sqrt{\frac{6\sqrt{3}\,Qq}{4\pi \in_0 m}\left[\frac{1}{a}-\frac{1}{(a^2+3x^2)^{1/2}}\right]}$ (B) $\sqrt{\frac{3\sqrt{3}\,Qq}{4\pi \in_0 m}\left[\frac{1}{a}-\frac{1}{(a^2+3x^2)^{1/2}}\right]}$

(C) $\sqrt{\frac{\sqrt{3}\,Qq}{4\pi \in_0 m}\left[\frac{1}{a}-\frac{1}{(a^2+3x^2)^{1/2}}\right]}$ (*D) $\sqrt{\frac{\sqrt{3}\,Qq}{2\pi \in_0 m}\left[\frac{1}{a}-\frac{1}{(a^2+3x^2)^{1/2}}\right]}$

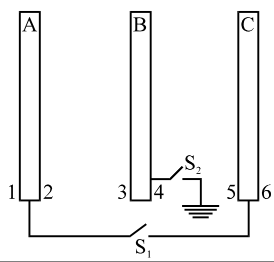
42. For small oscillation the period of oscillation of fourth particle is

(*A) $2\pi\sqrt{\frac{4\pi \in_0 ma^3}{9\sqrt{3}\,Q}}$ (B) $\pi\sqrt{\frac{4\pi \in_0 ma^3}{9\sqrt{3}\,Q}}$

(C) $2\pi\sqrt{\frac{2\pi \in_0 ma^3}{9\sqrt{3}\,Q}}$ (D) $2\pi\sqrt{\frac{\pi \in_0 ma^3}{9\sqrt{3}\,Q}}$

Passage # 2

Three large plates A, B and C are placed a distance d apart in air. The plates are numbered as shown. The plates A, B and C are given charges 7Q, 3Q and 2Q respectively. Area of each surfaces of plates is ‘S’
Following operations are conducted



X – Switch S_1 is closed
Y – Switch S_2 is closed
Z – Plate B is moved a distance d/2 right
W – Gap between A and B is filled completely with a dielectric slab of dielectric constant K = 2

43. When operation X is conducted, charge that flows through switch S_1 is
(*A) 2.5Q from plate A to B (B) 4Q from plate A to B
(C) 2.5Q from plate B to A (D) 4Q from plate B to A

44. When operation XY (X then Y) is performed charge on surface 4 is
(A) 2.5 Q (B) –2.5 Q (C) 4.5 Q (*D) –4.5 Q

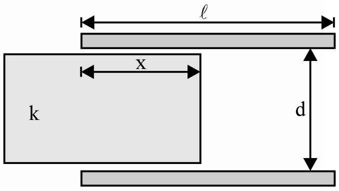
44. Work done in conducting operation Z (only) is
(A) $\frac{5Qd}{4\epsilon_0 S}$ (B) $-\frac{5Qd}{4\epsilon_0 S}$ (C) $\frac{15Qd}{4\epsilon_0 S}$ (*D) $-\frac{15Qd}{4\epsilon_0 S}$

45. Charge on plate 2 after operation XW (X then W) is
(A) +2Q (*B) –2Q (C) Q (D) –Q

46. Charge on plate 5 after operation YW (Y then W) is
(*A) 2Q (B) –2Q (C) Q (D) –Q

Passage # 3

A capacitor is constructed from two square plates of sides ℓ and separation d, as suggested in figure. You may assume that d is much less than ℓ. The plates carry charges $+Q_0$ and $-Q_0$. A block of metal has width ℓ, length ℓ, and thickness slightly less than d. It is inserted a distance x into the capacitor. The charges on the plates are not disturbed as the block slides in. The plates having a thin insulation over it.



47. The stored energy as a function of x is
(*A) $Q_0^2 d(\ell - x)/(2\ell^3 \epsilon_0)$ (B) $Q_0^2 d(\ell - x)/(\ell^3 \epsilon_0)$
(C) $Q_0^2 d/[2\ell \epsilon_0 (\ell - x)]$ (D) $Q_0^2 d/[\ell \epsilon_0 (\ell - x)]$

48. The direction and magnitude of the force that acts on the metallic block is
(A) $Q_0^2 d/(2\ell^3 \epsilon_0)$ to the left (*B) $Q_0^2 d/(2\ell^3 \epsilon_0)$ to the right
(C) $Q_0^2 d/(2\ell \epsilon_0)$ to the left (D) $Q_0^2 d/(2\ell \epsilon_0)$ to the right

49. The area of the advancing front face of the block is essentially equal to ℓd. Considering the force on the block as acting on this face, then the stress (force per area) on it is
(*A) $Q_0^2/(2\ell^4 \epsilon_0)$ (B) $Q_0^2/(2\ell^2 \epsilon_0)$ (C) $Q_0^2/(2\ell \epsilon_0)$ (D) $2Q_0^2/(2\ell \epsilon_0)$

50. If a metal block is released with its ℓ_0 length between the plates it starts oscillating its period of oscillation is (mass of block is m, there are no friction between plates)
(*A) $4\sqrt{\frac{4(\ell - \ell_0)\ell^3 \epsilon_0 m}{Q_0^2 d}}$ (B) $2\sqrt{\frac{4(\ell - \ell_0)\ell^3 \epsilon_0 m}{Q_0^2 d}}$
(C) $\sqrt{\frac{4(\ell - \ell_0)\ell^3 \epsilon_0 m}{Q_0^2 d}}$ (D) $\frac{1}{2}\sqrt{\frac{(\ell - \ell_0)\ell^3 \epsilon_0 m}{Q_0^2 d}}$

Passage # 4

In a T. V. serial "Son Pari". A pari has stick carrying positive charge Q over it and a white beam of positive charge q partical revolving around it. Ring remains in a stretched position around the stick. The radius of the ring is R, mass m and cross-sectional area is A. The length of the stick is L where L = R. Neglect the effect of gravity. (Given : Modulus of elasticity is Y)

51. Find the force required by the pari to hold the stick

(A) $\frac{kqQ(\sqrt{2}-1)}{\sqrt{2}\ R^2}$ (B) $\frac{kqQ(\sqrt{2}+1)}{\sqrt{2}\ R^2}$ (C) $\frac{kqQ}{2\sqrt{2}\ \pi R^2}$ (D) $\frac{kqQ(\sqrt{2}+1)}{2\sqrt{2}\ R^2}$

52. The Tension force acting on the particle of the ring to make it in the equilibrium.

(A) $\frac{kqQ}{\sqrt{2}\ \pi R^2}$ (B) $\frac{kqQ}{2\sqrt{2}\ \pi R^2}$ (C) $\frac{kqQ(\sqrt{2}-1)}{\sqrt{2}\ R^2}$ (D) $\frac{kqQ(\sqrt{2}+1)}{2\sqrt{2}\ R^2}$

53. The initial acceleration of the ring is

(A) $\frac{kqQ(\sqrt{2}-1)}{\sqrt{2}\ mR^2}$ (B) $\frac{kqQ}{2\sqrt{2}\ \pi mR^2}$ (C) $\frac{kqQ(\sqrt{2}+1)}{2\sqrt{2}\ mR^2}$ (D) $\frac{kqQ}{\sqrt{2}\ \pi mR^2}$

54. The change in the radius of the ring due to electrostatic repulsion force given that Young's modulus of elasticity is Y

(A) $\frac{kqQ}{2\sqrt{2}\ \pi RAY}$ (B) $\frac{kqQ}{\sqrt{2}\ \pi RAY}$ (C) $\frac{\sqrt{2}\ kqQ}{\pi RAY}$ (D) $\frac{2\sqrt{2}\ kqQ}{\pi RAY}$

55. The velocity of the ring when the ring reaches the infinity

(A) $\sqrt{\frac{kqQ}{mR}\log\left(\sqrt{2}\right)}$ (B) $\sqrt{\frac{2kqQ}{mR}\log\left(\sqrt{2}\right)}$ (C) $\sqrt{\frac{2kqQ}{mR}\log\left(2\sqrt{2}\right)}$ (D) None of these

EXERCISE - III (IIT JEE ASKED PROBLEMS)

1. Two fixed charges –2Q and Q are located at the points with coordinates (–3a, 0) and (+3a, 0) respectively in the x-y plane.

(a) Show that all points in the x-y plane where the electric potential due to the two charges is zero, lie on a circle. Find its radius and the location of its centre.

(b) Give the expression V(x) at a general point on the x-axis and sketch the function V(x) on the whole x-axis.

(c) If a particle of charge +q starts form rest at the centre of the circle, show by a short qualitative argument that the particle eventually crosses the circle. Find its speed when it does so.

[1991, 4+2+2M]

2. A parallel plate capacitor of plate area A & plate separation d is charged to a potential difference V & then the battery is disconnected. A slab of dielectric constant K is then inserted between the plates of the capacitor so as to fill the space between the plates. If Q, E & W denote respectively, the magnitude of the charge on each plate, the electric field between the plates (after the slab is inserted) & work done on the system, in the process of inserting the slab, then : **[1991, 2M]**

(A) $Q = \dfrac{\epsilon_0 AV}{d}$ (B) $Q = \dfrac{\epsilon_0 KAV}{d}$ (C) $E = \dfrac{V}{Kd}$ (D) $W = -\dfrac{\epsilon_0 AV^2}{2d}\left(1-\dfrac{1}{K}\right)$

3. (a) A charge of Q coulomb is uniformly distributed in a spherical volume of radius R metres. Obtain an expression for the energy of the system

(b) What will be the corresponding expression for the energy needed to completely disassemble the planet earth against the gravitational pull amongst its constituent particles?

Assume the earth to be a sphere of uniform mass density. Calculate this energy, given the product of the mass and the radius of the earth to be 2.5×10^{31} kg.m.

(c) If the same charge of Q coul. as in part (a) above is given to a spherical conductor of the same radius R, what will be energy of the system? **[1992, 10M]**

4. Two identical thin rings, each of radius R metres, are coaxially placed a distance R metres apart. If Q_1 coul. and Q_2 coul. are respectivley the charges uniformly spread on the two rings, the work done in moving a charge q from the centre of one ring to that of the other is **[1992, 2M]**

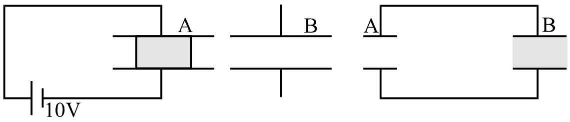
(A) zero (B) $\dfrac{q(Q_2-Q_1)(\sqrt{2}-1)}{(4\sqrt{2}\,\pi\epsilon_0 R)}$

(C) $\dfrac{q\sqrt{2}(Q_1+Q_2)}{(4\pi\epsilon_0 R)}$ (D) $\dfrac{q(Q_1+Q_2)(\sqrt{2}+1)}{(4\sqrt{2}\,\pi\epsilon_0 R)}$

5. The electric potential V at any point x, y, z (all in metres) in space is given by $V = 4x^2$ volts. The electric field at the point (1m, 0.2m) is .... V/m. **[1992, 1M]**

6. A circular ring of radius R with uniform positive density λ per unit length is located in the y-z plane with its centre at the origin O. A particle of mass m and positive charge q is projected from the point $P\left(R\sqrt{3},\ 0,\ 0\right)$ on the positive x-axis directly towards O with an initial speed v. Find the smallest (non zero) value of the speed v, such that the particle does not return to P. **[1993, 4M]**

7. Two parallel plate capacitors A & B have the same separation $d = 8.85 \times 10^{-4}$ m between the plates. The plate areas of A & B are $0.04\ m^2$ & $0.02\ m^2$ respectively. A slab of dielectric constant (relative permittivity) K = 9 has dimensions such that it can exactly fill the space between the plates of capacitor B.



(i) the di-electric slab is placed inside A as shown in the figure. A is then charged to a potential difference of 110 volt. Calculate the capacitance of A and the energy stored in it.
(ii) the battery is disconnected & then the dielectric slab is removed from A. Find the work done by the external agency in removing the slab from A. **[1993, 7M]**
(iii) the same di-electric slab is now placed inside B, filling it completely. The two capacitors A and B are then connected as shown in figure. Calculate the energy stored in the system.

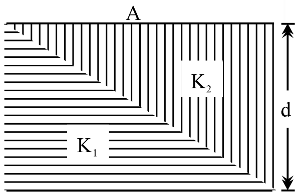
8. Two square metallic plates of 1 m side are kept 0.01 m apart, like a parallel plate capacitor, in air in such a way that one of their edges is perpendicular, to an oil surface in a tank filled with an insulating oil. The plates are connected to a battery of e.m.f. 500 volt. the plates are then lowered vertically into the oil at a speed of 0.001 m/s. Calculate the current drawn from the battery during the process.
[di-electric constant of oil = 11, $\in_0 = 8.85 \times 10^{-12}\ C^2/N^2\ m^2$] **[1994, 6M]**

9. Two point charges +q and –q are held fixed at (–d, 0) and (d, 0) respectively. For (x, y) co-ordinate system
(A) the electric field E at all point on the x-axis has the same direction
(B) E at all points of y-axis is along i
(C) work has to be done in bringing a test charge from infinity to the origin
(D) the dipole moment is 2qd along –i. **[1995, 2M]**

10. A parallel plate capacitor C is connected to a battery & is charged to a potential difference V. Another capacitor of capacitance 2C is similarly charged to a potential difference 2V volt. The charging battery is now disconnected & the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of other. The final energy of the configuration is - **[1995, 1M]**

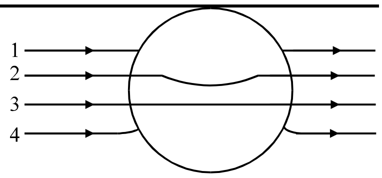
(A) zero (B) $\frac{3}{2}CV^2$ (C) $\frac{25}{6}CV^2$ (D) $\frac{9}{2}CV^2$

11. The capacitance of a parallel plate capacitor with plate area 'A' & separation d is C. The space between the plates is filled with two wedge of di-electric constant K_1 & K_2 respectively. Find the capacitance of the resulting capacitor. **[1996, 2M]**



12. The magnitude of electric field $\vec{E}$ in the annular region of charged cylindrical capacitor
(A) Is same throughout
(B) Is higher near the outer cylinder than near the inner cylinder
(C) Varies as (1/r) where r is the distance from the axis
(D) Varies as $(1/r^2)$ where r is the distance from the axis
[1996, 2M]

13. A metallic solid sphere is placed in a uniform electric field. The lines of force follow the path (s) shown in figure as : **[1996, 5M]**



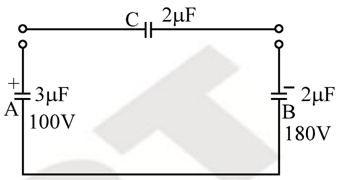
(A) 1 (B) 2
(C) 3 (D) 4

14. A non-conducting ring of radius 0.5 m carries a total charge of 1.11×10^{-10} C distributed non-uniformly on its circumference producing an electric field E every where in space. The value of the line integral

$\int_{\ell=\infty}^{\ell=0} -\text{E}.d\ell$ ($\ell = 0$ being centre of the ring) in volts is : **[1997, 1M]**

(A) + 2 (B) –1 (C) –2 (D) zero

15. Two capacitors A and B with capacities 3μF and 2μF are charged to a potential difference of 100V and 180V respetively. The plates of the capacitors are connected as shown in figure with one wire from each capacitor free. The upper plate of A is positive and that of B is negative. An uncharged 2μF capacitor C with lead wires falls on the free ends to complete the circuit. Calculate :



(i) the final charges on the three capacitors
(ii) the amount of electrostatic energy stored in the system before and after the completion of the circuit. **[1997(C), 4M]**

16. An electron enters the region between the plates of a parallel plate capacitor at a point equidistant from either plate. The capacitor plates are 2×10^{-2} m apart & 10^{-1} m long. A potential difference of 300 volt is kept across the plates. Assuming that the initial velocity of the electron is parallel to the capacitor plates, calculate the largest value of the velocity of the electron so that they do not fly out of the capacitor at the other end. **[1997, 5M]**

17. Select the correct alternative :
(A) A positively charged thin metal ring of radius R is fixed in the xy-plane with its centre at the origin O. A negatively charged particle P is released from rest at the point $(0, 0, z_0)$ where $z_0 > 0$. Then the motion of P is **[1998, 2+2+2M]**

(i) (A) Periodic, for all values of z_0 satisfying $0 < z_0 < \infty$
(B) simple harmonic, for all values of z_0 satisfying $0 < z_0 \le R$
(C) approximately simple harmonic, provided $z_0 << R$
(D) such that P crosses O and continues to move along the –ve z-axis towards $x = -\infty$

(ii) A charge +q is fixed at each of the point $x = x_0, x = 3x_0, x = 5x_0, \ldots\ldots \infty$ on the x-axis and a charge –q is fixed at each of the points $x = 2x_0, x = 4x_0, x = 6x_0, \ldots\ldots \infty$. Here x_0 is +ve constant. Take the electric potential at a point due to a charge Q at a distance r from it to be $\frac{Q}{4\pi\varepsilon_0 r}$. Then the potential at the origin due to the above system of charges is :

(A) 0 (B) $\frac{q}{4\pi\varepsilon_0 x_0 \ln 2}$ (C) ∞ (D) $\frac{q \ln 2}{4\pi\varepsilon_0 x_0}$

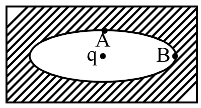
(iii) A non-conducting solid sphere of radius R is uniformly charged. The magnitude of the electric field due to the sphere at a distance r from its centre :
(A) increases as r increases, for r < R
(B) decreases as r increases, for $0 < r < \infty$
(C) decreases as r increases, for $R < r < \infty$
(D) is discontinuous at r = R

18. A conducting sphere S_1 of radius r is attached to an insulating handle. Another conducting sphere S_2 of radius R is mounted on an insulating stand. S_2 is initially uncharged. S_1 is given a charge Q, brought into contact with S_2 & removed, S_1 is recharged such that the charge on it is again Q and it is again brought into contact with S_2 and removed. This procedure is repeated n times.

(a) Find the electrostatic energy of S_2 after n such contacts with S_1

(b) What is the limiting value of this energy as $n \to \infty$? **[1998, 7 + 1M]**

19. (i) An ellipsoidal cavity is carved within a perfect conductor. A positive charge q is placed at the centre of the cavity. The points A and B are on the cavity surface as shown in figure. Then :



(A) electric field near A in the cavity = electric field near B in the cavity

(B) charge density at A = charge density at B

(C) potential at A = potential at B

(D) total electric flux through the surface of the cavity is q/ε_0.

(ii) A non-conducting disc of radius a and uniform positive surface density σ is placed on the ground, with its axis vertical,. A particle of mass m and positive charge q is dropped, along the axis of the disc, from a height H with zero initial velocity. The particle has $\frac{q}{m} = \frac{4\varepsilon_0 g}{\sigma}$.

(a) Find the value of H if the particle just reaches the disc.

(b) Sketch the potential energy of the particle as a function of its height and find its equilibrium position. **[1999, 3+5+5M]**

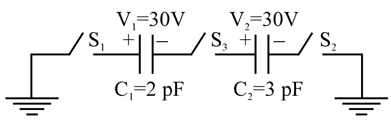
20. For the circuit shown, which of the following statements is true ? **[1999, 2M]**

(A) with S_1 closed $V_1 = 15$ V, $V_2 = 20$ V

(B) with S_3 closed, $V_1 = V_2 = 25$ V

(C) with S_1 & S_2 closed, $V_1 = V_2 = 0$

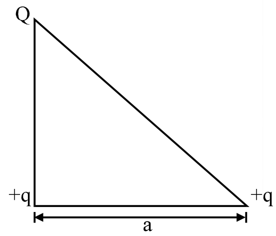
(D) with S_1 & S_2 closed, $V_1 = 30$ V, $V_2 = 20$ V



21. (a) The dimension of $\left(\frac{1}{2}\right)\varepsilon_0 E^2$ (ε_0 : permittivity of free space; E : electric field) is :

(A) MLT^{-1} (B) ML^2T^{-2} (C) MLT^{-2} (D) ML^2T^{-1} (E) $ML^{-1}T^{-2}$

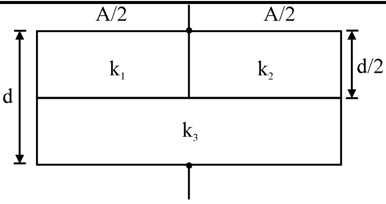
(b) Three charges Q, +q and +q are placed at the vertices of a right-angled isosceles triangle as shown. The net electrostatic energy of the configuration is zero if Q is equal to :



(A) $-\frac{q}{1+\sqrt{2}}$ (B) $-\frac{2q}{2+\sqrt{2}}$ (C) –2q (D) +q

(c) Four point charges + 8μC, –1μC, –1μC and +8 μC, are fixed at the points, $-\sqrt{\frac{27}{2}}$ m, $-\sqrt{\frac{3}{2}}$ m, $+\sqrt{\frac{3}{2}}$ m and $+\sqrt{\frac{27}{2}}$ m respectively on the y-axis. A particle of mass 6×10^{-4} kg and of charge +0.1 μC moves along the –x direction. Its speed at $x = +\infty$ is v_0. Find the least value of v_0 for which the particle will cross the origin. Find also the kinetic energy of the particle at the origin. Assume that space is gravity free. (Given : $1/(4\pi\varepsilon_0) = 9 \times 10^9$ Nm^2/C^2) **[2000, 10M]**

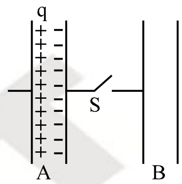
22. A parallel plate capacitor of area A, placed at a separation d and capacitance C is filled with three different dielectric materials having dielectric constants k_1, k_2 and k_3 as shown. If a single dielectric material is to be used to have the same capacitance C in this capacitor then its dielectric constant k is given by **[2000, 3M]**



(A) $\frac{1}{k}=\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{2k_3}$ (B) $\frac{1}{k}=\frac{1}{k_1+k_2}+\frac{1}{2k_3}$

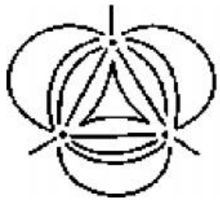
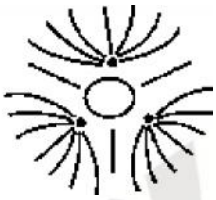
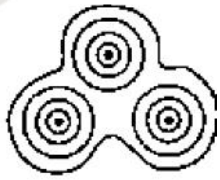
(C) $k=\frac{k_1k_2}{k_1+k_2}+\frac{1}{2k_3}$ (D) $k=\frac{k_1k_3}{k_1+k_3}+\frac{k_2k_3}{k_2+k_3}$

23. Consider the situation shown in the figure. The capacitor A has a charge q on it whereas B is uncharged. The charge appearing on the capacitor B a long time after the switch is closed is **[2001(S), 3M]**



(A) zero (B) q/2 (C) q (D) 2q

24. Three positive charges of equal value q are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in **[2001(S), 3M]**

(A)  (B)  (C)  (D) 

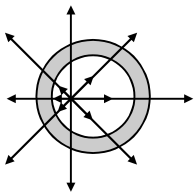
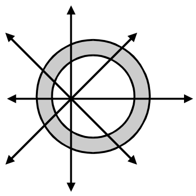
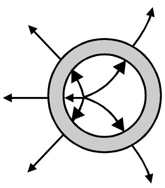
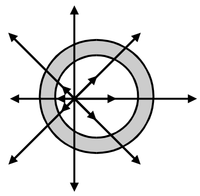
25. Two equal point charges are fixed at x = –a and x = +a on the x-axis. Another point charge Q is placed at the origin. The change in the electrical potential energy of Q, when it is displaced by a small distance x along the x-axis, is approximately proportional to **[2002(S), 3M]**

(A) x (B) x^2 (C) x^3 (D) 1/x

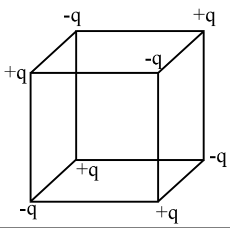
26. Two identical capacitors, have the same capacitance C. One of them is charged to potential V_1 and the other to V_2. The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the combined system is - **[2002(S), 3M]**

(A) $\frac{1}{4}C(V_1^2-V_2^2)$ (B) $\frac{1}{4}C(V_1^2+V_2^2)$ (C) $\frac{1}{4}C(V_1-V_2)^2$ (D) $\frac{1}{4}C(V_1+V_2)^2$

27. A point charge 'q' is placed at a point inside a hollow conducting sphere. Which of the following electric force pattern is correct? **[2003(S), 3M]**

(A)  (B)  (C)  (D) 

28. Charges +q and –q are located at the corners of a cube of side a as shown in the figure. Find the work done by electric force to separate the charges to infinite distance. **[2003, 2M]**

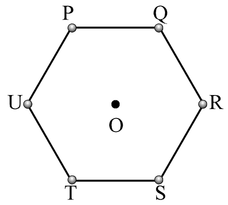


29. A charge +Q is fixed at the origin of the co-ordinate system while a small electric dipole of dipole-moment pointing away from the charge along the x-axis is set free from a point far away from the origin. **[2003, 4M]**

(a) calculate the K.E. of the dipole when it reaches to a point (d, 0)

(b) calculate the force on the charge +Q at this moment.

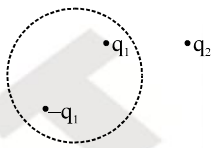
30. Six charges of equal magnitude, 3 positive and 3 negative are to be placed on PQRSTU corners of a regular hexagon, such that field at the centre is double that of what it would have been if only one +ve charge is placed at R. **[2004, 3M]**



(A) +, +, +, –, –, – (B) –, +, +, +, –, –

(C) –, +, +, –, +, – (D) +, –, +, –, +, –

31. A Gaussian surface in the figure is shown by dotted line. The electric field on the surface will be



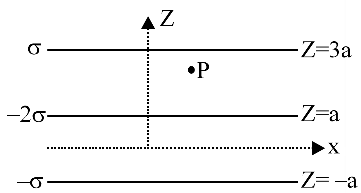
(A) due to q_1 and q_2 only (B) due to q_2 only

(C) zero (D) due to all

32. Two uniformly charged large plane sheets S_1 and S_2 having charge densities σ_1 and σ_2 $(\sigma_1 > \sigma_2)$ are placed at a distance d parallel to each other. A charge q_0 is moved along a line of length a (a < d) at an angle 45° with the normal to S_1. Calculate the work done by the electric field. **[2004, 4M]**

33. A conducting liquid bubble of radius a and thickness t (t << a) is charged to potential V. If the bubble collapses to a droplet, find the potential on the droplet. **[2005, 2M]**

34. Three infinitely long charge sheets are placed as shown in figure. The electric field at point P is **[2005S, 3M]**



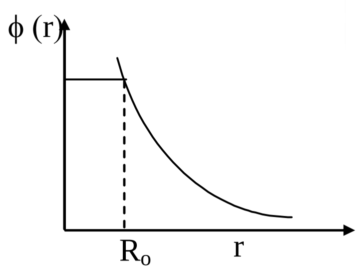
(A) $\frac{2\sigma}{\epsilon_0}\hat{k}$ (B) $\frac{4\sigma}{\epsilon_0}\hat{k}$

(C) $-\frac{2\sigma}{\epsilon_0}\hat{k}$ (D) $-\frac{4\sigma}{\epsilon_0}\hat{k}$

35. The following graph represents the variation of potential $\phi(r)$ with radius r of a sphereical region.

Given $\phi = \frac{Q}{4\pi \epsilon_0 R_0}$ $r \le R_0$

$\phi = \frac{Q}{4\pi \epsilon_0 r}$ $r > R_0$



Which of the following is/are correct?

(A) A spherical symmetry at $r = 2R_0$ encloses a net charge Q

(B) Electric field is discontinued at $r = R_0$

(C) Change is only present at $r = R_0$

(D) Electrostatic energy is zero for $r < R_0$