JEE EXPERT

PRACTICE TEST – 02 (30 MARCH 2020)

ANSWER KEY & SOLUTION

Physics [PART-I]

1.	**B**	2.	**B**	3.	**B**	4.	**B**
5.	**C**	6.	**D**	7.	**D**	8.	**A, D**
9.	**A, C**	10.	**A, B**	11.	**A, D**	12.	**B**
13.	**A**	14.	**A**	15.	**D**	16.	**C**
17.	**2**	18.	**2**	19.	**7**	20.	**1**
21.	**4**	22.	**4**	23.	**3**		

Chemistry [PART-II]

1.	**C**	2.	**D**	3.	**C**	4.	**A**
5.	**D**	6.	**B**	7.	**C**	8.	**A, B, C**
9.	**B, C, D**	10.	**A, C, D**	11.	**C, D**	12.	**B**
13.	**D**	14.	**B**	15.	**A**	16.	**D**
17.	**8**	18.	**1**	19.	**8**	20.	**5**
21.	**3**	22.	**4**	23.	**2**		

Mathematics [PART-III]

1.	**A**	2.	**C**	3.	**D**	4.	**A**
5.	**B**	6.	**B**	7.	**B**	8.	**A, B**
9.	**A, C**	10.	**B, C**	11.	**B, C**	12.	**B**
13.	**A**	14.	**B**	15.	**B**	16.	**D**
17.	**2**	18.	**3**	19.	**2**	20.	**3**
21.	**3**	22.	**1**	23.	**0**		

HINTS AND SOLUTIONS

PHYSICS

1. We have to calculate the work done by F, and not the work done by total external force.

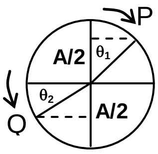
2. Angular momentum of the system about centre of mass of rod should be zero before and after collision.

3. $F_{avg} = \dfrac{\int_0^t F_{inst}\,dt}{\int_0^t dt}$

Acceleration will be zero when mg = kx
Use SHM equation for simplicity

4. Take the component of velocity perpendicular to incline and apply doppler's effect

6. Plot the phase diagram



7. Assume temperature of B as T_B, and use heat flow equation from A $\rightarrow$ B $\rightarrow$ C

9. Since the collision is elastic, for the ball to return to its original state, X should be half of the total horizontal range

10. Maximum velocity of A and B = ωA

$\nu_{max} = \left(\dfrac{v+v_D}{v-v_s}\right)\nu_0$ and $\nu_{min} = \left(\dfrac{v-v_D}{v+v_s}\right)\nu_0$

11. Use PV = nRT and VP^2 = constant to get the relations

12-13. In lower $h_0/2$, force on block will be upward and in upper $h_0/2$, force will be downward.
Find if force is proportional to x (displacement from mean position) and use, SHM equations to get time period.

17. Use trigonometry to add (superimpose) the waves, and get the resultant equation.

19. No sleeping means relative acceleration between the contact point is zero. For sphere the acceleration of top point is a_1 and bottom is a_0.

20. Centre of mass of the combined system does not change.

21. Draw the standing wave diagram, with nodes and antinodes according to condition given.

22. Assume the maximum displacement to be x, and conserve initial and final energy
Surface energy initial + KE_i = Surface energy final + KE_f

23. Conserve energy at initial and final points.

CHEMISTRY

3. $K_1 = \dfrac{A^2}{[A_2]} \Rightarrow A^2 = K_1\,A_2$ --------------- (i)

$$K_2 = \frac{B^2}{[B_2]} \Rightarrow B^2 = K_2\ B_2 \quad \text{--------------- (ii)}$$

$$\text{Rate} = K\ A\ B = K_1^{\frac{1}{2}}\ A_2^{\frac{1}{2}} . K_2^{\frac{1}{2}}\ B_2^{\frac{1}{2}} = K'\ A_2^{\frac{1}{2}}\ B_2^{\frac{1}{2}}$$

4. As the +M power increases the pKa value of phenol increases

5. $Ag_2CrO_4 \rightleftharpoons 2Ag^+ + CrO_4^{2-}$

$$K_{sp} = \left[Ag^+\right]^2 \left[CrO_4^{2-}\right]$$

$$\Rightarrow 2.4 \times 10^2 = \left(\frac{20 \times 0.001}{1020}\right)^2 \left[CrO_4^{2-}\right]$$

$$\left[CrO_4^{2-}\right] = 6.24 \times 10^{-3} M$$

After solving $[Ag^{+1}] = 1.96 \times 10^{-5}$ M

6. $I_2 + 2Na_2S_2O_3 \longrightarrow 2NaI + Na_2S_4O_6$
Moles of I_2 liberated = 1.7 m mole
Moles of OCl^- = 1.7 m mole
Moles of Cl in bleaching powder = 3.4 m mole = 0.12 gm Cl

$$\% \text{ of Cl} = \frac{0.12}{0.6} \times 100 = 20$$

10. P^{OH} of basic buffer solution is

$$P^{OH} = P^{kb} + \log\frac{[salt]}{[Base]}$$

P^H of acidic buffer solution is

$$P^H = 14 - P^{kb} - \log\frac{[salt]}{[Base]}$$

So, (A, C, D) incorrect option.

14. Bond angle
$H_2O = 104.5°$, $NH_3 = 107°$, $OF_2 = 103°$

17. CO :- $K\ K\sigma_{2s}^2\ \overset{*}{\sigma}_{2s}^2\ \pi_{2p_x}^2\ \pi_{2p_y}^2\ \sigma_{2p_z}^2$

$$\text{Bond order} = \frac{1}{2}\ 8 - 2 = 3$$

CN^- :- $K\ K\sigma_{2s}^2\ \overset{*}{\sigma}_{2s}^2\ \pi_{2p_x}^2\ \pi_{2p_y}^2\ \sigma_{2p_z}^2$

$$\text{Bond order} = \frac{1}{2}\ 8 - 2 = 3$$

O_2 : Bond order = 2
$\therefore$ sum of the bond orders = 3+3+2 = 8.

18. In I order reaction, the time taken for completion of 75% is equal to 2 times to the half life period.
$t_{75\%} = 2t_{1/2} = 2 \times 0.5 = 1$ min.

19. Use $r \propto \frac{P}{\sqrt{M}}$

20.

	$N_2O_4 \rightleftharpoons$	$2NO_2$
Initial moles	1	0
After reaction	1 - α	2α

$\therefore$ Total moles at equilibrium = 1 + α = 1 + 0.66 = 1.66
1 mole of $N_2O_4 \rightarrow$ 1.66 mole at equilibrium

$\frac{10}{92}$ mole of $N_2O_4 \rightarrow \frac{1.66\times10}{92}$ = 0.18 mole at equilibrium

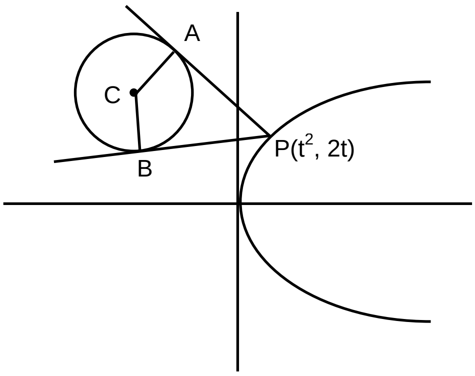
$PV = \frac{w}{m}RT$

$\Rightarrow V = \frac{nRT}{P} = \frac{0.18\times0.082\times340}{1} = 5$ lit.

21. Rate = K $[A]^a$ $[B]^b$
1×10^{-4} = K $(0.1)^a$ $(0.1)^b$ ----------------- (i)
8×10^{-4} = K $(0.2)^a$ $(0.2)^b$ ------------------ (ii)
2×10^{-4} = K $(0.1)^a$ $(0.2)^b$ ------------------ (iii)
Solve equation (i), (ii) & (iii) then
a = 2, b = 1
$\therefore$ order = a + b = 2 + 1 = 3.

MATHEMATICS

1. Centre of circle C $\equiv$ (– 3, 2). Let the co-ordinate of P $\equiv$ (t^2, 2t), then APBC is a cyclic quadrilateral, hence circumcentre of ΔPAB is the mid-point of CP hence



$h = \frac{t^2-3}{2} \Rightarrow t^2 = 2h + 3$

$k = \frac{2t+2}{2} \Rightarrow t = k - 1.$

Hence locus is $(y - 1)^2 = 2x + 3$.

3. We have $\frac{1}{^{3n}C_r} = \frac{(3n-r)!\,r!}{(3n)!} = \frac{3n+1}{3n+2}\left[\frac{\{(3n+1-r)+(r+1)\}\{(3n-r)!\,r!\}}{(3n+1)!}\right]$

$= \frac{3n+1}{3n+2}\left[\frac{(3n+1-r)!\,r! + (3n-r)!\,(r+1)!}{(3n+1)!}\right]$

$= \left[\frac{1}{^{3n+1}C_r} + \frac{1}{^{3n+1}C_{r+1}}\right]\frac{3n+1}{3n+2}.$

So, $\sum_{r=1}^{3n-1}\frac{(-1)^{r-1}r}{^{3n}C_r} = \sum_{r=1}^{3n-1}(-1)^{r-1}r\left[\frac{1}{^{3n+1}C_r} + \frac{1}{^{3n+1}C_{r+1}}\right]\left(\frac{3n+1}{3n+2}\right)$

$= \left[\left(\frac{1}{C_1}+\frac{1}{C_2}\right) - 2\left(\frac{1}{C_2}+\frac{1}{C_3}\right) + 3\left(\frac{1}{C_3}+\frac{1}{C_4}\right) - \ldots + (3n-1)\left(\frac{1}{C_{3n-1}}+\frac{1}{C_{3n}}\right)\right]\left(\frac{3n+1}{3n+2}\right)$

[where $C_r = {}^{3n+1}C_r$]

$= \left(\frac{3n+1}{3n+2}\right)\left[\frac{1}{C_1} - \frac{1}{C_2} + \frac{1}{C_3} - \frac{1}{C_4} + \ldots + \frac{1}{C_{3n-1}} - \frac{1}{C_{3n}} + \frac{3n}{C_{3n}}\right]$

$= \frac{3n+1}{3n+2}\cdot\frac{3n}{3n+1} = \frac{3n}{3n+2}.$

4. $\left(y+\frac{2}{3}\right)^2 = 2\left(x - \frac{10}{9}\right).$

Let Y = y + 2/3 ; X = x – 10/9
$Y^2 = 2X$ becomes the equation of parabola with reference to the new origin.
Hence equation of normal
$Y = mX - m - m^3/2$
(Since the three normals are drawn from point on the axis (H, 0) (say))

$H = 1 + \frac{m^2}{2} \Rightarrow m = \pm\sqrt{2H-2}$

i.e. H > 1

i.e. h – 10/9 > 1 $\Rightarrow$ h > 19/9 (h being the abscissa w. r.t the previous coordinate system)

Hence the points is $\left(h, -\frac{2}{3}\right)$ where $h > \frac{19}{9}$.

5. $\frac{z+i}{z-i}$ is purely imaginary $\Rightarrow$ arg (z + i) – arg (z –i) = $\pm\frac{\pi}{2}$

$\Rightarrow$ z lies on the circle whose diameter has the end points i and –i.

6. $B^2 = A - 2 \Rightarrow A = B^2 + 2$ or $\frac{A}{B} = B + \frac{2}{B} \geq 2\sqrt{B\frac{2}{B}} = 2\sqrt{2}$.

7. Centre of the required circle is the reflection of the point (0, 0) in the line y = mx + m.

Let C (h, k) be the centre of the reflected circle

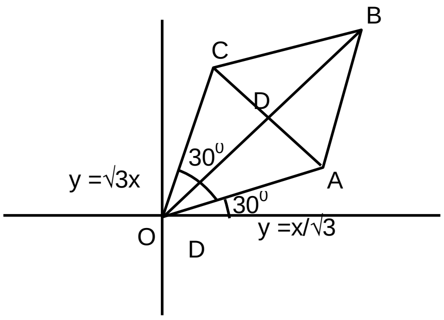
$\Rightarrow \frac{k}{h} = -\frac{1}{m}$ ……(1)

and $\frac{k}{2} = m\frac{h}{2} + m$ ……(2)

$\Rightarrow k = m(-km) + 2m \Rightarrow k = \frac{2m}{1+m^2}$

$\therefore$ C (h, k) is $\left(-\frac{2m^2}{1+m^2}, \frac{2m}{1+m^2}\right)$.

8. $\angle COA = 30^0$



Area of rhombus = $2 \cdot \frac{1}{2}$ OA · OC sin 30^0

$2 = 2 \cdot \frac{1}{2} x^2 \frac{1}{2}$

OA = OC = 2, $\angle OAB = 150^0$

$\cos 150^0 = \frac{OA^2 + AB^2 - OB^2}{2OA \cdot OB}$

$OB^2 = 4 + 4 + 2 \cdot 2 \cdot 2\frac{\sqrt{3}}{2} = 8 + 4\sqrt{3}$

$OB = \sqrt{2}\left(\sqrt{3}+1\right)$.

Coordinates of $B\left(\pm\sqrt{2}\left(\sqrt{3}+1\right)\cos 45^0, \pm\sqrt{2}\left(\sqrt{3}+1\right)\sin 45^0\right)$.

9. $(4a - 5b)^2 - c^2 = 0$

$\Rightarrow$ (4a – 5b +c) (4a –5b –c) = 0

either 4a –5b + c = 0, or, 4a – 5b – c = 0 $\Rightarrow$ – 4a + 5b + c = 0

10. Equation of tangent at $\left(4\cos\theta, \frac{16}{\sqrt{11}}\sin\theta\right)$ is

$16x(4\cos\theta) + 11y\left(\frac{16}{\sqrt{11}}\sin\theta\right) = 256$

It touches $(x - 1)^2 + y^2 = 4^2$ if

$\left|\frac{4\cos\theta - 16}{\sqrt{16\cos^2\theta + 11\sin^2\theta}}\right| = 4 \Rightarrow (\cos\theta - 4)^2 = 16\cos^2\theta + 11\sin^2\theta$

$\therefore 4\cos^2\theta + 8\cos\theta - 5 = 0$

$\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3} \quad \therefore \theta = \frac{\pi}{3}, \frac{5\pi}{3}.$

11. Point R(α, β) satisfies $\alpha^2 + \beta^2 - 1 \le 0$
$\Rightarrow$ R lies in the circle $x^2 + y^2 = 1$.
Area of triangle PQR will be maximum/minimum if tangent to circle is parallel to the chord PQ.

12. We can select three different letters in ${}^8C_3$ ways. Suppose we select A, L, M
Let X = set of words in which A is absent,
Y = set of words in which L is absent and
Z = set of words in which M is absent.
Then $n(X\cup Y\cup Z) = (2^6 + 2^6 + 2^6) - (1^6 + 1^6 + 1^6) + 0 = 189$.
So, the number of six letter words formed by using A, L and M = $3^6 - 189 = 540$.
$\therefore$ The desired number of words = ${}^8C_3 \times 540 = 30240$.

13. Desired number = $2985984 - \{\sqrt{2985984} + \sqrt[3]{2985984} - \sqrt[6]{2985984}\}$
$= 2985984 - 1860 = 2984124$

14–16. $(z^2 - a^2)(\bar{z}^2 - a^2) = (2az + b)(2a\bar{z} + b)$
$\Rightarrow |z - a|^2 = 2a^2 + b$ or $|z + a|^2 = 2a^2 - b$.

17. A cos x = cos x cos λ – sin x sin λ + B.
Comparing
A = cos λ and sin λ = B = 0
$\Rightarrow$ cos λ = 1, – 1.

18. $\cot\left(\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \tan^{-1}\left(\frac{1}{21}\right)\right)$
$= \cot(\tan^{-1} 2 - \tan^{-1} 1 + \tan^{-1} 3 - \tan^{-1} 2 + \ldots\ldots + \tan^{-1} 5 - \tan^{-1} 4)$
$= \cot[\tan^{-1} 5 - \tan^{-1} 1] = \frac{3}{2}$.

19. $676 = 26^2 = 13^2.2^2 = n_1.n_2$ (let).
Total number of ways in which n_1 and n_2 can be co-prime is equal to $\frac{2^2}{2} = 2$.

20. For x < – 1,

$$\cos^{-1}\frac{x^2-1}{x^2+1} = \cos^{-1}\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^2}} = -2\tan^{-1}\frac{1}{x}$$

$$\sin^{-1}\frac{2x}{x^2+1} = \sin^{-1}\frac{2/x}{1+1/x^2} = 2\tan^{-1}\frac{1}{x}$$

and $\tan^{-1}\frac{2x}{x^2-1} = \tan^{-1}\frac{2/x}{1-1/x^2} = 2\tan^{-1}\frac{1}{x}$.

Hence $\cos^{-1}\frac{x^2-1}{x^2+1} + \sin^{-1}\frac{2x}{1+x^2} - \tan^{-1}\frac{2x}{x^2-1} = \frac{\pi}{3}$

$-2\tan^{-1}\frac{1}{x} = \frac{\pi}{3} \Rightarrow x = -\sqrt{3}$.

21. $\sin^4 x - \cos^2 x \sin x + 2\sin^2 x + \sin x = 0$
$= (\cos^2 x - 1)(\sin^2 x + \sin x + 2) = 0$
$= \cos x = \pm 1 \Rightarrow x = n\pi$.

22. $\cos^{-1}\sqrt{1-x^2} = \pi - \cos^{-1}x = \cos^{-1}(-x) \Rightarrow \sqrt{1-x^2} = -x$.