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PRACTICE TEST – 01

Time: 3 Hours ***Maximum Marks: 240***

A. Question Paper Format

1. The question paper consists of **3 parts** (Chemistry-**Part-I**, Physics-**Part-II** and Mathematics-**Part-III**) and each part consists of **four sections**.
2. **Section - I** contains **7** multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **only one is correct.**
3. **Section - II** contains **4** multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **one or more is/are correct.**
4. **Section – III** contains **2** paragraphs. Based upon one of the paragraphs **2 multiple choice questions** and based on the other paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of the **only one** is correct.
5. **Section - IV** contains **7** questions. The answer to each of the questions is a **single – digit integer,** ranging from 0 to 9. The answer will have to be appropriately bubbled in the ORS as per the instructions given at the beginning of the section.

B. Marking scheme:

6. For each question in **Section I**, you will be awarded **3 marks** if you darken only the bubble corresponding to the correct answer and **zero mark** if no bubbles are darkened. In all other cases, **minus one (–1) mark** will be awarded.
7. For each question in **Section II**, you will be awarded **4 marks** if you darken **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY** and **zero marks** other wise. **No negative marks** will be awarded in this Section.
8. For each question in **Section III**, you will be awarded **3 marks** if you darken only the bubble corresponding to the correct answer and zero mark if no bubbles are darkened. In all other cases, minus one **(–1) mark** will be awarded.
9. For each question in **Section IV**, you will be awarded **4 marks** if you darken the bubble corresponding to the correct answer and zero mark if no bubble is darkened. **No negative marks** will be awarded for in this Section

Enrolment No. :	
Name :	
Batch :	**Date:**. . . . .

PART - I: CHEMISTRY

SECTION – I (Total Marks: 21)
(Single Correct Answer Type)

This section contains **7 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

1. An electron in a hydrogen atom in its ground state absorbs 1.50 times as much energy as the minimum required for its escape from the atom. What is the wavelength of the emitted electron? ($m_e = 9.1 \times 10^{-31}$ kg)
(A) 4.7Å (B) 4.7 nm
(C) 9.4Å (D) 9.40 nm

2. The correct order of radii is
(A) N < Be < B (B) $F^- < O^{2-} < N^{3-}$
(C) Na < Li < K (D) $Fe^{3+} < Fe^{2+} < Fe^{4+}$

3. In the preparation of quick lime from limestone the reaction is $CaCO_3(s) \rightleftharpoons CaO(s) + CO_2 \uparrow$ experiments carried out between 850 and 950°C led to a set of K_p values filling an empirical equation
$$\log K_p = 7.282 - \frac{8500}{T}$$
Where T is absolute temperature. If the reaction is carried out in quiet air saturated with CO_2, what temperature would be predicted for the complete decomposition of the limestone?
(A) 1167 K (B) 1127 K
(C) 1267 K (D) 1227 K

4. The $[H^+]$ that must be maintained in a saturated H_2S solution to precipitate Pb^{2-}, but not Zn^{2+} from a solution in which each ion is present at a counteraction of 0.01 M (K_{sp} of $H_2S = 1.1 \times 10^{-23}$ and K_{sp} of $ZnS = 1.0 \times 10^{-21}$) is
(A) 3.3×10^{-2} (B) 3.3×10^{-4}
(C) 3.3×10^{-6} (D) 3.3×10^{-8}

5. The pH at the second equivalent point of 30 ml of 0.124 M H_3PO_3 $\left(K_{a_2} = 2.6 \times 10^{-7}\right)$ is tritated with 0.1 M NaOH solution is [Given log 26 = 1.415, log 35.6 = 1.5514]
(A) 10.57 (B) 11.57
(C) 9.57 (D) 12.57

6. Consider the following reactions at 300 K.
$A \rightarrow B$ (uncatalysed reaction); $A \xrightarrow{\text{catalyst}} B$ (catalysed reaction)
The activation energy is lowered by 8.314 $kJmol^{-1}$ for the catalysed reaction. The rate of this reaction is
(A) 15 times (B) 38 times
(C) 22 times (D) 28 times
that of the uncatalyzed reaction.

7. The solubility (in water) order of the following is
$MgCO_3$, $CaCO_3$, $BaCO_3$, $SrCO_3$
(A) $MgCO_3 > CaCO_3 > BaCO_3 > SrCO_3$ (B) $CaCO_3 > MgCO_3 > BaCO_3 > SrCO_3$
(C) $SrCO_3 > CaCO_3 > BaCO_3 > MgCO_3$ (D) $MgCO_3 > CaCO_3 > SrCO_3 > BaCO_3$

Space for rough work

SECTION – II (Total Marks: 16)
(Multiple Correct Answers Type)

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

8. Choose the incorrect options amongst the following.
(A) More the bond angle between the bonds on an atom, more will be the dipole moment
(B) The correct order for electron affinity of halogens is F > Cl > Br > I
(C) In a quantum level the energies of the orbitals is in order s < p < d < f
(D) The concentration term 'Molality' is independent of temperature.

9. Which of the following buffers have a pH greater than 7?
(A) $NaHCO_3 + Na_2CO_3$
(B) $CH_3COOH + CH_3COONa$
(C) $Na_3PO_4 + NaH_2PO_4$
(D) NH_3

10. The table give below gives the kinetic data for the reaction $OCl^- + I^- \rightarrow OI^- + Cl^-$ at 298 K

Expt. No	$[OCl^-]$ mole/litre	$[I^-]$ mole/litre	$[OH^-]$ mole/litre	$\frac{+d[IO^-]}{dt}$ moleLitre^{-1}sec^{-1}
1	0.0017	0.0017	1.0	1.75×10^{-4}
2	0.0034	0.0017	1.0	3.50×10^{-4}
3	0.0017	0.0034	1.0	3.50×10^{-4}
4	0.0017	0.0017	0.5	3.50×10^{-4}

Which are correct statements?
(A) The rate law for the reaction is rate = $K[OCl^-][I^-][OH^-]$
(B) The rate law for the reaction is rate $= \frac{K[OCl^-][I^-]}{[OH^-]}$
(C) The overall order of the reaction is 1
(D) The unit of rate is mole litre^{-1}sec^{-1}

11. Which of the following is correct for alkaline earth metals?
(A) None of these forms superoxide
(B) Be and Mg impart colour to flame
(C) $BeCO_3$ is more soluble and $BaCO_3$ is least soluble in water
(D) $Be(OH)_2$ is least basic and $Ba(OH)_2$ is most basic

Space for rough work

SECTION –III (Total Marks: 15)
(Paragraph Type)

This section contains **2 paragraphs.** Based upon the paragraphs **2 multiple choice questions** and **3 multiple choice questions** have to be answered. Each of these questions has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

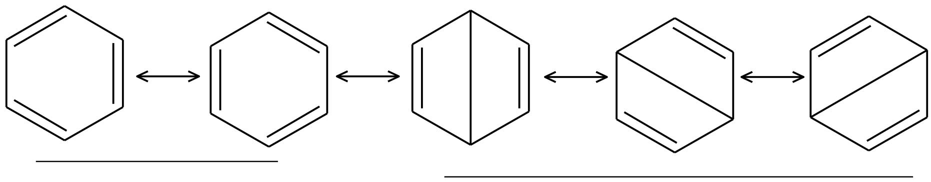
Paragraph for Questions 12 to 13

According to the concept of resonance, whenever a single Lewis structure cannot describe a molecule accurately, a number of structures with almost same energy and similar positions of nuclei, bonding and non-bonding pairs of electrons are taken as canonical structures of the resonance hybrid which describes the molecule accurately. In general, the resonance stabilizes the molecule as the energy of resonance hybrid is less than the energy of any single canonical structure and resonance averages the bond characteristics as a whole. Actually resonance hybrid does not oscillate between the canonical forms but it is a definite form and has a definite structure which cannot be written on paper. The contributing structures should be such that negative charge resides on more electronegative and positive charge on the electropositive. Like charges should not reside on atoms close together in the canonical forms.

As a result of resonance, the bond order changes in many molecules or ions.

$$\text{Bond order } = \frac{\text{Total number of bonds between two atoms is all the structures}}{\text{Total number of resonating structures}}$$

12. Consider the following statements about the resonating structures of benzene.



Kekule's Dewar's

The incorrect statement(s) is/are

(A) Two Kekule's structure contribute 80% while remaining three Dewar's structure 20% only
(B) Because of the resonance, all C–C bonds in benzene become identical
(C) Resonance decreases the bond length of C–C single bond and increases the bond length of C=C bond
(D) Benzene becomes less stable and more reactive due to resonance

13. The bond order of C–O bond for CO_3^{2-} ion is

(A) 1.25 (B) 1.33
(C) 1.5 (D) 1.0

Space for rough work

Paragraph for Questions 14 to 16

When sucrose hydrolysed it produced dextro–rotatory "glucose" and leavo–rotatory "fructose" whereas reactant itself is a dextro–rotatory compound. The above conversion process follows first order kinetics. Similarly an optically active compound A hydrolysed as follows:

$$A + H_2O \xrightarrow{H^+} 2B + C$$

The observed rotation of compound A, B and C are 60^o, 50^o and -80^o per mole respectively. The angle of rotation after 40 minutes and after the completion of reaction were 26^o and 10^o respectively at 27^oC and activation energy for conversion is 27 kJ mol^{-1}.

14. The value of rate constant of above reaction at 27^oC is:
(A) $5.57 \times 10^{-3}\ min^{-1}$
(B) $1.94 \times 10^{-3}\ min^{-1}$
(C) $1.37 \times 10^{-6}\ min^{-1}$
(D) $6.13 \times 10^{-3}\ min^{-1}$

15. The value of $t_{1/2}$ for above process is at 127^oC:
(A) 5.68 minutes
(B) 1.2 hours
(C) 8.4 minutes
(D) 1.5 hours

16. At what temperature half life of above process is equal to 31.1 minutes.
(A) 350 K
(B) 280 K
(C) 310.8 K
(D) 390 K

Space for rough work

SECTION –IV (Total Marks: 28)
(Integer Answer Type)

This section contains **7 questions**. The answer to each question is a **single-digit integer**, ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

17. Find the quantum number 'n' corresponding to the excited state of He^+ ion if on transition to the ground state that ion emits two photons in secession with the wavelengths 108.5 and 30.4 nm respectively.

18. By absorbing 12.75 eV of energy electrons in ground state of H-atoms are excited to a higher energy state. The maximum number of spectral lines possible in visible region during their de-excitation is

19. How many of the following molecules are having this bond order = 2?
$N_2, N_2^-, N_2^{2-}, O_2, CN^+, C_2, CO, NO, F_2, B_2$

20. For the reaction
$A \rightleftharpoons B + C; K_c = 2$
$A \rightleftharpoons D; K_c = 4$
K_c for the reaction $A + D \rightleftharpoons 2B + 2C$ is

21. Number of neutrons in a parent nucleus X, which gives ${}^{14}_{7}N$ after two successive β - emission would be:

22. The concentration of Fe^{+2} in a solution containing 0.2 M $[Fe(CN)_6]^{-4}$ and 0.1 M CN^- is [K_b for $[Fe(CN)_6]^{4-} = 1 \times 10^{24}$] is $x \times 10^{-19}$ M. The value of x will be

23. The solubility product of sparingly soluble salt Ag_2CrO_4 is 4×10^{-12}. The molar solubility of the salt is 10^{-x} mol/L? What is the value of x?

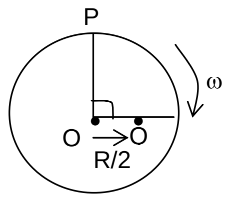
Space for rough work

PART - II: PHYSICS

SECTION – I (Total Marks: 21)
(Single Correct Answer Type)

This section contains **7 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

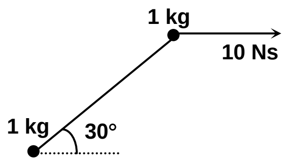
1. A circular disc of radius R is rotating about its axis O with a uniform angular velocity ω rad/s as shown in the figure. P and Q are two points on the disc. At any instant of time the magnitude of the relative velocity of P with respect to Q is



(A) 0 (B) $\frac{R\omega}{2}$

(C) $\sqrt{3}\frac{R\omega}{2}$ (D) $\frac{\sqrt{5}R\omega}{2}$

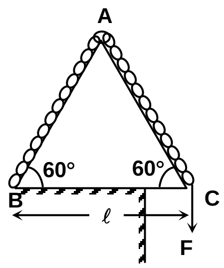
2. Two balls of masses 1 kg each are connected by an inextensible massless string. The system is resting on a smooth horizontal surface. An impulse of 10 Ns is applied to one of the balls at an angle 30° with the line joining two balls in horizontal direction as shown in the figure. Assuming that the string remains taut after the impulse, the magnitude of impulse of tension is:



(A) 6 Ns (B) $\frac{5}{2}\sqrt{3}$ Ns

(C) 5 Ns (D) $\frac{5}{\sqrt{3}}$ Ns

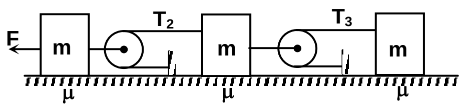
3. A fixed wedge ABC is in the shape of an equilateral triangle of side ℓ. Initially, a chain of length 2ℓ and mass m rests on the wedge as shown. The chain is slowly being pulled down by the application of a force F as shown. Work done by gravity till the time, the chain leaves the wedge will be:



(A) $-\left(\frac{(\sqrt{3}+1)mg\ell}{2}\right)$ (B) $\left(\frac{(\sqrt{3}+2)mg\ell}{2}\right)$

(C) $-\left(\frac{(\sqrt{3}+2)mg\ell}{4}\right)$ (D) $\left(\frac{(\sqrt{3}+4)mg\ell}{4}\right)$

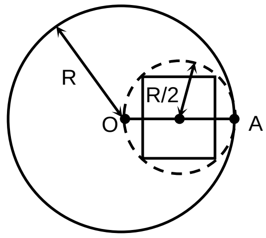
4. On a rough table, three blocks (including the first block) are placed as shown in the figure. Mass of each block is m and coefficient of friction for each block is μ. A force F is applied on the first block so as to move the system. The minimum value of F should be:



(A) 8 μmg (B) 9 μmg
(C) 7 μmg (D) 5 μmg

Space for rough work

5. There is a thin uniform disc of radius R and mass per unit area σ, in which a hole of radius R/2 has been cut out as shown in the figure. Inside the hole a square plate of same mass per unit area σ is inserted so that its corners touch the periphery of the hole. Find centre of mass of the system from the centre O.

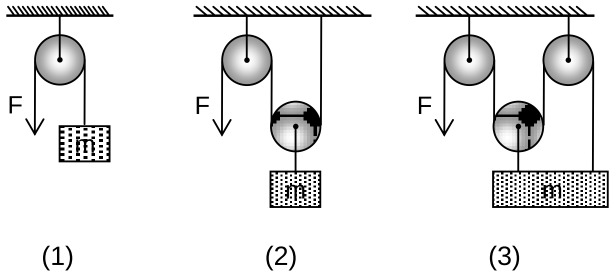


(A) $\frac{R[2-\pi]}{2[3\pi+2]}$ (B) $\frac{R[1-\pi]}{2[2\pi+1]}$

(C) $\frac{2R\pi}{2[3\pi+2]}$ (D) $\frac{3R\pi}{2[2\pi+1]}$

6. The accelerations of a particle as seen from two frames S_1 and S_2 have equal magnitude 4 m/s^2.
(A) The frames must be at rest with respect to each other.
(B) The frames may be moving with respect to each other but neither should be accelerated with respect
(C) The acceleration of S_2 with respect to S_1 may either be zero or 8 m/s^2.
(D) The acceleration of S_2 with respect to S_1 may be anything between zero and 8 m/s^2.

7. Equal force F (> mg) is applied to string in all the 3 cases. Starting from rest, the point of application of force moves a distance of 2 m down in all cases. In which case the block has maximum kinetic energy?
(A) 1
(B) 2
(C) 3
(D) equal in all 3 cases

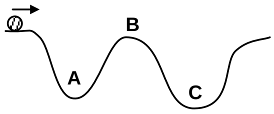


Space for rough work

SECTION – II (Total Marks: 16)
(Multiple Correct Answers Type)

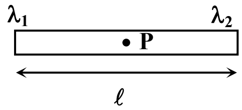
This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

8. A body moves along an uneven horizontal road surface with constant speed at all points. The normal reaction of the road on the body is



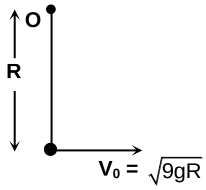
(A) maximum at A
(B) maximum at B
(C) minimum at B
(D) the same at A, B and C

9. Linear mass density of the rod vary linearly from λ_1 to λ_2. P is the mid point of the road. The position of centre of mass of the rod lie.



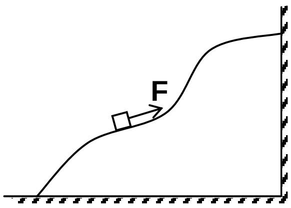
(A) Left to P if $\lambda_1 > \lambda_2$
(B) Right to P if $\lambda_1 > \lambda_2$
(C) Left to P if $\lambda_1 < \lambda_2$
(D) Right to P if $\lambda_1 < \lambda_2$

10. A ball is projected with horizontal velocity $v_0 = \sqrt{9gR}$ at the bottom most point attached with inextensible string of length R & fixed at O as shown. Tension in the string in horizontal position



(A) Net acceleration when string is in horizontal position is $5\sqrt{2}g$.
(B) Tension in the string when string is in horizontal position 7 mg
(C) Radial acceleration when string is in horizontal position is 7g.
(D) Tangential acceleration when string is in horizontal position is g.

11. A body of mass m was slowly hauled up the rough hill by a force f which at each point was directed along tangent to the hill. Work done by the force.



(A) Is independent of shape of trajectory
(B) Depends upon vertical component of displacement but independent of horizontal component
(C) Depends upon both the components of displacement
(D) Does not depend upon coefficient of friction

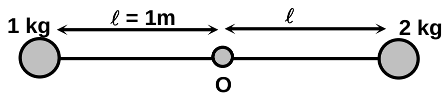
Space for rough work

SECTION –III (Total Marks: 15)
(Paragraph Type)

This section contains **2 paragraphs.** Based upon the paragraphs **2 multiple choice questions** and **3 multiple choice questions** have to be answered. Each of these questions has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Questions 12 to 13

Two balls of mass 1 kg and 2 kg are tied with massless inextensible strings. The other end of string is tied at same fixed point O. The two strings are made horizontal as shown. The length of string is 1m. The coefficient of restitution between the balls is 1. The balls are released at the same instant.



12. Velocity of 1 kg ball just after collision?

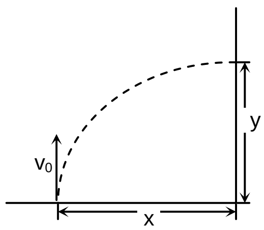
(A) 7 m/s
(B) $\frac{5\sqrt{20}}{3}$ m/s
(C) $\frac{4\sqrt{20}}{3}$ m/s
(D) $\frac{2}{3}\sqrt{20}$ m/s

13. The height attained by 2 kg ball after the collision from the point of collision is

(A) 1 m
(B) 1/4 m
(C) 1/9 m
(D) 1/8 m

Paragraph for Questions 14 to 16

A particle is projected vertically with velocity v_0. Wind is blowing and is providing a constant horizontal acceleration a_0. There is a vertical wall at some distance from point of projection. If particle strikes the vertical wall perpendicularly then calculate



14. The time taken by the particle to hit the ball is

(A) v_0/g
(B) $2v_0/g$
(C) $3v_0/g$
(D) $v_0/2g$

15. Horizontal component of velocity with which particle strikes the vertical wall is

(A) a_0v_0/g
(B) $2\ a_0v_0/g$
(C) $3\ a_0v_0/g$
(D) $a_0v_0/2g$

16. Distance x is given by

(A) $\frac{v_0^2}{2g}$
(B) $\frac{v_0^2}{g}$
(C) $\frac{a_0v_0^2}{2g^2}$
(D) $\frac{a_0v_0^2}{g}$

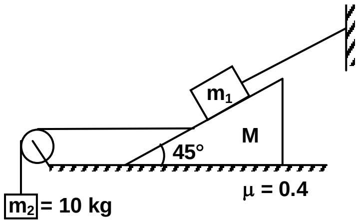
Space for rough work

SECTION –IV (Total Marks: 28)
(Integer Answer Type)

This section contains **7 questions**. The answer to each question is a **single-digit integer**, ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

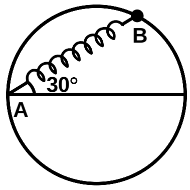
17. Two blocks of masses 10 kg and 4 kg and are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 m/s to the heavier block in the direction of the lighter block. The velocity of the centre of mass is 5K m/s. Then find the value of K.

18. For the arrangement shown in the figure, if m_1 = 10 kg, m_2 = 10 kg, M = 20 kg and the coefficient of friction between the wedge and the floor is μ = 0.4. there is no friction between m_1 and M. The tension in the string connecting m_2 to wedge is 25λ Newton. Then find the value of λ



19. A particle moves, along a straight line OX. At a time t (in second) the x coordinate (in metre) of the particle from O is given by $x = t^3 - 3t^2$. Find the distance (in meter) travelled by particle in 3sec.

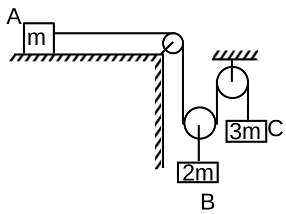
20. A bead of mass m is attached to one end of a spring of natural length R and spring constant $k = \frac{(\sqrt{3}+1)mg}{R}$. The other end of the spring is fixed at a point A on a smooth vertical ring of radius R as shown in the figure. If the normal reaction at B just after it released to move is $\frac{\lambda\sqrt{3}mg}{2}$. Then find the value of λ.



21. A body of mass 3 kg acted upon by a constant force is displaced by S metre, given by relation $S = \frac{1}{3}t^2$, where t is in second. Work done by the force in 2 second is $\frac{\lambda}{3}$ Joule, find the value of λ.

22. A particle slides down from the top outside smooth surface of a fixed sphere of radius a = 10 m. The initial horizontal velocity to be imparted to the particle 'at the top' is 5K m/s, if it leaves the surface at a point whose vertical height above the centre of sphere is 3a/4. Find the value of K.

23. In the pulley – block system shown, assume the friction to be negligible and the string to be light and inextensible. The masses of the blocks are m, 2m and 3m respectively. The tension in the string connecting the mass C is given as $T = k\frac{mg}{10}$, then value of k is



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PART - III: MATHEMATICS

SECTION – I (Total Marks : 21)
(Single Correct Answers Type)

This section contains **7 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

1. If f : R $\rightarrow$ R satisfies f (x + y) = f (x) + f (y), for all x, y $\in$ R and f (1) = 7, then $\sum_{r=1}^{n} f(r)$ is

(A) $\frac{7n}{2}$ (B) $\frac{7(n+1)}{2}$
(C) $\frac{7n(n+1)}{2}$ (D) 7n

2. If g (x) = x^2 + x – 2 and $\frac{1}{2}$ gof (x) = $2x^2$ – 5x + 2, then f (x) may be

(A) 2x – 3 (B) 2x + 3
(C) $2x^2$ + 3x + 1 (D) $2x^2$ – 3x – 1

3. Let f : R $\rightarrow$ R be a differential function and f (5) = 2, then $\lim_{x\to 5} \int_{2}^{f(x)} \frac{t^2 - 2t}{2x - 10} dt$ is equal to

(A) 8f′ (5) (B) $\frac{1}{2}f'(5)$
(C) $\frac{1}{4}f'(5)$ (D) 0

4. Three differential function f, g, h are such that f′(x) = g (x), g′(x) = h (x), h′(x) = f (x), f (0) = 1, g (0) = 0 and h (0) = 0. The value of f^3 (x) + g^3 (x) + h^3 (x) – 3f (x) g (x) h (x) at x = 5 is

(A) 0 (B) 1
(C) 2 (D) 3

Space for rough work

5. f (x) is a polynomial of degree four having relative maximum/minimum at x = 0, x = ± 1, f (0) = 3 and $\int_{-2}^{2} f(x)dx = \frac{212}{15}$. The function f (x) is

(A) $x^4 - 2x^2 + 3$
(B) $\frac{1}{6}\left(x^4 - 2x^2 + 18\right)$
(C) $\frac{x^5}{10} - \frac{x^3}{3} + 3$
(D) $\frac{1}{4}\left(x^4 - 2x^2 + 12\right)$

6. If $I_n = \int_{0}^{\pi/4} \tan^n x dx$, then $\lim_{n\to\infty} n\left[I_n + I_{n-2}\right]$ equals

(A) 1/2
(B) 1
(C) ∞
(D) 0

7. Let y be a function of x such that the curve y = f (x) passes through (1, 2) having slope (2x + 1). The area bounded between the curve and x-axis is

(A) 6 sq. units
(B) 1/3 sq. unit
(C) 1/6 sq. unit
(D) 5/6 sq. unit

SECTION – II (Total Marks : 16)
(Multiple Correct Answers Type)

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

---

8. $\int \frac{x^4 + 1}{x^6 + 1} dx$ is equal to

(A) $\tan^{-1} x - \frac{1}{3}\cot^{-1} x^3 + c$
(B) $\tan^{-1} x + \frac{1}{3}\tan^{-1} x^3 + c$
(C) $\frac{1}{3}\tan^{-1} x^3 - \cot^{-1} x + c$
(D) $-\tan^{-1} x - \frac{1}{3}\tan^{-1} x^3 + c$

9. Let f (x) and g (x) (f (x) ≠ g (x) for any x ∈ R) are two functions continuous everywhere and h (x) = $\lim_{n\to\infty} \frac{f(x) + x^{2n} g(x)}{1 + x^{2n}}$, then

(A) h (x) is continuous everywhere
(B) h (x) is discontinuous at exactly two point
(C) h (x) is discontinuous at 1
(D) h (x) is discontinuous at 0

Space for rough work

10. Let f (x) be defined in the interval [– 2, 2] such that f (x) = $\begin{cases} -1, & -2 \le x \le 0 \\ x-1, & 0 < x \le 2 \end{cases}$ and g (x) = f (|x|) + |f (x)|, then
(A) g (x) is not differentiable at x = 1
(B) g (x) is not differentiable at x = 0
(C) f (x) is not differentiable at x = 0
(D) f (x) is continuous at x = 0

11. If the tangent at the point (p, q) to the curve $x^3 + y^3 = k$ meets the curve again at the point (a, b), then
(A) $\frac{q-b}{p-a} = -\frac{p^2}{q^2}$
(B) $\frac{q-b}{p-a} = -\frac{p^2+ap+a^2}{q^2+bq+b^2}$
(C) $\frac{a}{p} + \frac{b}{q} = -1$
(D) $\frac{a}{p} + \frac{b}{q} = 1$

SECTION –III (Total Marks : 15)
(Paragraph Type)

This section contains **2 paragraphs.** Based upon the paragraphs **2 multiple choice questions** and **3 multiple choice questions** have to be answered. Each of these questions has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Questions 12 to 13

Let f : R → R $f(x) = \int_{\alpha}^{\beta} \frac{dt}{|t-x|+1}$ α, β is constant (α < β) is defined as:

$$f(x) = \begin{cases} \int_{\alpha}^{\beta} \frac{dt}{t-x+1}, & x < \alpha \\ \int_{\alpha}^{\beta} \frac{dt}{x-t+1}, & x > \beta \\ \int_{\alpha}^{x} \frac{dt}{x-t+1} + \int_{x}^{\beta} \frac{dt}{t-x+1}, & \alpha < x < \beta \end{cases}$$

Now if f : R → R be defined by $f(x) = \int_{1}^{3} \frac{dt}{|t-x|+1}$, then

12. For x > 3 f(x) is equal to
(A) $\ln\left(\frac{x}{x-2}\right)$
(B) $\ln\left(\frac{x}{x+2}\right)$
(C) $2\ln\left(\frac{x}{x-2}\right)$
(D) none of these

Space for rough work

13. For $x \in (1, 3)$ f(x) is equal to
(A) ln $(3 - x)x$
(B) ln $(4 - x)x$
(C) 2ln $(4 - x)x$
(D) none of these

Paragraph for Questions 14 to 16

Let y = f(x) be a continuous function and
$g_1(x) = \text{min.} \{|f(x-1)|, |f(x)|\}$
$g_2(x) = f(|x|)$
$g_3(x) = -f(|x|)$
Now if $f(x) = x - 1$. Then

14. The area bounded by $y = g_1(x)$, x-axis and lines x = 0 and x = 3 is equal to
(A) $\frac{4}{5}$
(B) $\frac{5}{4}$
(C) $\frac{2}{5}$
(D) none of these

15. The area bounded by $y = g_2(x)$ and $y = g_3(x)$ is equal to
(A) 2
(B) 3
(C) 4
(D) none of these

16. The area bounded by $y = g_3(x)$ and $y = \ln(|x|)$ is equal to
(A) 4
(B) 2
(C) 3
(D) none of these

Space for rough work

SECTION –IV (Total Marks : 28)
(Integer Answer Type)

This section contains **7 questions**. The answer to each question is a **single-digit integer**, ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

17. The function 'f' stisfies the functional equaiton 3f (x) + 2f $\left(\frac{x+59}{x-1}\right)=10x+30$ for all real $x \neq 1$. The value of f (7) is ______

18. $\int_{-2}^{2}\left|1-x^2\right|dx$ is equal to ________

19. Let f be a positive function. Also let $I_1 = \int_{1-k}^{k} xf\left(x(1-x)\right)dx$, $I_2 = \int_{1-k}^{k} f\left(x(1-x)\right)dx$, when 2k – 1 > 0. Then $\frac{I_2}{I_1}$ is equal to ________

20. If A is the area enclosed between the curve y = 1+ x^2, y-axis and the straight line y = 5 in first quadrant, then $\frac{3A}{2}$ is equal to ______

21. If F (x) = $f(x)+f\left(\frac{1}{x}\right)$, where f (x) = $\int_{1}^{x}\frac{2\ln t}{1+t}dt$, then 2F (e) equals ________

22. Number of non-negative integral values of a satisfying $\int_{0}^{\pi/2}\left[a^2\left(\frac{\cos 3x}{4}+\frac{3}{4}\cos x\right)+a\sin x-20\cos x\right]dx \le -\frac{a^2}{3}$ is ______

23. If P be the period of the function f (x) which satisfies the equation f (x + 1) + f (x – 1) = $\sqrt{3}$ f (x) $\forall x \in R$, then $\frac{P}{3}$ is _______

Space for rough work

ANSWERS

Chemistry [PART-I]

1.	**A**	2.	**B**	3.	**A**	4.	**A**
5.	**A**	6.	**D**	7.	**A**	8.	**A, B**
9.	**A, C**	10.	**B, C, D**	11.	**A, C, D**	12.	**D**
13.	**B**	14.	**A**	15.	**C**	16.	**A**
17.	**5**	18.	**2**	19.	**4**	20.	**1**
21.	**9**	22.	**2**	23.	**4**		

Physics [PART-II]

1.	**D**	2.	**B**	3.	**D**	4.	**C**
5.	**A**	6.	**D**	7.	**C**	8.	**A, C**
9.	**A, D**	10.	**A, B, C, D**	11.	**A, C**	12.	**B**
13.	**C**	14.	**A**	15.	**A**	16.	**C**
17.	**2**	18.	**4**	19.	**8**	20.	**3**
21.	**8**	22.	**1**	23.	**9**		

Mathematics [PART-III]

1.	**C**	2.	**A**	3.	**D**	4.	**B**
5.	**A**	6.	**B**	7.	**C**	8.	**A, B, C**
9.	**B, C**	10.	**A, B, C, D**	11.	**A, B, C**	12.	**A**
13.	**B**	14.	**B**	15.	**A**	16.	**C**
17.	**4**	18.	**4**	19.	**2**	20.	**8**
21.	**2**	22.	**5**	23.	**4**		

HINTS & SOLUTIONS

PHYSICS

1. $\vec{v}_{PQ} = \vec{v}_p - \vec{v}_Q$

$$|\vec{v}_{PQ}| = \sqrt{(R\omega)^2 + \left(\frac{R\omega}{2}\right)^2} = \frac{\sqrt{5}}{2}R\omega$$

4. For minimum value of F

$T_3 = \mu mg$

$T_2 = 2T_3 + \mu mg = 3\mu mg$

$F = 2T_2 + \mu mg = 7\mu mg$

7. Apply work energy theorem

$W = \Delta K$

$W_g + W_F = K_f$

12. Apply conservation of momentum

$1 \times v_0 - 2v_0 = -1 \times v_1 + 2v_2$ ...(i)

$1 = \dfrac{2v_0}{v_1 + v_2}$ ...(ii)

$v_0 \rightarrow$ velocity of blocks before collision

v_1 & $v_2 \rightarrow$ velocity of blocks after collision

$v_1 = \dfrac{5\sqrt{20}}{3}$, $v_2 = \dfrac{\sqrt{20}}{3}$

13. From energy conservation

$\dfrac{1}{2}(2)\left(\dfrac{\sqrt{20}}{3}\right)^2 = 2 \times 10 \times h$ $h = \dfrac{1}{9}$m.

14. $0 = v_0 - gt$

$t = \dfrac{v_0}{g}$

15. Velocity of particle at the time of hit

$v = 0 + a_0 t = \dfrac{a_0 v_0}{g}$

16. Distance $x = 0 + \dfrac{1}{2}a_0\left(\dfrac{v_0}{g}\right)^2 = \dfrac{a_0 v_0^2}{2g}$

17. $10 \times 14 + 4 \times 0 = (10 + 4)V_{cm}$

$V_{cm} = 10\,m/s$

18. Friction is sufficient to prevent the slipping so system remain in equilibrium

$T = 10g = 100N$

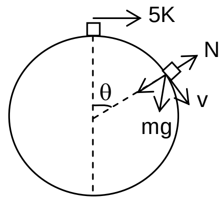
21. $S = \dfrac{t^2}{3}$

$v = \dfrac{ds}{dt} = \dfrac{2t}{3}$

$a = \dfrac{dv}{dt} = \dfrac{2}{3}$

$W = F \times s = \dfrac{8}{3}$

22. $mg\cos\theta - N = \frac{mv^2}{a}$



$v^2 = ag\cos\theta \qquad (N = 0)$

$v^2 = \frac{3ag}{4} \qquad \left\{\cos\theta = \frac{3}{4}\right\}$

from conservation of energy

$\frac{1}{2}m(5k)^2 + mg\left(\frac{a}{4}\right) = \frac{1}{2}m\left(\frac{3ag}{4}\right)$

K = 1

MATHS

1. $x = y = 0 \to f(0) = 0$
$x = 1, y = 1 \to f(2) = 2f(1) = 2 \cdot 7$
$x = 2, y = 1 \to f(3) = f(2) + f(1) = 3f(1) = 3 \cdot 7$
$x = 2, y = 2 \to f(4) = f(2) + f(2) = 4f(1) = 4 \cdot 7$ and so on
Hence, $f(r) = 7r$
$\Rightarrow \sum_{r=1}^{n} f(r) = 7\sum_{r=1}^{n} r = \frac{7n(n+1)}{2}$.

2. $\frac{1}{2}g[f(x)] = 2x^2 - 5x + 2 \Rightarrow f^2(x) + f(x) - 2 = 4x^2 - 10x + 4$
or $f^2(x) + f(x) - (4x^2 - 10x + 6) = 0$
$\Rightarrow f(x) = \frac{-1 \pm \sqrt{1 + 4(4x^2 - 10x + 6)}}{2} = \frac{-1 \pm (4x-5)}{2}$.

3. $\lim_{x\to5} \int_2^{f(x)} \frac{t^2 - 2t}{2x - 10}dt = \lim_{x\to5} \frac{\int_2^{f(x)} (t^2 - 2t)dt}{2x-10} = \lim_{x\to5} \frac{(f^2(x) - 2f(x))f'(x)}{2} = \frac{(4-4)\cdot f'(5)}{2} = 0$.

4. $f^3(x) + g^3(x) + h^3(x) - 3f(x)g(x)h(x) = \phi(x)$ (say)
$\phi'(x) = 3f^2(x)f'(x) + 3g^2(x)g'(x) + 3h^2(x)h'(x) - 3f'(x)g(x)h(x)$
$- 3f(x)g'(x)h(x) - 3f(x)g(x)h'(x) = 0$ (using given relations)
Hence $\phi(x)$ is a constant function.
Hence $\phi(5) = \phi(0) = 1 + 0 + 0 - 0 = 1$.

5. Clearly, $f'(x) = 0$ has roots $x = 0, \pm 1$
$\Rightarrow f(x) = \int ax(x-1)(x+1)dx = a\int(x^3 - x)dx + c = a\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + c$
$f(0) = 3 \Rightarrow c = 3$. Hence $f(x) = a\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + 3 \quad \ldots (1)$
$\int_{-2}^{2} f(x)dx = \frac{212}{15} \Rightarrow \frac{212}{15} = \left[a\left(\frac{x^5}{20} - \frac{x^3}{6}\right) + 3x\right]_{-2}^{2} = a\left(\frac{32}{10} - \frac{8}{3}\right) + 12 \Rightarrow \frac{8}{15}a + 12 = \frac{212}{15} \Rightarrow a = 4$
Hence $f(x) = x^4 - 2x^2 + 3$ (using (1))

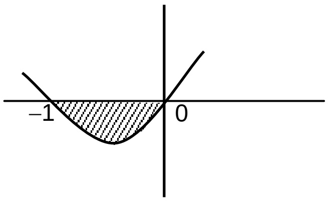
6. $I_n = \int_0^{\pi/4} \tan^n x dx = \int_0^{\pi/4} (\sec^2 x - 1)\tan^{n-2} x dx = \int_0^{\pi/4} \sec^2 x \tan^{n-2} x dx - \int_0^{\pi/4} \tan^{n-2} x dx$

$\Rightarrow I_n = \left[\dfrac{\tan^{n-1}x}{n-1}\right]_0^{\pi/4} - I_{n-2} \Rightarrow I_n + I_{n-2} = \dfrac{1}{n-1}$.

Hence $\lim\limits_{n\to\infty} n\left(I_n + I_{n-2}\right) = \lim\limits_{n\to\infty}\dfrac{n}{n-1} = \lim\limits_{n\to\infty}\dfrac{1}{1-1/n} = 1$.

7. We have $\dfrac{dy}{dx} = 2x + 1 \Rightarrow y = x^2 + x + c$

$\Rightarrow y = x^2 + x$ (as the curve passes through (1, 2))
Solving with x-axis, we get
$x = -1, 0$



$\therefore$ Required area = $\left|\int_{-1}^{0}\left(x^2+x\right)dx\right| = \left|\left(\dfrac{x^3}{3}+\dfrac{x^2}{2}\right)_{-1}^{0}\right| = \dfrac{1}{6}$ sq. unit.

8. $\displaystyle\int\frac{x^4+1}{x^6+1}dx = \int\frac{\left(x^4-x^2+1\right)+x^2}{\left(x^2+1\right)\left(x^4-x^2+1\right)}dx = \int\frac{dx}{x^2+1} + \int\frac{x^2}{\left(x^3\right)^2+1}dx$

$= \tan^{-1}x + \dfrac{1}{3}\tan^{-1}\left(x^3\right) + c$. (Check other options also using relation $\tan^{-1}x + \cot^{-1}x = \pi/2$)

9. As $n \to \infty$, $\left(x^2\right)^n \to 0$ for $x^2 < 1$, 1 for $x^2 = 1$ and ∞ for $x^2 > 1$

Case I: $x^2 > 1$, h (x) = $\lim\limits_{n\to\infty}\dfrac{\dfrac{f(x)}{x^{2n}} + g(x)}{\dfrac{1}{x^{2n}} + 1} = g(x)$

Case II: $x^2 = 1$, h (x) = $\dfrac{f(x)+g(x)}{2}$

Case III: $x^2 < 1$, h (x) = $\lim\limits_{n\to\infty}\dfrac{f(x)+x^{2n}g(x)}{1+x^{2n}} = f(x)$

Hence, h (x) = $\begin{cases} f(x), & x^2 < 1 \\ \dfrac{f(x)+g(x)}{2}, & x^2 = 1 \\ g(x), & x^2 > 1 \end{cases}$

Hence, h (x) is discontinuous for $x^2 = 1$ (Since f (x) $\neq$ g (x) for any x)
Hence h (x) is discontinuous at x = – 1 and 1.

10. g (x) = $\begin{cases} -x-1+1, & -2 \le x \le 0 \\ 0, & 0 < x < 1 \\ (x-1)+(x-1), & 1 \le x \le 2 \end{cases}$

Hence g (x) = $\begin{cases} -x, & -2 \le x \le 0 \\ 0, & 0 < x < 1 \\ 2(x-1), & 1 \le x \le 2 \end{cases}$

$g'(0^-) = -1$, $g'(0^+) = 0$. Hence g (x) is not differentiable at x = 0.
Also, $g'(1^-) = 0$, $g'(1^+) = 2$. Hence g (x) is not differentiable at x = 1.
Similarly, f (x) is continuous but not differentiable at x = 0.

11. $\dfrac{dy}{dx} = -\dfrac{3x^2}{3y^2} = -\dfrac{p^2}{q^2}$ (Slope of tangent)

Since this is the line joining (p, q) and (a, b)

$\therefore \dfrac{q-b}{p-a} = -\dfrac{p^2}{q^2}$ … (1)

As these two points line on the curve.
$p^3 + q^3 = a^3 + b^3 = k$.
Hence $p^3 - a^3 + q^3 - b^3 = 0 \Rightarrow (p-a)(p^2 + ap + a^2) + (q-b)(q^2 + bq + b^2) = 0$

$\Rightarrow \dfrac{q-b}{p-a} = -\dfrac{p^2+ap+a^2}{q^2+bq+b^2}$ … (2)

From (1) and (2), $\dfrac{p^2}{q^2} = \dfrac{p^2+ap+a^2}{q^2+bq+b^2}$

$\Rightarrow p^2(bq + b^2) = q^2(ap + a^2) \Rightarrow pq(pb - aq) = -(p^2b^2 - q^2a^2) \Rightarrow pq = -(pb + aq)$

$\Rightarrow \dfrac{b}{q} + \dfrac{a}{p} = -1$ … (3)

12. $f(x) = \displaystyle\int_1^3 \frac{dt}{x-t+1} = -\left[\ln(x+1-t)\right]_1^3 = \left[\ln(x-2) - \ln x\right] = \ln\left(\frac{x}{x-2}\right)$.

13. For $x \in (1, 3)$

$f(x) = \displaystyle\int_1^x \frac{dt}{x-t+1} + \int_x^3 \frac{dt}{t-x+1}$

$= \left[-\ln(x+1-t)\right]_1^x + \left[\ln(t-x+1)\right]_x^3$

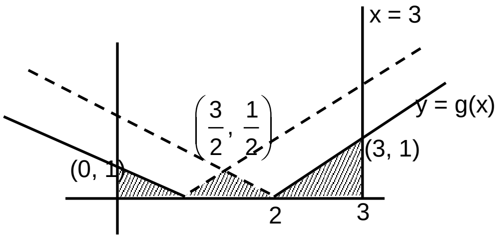
$= -(\ln 1 - \ln x) + [\ln(4-x) - \ln 1]$

$= \ln(4-x)x$.

14. $g_1(x) = \min\{|x-2|, |x-1|\}$

$\Rightarrow$ required area

$= \dfrac{1}{2}(1\times 1) + \dfrac{1}{2}\left(1+\dfrac{1}{2}\right) + \dfrac{1}{2}(1\times 1) = 1 + \dfrac{1}{4} = \dfrac{5}{4}$.



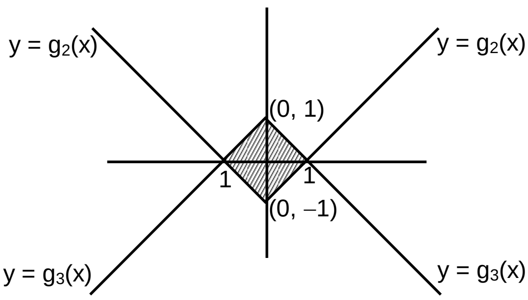
15. $g_2(x) = |x| - 1$

$= \begin{cases} x-1, & x \ge 0 \\ -x-1, & x < 0 \end{cases}$

$g_3(x) = -|x| + 1$

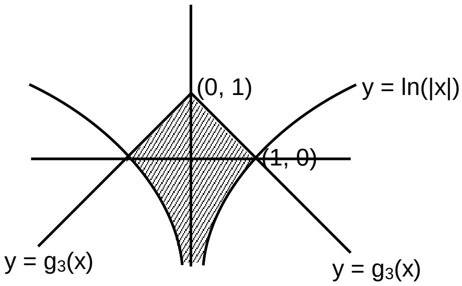
$= \begin{cases} -x+1, & x \ge 0 \\ x+1, & x < 0 \end{cases}$

$\Rightarrow$ required area = $\left(\sqrt{2}\right)^2 = 2$ sq. units



16. Required area $= 2\left[\displaystyle\int_{\infty}^{0} e^y\,dy + \frac{1}{2}(1\times 1)\right]$

$= 2\left[1 + \dfrac{1}{2}\right] = 3$ sq. units



17. $3f(x) + 2f\left(\dfrac{x+59}{x-1}\right) = 10x + 30$

For x = 7, 3f (7) + 2f (11) = 100
For x = 11, 3f (11) + 2f (7) = 140
Solving, we get f (7) = 4.

18. $\int_{-2}^{2}\left|1-x^2\right|dx = \int_{-2}^{-1}\left|1-x^2\right|dx + \int_{-1}^{1}\left|1-x^2\right|dx + \int_{1}^{2}\left|1-x^2\right|dx$

$= -\int_{-2}^{-1}\left(1-x^2\right)dx + \int_{-1}^{1}\left(1-x^2\right)dx - \int_{1}^{2}\left(1-x^2\right)dx = \frac{4}{3}+\frac{4}{3}+\frac{4}{3} = 4.$

19. $I_1 = \int_{1-k}^{k} xf\left(x(1-x)\right)dx$

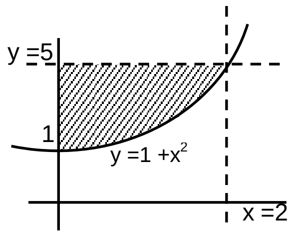
$= \int_{1-k}^{k}(1-k+k-x)f\left[(1-k+k-x)\left(1-(1-k+k-x)\right)\right]dx$ $\left[\text{using } \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right]$

$= \int_{1-k}^{k}(1-x)f\left(x(1-x)\right)dx = I_2 - I_1 \Rightarrow 2I_1 = I_2 \Rightarrow \frac{I_2}{I_1} = 2.$

20. y = 5 gives $x^2 = 4 \Rightarrow x = 2$

Hence required area = $10 - \int_0^2 y\,dx = 10 - \int_0^2\left(1+x^2\right)dx$

$\Rightarrow A = 10 - \left[x+\frac{x^3}{3}\right]_0^2 = 10 - \frac{14}{3} = \frac{16}{3} \Rightarrow \frac{3A}{2} = 8.$



21. F (x) = $f(x) + f\left(\frac{1}{x}\right)$

F (e) = $\int_1^e \frac{2\ln t}{1+t}dt + \int_1^{1/e}\frac{2\ln t}{1+t}dt = \int_1^e \frac{2\ln t}{1+t}dt + \int_1^e \frac{2\ln(1/u)}{1+1/u}\left(-\frac{1}{u^2}\right)du$

$= \int_1^e \frac{2\ln t}{1+t}dt + \int_1^e \frac{2\ln t}{t(1+t)}dt = \int_1^e \frac{2\ln t}{t}dt = 2\int_0^1 z\,dz = 1 \Rightarrow 2F(e) = 2.$

22. $\left[\frac{a^2}{4}\frac{\sin 3x}{3} + \frac{3a^2}{4}\sin x - a\cos x - 20\sin x\right]_0^{\pi/2} \le -\frac{a^2}{3} \Rightarrow \left(-\frac{a^2}{12}+\frac{3a^2}{4}-20\right)-(-a) \le -\frac{a^2}{3}$

$\Rightarrow a^2 + a - 20 \le 0 \Rightarrow a \in [-5, 4]$.
Hence non-negative integral values of a are 0, 1, 2, 3, 4.
Hence number of such values is 5.

23. f (x + 1) + f (x – 1) = $\sqrt{3}$ f (x) … (1)
Replace x by x +1, we get, f (x + 2) + f (x) = $\sqrt{3}$ f (x + 1)
Replacing x by x – 1, f (x) + f (x – 2) = $\sqrt{3}$ f (x – 1)
Adding f (x + 2) + 2f (x) + f (x – 2) = 3f (x) (using equation (1))
$\Rightarrow$ f (x + 2) + f (x – 2) = f (x) … (2)
Replace x by x + 2 in (2), we get, f (x + 4) + f (x) = f (x + 2) … (3)
Adding (2) and (3), f (x + 4) + f (x – 2) = 0 … (4)
Replace x by x + 2 in (4), f (x + 6) + f (x) = 0
Hence f (x + 6) = – f (x)
Now, replacing x by x + 6, f (x + 12) = – f (x + 6) = f (x).
Hence, f (x) is periodic with period 12 $\Rightarrow$ P = 12.