JEE EXPERT

ANSWER KEY

REGULAR TEST SERIES - (RTS-04)

Batch : 12TH Pass (Desire A01)

Date 01.09.2019

PHYSICS

1	(C)	2	(C)	3	(A)	4	(B)	5	(A)
6	(A)	7	(A)	8	(B)	9	(A)	10	(C)
11	(D)	12	(C)	13	(B)	14	(C)	15	(D)
16	(C)	17	(C)	18	(C)	19	(C)	20	(C)
21	(D)	22	(C)	23	(C)	24	(B)	25	(C)
26	(B)	27	(B)	28	(A)	29	(D)	30	(A)

CHEMISTRY

31	(C)	32	(A)	33	(C)	34	(A)	35	(D)
36	(A)	37	(D)	38	(B)	39	(C)	40	(C)
41	(A)	42	(D)	43	(B)	44	(B)	45	(B)
46	(C)	47	(D)	48	(D)	49	(D)	50	(D)
51	(B)	52	(D)	53	(C)	54	(B)	55	(B)
56	(D)	57	(B)	58	(A)	59	(B)	60	(A)

MATHEMATICS

61	(A)	62	(C)	63	(D)	64	(C)	65	(C)
66	(B)	67	(D)	68	(B)	69	(C)	70	(A)
71	(A)	72	(A)	73	(C)	74	(C)	75	(C)
76	(A)	77	(C)	78	(B)	79	(C)	80	(B)
81	(C)	82	(B)	83	(D)	84	(A)	85	(C)
86	(C)	87	(B)	88	(A)	89	(C)	90	(D)

JEE EXPERT

SOLUTIONS

REGULAR TEST SERIES - (RTS-04)

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PART - I [PHYSICS]

1. Sol. (C)

$F_{net} = \frac{d}{dt}(mv)$

$F - mg = \frac{dm}{dt} v + 0$

$F = mg + v\frac{dm}{dt}$, $(m = vt\lambda)$

$= vt\lambda g + v^2\lambda$

$P = Fv = v^2 \lambda gt + \lambda v^3$

$<P> = \frac{\int_0^T P\,dt}{T} = v^2\lambda g \left(\frac{T}{2}\right) + \lambda v^3$ $[\because T = \ell/v]$

$<P> = \frac{\lambda \ell vg}{2} + \lambda v^3$

2. Sol. (C)

$F_{net} = \frac{d}{dt}(mv)$

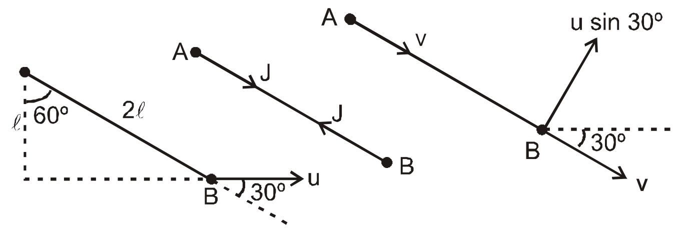
$F - mg = \frac{dm}{dt} v + 0$

$F = mg + v\frac{dm}{dt}$, $(m = vt\lambda)$

$= vt\lambda g + v^2\lambda$

$P = Fv = v^2 \lambda gt + \lambda v^3$, $P \uparrow$ as $t\uparrow$ so $P_{max} = v^2\lambda g\left(\frac{\ell}{v}\right) + \lambda v^3 = \ell\lambda gv + \lambda v^3$

3. Sol. (A)



When the string becomes tight, both particles begin to move with velocity components v in the direction AB. Using conservation of momentum in the direction AB

$$mu\cos 30^\circ = mv + mv$$

or $$v = \frac{u\sqrt{3}}{4}$$

Hence the velocity of ball A just after the jerk is $v = \frac{u\sqrt{3}}{4}$.

4. Sol. (B)

Centre of mass will move in a vertical line if $v_1 \cos\theta_1 = v_2 \cos\theta_2$. Otherwise for any other values it will follow a parabolic path.

5. Sol. (A)

Velocity of the system just after the collision

$$mv_o = (m + M)V' \quad \Rightarrow \quad V' = \frac{mv_0}{(m+M)}$$

Using work energy theorem.

$\Delta K = W_{All} = W_g + W_N + W_S$ (Assume friction force is absent)

$$0 - \frac{1}{2}(m + M)V'^2 = 0 + 0 - \frac{1}{2}KX^2_{max}$$

$$\frac{m_o^2 v_o^2}{(m+M)} = KX^2_{max} \quad \Rightarrow \quad X_{max} = \frac{m_o v_o}{\sqrt{K(M+m)}} = \sqrt{\frac{m_o^2 v_o^2}{K(M+m)}}$$

6. Sol. (A)

In the centre of mass frame

$$\frac{1}{2}kx^2 = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(\vec{u}_1 - \vec{u}_2)^2$$

$$200x^2 = \left(\frac{3\times 6}{3+6}\right)(2+1)^2$$

$$x = \frac{3}{10} = 0.3 \text{ m}$$

$= 30$ cm

7. **Sol. (A)**

$v_2 = 2v_1$

$(1 + e)\, u_1 = 2(1 - e)u_1$

$e = \dfrac{1}{3}$

8. **Sol. (B)** $\vec{a}_{cm} = \dfrac{m_1 \vec{a}_1 + m_2 \vec{a}_2}{m_1 + m_2} = \dfrac{m\vec{O} + m\vec{a}}{(m+m)} = \dfrac{\vec{a}}{2}$

9. **Sol. (A)**

$S = (at^2 + 2bt + c)^{1/2}$

Differentiating, $\dfrac{dS}{dt} = \dfrac{1}{2}(at^2 + 2bt + c)^{-1/2} \times (2at + 2b) = \dfrac{at + b}{\sqrt{at^2 + 2bt + c}}$

$$\frac{d^2S}{dt^2} = \frac{\left(\sqrt{at^2 + 2bt + c}\right) \times a - \dfrac{(at+b)(at+b)}{\sqrt{at^2 + 2bt + c}}}{(at^2 + 2bt + c)}$$

$$= \frac{a(at^2 + 2bt + c) - (at+b)^2}{\sqrt{at^2 + 2bt + c} \times \left(at^2 + 2bt + c\right)} = \frac{(ac - b^2)}{S \times S^2}$$

$\therefore \quad \dfrac{d^2S}{dt^2} \propto \dfrac{1}{S^3} \quad \Rightarrow \quad$ acceleration $\propto S^{-3}$

10. **Sol. (C)**

$m = 100$ kg

$M = 5 \times 10^3$ kg

initially system is at rest $\vec{F}_{ext} = 0$, $\vec{V}_{Cm} = 0$, $\Delta\vec{P} = 0$

$\vec{P}_{shell} = -\vec{P}_{Gun} \quad \Rightarrow P^2_{shell} = P^2_{Gun}$

$K_{shell} = \dfrac{P_S^2}{2m} \qquad K_{Cannon} = \dfrac{P_C^2}{2M}$

11. **Sol. (D)**

For W to be maximum ; $\dfrac{dW}{dx} = 0$;

i.e. $F(x) = 0 \quad \Rightarrow x = \ell,\ x = 0$

Clearly for d = l ,the work done is maximum.

d = l

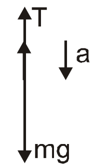
12. **Sol.(C)**

13. **Sol. (B)**

For A :

$T - m\omega^2 r - ma = 0$ ....(i)

Seen from object itself



$T - \frac{mg}{3} = ma$ ....(i)

For B :

$mg - T = ma$ ....(ii)

(i) – (ii)

$2T = \frac{4}{3} mg$

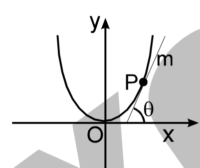
$T = \frac{2}{3} mg$

14. **Sol. (C)**

$x^2 = 4ay$

Differentiating w.r.t. y, we get

y ds lkis { k vodyu djus ij



$\frac{dy}{dx} = \frac{x}{2a}$

$\therefore$ At (2a, a), $\frac{dy}{dx} = 1$ $\Rightarrow$ hence vr% $\theta = 45°$

the component of weight along tangential direction is mg sin θ.

hence tangential acceleration is $g \sin\theta = \frac{g}{\sqrt{2}}$

15. **Sol. (D)**

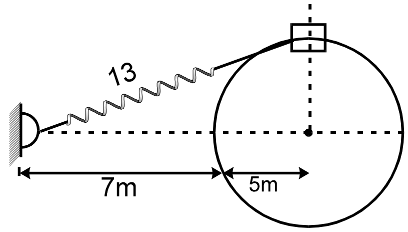
Since $\vec{F} \perp \vec{V}$, the particle will move along a circle.

$\therefore$ $F = \frac{mv^2}{R}$ & $\theta = \frac{S}{R}$ $\Rightarrow$ $\theta = \frac{FS}{mv^2}$

16. **Sol. (C)**

Initial extension will be equal to 6 m.

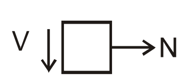
$\therefore$ Initial energy = $\frac{1}{2}$ (200) $(6)^2$ = 3600 J.



Reaching A : $\frac{1}{2} mv^2 = 3600$ J

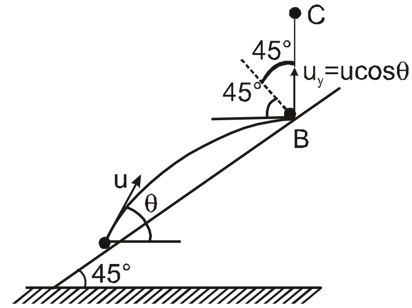
$\Rightarrow$ $mv^2 = 7200$ J

From F.B.D. at A :



$N = \frac{mv^2}{R} = \frac{7200}{5} = 1440$ N

17. **Sol.(C)**



After the elastic collision with inclined plane the projectile moves in vertical direction.

The inclination of plane with horizontal is 45°, hence velocity of particle just before collision should be horizontal.

$\therefore$ Time required to reach maximum height

$$= t_{AB} + t_{BC} = \frac{u\sin\theta}{g} + \frac{u\cos\theta}{g}$$

18. **Sol. (C)**

Let v be the speed of particle at B, just when it is about to loose contact.

From application of Newton's second law to the particle normal to the spherical surface.

$$\frac{mv^2}{r} = mg\sin\beta \qquad .... (1)$$

Applying conservation of energy as the block moves from A to B..

$$\frac{1}{2}mv^2 = mg(r\cos\alpha - r\sin\beta) \qquad .... (2)$$

Solving 1 and 2 we get

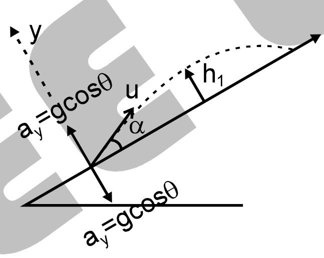
$$3\sin\beta = 2\cos\alpha$$

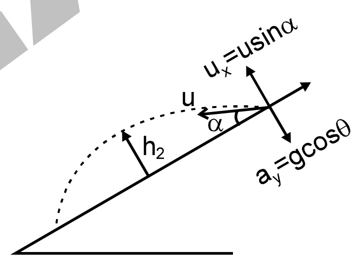
19. **Sol. (C)**

If component of velocity normal to incline are equal, time of flight is same. Also if horizontal components are equal, range on inclined plane will be equal for both.

20. **Sol. (C)**

For both the particles





$u_y = u\sin\theta$ and $a_y = g\cos\theta$

So y motion will be similar for both the particles.

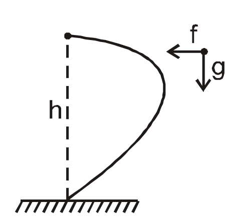
$\Rightarrow$ Maximum height and time of flight will be same for the both. $\Rightarrow h_1 = h_2$

21. **Sol. (D)**

Time taken to reach the ground is given by $h = \frac{1}{2}gt^2$ .... (1)

Since horizontal displacement in time t is zero

$$h = \frac{1}{2}gt^2 \qquad .... (1)$$



$$\therefore t = \frac{2v}{f} \qquad .... (2)$$

$$h = \frac{2gv^2}{f^2}$$

22. Sol. (C)

Blocks A and C both move due to friction. But less friction is available to A as compared to C because normal reaction between A and B is less. Maximum friction between A and B can be :

$$f_{max} = \mu m_A g = \left(\frac{1}{2}\right)mg$$

$\therefore$ Maximum acceleration of A can be :

$$a_{max} = \frac{f_{max}}{m} = \frac{g}{2}$$

further $a_{max} = \dfrac{m_D g}{3m + m_D}$

or $\dfrac{g}{2} = \dfrac{m_D g}{3m + m_D}$

23. Sol. (C)

Since $mg\sin 30° > \mu mg\cos 30°$

the block has a tendency to slip downwards. Let F be the minimum force applied on it, so that it does not slip. Then

$N = F + mg\cos 30°$

$\therefore$ $mg\sin 30° = \mu N = \mu(F + mg\cos 30°)$

or $F = \dfrac{mg\sin 30°}{\mu} - mg\cos 30° = \dfrac{(2)(10)(1/2)}{0.5} - (2)(10)\left(\dfrac{\sqrt{3}}{2}\right)$

or $F = 20 - 17.32 = 2.68$ N

24. Sol. (B)

25. Sol. (C)

By law of Conservation of Linear momentum

mu = mv +MV ...(1)

where m = mass of bullet

M = mass of block

u = velocity of bullet before collision

v = velocity of bullet after collision

V = velocity of block after collision

By law of Conservation of Energy

Mgh =

V =

V = 1.4 ms^{-1}

Put in (1), we get

$5 = 0.01v + 2(1.4)$

$v = 220\ ms^{-1}$.

26. Sol. (B)

27. Sol. (B)

As string is massless and pulley is frictionless therefore F = T

28. Sol. (A)

As acceleration is zero on the inclined plane this means f = mgsinθ.

For upward motion net downward force is f + mgsinθ = 2mgsinθ

⇒ a = 2gsinθ

⇒ using $v^2 = u^2 + 2as$

$0 = u^2 - 4g\sin\theta\, s$

⇒ $\frac{u^2}{4g\sin\theta} = s$

29. Sol. (D)

$$F_{buoyant} = \left[\frac{m}{\rho}\right]\rho_w g = \frac{mg}{3}$$

$$F_{applied} = mg - \frac{mg}{3} = \left(\frac{2mg}{3}\right)$$

work done by applied force = $\left(\frac{2mg}{3}\right)3 = 100J$

30. Sol. (A)

$$p = \vec{F}.\vec{V} = mg\sin\theta\sqrt{2g\ell\sin\theta} = \sqrt{2m^2g^3\ell\sin^3\theta}$$

PART - III [MATHEMATICS]

61. **(A)** $f'(x) = \frac{1}{2+x^4}$

By LMVT $f'(c) = \frac{f(2) - f(1)}{2-1}$ for some $c \in (1, 2)$

$\Rightarrow \quad f(2) = \frac{1}{2+c^4}$ as $f(1) = 0 \Rightarrow 1 < c < 2 \Rightarrow 3 < 2 + c^4 < 18 \Rightarrow f(2) < \frac{1}{3}$

62. **(C)** $I_2 = \frac{1}{3}\int_0^1 \frac{3x^2 dx}{e^{x^3}(2-x^3)}$ put $t = x^3$

$I_2 = \frac{1}{3}\int_0^1 \frac{dt}{e^t(2-t)}$; $\quad I_2 = \frac{1}{3}\int_0^1 \frac{dt}{e^{1-t}(1+t)}$

$\Rightarrow \quad \frac{I_1}{I_2} = 3e$

63. **(D)** $I_2 = \int_1^{\operatorname{cosec}\theta} \frac{dx}{x(x^2+1)} = -\int_1^{\sin\theta} \frac{t}{1+t^2} dt$

$I_2 = -I_1$

$\therefore \begin{vmatrix} I_1 & I_1^2 & I_2 \\ e^{I_1+I_2} & I_2^2 & -1 \\ 1 & I_1^2 + I_2^2 & -1 \end{vmatrix} = 0$

64. **(C)** $f(x) = \cos x - \int_0^x xf(t)dt + \int_0^x t\ f(t)dt$

$f(x) = \cos x - x\int_0^x f(t)dt + \int_0^x t\ f(t)dt$

$f'(x) = -\sin x - \int_0^x f(t)dt - xf(x) + xf(x)$

$f'(x) = -\sin x - \int_0^x f(t)dt$

$f''(x) = -\cos x - f(x)$

$f''(x) + f(x) = -\cos x$

65. **(C)** Differentiating the given equation

$$2008(f(x))^{2007} f'(x) = f(x) \quad \Rightarrow \quad 2008(f(x))^{2006} f'(x) = 1$$

Integrating $\dfrac{2008}{2007}(f(x))^{2007} = x + c$

$$f(0) = 1$$

$\Rightarrow \quad \dfrac{2008}{2007}\times 1 = 0 + c$. Hence $(f(x))^{2007} = \dfrac{2007}{2008}x + 1$

$$(f(2008))^{2007} = 2008$$

66. **(B)** $I = \displaystyle\int_1^e f''(x)\ln x dx = \ln x f'(x)\Big|_1^e - \int_1^e \frac{f'(x)}{x}dx$

$$I = 1 - \left\{\frac{1}{x}f(x)\Big|_1^e + \int_1^e \frac{f(x)}{x^2}dx\right\} = 1 - \left(\frac{1}{e} - \frac{1}{2}\right) = \frac{3}{2} - \frac{1}{e}$$

67. **(D)** Put $nx = t$ in I_n

$$I_n = \frac{1}{n}\int_{n/n+1}^{1} \frac{\tan^{-1}t}{\sin^{-1}t}dt$$

now $L = \lim\limits_{n\to\infty} n^2 I_n = \lim\limits_{n\to\infty} n\displaystyle\int_{n/n+1}^{1}\frac{\tan^{-1}t}{\sin^{-1}t}dt$ $\quad \{\infty \times 0 \text{ form}\}$

$$L = \lim_{n\to\infty}\left(\frac{\displaystyle\int_{n/n+1}^{1}\frac{\tan^{-1}t}{\sin^{-1}t}dt}{1/n}\right) = \lim_{n\to\infty}\left(\frac{\dfrac{-\tan^{-1}\left(\dfrac{n}{n+1}\right)}{\sin^{-1}\left(\dfrac{n}{n+1}\right)}\left(\dfrac{1}{(n+1)^2}\right)}{-\dfrac{1}{n^2}}\right) = \frac{1}{2}$$

68. **(B)** For $x \in [0,1]$, $f'(x) > f'(1)$ $\quad \{\because f'(x)$ is decreasing]

$$\Rightarrow \quad \frac{f'(x)}{f^2(x)+1} > \frac{f'(1)}{f^2(x)+1}$$

Integrating both sides w.r.t x between 0 to 1.

$$\Rightarrow \quad \int_0^1 \frac{f'(x)}{f^2(x)+1}dx > f'(1)\int_0^1 \frac{dx}{f^2(x)+1}$$

$$\Rightarrow \quad [\tan^{-1} f(x)]_0^1 > f'(1)\int_0^1 \frac{dx}{f^2(x)+1} \quad \Rightarrow \int_0^1 \frac{dx}{f^2(x)+1} < \frac{\tan^{-1} f(1)}{f'(1)}$$

$$\because \quad \tan^{-1}\alpha < \alpha \ \forall \alpha > 0 \Rightarrow \frac{\tan^{-1} f(1)}{f'(1)} < \frac{f(1)}{f'(1)}$$

$$\Rightarrow \quad \int_0^1 \frac{dx}{f^2(x)+1} < \frac{f(1)}{f'(1)}$$

69. **(C)** $f(x) = \sqrt{\left(\sqrt{x^3+1}-1\right)^2} + \sqrt{\left(\sqrt{x^3+1}-3\right)^2} = \left|\sqrt{x^3+1}-1\right| + \left|\sqrt{x^3+1}-3\right|$

when $x \in [0, 2] \Rightarrow 1 \le \sqrt{x^3+1} \le 3$

$$\Rightarrow \quad f(x) = \sqrt{x^3+1} - 1 + 3 - \sqrt{x^3+1} = 2 \ \Rightarrow \int_0^2 f(x)dx = 2x = 4$$

70. **(A)** $\sin x < x$, for $x > 0$

$\Rightarrow \sin(\cos x) < \cos x$ for $0 < x < \frac{\pi}{2} \Rightarrow \int_0^{\pi/2} \sin(\cos x)dx < \int_0^{\pi/2} \cos x \, dx \quad \Rightarrow \quad I_2 < I_3$

Now, $\cos x < \cos\alpha$

If $x > \alpha$, $x, \alpha \in \left[0, \frac{\pi}{2}\right]$

Now $x > \sin x$

$\Rightarrow \cos x < \cos(\sin x) \Rightarrow \int_0^{\pi/2} \cos x \, dx < \int_0^{\pi/2} \cos(\sin x) \, dx \Rightarrow I_3 < I_1$

So, $I_1 > I_3 > I_2$

71. **(C)** $\int_0^1 [[2x] - [3x]]\,dx \ge \frac{3\cos^{-1}\alpha}{\pi} - 1$

$$\Rightarrow \ -\frac{1}{2} \ge \frac{3\cos^{-1}\alpha}{\pi} - 1 \ \Rightarrow \ \frac{3\cos^{-1}\alpha}{\pi} \le \frac{1}{2} \ \Rightarrow \ \cos^{-1}\alpha \le \frac{\pi}{6} \Rightarrow \ \alpha \in \left[\frac{\sqrt{3}}{2}, 1\right]$$

72. **(A)** $f''(x) = f'(x) \Rightarrow \quad \frac{f''(x)}{f'(x)} = 1$

On integrating,

$$f'(x) = Ce^x$$

Which gives f (x) = $Ce^x + D$

But f (0) = 1 $\Rightarrow$ C + D = 1

$\therefore$ f (x) = $Ce^x + 1 - C$

So, $f'(x) = Ce^x$

Putting it in $f'(x) = f(x) + \int_0^1 f(x)\,dx$

$\Rightarrow$ $Ce^x = Ce^x + 1 - C + \int_0^1 (Ce^x + 1 - C)\,dx$ $\Rightarrow$ $C = \frac{2}{3-e}$

So, $f(x) = \frac{2e^x - e + 1}{3 - e}$

73. **(C)** We have $g(x+2) = \int_0^{x+2} f(t)dt = \int_0^2 f(t)dt + \int_2^{x+2} f(t)dt = g(2) + \int_0^x f(t)dt$

$\therefore$ $g(x+2) = g(2) + g(x)$

Also, $g(2) = \int_0^2 f(t)dt = \int_0^1 f(t)dt + \int_1^2 f(t)dt = \int_0^1 f(t)dt + \int_{-1}^0 f(t)dt = \int_{-1}^1 f(t)dt = 0$

[$\therefore$ f is odd function]

$\because$ $f(x)$ is odd

$\therefore$ $g(2n) = 0$ (is periodic with period = 2).

74. **(C)** $I = \int_0^a \ln \frac{\cos(a-x)}{\sin a \cos x} dx = \int_0^a \ln(\cos(a-x))dx - \int_0^a \ln(\cos x)dx - \int_0^a \ln(\sin a)dx$

75. **(C)** $\int_{10}^{19} \frac{\sin x}{1+x^8}\,dx \le \int_{10}^{19} \frac{|\sin x|}{1+x^8}\,dx \le \int_{10}^{19} \frac{1}{x^8}\,dx = -\frac{x^{-7}}{7}\Big|_{10}^{19} = -\frac{1}{7}(19)^{-7} + \frac{1}{7}(10)^{-7} \le 10^{-7}$.

76. **(A)** $I = \int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x}\,dx = \int_0^{\pi/2} \left(\frac{\cos x}{1+e^x} + \frac{\cos(-x)}{1+e^{-x}} \right) dx$ $= \int_0^{\pi/2} \cos x\,dx = \sin x \Big|_0^{\pi/2} = 1$.

77. **(C)** Given $I_1 = \int_{1-k}^{k} x\, f(x(1-x))\,dx$ …(i)

$I_2 = \int_{1-k}^{k} x\, f(x(1-x))\,dx$ …(ii)

Here $I_1 = \int_{1-k}^{k} x\, f(x(1-x))\,dx$

$$= \int_{1-k}^{k} (k+1-k-x)\ f(x+1-k-x)\,(1-(k+1-k-x))\,dx$$

$$= \int_{1-k}^{k} f((1-x)x)\,dx - \int_{1-k}^{k} x\,f(x(1-x))\,dx \qquad = \int_{1-k}^{k} f(x(1-x))\,dx - I_1$$

or $\quad 2I_1 = I_2$ [using (ii)]

or $\quad \dfrac{I_1}{I_2} = \dfrac{1}{2}$.

78. **(B)** $f(x) = \int \dfrac{x^2(\sqrt{1+x^2}-1)}{(1+x^2)(1+x^2-1)}dx = \int \dfrac{\sqrt{1+x^2}-1}{1+x^2}dx = \int \dfrac{dx}{\sqrt{1+x^2}} - \int \dfrac{dx}{1+x^2}$

$= \log(x+\sqrt{1+x^2}) - \tan^{-1} x + k$

$\therefore \quad f(0) = \log 1 - \tan^{-1} 0 + k = k = 0$

$\therefore \quad f(x) = \log(x+\sqrt{1+x^2}) - \tan^{-1} x$

$\therefore \quad f(1) = \log(1+\sqrt{2}) - \dfrac{\pi}{4}$

79. **(C)** $I = \int \dfrac{(x^2+\cos^2 x)}{(1+x^2)} \cdot \operatorname{cosec}^2 x\,dx$

$= \int \dfrac{(1+x^2-\sin^2 x)}{(1+x^2)} \cdot \operatorname{cosec}^2 x\,dx \qquad = \int \operatorname{cosec}^2 x\ dx - \int \dfrac{dx}{1+x^2} = -\cot x + \tan^{-1} x + C$

80. **(B)** $I = \int \sin x \cdot \cos x \cdot \cos 2x \cdot \cos 4x \cdot \cos 8x\,dx$

$= \dfrac{1}{2}\int \sin 2x \cdot \cos 2x \cdot \cos 4x \cdot \cos 8x\,dx \qquad = \dfrac{1}{4}\int \sin 4x \cdot \cos 4x \cdot \cos 8x\,dx$

$= \dfrac{1}{8}\int \sin 8x \cdot \cos 8x\,dx = \dfrac{1}{16}\int \sin 16x\,dx = \dfrac{1}{256}\cos 16x + C$

81. **(C)** Let $I = \int \dfrac{dx}{\cos^6 x + \sin^6 x} = \int \dfrac{\sec^6 x}{1+\tan^6 x}\,dx$

$= \int \dfrac{(1+\tan^2 x)^2 \sec^2 x\,dx}{1+\tan^6 x} \qquad$ If tanx = p, then $\sec^2$x dx = dp

$$\Rightarrow \quad I = \int \frac{(1+p^2)^2\,dp}{1+p^6} = \int \frac{(1+p^2)}{p^4 - p^2 + 1}\,dp = \int \frac{p^2\left(1+\frac{1}{p^2}\right)}{p^2\left(p^2+\frac{1}{p^2}-1\right)}\,dp$$

$$= \int \frac{dk}{k^2+1} = \tan^{-1}(k) + c \qquad \left(p - \frac{1}{p} = k, \left(1+\frac{1}{p^2}\right)dp = dk\right)$$

$$= \tan^{-1}\left(p - \frac{1}{p}\right) + c = \tan^{-1}(\tan x - \cot x) + c$$

82. **(B)** Let $I = \int \frac{-dx}{(x+a)^{8/7}(x-b)^{6/7}}$

$$= \int \frac{dx}{(x+a)^2\left(\frac{x-b}{x+a}\right)^{6/7}} \qquad \text{If } \left(\frac{x-b}{x+a}\right) = p \text{, then } \frac{a+b}{(x+a)^2}\,dx = dp$$

$$\Rightarrow \quad I = \frac{1}{a+b}\int \frac{dp}{p^{6/7}} = \frac{7}{a+b}(p^{1/7}) = \left(\frac{7}{a+b}\right)\left(\frac{x-b}{x+a}\right)^{1/7} + c$$

83. **(D)** Put tanx = t, where $I = 2\int \frac{\sqrt{\cot x}}{\sin 2x}\,dx = \int t^{-3/2}\,dt = -2\sqrt{\cot x} + c$

84. **(A)** Put logx = t $\Rightarrow$ dx = e^t dt

Hence $I = \int e^t\left(\frac{1}{t} - \frac{1}{t^2}\right)dt = \frac{e^t}{t} + c = \frac{x}{\log x} + c$

85. **(C)** $\frac{\cos 5x + \cos 4x}{1 - 2\cos 3x} = \frac{\sin 3x(\cos 5x + \cos 4x)}{\sin 3x - \sin 6x}$

$$= \frac{\sin 3x \cdot 2\cos\frac{9x}{2}\cos\frac{x}{2}}{-2\cos\frac{9x}{2}\sin\frac{3x}{2}} = \frac{2\sin\frac{3x}{2}\cos\frac{3x}{2}\cos\frac{x}{2}}{-\sin\frac{3x}{2}}$$

$$= -\left(2\cos\frac{3x}{2}\cos\frac{x}{2}\right) = -(\cos 2x + \cos x)$$

$$\therefore \text{ given integral} = -\int(\cos 2x + \cos x)\,dx = -\frac{\sin 2x}{2} - \sin x + c$$

86. **(C)** $\dfrac{dx}{dt} = f'''(t)\cos t - f''(t)\sin t + f''(t)\sin t + f'(t)\cos t = [f'''(t) + f'(t)]\cos t$

$\dfrac{dy}{dt} = -f'''(t)\sin t - f''(t)\cos t + f''(t)\cos t - f'(t)\sin t \quad = -[f'''(t) + f'(t)]\sin t$

y $\left[\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2\right]^{1/2} = \left[(f'''(t) + f'(t))^2(\cos^2 t + \sin^2 t)\right]^{1/2} = f'''(t) + f'(t)$

y $\displaystyle\int\left[\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2\right]^{1/2} dt = f''(t) + f(t) + c$

87. **(B)** $I = \displaystyle\int \log\frac{\phi(x)}{f(x)} d\left\{\log\frac{\phi(x)}{f(x)}\right\} = \frac{1}{2}\left\{\log\frac{\phi(x)}{f(x)}\right\}^2 + k$

88. **(A)** Substituting $x = p^6$, $dx = 6p^5\, dp$, we have

$$I = \int \frac{6p^5(p^5 + p^4 + p)}{p^6(1 + p^2)}\, dp = \int \frac{6(p^5 + p^3 + 1)}{(p^2 + 1)}\, dp = \int 6p^3 dp + \int \left(\frac{6}{p^2 + 1}\right) dp$$

$$= \frac{6p^4}{4} + 6\tan^{-1} p = \frac{3}{2}x^{2/3} + 6\tan^{-1}(x^{1/6}) + c$$

89. **(C)** Put $\ln x = t$

$$I = \int e^t \left(\frac{t-1}{t^2+1}\right)^2 dt = \int e^t \left(\frac{1}{t^2+1} - \frac{2t}{(t^2+1)^2}\right) dt \quad = \frac{e^t}{t^2+1} + c = \frac{x}{(\ln x)^2 + 1} + c.$$

90. **(D)** Let $I = \displaystyle\int \frac{dx}{(1+\sqrt{x})\sqrt{(x - x^2)}}$

If $\sqrt{x} = \sin p$, then $\dfrac{1}{2\sqrt{x}}\, dx = \cos p\, dp$

$$I = \int \frac{2\sin p\cos p\; dp}{(1 + \sin p)\sin p\cos p} = 2\int \frac{dp}{(1 + \sin p)} = 2\int \frac{(1 - \sin p)\, dp}{\cos^2 p}$$

$$= 2\int \sec^2 p\; dp - \int (\tan p \sec p)\; dp$$

$$= 2(\tan p - \sec p) = 2\left(\sqrt{\frac{x}{(1-x)}} - \frac{1}{\sqrt{(1-x)}}\right) + c = \frac{2(\sqrt{x} - 1)}{\sqrt{(1-x)}} + c$$