ALTERNATING CURRENT SOLUTIONS

Q.1. Given current in a mixture of a d.c. component of 10 A and an alternating current of maximum value 5A.

R.M.S. value $= \sqrt{(\text{d.c current})^2 + (\text{rms value of a.c.current})^2}$

$= \sqrt{(10)^2 + (5/\sqrt{2})^2} = \frac{15}{\sqrt{2}}$.

Q.2. $X_L = 2\pi fL = 2\pi \times 50 \times 0.7 = 220\ \Omega$.

$z = \sqrt{R^2 + X_L^2} = 220\sqrt{2}\ \Omega$

Power factor, $\cos\phi = \frac{R}{z} = \frac{220}{220\sqrt{2}} = \frac{1}{\sqrt{2}}$.

Q.3. (a) $v_{avg} = \frac{1}{2\pi}\int_0^{\pi} v_m \sin\alpha d\alpha = \frac{v_m}{2\pi}[-\cos\alpha]_0^{\pi} = \frac{v_m}{\pi}$

$v_{eff} = \frac{1}{2\pi}\left[\int_0^{\pi}(v_m \sin\alpha)^2 d\alpha + 0\right] = \frac{v_m^2}{4}$

$\Rightarrow\ v_{eff} = \frac{v_m}{2}$

(b) When switch is closed

2 & 3 – series $\Rightarrow$ 2 + 3 = 5 and 6 & 9 - series $\Rightarrow$ 6 + 9 = 15

5 & 12 – parallel $\Rightarrow \frac{5\times12}{5+12} = \frac{60}{17}$

15 & $\frac{60}{17}$ series $\Rightarrow 15 \times \frac{60}{17} = \frac{315}{17}$

5 & 10 parallel $\Rightarrow \frac{10}{3}$

$\frac{10}{3}$ & $\frac{315}{17}$ $\Rightarrow$ are parallel $\Rightarrow \frac{(10/3)\times(315/17)}{\frac{10}{3}+\frac{315}{17}}$ = 2.825H

Q.4. $I(t) = \frac{1}{R}(E_1 + E_2)$

$= \frac{1}{50}(25\sqrt{3} + 25\sqrt{6}\sin\omega t)$

$I(t) = \frac{\sqrt{3}}{2}(1 + \sqrt{2}\sin\omega t)$

Heat produced in one cycle of AC.

$= \int_0^{2\pi/\omega} I^2(t)R\,dt = \frac{3}{4}\times 50\int_0^{2\pi/\omega}\left(1+2\sin^2\omega t+2\sqrt{2}\sin\omega t\right)dt$

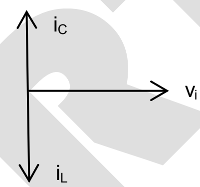
$= \frac{75}{2}\left[\frac{2\pi}{\omega}+\frac{2\pi}{\omega}\right] = \frac{3}{2}\text{J}$

No. of cycle in 14 minute is N = $14 \times 60 \times 50$

Total heat produced = $\frac{3}{2} \times 14 \times 60 \times 50$

Total heat produced = $3/2 \times 14 \times 60 \times 50 = 63000$ J

Q.5. (a) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{50}{\pi^2}\times 10^{-2}\times 200\times 10^{-6}}}$



$\omega_0 = 100\,\pi$ (= frequency of impressed voltage)
hence net current through resistance is zero.

(b) v = 5 sin 100 πt.

Q.6. Comparing, E = 100 $\sqrt{2}$ sin (100 t) with E = E_{max} sin ωt,
we get, $E_{max} = 100\sqrt{2}$ V and ω = 100 E_{max} (rad/sec.)

$X_C = \frac{1}{\omega C} = \frac{1}{100\times 10^{-6}} = 10^4\,\Omega$

as ac instrument reads rms value, the reading of ammeter will be,

$I_{rms} = \frac{E_{rms}}{2} = \frac{E_{max}}{\sqrt{2}X_c} = 10\,\text{mA}$

Q.7. Charged stored in the capacitor oscillates simple harmonically as
$Q = Q_0 \sin(\omega t \pm \phi)$
$Q_0 = 2 \times 10^{-4}$ C

$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2\times 10^{-3}\times 5\times 10^{-6}}} = 10^4\,s^{-1}$

Let at t = 0, Q = Q_0 then,

$Q(t) = Q_0 \cos\omega t$ ..... (i)

$i(t) = \frac{dQ}{dt} = -Q_0\omega\sin\omega t$ ..... (ii)

and $\frac{di(t)}{dt} = -Q_0\,\omega^2\cos\omega t$ ..... (iii)

(a) Q = 100 μC or $\frac{Q_0}{2}$

from (i) equation $\cos\omega t = \frac{1}{2}$

from equation (iii)

$\left(\frac{di}{dt}\right) = Q_0\omega^2\cos\omega t$

$= 2 \times 10^{-4}\,(10^4)^2 \times \frac{1}{2}$

$\frac{di}{dt} = 10^4$ A/s

(b) Q = 200 μC or Q_0 then cos $\omega t = 1$ i.e., $\omega t = 0, 2\pi$ ……………
from equation (ii)
$I(t) = -Q_0\,\omega \sin \omega t$
$I(t) = 0$
$[\sin 0 = \sin 2\pi = 0]$

Q.8. In one complete cycle
$I_{av} = 0$

$$I_{rms} = \sqrt{\frac{\int_0^{\tau/2}\left[\left(\frac{2I_0}{\tau}\right)t\right]^2}{\int_0^{\tau/2} dt}}$$

$$I_{rms} = \frac{I_0}{\sqrt{3}}.$$

Q.9. When 'S' is closed
Applying Kirchoff's law
$0 = iR + (1/C)\int i\,dt$
$i = -(q_0 / RC)\, e^{-t/RC} = -(40/R)e^{-t/RC}$
but $RC = 0.2 \times 10^6 \times 10 \times 10^{-6} = 2$ sec.

$\therefore\ i = \frac{-40}{0.2 \times 10^6} e^{-t/2} = -2 \times 10^{-4} e^{-t/2}$ amp.

where –ve sign indicates current is flowing in opposite direction. to our convention.

$$\therefore\ W_{dissipated} = \int_0^{\infty} i^2 R\,dt = \int_0^{\infty} (4 \times 10^{-8})(2 \times 10^5) e^{-t}\,dt$$

$$= 8 \times 10^{-3}\left[e^{-t}\right]_0^{\infty} = 8 \times 10^{-3} \text{ J.}$$

Q.10. $v = \sqrt{(80)^2 + (60)^2} = 100$ volt,

p.f. $= \frac{80}{100} = 0.8$ lagging

Q.11. The rms value of the current is 1.5 mA

The impedance of the capacitor is given by $|Z_C| = \frac{1}{\omega C} = \frac{1}{300 \times 0.5 \times 10^{-6}}\Omega = 6.67\,k\Omega$

The rms voltage across the capacitor is $1.5 \times 10^{-3} \times \frac{20}{3} \times 10^3 = 10$ V

The impedance of the circuit is

$$|Z| = \sqrt{\left(10\times10^3\right)^2+\left(\frac{20}{3}\times10^3\right)^2} = 10^3\times\sqrt{100+\frac{400}{9}} = 1.2\times10^4\Omega$$

Q.12. $2\pi f\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.5\times10^{-3}\times5\times10^{-6}}}$

$\Rightarrow$ $f = \frac{10^4}{\pi}Hz$

$\frac{1}{2}cV^2 = \frac{1}{2}Li^2$

$5\times10^{-6}\times4\times10^4 = 0.5\times10^{-7}\, i^2$

$i = 20$ Amp.

Q.13. $X_L = \omega L = \pi\ \Omega$

Impedance $= Z = 3.3\ \Omega$

Power factor $= R/Z = 1/3.3$

Q.14. $C = 0.014\times200\ \mu F$

For minimum impedance, $\omega L = 1/\omega C$

$L = 0.36$ mH.

Q.15. The loop equation

$$\square\square\square L\frac{di}{dt} - iR = 0$$

At $t = 0$, $i = 0$,

$$60 - 0.008\,\frac{di}{dt} = 0$$

$$\frac{di}{dt} = \frac{60}{0.008} = 7500\text{ A/s}$$

Where $\frac{di}{dt} = 500$A/s, an equation yields,

$60 - (0.008)(500) - 30i = 0$

$30i = 60 - 4 \Rightarrow i = 1.867$ A

For the final current $\frac{di}{dt} = 0$, and

$\square\square\square L(0) - 30\,I_F = 0$

$I_F = 2A$

Q.16. Applying KVL to the circuit at time 't'

$$2i + \frac{0.2}{10}\frac{di}{dt} = 20$$

solving this differential equation :

$i = 10\,(1+9e^{-100t})$

Q.17. Current in inductance will lag the applied voltage while across the capacitor will lead.
$I_L = I_{max} \sin(\omega t - \pi/2) = -0.4 \cos \omega t$ for inductor
$I_C = I_{max} \sin(\omega t + \pi/2) = +0.3 \cos \omega t$ for capacitor
so current drawn from the source.
$I = I_L + I_C = -0.1 \cos \omega t$
$I_{max} = |I_0| = 0.1$ A.

Q.18. Constant value in the cycle therefore
$v_{avg} = v_0$
$v_{rms} = v_0$

Q.19. $v = \sqrt{(80)^2 + (60)^2} = 100$ volt,

p.f. $= \frac{80}{100} = 0.8$ lagging

Q.20. When connected to d.c. source
$R = \frac{V}{I} = \frac{12}{4} = 3\,\Omega$ …(1)
When connected to a.c. source
$Z = \frac{V_{rms}}{I_{rms}} = \frac{12}{2.4} = 5\,\Omega$ …(2)
$Z = \sqrt{R^2 + (\omega L)^2}$ …(3)
From (1), (2) and (3)
L = 0.08 H
with condenser

$$Z' = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \sqrt{3^2 + \left(50 \times 0.08 - \frac{1}{50 \times 2500 \times 10^{-6}}\right)^2}$$

$= 5\,\Omega$

$\cos\phi = \frac{R}{Z'} = 0.6$
$\therefore$ P = $V_{rms}.I_{rms} \cos\phi$
$= 12 \times 2.4 \times 0.6 = 17.8$ volt

Q.21. (a) $\tau_L = \frac{L}{R} = \frac{100 \times 10^{-3}}{50} = 2 \times 10^{-3}$ sec.

$\therefore\ \tau = I_0 (1 - e^{t/\tau}) = \frac{v}{R}(1 - e^{t/\tau})$

$\frac{dI}{dt} = \frac{v}{\tau.R}e^{-t/\tau} = \frac{100}{2\times10^{-3}\times50}.e^{-\frac{0.001}{2\times10^{-3}}}$

$= 10^3$ (0.606) = 606 amp/sec.

(b) Heat produced, H = $\int_0^{\tau} I_0^2(t/\tau)^2 R dt = \frac{1}{3}I_0^2 R\tau$

$\therefore$ $I_{rms} = \sqrt{\frac{(1/3)I_0^2 R\tau}{R\tau}} = \frac{I_0}{\sqrt{3}}$.

Q.22. Let effective resistance is r.

$\sqrt{3} = \frac{V_c}{rI}$ … (i)

$v_c = \sqrt{3}\, rI$

$\frac{1}{\sqrt{3}} = \frac{v_c}{(rI + 10I)}$

3r = r + 10

r = 5 Ω

$\therefore$ capacitive nature of box.

Q.23. The angular resonance frequency of the circuit is given by:

$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10\times10^{-3}\times10^{-6}}} = 10^4\,\text{rad/s}$

At a frequency that is 10% lower than the resonance frequency, the reactance X_C, of the capacitor is:

$X_C = \frac{1}{\omega C} = \frac{1}{10^4\times10^{-6}\times0.9}\Omega = 111\Omega$

And that of the inductance is

$X_L = \omega L = 10^4 \times 0.9 \times 10^{-2} = 90\Omega$

The impedance of the circuit is

$|Z| = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{3^2 + (21)^2}\Omega = 21.2\Omega$

The current amplitude, i = $\frac{15}{21.2}$A $\cong 0.7$A

The average power dissipated is:

$\frac{1}{2}i^2R = \frac{1}{2}(0.7)^2 \times 3 = 0.735$ W

The average energy dissipated in 1 cycle is

$0.735\times\frac{2\pi}{0.9\times10^4}$ J

or 5.13×10^{-4} J

Q.24. $\frac{1}{2}Li_0^2 = \frac{1}{2}Cv_0^2$

$i_0 = v_0\sqrt{\frac{C}{L}}$

$v_0 = \frac{q_0}{C} = \frac{5.0\times10^{-6}}{4.0\times10^{-4}} = 1.25\times10^{-2}$ volt

$i_0 = 1.25 \times 10^{-2}\sqrt{\frac{4.0\times10^{-4}}{0.09}} = 8.33\times10^{-4}$ A

$u_{max} = \frac{1}{2}Li_0^2 = \frac{1}{2}Cv_0^2 = 3.125\times10^{-8}$ J

$3.125 \times 10^{-8} = \frac{1}{2}(0.09)\left(\frac{8.33\times10^{-4}}{2}\right)^2 + \frac{1}{2}\frac{q^2}{(4.0\times10^{-4})}$

$q = 4.33 \times 10^{-6}$ C.

Q.25. At resonance frequency $f_0 = \frac{1}{2\pi\sqrt{LC}}$

$i_0 = v_0/R$

From given condition

$$\frac{1}{\sqrt{2}}\frac{v_0}{R} = \frac{v_0}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

$\omega L - \frac{1}{\omega C} = \pm R.$

$2\pi f_1 L - \frac{1}{2\pi f_1 C} = R$ … (i)

$2\pi f_2 L - \frac{1}{2\pi f_2 C} = -R$ … (ii)

solving equations (i) and (ii) we get,

$f_1 - f_2 = \frac{R}{2\pi L}$.

Q.26. $v_{rms} = \sqrt{\frac{\int_0^T V^2 dt}{\int_0^T dt}} = \sqrt{\frac{\int_0^T \left(\frac{V}{T}\right)^2 t^2 dt}{\int_0^T dt}} = \frac{V}{\sqrt{3}}$

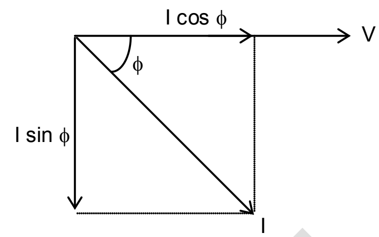
$v_{mean} = \frac{\int_0^T \frac{V}{T} t\,dt}{T}$

$v_{mean} = \frac{V}{2}$

Q.27. Impedance Z= $\sqrt{(R^2 + \omega^2 L^2)}$

$= \sqrt{(50)^2 + (2\pi \times 50 \times 0.7)^2}$

= 225.4 ohms



$\therefore$ Current $I = \frac{V}{Z} = \frac{200}{225.4}$ = 0.887 amp

and tan ϕ = $\frac{\omega L}{R} = \frac{2\pi \times 50 \times 0.7}{50}$ = 4.4

or ϕ = 77° 12'.

$\therefore$ Power component of current IR = I cos ϕ = 0.887×cos 77° 12' = 0.197 amp.

and wattless components of the current I_L

= I sinϕ= 0.887×sin77°12' = 0.865 amp.

Q.28. (i) Applying Kirchhoff's law to meshes (1) and (2)

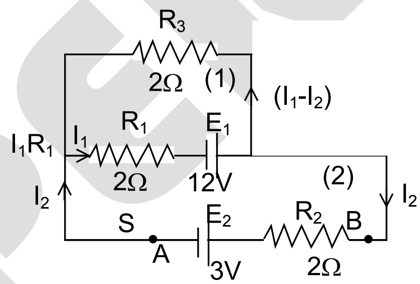
$2I_2 + 2I_1 = 12 - 3 = 9$

and $2I_1 + 2(I_1 - I_2) = 12$

solving $I_1 = 3.5$ A, $I_2 = 1$A

P.D.between AB = $2I_2 + 3 = 5$ volt

Rate of production of heat $\left(\frac{dQ}{dt}\right) = i_1^2 R_1 = 24.5$ (J)



(ii) i = $\frac{E}{R}\left[1 - e^{-Rt/L}\right] = i_0 \left[1 - e^{-Rt/L}\right]$

$i = i_0/2 \Rightarrow \frac{Rt}{L} = \ln 2$

$\Rightarrow$t = 0.0014 sec.

Energy stored = $\frac{1}{2}$ Li^2

= 0.00045 (J).