JEE EXPERT

PRACTICE TEST – 7 (04 APRIL 2020)

ANSWER KEY & SOLUTION

PART-I : PHYSICS

SECTION –A
(Single Correct Choice Type)

Sol 1. (B)

If R is normal reaction and θ is half angle of the cone.

$$R\cos\theta = \frac{mv^2}{r}\ ,\ R\sin\theta = mg$$

Dividing,

$$\tan\theta = \frac{gr}{v^2} = \frac{r}{h}$$

$$v^2 = gh$$

$$v = \sqrt{gh} = \sqrt{9.8\times 9.8\times\frac{1}{100}} = 0.98\ ms^{-1}$$

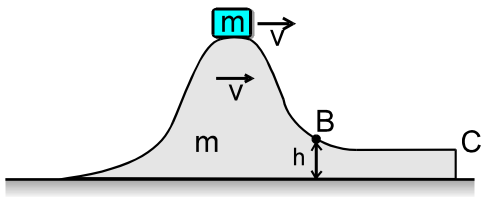
The answer is (B).

Sol 2. (D)

Reaction of force mg is mg and of friction is friction reaction of mg and friction will be on earth.
Since block is moving so these is only kinetic friction

Net force on block is E – f = ma $\Rightarrow a = \frac{E-f}{m}$

Sol 3. (A)



(At highest point)

Let 'u' be the required minimum velocity. By momentum conservation :

$mu = (m + m)v \qquad \Rightarrow v = u/2.$

Energy equation :

$$\frac{1}{2}mu^2 = \frac{1}{2}(2m)v^2 + mgH.$$

Substituting v = u/2 :

$$u = 2\sqrt{gH}$$

Sol 4. (A)

By work energy theorem at A and B

$$0 + F\times l\sin 45^\circ - mgl(1-\cos 45^\circ) = \frac{1}{2}mv_B^2$$

$$\frac{10\times 2}{\sqrt{2}} - \sqrt{2}\times 10\times 2\left[1-\frac{1}{\sqrt{2}}\right] = \frac{1}{2}\times\sqrt{2}\,v_B^2$$

$$10\sqrt{2} - 20(\sqrt{2}-1) = \frac{1}{\sqrt{2}}v_B^2$$

$$-10\sqrt{2}+20=\frac{1}{\sqrt{2}}v_B^2$$
$$-10\times 2+20\sqrt{2}=v_B^2$$
$$\sqrt{20[\sqrt{2}-1]}=v_B$$

Sol.5. (C)

The work done on the object $W=\vec{F_1}.\vec{D}+\vec{F_2}.\vec{D}$ where $\vec{D}$ is the displacement vector

$\therefore$ W = 4(1) + (1) (1) + (–1) (6) = –1J

From work energy theorem

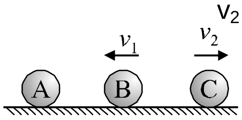
$$W=KE_f-KE_i=-1J$$

$\Rightarrow$ K.E. decreases by 1J

Sol.6. (B)

For first collision,

$$e = 1$$
$$v = v_1 + v_2$$
$$v_2 = v - v_1 \quad \text{...(i)}$$



By momentum conservation

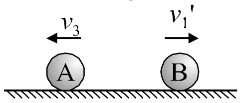
$$m_Bv = -m_Bv_1 + m_Cv_2$$
$$m_Bv = -m_Bv_1 + 4m_Bv_2 \quad \text{...(ii)}$$
$$v_2 = \frac{v_1+4}{4}$$

By eqn. (i) & (ii)

$$v_1 = \frac{3V}{5} \;\&\; v_2 = \frac{2V}{5}$$

For second collision, e = 1

$$v_1 = v_1' + v_3$$
$$v_3 = v_1 - v_1 \quad \text{...(iii)}$$



By momentum conservation,

$$-m_Bv_1 = m_Bv_1' - m_Av_3$$
$$-m_Bv_1 = m_Bv_1' - 4m_Bv_3 \quad (\because m_A = 4m_B)$$
$$v_3 = \frac{v_1'+v_1}{4} \quad \text{...(iv)}$$

By eqs. (iii) and (iv)

$$v_1' = \frac{3}{5}v_1 = \frac{3}{5}\left(\frac{3}{5}v\right) = \frac{9}{25}v < \frac{2}{5}v$$

So B can not collide with C 2^{nd} time.

Sol 7. (B)

Let P be a particle of mass m lying midway between the centres of the earth and the moon, and v be the minimum velocity of projection of the particle to escape to infinity. Potential energy of P due to earth

$$= -\frac{GM_1m}{d/2}$$
$$= -\frac{2GM_1m}{d}$$

Potential energy of P due to moon

$$= -\frac{GM_2m}{d/2}$$

$$= -\frac{2GM_2m}{d}$$

Kinetic energy of $P = \frac{1}{2}mv^2$

The total initial energy of P = $E_i = -\frac{2Gm}{d}(M_1+M_2)+\frac{1}{2}mv^2$

If the particle P is to escape to infinity, its final kinetic energy and potential energy is zero, the final energy $E_f = 0$.

From the law of conservation of energy, $E_i = E_f$

$$= -\frac{2Gm}{d}(M_1+M_2)+\frac{1}{2}mv^2 = 0$$

$$v = 2\left[\frac{G(M_1+M_2)}{d}\right]^{1/2}$$

Sol.8. (D)

$m_2v_0 = (m_1 + m_2)v$

$\Rightarrow v = \frac{m_2}{m_1+m_2}v_2$

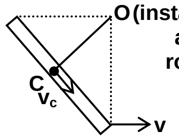
From work-energy conservation,

$$\frac{1}{2}m_2v_0^2 - \frac{1}{2}m_1+m_2\frac{m_2^2v_0^2}{(m_1+m_2)^2} = \frac{1}{2}kx^2$$

$$x = v_0\sqrt{\frac{m_1m_2}{k(m_1+m_2)}}$$

(Multi Correct Choice Type)

Sol 9. (A, B, C)



$$v_C = \omega\frac{r}{\sqrt{2}} = \frac{v}{\sqrt{2}}$$

$$\omega = \frac{v}{r}$$

$$L_0 = I\omega + \frac{mv}{\sqrt{2}}\times\frac{r}{\sqrt{2}} = \frac{2mvr}{3}$$

Sol 10. (B)

$$R = \frac{v^2}{g\cos\theta} = \frac{u^2}{4g\cos^3\theta}$$

$$R_{min} = \frac{u^2}{4g}$$

$$R = \frac{8}{3\sqrt{3}}R\,min$$

$\Rightarrow$ $\theta = 30°$

$$v = \frac{u}{2}$$

Sol.11. (A, B)

Area under F–t graphs give the impulse.
Hence, (A) & (B) are correct.

Sol 12. (A, B & C)

Average speed = $\frac{\text{Total distance travelled}}{\text{Total time taken}}$

Average velocity = $\frac{\text{Total displacement}}{\text{Total time taken}}$

for a given motion distance travelled is always greater than or equal to displacement
hence Average speed $\geq$ Average velocity.

(B) In a uniform circular motion the speed does not change but velocity changes as direction changes.

$\therefore \frac{d|v|}{dt} = 0$ rate of change of speed is zero.

$\frac{d|v|}{dt} \neq 0$ acceleration is not zero.

(C) In uniform circular motion velocity is never zero but average velocity becomes zero for a revolution.

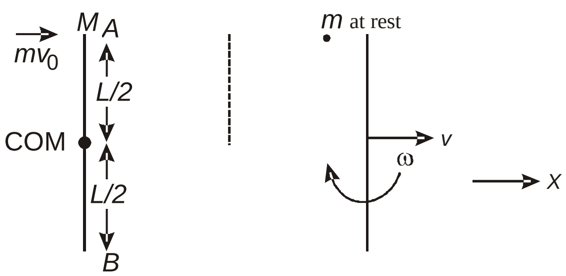
(Paragraph Type)

Ans.13.(A)

Ans.14.(C)

Ans.15.(B)

Sol.13-15.



By conservation of momentum

$$mv_0 = mv \qquad .....(1)$$

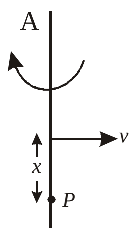
By conservation of angular momentum

$$mv_0 \frac{L}{2} \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{ML^2}{12}\omega \qquad ....(2)$$

Since collision is elastic 4t is also conserved

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \qquad ....(3)$$

By solving (1), (2) and (3)



$\frac{m}{M} = \frac{1}{4}$

point P will be at rest, if

$x\omega = v$

$\Rightarrow x = \frac{v}{\omega} = \frac{L}{6}$

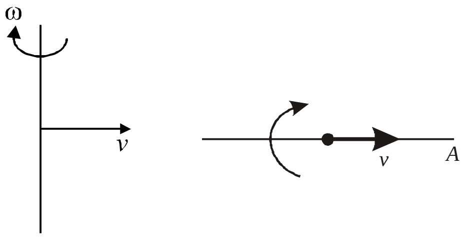
Distance from A

$AP = \frac{L}{2} + \frac{L}{6} = \frac{2L}{3}$

After time $t = \dfrac{\pi L}{3v_0}$

Angle rotated by rod

$$\theta = wt$$
$$= 2\pi\left(\frac{m}{M}\right) = 2\pi \times \frac{1}{4} = \frac{\pi}{2}$$



$$v_P = \sqrt{2}v = \sqrt{2}\frac{m}{M}v_0 = \frac{\sqrt{2}}{4}v_0 = \frac{v_0}{2\sqrt{2}}$$

Sol 16. (D)

$$Mg\left(\frac{a}{\sqrt{2}} - \frac{a}{2}\right) = \frac{1}{2}I_A\omega_0^2 \quad \omega_0\sqrt{\frac{3g\left(\sqrt{2}-1\right)}{2\sqrt{2}a}}$$

Sol 17. (C)

Conserving angular momentum about B

$$\frac{Ma^2}{6}\omega_0 = \frac{2Ma^2\omega}{3}$$

$$\omega = \frac{\omega_0}{4}$$

SECTION – B
(Matrix Type)

Sol 1. (A)- (p), (s); (B) –(r); (C) –(p); (D)– (s)

By Fact Base

SECTION – C
(Integer Type)

Sol 1. (4)

In the figures S $\rightarrow$ station. F $\rightarrow$ Factory and ‘P’ is the place where he meets the car.

usual day :

car starts from F at t = 0, reaches station at T and again reaches at the factory at time 2T.

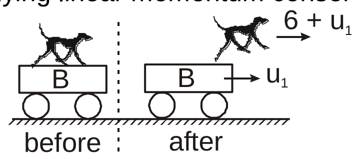
This day :

Person reaches ‘S’ at T – 30. Car starts at t = 0 from F. Person walks for time t and reaches point P at time T – 30 + t. At this time car also reaches ‘P’. Car comes back at ‘F’ at time (2T – 20). That means car takes time T – 10 from F to P. That means car reach at ‘P’ at time T – 5.

Now T – 10 = T – 30 + t $\Rightarrow$ t = 20 min.

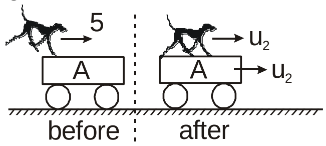
Sol 2. **(5)**

For dog and cart B system, applying linear momentum conservation.



$0 = 4(6 + u_1) + 20\ u_1 \quad \Rightarrow u_1 = -1$ m/s $\qquad$ or $u_1 = 1$ m/s $(\leftarrow)$

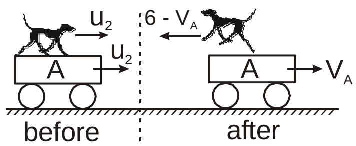
For dog and car A system applying linear momentum conservation



$20 = (20 + 4)\ u_2$

$\Rightarrow \qquad u_2 = 5/6$ m/s $(\rightarrow)$

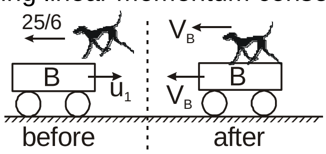
for dog and cart A system



$20 = 20\ V_A + 4\ (V_A - 6)$

$\Rightarrow V_A = \frac{11}{6}$ m/s $(\rightarrow)$ **....Ans**

For dog and cart B system, applying linear momentum conservation



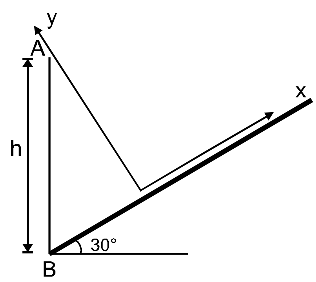
$20(-1) + 4\left(\frac{-25}{6}\right) = 24\ (-V_B) \quad \Rightarrow \quad V_B = \frac{55}{36}$ m/s $(\leftarrow)$

[Ans: x = 5]

Sol.3. **(4)**

$I_0 = \left[\frac{MR^2}{2}\right]\omega = I_E = \left[\frac{2\times 2^2}{2}\right]\times 10 = 40$

Sol.4.(3)



Assume the x-axis up the inclined surface and +y axis perpendicular to inclined surface.

Particle collides inclined plane perpendicular so $(V_{rel})_x =$ 0

$\Rightarrow \left(\frac{v_1}{\sqrt{2}} - v_2\cos 60°\right) - \frac{g}{2}t = 0$

....(1)

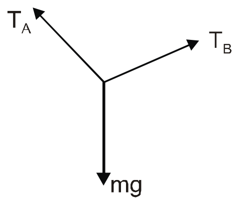
$\Delta y_r = (\Delta y_0)_r + (u_{ry})t + \frac{1}{2}(a_r)_y t^2$

$0 = \frac{h\sqrt{3}}{2} + \left(\frac{v_1}{\sqrt{2}} - \frac{v_2\sqrt{3}}{2}\right)t - \frac{1}{2}\frac{g\sqrt{3}}{2}t^2$

....(2)

From (1) & (2)

$h = \frac{15\sqrt{3} - 20}{\sqrt{3}} \Rightarrow$ So K = 3



Sol.5. (2)

$\vec{T}_A + \vec{T}_B + \vec{mg} = 0 \Rightarrow m = 5$ kg

$\vec{T'}_A + \vec{T'}_B + \vec{mg} + \vec{F} = 0 \Rightarrow T'_B = 78N$

PART-II : CHEMISTRY

SECTION –A
(Single Correct Choice Type)

Sol 1. (A)
Orbital angular momentum depends upon the type of the orbital.

Sol 2. (B)
X – CaC_2,   Y – $CaCN_2$,   Z – NH_4OH.

Sol 3. (A)
More the lone pairs on the central atom, lesser the bond angle.

Sol 4. (B)
Substitution of values in PV = nRT gives Moles = 0.178

Sol 5. (D)

$$\frac{k_t}{k_0} = (TC)^{t-0/10}$$

Taking log gives $\log_e k_t - \log_e k_0$

$= \frac{t}{10} \log_e (TC) \Rightarrow \ln k_t = \ln k_0 + \left(\frac{\ln (TC)}{10}\right) t$

Comparision indicates $\frac{\ln (TC)}{10} = 0.0693$

$\Rightarrow \ln (TC) = 0.693 \Rightarrow TC = 2$

Sol 6. (C)
According to Arrhenius equation

$$k = A \,.\, e^{-E_a/RT}$$

At temp., T the equation will be

$$= A \,.\, e^{-E_{a'}/RT}$$

Sol 7. (D)

$[CaO] = \frac{\rho_{CaO(s)}}{M_{CaO(s)}} = \frac{1.12}{56} \times 1000$

Sol 8. (B)

$3.6 \times \frac{10^{-3}}{6 \times 10^{-3}}$

(Multi Correct Choice Type)

Sol 9. (B, C & D)
Since reaction is exothermic therefore % yield will decreases with increase in temperature.

Sol.10. (A, B)
UV and visible radiations have less energy than IR.

Sol.11. (B)

Kinetic energy does not depend upon intensity.

Sol.12. (A,B)

In A & B electrons occupy higher energy orbitals, while lower energy orbitals are still available.

(Paragraph Type)

Sol 13. (B)

The chemical equation for dimerisation of vaporised acetic acid be following

$2A_{(g)} \rightleftharpoons (HA)_{2(g)}$.

The average molecules mass of the vapour at 160°C and 200°C can be determined from the following equation.

$$M_{160} = \frac{W.R.T}{P.V} = \frac{40.7 \times 0.0821 \times (273+160)}{1.0 \times 20} = 72.3 \text{ g mol}^{-1}$$

$$M_{200} = \frac{W.R.T}{P.V} = \frac{33.4 \times 0.0821 \times (273+200)}{1.0 \times 20} = 64.8 \text{ g mol}^{-1}$$

$$M_{(CH_3COOH)} = 60\,\text{g}\,\text{mol}^{-1}$$

Let the mole fraction of dimmer in equilibrium mixture at 160°C is x. Then

$120x + 60(1 - x) = 72.3$, which gives $x = 0.205$

$$K_{P(160)} = \frac{x}{P(1-x)^2} = 0.324 \text{ atm}^{-1}$$

Sol 14. (A)

Let the more fraction of dimmer at 200°C be y.

$120y + 60(1 - y) = 64.8$

$y = 0.08$

$$K_{P(200)} = 0.\frac{0.08}{(1-0.08)^2} = 0.095 \text{ atm}^{-1}$$

Sol 15. (A)

$$\log\frac{K_1}{K_2} = \frac{\Delta H}{2.303R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

$$\log\frac{0.324}{0.095} = \frac{\Delta H}{2.303 \times 8.314}\left(\frac{1}{473} - \frac{1}{433}\right) \times 10^{-3}$$

$= -52.2 \text{ KJ mol}^{-1}$

Sol 16. (A)

$xA \longrightarrow yB$

$$\frac{-1}{x}\frac{dA}{dt} = \frac{1}{y}\frac{d_B}{dt}$$

$$-\frac{dA'}{dt} = \frac{x'}{y}\frac{dB'}{dt}$$

Taking log

$$-\log\frac{d_A}{dt} = \log\frac{O_B}{dt} + \log\frac{x'}{y} \qquad ...(1)$$

$$-\log\frac{dA'}{dt} = \log\frac{dB'}{dt} + 0.3010 \qquad ...(2)$$

Comparing equation (1) and (2)

$x = 2$, $y = 1$

Sol 17. (B)

$$-\frac{do_1}{dt} = \frac{1}{2}\frac{dso_3}{dt}$$

$$\frac{-dO_2}{dt} = \frac{1}{2} \times \frac{100}{80} \times 32$$

$= 20$ g/m.

SECTION – B
(Matrix Type)

Ans. **(A)-p,r; (B)-r,s; (C)-p,r,s; (D)-q,s**

Sol. $\Delta H = E_f - E_b$

SECTION – C
(Integer Type)

Sol 1. **(4)**

$$\underset{\text{At Eq.}}{}\quad \underset{2-2x}{2A(g)} + \underset{1-x}{B(g)} \rightleftharpoons \underset{7+x}{C(g)} + \underset{3+x}{D(g)}$$

Let y moles of B and 2y moles of A respectively are left.

$(1 - x) = y$, $(2 - 2x) = 2y \Rightarrow x = 1$

$$10^{12} = \frac{8 \times 4}{(2y)^2 y}$$

$$\Rightarrow y^3 = 8 \times 10^{-12}$$

$$y = 2 \times 10^{-4}$$

$A = 2y = 4 \times 10^{-4}$

Sol 2. **(5)**

$N_2O_4 \rightleftharpoons 2NO_2$

At eq. $1 - \alpha$ $\quad 2\alpha$

Total moles = $1 + \alpha = 1.66$

1 mole of N_2O_4 is taken, mole at eq. = 1.66

$\frac{10}{92}$ mole of N_2O_4 is taken, mole at eq. = $\frac{1.66 \times 10}{92} = 0.18$

$PV = \frac{\omega}{m}RT$

$1 \times V = 0.18 \times 0.0821 \times 340 = 5.02$ L

Sol 3. **(1)**

$PCl_5 \rightleftharpoons PCl_3 + Cl_2$

Mole diff. dissociation

$1 - \alpha \qquad \alpha \qquad \alpha$

$$K_P = \frac{n_{PCl_3} \times n_{Cl_2}}{n_{PCl_6}} \times \left(\frac{P}{\Sigma}\right)^{\Delta n}$$

$$= \frac{\alpha.\alpha}{1-\alpha}\left[\frac{P}{1+\alpha}\right]^1 = \frac{4 \times (0.1)^2}{1-(0.1)^2} = 0.040\,\text{atm}$$

Again when $\alpha = 0.2$, K_p = constant

$$K_P = \frac{P\alpha^2}{1-\alpha^2}$$

P = 0.96 atm

Sol 4. **(4)**

The equilibrium $N_2 + O_2$ ....of nitric oxide is

$$N_{2\,(g)} + O_{2\,(g)} \rightleftharpoons 2NO_{(g)}$$

Initial $\quad$ 2 moles $\quad$ 4moles

At Eqm, $2 - \frac{1}{2}$ $\quad 4 - \frac{1}{2}$ $\quad 2 \times \frac{1}{2} = 1$ mol

Molar conc. of NO at eqm. = $\frac{1}{0.25} = 4$

Sol 5. **(5)**

Decomposition of A ....A is present initially.

$$4A(g) \longrightarrow B(g) + 2C(g)$$

t = 0   800 – –

t = 10 minutes,   800 – 4p   p   2p

800 – p = 650

∴ p = 150 Pressure of A = 200, so

∴ 2 x $t_{1/2}$ = 10 minutes

$t_{1/2}$ = 5 minutes

PART-III : MATHEMATICS

Sol 1. (A)

$2^{(\log_2 3)^x} = 3^{(\log_3 2)^x}$

Taking log to the base 2 on both the sides, we get

$(\log_2 3)^x.\log_2 2 = (\log_3 2)^x \log_2 3$

$(\log_2 3)^{x-1} = (\log_3 2)^x$

$\Rightarrow \dfrac{(\log_2 3)^{x-1}}{(\log_3 2)^x} = 1$

$(\log_2 3)^{2x-1} = 1 = (\log_2 3)^0$

$\Rightarrow 2x - 1 = 0$

$\Rightarrow x = \dfrac{1}{2}$

Sol 2. (D)

As, $\tan x = \dfrac{\cos 25^o + \cos 85^o}{\sin 25^o - \sin 85^o} = \dfrac{-2\cos 55^o \cos 30^o}{2\cos 55^o \sin 30^o}$ = cot 30º = – $\sqrt{3}$ ⇒ tan x = – $\sqrt{3}$

⇒ x = 120º

Sol 3. (C)

Using $a^{\log_a N}$ = N repeatedly, we get $(2)^{\log_2 x} = 5 \Rightarrow 2^{\log_2 \sqrt{x}} = 5$

$\Rightarrow \sqrt{x} = 5$

⇒ x = 25

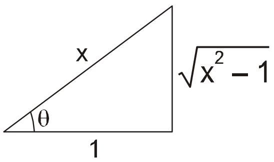
Sol 4. (C)

We have, $\sin^2\left(\dfrac{\theta}{2}\right) = \dfrac{x-1}{2x}$

$\Rightarrow 2\sin^2\left(\dfrac{\theta}{2}\right) = \dfrac{x-1}{x}$

$\Rightarrow 1 - \cos\theta = \dfrac{x-1}{x}$

$\Rightarrow \cos\theta = 1 - \left(\dfrac{x-1}{x}\right) = \dfrac{1}{x}$



Hence, $\tan\theta = \sqrt{x^2 - 1}$.

Sol 5. (B)

We have, $\log_2(-3\sin\theta) = \log_2(\cos^2\theta) + \log_2 2 = \log_2(2\cos^2\theta)$

$\Rightarrow \log_2(-3\sin\theta) = \log_2(2\cos^2\theta)$

$\Rightarrow 2(1 - \sin^2\theta) = -3\sin\theta$

$\Rightarrow 2\sin^2\theta - 3\sin\theta - 2 = 0$

$\Rightarrow (2\sin\theta + 1)(\sin\theta - 2) = 0$

$\therefore \sin\theta = \dfrac{-1}{2}$

⇒ θ = 330º (θ = 210º to be rejected)

Sol 6. (A)

Let $E = \sin^2\alpha + \sin^2\beta + \cos^2(\alpha+\beta) + 2 . \sin\alpha . \sin\beta . \cos(\alpha+\beta)$
$= \sin^2\alpha + \sin^2\beta + \cos^2(\alpha+\beta) + [\cos(\alpha-\beta) - \cos(\alpha+\beta)]. \cos(\alpha+\beta)$
$= \sin^2\alpha + \sin^2\beta + (\cos^2\alpha - \sin^2\beta) = 1$

Aliter : $E = \sin^2\alpha + \sin^2\beta + \cos^2(\alpha+\beta) + 2\sin\alpha\sin\beta\cos(\alpha+\beta)$
$= \sin^2\alpha - \sin^2(\alpha+\beta) + \sin^2\beta + 1 + 2\sin\alpha\sin\beta\cos(\alpha+\beta)$
$= -\sin(2\alpha+\beta) . \sin\beta + \sin^2\beta + 1 + 2\sin\alpha\sin\beta\cos(\alpha+\beta)$
$= 1 - \sin\beta[\sin(2\alpha+\beta) - \sin\beta] + 2\sin\alpha\sin\beta\cos(\alpha+\beta)$
$= 1 - \sin\beta[2\cos(\alpha+\beta)\sin\alpha] + 2\sin\alpha\sin\beta\cos(\alpha+\beta)$
$= 1 - 2\sin\alpha\sin\beta\cos(\alpha+\beta) + 2\sin\alpha\sin\beta\cos(\alpha+\beta)$

$\therefore$ $E = 1$

Sol 7. (A)

$$T_1 = \left(\log_{\frac{a}{b}} p\right)^2 = \frac{1}{\left(\log_p \frac{a}{b}\right)^2} = \frac{1}{\left((\log_p a) - (\log_p b)\right)^2}; \ \log_p a = x;\ \log_p b = y \text{ and } \log_p c = z$$

$\Rightarrow$ $$T_1 = \frac{1}{(x-y)^2};\ T_2 = \frac{1}{(y-z)^2} \text{ and } T_3 = \frac{1}{(z-y)^2}$$

Hence $$E = \frac{\frac{1}{(x-y)^2} + \frac{1}{(y-z)^2} + \frac{1}{(z-x)^2}}{\left(\frac{1}{x-y} + \frac{1}{y-z} + \frac{1}{z-x}\right)^2} = 1$$

Sol 8. (B)

$T_r = (2r-1)(2r+1)(2r+3)$

$$S_n = \sum_{r=1}^{n} T_r$$

(Multi Correct Choice Type)

Sol 9. (B,D)

Dividing by cos (2012º), we get

$$\tan n^\circ = \frac{1 + \tan 2012^\circ}{1 - \tan 2012^\circ}$$

$\tan n^\circ = \tan(2012^\circ + 45^\circ) = \tan 2057^\circ$

Hence $n = k(180^\circ) + 2057^\circ$

Put $k = -10$

$n = 2057^\circ - 1800^\circ = 257^\circ$

if $k = -11$

$\Rightarrow$ $n = 77^\circ$

Sol 10. (A,C)

We have, $$E = \frac{(3 + \log_3 5)}{(1 + \log_5 3)} + \log_{15} 3 + \frac{1}{(1 + \log_3 5)}$$

$$= \frac{\log_3 5(3 + \log_3 5)}{(1 + \log_3 5)} + \frac{1}{(1 + \log_3 5)} + \frac{1}{(1 + \log_3 5)}$$

$$= \frac{(\log_3 5)^2 + 3(\log_3 5) + 2}{(1 + \log_3 5)} = \frac{x^2 + 3x + 2}{1 + x} = \frac{(x+2)(x+1)}{x+1}$$

$= x + 2 = 2 + \log_3 5 = \log_3 45$

Which is an irrational number.

Also, $27 < 45 < 81$

$\Rightarrow$ $3 < \log_3 45 < 4$.

Sol 11. (A,B,C)

(A) $\log_5 \sqrt{25.\sqrt[4]{2^{-5}.2^{-3}}} = \log_5 \sqrt{25.2^{-2}} = \log_5\left(\frac{5}{2}\right) > 0$

(As base and number are located on the same side of unity)

(B) $\log_{\cos\frac{7\pi}{4}}\left(\sin\frac{5\pi}{6}\right) = \log_{\frac{1}{\sqrt{2}}}\left(\frac{1}{2}\right) > 0$

(C) $\log_{\tan\frac{4\pi}{3}}\left(\cot\frac{7\pi}{6}\right) = \log_{\sqrt{3}} = 1 > 0$

(D) $\log_2 \sqrt{9\sqrt[3]{3^{-5}.3^{-7}}} = \log_2 \sqrt{9.3^{-4}} = \log_2\left(\frac{1}{3}\right) =$ negative number.

Sol 12. (A,B,D)

$$P(x) = \left(1+\cos\frac{\pi}{6x}\right)\left(1+\cos\left(\frac{\pi}{2}-\frac{\pi}{6x}\right)\right)\left(1+\cos\left(\frac{\pi}{2}+\frac{\pi}{6x}\right)\right)\left(1+\cos\left(\pi-\frac{\pi}{6x}\right)\right)$$

$$P(x) = \left(1+\cos\frac{\pi}{6x}\right)\left(1+\sin\frac{\pi}{6x}\right)\left(1-\sin\frac{\pi}{6x}\right)\left(1-\cos\frac{\pi}{6x}\right) = \frac{4\sin^2\frac{\pi}{6x}.\cos^2\frac{\pi}{6x}}{4} = \frac{1}{4}\sin^2\left(\frac{\pi}{3x}\right)$$

$$P(1) = \frac{1}{4}\sin^2\left(\frac{\pi}{3}\right) = \frac{1}{4}\times\frac{3}{4} = \frac{3}{16}$$

$$P(2) = \frac{1}{4}\sin^2\left(\frac{\pi}{6}\right) = \frac{1}{4}\times\frac{1}{4} = \frac{1}{16}$$

$$P(3) = \frac{1}{4}\sin^2\left(\frac{\pi}{9}\right) \neq 0$$

$$P(4)\ \frac{1}{4}\sin^2\left(\frac{\pi}{12}\right) = \frac{1}{4}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2 = \frac{4-2\sqrt{3}}{4\times 8} = \frac{2-\sqrt{3}}{16}$$

(Paragraph Type)

Ans.13.(A)

Ans.14.(B)

Ans.15.(C)

Sol.13-15.

$z = -\lambda \pm \sqrt{\lambda^2 - 1}$

Case-1 : $-1 < l < 1 \Rightarrow z = -\lambda \pm i\sqrt{1-\lambda^2}$

$\Rightarrow x^2 + y^2 = 1$

Case-2 : $\lambda > 1$, $z = \lambda \pm \sqrt{\lambda^2 - 1}$

$\Rightarrow x = -\lambda \pm \sqrt{\lambda^2 - 1}, y = 0$

One root inside the unit circle and the other root lies outside the unit circle.

Case-3 : When λ is very large

$z = -\lambda - \sqrt{\lambda^2 - 1} \approx -2\lambda$

$$z = -\lambda + \sqrt{\lambda^2 - 1} = \frac{1}{-\lambda - \sqrt{\lambda^2 - 1}} \approx -\frac{1}{2\lambda}$$

Ans.16.(A)

Ans.17.(B)

Sol.16-17.

Assume $x_1 = a - d$, $x_2 = a$, and $x_3 = a + d$, then $x_1 + x_2 + x_3 = 1$

$\Rightarrow$ a = 1/3

Also, $x_1x_2 + x_2x_3 + x_3x_1 = \beta$

and $x_1x_2x_3 = \gamma$

From the above we get $\beta = \frac{1}{3} - d^2 < \frac{1}{3}$ and g = $\frac{1}{3}\left(d^2 - \frac{1}{9}\right) > -\frac{1}{27}$

SECTION – B
(Matrix Type)

Ans. 13(A)-Q; (B)-Q,S; (C)-P; (D)-R

Sol. (A) $(\alpha + 4)\,\alpha = K$

$\alpha + \alpha + 4 = 8$

$2\alpha = 4$

$\alpha = 2$

$K = 2 \cdot 6 = 12$

(B) roots = α, 3α

$4\alpha = -K$

$3\alpha^2 = 27$

$\alpha^2 = 9$

$\alpha = \pm 3 \Rightarrow \alpha = 3$

$K = -12$

(C) $3 + 4i + 3 - 4i = -p$

$(3 + 4i)(3 - 4i) = q$

(D) $x^2 - (a-1)n + \left(a + \frac{1}{4}\right) + \left(\frac{a-1}{2}\right)^2 - \left(\frac{a-1}{2}\right)^2$

$$= \left(x - \left(\frac{a-1}{2}\right)\right)^2 + \left[\left(a + \frac{1}{4}\right) - \left(\frac{a-1}{2}\right)^2\right]$$

$\left(a + \frac{1}{4}\right) - \left(\frac{a-1}{2}\right)^2 = 0 \Rightarrow \boxed{a = 6}$

SECTION – C
(Integer Type)

Sol 1. (1)

$\arg(z^2 + \bar{z}z^{1/3}) - \arg z^{2/3} = 0$

$\arg\left(\frac{z^2 + \bar{z}z^{1/3}}{z^{2/3}}\right) = 0$

$\arg\left(z^{4/3} + \frac{\bar{z}}{z^{1/3}}\right) = 0$

$\Rightarrow z^{4/3} + \frac{\bar{z}}{z^{1/3}} = \bar{z}^{4/3} + \frac{z}{(\bar{z})^{1/3}}$

$\Rightarrow z^{4/3} - \bar{z}^{4/3} = \frac{z.z^{1/3}}{|z|^{2/3}} - \frac{\bar{z}.\bar{z}^{1/3}}{|z|^{2/3}}$

$\Rightarrow (z)^{4/3} - (\bar{z})^{4/3} = \frac{\left(z^{4/3} - (\bar{z})^{4/3}\right)}{(|z|)^{2/3}}$

$\Rightarrow (z^{4/3} - (\bar{z})^{4/3}\left(1 - \frac{1}{|z|^{2/3}}\right) = 0$

$\Rightarrow |z|^{2/3} = 1$

$\Rightarrow |z| = 1$

Sol 2. **(3)**

Given $\underbrace{4x^2 - (5p + 1)x + 5p = 0}_{\alpha \quad \beta = 1 + \alpha}$

Clearly, sum of roots = a + (1 + a) = $\frac{5p+1}{4}$

$\Rightarrow \quad a = \frac{\left(\frac{5p+1}{4} - 1\right)}{2} = \frac{5p-3}{8}$ …. (1)

Also, product of roots = $\alpha (1 + \alpha) = \frac{5p}{4}$ …. (2)

$\therefore$ From (1) and (2), we get

$$\alpha^2 + \alpha = \frac{5p}{4}$$

$\Rightarrow \quad \left(\frac{5p-3}{8}\right)^2 + \left(\frac{5p-3}{8}\right) = \frac{5p}{4}$

$\Rightarrow \quad (5p - 3)^2 + 8(5p - 3) = 80\,p$

$\Rightarrow \quad 25p^2 - 70p - 15 = 0$

$\Rightarrow \quad 5p^2 - 14p - 3 = 0$

$\Rightarrow \quad (5p + 1)(p - 3) = 0$

$\therefore \quad p = \frac{-1}{5}, 3$

Hence, integral value of p= 3.

Sol 3. **(3)**

$(x - 1)^3 + 8 = 0$

$\alpha - 1 = (-8)^{1/3}$

$\Rightarrow \alpha - 1 = -2$

$\beta - 1 = -2\omega$

$\gamma - 1 = -2\omega^2$

Hence $z = \frac{\alpha-1}{\beta-1} + \frac{\beta-1}{\gamma-1} + \frac{\gamma-1}{\alpha-1} = \frac{1}{\omega} + \frac{1}{\omega} + \omega^2 = 3\omega^2$

Hence $|z| = |3\omega^2| = 3$

Sol 4. **(4)**

Let $\frac{\pi}{24} = \theta$ so that $12\theta = \frac{\pi}{2}$

$S = \sin^4\theta + \sin^4 2\theta + \ldots\ldots + \underbrace{\sin^4 6\theta}_{\frac{1}{4}} + \ldots\ldots + \sin^4(11\theta)$

$= \frac{1}{4} + \sum_{r=1}^{5}(\sin^4 r\theta + \cos^4 r\theta) = \frac{1}{4} + \sum_{r=1}^{5}\left((\sin^2 r\theta + \cos^2 r\theta)^2 - 2\sin^2 r\theta\cos^2 r\theta\right)$

$= \frac{1}{4} + \sum_{r=1}^{5}(1 - 2\sin^2 r\theta\cos^2 r\theta) = \frac{21}{4} - \frac{1}{2}\sum_{r=1}^{5} 4\sin^2 r\theta\cos^2 r\theta$

$= \frac{21}{4} - \frac{1}{2}\sum_{r=1}^{5}\sin^2 2r\theta = \frac{21}{4} - \frac{1}{4}\sum_{r=1}^{5} 2\sin^2 2r\theta = \frac{21}{4} - \frac{1}{4}\sum_{r=1}^{5}(1 - \cos 4r\theta)$

$= \frac{21}{4} - \frac{5}{4} + \underbrace{\sum_{r=1}^{5}\cos 4r\theta}_{\text{zero}} = \frac{16}{4} = 4$

Sol 5. **(5)**

Given $y = ax^2 + bx + c$ does not intersect the x-axis.

$\Rightarrow$ $b^2 - 4ac < 0$

also $1 = 4a - 2b + c$ .... (2)

and $9 = 4a + 2b + c$ .... (3)

Now, $(3) - (2) \Rightarrow 4b = 8 \Rightarrow b = 2$

$\therefore$ $4a + c = 5$ .... (4)

$\Rightarrow$ $c = (5 - 4a)$

Using $b^2 - 4ac < 0$

$\Rightarrow$ $4 - 4a(5 - 4a) < 0 \Rightarrow 4 - 20a + 16a^2 < 0 \Rightarrow 1 - 5a + 4a^2 < 0 \Rightarrow 4a^2 - 5a + 1 < 0$

$\Rightarrow$ $4a^2 - 4a - a + 1 < 0 \Rightarrow 4a(a - 1) - (a - 1) < 0 \Rightarrow (a - 1)(4a - 1) < 0 \Rightarrow \frac{1}{4} < a < 1$

$\Rightarrow$ $\frac{1}{2} < 2a < 2 \Rightarrow 1 < \frac{b}{2a} < 4 \Rightarrow -4 < \frac{-b}{2a} < -1 \Rightarrow \frac{-b}{2a} \in (-4, -1) = (x_1, x_2)$

Hence, $|x_1 + x_2| = 5$.