CLASSROOM STUDY PACKAGE

MATHEMATICS

Polynomial

JEE EXPERT

POLYNOMIAL

CONTENTS

KEY-CONCEPTS

An algebraic expression of the form

$p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots\ldots + a_1 x^1 + a_0 x^0$, where

(i) $a_n \neq 0$

(ii) $a_0, a_1, a_2, \ldots\ldots\ldots a_n$ are real numbers

(iii) n is whole number, is called a polynomial.

a_n, a_{n-1}, a_{n-2}, are coefficients of $x^n, x^{n-1} \ldots\ldots x^0$ respectively and $a_n x^n, a_{n-1} x^{n-1}, a_{n-2} x^{n-2}, \ldots\ldots\ldots$ are terms of the polynomial. Here the term $a_n x^n$ is called the **leading term** and its coefficient a_n, the **leading coefficient**.

For example : $p(x) = 3x^3 - 4x^2 + 5x + 9$ is a polynomial in variable x.

$3x^3, -4x^2, 5x, 9$ are known as terms of polynomial and 3,–4, 5, 9 are their respective coefficients.

Types of Polynomials : Generally we divide the polynomials in three categories.

Types of polynomials

Based on number of distinct variables | Based on degree | Based on number of terms

Polynomials classified by number of distinct variables

Number of distinct variables	Name	Example
1	Univariate	x+2
2	Bivariate	x+y=2
3	Trivariate	x+y+z+2

Generally, a polynomial in more than one variable is called a multivariate polynomial. A second major way of classifying polynomials is by their degree. Recall that the degree of a term is the sum of the exponents on variables, and that the degree of a polynomial is the largest degree of any one term.

Polynomials classified by degree

Degree	Name	Example
0	(non-zero) constant	1
1	Linear	x+4
2	Quadratic	x^2+4
3	Cubic	x^3+4

Usually, a polynomial of degree n, for n greater than 3, is called a polynomial of degree n.

Polynomials classified by number of non-zero terms

Number of non - zero terms	Name	Example
0	zero polynomial	0
1	monomial	x^2
2	binomial	x^2+2
3	trinomial	x^2+x+2

If a polynomial has only one variable, then the terms are usually written either from highest degree to lowest degree ("descending powers") or from lowest degree to highest degree ("ascending powers").

Value of Polynomial : If p(x) is a polynomial in variable x and α is any real number, then the value obtained by replacing x by α in p(x) is called value of p(x) at x = α and is denoted by p(α).

Example : Find the value of $p(x) = x^3 - 6x^2 + 11x - 6$ at $x = -2$

$\Rightarrow$ $p(-2) = (-2)^3 - 6(-2)^2 + 11(-2) - 6 = -8 - 24 - 22 - 6$

$\Rightarrow$ $p(-2) = -60$

Then the value of p(x) is – 60

Zero of a Polynomial : A real number α is a zero of the polynomial p(x) if p(α) = 0.

Example : Consider $p(x) = x^3 - 6x^2 + 11x - 6$.

$\Rightarrow$ $p(1) = (1)^3 - 6(1)^2 + 11(1) - 6 = 1 - 6 + 11 - 6 = 0$

$\Rightarrow$ $p(2) = (2)^3 - 6(2)^2 + 11(2) - 6 = 8 - 24 + 22 - 6 = 0$

$\Rightarrow$ $p(3) = (3)^3 - 6(3)^2 + 11(3) - 6 = 27 - 54 + 33 - 6 = 0$

Thus, 1,2 and 3 are called the zeros of polynomial p(x).

GEOMETRICAL MEANING OF THE ZEROES OF A POLYNOMIAL :

Geometrically the zeros of a polynomials f(x) are the x-co-ordinates of the points where the graph y = f(x) intersects x-axis. To understand it, we will see the geometrical representations of linear and quadratic polynomials.

Geometrical Representation of the zero of a Linear Polynomial

consider a linear polynomial, y = 2x – 5.

The following table lists the values of y corresponding to different values of x.

x	1	4
y	–3	3

Table-1

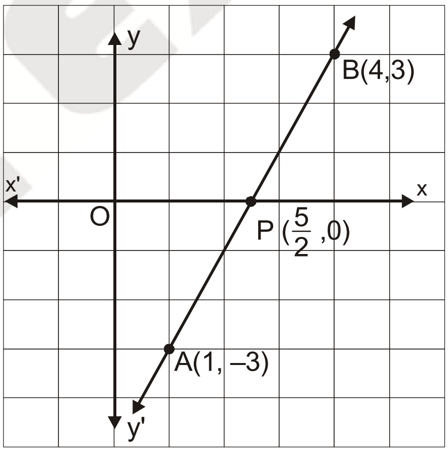


Figure-1

On plotting the points A(1, –3) and B(4, 3) and joining them, a straight line is obtained.

From, graph (see fig. 1) we observe that the graph of y = 2x – 5 intersects the x-axis at $\left(\frac{5}{2}, 0\right)$ whose x-coordinate is $\frac{5}{2}$. Also, zero of 2x – 5 is $\frac{5}{2}$. Therefore, we conclude that the linear polynomial ax + b has one and only one zero, which is the x-coordinate of the point where the graph of y = ax + b intersects the x-axis

Geometrical representation of the zero of a quadratic polynomial

x	$y = x^2 - 2x - 8$
– 4	16
– 3	7
– 2	0
– 1	–5
0	–8
1	–9
2	–8
3	–5
4	0
5	7
6	16

Table-2

Consider a quadratic polynomial, $y = x^2 - 2x - 8$,
The table-2 gives the values of y or f(x) for various values of x.
On plotting the points (–4, 16), (–3,7) (–2, 0), (–1, –5), (0, –8), (1,–9) (2, –8), (3, –5), (4, 0), (5, 7) and (6, 16) on a graph paper and drawing a smooth free hand curve passing through these points, the curve thus obtained represents the graph of the polynomial $y = x^2 - 2x - 8$. This is called a parabola.

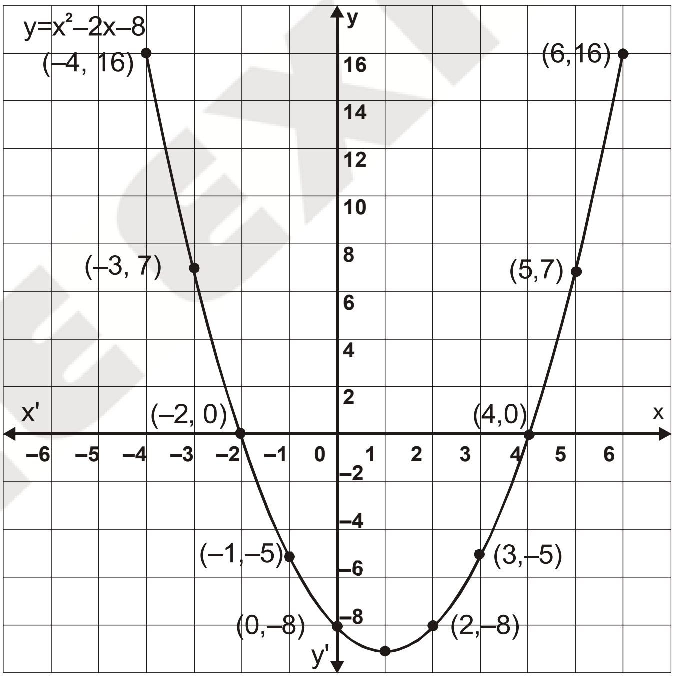


Figure-2

It is clear from the table that –2 and 4 are the zeros of the quadratic polynomial $(x^2 - 2x - 8)$. Also, we observe that –2 and 4 are the x-coordinates of the points where the graph of $y = x^2 - 2x - 8$ intersects the x-axis.

Relationship between zeros and coefficients of a quadratic polynomial

Let α and β be the zeros of a quadratic polynomial $f(x) = ax^2 + bx + c$. By factor theorem $(x-\alpha)$ and $(x-\beta)$ are the factors of f(x).

$\therefore$ $f(x) = k(x-\alpha)(x-\beta)$ are the factors of f(x)

$\Rightarrow$ $ax^2 + bx + c = k\{x^2 - (\alpha + \beta)x + \alpha\beta\}$

$\Rightarrow$ $ax^2 + bx + c = kx^2 - k(\alpha + \beta)x + k\alpha\beta$

comparing the coefficients of x^2, x and constant terms on both sides, we get

$a = k$, $b = -k(\alpha + \beta)$ and $c = \alpha\beta$

$\Rightarrow \quad \alpha + \beta = -\dfrac{b}{a}$ and $\alpha\beta = \dfrac{c}{a}$

$\Rightarrow \quad \alpha + \beta = -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

and $\quad \alpha\beta = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

Hence,

Sum of the zeros $= -\dfrac{b}{a} = -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= \dfrac{c}{a} = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

- **Remarks :**

If α and β are the zeros of a quadratic polynomial f(x). Then, the polynomial f(x) is given by

$f(x) = k\{x^2 - (\alpha + \beta)x + \alpha\beta\}$

or, $\quad f(x) = k[x^2 - \{\text{Sum of the zeros})x + \text{Product of the zeros}\}$

Ex. Find the zeros of the quadratic polynomial $f(x) = x^2 - 2x - 8$ and verify the relationship between the zeros and their coefficients.

Sol. $f(x) = x^2 - 2x - 8$

$\Rightarrow \quad f(x) = x^2 - 4x + 2x - 8$

$\Rightarrow \quad f(x) = x(x-4) + 2(x-4)$

$\Rightarrow \quad f(x) = (x - 4)(x + 2)$

Zeros of f(x) are given by f(x) = 0

$\Rightarrow \quad x^2 - 2x - 8 = 0$

$\Rightarrow \quad (x - 4)(x + 2) = 0$

$\Rightarrow \quad x = 4$ or $x = -2$

So, $\alpha = 4$ and $\beta = -2$

$\therefore \quad$ sum of zeros $= \alpha + \beta = 4 - 2 = 2$

Also, sum of zeros $= -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$= \dfrac{-(-2)}{1} = 2$

Now, product of zeros $= \alpha\beta = (4)(-2) = -8$

Also, product of zeros $= \dfrac{\text{Constant term}}{\text{Coefficient of } x^2} = \dfrac{-8}{1} = -8$

$\therefore \quad$ Product of zeros $= \dfrac{\text{Constant term}}{\text{Coefficient of } x^2} = \alpha\beta.$

Ex. Find a quadratic polynomial whose zeros are $5 + \sqrt{2}$ and $5 - \sqrt{2}$

Sol. Given, $\alpha = 5 + \sqrt{2}$, $\beta = 5 - \sqrt{2}$

$\therefore$ $f(x) = k\{x^2 - x(\alpha + \beta) + \alpha\beta\}$

Here, $\alpha + \beta = 5 + \sqrt{2} + 5 - \sqrt{2} = 10$

and $\alpha\beta = (5 + \sqrt{2})(5 - \sqrt{2}) = 25 - 2 = 23$

$\therefore$ $f(x) = k\{x^2 - 10x + 23\}$, where, k is any non-zero real number.

Ex. Sum and product of zeros of quadratic polynomial are 5 and 17 respectively. Find the polynomial.

Sol. Given : Sum of zeros = 5 and product of zeros = 17

So, quadratic polynomial is given by

$\Rightarrow$ $f(x) = k(x^2 - x$ (sum of zeros) + product of zeros)

$\Rightarrow$ $f(x) = k\{x^2 - 5x + 17\}$, where, k is any non-zero real number.

Relationship between zeros and coefficients of a cubic polynomial

Let α, β, γ be the zeros of a cubic polynomial $f(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$. Then, by factor theorem, $x - \alpha$, $x - \beta$ and $x - \gamma$ are factors of f(x). Also, f(x) being a cubic polynomial, cannot have more than three linear factors.

$\therefore$ $f(x) = k(x-\alpha)(x-\beta)(x-\gamma)$

$\Rightarrow$ $ax^3 + bx^2 + cx + d = k(x-\alpha)(x-\beta)(x-\gamma)$

$\Rightarrow$ $ax^3 + bx^2 + cx + d = k\{x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma\}$

$\Rightarrow$ $ax^3 + bx^2 + cx + d = kx^3 - k(\alpha + \beta + \gamma)x^2 + k(\alpha\beta + \beta\gamma + \gamma\alpha)x - k\alpha\beta\gamma$

Comparing the coefficients of x^3, x^2, x and constant terms on both sides, we get

$a = k$, $b = -k(\alpha + \beta + \gamma)$, $c = k(\alpha\beta + \beta\gamma + \gamma\alpha)$ and $d = -k(\alpha\beta\gamma)$

$\Rightarrow$ $\alpha + \beta + \gamma = -\dfrac{b}{a}$

$\Rightarrow$ $\alpha\beta + \alpha\gamma + \gamma\alpha = \dfrac{c}{a}$

and $\alpha, \beta, \gamma = -\dfrac{d}{a}$

$\Rightarrow$ Sum of the zeros $= -\dfrac{b}{a} = -\dfrac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

$\Rightarrow$ Sum of the products of the zeros taken two at a time $= \dfrac{c}{a} = \dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$

$\Rightarrow$ Product of the zeros $= -\dfrac{d}{a} = -\dfrac{\text{Constant term}}{\text{Coefficient of } x^3}$

• Remarks:

Cubic polynomial having α, β and γ as its zeros is given by

$f(x) = k(x - \alpha)(x-\beta)(x-\gamma)$

$f(x) = k\{x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma\}$,

where k is any non-zero real number.

Ex. Verify that $\frac{1}{2}$, 1, –2 are zeros of cubic polynomial

$2x^3 + x^2 – 5x + 2$. Also verify the relationship between, the zeros and their coefficients.

Sol. $f(x) = 2x^3 + x^2 – 5x + 2$

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$$

$$= \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2 = 0$$

$f(1) = 2(1)^3 + (1)^2 – 5(1) + 2 = 2 + 1 – 5 + 2 = 0$,

$f(–2) = 2(–2)^3 + (–2)^2 – 5(–2) + 2 = – 16 + 4 + 10 + 2 = 0$.

Let $\alpha = \frac{1}{2}$, $\beta = 1$, and $\gamma = – 2$

Now, Sum of zeros $= \alpha + \beta + \gamma = \frac{1}{2} + 1 – 2 = – \frac{1}{2}$

Also, sum of zeros $= – \frac{(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3} = – \frac{1}{2}$

So, sum of zeros $= \alpha + \beta + \gamma = – \frac{(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3}$

Sum of product of zeros taken two at a time $= \alpha\beta + \beta\gamma + \gamma\alpha = \frac{1}{2} \times 1 + 1 \times (–2) + (–2) \times \frac{1}{2} = \frac{-5}{2}$

Also, $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \frac{-5}{2}$

So, sum of product of zeros taken two at a time $= \alpha\beta + \beta\gamma + \gamma\alpha = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$

Now, Product of zeros $= \alpha\beta\gamma = \left(\frac{1}{2}\right)(1)(–2) = –1$

Also, product of zeros $= – \frac{\text{Constant term}}{\text{Coefficient of } x^3} = -\frac{2}{2} = –1$

$\therefore$ Product of zeros $= \alpha\beta\gamma = – \frac{\text{Constant term}}{\text{Coefficient of } x^3}$.

Ex. Find a polynomial with the sum, sum of the product of its zeros taken two at a time, and product of its zeros as 3, –1 and –3 respectively.

Sol. Given, $\alpha + \beta + \gamma = 3$, $\alpha\beta + \beta\gamma + \gamma\alpha = –1$ and $\alpha\beta\gamma = –3$ So, polynomial $f(x) = k \{x^3 – x^2 (\alpha + \beta + \gamma) + x (\alpha\beta + \beta\gamma + \gamma\alpha) – \alpha\beta\gamma\}$

$f(x) = k \{x^3 – 3x^2 – x + 3\}$, where k is any non-zero real number.

Remainder Theorem

Let p(x) be any polynomial of degree greater than or equal to one and 'a' be any real number. If p(x) is divided by (x–a), then the remainder is equal to p(a).

Ex. Find the remainder when $f(x) = x^3 - 6x^2 + 2x - 4$ is divided by $g(x) = 1 - 2x$.

Sol. $1 - 2x = 0 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 - 6\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) - 4 = \frac{1}{8} - \frac{3}{2} + 1 - 4$$

$$= \frac{1-12+8-32}{8} = -\frac{35}{8}$$

FACTOR THEOREM

Let p(x) be a polynomial of degree greater than or equal to 1 and 'a' be a real number such that p(a) = 0, then (x–a) is a factor of p(x). conversely, if (x–a) is a factor of p(x), then p(a) = 0.

Ex. Show that x + 1 and 2x – 3 are factors of $2x^3 - 9x^2 + x + 12$.

Sol. To prove that (x + 1) and (2x – 3) are factors of $p(x) = 2x^3 - 9x^2 + x + 12$ it is sufficient to show that

p(–1) and $p\left(\frac{3}{2}\right)$ both are equal to zero.

$p(-1) = 2(-1)^3 - 9(-1)^2 + (-1) + 12 = -2 - 9 - 1 + 12$

$= -12 + 12 = 0$

And $p\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 - 9\left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right) + 12$

$$= \frac{27}{4} - \frac{81}{4} + \frac{3}{2} + 12 = \frac{27-81+6+48}{4} = \frac{-81+81}{4} = 0.$$

Ex. Find α and β if x +1 and x + 2 are factprs $p(x) = x^3 + 3x^2 - 2\alpha x + \beta$.

Sol. x + 1 and x + 2 are the factor of p(x).

Then, p(–1) = 0 & p(–2) = 0

Therefore, $p(-1) = (-1)^3 + 3(-1)^2 - 2\alpha(-1) + \beta = 0$

$\Rightarrow \quad -1 + 3 + 2\alpha + \beta = 0$

$\Rightarrow \quad \beta = -2\alpha - 2$ ...(i)

$\Rightarrow \quad p(-2) = (-2)^3 + 3(-2)^2 - 2\alpha(-2) + \beta = 0$

$\Rightarrow \quad -8 + 12 + 4\alpha + \beta = 0$

$\Rightarrow \quad \beta = -4\alpha - 4$ ...(ii)

From equation (1) and (2)

$-2\alpha - 2 = -4\alpha - 4$

$\Rightarrow \quad 2\alpha = -2$

$\Rightarrow \quad \alpha = -1$

Put $\alpha = -1$ in equation (1)

$\Rightarrow \quad \beta = -2(-1) - 2 = 2 = 2 - 2 = 0.$

Hence $\alpha = -1, \beta = 0$.

DIVISION ALGORITHM FOR POLYNOMIALS

If p(x) and g(x) are any two polynomials with $g(x) \neq 0$.

then we can find polynomials r(x) and q(x) such that

p(x) = g(x) + q(x) + r(x)

i.e. Dividend = (Divisor × Quotient) + Remainder

where r(x) = 0 or degree of r(x) < degree of g(x).

(i) if r(x) = 0, g(x) is a factor of p(x)

(ii) if deg (p(x)) > deg (g(x)), then deg (q(x)) = deg (p(x)) – deg(g(x))

(iii) if deg (p(x)) = deg (g(x)), then deg (q(x)) = 0 and deg (r(x)) < deg (g(x))

Ex. What must be added to $3x^3 + x^2 - 22x + 9$ so that the result is exactly divisible by $3x^2 + 7x - 6$

Sol. Let p(x) $3x^3 + x^2 - 22x + 9$ and $q(x) = 3x^2 + 7x - 6$

We know that if p(x) is divided by q(x) which is quadratic polynomial then the remainder be r(x) and degree of r(x) is less than q(x) or Divisor.

$\therefore$ By long division method

Let we added ax + b (linear polynomial) in p(x). so that p(x) + ax + b is exactly divisible by $3x^2 + 7x - 6$

Hence, $p(x) + ax + b = s(x) = 3x^3 + x^2 - 22x + 9 + ax + b = 3x^3 + x^2 - x(22 - a) + (9 + b)$.

$$
\begin{array}{r|l}
 & \quad x-2 \\
\hline
3x^2+7x-6 & 3x^3+x^2-x(22-a)+9+b \\
 & \underset{-}{}3x^3\underset{-}{+}7x^2\underset{+}{-}6x \\
\cline{2-2}
 & -6x^2+6x(22-a)+9+b \\
 & \qquad \text{or} \\
 & -6x^2+x(-16+a)+9+b \\
 & \underset{+}{-}6x^2\underset{+}{-}14x\underset{-}{+}12 \\
\cline{2-2}
 & x(-2+a)+(b-3)=0
\end{array}
$$

Hence, $x(a-2) + b - 3 = 0 . x + 0$

$\Rightarrow$ $a - 2 = 0$ & $b - 3 = 0$

$\Rightarrow$ $a = 2$ and $b = 3$

hence if in p(x) we added 2x + 3 then it is exactly divisible by $3x^2 + 7x - 6$

SOLVED EXAMPLES

1. Find the zeros of the quadratic polynomial $9x^2 – 5$ and the verify the relation between the zeros and its coefficients.

Sol. We have,

$9x^2 – 5 = (3x)^2 – (\sqrt{5})^2 = (3x - \sqrt{5})\left(3x + \sqrt{5}\right)$

So, the value of $9x^2 – 5$ is 0, when

$3x - \sqrt{5} = 0$ or $3x + \sqrt{5} = 0$

i.e., when $x = \dfrac{\sqrt{5}}{3}$ or $x = \dfrac{-\sqrt{5}}{3}$

Therefore, the zeros of $9x^2 – 5$ are $\dfrac{\sqrt{5}}{3}$ and $\dfrac{-\sqrt{5}}{3}$.

Sum of the zeros

$= \dfrac{\sqrt{5}}{3} - \dfrac{\sqrt{5}}{3} = 0 = \dfrac{-(0)}{9} = \dfrac{\text{coefficient of x}}{\text{coefficient of } x^2}$

Product of the zeros

$= \left(\dfrac{\sqrt{5}}{3}\right)\left(\dfrac{-\sqrt{5}}{3}\right) = \dfrac{-5}{9} = \dfrac{\text{constant term}}{\text{coefficient of } x^2}$

2. Verify that the number given along sides of the cubic polynomials are their zeros. Also verify the relationship between the zeros and the coefficients,

$x^3 + 2x^2 – x – 2$; 1 –1 , –2

Sol. Here the polynomial p(x) is $x^3 + 2x^2 – x – 2$

Value of the polynomial $x^3 + 2x^2 – x – 2$

when x = 1

$P(1) = (1)^3 + 2(1)^2 – 1 – 2 = 1 + 2 – 1 – 2 = 0$

So, 1 is a zero of p(x).

on putting (x = – 1) in the cubic polynomial

$x^3 + 2x^2 – x – 2$

$P(–1) = (–1)^3 + 2 (–1)^2 –(– 1) – 2$

$= –1 + 2 + 1 – 2 = 0$

So, –1 is a zero of p(x).

On putting x = – 2 in the cubic polynomial

$x^3 + 2x^2 – x – 2$

$P (–2) = (–2)^3 + 2 (–2)^2 – (–2) – 2 = – 8 + 8 + 2 – 2 = 0$

So, –2 is zero of p(x).

Hence, 1, –1 and –2 are the zeros of the given polynomial.

Sum of the zeros of p(x) = $1 –1 –2 = \dfrac{-2}{1} = \dfrac{-\text{coefficient of } x^2}{\text{coefficient of } x^3}$

Sum of the products of two zeros taken at a time = (1) (–1) + (–1) (–2) + (1) (–2)

$= – 1 + 2 – 2 = – 1 = \dfrac{-1}{1} = \dfrac{\text{coefficient of x}}{\text{coefficient of } x^3}$

Product of all three zeros = (1) (–1) (–2) = 2 = $\dfrac{-(-2)}{1} = -\dfrac{\text{constant term}}{\text{coefficient of } x^3}$

3. Verify that the numbers given along side of the polynomial are their zeros.
$x^4 + 2x^3 - 7x^2 - 8x + 12; -3, -2, 1, 2$

Sol. Here the polynomial p(x) is
$x^4 + 2x^3 - 7x^2 - 8x + 12$
Value of the polynomial $x^4 + 2x^3 - 7x^2 - 8x + 12$
when $x = -3$
$P(-3) = (-3)^4 + 2(-3)^3 - 7(-3)^2 - 8(-3) + 12 = 81 - 54 - 63 + 24 + 12 = 0$
So, -3 is a zero of p(x).
On putting $x = -2$ is the given polynomial, we have
$P(-2) = (-2)^4 + 2(-2)^3 - 7(-2)^2 - 8(-2) + 12 = 16 - 16 - 28 + 16 + 12 = 0$
So, -2 is a zero of p(x).
On putting $x = 1$ in the given polynomial, we have
$P(1) = (1)^4 + 2(1)^3 - 7(1)^2 - 8(1) + 12 = 1 + 2 - 7 - 8 + 12 = 0$
So, 1 is a zero of p(x).
On putting $x = 2$ in the given polynomial, we have
$P(2) = (2)^4 + 2(2)^3 - 7(2)^2 - 8(2) + 12 = 16 + 16 - 28 - 16 + 12 = 0$
So, 2 is a zero of p(x).
Hence, $-3, -2, 1, 2$ are the zeros of given polynomial.

4. Find the cubic polynomial with the sum, sum of the products of its zeros taken two at a time and product of its zeros as 0, –7 and –6 respectively.

Sol. Le the cubic polynomial be
$ax^3 + bx^2 + cx + d$

$\Rightarrow \quad x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a}$ .....(1)

and its zeros are α, β and γ. Then

$\alpha + \beta + \gamma = 0 = -\frac{b}{a}$

$\alpha\beta + \beta\gamma + \alpha\gamma = -7 = \frac{c}{a}$

$\alpha\beta\gamma = -6 = \frac{-d}{a}$

Putting the values of $\frac{b}{a}, \frac{c}{a}$ and $\frac{d}{a}$ in (1), we get $x^3 - (0)x^2 + (-7)x - (-6)$ or $x^3 - 7x + 6$.

5. If α and β are the zeros of the polynomial $x^2 + 4x + 3$, form the polynomial whose zeros are

$1 + \frac{\beta}{\alpha}$ and $1 + \frac{\alpha}{\beta}$.

Sol. Since α and β are the zeros of the quadratic polynomial
$x^2 + 4x + 3$
Then, $\alpha + \beta = -4$, $\alpha\beta = 3$

sum of the zeros $= 1 + \frac{\beta}{\alpha} + 1 + \frac{\alpha}{\beta} = \frac{\alpha\beta + \beta^2 + \alpha\beta + \alpha^2}{\alpha\beta} = \frac{\alpha^2 + \beta^2 + 2\alpha\beta}{\alpha\beta} = \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(-4)^2}{3} = \frac{16}{3}$

Product of the zeros

$$= \left(1+\frac{\beta}{\alpha}\right)\left(1+\frac{\alpha}{\beta}\right) = 1+\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+\frac{\alpha\beta}{\alpha\beta} = 2+\frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{2\alpha\beta+\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(-4)^2}{3} = \frac{16}{3}$$

But required polynomial is

x^2 – (sum of the zeros) x + product of the zeros

or $\quad x^2 - \frac{16}{3}x + \frac{16}{3}$ or $k\left(x^2 - \frac{16}{3}x + \frac{16}{3}\right)$

or $\quad 3\left(x^2 - \frac{16}{3}x + \frac{16}{3}\right)$ (if k = 3)

$\Rightarrow \quad 3x^2 - 16x + 16$

6. Check whether the first polynomial is a factor of the second polynomial by applying the Division Algorithm;
$x^2 - 2x + 1$ and $2x^4 - 3x^3 - 3x^2 + 7x - 3$

Sol. We divide

$$\begin{array}{r|l} & 2x^2 + x - 3 \\ \hline x^2-2x+1 & 2x^4 - 3x^3 - 3x^2 + 7x - 3 \\ & 2x^4 - 4x^3 - 2x^2 \\ & \underline{- \quad + \quad +} \\ & \quad x^3 - 5x^2 + 7x \\ & \quad \underline{x^3 - 2x^2 + x} \\ & \quad -3x^2 + 6x - 3 \\ & \quad -3x^2 + 6x - 3 \\ & \quad \underline{+ \quad - \quad +} \\ & \qquad 0 \end{array}$$

Since here remainder is zero, so $x^2 - 2x + 1$ is a factor of the polynomial $2x^4 - 3x^2 + 7x - 3$

7. Check whether the first polynomial is a factor of the second polynomial by applying the division algorithm.
$x^3 - 3x + 1$, $x^5 - 4x^3 + x^2 + 3x + 1$

Sol. We divide
$x^5 - 4x^3 + x^2 + 3x + 1$ by $x^3 - 3x + 1$

$$\begin{array}{r|l} & x^2 - 1 \\ \hline x^3-3x+1 & x^5 - 4x^3 + x^2 + 3x + 1 \\ & x^5 - 3x^3 + x^2 \\ & \underline{- \quad + \quad -} \\ & \quad -x^3 + 3x + 1 \\ & \quad -x^3 + 3x - 1 \\ & \quad \underline{+ \quad - \quad +} \\ & \qquad 2 \end{array}$$

Here, remainder is 2, so $x^3 - 3x + 1$ is not a factor of $x^5 - 4x^3 + x^2 + 3x + 1$

8. What must be subtracted from or added to $8x^4 + 14x^3 - 2x^2 + 8x - 12$ so that it may be exactly divisible by $4x^2 + 3x - 2$?

Sol. We divide

$$
\begin{array}{r|l}
 & 2x^2 + 2x - 1 \\
\hline
4x^2+3x-2 & 8x^4 + 14x^3 - 2x^2 + 8x - 12 \\
 & 8x^4 + 6x^3 - 4x^2 \\
 & \underline{-\quad\ -\qquad +\qquad} \\
 & \qquad 8x^3 + 2x^2 + 8x - 12 \\
 & \qquad 8x^3 + 6x^2 - 4x \\
 & \qquad \underline{-\quad\ -\quad\ +\qquad} \\
 & \qquad -4x^2 + 12x - 12 \\
 & \qquad -4x^2 - 3x + 2 \\
 & \qquad \underline{+\quad\ +\quad\ -\qquad} \\
 & \qquad\qquad\quad 15x - 14
\end{array}
$$

$\therefore$ The expression that must be subtracted = $15x - 14$

and the expression that must be added = $-(15x - 14) = -15x + 14$

9. Find the value of b for which the polynomial $2x^3 + 9x^2 - x - b$ is exactly divisible by $2x + 3$.

Sol. We divide

$$
\begin{array}{r|l}
 & x^2 + 3x - 5 \\
\hline
2x + 3 & 2x^3 + 9x^2 - x - b \\
 & 2x^3 + 3x^2 \\
 & \underline{-\qquad -\qquad\qquad} \\
 & \quad 6x^2 - x - b \\
 & \quad 6x^2 + 9x \\
 & \quad \underline{-\qquad -\qquad\qquad} \\
 & \quad -10x - b \\
 & \quad -10x - 15 \\
 & \quad \underline{+\qquad +\qquad\qquad} \\
 & \qquad -b + 15
\end{array}
$$

For the dividend to be exactly divisible by the divisor, the remainder must be zero.

$\therefore$ $-b + 15 = 0$

or $b = 15$

10. If the zeros of the polynomial $x^3 - 3x^2 + x + 1$ are $a-b$, a, $a + b$, find a and b.

Sol. The given polynomial is $x^3 - 3x^2 + x + 1$

As $(a-b)$, a and $(a+b)$ are the zeros of the given polynomial

$\therefore$ $(a-b)^3 - 3(a-b)^2 + (a-b) + 1 = 0$ .....(1)

$a^3 - 3a^2 + a + 1 = 0$ .....(2)

and $(a+b)^3 - 3(a+b)^2 + (a+b) + 1 = 0$ .....(3)

Putting $a = 1$ in equation (2), we get

$1 - 3 + 1 + 1 = 0$ or $0 = 0$

$\Rightarrow$ Equation (2) is satisfied, when $a = 1$

$\therefore$ 1 is a zero of the given polynomial

Putting $a = 1$ in (1), we get

$(1-b)^3 - 3(1-b)^2 + (1-b) + 1 = 0$ or $1 - b^3 - 3b + 3b^2 - 3(1 - 2b + b^2) + 1 - b + 1 = 0$

or $3 - 4b + 3b^2 - b^3 - 3 + 6b - 3b^2 = 0$ or $-b^3 + 2b = 0$ or $-b(b^2 - 2) = 0$

Either $b = 0$ but b can not be zero or $b^2 - 2 = 0$ $\therefore$ $b^2 = 2$ or $b = \pm\sqrt{2}$

Thus $a = 1$ and $b = \pm\sqrt{2}$.

EXCERICE - I

(OBJECTIVE TYPE QUESTIONS)

1. Quadratic polynomial having zeros 1 and –2 is -

(A) $x^2 - x + 2$ (B) $x^2 - x - 2$ (C) $x^2 + x - 2$ (D) None of these

2. If $(x - 1)$ is a factor of $k^2x^3 - 4kx + 4k - 1$, then the value of k is –

(A) 1 (B) – 1 (C) 2 (D) – 2

3. For what value of a is the polynomial $2x^4 - ax^3 + 4x^2 + 2x + 1$ divisible by $1 - 2x$?

(A) a = 25 (B) a = 24 (C) a = 23 (D) a = 22

4. If one of the factors of $x^2 + x - 20$ is $(x + 5)$, then other factor is -

(A) $(x - 4)$ (B) $(x - 5)$ (C) $(x - 6)$ (D) $(x - 7)$

5. If α, β be the zeros of the quadratic polynomial $2x^2 + 5x + 1$, then value of $\alpha + \beta + \alpha\beta$

(A) – 2 (B) – 1 (C) 1 (D) None of these

6. If α, β be the zeros of the quadratic polynomial $2 - 3x - x^2$, then $\alpha + \beta =$

(A) 2 (B) 3 (C) 1 (D) None of these

7. Quadratic polynomial having sum of it's zeros 5 and product of it's zeros – 14 is-

(A) $x^2 - 5x - 14$ (B) $x^2 - 10x - 14$ (C) $x^2 - 5x + 14$ (D) None of these

8. If x = 2 and x = 3 are zeros of the quadratic polynomial $x^2 + ax + b$, the values of a and b respectively are:

(A) 5, 6 (B) – 5 , – 6 (C) –5, 6 (D) 5, 6

9. If 3 is a zero of the polynomial $f(x) = x^4 - x^3 - 8x^2 + kx + 12$, then the value of k is -

(A) – 2 (B) 2 (C) – 3 (D) $\frac{3}{2}$

10. The sum and product of zeros of the quadratic polynomial are – 5 and 3 respectively the quadratic polynomial is equal to -

(A) $x^2 + 2x + 3$ (B) $x^2 - 5x + 3$ (C) $x^2 + 5x + 3$ (D) $x^2 + 3x - 5$

11. On dividing $x^3 - 3x^2 + x + 2$ by polynomial g(x), the quotient and remainder were $x - 2$ and $4 - 2x$ respectively then g(x) :

(A) $x^2 + x + 1$ (B) $x^2 + x - 1$ (C) $x^2 - x - 1$ (D) $x^2 - x + 1$

12. If the polynomial $3x^2 - x^3 - 3x + 5$ is divided by another polynomial $x - 1 - x^2$, the remainder comes out to be 3, then quotient polynomial is –

(A) $2 - x$ (B) $2x - 1$ (C) $3x + 4$ (D) $x - 2$

13. If sum of zeros = $\sqrt{2}$, product of its zeros = $\frac{1}{3}$. The quadratic polynomial is –

(A) $3x^2 - 3\sqrt{2}x + 1$ (B) $\sqrt{2}x^2 + 3x + 1$ (C) $3x^2 - 2\sqrt{3}x + 1$ (D) $\sqrt{2}x^2 + x + 3$

14. If $-\frac{1}{3}$ is the zeros of the cubic polynomial $f(x) = 3x^3 - 5x^2 - 11x - 3$ the other zeros are :

(A) – 3 , –1 (B) 1, 3 (C) 3, –1 (D) –3, 1

15. If α and β are the zeros of the polynomial $f(x) = 6x^2 - 3 - 7x$ then $(\alpha + 1)(\beta + 1)$ is equal to –

(A) $\frac{5}{2}$ (B) $\frac{5}{3}$ (C) $\frac{2}{5}$ (D) $\frac{3}{5}$

16. Let $p(x) = ax^2 + bx + c$ be quadratic polynomial. It can have at most –
(A) One zero (B) Two zeros (C) Three zeros (D) None of these

17. The graph of the quadratic polynomial $ax^2 + bx + c$, $a \neq 0$ is always-
(A) Straight line (B) Curve (C) Parabola (D) None of these

18. If 2 and $-\frac{1}{2}$ as the sum and product of its zeros respectively then the quadratic polynomial f(x) is –
(A) $x^2 - 2x - 4$ (B) $4x^2 - 2x + 1$ (C) $2x^2 + 4x - 1$ (D) $2x^2 - 4x - 1$

19. If α and β are the zeros of the polynomial $f(x) = 16x^2 + 4x - 5$ then $\frac{1}{\alpha} + \frac{1}{\beta}$ is equal to –
(A) $\frac{2}{5}$ (B) $\frac{5}{2}$ (C) $\frac{3}{5}$ (D) $\frac{4}{5}$

20. If α and β are the zeros of the polynomial $f(x) = 15x^2 - 5x + 6$ then $\left(1+\frac{1}{\alpha}\right)\left(1+\frac{1}{\beta}\right)$ is equal to –
(A) $\frac{13}{3}$ (B) $\frac{13}{2}$ (C) $\frac{16}{3}$ (D) $\frac{15}{2}$

22. The number of polynomials having zeros as –2 and 5 is :
(A) 1 (B) 2 (C) 3 (D) more than 3

23. The zeros of the quadratic polynomial $x^2 + 99x + 127$are
(A) both positive (B) both negative
(C) one positive and one negative (D) both equal

24. The zeros of the quadratic polynomial $x^2 + kx + k$, $k \neq 0$,
(A) cannot both be positive (B) cannot both be negative
(C) are always unequal (D) are always equal

25. If the zeros of the quadratic polynomial $ax^2 + bx + c$, $c \neq 0$ are equal, then
(A) c and a have opposite signs (B) c and b have opposite signs
(C) c and a have the same sign (D) c and b have the same sign

26. If one of the zeros of the cubic polynomial $x^3 + ax^2 + bx + c$ is –1, then the product of the other two zeros is
(A) $b - a + 1$ (B) $b - a - 1$ (C) $a - b + 1$ (D) $a - b - 1$

27. If one of the zeros of a quadratic polynomial of the form $x^2 + ax + b$ is the negative of the other, then it
(A) has no linear term and the constant term is negative
(B) has no linear term and the constant term positive
(C) can have a linear term but the constant term negative
(D) can have a linear term but the constant term positive

28. If 1 is zero of the polynomial $f(x) = a^2x^2 - 3ax + 3x - 1$, then a =
(A) –1 (B) 2 (C) –2 (D) 0

29. If 1 is zero of $3x^3 - x^2 - 3x + 1$, then the other two zeros are
(A) 1 and $\frac{-2}{3}$ (B) 1 and –1 (C) –1 and $\frac{-1}{3}$ (D) –1 and $\frac{1}{3}$

31. If α, β are the zeros of $f(x) = 2x^2 + 6x - 6$, then
(A) $\alpha + \beta = \alpha\beta$ (B) $\alpha + \beta > \alpha\beta$ (C) $\alpha + \beta < \alpha\beta$ (D) $\alpha + \beta + \alpha\beta$

32. If p(x) and g(x) are any two polynomials with $g(x) \neq 0$, then we can find polynomial q(x) and r(x) such that $p(x) = q(x)\, g(x) + r(x)$ where
(A) r(x) = 0 always (B) deg. r(x) < deg. q(x) (C) r(x) = 0 or deg r(x) < deg g(x) (D) $r(x) \neq 0$ always

33. When $x^{200} + 1$ is divided by $x^2 + 1$, the remainder is equal to –
(A) x + 2 (B) 2x – 1 (C) 2 (D) - 1

34. If 2 and 3 are the zeros of $f(x) = 2x^3 + mx^2 - 13x + n$, then the values of m and n are respectively –
(A) -5 , - 30 (B) -5, 30 (C) 5, 30 (D) 5, -30

35. If α, β are the zeros of the polynomial $6x^2 + 6px + p^2$, then the polynomial whose zeros are $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$ is –
(A) $3x^2 + 4p^2x + p^4$ (B) $3x^2 + 4p^2x - p^4$
(C) $3x^2 - 4p^2x + p^4$ (D) None of theses

36. If c, d are zeros of $x^2 - 10ax - 11b$ and a, b are zeros of $x^2 - 10cx - 11d$, then value of a + b + c + d is –
(A) 1210 (B) -1 (C) 2530 (D) -11

37. If the sum of the two zeros of $x^3 + px^2 + qx + r$ is zero, then pq =
(A) – r (B) r (C) 2r (D) –2r

38. If one root of the polynomial $x^2 + px + q$ is square of the other root, then
(A) $p^3 - q(3p - 1) + q^2 = 0$ (B) $p^3 - q(3p + 1) + q^2 = 0$
(C) $p^3 + q(3p - 1) - q^2 = 0$ (D) $p^3 + q(3p + 1) - q^2 = 0$

39. The quadratic polynomial whose zeros are twice the zeros of $2x^2 - 5x + 2 = 0$ is –
(A) $8x^2 - 10x + 2$ (B) $x^2 - 5x + 4$ (C) $2x^2 - 5x + 2$ (D) $x^2 - 10x + 6$

40. If $\alpha + \beta = 4$ and $\alpha^2 + \beta^2 = 44$, then α, β are the zeros of the polynomial .
(A) $2x^2 - 7x + 6$ (B) $3x^2 + 9x + 11$ (C) $9x^2 - 27x + 20$ (D) $3x^2 - 12x + 5$

41. If α, β are the zeros of the quadratic polynomial $4x^2 - 4x + 1$, then $\alpha^3 + \beta^3$ is –
(A) $\frac{1}{4}$ (B) $\frac{1}{8}$ (C) 16 (D) 32

42. The value of λ for which one zero of $3x^2 - (1 + 4\lambda)x + \lambda^2 + 2$ may be one-third of the other is –
(A) 4 (B) $\frac{33}{8}$ (C) $\frac{17}{4}$ (D) $\frac{31}{8}$

43. If the sum of the zeros of the polynomial $x^2 + px + q$ is equal to the sum of their squares, then –
(A) $P^2 - q^2 = 0$ (B) $p^2 + q^2 = 0$ (C) $p^2 + p = 2q$ (D) None of these

44. Let α, β be the zeros of the polynomial $(x - a)(x - b) - c$ with $c \neq 0$. then the zeros of the polynomial $(x - \alpha)(x - \beta) + c$ are :
(A) a, c (B) b, c (C) a, b (D) a + c, b + c

45. If $\alpha \neq \beta$ and $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$, then the polynomial whose zeros are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is :
(A) $3x^2 - 25x + 3$ (B) $x^2 - 5x + 3$
(C) $x^2 + 5x - 3$ (D) $3x^2 - 19x + 3$

46. The minimum value of the expression $4x^2 + 2x + 1$ ($x \in R$) is -

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1

47. If x be real , the maximum value of $7 + 10x - 5x^2$ is –

(A) 12 (B) 15 (C) 16 (D) 18

48. If x is real, the minimum value of $x^2 - 8x + 17$ is –

(A) – 1 (B) 0 (C) 1 (D) 2

True or False

49. If the zeros of a quadratic polynomial $ax^2 + bx + c$ are both positive, then a, b and c all have the same sign.

50. If the graph of a polynomial intersect the x-axis at only one point, it cannot be a quadratic polynomial.

51. If the graph of a polynomial intersect the x-axis at exactly two points, it need not be a quaeratic polynomial

52. If two of the zeros of a cubic polynomial are zero, then it does not have linear and constant terms,

Match the Column

53.

	Column-I	Column-II	
(A)	If α and β are the zeros of the polynomial $f(x)=x^2 - 5x + k$ such that $\alpha - \beta = 1$ then k =	(1)	–5
(B)	If one zero of $f(x) = 4x^2 - 8kx - 9$ is negative of the other, then k =	(2)	1
(C)	If α , β , γ are the zeros of the polynomial $x^3 + px^2 + qx + 2$ such that $\alpha\beta + 1 = 0$, then 2p + q =	(3)	0
(D)	If x + 1 is a factor of polynomial $x^2 - 3ax + 3a - 7$, then a =	(4)	6

54.

	Column-I	Column-II	
(A)	If sum of the square of zeros of a quadratic polynomial $f(x) = x^2 - 8x + k$ is 40, then k =	(1)	2
(B)	If α and β are the zeros of the polynomial $f(x) = x^2 - 5x^2 + k$, where $\alpha - \beta = 1$, then k =	(2)	–2/3
(C)	If α, β be the zeros of the polynomial $f(x) = 2x^2 + 5kx + k$ such that $\alpha^2 + \beta^2 + \alpha\beta = 24$, then k	(3)	1
(D)	If the sum of the zeros of the qudratic polynomial $f(x) = kx^2 + 2x + 3k$ is equal to their product, then k =	(4)	12

55.

	Column-I	Column-II	
(A)	If one zero of thequadratic polynomial $f(x) = x^2 - kx - 9$ is negative of the other, then k =	(1)	–28
(B)	If the zeros of the polynomial $f(x) = x^3 - 3x^2 + x + 1$ are a–d, a, a + d, then a =	(2)	5
(C)	If the zeros ofthe polynomial $f(x) = x^3 - 12x^2 + 39x + k$ are a–d, a , a + d, then k =	(3)	1
(D)	If the zeros of the polyniomial $f(x) = 5x^2 + 13x + k$ is reciprocal of the other, then k =	(4)	0

56.

	Column-I		Column-II
(A)	If α, β are zeros of the polynomial $x^2 + 4x + 3$, then the polynomial whose zeros are $1 + \beta/\alpha$ and $1 + \alpha/\beta$ is.	(1)	$cx^2 + bx + a$
(B)	If α, β are zeros of the polynomial $ax^2 + bx + c$, then the polynomial whose zeros are $1/\alpha$, $1/\beta$ is	(2)	$3x^2 – 16x + 16$
(C)	If α , β arethe zeros of the polynomial $x^2 + 8x + 6$, then the polynomial whose zeros are 2α, 2β is	(3)	$x^2 – 12x + 45$
(D)	If α, β are the zeros of the polynomial $x^2 – 4x + 5$, then the polynomial whose zeros are 3α, 3β is.	(4)	$x^2 + 16x + 24$

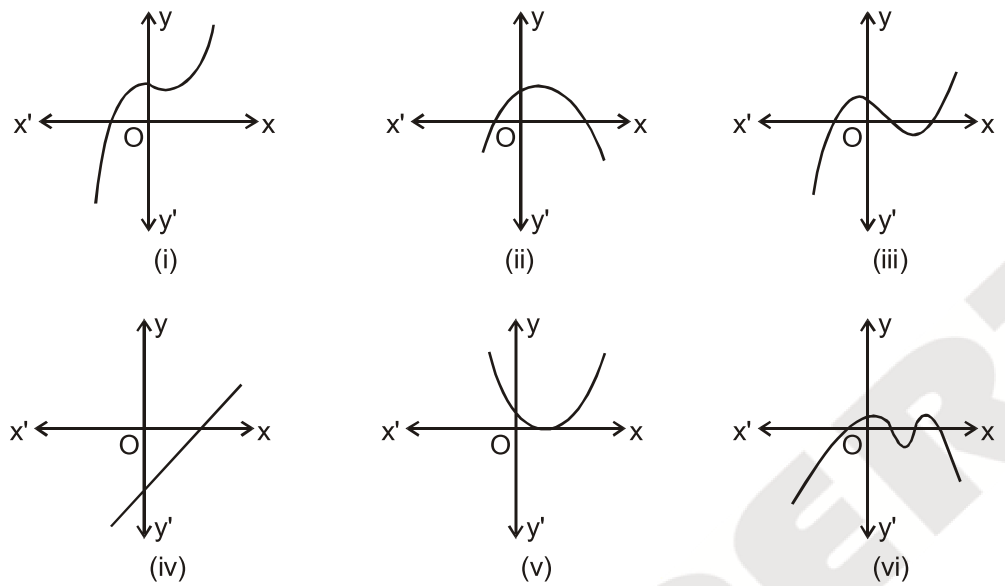
57.

	Column-I		Column-II
(A)	Polynomial whose zeros are 2/3, –1/3 is	(1)	$x^2 – 4x + 1$
(B)	Polynomial whose zeros are 3 and –2 is	(2)	$x^2 – 10x + 23$
(C)	Polynomial whose zeros are $2 + \sqrt{3}$, $2 - \sqrt{3}$ is	(3)	$x^2 – x – 6$
(D)	Polynomial with zeros $5 + \sqrt{2}$ and $5 - \sqrt{2}$ is	(4)	$9x^2 – 3x – 2$

EXCERICE - II

(Subjective type Question)

1. Look at the graph in fig given below. Each is the graph of y = p(x) , where p(x) is a polynomial. For each of the graph, find the number of zeros of p(x).



2. Find the zeros of the following quadratic polynomial and verify the relationship between the zeros and their coefficients.

(i) $6x^2 - x - 1$ (ii) $48y^2 - 13y - 1$ (iii) $63 - 2x - x^2$ (iv) $49x^2 - 81$

3. Find a quadratic polynomial each with the given numbers as the zeros of the polynomial .

(i) $3+\sqrt{7}$, $3-\sqrt{7}$ (ii) $2\sqrt{3}$, $-2\sqrt{3}$ (iii) $\frac{8}{3}, \frac{5}{2}$

4. If α and β are the zeros of the polynomial $f(x) = 5x^2 + 4x - 9$ then evaluate the following :

(i) $\alpha - \beta$ (ii) $\alpha^2 - \beta^2$ (iii) $\alpha^3 - \beta^3$ (iv) $\alpha^4 - \beta^4$

5. If one of the zeros of the quadratic polynomial $2x^2 + px + 4$ is 2, find the other zero. Also find the value of p.

6. If one zero of the polynomial $(a^2 + 9)x^2 + 13x + 6a$ is the reciprocal of the other, find the value of a .

7. Find the zeros of the quadratic polynomial $5x^2 - 4 - 8x$ and verify the relationship between the zeros and the coefficients of the polynomial .

8. Determined if 3 is a zero of p(x) = $\sqrt{x^2-4x+3} + \sqrt{x^2-9} - \sqrt{4x^2-14x+6}$

9. If α and β be two zeros of the quadratic polynomial $ax^2 + bx + c$, then evaluate :

(i) $\alpha^2 + \beta^2$ (ii) $\alpha^3 + \beta^3$ (iii) $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$ (iv) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

10. Find the value of k :

(i) If α and β are the zeros of the polynomial $x^2 - 8x + k$ such that $\alpha^2 + \beta^2 = 40$

(ii) If α and β are the zeros of the polynomial $x^2 - 6x + k$ such that $3\alpha + 2\beta = 20$

11. If 2 and 3 are zeros of polynomial $3x^2 - 2kx + 2m$, find the values of k and m.

12. If α and β are the zeros of the polynomial $2x^2 - 4x + 5$. Form the polynomial where zeros are :

(i) $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ (ii) $1+\frac{\beta}{\alpha}$ and $1+\frac{\alpha}{\beta}$

13. If α and β are the zeros of the quadratic polynomial $x^2 - 3x + 2$, find a quadratic polynomial whose zeros are :

(i) $\frac{1}{2\alpha+\beta}$ and $\frac{1}{2\beta+\alpha}$ (ii) $\frac{\alpha-1}{\alpha+1}$ and $\frac{\beta-1}{\beta+1}$

14. If the sum of the squares of zeros of the polynomial $5x^2 + 3x + k$ is $-\frac{11}{25}$, find the value of k .

15. If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - px + q$, prove that $\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}=\frac{p^4}{q^2}-\frac{4p^2}{q}+2$

16. (a) Find all the zeros of $3x^3 + 16x^2 + 23x + 6$ if two its zeros are -3 and -2 .

(b) Determine all the zeros of $4x^3 + 12x^2 - x - 3$ if two of its zeros are $-\frac{1}{2}$ and $\frac{1}{2}$.

(c) Determine all the zeros of $x^3 + 5x^2 - 2x - 10$ if two of its zeros are $\sqrt{2}$ and $-\sqrt{2}$

(d) Determine all the zeros of $4x^3 + 12x^2 - x - 3$ if one of its zeros is $\frac{5}{2}$

(e) Determine all the zeros of $4x^3 + 5x^2 - 180x - 225$ if one of its zeros is $-\frac{5}{4}$

17. (A) On dividing $f(x) = 3x^3 + x^2 + 2x + 5$ by a polynomial $g(x) = x^2 + 2x + 1$, the remainder $r(x) = 9x + 10$. Find the quotient polynomial q(x).
(B) On dividing f(x) be a polynomial $x - 1 - x^2$, the quotient q(x) and remainder r(x) are $(x - 2)$ and 3 respectively . Find f(x).
(C) On dividing $x^5 - 4x^3 + x^2 + 3x + 1$ by polynomial g(x), the quotient and remainder are $(x^2 - 1)$ and 2 respectively. Find g(x).
(D) On dividing $f(x) = 2x^5 + 3x^4 + 4x^3 + 4x^2 + 3x + 2$ by a polynomial g(x), where $g(x) = x^3 + x^2 + x + 1$, the quotient obtained as $2x^2 + x + 1$. Find the remainder r(x).

18. Find all the zeros of the polynomial $2x^3 + x^2 - 6x - 3$, if two of its zeros are $-\sqrt{3}$ and $\sqrt{3}$.

19. Can (x–1) be the remainder on division of a polynomial p(x) by 2x + 3? Justify your answer.

20. Find the values of a and b so that $x^4 + x^3 + 8x^2 + ax + b$ is divisible by $x^2 + 1$.

21. What must be added to $f(x) = 4x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is divisible by $g(x) = x^2 + 2x - 3$.

22. If α, β, γ are the zeros of the polynomial $x^3 + px^2 + qx + 2$ such that $\alpha\beta + 1 = 0$, find the value of $2p + q + 5$.

23. If the polynomial $f(x) = x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find k and a.

24. For which values of a and b, are the zeros of $q(x) = x^3 + 2x^2 + a$ also the zeros of the polynomial $p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$? Which zeros of p(x) are not the zeros of q(x)?

25. Find a cubic polynomial with the sum, sum of the product of its zeros taken two at a time and product of its zeros as 2, –7, –14 respectively.

26. Given that the zeros of the cubic polynomial $x^3 - 6x^2 + 3x + 10$ are of the form a, a +b, a + 2b for some real numbers a and b, find the values of a and b as well as the zeros ofthe given polynomial.

27. Find k so that $x^2 + 2x + k$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$. Also find all the zeros of the two polynomials.

28. What should be added to $8x^4 + 14x^3 - 2x^2 + 7x - 8$ so that the resulting polynomial is exactly divisible by $4x^2 + 3x - 2$.

ANSWER KEY

EXERCISE -1

1	C	2	A	3	A	4	A	5	A	6	D	7	A
8	C	9	B	10	C	11	D	12	D	13	A	14	C
15	B	16	B	17	C	18	D	19	D	20	A	21	D
22	B	23	A	24	C	25	A	26	A	27	B	28	D
29	B	30	A	31	C	32	C	33	B	34	C	35	A
36	B	37	A	38	B	39	D	40	A	41	D	42	C
43	C	44	D	45	C	46	A	47	C				

True or False

48	F	49	F	50	T	51	T

Match the Column

52 (A→ 4), (B→ 3), (C→ 1), (D → 2)

53 (A→ 4), (B→ 3), (C→ 1), (D → 2)

54 (A→ 4), (B→ 3), (C→ 1), (D → 2)

55 (A→ 2), (B→ 1), (C→ 4), (D → 3)

56 (A→ 4), (B→ 3), (C→ 1), (D → 2)

EXERCISE -2

1. (i) One zeros, (ii) Two zeros, (iii) Three zeros, (iv) No zeros, (v) One zeros (vi) Four zeros

2. (i) $-\frac{1}{3}, \frac{1}{2}$ (ii) $\frac{1}{3}, \frac{-1}{16},$ (iii) 7, –9 (iv) $\frac{9}{7}, -\frac{9}{7}$

3. (i) $x^2 - 6x + 2$, (ii) $x^2 - 12$, (iii) $6x^2 - 31x + 40$

4. (i) $\frac{14}{5}$ (ii) $-\frac{56}{25}$ (iii) $\frac{854}{125}$ (iv) $\frac{-5936}{125}$

5. p = - 6 , other zero = 1

6. a = 3 **7.** 2 and $\frac{-2}{5}$ **8.** Yes

9. (i) $\frac{b^2 - 2ac}{a^2}$ (ii) $\frac{3abc - b^3}{a^3}$ (iii) $\frac{3abc - b^3}{c^3}$ (iv) $\frac{3abc - b^3}{a^2c}$ **10.** (i) 12 (ii) – 16

11. k = $\frac{15}{2}$, m = 9

12. (i) $\frac{1}{25}(25x^2 + 4x + 4)$ (ii) $\frac{1}{5}(5x^2 - 8x + 8)$ **13.** (i) $\frac{1}{20}(20x^2 - 9x + 1)$ (ii) $3x^2 - x$

14. K = 2 **16.** (a) $-2, -3, -\frac{1}{3},$ (b) $\frac{1}{2}, -\frac{1}{2}, 3,$ (c) $\sqrt{2}, -\sqrt{2}, -5,$ (d) $\frac{5}{2}, \frac{3}{2}, \frac{1}{2},$ (e) $-\frac{5}{4}, 3\sqrt{5}, -3\sqrt{5}$

17. (a) q(x) = 3x – 5 , (b) f(x) = $-x^3 + 3x^2 - 3x + 5$, (c) q(x) = $x^3 - 3x + 1$, (d) r(x) = x + 1,

18. Third zero = $-\frac{1}{2}$ **19.** NO **20.** a = 1 and b = 7

21. 61x – 65 **22.** 0 **23.** k = 5 & a = –5 **24.** a = –1, b = –2; 1, 2

25. $k(x^3 - 2x^2 - 7x + 14)$ **26.** a = –1 and b = 3 or a = 5, b = –3 zeros are –1, 2, 5

27. k = –3, zeroes of $2x^4 + x^3 - 14x^2 + 5x + 6$ are 1, –3, 2, –1/2 and zeroes of $x^2 + 2x - 3$ are 1, –3

28. 10 – 14x

JEE EXPERT