CLASSROOM STUDY PACKAGE

PHYSICS

CAPACITANCE

JEE EXPERT

CAPACITANCE

CONTENTS

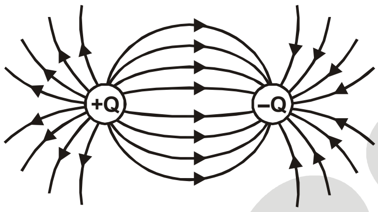
CAPACITANCE

Syllabus in IIT-JEE :

Capacitance; Parallel plate capacitor with and without dielectrics; Capacitors in series and parallel; Energy stored in a capacitor.

1. INTRODUCTION :

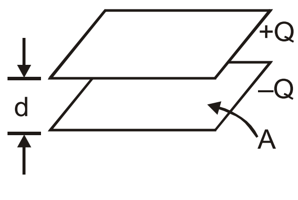
A capacitor is a device which stores electric charge. Capacitors vary in shape and size, but the basic configuration is two conductors carrying equal but opposite charges. Capacitors have many important applications in electronics. Some examples include storing electric potential energy, delaying voltage changes when coupled with resistors, filtering out unwanted frequency signals, forming resonant circuits and making frequency-dependent and independent voltage dividers when combined with resistors. Some of these applications will be discussed in latter chapters.



Basic configuration of a capacitor

In the uncharged state, the charge on either one of the conductors in the capacitor is zero. During the charging process, a charge A is moved from one conductor to the other one, giving one conductor a charge +Q, and the other one a charge –Q. A potential difference ΔV is created, with the positively charged conductor at a higher potential than the negatively charged conductor. Note that whether charged or uncharged, the net charge on the capacitor as a whole is zero.

The simplest example of a capacitor consists of two conducting plates of area ,4, which are parallel to each other, and separated by a distance d, as shown in Figure.



A parallel-plate capacitor

Experiments show that the amount of charge Q stored in a capacitor is linearly proportional to ΔV, the electric potential difference between the plates. Thus, we may write

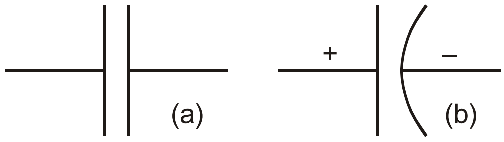
$$Q = C\,|\Delta V|$$

where C is a positive proportionality constant called capacitance. Physically, capacitance is a measure of the capacity of storing electric charge for a given potential difference ΔV. The SI unit of capacitance is the farad (F) :

$$1\text{ F} = 1\text{ farad} = 1\text{ coulomb/volt} = 1\text{C/V}$$

A typical capacitance is in the picofarad (1 pF = 10^{-12} F) to millifarad range, (1 mF = 10^{-3} F = 1000 μF ; 1 μF = 10^{-6} F).

Figure (a) shows the symbol which is used to represent capacitors in circuits. For a polarized fixed capacitor which has a definite polarity, Figure (b) is sometimes used.



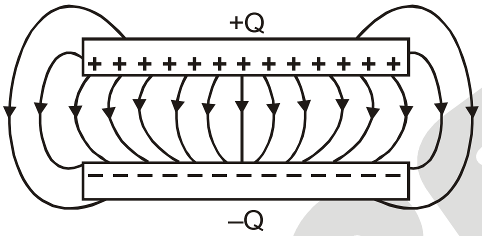
Capacitor symbols

2. CALCULATION OF CAPACITANCE :

Let's see how capacitance can be computed in systems with simple geometry.

Ex. Parallel-Plate Capacitor

Consider two metallic plates of equal area A separated by a distance d, as shown in Figure below. The top plate carries a charge +Q while the bottom plate carries a charge –Q. The charging of the plates can be accomplished by means of a battery which produces a potential difference. Find the capacitance of the system.



The electric field between the plates of a parallel-plate capacitor

Solution :

To find the capacitance C, we first need to know the electric field between the plates. A real capacitor is finite in size. Thus, the electric field lines at the edge of the plates are not straight lines, and the field is not contained entirely between the plates. This is known as edge effects, and the non-uniform fields near the edge are called the fringing fields. In figure the field lines are drawn by taking into consideration edge effects. However, in what follows, we shall ignore such effects and assume an idealized situation, where field lines between the plates are straight lines.

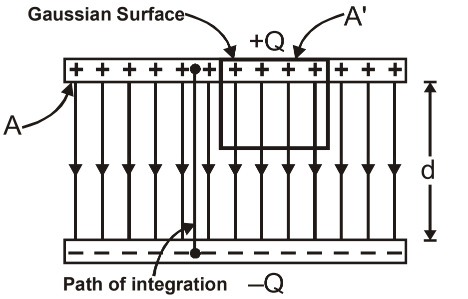
In the limit where me plates are infinitely large, the system has plannar symmetry and we can calculate the electric field everywhere using Gauss's law given in Eq.:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

By choosing a Gaussian "pillbox" with cap area A' to enclose the charge on the positive plate (see Figure), the electric field in the region between the plates is

$$EA' = \frac{q_{ene}}{\varepsilon_0} = \frac{\sigma A'}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{\varepsilon_0}$$

The same result has also been obtained in Section 4.8.1 using superposition principle.



Gaussian surface for calculating the electric Geld between the plates

The potential difference between the plates is

$$\Delta V = V_- - V_+ = -\int_+^- \vec{E}\cdot d\vec{s} = -Ed$$

where we have taken the path of integration to be a straight line from the positive plate to the negative plate following the field lines (Figure). Since the electric field lines are always directed from higher potential to lower potential, $V_- < V_+$. However, in computing the capacitance C, the relevant quantity is the magnitude of the potential difference:

$$|\Delta V| = Ed$$

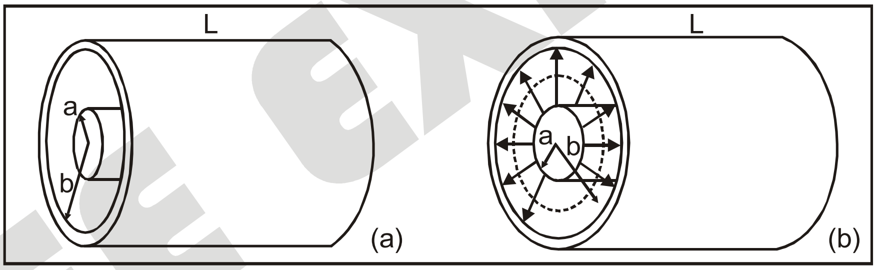
and its sign is immaterial. From the definition of capacitance, we have

$$C = \frac{Q}{|\Delta V|} = \frac{\varepsilon_0 A}{d} \text{ (parallel plate)}$$

Note that C depends only on the geometric factors A and d. The capacitance C increases linearly with the area A since for a given potential difference ΔV, a bigger plate can hold more charge. On the other hand, C is inversely proportional to the distance of separation because the smaller the value of d, the smaller the potential difference $|\Delta V|$ for a fixed Q.

Ex. : Cylindrical Capacitor

Consider next a solid cylindrical conductor of radius a surrounded by a coaxial cylindrical shell of inner radius b, as shown in Figure. The length of both cylinders is L and we take this length to be much larger than b-a, the separation of the cylinders, so that edge effects can be neglected. The capacitor is charged so that the inner cylinder has charge +Q while the outer shell has a charge –Q. What is the capacitance?



(a) A cylindrical capacitor, (b) End view of the capacitor.
The electric field is non-vanishing only in the region a < r < b.

Solution :

To calculate the capacitance, we first compute the electric field everywhere. Due to the cylindrical symmetry of the system, we choose our Gaussian surface to be a coaxial cylinder with length $\ell < L$ and radius r where $a < r < b$. Using Gauss's law, we have

$$\oint \vec{E}\cdot d\vec{A} = EA = E(2\pi r\ell) \Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

where $\lambda = Q/L$ is the charge per unit length. Notice that the electric field is non-vanishing only in the region $a < r < b$. For $r < a$, the enclosed charge is $q_{enc} = 0$ since any net charge in a conductor must reside on its surface. Similarly, for $r > b$, the enclosed charge is $q_{enc} = \lambda\ell - \lambda\ell = 0$ since the Gaussian surface encloses equal but opposite charges from both conductors.

The potential difference is given by

$$\Delta V = V_b - V_a = -\int_a^b E_r dr = -\frac{\lambda}{2\pi\varepsilon_0}\int_a^b \frac{dr}{r} = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$$

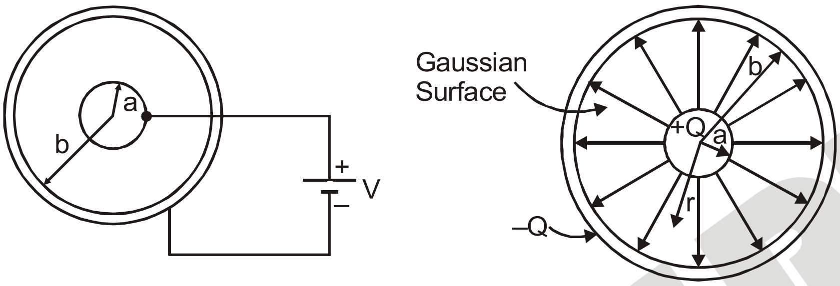
where we have chosen the integration path to be along the direction of the electric field lines. As expected, the outer conductor with negative charge has a lower potential. This gives

$$C = \frac{Q}{|\Delta V|} = \frac{\lambda L}{\lambda \ln(b/a)/r\pi\varepsilon_0} = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}$$

Once again, we see that the capacitance C depends only on the geometrical factors, L, a and b.

Ex.: Spherical Capacitor

As a third example, let's consider a spherical capacitor which consists of two concentric spherical shells of radii a and b, as shown in Figure. The inner shell has a charge +Q uniformly distributed over its surface, and the outer shell an equal but opposite charge –Q. What is the capacitance of this configuration?



(a) spherical capacitor with two concentric spherical shells of radii a and b.
(b) Gaussian surface for calculating the electric field

Solution : The electric Geld is non-vanishing only in the region a < r < b. Using Gauss's law, we obtain

$$\oint \vec{E}\cdot d\vec{A} = E_r A = E_r(4\pi r^2) = \frac{Q}{\varepsilon_0} \text{ or } E_r = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

Therefore, the potential difference between the two conducting shells is :

$$\Delta V = V_b = V_a = -\int_a^b E_r dr = -\frac{Q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{r^2} = -\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right) = -\frac{Q}{4\pi\varepsilon_0}\left(\frac{b-a}{ab}\right)$$

which yields

$$C = \frac{Q}{|\Delta V|} = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$$

Again, the capacitance C depends only on the physical dimensions, a and b.

An 'isolated" conductor (with the second conductor placed at infinity) also has a capacitance. In the limit where $b \to \infty$, the above equation becomes.

$$\lim_{b\to\infty} C = \lim_{b\to\infty} 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)\lim_{b\to\infty} 4\pi\varepsilon_0 \frac{a}{\left(1-\frac{a}{b}\right)} = 4\pi\varepsilon_0 a$$

Thus, for a single isolated spherical conductor of radius R, the capacitance is

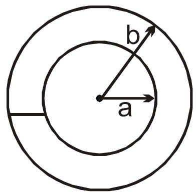
$$C = 4\pi\varepsilon_0 R$$

The above expression can also be obtained by noting that a conducting sphere of radius R with a charge Q uniformly distributed over its surface has $V = Q/4\pi\varepsilon_0 R$, using infinity as the reference point having zero potential, $V(\infty) = 0$. This gives

$$C = \frac{Q}{|\Delta V|} = \frac{Q}{Q/4\pi\varepsilon_0 R} = 4\pi\varepsilon_0 R$$

As expected, the capacitance of an isolated charged sphere only depends on its geometry, namely, the radius R.

Ex. The conducting spherical shells shown in the figure are connected by a conductor. The capacitance of the system is



(A) $4\pi\varepsilon_0 \dfrac{ab}{b-a}$ (B) $4\pi\varepsilon_0 a$ (C) $4\pi\varepsilon_0 b$ (D) $4\pi\varepsilon_0 \dfrac{a^2}{b-a}$

Sol. Hence, the capacitance of the system is the capacitance due to outer sphere of radius b.

$\therefore \quad C = 4\pi\varepsilon_0 b$

4. STORING ENERGY IN A CAPACITOR :

As discussed in the introduction, capacitors can be used to stored electrical energy. The amount of energy stored is equal to the work done to charge it. During the charging process, the battery does work to remove charges from one plate and deposit them onto the other.

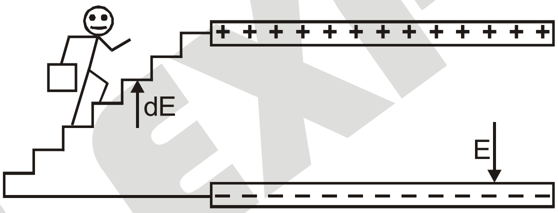


Figure : Work is done by an external agent in bringing +dq from the negative plate and depositing the charge on the positive plate

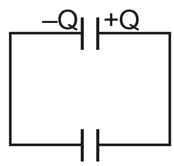
Let the capacitor be initially uncharged. In each plate of the capacitor, there are many negative and positive charges, but the number of negative charges balances the number of positive charges, so that there is no net charge, and therefore no electric field between the plates. We have a magic bucket and a set of stairs from the bottom plate to the top plate (Figure).

We start out at the bottom plate, fill our magic bucket with a charge +dq, carry the bucket up the stairs and dump the contents of the bucket on the top plate, charging it up positive to charge +dq. However, in doing so, the bottom plate is now charged to –dq. Having emptied the bucket of charge, we now descend the stairs, get another bucketful of charge +dq, go back up the stairs and dump that charge on the top plate. We then repeat this process over and over. In this way we build up charge on the capacitor, and create electric field where there was none initially.

Suppose the amount of charge on the top plate at some instant is +q, and the potential difference between the two plates is $|\Delta V| = q/C$. To dump another bucket of charge +dq on the top plate, the amount of work done to overcome electrical repulsion is $dW = |\Delta V|\, dq$. If at the end of the charging process, the charge on the top plate is +Q, then the total amount of work done in this process is

$$W = \int_0^Q dq\,|\Delta V| = \int_0^Q dq\frac{q}{C} = \frac{1}{2}\frac{Q^2}{C}$$

This is equal to the electrical potential energy U_E of the system :



$$U_E = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}Q\,|\Delta V| = \frac{1}{2}C\,|\Delta V|^2$$

Energy Density of the Electric Field :

One can think of the energy stored in the capacitor as being stored in the electric field itself. In the case of a parallel-plate capacitor, with $C = \varepsilon_0 A/d$ and $|\Delta V| = Ed$, we have

$$U_E = \frac{1}{2}C\,|\Delta V|^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d}(Ed)^2 = \frac{1}{2}\varepsilon_0 E^2(Ad)$$

Since the quantity Ad represents the volume between the plates, we can define the electric energy density as

$$u_E = \frac{U_E}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E^2$$

Note that u_E is proportional to the square of the electric field. Alternatively, one may obtain the energy stored in the capacitor from the point of view of external work. Since the plates are oppositely charged, force must be applied to maintain a constant separation between them. From Eq., we see that a small patch of charge $\Delta q = \sigma(\Delta A)$ experiences an attractive force $\Delta F = \sigma^2(\Delta A)/2\varepsilon_0$. If the total area of the plate is A, then an external agent must exert a force = $F_{ext} = \sigma^2 A/2\varepsilon_0$ to pull the two plates apart. Since the electric field strength in the region between the plates is given by $E = \sigma/\varepsilon_0$, the external force can be rewritten as

$$F_{ext} = \frac{\varepsilon_0}{2}E^2 A$$

Note that F_{ext} is independent of d. The total amount of work done externally to separate the plates by a distance d is then

$$W_{ext} = \int \vec{F}_{ext}\cdot d\vec{s} = F_{ext}d = \left(\frac{\varepsilon_0 E^2 A}{2}\right)d$$

consistent with Eq. Since the potential energy of the system is equal to the work done by the external agent, we have $u_E = W_{ext}/Ad = \varepsilon_0 E^2/2$. In addition, we note that the expression for u_E is identical to Eq. in. Therefore, the electric energy density u_E can also be interpreted as electrostatic pressure P.

Ex. : Electric Energy Density of Dry Air

The breakdown field strength at which dry air loses its insulating ability and allows a discharge to pass through is $E_b = 3 \times 10^6$ V/m. At this field strength, the electric energy density is :

$$u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\,(8.85 \times 10^{-12}\ C^2/N\cdot m^2)\,(3 \times 10^6\ V/m)^2 = 40\ J/m^3$$

Ex. : Energy Stored in a Spherical Shell

Find the energy stored in a metallic spherical shell of radius a and charge Q.

Solution : The electric Geld associated of a spherical shell of radius a is (Example 4.3)

$$\vec{E} = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r}, & r > a \\ 0, & r < a \end{cases}$$

The corresponding energy density is :

$$u_E = \frac{1}{2}e_0E^2 = \frac{Q^2}{32\pi^2\varepsilon_0 r^4}$$

outside the sphere, and zero inside. Since the electric field is non-vanishing outside the spherical shell, we must integrate over the entire region of space from r = a to r = ∞. In spherical coordinates, with, $dV = 4\pi r^2 dr$, we have

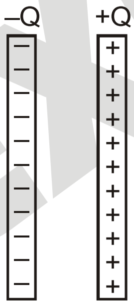
$$U_E = \int_a^{\infty}\left(\frac{Q^2}{32\pi^2\varepsilon_0 r^4}\right)4\pi r^2 dr = \frac{Q^2}{8\pi\varepsilon_0}\int_a^{\infty}\frac{dr}{r^2} = \frac{Q^2}{8\pi\varepsilon_0 a} = \frac{1}{2}QV$$

where $V = Q/4\pi\varepsilon_0 a$ is the electric potential on the surface of the shell, with $V(\infty) = 0$. We can readily verify that the energy of the system is equal to the work done in charging the sphere. To show this, suppose at some instant the sphere has charge q and is at a potential $V = q/4\pi\varepsilon_0 a$. The work required to add an additional charge dq to the system is dW = Vdq. Thus, the total work is

$$W = \int dW = \int Vdq = \int_0^Q dq\left(\frac{q}{4\pi\varepsilon_0 a}\right) = \frac{Q^2}{8\pi\varepsilon_0 a}$$

Force on the Plates of a Capacitor

The plates of a parallel-plate capacitor have area A and carry total charge ±Q (see Figure). We would like to show that these plates attract each other with a force given by $F = Q^2(2\varepsilon_0 A)$.



(a) Calculate the total force on the left plate due to the electric field of the right plate, using Coulomb's Law. Ignore fringing fields.

(b) If you pull the plates apart, against their attraction, you are doing work and that work goes directly into creating additional electrostatic energy. Calculate the force necessary to increase the plate separation from x to x + dx by equating the work you do, $\vec{F}\cdot\vec{dx}$ to the increase in electrostatic energy, assuming that the electric energy density is $\varepsilon_0E^2 / 2$ and that the charge Q remains constant.

(c) Using this expression for the force, show that the force per unit area (the electrostatic stress) acting on either capacitor plate is given by $e_0E^2 / 2$. This result is true for a conductor of any shape with an electric field $\vec{E}$ at its surface. E

(d) Atmospheric pressure is 14.7 lb/In^2 or 101, 341 N/m^2. How large would E have to be to produce this force per unit area? [Ans : 151 MV/m. Note that Van de Graff accelerators can reach Gelds of 100 MV/m maximum before breakdown, so that electrostatic stresses are on the same order as atmospheric pressures in this extreme situation, but not much greater].

Ex. Each plate of a parallel-plate air capacitor has an area A. What amount of work has to be performed to slowly increase the distance between the plates from x_1 to x_2 if
(a) the charge on the capacitor, which is equal to q, or (b) the voltage across the capacitor, which is equal to V, is kept constant in the process?

Ans. (a) $A = q^2(x_2 - x_,)/2\varepsilon_0 A$; (b) $A = \varepsilon_0 AV^2(x_2 - x_1) / 2x_1x_2]$

Sol. (a) Sought work is equivalent to the work performed against the electric field created by one plate, holding at rest and to bring the other plate away. Therefore the required work,

$$A_{agent} = qE(x_2 - x_1),$$

where $E = \dfrac{\sigma}{2\varepsilon_0}$ is the intensity of the field created by one plate at the location of other.

So, $A_{agent} = q\dfrac{\sigma}{2\varepsilon_0}(x_2 - x_1) = \dfrac{q^2}{2\varepsilon_0 S}(x_2 - x_1)$

Alternate : $A_{ext} = \Delta U$ (as field is potential)

$$= \frac{q^2}{2\varepsilon_0 S}x_2 - \frac{q^2}{2\varepsilon_0 S}x = \frac{q^2}{2\varepsilon_0 S}(x_2 - x_1)$$

(b) When voltage is kept const., the force acting on each plate of capacitor will depend on the distance between the plates.

So, elementary work done by agent, in its displacement over a distance dx, relative to the other,

$$dA = -F_x\,dx$$

But, $F_x = -\left(\dfrac{\sigma(x)}{2\varepsilon_0}\right)S\sigma(x)$ and $\sigma(x) = \dfrac{\varepsilon_0 V}{x}$

Hence, $A = \int dA \int \dfrac{1}{2}\varepsilon_0 \dfrac{SV^2}{x^2}dx = \dfrac{\varepsilon_0 SV^2}{2}\left[\dfrac{1}{x_1} - \dfrac{1}{x_2}\right]$

Alternate : From energy Conservation,

$$U_f - U_i = A_{cell} + A_{agent}$$

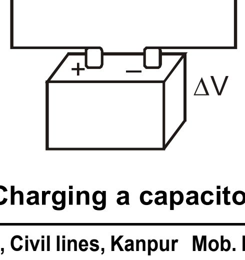
or $\dfrac{1}{2}\dfrac{\varepsilon_0 S}{x_2}V^2 - \dfrac{1}{2}\dfrac{\varepsilon_0 S}{x_1}V^2 = \left[\dfrac{\varepsilon_0 S}{x_2} - \dfrac{\varepsilon_0 S}{x_1}\right]V^2 + A_{agent}$

(as $A_{cell} = (q_f - q_i)V = (C_f - C_i)V^2$)

So, $A_{agent} = \dfrac{\varepsilon_0 SV^2}{2}\left[\dfrac{1}{x_1} - \dfrac{1}{x_2}\right]$

3. CAPACITORS IN ELECTRIC CIRCUITS

A capacitor can be charged by connecting the plates to the terminals of a battery, which are maintained at a potential difference ΔV called the terminal voltage.

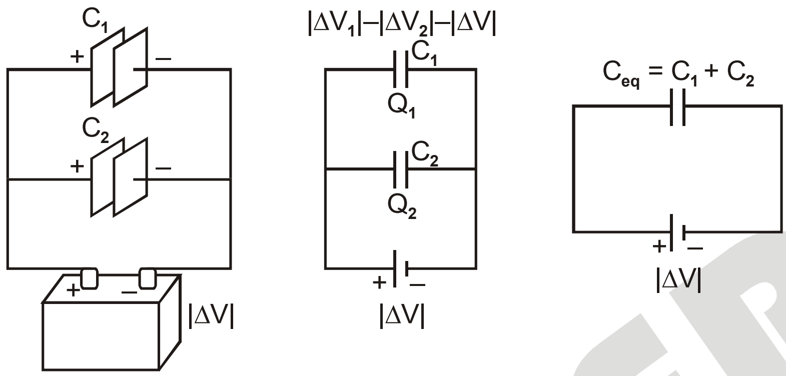


Charging a capacitor

The connection results in sharing the charges between the terminals and the plates. For example, the plate that is connected to the (positive) negative terminal will acquire some (positive) negative charge. The sharing causes a momentary reduction of charges on the terminals, and a decrease in the terminal voltage. Chemical reactions are then triggered to transfer more charge from one terminal to the other to compensate for the loss of charge to the capacitor plates, and maintain the terminal voltage at its initial level. The battery could thus be thought of as a charge pump that brings a charge Q from one plate to the other.

Parallel Connection :

Suppose we have two capacitors C_1 with charge Q_1 and C_2 with charge $2Q_2$ that are connected in parallel, as shown in Figure.



Capacitors in parallel and an equivalent capacitor

The left plates of both capacitors C_1 and C_2 are connected to the positive terminal of the battery and have the same electric potential as the positive terminal. Similarly, both right plates are negatively charged and have the same potential as the negative terminal. Thus, the potential difference $|\Delta V|$ is the same across each capacitor. This gives

$$C_1 = \frac{Q_1}{|\Delta V|}, C_2 = \frac{Q_2}{|\Delta V|}$$

These two capacitors can be replaced by a single equivalent capacitor with a total charge Q_{eq} supplied by the battery. However, since Q is shared by the two capacitors, we must have

$$Q = Q_1 + Q_2 = C_1 |\Delta V| + C_2 |\Delta V| = (C_1 + C_2) |\Delta V|$$

The equivalent capacitance is then seen to be given by

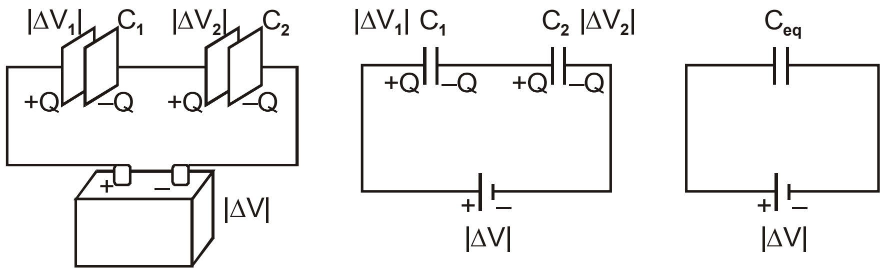
$$C_{eq} = \frac{Q}{|\Delta V|} = C_1 + C_2$$

Thus, capacitors that are connected in parallel add. The generalization to any number of capacitors is

$$C_{eq} = C_1 + C_2 + C_3 + ... + C_N = \sum_{i=1}^{N} C_i \quad \text{(parallel)}$$

Series Connection :

Suppose two initially uncharged capacitors C_1 and C_2 are connected in series, as shown in Figure. A potential difference $|\Delta V|$ is then applied across both capacitors. The left plate of capacitor 1 is connected to the positive terminal of the battery and becomes positively charged with a charge +Q, while the right plate of capacitor 2 is connected to the negative terminal and becomes negatively charged with charge –Q as electrons flow in. What about the inner plates? They were initially uncharged; now the outside plates each attract an equal and opposite charge. So the right plate of capacitor 1 will acquire a charge –Q and the left plate of capacitor +Q.



Capacitor in series and an equivalent capacitor

The potential differences across capacitors C_1 and C_2 are

$$|\Delta V_1| = \frac{Q}{C_1}, |\Delta V_2| = \frac{Q}{C_2}$$

respectively. From figure, we see that the total potential difference is simply the sum of the two individual potential differences :

$$|\Delta V| = |\Delta V_1| + |\Delta V_2|$$

In fact, the total potential difference across any number of capacitors in series connection is equal to the sum of potential differences across the individual capacitors. These two capacitors can be replaced by a single equivalent capacitor $C_{eq.} = \frac{Q}{|\Delta V|}$. Using the fact that the potentials add in series,

$$\frac{Q}{C_{eq.}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

and so the equivalent capacitance for two capacitors in series becomes

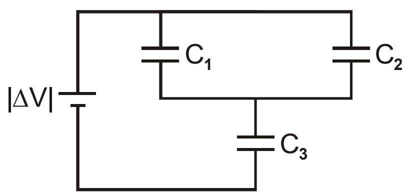
$$\frac{1}{C_{eq.}} = \frac{2}{C_1} + \frac{1}{C_2}$$

The generalization to any number of capacitors connected in series is

$$\frac{1}{C_{eq.}} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_N} = \sum_{i=1}^{N} \frac{1}{C_i} \text{ (series)}$$

Ex.: Equivalent Capacitance :

Find the equivalent capacitance for the combination of capacitors shown in figure.



Capacitors connected in series and in parallel

Solution : Since C_1 and C_2 are connected in parallel, their equivalent capacitance C_{12} is given by $C_{12} = C_1 + C_2$

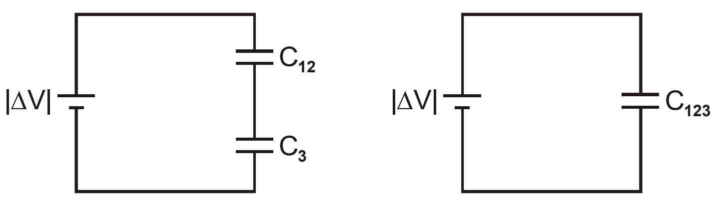


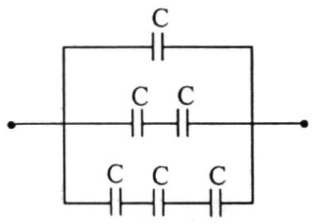
Figure (a) and (b) Equivalent circuits

Now capacitor C_{12} is in series with C_3, as seen from Figure (b). So, the equivalent capacitance C_{123} is given by

$$\frac{1}{C_{123}}=\frac{1}{C_{12}}+\frac{1}{C_3} \qquad \text{or} \qquad C_{123}=\frac{C_{12}C_3}{C_{12}+C_3}=\frac{(C_1+C_2)C_3}{C_1+C_2+C_3}$$

Equivalent Capacitance

Consider the configuration shown in figure. Find the equivalent capacitance, assuming that all the capacitors have the same capacitance C.



Combination of capacitors

Solution :

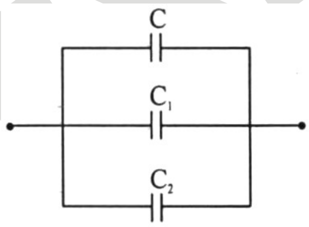
For capacitors that are connected in series, the equivalent capacitance is

$$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}+......=\sum_i C_i \quad \text{(series)}$$

On the other hand, for capacitors that are connected in parallel, the equivalent capacitance is

$$C_{eq}=C_1+C_2+......=\sum_i C_i \quad \text{(parallel)}$$

Using the above formula for series connected, the equivalent configuration is shown in figure.

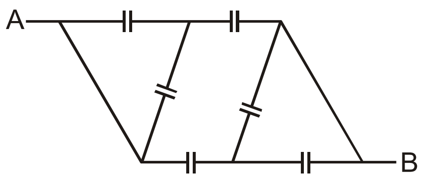


Now we have three capacitors connected in parallel. The equivalent capacitance is given by

$$C_{eq}=C\left(1+\frac{1}{2}+\frac{1}{3}\right)=\frac{11}{6}C$$

Ex. A network of six identical capacitors, each of value C is made as shown in the figure. Equivalent capacitance between points A and B is :

(A) C/4 (B) 3C/4 (C) 4C/3 (D) 3C



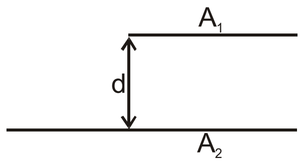
Ans. (C)

Sol. The network is equivalent to

Therefore equivalent capacitance = [2C series C] // [C series 2C]

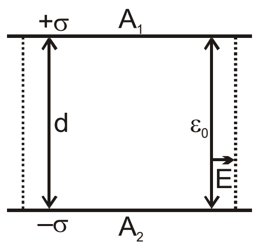
$$= 2\left(\frac{2C\times C}{2C+C}\right)=\frac{4C}{3}$$

Ex. The capacitance of the system of parallel plate capacitor shown in the figure is :



(A) $\dfrac{2\varepsilon_0 A_1 A_2}{(A_1 + A_2)d}$ (B) $\dfrac{2\varepsilon_0 A_1 A_2}{(A_2 - A_1)d}$ (C) $\dfrac{\varepsilon_0 A_1}{d}$ (D) $\dfrac{\varepsilon_0 A_2}{d}$

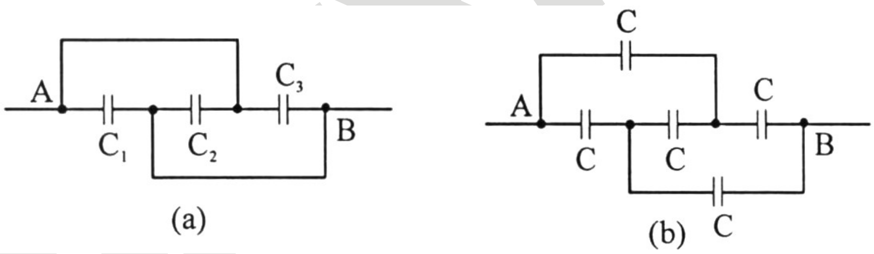
Sol.



Since the electric field between the parallel charge plates is uniform and independent of the distance, neglecting the edge effect, the effective area of the plate of area A_2 is A_1. Thus the capacitance between the plates is

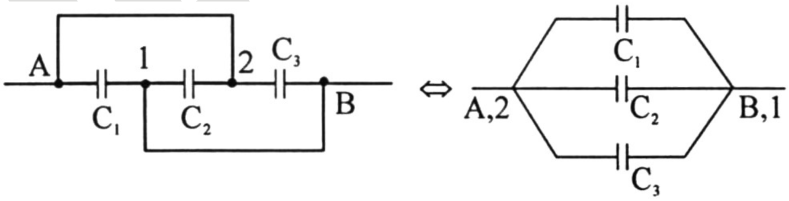
$C = \dfrac{\varepsilon_0 A_1}{d}$

Ex. Find the capacitance of a system of identical capacitors between points A and B shown in (a) in Fig. (a), (b) in Fig.(b)



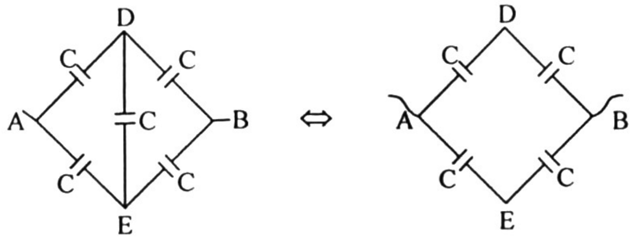
Ans. (a) $C_{total} = C_1 + C_2 + C_3$; (b) $C_{total} = C$

Sol.



(a) Since $\varphi_1 = \varphi_B$ and $\varphi_2 = \varphi_A$

The arrangement of capacitors shown in the problem is equivalent to the arrangement shown in the fig.



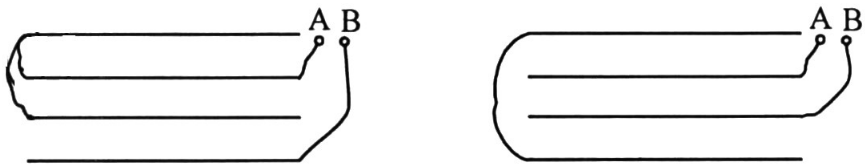
and hence the capacitance between A and B is,

$$C = C_1 + C_2 + C_3$$

(b) From the symmetry of the problem, there is no. P.d. between D and E. So, the combination reduces to a simple arrangement shown in the fig. and hence the net capacitance,

$$C_0 = \frac{C}{2} + \frac{C}{2} = C$$

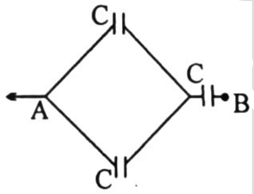
Ex. Four identical metal plates are located in air at equal distances d from one another. The area of each plate is equal to A. Find the capacitance of the system between points A and B if the plates are interconnected as shown (a) in Fig. (a) (b)inFig.(b)



(a) (b)

Ans. (a)$C = 2\varepsilon_0 A/3d$; (b) $C = 3\varepsilon_0 A/2d$

Sol. (a) In the given arrangement, we have three capacitors of equal capacitance $C = \dfrac{\varepsilon_0 A}{d}$ and the first and third plates are at the same potential.



Hence, we can resolve the network into a simple form using series and parallel grouping of capacitors, as shown in the figure.
Thus the equivalent capacitance

$$C_0 = \frac{(C+C)C}{(C+C)+C} = \frac{2}{3}C$$

(b) Let us mentally impart the charges + q and - q to the plates 1 and 2 and then distribute them to other plates using charge conservation and electric induction (fig.). As the potential difference between the plates 1 and 2 is zero,

$$-\frac{q_1}{C} + \frac{q_2}{C} - \frac{q_1}{C} = 0, \left(\text{where } C = \frac{\varepsilon_0 A}{d}\right)$$

or, $q_2 = 2q_1$,

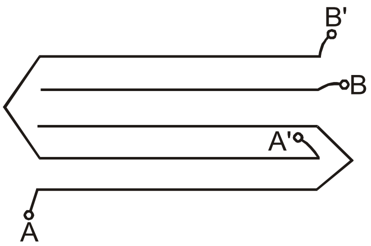
The potential difference between A and B,

$V = V_A - V_B = q_2/C$,

Hence the sought capacitance,

$$C_0 = \frac{q}{V} = \frac{q_1 + q_2}{q_2/C} = \frac{3q_1}{2q_1/C} = \frac{3}{2}C = \frac{3\varepsilon_0 A}{2d}$$

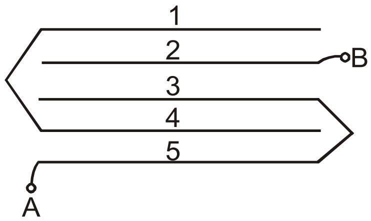
Ex. The equivalent capacitance between A and B is (each of the capacitors obtained is of capacitance equal to C).



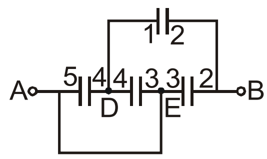
(A) $\frac{1}{2}C$ (B) $\frac{3}{5}C$ (C) $\frac{5}{3}C$ (D) $\frac{2}{5}C$

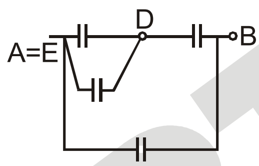
Ans. (C)

Sol.



$\Rightarrow$





$\Rightarrow \qquad C_{AB} = \frac{2C \times C}{3C} + C = \frac{5C}{3}$

Ex. A capacitor of capacitance $C_1 = 1.0\ \mu F$ with stands the maximum voltage $V_1 = 6.0$ kV while a capacitor of capacitance $C_2 = 2.0\ \mu F$, the maximum voltage $V_2 = 4.0$ kV. What voltage will the system of these two capacitors withstand if they are connected in series?

Ans. $V \leq V_1(1 + C_1/C_2) = 9kV$

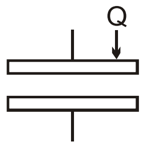
Sol. Amount of charge, that the capacitor of capacitance C_1 can withstand, $q_1 = C_1V_1$ and similarly the charge, that the capacitor of capacitance C_2 can withstand, $q_2 = C_2V_2$. But in series combination, charge on both the capacitors will be same, so, q_{max}, that the combination can withstand = C_1V_1,

as $C_1V_1 < C_2V_2$, from the numerical data, given.

Now, net capacitance of the system,

$$C_0 = \frac{C_1C_2}{C_1 + C_2} \quad \text{and hence,} \quad V_{max} \frac{q_{max}}{C_0} = \frac{C_1V_1}{C_1C_2/C_1 + C_2} = V_1\left(1 + \frac{C_1}{C_2}\right) = 9kV$$

Ex. Charge Q is given to the upper plate of an isolated parallel plate capacitor of capacitance C. The potential difference between the plates.



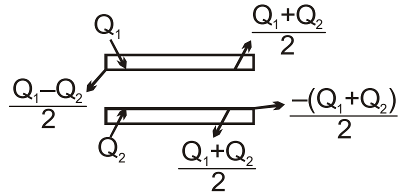
(A) $\frac{Q}{C}$ (B) $\frac{Q}{\frac{C}{2}}$ (C) $\frac{Q/2}{C}$ (D) Zero

Ans. (C)

Sol. In general, for charge Q_1 and Q_2 on upper and lower plate respectively the charge distributions on outer and inner part of the plates are shown in figure.

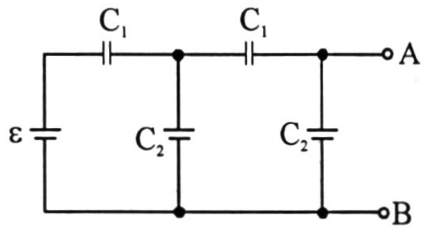
Here $Q_1 = Q$, $Q_2 = 0$

$\therefore$ Charge on inner side of plate are $\frac{Q}{2}$ and $-\frac{Q}{2}$ respectively.



Hence V = $\frac{Q/2}{C}$

Ex. Find the potential difference between points A and B of the system shown in Fig. if the emf is equal to ξ = 110 V and the capacitance ratio C_2/C_1 = 2.0.



Ans. U = 10V

Sol. Let us distribute the charge, as shown in the figure.

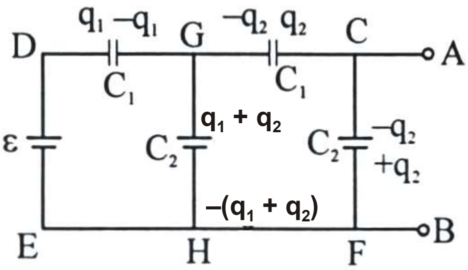
Now, we know that in a closed circuit, $-\Delta\varphi = 0$

So, in the loop,DCFED,

$$\frac{q_1}{C_1}-\frac{q_2}{C_1}-\frac{q_2}{C_2}=\xi \text{ or, } q_1 = C_1\left[\xi + q_2\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\right] \quad .....(1)$$

Again in the loop DGHED,

$$\frac{q_1}{C_1}+\frac{q_1+q_2}{C_2}=\xi \quad .....(2)$$



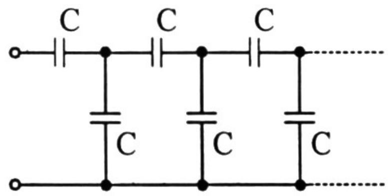
Using equation (1) and (2), we get

$$q_2\left(\frac{1}{C_1}+\frac{3}{C_2}+\frac{C_1}{C_2}\right)=-\frac{\xi C_1}{C_2}$$

Now, $$V_A - V_B = \frac{-q_2}{C_2} = \frac{\xi}{C_2^2/C_1}\frac{1}{\left[\frac{1}{C_1}+\frac{3}{C_2}+\frac{C_1}{C_2^2}\right]}$$

or $$V_A - V_B = \frac{1}{\left[\frac{C_2^2}{C_1^2}+\frac{3C_2}{C_1}+1\right]} = \frac{\xi}{\eta^2+3\eta+1} = 10V$$

Ex. Find the capacitance of an infinite circuit formed by the repetition of the same link consisting of two identical capacitors, each with capacitance C.



Ans. $C_x = \frac{C(\sqrt{5}-1)}{2} = 0.62C$. Since the chain is infinite, all the links beginning with the second can be replaced by the capacitance C_x equal to the sought one.

Sol. The infinite circuit, may be reduced to the circuit, shown in figure, where, C_0 is the net capacitance of the combination.

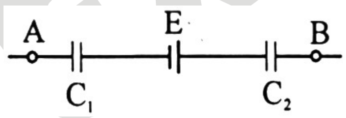
So, $$\frac{1}{C+C_0} + \frac{1}{C} = \frac{1}{C_0}$$

Solving the quadratic,

$$CC_0 + C_0^2 - C^2 = 0,$$

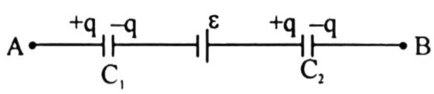
$$C_0 = \frac{(\sqrt{5}-1)}{2}C,$$ taking only +ve as C_0 can not be negative.

Ex. A circuit has a section AB shown in Fig. The emf of the source equals E = 10V, the capacitor capacitances are equal to $C_1 = 1.0\ \mu F$ and $C_2 = 2.0\ \mu F$, and the potential difference $V_A - V_B = 5.0$ V. Find the voltage across each capacitor.



Ans. $V_1 = q/C_1 = 10$ V, $V_2 = q/C_2 = 5$V, where $q = (V_A - V_B + E)C_1C_2 / (C_1 + C_2)$

Sol. Let, us make the charge distribution, as shown in the figure.



Now, $$V_A - V_B = \frac{q}{C_1} - \xi + \frac{q}{C_2}$$

or, $$q = \frac{(V_A - V_B) + \xi}{C_1 + C_2}C_1C_2$$

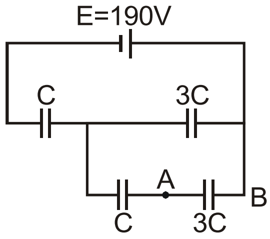
Hence, voltage across the capacitor C_1

$$= \frac{q}{C_1} = \frac{(V_A - V_B) + \xi}{C_1 + C_2}C_2 = 10V$$

and voltage across the capacitor, C_2

$$= \frac{q}{C_2} = \frac{(V_A - V_B) + \xi}{C_1 + C_2}C_1 = 5V$$

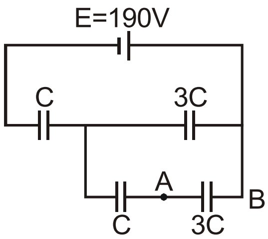
Ex. In the circuit shown in figure potential difference between A and B



(A) 30 V (B) 60 V (C) 10 V (D) 90 V

Ans. (C)

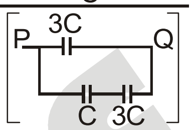
Sol. Potential difference is divided among two capacitance C_1 and C_2 in the inverse ratio of their capacities when it is final in series. Therefore



Voltage across P and Q is

$$= 190 \times \frac{C}{C + \text{Capacitance across P and Q}}$$

$$= 190 \times \frac{C}{C + [\ \ ]}$$



equivalent capacitance between P and Q

$$= 190 \times \frac{C}{C + \frac{15C}{4}} = 40 \text{ volt}$$

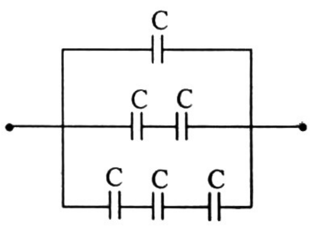
Potential difference between P and Q (i.e. 40 volt)

Will divide between two capacitor C and 3C which is in series. Therefore voltage across 3C capacitance.

$$\text{volt} = 40 \times \frac{C}{C + 3C} = 10 \text{ Volt}$$

Equivalent Capacitance

Consider the configuration shown in figure. Find the equivalent capacitance, assuming that all the capacitors have the same capacitance C.



Combination of capacitors

Solution :

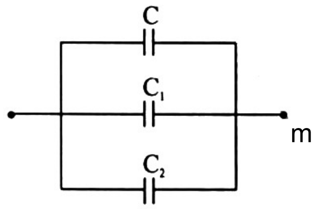
For capacitors that are connected in series, the equivalent capacitance is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + ... = \sum_i C_i \text{ (series)}$$

On the other hand, for capacitors that are connected in parallel, the equivalent capacitance is

$$C_{eq} = C_1 + C_2 + ... = \sum_i C_i \text{ (parallel)}$$

Using the above formula for series connected, the equivalent configuration is shown in figure.



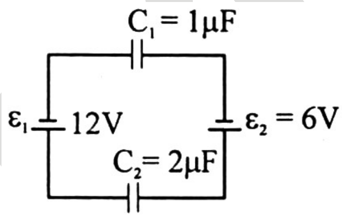
Now we have three capacitors connected in parallel. The equivalent capacitance is given by

$$C_{eq} = C\left(1 + \frac{1}{2} + \frac{1}{3}\right) = \frac{11}{6}C$$

Useof Kirchof's Laws in Solving capacitor newtorks :

1. Nodal teachnique
2. Energy loss calculation

Ex. In a circuit shown in Fig. find the potential difference between the left and right plates of each capacitor.

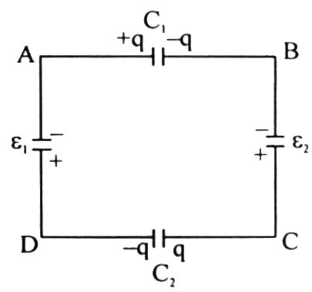


Ans. $V_1 = (\xi_2 - \xi_1) / (1 + C_1/C_2)$, $V_2 = (\xi_1 - \xi_2) / (1 + C_2/C_1)$.

$V_1 = -4V$

$V_2 = 2V$

Sol. Let $\xi_2 > \xi_1$, then using $-\Delta\varphi = 0$ in the closed circuit, (fig.)



$$\frac{-q}{C_1} + \xi_2 - \frac{q}{C_2} - \xi_1 = 0$$

or $$q = \frac{(\xi_2 - \xi_1)C_1C_2}{C_1 + C_2}$$

Hence the P.D. across the left and right plates of capacitors,

$$\varphi_1 = \frac{q}{C_1} = \frac{(\xi_2 - \xi_1)C_2}{C_1 + C_2}$$

and similarly

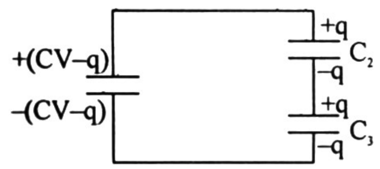
$$\varphi_2 = \frac{-q}{C_2} = \frac{(\xi_1 - \xi_2)C_1}{C_1 + C_2}$$

Ex. A capacitor of capacitance C_1 = 1.0 μF charged up to a voltage V = 110V is connected in parallel to the terminals of a circuit consisting of two uncharged capacitors connected in series and possessing the capacitances C_2 = 2.0 μF and C_3 = 3.0 μF. What charge will flow through the connecting wires?

Ans. $q = \dfrac{v}{1/C_1 + 1/C_2 + 1/C_3} = 0.06 \text{ mC}$

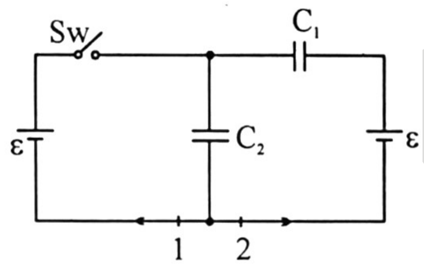
Sol. Let, the charge q flows through the connecting wires, then at the state of equilibrium, charge distribution will be as shown in the fig. In the closed circuit 12341, using $-\Delta\varphi = 0$

$$-\frac{(C_1V - q)}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} = 0$$



or $$q = \frac{V}{(1 + C_1 + 1/C_2 + 1/C_3)} = 0.06 \text{ mC}$$

Ex. What charges will flow after the shorting of the switch S w in the circuit illustrated in fig. through sections 1 and 2 in the directions indicated by the arrows?



Ans. $q_1 = \xi C_2, q_2 = -\xi C_1 C_2 / (C_1 + C_2)$

Sol. Initially, charge on the capacitor C_1 or C_2,

$q = \dfrac{\xi C_1 C_2}{C_1 + C_2}$, as they are in series combination (fig. a)

when the switch is closed, in the circuit CDEFC from $-\Delta V = 0$, (fig,, (b)

$$\xi - \frac{q_2}{C_2} = 0 \text{ or } q_2 = C_2\xi \qquad ......(1)$$

And in the closed loop BCFAB from $-\Delta\varphi = 0$

$$\frac{-q_1}{C_1} + \frac{-q_2}{C_2} - \xi = 0 \qquad ....(2)$$

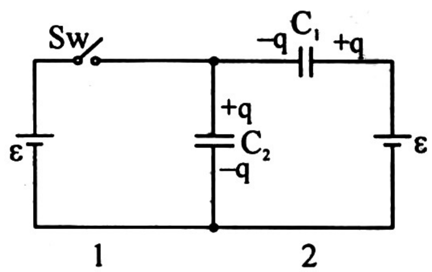


fig. (a)

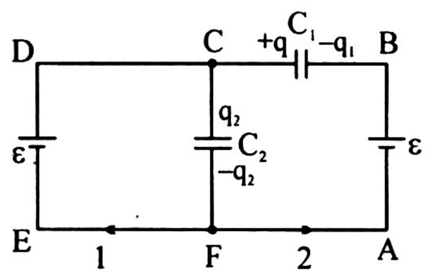


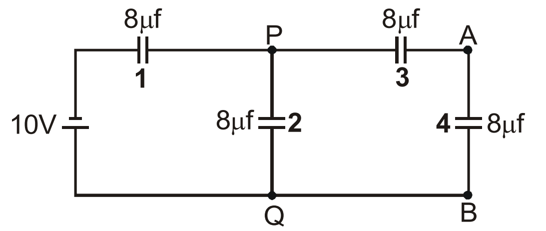
fig. (b)

From (1) and (2) $q_1 = 0$

Now, charge flown through section 1 = $(q_1 + q_2) - 0 = C_2\xi$

and charge flown through section 2 = $-q_1 - q = -\dfrac{\xi C_1 C_2}{C_1 + C_2}$

Ex. In the above circuit, find the potential difference across AB.



Sol. Let us mark the capacitors as 1, 2, 3 and 4 for identifications. As is clear, 3 and 4 are in series, and they are in parallel with 2. Then 2, 3, 4 combine is in series with 1.

$C_{34} = \dfrac{C_3.C_4}{C_3 + C_4} = 4\mu f$, $C_{2.34} = 8 + 4 = 12\mu f$

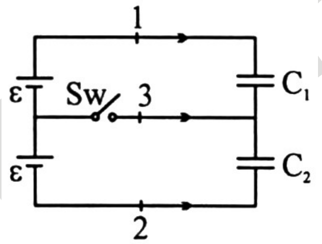
$C_{eq} = \dfrac{8 \times 12}{8 + 12} = 4.8\mu f$, $q = C_{eq.} V = 4.8 \times 10 = 48\mu C$

The 'q' on 1 is 48 μC, thus $V_1 = q/c = 6v$ $\left[v_1 = \dfrac{48\mu c}{8\mu f} = 6v\right]$

$\Rightarrow$ $V_{PQ} = 10 - 6 = 4V$

By symmetry of 3 and 4, we say, $V_{AB} = 2V$.

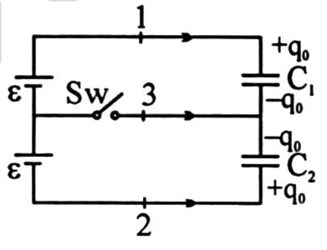
Ex. In the circuit shown in fig. the emf of each battery is equal to $\xi = 60$ V, and the capacitor capacitances are equal to $C_1 = 2.0$ μF and $C_2 = 3.0$ μF. Find the charges which will flow after the shorting of the switch Sw through sections 1, 2 and 3 in the directions indicated by the arrows.

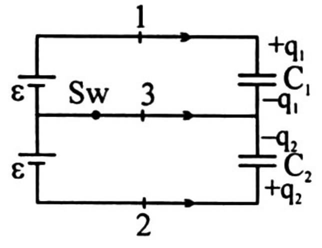


Ans. $q_1 = \xi C_1(C_1 - C_2) / (C_1 + C_2) = -24\mu C$, $q_2 = \xi C_2(C_1 - C_2) / (C_1 + C_2) = -36\mu C$, $q_3 = \xi(C_2 - C_1) + 60\mu C$

Sol. When the switch is open, (fig (a)

$$q_0 = \frac{2\xi C_1 C_2}{C_1 + C_2}$$





and when the switch is closed,

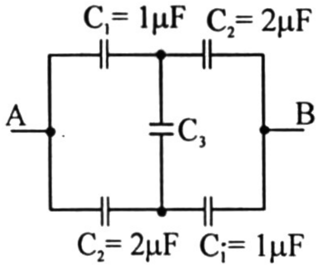
$q_1 = \xi C_1$ and $q_2 = \xi C_2$

Hence section $1 = q_1 - q_0 = \xi C_1 \left[\dfrac{C_1 - C_2}{C_1 + C_2}\right] = -24\mu C$

through the section $2 = -q_2 - (q_0) = \xi C_2 \left[\dfrac{C_1 - C_2}{C_1 + C_2}\right] = -36\mu C$

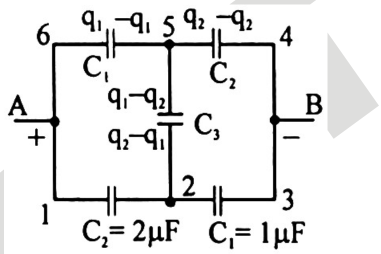
through the section 2 = $q_2 - (q_2 - q_1) - 0 = \xi(C_2 - C_1) = -60$ μC

Ex. Find the capacitance of the circuit shown in Fig. between points A and B.



Ans. $C_{total} = \dfrac{2C_1C_2 + C_3(C_1+C_2)}{C_1+C_2+2C_3} = \dfrac{2\times1\times2+1(3)}{1+2+2} = \dfrac{7}{5}\mu F$

Sol. Taking the advantage of symmetry of the problem charge distribution may be made, as shown in the figure. In the loop, 12561, $-\Delta\varphi = 0$



or $\quad \dfrac{q_2}{C_2} + \dfrac{q_2-q_1}{C_3} - \dfrac{q_1}{C_1} = 0$

or $\quad \dfrac{q_1}{q_2} = \dfrac{C_1(C_3+C_2)}{C_2(C_1+C_3)}$ ......(1)

Now, capacitance of the network,

$$C_0 = \frac{q_1+q_2}{V_A - V_B} = \frac{q_1+q_2}{q_2/C_2 + q_1/C_1}$$

$$= \frac{(1+q_1/q_2)}{\left(\dfrac{1}{C_2} + \dfrac{q_1}{q_2C_1}\right)} \quad .....(2)$$

From eqs.(1) and (2)

$$C_0 = \frac{2C_1C_2 + C_3(C_1+C_2)}{C_1+C_2+2C_3}$$

Ex. A capacitor of capacitance C_1 = 1.0 μF carrying initially a voltage V = 300 V is connected in parallel with an uncharged capacitor of capacitance C_2 = 2.0 μF. Find the loss of the electric energy of this system by the moment equilibrium is reached. Explain the result obtained.

Ans. $\Delta W = -1/2\ V^2C_1C_2 / (C_1 + C_2)$ 0.03 mJ

Sol. Charge contained in the capacitor of capacitance C_1 is $q = C_1\varphi$ and the energy, stored in it :

$$U_i = \frac{q^2}{2C_1} = \frac{1}{2}C_1V^2$$

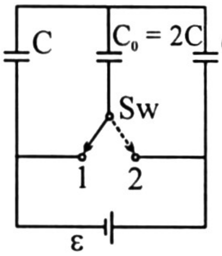
Now, when the capacitors are connected in parallel, equivalent capacitance of the system, $C = C_1 + C_2$ and hence, energy stored in the system :

$U_f = \dfrac{C_1^2V^2}{2(C_1+C_2)}$, as charge remains conserved during the process.

So, increment in the energy,

$$\Delta U = \frac{C_1^2V^2}{2}\left(\frac{1}{C_1+C_2} - \frac{1}{C_1}\right) = \frac{-C_2C_1V^2}{2(C_1+C_2)} = -0.03 \text{ mJ}$$

Ex. What amount of heat will be generated in the circuit shown in Fig. after the switch Sw is shifted from position 1 to position 2?.



Ans. $Q = \xi^2 CC_0 / (2C + C_0)$
for $C_0 = 2C$

$$Q = \frac{\varepsilon^2 C2C}{(2C+2C)} = \frac{\varepsilon^2 2C^2}{4C}$$

$$Q = \frac{C\varepsilon^2}{2}$$

Sol. The charge on the condensers in position 1 are as shown. Here

$$\frac{q}{C} = \frac{q_0}{C_0} = \frac{q+q_0}{C+C_0}$$

and $$(q+q_0)\left(\frac{1}{C+C_0} + \frac{1}{C}\right) = \xi$$

or, $$q + q_0 = \frac{C(C+C_0)\xi}{C_0+2C}$$

hence, $$q = \frac{C^2\xi}{C_0+2C} \text{ and } q_0 = \frac{CC_0\xi}{C_0+2C}$$

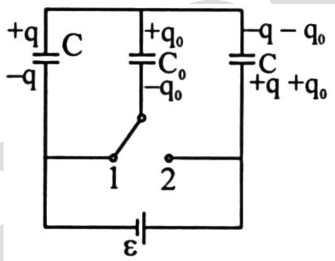


Fig. (a)

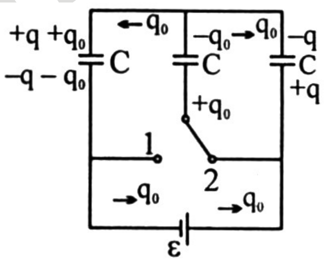


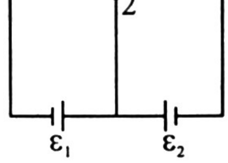
Fig. (b)

After the switch is thrown to position 2, the charges change as shown in (fig. b). A charge q_0 has flown in the right loop through the two condensers and a charge q_0 through the cell, Because of the symmetry of the problem there is no change in the energy stored in the condensers. Thus.

H (Heat produced) = Energy delivered by the cell

$$= \Delta q\xi = q_0\xi = \frac{CC_0\xi^2}{C_0+2C}$$

Ex. What amount of heat will be generated in the circuit shown in Fig. after the switch Sw is shifted from position 1 to position 2?



Ans. $Q = 1/2C\xi_2^2$. It is remarkable that the result obtained is independent of ξ_1.

Sol. Initially, the charge on the right plate of the capacitor q = C ($\xi_1 - \xi_2$) and finally, when switched to the position, 2. charge on the same plate of capacitor ;

$q' = C\xi_1$

So, $\Delta q = q' = C\xi_2$

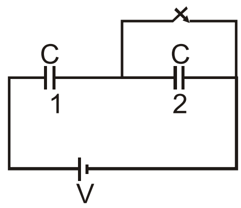
Now, from energy conservation,

ΔU + Heat liberated = A_{cell}, where ΔU is the electrical energy.

$$\frac{1}{2}C\xi_1^2 - \frac{1}{2}C(\xi_1 - \xi_2)^2 + \text{Heat liberated} = C\xi_2\xi_1$$

Hence heat liberated = $\frac{1}{2}C\xi_2^2$

Ex. The charge flowing across the cell on closing the key k is equal to



(A) CV (B) CV/2 (C) 2CV (D) Zero

Ans. (B)

Sol. When the key is kept open, the charge drawn from the source is

$Q = C_{eq}V = \frac{C}{2}V$

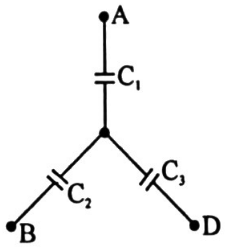
When the key is closed the capacitor 2 gets short circuited

And $C'_{eq} = C$

$\therefore$ $Q' = CV$

charge flown through cell $Q' - Q = \frac{C}{2}V$

Ex. Three uncharged capacitors of capacitance C_1, C_2 and C_3 are connected as shown in figure to one another and to points A, B and D at potentials ϕ_A , ϕ_B and ϕ_D. Determine the potential ϕ_0 at point O.



Sol. Taking into account the relation between the capacitance, voltage and charge of a capacitor, we can write the following equations for the three capacitors :

$$V_A - V_0 = \frac{q_1}{C_1}, V_B - V_0 = \frac{q_2}{C_2}, V_D - V_0 = \frac{q_3}{C_3},$$

where C_1, C_2 and C_3 are the capacitances of the corresponding capacitors, and q_1, q_2 and q_3 are the charges on their plates. According to the charge conservation law, $q_1 + q_2 + q_3 = 0$, and hence the potential of the common point O is :

$$V_0 = \frac{V_AC_1 + V_BC_2 + V_DC_2}{C_1 + C_2 + C_3}$$

The work done upon moving capacitor plates apart

A parallel-plate air capacitor has the plates of area S each. Find the work A' against the electric forces, done to increase the distance between the plates from x_1 to x_2, if (1) the charge q of the capacitor and (2) its voltage U are maintained constant. Find the increments of the electric energy of the capacitor in the two cases.

$$A' = qE_1 = (x_2 - x_1) = \frac{q^2}{2\varepsilon_0 S}(x_2 - x_1)_1$$

Where E_1 is the intensity of the field created by one plate ($E = \sigma/2\varepsilon_0$). It is in this field that the charge located on the other plate moves. This work is completely spent for increasing the electric energy : $\Delta W = A'$.

(2) In this case, the force acting on each capacitor plate will depend on the distance between the plates. Let us write the elementary work of the force acting on a plate during its displacement over a distance dx relative to the other plate :

$$\delta A' = qE_1 dx = \frac{\varepsilon_0 SU^2}{2}\frac{dx}{x^2},$$

where we took into account that q = CU, $E_1 = U/2x$, and $C = e_0 S/x$

After integration, we obtain

$$A' = \frac{\varepsilon_0 SU^2}{2}\left(\frac{1}{x_1} - \frac{1}{x_2}\right) > 0$$

The increment of the electric energy of the capacitor is

$$\Delta W = \frac{(C_2 - C_1)U^2}{2} = \frac{\varepsilon_0 SU^2}{2}\left(\frac{1}{x_2} - \frac{1}{x_1}\right) < 0\ .$$

It should be noted that $\Delta W = -A'$.

Thus, by moving the plates apart, we perform a positive work (against the electric forces). The energy of the capacitor decreases in this case. In order to understand this, we must consider a source maintaining the potential difference of the capacitor at a constant value. This source also accomplishes the work A. According to the law of conservation of energy, $A_s + A' = \Delta W$, where $A_s = \Delta W - A' = -2A' < 0$.

DIELECTRICS

In many capacitors there is an insulating material such as paper or plastic between the plates. Such material, called a dielectric, can be used to maintain a physical separation of the plates. Since dielectrics break down less readily than air, charge leakage can be minimized, especially when high voltage is applied. Experimentally it was found that capacitance C increases when the space between the conductors is filled with dielectrics. To sec how this happens, suppose a capacitor has a capacitance when there is no material between the plates. When a dielectric material is inserted to completely fill the space between the plates, the capacitance increases to :

$$C = K_e C_0$$

where K_e is called the dielectric constant. In the Table below, we show some dielectric materials with their dielectric constant. Experiments indicate that all dielectric materials have $K_e > 1$. Note that every dielectric material has a characteristic dielectric strength which is the maximum value of electric field before breakdown occurs and charges begin to flow.

Material	K_e	Dielectric strength (10^6 V/m)
Air	1.00059	3
Paper	3.7	16
Glass	4 – 6	9
Water	80	–

The fact that capacitance increases in the presence of a dielectric can be explained from a molecular point of view. We shall show that K_e is a measure of the dielectric response to an external electric Geld. There are two types of dielectrics. The first type is polar dielectrics, which are dielectrics that have permanent electric dipole moments. An example of this type of dielectric is water.

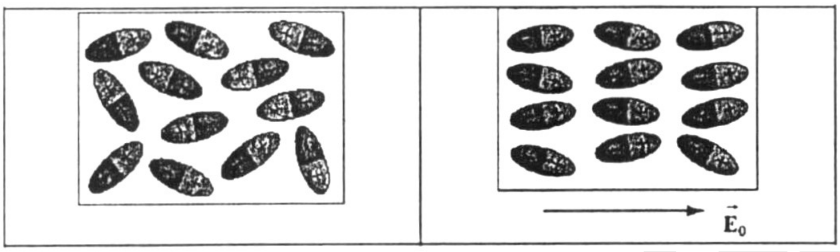


Figure : Orientations of polar molecules when (a) $\vec{E}_0 = \vec{0}$ and (b) $\vec{E}_0 \neq 0$

As depicted in Figure, the orientation of polar molecules is random in the absence of an external field. When an external electric field $\vec{E}_0$ is present, a torque is set up and causes the molecules to align with $\vec{E}_0$. However, the alignment is not complete due to random thermal motion. The aligned molecules then generate an electric field that is opposite to the applied field but smaller in magnitude.

The second type of dielectrics is the non-polar dielectrics, which are dielectrics that do not possess permanent electric dipole moment. Electric dipole moments can be induced by placing the materials in an externally applied electric field.

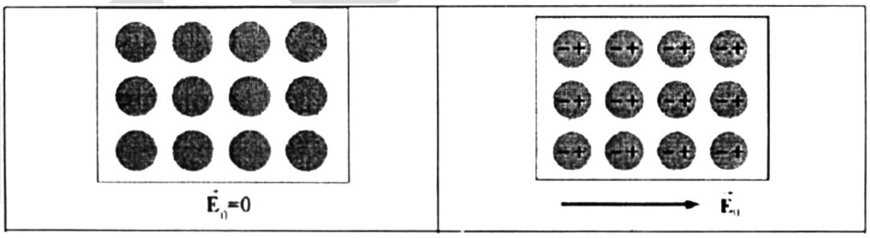


Figure : Orientations of non-polar molecules when (a) $\vec{E}_0 = \vec{0}$ and (b) $\vec{E}_0 \neq 0$

Figure illustrates the orientation of non-polar molecules with and without an external field $\vec{E}_0$. The induced surface charges on the faces produces an electric field $\vec{E}_P$ in the direction opposite to $\vec{E}_0$, leading to $\vec{E} = \vec{E}_0 + \vec{E}_P$, with $|\vec{E}| < |\vec{E}|$. Below we show how the induced electric field $\vec{E}_P$ is calculated.

Let us now examine the effects of introducing dielectric material into a system. We shall first assume that the atoms or molecules comprising the dielectric material have a permanent electric dipole moment. If left to themselves, these permanent electric dipoles in a dielectric material never line up spontaneously, so that in the absence of any applied external electric field $\vec{P} = \vec{0}$ due to the random alignment of dipoles, and the average electric field $\vec{E}_P$ is zero as well. However, when we place the dielectric material in an external field $\vec{E}_0$, the dipoles will experience a torque $\vec{\tau} = \vec{p} \times \vec{E}_0$, that tends to align the dipole vectors $\vec{P}$ with $\vec{E}_0$.

The effect is a net polarization $\vec{P}$ parallel to $\vec{E}_0$, and therefore an average electric field of the dipoles $\vec{E}_P$ anti-parallel to $\vec{E}_0$, i.e., that will tend to reduce the total electric field strength below $\vec{E}_0$. The total electric field $\vec{E}$ is the sum of these two fields :

$$\vec{E} = \vec{E}_0 + \vec{E}_P = \vec{E}_0 - \vec{P}/\varepsilon_0$$

In most cases, the polarization $\vec{P}$ is not only in the same direction as $\vec{E}_0$, but also linearly proportional to $\vec{E}_0$ (and hence $\vec{E}$). This is reasonable because without the external field $\vec{E}_0$ there would be no alignment of dipoles and no polarization $\vec{P}$. We write the linear relation between $\vec{P}$ and $\vec{E}$ as:

$$\vec{P} = \varepsilon_0 X_e \vec{E}$$

where X_e is called the electric susceptibility. Materials they obey this relation are linear dielectrics. Combing Eqs. and gives

$$\vec{E}_0 = (1 + X_e)\vec{E} = K_e \vec{E}$$

where $K_e = (1 + X_e)$

is the dielectric constant. The dielectric constant e. is always greater than one since $X_e > 0$. This implies

$$E = \frac{E_0}{K_e} < E_0$$

Thus, we see that the effect of dielectric materials is always to decrease the electric field below what it would otherwise be.

In the case of dielectric material where there are no permanent electric dipoles, a similar effect is observed because the presence of an external field $\vec{E}_0$ induces electric dipole moments in the atoms or molecules. These induced electric dipoles are parallel to $\vec{E}_0$, again leading to a polarization $\vec{P}$ parallel to $\vec{E}_0$, and a reduction of the total electric field strength.

Dielectrics without Battery :

As shown in Figure, a battery with a potential difference $|DV_0|$ across its terminals is first connected to a capacitor C_0, which holds a charge $Q_0 = C_0|\Delta V_0|$. We then disconnect the battery, leaving Q_0 = const.

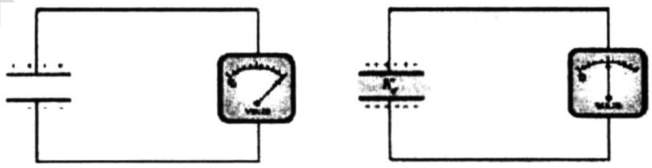


Figure : Inserting a dielectric material between the capacitor plates while keeping the charge Q_0 constant

If we then insert a dielectric between the plates, while keeping the charge constant, experimentally it is found that the potential difference decreases by a factor of K_e.

$$|DV| = \frac{|\Delta V_0|}{K_e}$$

This implies that the capacitance is changed to

$$C = \frac{Q}{|\Delta V|} = \frac{Q_0}{|\Delta V_0|/K_e} = K_e \frac{Q_0}{|\Delta V_0|} = K_e C_0$$

Thus, we see that the capacitance has increased by a factor of K_e. The electric field within the dielectric is now

$$E = \frac{|\Delta V|}{d} = \frac{|\Delta V_0|/K_e}{d} = \frac{1}{K_e}\left(\frac{|\Delta V_0|}{d}\right) = \frac{E_0}{K_e}$$

We see that in the presence of a dielectric, the electric field decreases by a factor of K_e.

Dielectrics with Battery :

Consider a second case where a battery supplying a potential difference remains connected as the dielectric is inserted. Experimentally, it is found (first by Faraday) that the charge on the plates is increased by a factor K_e.

$$Q = K_e Q_0$$

where Q_0 is the charge on the plates in the absence of any dielectric.

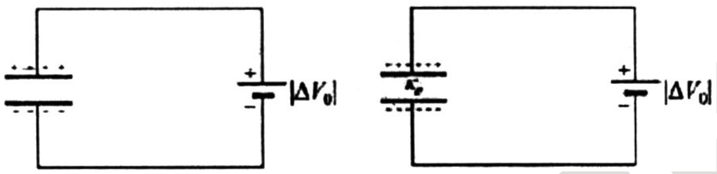


Figure : Inserting a dielectric material between the capacitor plates while maintaining a constant potential difference $|V_0|$

The capacitance becomes

$$C = \frac{Q}{|\Delta V_0|} = \frac{K_e Q_0}{|\Delta V_0|} = K_e C_0$$

which is the same as the first case where the charge Q_0 is kept constant, but nowthe charge has increased.

Calculation of induced charge on dielectric :

However, we have just seen that the effect of the dielectric is to weaken the original field E_0 by a factor K_e. Therefore,

$$E = \frac{E_0}{K_e} = \frac{Q}{K_e \varepsilon_0 A} = \frac{Q - Q_P}{\varepsilon_0 A}$$

from which the induced charge Q_P. can be obtained as

$$Q_P = Q\left(1 - \frac{1}{K_e}\right)$$

In terms of the surface charge density, we have

$$\sigma_P = \sigma\left(1 - \frac{1}{K_e}\right)$$

Note that in the limit, K_e = 1, Q_P = which corresponds to the case of no dielectric material.

Substituting Eq. into Eq. we see that Gauss's law with dielectric can be rewritten as

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{K_e \varepsilon_0} = \frac{Q}{\varepsilon}$$

where $\varepsilon = K_e \varepsilon_0$ is called the dielectric permittivity. Alternatively, we may also write

Ex. Capacitance with Dielectrics :

A non-conducting slab of thickness t, area A and dielectric constant is inserted into the space between the plates of a parallel-plate capacitor with spacing d charge Q, and area A, as shown in Figure(a). The slab is not necessarily halfway between the capacitor plates. What is the capacitance of the system?

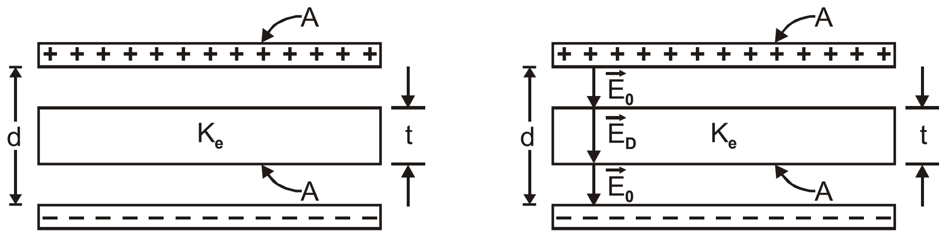


Figure : (a) Capacitor with a dielectric, (b) Electric Geld between the plates

Sol. To find the capacitance C, we first calculate the potential difference ΔV. We have already seen that in the absence of a dielectric, the electric Geld between the plates is given by $E_0 = Q/\varepsilon_0 A$ and $E_D = E_0/K_e$ when a dielectric of dielectric constant K_e is present, as shown in Figure (b). The potential can be found by integrating the electric Geld along a straight line from the top to the bottom plates :

$$\Delta V = -\int_{+}^{-} E\,d\ell = -\Delta V_0 - \Delta V_D = -E_0(d-t) - E_D t = -\frac{Q}{A\varepsilon_0}(d-t) - \frac{Q}{A\varepsilon_0 K_e}t = -\frac{Q}{A\varepsilon_0}\left[d - t\left(1 - \frac{1}{K_e}\right)\right]$$

where $\Delta V_D = E_D t$ is the potential difference between the two faces of the dielectric. This gives

$$C = \frac{Q}{|\Delta V|} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K_e}\right)}$$

It is useful to check the following limits :

(i) As $t \to 0$ i.e., the thickness of the dielectric approaches zero, we have $C = e_0A/d = C_0$, which is the expected result for no dielectric.

(ii) As, $K_e \to 1$, we again have $C \to \varepsilon_0 A/d = C_0$, and the situation also correspond to the case where the dielectric is absent.

(iii) In the limit where $t \to d$, the space is filled with dielectric, we have. $C \to K_e\varepsilon_0 A/d = K_e C_0$.

We also comment that the configuration is equivalent to two capacitors connected in series, as shown in Figure.

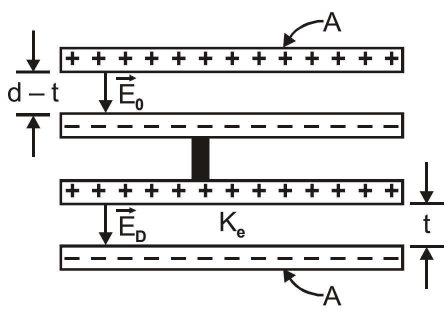


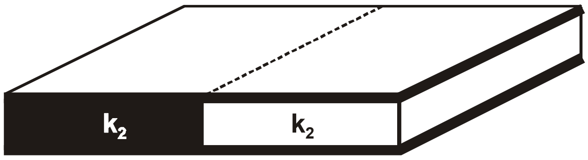
Figure : Equivalent configuration

Using Eq. for capacitors connected in series, the equivalent capacitance is

$$\frac{1}{C} = \frac{d-t}{\varepsilon_0 A} + \frac{t}{K_e\varepsilon_0 A}$$

Capacitor filled with two different dielectrics

Two dielectric with dielectric constant k_1 and k_2 each fill half the space between the plates of a parallel-plate capacitor as shown in figure.



Capacitor filled with two different dielectrics

Each plate has an area A and the plates are separated by a distance d. Compute the capacitance of the system.

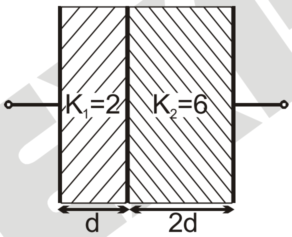
Sol. Since the potential difference on each half of the capacitor is the same, we may treat the system as being composed of two capacitors connected in parallel. Thus, the capacitance of the system is

$$C = C_1 + C_2$$

with $$C_i = \frac{k_i \varepsilon_0 (A/2)}{d}, i = 1, 2$$

we obtain $$C = \frac{k_1 \varepsilon_0 (A/2)}{d} + \frac{k_2 \varepsilon_0 (A/2)}{d} = \frac{\varepsilon_0 A}{2d}(k_1 + k_2)$$

Ex. A parallel plate capacitor has two layers of dielectrics as shown in figure. This capacitor is connected across a battery, then the ratio of potential difference across the dielectric layers is :



(A) 4/3 (B) 1/2 (C) 1/3 (D) 3/2

Ans. (D)

Capacitance (for k = 2)

$C_1 = \frac{\varepsilon_0 kA}{d}$

$C_1 = 2C$ Where $C = \frac{\varepsilon_0 A}{d}$

Capacitance (for k = 6)

$C_2 = \frac{6\varepsilon_0 A}{2d}$

$C_2 = 3C$

Therefore ratio of potential difference across the dielectric layer is = 3/2

Ex. A capacitor stores $10\mu C$ charge when connected across a battery. When the gap between the plates is filled with a dielectric, a charge of $20\mu C$ flows through the battery. Find the dielectric constant of the dielectric.

(A) k = 2 (B) k = 4 (C) k = 3 (D) k = 1

Ans. (C)

Sol. In absence of dielectric

$Q = CV = 10\mu C$ .....(1)

With dielectric

$Q' = kCV = 30\mu C$ .....(2)

From (1) and (2)

k = 3

Ex. The capacitance of an filled parallel plate capacitor is $20\mu\mu F$. The separation between the plates is doubled and the space between the plates is then fillded with was giving the capacitance a new value of 40×10^{-12} farads. The dielectric constant of wax is

(A) 12.0 (B) 10.0 (C) 8.0 (D) 4.2

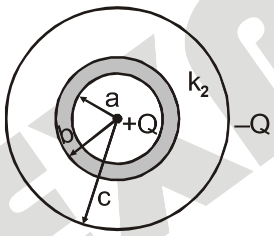
Ans. (C)

Sol. $10 \times 10^{-12} = \frac{\varepsilon_0 A}{d}.40\times10^{-12} = \frac{K\varepsilon_0 A}{2d}$

$\therefore$ K = 8

Ex. **Capacitor with dielectrics**

Consider a conducting spherical shell with an inner radius a and outer radius c. Let the space between two surfaces be filed with two different dielectric materials so that the dielectric constant is k_1 between a and b, and k_2 between b and c, as shown in Figure. Determine the capacitance of this system.



Spherical capacitor filled with dielectric

Solution

The system can be treated as two capacitors connected in series, since the total potential difference across the capacitors is the sum of potential differences across individual capacitors. The equivalent capacitance for a spherical capacitor of inner radius r_1 and outer radius r_2 filled with dielectric with dielectric constant k is given by

$$C = \pi\varepsilon_0 k_e\left(\frac{r_1 r_2}{r_2 - r_1}\right)$$

Thus, the equivalent capacitance of this system is

$$\frac{1}{C} = \frac{1}{\frac{4\pi\varepsilon_0 k_1 ab}{(b-a)}} + \frac{1}{\frac{4\pi\varepsilon_0 k_2 bc}{(c-b)}} = \frac{k_2 c(b-a) + k_1 a(c-b)}{4\pi\varepsilon_0 k_1 k_2 abc}$$

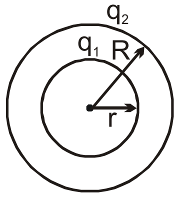
or $$C = \frac{4\pi\varepsilon_0 k_1 k_2 abc}{k_2 c(b-a) + k_1 a(c-b)}$$

It is instructive to check the limit where $k_1, k_2 \to 1$. In this case, the above expression reduces to

$$C = \frac{4\pi\varepsilon_0 abc}{c(b-a) + a(c-b)} = \frac{4\pi\varepsilon_0 abc}{b(c-a)} = \frac{4\pi\varepsilon_0 ac}{(c-a)}$$

which agrees with Eq. $C = \frac{Q}{|\Delta V|} = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$ for a spherical capacitor of inner radius a and outer radius c.

Ex. A spherical condenser has 10cm and 12cm as the radii of inner and outer spheres. The space between the two is filled with a dielectric of dielectric constant 3. Find the capacity when the outer sphere is earthed.

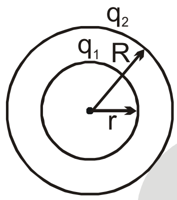


(A) 4×10^{-10}F (B) 1×10^{-10}F (C) 2×10^{-10}F (D) 6×10^{-10}F

Ans. (C)

Sol. $C = 4\pi k\varepsilon_0 \left(\frac{ab}{b-a}\right) = \frac{3}{9\times10^9} \times \frac{0.1\times0.12}{0.12-0.1} = 2 \times 10^{-10}$ F

Ex. A spherical condenser has 10 cm and 12 cm as the radii of inner and outer spheres. The space between the two is filled with a dielectric of dielectric constant 3. Find the capacity when the inner sphere is earthed.



(A) $\frac{15}{32} \times 10^{-10}$ F (B) $\frac{32}{15} \times 10^{+10}$ F (C) $\frac{32}{15} \times 10^{-9}$ F (D) $\frac{32}{15} \times 10^{-10}$ F

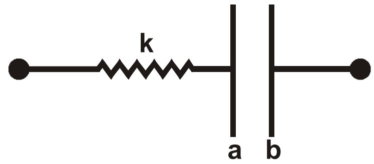
Ans. (D)

Sol. By choice of reference potential at infinity and the potential of earthed conductor zero, the given system can be visualised as combination of two spherical capacitors, both being at same potential difference. Connection wise these may be considered to be in parallel connection.

$\therefore \quad C = 4\pi\varepsilon_0 b + 4\pi k\varepsilon_0 \left(\frac{ab}{b-a}\right) = \frac{32}{15} \times 10^{-10}$ F

Capacitor Connected to a Spring

Consider an air-filled parallel-plate capacitor with one plate connected to a spring having a force constant k, and another plate held fixed. The system rests on a table top as shown in Figure



Capacitor connected to a spring

If the charges placed on plates a and b are +Q and –Q, respectively, how much does the spring expand?

Sol. The spring force $\vec{F}_5$ acting on plate a is given by

$$\vec{F}_5 = -kx\hat{i}$$

Similarly, the electrostatic force $\vec{F}_e$ due to the electric field created by plate b is

$$\vec{F}_e = QE\hat{i} = Q\left(\frac{\sigma}{2\varepsilon_0}\right)\hat{i} = \frac{Q^2}{2A\varepsilon_0}\hat{i}$$

where A is the area of the plate. Notice that charges on plate a cannot exert a force on itself, as required by Newton's third law. Thus, only the electric field due to plate b is considered. At equilibrium the two forces cancel and we have

$$kx = Q\left(\frac{Q}{2A\varepsilon_0}\right)$$

which gives

$$x = \frac{Q^2}{2kA\varepsilon_0}$$

Problem :

1. The charges on the plates of a parallel-plate capacitor are of opposite sign, and they attract each other. To increase the plate separation, is the external work done positive or negative? What happens to the external work done in this process?

2. How does the stored energy change if the potential difference across a capacitor is tripled?

3. Does the presence of a dielectric increase or decrease the maximum operating voltage of a capacitor? Explain.

4. If a dielectric-filled capacitor is cooled down, what happens to its capacitance?

Additional Problems

Capacitors in Series and in Parallel

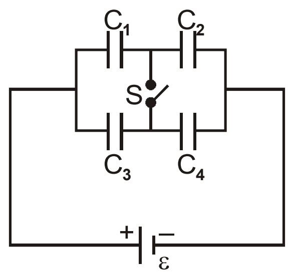
A12-Volt battery charges the four capacitors shown in figure.

Let C_1 = μF, C_2 = 2μF, C_3 = μF, and C_4 = 4μF.

(a) What is the equivalent capacitance of the group C_1 and C_2 if switch S is open (as shown)?

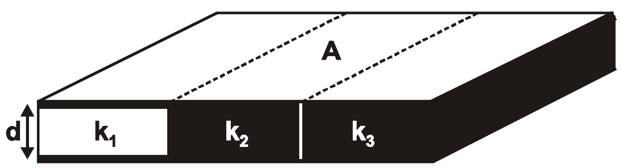
(b) What is the charge on each of the four capacitors if switch S is open?

(c) What is the charge on each of the four capacitors if switch S is closed?



Capacitors and Dielectrics

(a) A parallel-plate capacitor of area A and spacing d is filled with three dielectrics as shown in Figure. Each occupies 1/3 of the volume. What is the capacitance of this system? [Hint : Consider an equivalent system to be three parallel capacitors, and justify this assumption.] Show that you obtain the proper limits as the dielectric constants approach unity $k_1 \to 1$]



(b) This capacitor is now filled as shown in Figure. What is its capacitance? Use Gauss's law to find the field in each dielectric, and then calculate ΔV across the entire capacitor. Again, check your answer as the dielectric constants approach unity, $k_1 \to 1$. Could you have assumed that this system is equivalent to three capacitors in series?

A Capacitor with a Dielectric

A parallel plate capacitor has a capacitance of 112 pF, a plate area of 96.5 cm^2, and a mica dielectric (k_e = 5.40). At a 55 V potential difference, calculate

(a) The electric field strength in a the mica. [Ans. 13.4 kV/m]

(b) The magnitude of the free charge on the plates. [Ans. 6.16 nC]

(c) The magnitude of the induced surface charge. [Ans. 5.02 nC]

Energy Density in a Capacitor with a Dielectric

Consider the case in which a dielectric material with dielectric constant k_e completely fills the space between the plates of a parallel-plate capacitor.

(a) Given the electric field and potential of such a capacitor with free charge q on it (problem 4-1 a above), calculate the work done to charge up the capacitor from q = 0 to, the final charge, q = Q, the final charge.

(b) Find the energy density u_E.

Ex. Two parallel-plate air capacitors, each of capacitance C, were connected in series to a battery with emf ξ. Then one of the capacitors was filled up with slab of dielectric constant k.

(a) What amount of charge flows through the battery?

(b) Find the factor by which electric field in each capacitor charges during the process. $\left(\text{i.e. } \frac{E_{after}}{E_{before}}\right)$

Ans. The strength decreased 1 /2 (ε + 1) times;

(a) q = 1/2Cξ (ε – 1)/(ε + 1)

Sol. From the symmetry of the problem, the voltage across each capacitor, $\Delta V = \xi/2$ and charge on each capacitor q = Cξ/2 in the absence of dielectric.

Now when the dielectric is filled up in one of the capacitors, the equivalent capacitance of the system,

$$C_0' = \frac{C\varepsilon}{1+\varepsilon}$$

and the potential difference across the capacitor, which is filled dielectric,

$$\Delta V' = \frac{q'}{\varepsilon C} = \frac{C\varepsilon}{(1+\varepsilon)}\frac{\xi}{C\varepsilon} = \frac{\xi}{(1+\varepsilon)}$$

but $\quad V \alpha E$

So, as φ decreases $\frac{1}{2}(1+\varepsilon)$ times, the field strength also decreases by the same factor and flow of charge,

$\Delta q = q' - q$

$$= \frac{C\varepsilon}{(1+\varepsilon)}\xi - \frac{C}{2}\xi = \frac{1}{2}C\xi\frac{(\varepsilon-1)}{(\varepsilon+1)}$$

Ex. Find the capacitance of a spherical capacitor whose conductors have radii R_1 and $R_2 > R_1$ which is filled with material whose dielectric constant varies as k = a/r, where a is a constant, and r is the distance from the centre of the capacitor.

Ans. $C = 4\pi\varepsilon_0 a / \ell n(R_2/R_1)$

Sol. Let, us mentally impart a charge q to the conductor. Now potential difference between the plates,

$$V_+ - V_- = \int_{R_1}^{R_2} \vec{E}\cdot d\vec{r} = \int_{R_1}^{R_2} \frac{q}{4\pi\varepsilon_0 a/r}\frac{1}{r^2}dr = \frac{q}{4\pi\varepsilon_0 a}\ln R_2/R_1$$

Hence, the sought capacitance,

$$C = \frac{q}{V_+ - V_-} = \frac{q4\pi\varepsilon_0 a}{q\ln R_2/R_1} = \frac{4\pi\varepsilon_0 a}{\ln R_2/R_1}$$

Ex. Between the plates of a parallel-plate capacitor there is a metallic plate whose thickness takes up $\eta = 0.60$ of the capacitor gap. When that plate is absent the capacitor has a capacity C = 20 μF. The capacitor is connected to a dc voltage source V = 100 V. The metallic plate is slowly extracted from the gap. Find: (a) the change in the energy of the capacitor; (b) the mechanical work performed in the process of plate extraction.

Ans. (a) $\Delta W = -1/2CV^2\,\eta / (1-\eta) = -0.15$ mJ ; (b) $A = 1/2\, CV^2\, \eta / (1-\eta) = 0.15$ mJ

Sol. When the plate is absent the capacity of the condenser is

$$C = \frac{\varepsilon_0 S}{d}$$

When it is present, the capacity is $C' = \frac{\varepsilon_0 S}{d(1-\eta)} = \frac{C}{1-\eta}$

(a) The energy increment is clearly, $\Delta U = \frac{1}{2}CV^2 - \frac{1}{2}C'V^2 = \frac{C\eta}{2(1-\eta)}V^2$

(b) The charge on the plate is $q_i = \frac{CV}{1-\eta}$ initially and q_f = CV finally.

A charge $\frac{CV\eta}{1-\eta}$ has flown thmugh me battery charging it and with drawing $\frac{CV^2\eta}{1-\eta}$ units of energy from the system into the battery. The energy of the capacitor has decreased by just half of this. The remaining half i.e. $\frac{1}{2}\frac{CV^2\eta}{1-\eta}$ must be the work done by the external agent in withdrawing the plate. This ensures conservation of energy.

Ex. A parallel-plate air capacitor has a plate area of 100cm^3 and separation 5mm. A potential difference of 300V is established between its plates by a battery. After disconnecting a battery, the space between the plates is filled by ebonite (K = 2.6). Find out initial and final surface-density of charge on the plates.

(A) $5.31 \times 10^{+7}$ Cm/m^2 (B) 6.31×10^{-7} C/m^2

(C) 5.31×10^{-7} C/m^2 (D) 9.31×10^{-7} C/m^2

Ans. (C)

Sol. Capacity of the parallel plate air capacitor

$= \frac{\varepsilon_0 A}{d} = \frac{8.86\times10^{-12}\times100\times10^{-4}}{5\times10^{-3}}$ = 1.77×10^{-11}F

Final capacity of the capacitor with dielectric between the plates is

C' = KC = $2.6 \times 1.77 \times 10^{-11}$, C' = 4.6×10^{-11}F

Initial charge on the capacitor $1.77 \times 10^{-11} \times 300 = 5.31 \times 10^{-9}$C

Since, the battery has been disconnected, the charge remains the same, therefore the new potential difference is

$V' = \frac{q}{C'} = \frac{5.31\times10^{-9}}{4.6\times10^{-11}}$ = 115V

The surface density of charge remains the same in both the cases, i.e.,

$\sigma = \frac{q}{A} = \frac{5.31\times10^{-9}}{100\times10^{-4}}$ = 5.31×10^{-7} C/m^2

Ex. A glass plate totally fills up the gap between the electrodes of a parallel-plate capacitor whose capacitance ' in the absence of that glass plate is equal to C = 20 μF. The capacitor is connected to a dc voltage source V = 100 V. The plate is slowly, and without friction, extracted from the gap. Find the change in energy of capacitor and the mechanical work performed in the process of plate extraction.

Ans. $\Delta W = -1/2(\varepsilon - 1)CV^2 = -0.5$mJ, $A_{mech} = 1/2(\varepsilon - 1)CV^2 = 0.5$ mJ

Sol. Initially, capacitance of the system = Cξ.

So, initial energy of the system : $U_i = \frac{1}{2}(C\varepsilon)V^2$

and finally, energy of the capacitor : $U_f = \frac{1}{2}CV^2$

Hence capacitance energy increment, $\Delta U = \frac{1}{2}CV^2 - \frac{1}{2}(C\varepsilon)V^2 = \frac{1}{2}CV^2(\varepsilon - 1) = -0.5$ mJ

From energy conservation $\Delta U = A_{cell} + A_{agent}$

(as there is no heat liberation)

But $A_{cell} = (C_f - C_i)V^2 = (C - C\varepsilon)V^2$

Hence $A_{agent} = \Delta U - A_{cell} = \frac{1}{2}C(1-\varepsilon)V^2 = 0.5$ mJ

Ex. In a parallel plate capacitor of capacitance C, a metal sheet is inserted between the plates, parallel to them. The thickness of the sheet is half of the separation between the plates. The capacitance now becomes.

(A) 4C (B) 2C (C) C/2 (D) C/4

Ans. (B)

Sol. Before the metal sheet is inserted, $C = \varepsilon_0 A/d$

After the sheet is inserted, the system is equivalent to two capacitor in series, each of capacitance C' =

$\frac{\varepsilon_0 A}{d/4} = 4C$

The equivalent capacity is now 2C.

Ex. **The work of an e.m.f. source :**

A glass plate completely fills the gap between the plates of a parallel-plate capacitor whose capacitance is equal to C_0 when the plate is absent. The capacitor is connected to a source of permanent voltage U. Find the mechanical work which must be done against electric forces for extracting the plate out of the capacitor.

Sol. According to the law of conservation of energy, we can write

$$A_m + A_s = \Delta w,$$

where A_m is the mechanical work accomplished by extraneous forces against electric forces, A_s is the work of the voltage source in this process, and ΔW is the corresponding increment in the energy of the capacitor (we assume that contributions of other forms of energy to the change in the energy of the system is negligibly small).

Let us find ΔW and A_s. It follows from the formula $W = Cu^2/2 = qU/2$ for the energy of a capacitor that for U = const.

$$\Delta W = \Delta C\, U^2 / 2 = \Delta q\, U/2$$

Since the capacitance of the capacitor decreases upon the removal of the plate ($\Delta C < 0$), the charge of the capacitor also decreases ($\Delta q < 0$). This means that the charge has passed through the source against the direction of the action of extraneous forces, and the source has done negative work.

$$A_s = \Delta q \bullet U$$

Comparing formulas (3) and (2), we obtain

$$A_s = 2\Delta W$$

Substitution of this expression into (1) gives

$$A_m = -\Delta W \text{ or } A_m = \frac{1}{2}(\varepsilon - 1)C^0U^2.$$

Thus, extracting the plate out of the capacitor, we (extraneous forces) do a positive work (against electric forces). The e.m.f. source in this case accomplishes a negative work, and the energy of the capacitor decreases :

$$A_m > 0,\ A_s < 0,\ \Delta W < 0$$

Energy method for calculating forces.

This method is the most general. It allows us to take into account automatically all force interactions (both electric and mechanical) ignoring their origin, and hence leads to a correct result.

Let us consider the essence of the energy method for calculating forces. The simplest case corresponds to a situation when charged conductors are disconnected from the power supply. In this case, the charges on the conductors remain uncharged, and we may state that the work A of all internal forces of the system upon slow displacements of the conductors and dielectrics is done completely at the expense of a decrease in the electric energy W of the system (or its field). Here we assume that these displacements do not cause the transformation of electric energy into other kinds of energy. To be more precise, it is assumed that such transformations are negligibly small. Thus, for infinitesimal displacements we can write

$$\delta A = -\Delta W|_q,$$

where the symbol q emphasizes that the decrease in the energy of the system must be calculated when charges on the conductors are constant.

Equation $\delta A = -dW|_q$, is the initial equation for determining the forces acting on conductors and dielectrics in the electric field. This can be done as follows. Suppose that we are interested in the force acting on a given body (a conductor or a dielectric). Let us displace this body by an infinitesimal distance dx in the direction X we are interested in. Then the work of the required force F over the distance dx is δA = Fxdx, where F_x is the projection of the force F onto the positive direction of the X-axis. Substituting this expression for δA into $\delta A = -dW|_q$, and dividing both parts of $\delta A = -dW|_q$, by dx, we obtain.

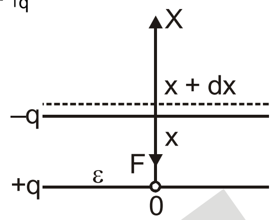
$$F_x = -\left.\frac{\partial W}{\partial x}\right|_q$$

We must pay attention to the following circumstance. It is well known that the force depends only on the position of bodies and on the distribution of charges at a given instant. It cannot depend on how the energy process will develop if the system starts to move under the action of forces. And this means that in order to calculate F_x by formula $F_x = -\left.\frac{\partial W}{\partial x}\right|_q$, we do not have to select condition under which all the charges of the conductor are necessarily constant (q = const). We must simply find the increment dW under the condition that q = const, which is a purely mathematical operation.

It should be noted that if a displacement is performed at constant potential on the conductors, the corresponding calculation leads to another expression for the force : $F_x = +\partial W / \partial x|_\varphi$. However (and it is important) the result of the calculation of F_x with the help of this formula or $F_x = -\left.\frac{\partial W}{\partial x}\right|_q$ is the same, as should be expected. Therefore, henceforth we shall confine ourselves to the application of only formula $F_x = -\left.\frac{\partial W}{\partial x}\right|_q$ and will use it for any conditions, including those where $q \neq$ const upon small displacement. We must not be confused; the derivative $\partial W / \partial x$ will be calculated at q = const in such cases as well.

Ex. Find the force acting on one of the plates of a parallel-plate capacitor in a liquid dielectric, if the distance between the plates is h, the capacitance of the capacitor under given conditions is C and the voltage U is maintained across its plates.

In this case, if we mentally moves the plates apart, the voltage U remains constant, while the charge q of the capacitor changes (this follows from the relation C = q/U). In spite of this, we shall calculate the force under the assumption that q = const, i.e. with the help of formula $F_x = -\left.\frac{\partial W}{\partial x}\right|_q$. Hence the most convenient expression for the energy of the capacitor is



$$W = \frac{q^2}{2C} = \frac{q^2}{2\varepsilon\varepsilon_0 S}x,$$

where c is the permittivity of the dielectric, S is the area of each plate, and x is the distance between them (x = h). Next, let us choose the positive direction of the X-axis as is shown in fig. According to $F_x = -\left.\frac{\partial W}{\partial x}\right|_q$ the force acting on the upper plate of the capacitor is

$$F_x = -\left.\frac{\partial W}{\partial x}\right|_q = -\frac{q^2}{2\varepsilon\varepsilon_0 S}$$

The minus sign in this formula indicates that vector F is directed towards the negative values on the X-axis, i.e. the force is attractive by nature. Considering that $q = \sigma S = DS = \varepsilon\varepsilon_0 ES$ and E = U/h, we transform (1) to

$$F_x = -CU^2 / 2h$$

Ex. Find the work that must be done against the electric forces in order to remove a dielectric plate of dielectric constant k from a parallel-plate charged capacitor. It is assumed that the charge q of the capacitor remains unchanged and that the dielectric fills the entire space between the capacitor plates. The capacitance of the capacitor in the absence of the dielectric is C.

The work against the electric forces in this system is equal to the increment of the electric energy of the system :

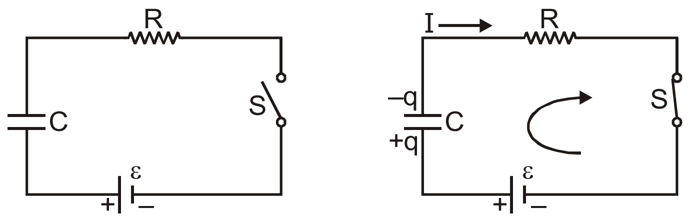
$$A = \Delta W = W_2 - W_1,$$

where W_1 is the energy of the field between the capacitor plates in the presence of the dielectric and W_2 is the same quantity in the absence of the dielectric.

$$A = W_2 - W_1 = \frac{q^2}{2C}\left(1 - \frac{1}{k}\right)$$

RC Circuts Charging of Capacitor

Consider the circuit shown below. The capacitor is connected to a DC voltage source of emf E. At time t = 0, the switch S is closed. The capacitor initially is uncharged, q (t – 0) = 0.

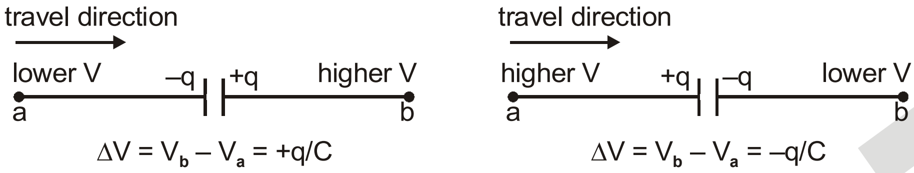


(a) RC circuit diagram for t < 0. (b) Circuit diagram for t > 0

In particular for t < 0, there is no voltage across the capacitor so the capacitor acts like a short circuit. At t = 0, the switch is closed and current begins to flow according to

$$I_e = \frac{\varepsilon}{R}$$

At this instant, the potential difference from the battery terminals is the same as that across the resistor. This initiates the charging of the capacitor. As the capacitor starts to charge, the voltage across the capacitor increases in time $V_c(t) = \frac{q(r)}{C}$



Using Kirchhoff's loop rule shown in for capacitors and traversing the loop clockwise, we obtain

$$0 = \varepsilon - I(t)\,R - V_c(t) = \varepsilon - \frac{dq}{dt}R - \frac{q}{C}$$

where we have substituted I = +dq /dt for the current. Since /must be the same in all parts of the series circuit, the current across the resistance is equal to the rate of increase of charge on the capacitor plates. The current flow in the circuit will continue to decrease because the charge already present on the capacitor makes it harder to put more charge on the capacitor. Once the charge on the capacitor plates reaches its maximum value Q, the current in the circuit will drop to zero. This is evident by rewriting the loop law as

$$I(t)R = \varepsilon - V_C(t)$$

Thus, the charging capacitor satisfies a first order differential equation that relates the rate of change of charge to the charge on the capacitor :

$$\frac{dq}{dt} = \frac{1}{R}\left(s - \frac{q}{C}\right)$$

This equation can be solved by the method of separation of variables. The first step is to separate terms involving charge and time, (this means putting terms involving dq and q on one side of the equality sign and terms involving on the other side),

$$\frac{dq}{\left(\varepsilon - \frac{q}{C}\right)} = \frac{1}{R}dt \Rightarrow \frac{dq}{q - C\varepsilon} = -\frac{1}{RC}dt$$

Now we can integrate both sides of the above equation,

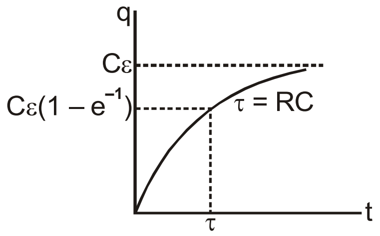
$$\int_0^q \frac{dq'}{q' - C\varepsilon} = -\frac{1}{RC}\int_0^t dt'$$

which yields $$\ln\left(\frac{q - Cs}{-Cs}\right) = -\frac{1}{RC}$$

This can now be exponentiated using the fact that exp(lnx) = x to yield

$$q(t) = Cs(1 - e^{-t/RC}) = Q\,(1 - e^{-t/RC})$$

where $Q = C\varepsilon$ is the maximum amount of charge stored on the plates. The time dependence of is plotted in figure below :



Charge as a function of time during the charging process

Once we know the charge on the capacitor we also can determine the voltage across the capacitor,

$$V_C(t) = \frac{q(t)}{C} = \varepsilon(1 - e^{-t/RC})$$

The graph of voltage as a function of time has the same form as figure. From the figure, we see that after a sufficiently long time the charge on the capacitor approaches the value.

$$q(1 - \infty) = C\varepsilon = Q$$

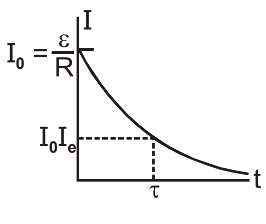
At that time, the voltage across the capacitor is equal to the applied voltage source and the charging process effectively ends,

$$V_C = \frac{q(t = \infty)}{CQ} = \frac{Q}{C} = \varepsilon$$

The current that flows in the circuit is equal to the derivative in time of the charge,

$$I(t) = \frac{dq}{dt} = \left(\frac{\varepsilon}{R}\right)e^{-t/RC} = I_0 e^{-t/RC}$$

The coefficient in front of the exponential is equal to the initial current that flows in the circuit when the switch was closed at t = 0. The graph of current as a function of time is shown in figure below :



Current as a function of time during the charging process

The current in the charging circuit decreases exponentially in time, $I(t) = I_0 e^{-t/RC}$. This function is often written as $I(t) = I_0 e^{-t/\tau}$ where $\tau = RC$ is called the time constant. The SI units of are seconds, as can be seen from the dimensional analysis :

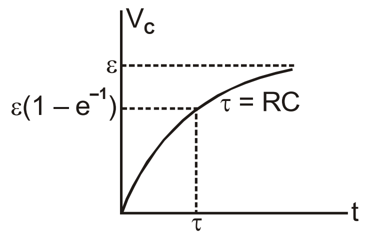
$$[\Omega]\ [F] = ([V] / [A])\ ([C] / [V]) = [C] / [A] = [C] / ([C] / [s]) = [s]$$

The time constant τ is a measure of the decay time for the exponential function. This decay rate satisfies the following property :

$$I(t + \tau) = I(t)e^{-1}$$

which shows that after one time constant has elapsed, the current falls offby a factor of, as indicated in figure above. Similarly, the voltage across the capacitor (Figure below) can also be expressed in terms of the time constant τ.

$$V_C(t) = \varepsilon(1 - e^{-t/\tau})$$



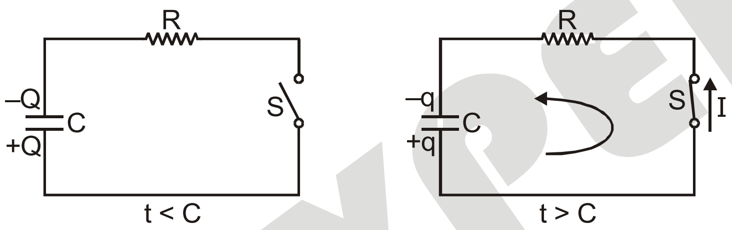
Voltage across capacitor as a function of time during the charging process

Notice that initially at time, t = 0, $V_C(t = 0) = 0$. After one time constant τ has elapsed, the potential difference across the capacitor plates has increased by a factor $(1 - e^{-1}) = 632$ of its final value :

$$V_C(\tau) = \varepsilon(1 - e^{-1}) = 0.632 \text{ s}$$

Discharging a Capacitor

Suppose initially the capacitor has been charged to some value Q. For t < 0, the switch is open and the potential difference across the capacitor is given by $V_C = Q/C$. On the other hand, the potential difference across the resistor is zero because there is no current flow, that is, I = 0. Now suppose at t = 0 the switch is closed (Figure). The capacitor will begin to discharge.



Discharging the RC circuit

The charged capacitor is now acting like a voltage source to drive current around the circuit. When the capacitor discharges (electrons flow from the negative plate through the wire to the positive plate), the voltage across the capacitor decreases. The capacitor is losing strength as a voltage source. Applying the Kirchhoff's loop rule by traversing the loop counterclockwise, the equation that describes the discharging process is given by

$$\frac{q}{C} - IR = 0$$

The current that flows away from the positive plate is proportional to the charge on the plate,

$$I = -\frac{dq}{dt}$$

The negative sign in the equation is an indication that the rate of change of the charge is proportional to the negative of the charge on the capacitor. This is due to the fact that the charge on the positive plate is decreasing as more positive charges leave the positive plate. Thus, charge satisfies a first order differential equation :

$$\frac{q}{C} + R\frac{dq}{dt} = 0$$

This equation can also be integrated by the method of separation of variables

$$\frac{dq}{q} = -\frac{1}{RC}dt$$

which yields

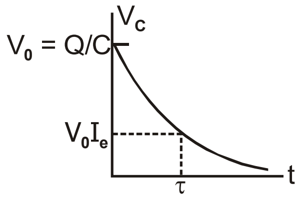
$$\int_Q^a \frac{dq'}{q'} = -\frac{1}{RC}\int_0^t dt' \Rightarrow \ln\left(\frac{q}{C}\right) = -\frac{1}{RC}$$

or $$q(t) = Qe^{-t/RC}$$

The voltage across the capacitor is then

$$V_C(t) = \frac{q(t)}{C} = \left(\frac{Q}{C}\right)e^{-t/RC}$$

A graph of voltage across the capacitor vs. time for the discharging capacitor is shown in figure.

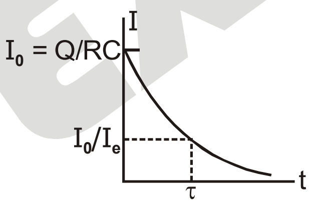


Voltage across the capacitor as a function of time for discharging capacitor

The current also exponentially decays in the circuit as can be seen by differentiating the charge on the capacitor

$$I = -\frac{dq}{dt} = \left(\frac{Q}{RC}\right)e^{-t/RC}$$

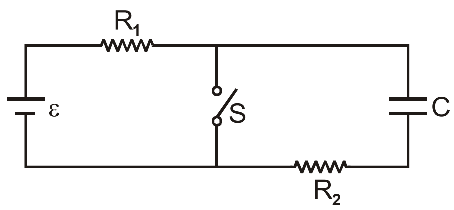
A graph of the current flowing in the circuit as a function of time also has the same form as the voltage graph depicted in figure.



Current as a function of time for discharging capacitor.

Illustration :

In the circuit in figure, suppose the switch has been open for a very long time. At time t=0, it is suddenly closed.



(a) What is the time constant before the switch is closed?

(b) What is the time constant after the switch is closed?

(c) Find the current through the switch as a function of time after the switch is closed.

Sol. (a) Before the switch is closed, the two resistors R_1 and R_2 are in series with the capacitor. Since the equivalent resistance is $R_{eq} = R_1 + R_2$, the time constant is given by

$$\tau = R_{eq}C = (R_1 + R_2)C$$

The amount of charge stored in the capacitor is

$$q(t) = C\varepsilon(1 - e^{-t/\tau})$$

(b) After the switch is closed, the closed loop on the right becomes a decaying RC circuit with time constant $t' = R_2C$. Charge begins to decay according to

$$q'(t) = C\varepsilon e^{-t/\tau'}$$

(c) The current passing through the switch consists of two sources: the steady current I_1 from the left circuit, and the decaying current I_2 from the RC circuit. The currents are given by

$$I_1 = \frac{\varepsilon}{R_1}$$

$$I'(t) = \frac{dq'}{dt} = -\left(\frac{C\varepsilon}{\tau'}\right)e^{-t/\tau'} = -\left(\frac{\varepsilon}{R_2}\right)e^{-t/R_2C}$$

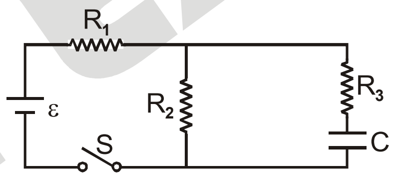
The negative sign in $I'(t)$ indicates that the direction of flow is opposite of the charging process. Thus, since both I_1 and I' move downward across the switch, the total current is

$$I(t) = I_1 + I'(t) = \frac{\varepsilon}{R_1} + \left(\frac{\varepsilon}{R_2}\right)e^{-t/R_2C}$$

Does the resistor in an RC circuit affect the maximum amount of charge that can be stored in a capacitor? Explain.

Problem

Consider the circuit shown in figure. Let $\varepsilon = 40$ V. $R_1 = 8.0\Omega$. $R_2 = 6.0\Omega$. $R_2 = 4.0\Omega$ and $C = 4.0\mu F$. The capacitor is initially uncharged.

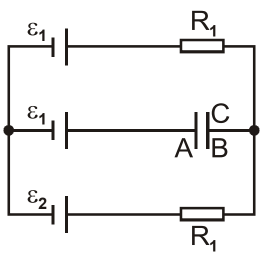


At t = 0, the which is closed.

(a) Find the current through each resistor immediately after the switch is closed.

(b) Find the final charge on the capacitor.

Ex. In the circuit shown in Fig. the sources have emf's $\xi_1 = 1.0$V and $\xi_2 = 2.5$ V the resistances have the values $R_1 = 10\Omega$ and $R_2 = 20\Omega$. The internal resistances of the sources are negligible. Find a potential difference between the plates A and B of the capacitor C.



Ans. $V_A - V_B = (\xi_1 - \xi_2)\, R_1 / (R_1 + R_2) = -0.5$ V

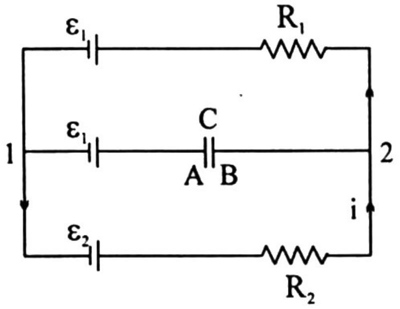
Sol. As the capacitor is fully charged, no current flows through it. So, current

$$i = \frac{\xi_2 - \xi_1}{R_1 + R_2} \qquad (\text{as } \xi_2 > \xi_1)$$

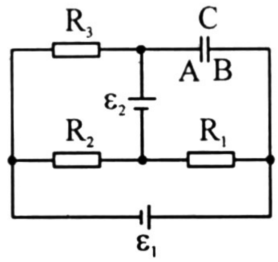
And hence, $V_A - V_B = \xi_1 - \xi_2 + iR_2$

$$= \xi_1 - \xi_2 + \frac{\xi_2 - \xi_1}{R_1 + R_2} R_2$$

$$= \frac{(\xi_1 - \xi_2)R_1}{R_1 + R_2} = -0.5\text{V}$$



Ex. Find a potential difference between the plates of a capacitor C in the circuit shown in Fig. if the sources have emf's ξ_1 = 4.0 V and ξ_2 = 1.0 V and the resistances are equal to $R_1 = 10\Omega$, $R_2 = 20\Omega$, and $R_3 = 30\Omega$. The internal resistances of the sources are negligible.



Ans. $\varphi_A - \varphi_B = [\xi_2R_3 (R_1 + R_2) - \xi_1R_1(R_2 + R_3) / (R_1R_2 + R_2R_3 + R_3R_1) = -1.0$ V

Sol. Indicate the currents in all the branches using charge conservation as shown in the figure. Applying the loop rule ($-\Delta V = 0$) in the loops 12341 and 15781, we get

$$-\xi_1 + i_3R_1 - (i_1 - i_3)R_2 = 0 \qquad .....(1)$$

and $\quad (i_2 - i_3)R_2 - \xi_2 + i_1R_3 = 0 \qquad .....(2)$

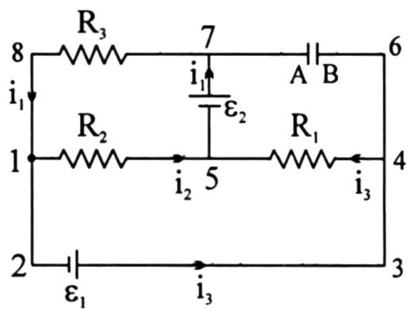
Solving eqs. (1) and (2), we get

$$i_3 = \frac{\xi_1(R_2 + R_3) + \xi_2R_2}{R_1R_2 + R_2 + R_3 + R_3R_1}$$

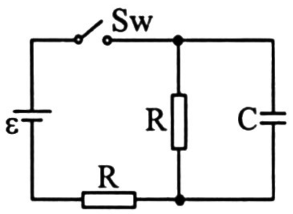
Hence, the sought p.d.

$$V_A - V_B = \xi_2 - i_3R_1$$

$$= \frac{\xi_2R_3(R_1 + R_2) - \xi_1(R_2 + R_3)}{R_1R_2 + R_2 + R_3 + R_3R_1} = -1\text{V}$$



Ex. Find how the voltage across the capacitor C varies with time t (Fig.) after the shorting of the switch Sw at the moment t = 0.



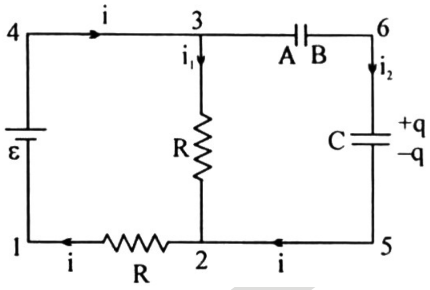
Ans. $V = 1/2\xi\,(1 - e^{-2t/RC})$

Sol. Let, at any moment of time, charge on the plates be +q and –q respectively, then voltage across the capacitor, $\varphi = q/C$ ....(1)

Now, from charge conservation,

$$i + i_1 + i_2 \text{ where } i_2 = \frac{dq}{dt} \quad ....(2)$$

In the loop 65146, using $-\Delta\varphi = 0$



$$\frac{q}{C} + \left(i_1 + \frac{dq}{dt}\right)R - \xi = 0 \quad .....(3)$$

(using (1) and (2))

In the loop 25632, using $-\Delta V = 0$

$$-\frac{q}{C} + i_1R = 0 \text{ or, } i_1R = \frac{q}{C} \quad .....(4)$$

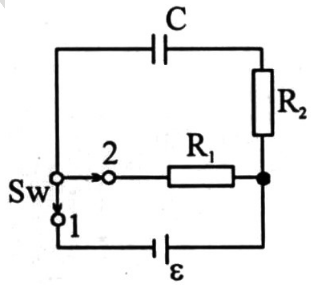
From (1) and (2)

$$\frac{dq}{dt}R = \xi_1 - \frac{2q}{C} \text{ or } \frac{dq}{\xi - \frac{2q}{C}} = \frac{dt}{R} \quad ....(5)$$

On integrating the expression (5) between suitable limits,

$$\int_0^q \frac{dq}{\xi - \frac{2q}{C}} = \frac{1}{R}\int_0^t dt \text{ or } -\frac{C}{2}\ln\frac{\xi - \frac{2q}{C}}{\xi} = \frac{t}{R}. \text{ Thus, } \quad \frac{q}{C} = V = \frac{1}{2}\xi(1 - e^{-2t/RC})$$

Ex. A capacitor of capacitance C = 5.00 μF is connected to a source of constant emf ξ = 200 V (Fig.). Then the switch Sw was thrown over from contact 1 to contact 2. Find the amount of heat generated in a resistance $R_1 = 500\Omega$ if $R_2 = 330\Omega$.

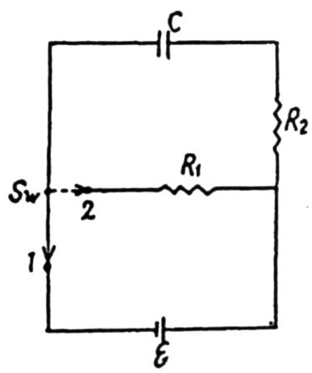


Ans. $Q = 1/2C\xi^2R_1/(R_1 + R_2) = 60$ mJ

Sol. When switch 1 is closed, maximum charge accumulated on the capacitor, $q_{max} = C\xi$ ......(1)

and when switch 2 is closed, at any arbitrary instant of time,

$$(R_1 + R_2)\left(-\frac{dq}{dt}\right) = \frac{q}{C}$$



because capacitor is discharging,

$$\text{or} \quad \int_{q_{max}}^{q} \frac{1}{q}dq = -\frac{1}{(R_1 + R_2)}\int_0^t dt$$

Integrating, we get

$$\ln\frac{q}{q_{max}} = \frac{-t}{(R_1 + R_2)} \text{ or, } q = q_{max}e^{\frac{-t}{(R_1+R_2)C}}$$

Differentiating with respect to time,

$$i(t)=\frac{dq}{dt}=q_{max}e^{\frac{-t}{(R_1+R_2)C}}\left(-\frac{1}{(R_1+R_2)C}\right)$$

or $$i(t)=\frac{C\xi}{(R_1+R_2)C}e^{\frac{-t}{(R_1+R_2)C}}$$

Negative sign is ignored, as we are not interested in the direction of the current.

thus, $$i(t)=\frac{\xi}{(R_1+R_2)}e^{\frac{-t}{(R_1+R_2)C}} \qquad ......(3)$$

When the switch (Sw) is at the position 1, the charge (maximum) accumulated on the capacitor is,

$q = C\xi$

When the Sw is thrown to position 2, the capacitor starts discharging and as a result the electric energy stored in the capacitor totally turns into heat energy tho' the resistors R_1 and R_2 (during a very long interval of time). Thus from the energy conservation, the total heat liberated tho* the resistors.

$$H=U_i=\frac{q^2}{2C}=\frac{1}{2}C\xi^2$$

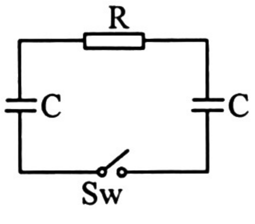
During the process of discharging of the capacitor, the current tho' the resistors R_1 and R_2 is the same at all the moments of time, thus

$H_1 \propto R_1$ and $H_2 \propto R_2$

So, $H_1=\frac{HR_1}{(R_1+R_2)}$ (as $H = H_1 + H_2$)

Hence $H_1=\frac{1}{2}\frac{CR_1}{R_1+R_2}\xi^2$

Ex. In a circuit shown in Fig. the capacitance of each capacitor is equal to C and the resistance, to R. One of the capacitors was connected to a voltage V_0 and then at the moment t = 0 was shorted by means of the switch Sw. Find :



(a) a current I in the circuit as a function of time t;

(b) the amount of generated heat provided a dependence I (t) is known.

Ans. (a) $I = (V_0/R)e^{-2t/RC}$; (b) $Q = 1/4\ CV_0^2$

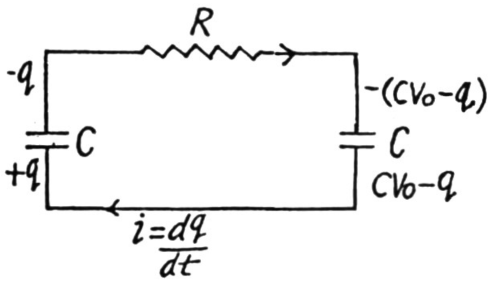
Sol. Let, at any moment of time, charge flown be q then current i = $\frac{dq}{dt}$

Applying loop rule in the circuit, $-\Delta V = 0$, we get :

$$\frac{dq}{dt}IR-\frac{(CV_0-q)}{C}+\frac{q}{C}=0$$

or, $\dfrac{dq}{CV_0 - 2q} = \dfrac{1}{RC}dt$

So, $\ln\dfrac{CV_0 - 2q}{CV_0} = -2\dfrac{1}{RC}$ for $0 \geq t \geq t$



or, $q = \dfrac{CV_0}{2}\left(1 - e^{\frac{-2t}{RC}}\right)$

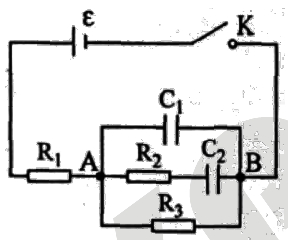
Hence, $i = \dfrac{dq}{dt} = \dfrac{CV_0}{2}\dfrac{2}{RC}e^{-2t/RC} = \dfrac{V_0}{R}e^{-2t/RC}$

Now, heat liberated,

$$Q = \int_0^{\infty} t^2 R dt = \frac{V_0^2}{R^2}R\int_0^{\infty} e^{\frac{-4t}{RC}}dt = \frac{1}{4}CV_0^2$$

Problem

Determine the current through the battery in the circuit shown in figure.

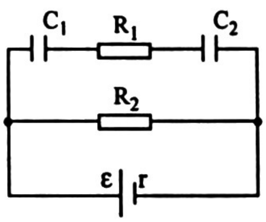


(i) immediately after the key K is closed and

(ii) in a long time interval, assuming that the parameters of the circuit are known.

Problem

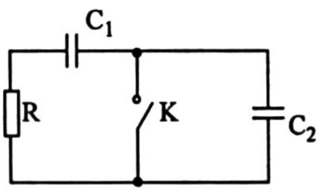
A circuit consists of a current source of emf ε and internal resistance r, capacitors of capacitance C_1 and C_2, and resistors of rsistance R_1 and R_2.



Determine the voltages U_1 and U_2 across each capacitor.

Problem

A capacitor of capacitance C_1 is discharged through a resistor of resistance R. When the discharge current attains the value I_0, the key K is opened (Figure).



Determine the amount of heat Q liberated in the resistor starting from this moment.

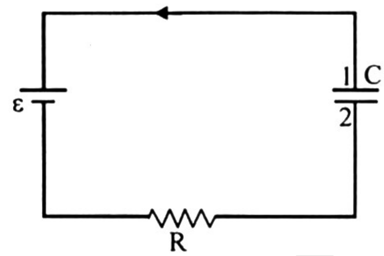
Ex. **Transient processes :** A circuit consists of a permanent source of e.m.f. E, and a resistor R and capacitor C connected in series. The internal resistance of the source is negligibly small. At the moment t = 0, the capacitance of the capacitor was abruptly (jump-wise) decreased by a factor of η. Find the current in the circuit as a function of time.

Sol. We write Ohm's law for the inhomogeneous part $1\varepsilon R^2$ of the circuit (figure).

$$RI = V_1 - V_2 - \varepsilon = U - \varepsilon$$

Considering that U = q/C', where C' = C/η, we obtain

$$RI = \eta q/C - \varepsilon$$



We differentiate this equation with respect to time, considering that in our case (q-decreases) dq/dt = –I :

$$R\frac{dI}{dt} = -\frac{\eta}{C}I, \frac{dI}{I} = -\frac{\eta}{RC}dt$$

Integration of this equation gives

$$\ln\frac{I}{I_0} = -\frac{\eta t}{RC}, \quad I = I_0 e^{-\eta t/RC},$$

Where I_0 is determined by condition (1). Indeed, we can write

$$RI_0 = \varepsilon q_0 / C - \varepsilon,$$

where $q_0 = \varepsilon C$ is the charge of the capacitor before its capacitance before its capacitance has changed. Therefore,

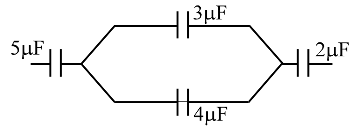
$$I_0 = (\eta - 1)\varepsilon/R$$

EXERCISE-I

ONLY ONE OPTION IS CORRECT

Take approx. 2 minutes for answering each question.

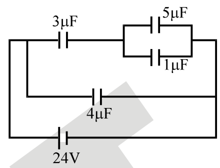
1. If charge on left plane of the $5\mu F$ capacitor in the circuit segment shown in the figure is $-20\mu C$, the charge on the right plate of $3\mu F$ capacitor is



(A) $+8.57\ \mu C$ (B) $-8.57\ \mu C$

(C) $+11.42\ \mu C$ (D) $-11.42\ \mu C$

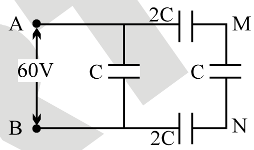
2. In the circuit shown, the energy stored in $1\mu F$ capacitor is



(A) $40\ \mu J$ (B) $64\ \mu J$

(C) $32\ \mu J$ (D) none

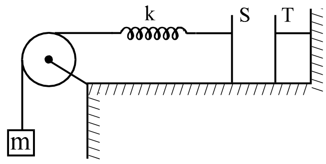
3. In the circuit shown, a potential difference of 60V is applied across AB. The potential difference between the point M and N is



(A) 10 V (B) 15 V

(C) 20 V (D) 30 V

4. The plates S and T of an uncharged parallel plate capacitor are connected across a battery. The battery is then disconnected and the charged plates are now connected in a system as shown in the figure. The system shown is in equilibrium. All the strings are insulating and massless. The magnitude of charge on one of the capacitor plates is: [Area of plates = A]



(A) $\sqrt{2mgA\in_0}$ (B) $\sqrt{\dfrac{4mgA\in_0}{k}}$

(C) $\sqrt{mgA\in_0}$ (D) $\sqrt{\dfrac{2mgA\in_0}{k}}$

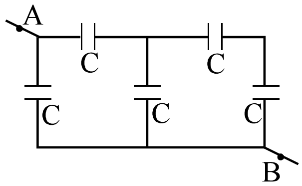
5. From a supply of identical capacitors rated $8\ \mu F$, 250 V, the minimum number of capacitors required to form a composite $16\ \mu F$, 1000 V is :

(A) 2 (B) 4 (C) 16 (D) 32

6. Two capacitor having capacitances $8\ \mu F$ and $16\ \mu F$ have breaking voltages 20 V and 80 V. They are combined in series. The maximum charge they can store individually in the combination is

(A) $160\ \mu C$ (B) $200\ \mu C$ (C) $1280\ \mu C$ (D) none of these

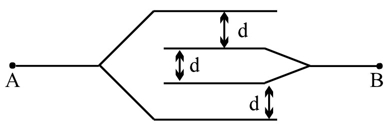
7. What is the equivalent capacitance of the system of capacitors between A & B



(A) $\dfrac{7}{6}C$ (B) 1.6 C

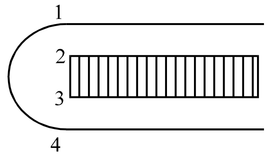
(C) C (D) None

8. Four metallic plates arearranged as shown in the figure. If the distance between each plate then capacitance of the given system between points A and B is (Given d << A)



(A) $\dfrac{\varepsilon_0 A}{d}$ (B) $\dfrac{2\varepsilon_0 A}{d}$ (C) $\dfrac{3\varepsilon_0 A}{d}$ (D) $\dfrac{4\varepsilon_0 A}{d}$

9. Four identical plates 1, 2, 3 and 4 are placed parallel to each other at equal distance as shown in the figure. Plates 1 and 4 are joined together and the space between 2 and 3 is filled with a dielectric of dielectric constant k = 2. The capacitance of the system between 1 and 3 & 2 and 4 are C_1 and C_2 respectively. The ratio C_1/C_2 is :

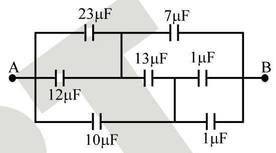


(A) $\frac{5}{3}$ (B) 1 (C) $\frac{3}{5}$ (D) $\frac{5}{7}$

10. A capacitor of capacitance C is initially charged to a potential difference of V volt. Now it is connected to a battery of 2V Volt with opposite polarity. The ratio of heat generated to the final energy stored in the capacitor will be

(A) 1.75 (B) 2.25 (C) 2.5 (D) 1/2

11. Find the equivalent capacitance across A & B



(A) $\frac{28}{3}$ μf (B) $\frac{15}{2}$ μF

(C) 15 μF (D) none

12. A parallel plate capacitor has an electric field of 10^5V/m between the plates. If the charge on the capacitor plate is 1μC, then the force on each capacitor plate is

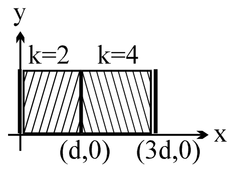
(A) 0.1 Nt (B) 0.05 Nt (C) 0.02 Nt (D) 0.01 Nt

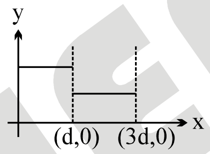
13. A capacitor is connected to a battery. The force of attraction between the plates when the separation between them is halved

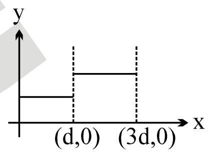
(A) remains the same (B) becomes eight times

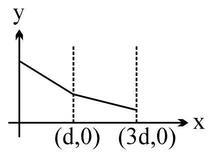
(C) becomes four times (D) becomes two times

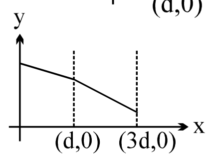
14. A parallel plate capacitor has two layers of dielectric as shown in figure. This capacitor is connected across a battery. The graph which shows the variation of electric field (E) and distance (x) from left plate.



(A) 

(B) 

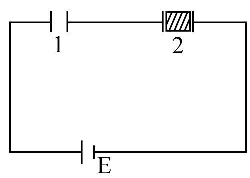
(C) 

(D) 

15. A capacitor stores 60μC charge when connected across a battery. When the gap between the plates is filled with a dielectric , a charge of 120μC flows through the battery. The dielectric constant of the material inserted is :

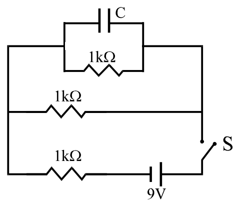
(A) 1 (B) 2 (C) 3 (D) none

16. Two identical capacitors 1 and 2 are connected in series to a battery as shown in figure. Capacitor 2 contains a dielectric slab of dielectric constant k as shown. Q_1 and Q_2 are the charges stored in the capacitors. Now the dielectric slab is removed and the corresponding charges are Q'_1 and Q'_2.
Then



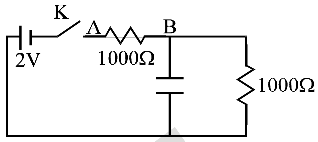
(A) $\frac{Q_1'}{Q_1}=\frac{k+1}{k}$ (B) $\frac{Q_2'}{Q_2}=\frac{k+1}{2}$ (C) $\frac{Q_2'}{Q_2}=\frac{k+1}{2k}$ (D) $\frac{Q_1'}{Q_1}=\frac{k}{2}$

17. A capacitor C = 100 μF is connected to three resistor each of resistance 1 kΩ and a battery of emf 9V. The switch S has been closed for long time so as to charge the capacitor. When switch S is opened, the capacitor discharges with time constant



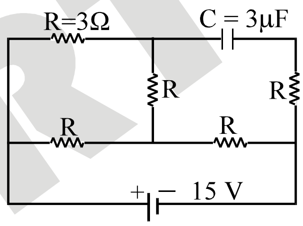
(A) 33 ms (B) 5 ms
(C) 3.3 ms (D) 50 ms

18. In the circuit shown, when the key k is pressed at time t = 0, which of the following statements about current I in the resistor AB is true



(A) I = 2mA at all t
(B) I oscillates between 1 mA and 2mA
(C) I = 1 mA at all t
(D) At t = 0, I = 2mA and with time it goes to 1 mA

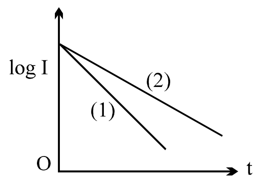
19. In the circuit shown, the cell is ideal, with emf = 15 V. Each resistance is of 3Ω. The potential difference across the capacitor is



(A) zero
(B) 9 V
(C) 12 V
(D) 15 V

20. Two capacitors C_1 and C_2 are connected in series, assume that $C_1 < C_2$. The equivalent capacitance of this arrangement is C, where
(A) $C < C_1/2$ (B) $C_1/2 < C < C_1$ (C) $C_1 < C < C_2$ (D) $C_2 < C < 2C_2$

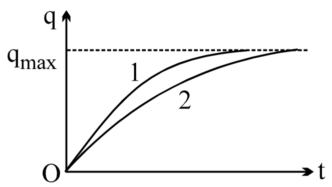
21. A capacitor of capacity C is charged to a steady potential difference V and connected in series with an open key and a pure resistor 'R'. At time t = 0, the key is closed. If I = current at time t, a plot of log I against 't' is as shown in (1) in the graph. Later one of the parameters i.e. V, R or C is changed keeping the other two constant, and the graph (2) is recorded. Then



(A) C is reduced (B) C is increased (C) R is reduced (D) R is increased

Question No.22 to 23 (2 questions)

The charge across the capacitor in two different RC circuits 1 and 2 are plotted as shown in figure.



22. Choose the correct statement(s) related to the two circuits.
(A) Both the capacitors are charged to the same charge.
(B) The emf's of cells in both the circuit are equal.
(C) The emf's of the cells may be different.
(D) The emf E_1 is more than E_2

23. Identify the correct statement(s) related to the R_1, R_2, C_1 and C_2 of the two RC circuits.
(A) $R_1 > R_2$ if $E_1 = E_2$ (B) $C_1 < C_2$ if $E_1 = E_2$ (C) $R_1C_1 > R_2C_2$ (D) $\frac{R_1}{R_2} < \frac{C_2}{C_1}$

24. A parallel plate capacitor of plate area A and plate seperation d is charged to potential difference V and then the battery is disconnected. A slab of dielectric constant K is then inserted between the plates of the capacitor so as to fill the space between the plates. If Q, E and W denote respectively, the magnitude of charge on each plate, the electric field between the plates (after the slab is inserted) and the work done on the system, in question, in the process of inserting the slab, then

(A) Q = $\dfrac{\varepsilon_0 AV}{d}$ (B) Q = $\dfrac{\varepsilon_0 KAV}{d}$ (C) E = $\dfrac{V}{Kd}$ (D) W = $-\dfrac{\varepsilon_0 AV^2}{2d}\left(1-\dfrac{1}{K}\right)$

25. A parallel-plate capacitor is connected to a cell. Its positive plate A and its negative plate B have charges +Q and –Q respectively. A third plate C, identical to A and B, with charge +Q, is now introduced midway between A and B, parallel to them. Which of the following are correct?

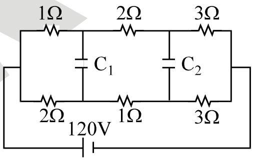
(A) The charge on the inner face of B is now $-\dfrac{3Q}{2}$

(B) There is no change in the potential difference between A and B.

(C) The potential difference between A and C is one-third of the potential difference between B and C.

(D) The charge on the inner face of A is now Q/2.

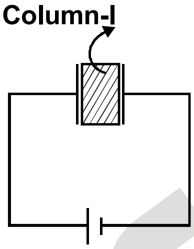
26. In the circuit shown in figure $C_1 = C_2 = 2\mu F$. Then charge stored in



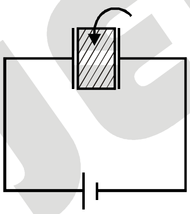
(A) capacitor C_1 is zero (B) capacitor C_2 is zero

(C) both capacitor is zero (D) capacitor C_1 is 40 μC

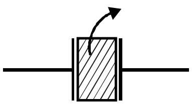
27. **Column-I** **Column-II**

(A)

Removal of dielectric when battery is present

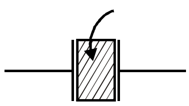
(P) Potential difference between plates increases.

(B)

Insertion of dielectric when battery is present

(Q) Capacitance increases

(C)

Removal of dielectric when battery is not present

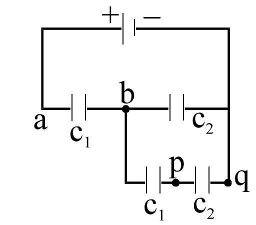
(R) Stored energy increases

(D)

Insertion of dielectric when battery is not present

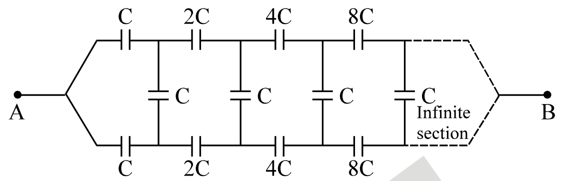
(S) Charge present on plates decreases

EXERCISE-II

1. In the given network if potential difference between p and q is 2V and $C_2 = 3C_1$. Then find the potential difference between a & b.

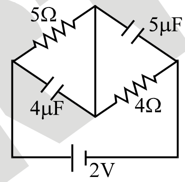


2. Find the equivalent capacitance of the circuit between point A and B.

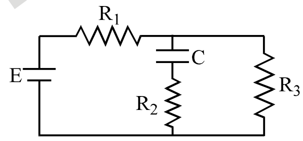


3. The plates of a parallel plate capacitor are given charges +4Q and –2Q. The capacitor is then connected across an uncharged capacitor of same capacitance as first one (= C). Find the final potential difference between the plates of the first capacitor.

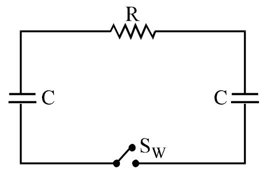
4. Find the ratio between the energy stored in 5 μF capacitor to the 4 μF capacitor in the given circuit in steady state.



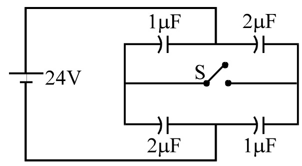
5. In the circuit shown here, at the steady state, the charge on the capacitor is ____.



6. In the circuit shown in figure the capacitance of each capacitor is equal to C and resistance R. One of the capacitors was charge to a voltage V_0 and then at the moment t = 0 was shorted by means of the switch S. Find:
(a) the current in the circuit as a function of time t.
(b) the amount of generated heat.



7. The connections shown in figure are established with the switch S open. How much charge will flow through the switch if it is closed?

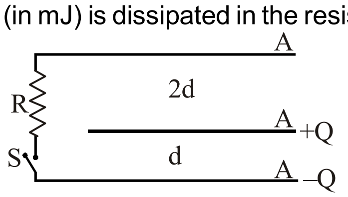


8. A parallel plate capacitor has plates with area A & separation d . A battery charges the plates to a potential difference of V_0. The battery is then disconnected & a di-electric slab of constant K & thickness d is introduced. Calculate the positive work done by the system (capacitor + slab) on the man who introduces the slab.

9. Two square metallic plates of 1 m side are kept 0.01 m apart, like a parallel plate capacitor, in air in such a way that one of their edges is perpendicular, to an oil surface in a tank filled with an insulating oil. The plates are connected to a battery of e.m.f. 500 volt . The plates are then lowered vertically into the oil at a speed of 0.001 m/s. Calculate the current drawn from the battery during the process.
[di-electric constant of oil = 11, $\epsilon_0 = 8.85 \times 10^{-12}$ $C^2/N^2\,m^2$]

10. Three identical large metal plates of area A are small at distances d and 2d from each other. Top metal plate is uncharged, while other metal plates have charges +Q and –Q. Top and bottom metal plates are connected by switch S through a resistor of unknown resistance. What energy (in mJ) is dissipated in the resistor when switch is closed ?

(Given : $\dfrac{\epsilon_0 A}{d}$ = 6μF , Q = 60μC)



EXERCISE-III

OLD AIEEE QUESTIONS

1. If there are n capacitores in parallel connected to V volt source, then the energy stored is equal to

(A) CV (B) $\frac{1}{2}nCV^2$ (C) CV^2 (D) $\frac{1}{2n}CV^2$ **[AIEEE-2002]**

2. Capacitance (in F) of a spherical conductor with radius 1 m is **[AIEEE-2002]**

(A) 1.1×10^{-10} (B) 10^{-6} (C) 9×10^{-9} (D) 10^{-3}

3. A parallel plate capacitor is made by stacking n equally spaced plates connected alternatively. If the capacitance between any two adjacent plates is C then the resultant capacitance is **[AIEEE-2005]**

(A) (n – 1)C (B) (n + 1)C (C) C (D) nC

4. A battery is used to charge a parallel plate capacitor till the potential difference between the plates becomes equal to the electromotive force of the battery. The ratio of the energy stored in the capacitor and the work done by the battery will be **[AIEEE-2007]**

(A) 1 (B) 2 (C) 1/4 (D) 1/2

5. A parallel plate condenser with a dielectric of dielectric constant K between the plates has a capacity C and is charged to a potential V volts. The delectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is : **[AIEEE-2007]**

(A) $\frac{1}{2}(K-1)CV^2$ (B) $CV^2(K-1)/K$ (C) $(K-1)CV^2$ (D) zero

6. A parallel plate capacitor with air between the plates has a capacitance of 9 pF. The separation between its plates is 'd'. The space between the plates is now filled with two dielectrics. One of the dielectrics has dielectric constant $k_1 = 3$ and thickness $\frac{d}{3}$ while the other one has dielectric constant $k_2 = 6$ and thickness $\frac{2d}{3}$. Capacitance of the capacitor is now **[AIEEE-2008]**

(A) 1.8 pF (B) 45 pF (C) 40.5 pF (D) 20.25 pF

7. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1/t_2 will be **[AIEEE-2010]**

(A) 1/4 (B) 2 (C) 1 (D) /2

8. A resistor 'R' and 2μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5s after the switch has been closed. ($\log_{10} 2.5 = 0.4$) **[AIEEE-2011]**

(A) $1.3 \times 10^4\Omega$ (B) $1.7 \times 10^5\Omega$ (C) $2.7 \times 10^6\Omega$ (D) $3.3 \times 10^7\Omega$

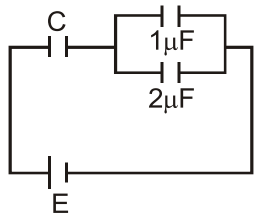
9. Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zero. Then : **[JEE (Mains)-2013]**

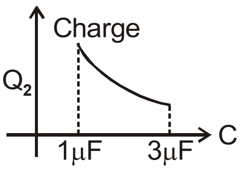
(A) $9C_1 = 4C_2$ (B) $5C_1 = 3C_2$ (C) $3C_1 = 5C_2$ (D) $3C_1 + 5C_2 = 0$

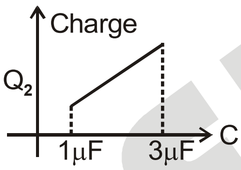
10. A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to : **[JEE (Mains)-2014]**

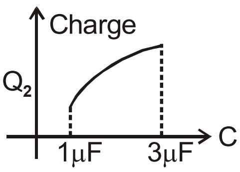
(A) 3×10^{-7} C/m^2 (B) 3×10^4 C/m^2 (C) 6×10^4 C/m^2 (D) 6×10^{-7} C/m^2

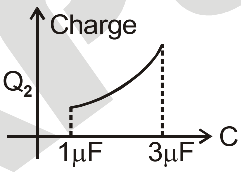
11. In the given circuit, charge Q_2 on the 2μF capacitor changes as C is varied from 1μF to 3μF. Q_2 as a function of 'C' is given peroperly by : (figures are drawn schematically and are not to scale). **[JEE Mains-2015]**



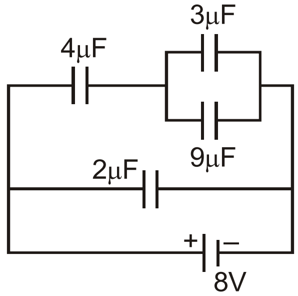
(A)


(B)


(C)


(D)


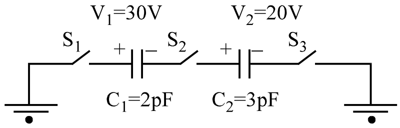
12. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4 μF and 9 μF capacitors), at a point distant 30 m from it, would equal **[JEE Mains-2016]**



(A) 240 N/C (B) 360 N/C (C) 420 N/C (D) 480 N/C

OLD IIT-JEE QUESTIONS

1. For the circuit shown, which of the following statements is true ? **[JEE-1999]**



(A) with S_1 closed, V_1 = 15 V, V_2 = 20 V (B) with S_3 closed, $V_1 = V_2$ = 25 V

(C) with S_1 & S_2 closed, $V_1 = V_2$ = 0 (D) with S_1 & S_2 closed, V_1 = 30 V, V_2 = 20 V

2. Calculate the capacitance of a parallel plate condenser, with plate area A and distance between plates d, when filled with a medium whose permittivity varies as ; **[REE-2000]**

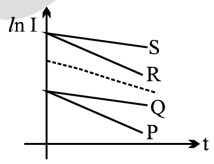
$\in(x) = \in_0 + \beta x \qquad 0 < x < \frac{d}{2}$

$\in(x) = \in_0 + \beta\,(d - x) \qquad \frac{d}{2} < x < d$.

3. Two identical capacitors, have the same capacitance C. One of them is charged to potential V_1 and the other to V_2. The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the combined system is **[JEE-2002 (Scr)]**

(A) $\frac{1}{4}C\left(V_1^2 - V_2^2\right)$ (B) $\frac{1}{4}C\left(V_1^2 + V_2^2\right)$ (C) $\frac{1}{4}C\left(V_1 - V_2\right)^2$ (D) $\frac{1}{4}C\left(V_1 + V_2\right)^2$

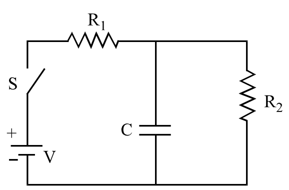
4. In an RC circuit while charging, the graph of *l*n I versus time is as shown by the dotted line in the adjoining diagram where I is the current. When the value of the resistance is doubled, which of the solid curves best represents the variation of *l*n I versus time? **[JEE-2004 (Scr)]**



(A) P (B) Q (C) R (D) S

5. In the given circuit, the switch S is closed at time t = 0. The charge Q on the capacitor at any instant t is given by $Q(t) = Q_0(1 - e^{-\alpha t})$.

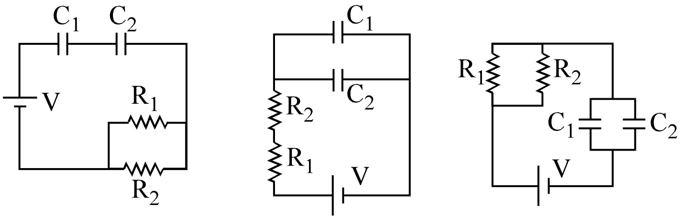
Find the value of Q_0 and α in terms of given parameters shown in the circuit. **[JEE 2005]**



6. An uncharged capacitor of capacitance 4μF, a battery of emf 12 volt and a resistor of 2.5 MΩ are connected in series. The time after which $v_c = 3v_R$ is (take *l*n2 = 0.693) **[JEE' 2005 (Scr)]**

(A) 6.93 sec. (B) 13.86 sec. (C) 20.52 sec. (D) none of these

7. Given : $R_1 = 1\Omega$, $R_2 = 2\Omega$, $C_1 = 2\mu F$, $C_2 = 4\mu F$. The time constants (in μS) for the circuits I, II, III are respectively **[JEE 2006]**



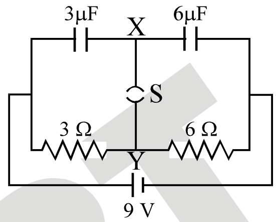
(I) (II) (III)

(A) 18, 8/9, 4 (B) 18, 4, 8/9 (C) 4, 8/9, 18 (D) 8/9, 18, 4

8. A circuit is connected as shown in the figure with the switch S open. When the switch is closed, the total amount of charge that flows from Y to X is **[JEE 2007]**

(A) 0 (B) 54 μC

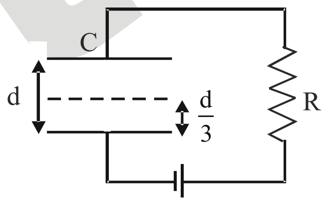
(C) 27 μC (D) 81 μC



9. A parallel plate capacitor C with plates of unit area and separation d is filled with a liquid of dielectric constant K = 2. The level of liquid is d/3 initially. Suppose the liquid level decreases at a constant speed V, the time constant as a function of time t is **[JEE 2008]**

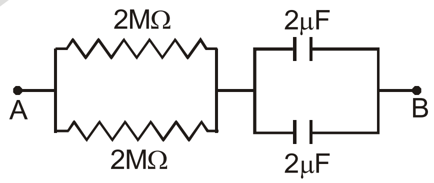
(A) $\dfrac{6\varepsilon_0 R}{5d + 3Vt}$ (B) $\dfrac{(15d + 9Vt)\varepsilon_0 R}{2d^2 - 3dVt - 9V^2t^2}$

(C) $\dfrac{6\varepsilon_0 R}{5d - 3Vt}$ (D) $\dfrac{(15d - 9Vt)\varepsilon_0 R}{2d^2 + 3dVt - 9V^2t^2}$



10. At time t = 0, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in seconds) does the voltage across them becomes 4 V ?

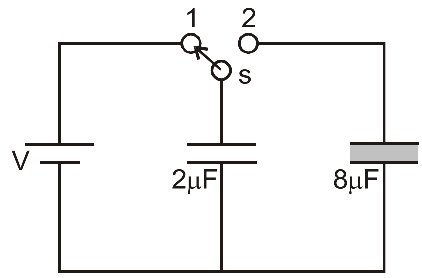
[take : ln 5 = 1.6, ln 3 = 1.1] **[JEE 2010]**



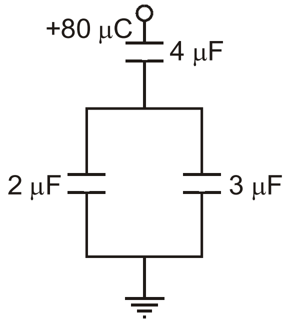
11. A 2 μF capacitor is charged as shown in figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is :

(A) 0% (B) 20% **[JEE - 2011]**

(C) 75% (D) 80%



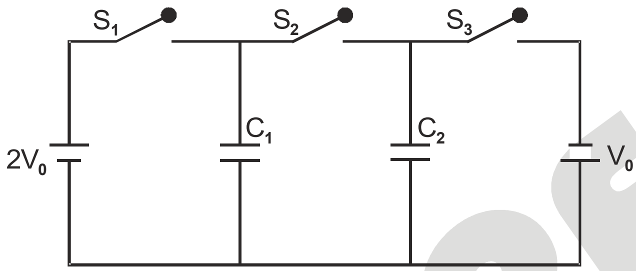
12. In the given circuit, a charge of +80 μC is given to the upper plate of the 4 μF capacitor. Then in the steady state, the charge on the upper plate of the 3 μF capacitor is **[JEE - 2012]**



(A) +32 μC (B) +40 μC

(C) +48 μC (D) +80 μC

13. In the circuit shown in the figure, there are two parallel plate capacitors ______ of capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After same time, S_2 is released and then S_3 is pressed. After same time

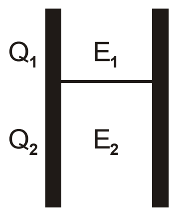


[JEE (Adv.)-2013]

(A) The charge on the upper plate of C_1 is $2CV_0$ (B) The charge on the upper plate of C_1 is CV_0

(C) The charge on the upper plate of C_2 is 0 (D) The charge on the upper plate of C_2 is $-CV_0$

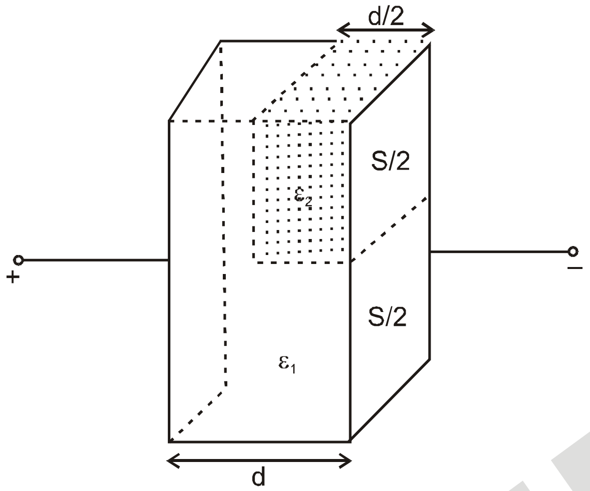
14. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options, ignoring edge effects **[JEE (Adv.)-2014]**



(A) $\frac{E_1}{E_2}=1$ (B) $\frac{E_1}{E_2}=\frac{1}{K}$

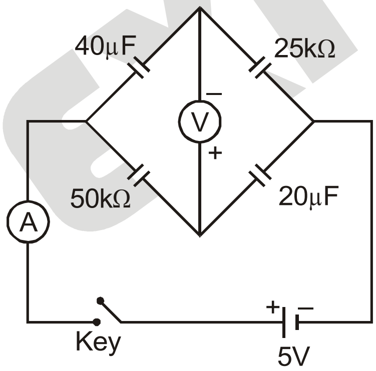
(C) $\frac{Q_1}{Q_2}=\frac{3}{K}$ (D) $\frac{C}{C_1}=\frac{2+K}{K}$

15. A parallel plate capacitor having plates of area S and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative permittivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2. The ratio $\frac{C_2}{C_1}$ is **[JEE Advance-2015]**



(A) 6/5 (B) 5/3 (C) 7/5 (D) 7/3

16. In the circuit shown below, the key is pressed at time t = 0. Which of the following statement(s) is(are) true?

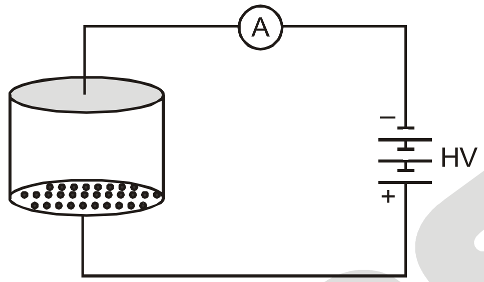


(A) The voltmeter displays –5 V as soon as the key is pressed, and displays +5 V after a long time.
(B) The voltmeter will display 0 V at time t = ln 2 seconds. **[JEE Advance-2016]**
(C) The current in the ammeter becomes 1/e of the initial value after 1 second.
(D) The current in the ammeter becomes zero after a long time.

Paragraph-2 (Q.17 to Q.18)

Consider an evacuated cylindrical chamber of height h having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a light weight and soft material and coated with a conducting material are placed on the bottom plate. The balls have a radius r << h. Now a high voltage source (HV) is connected across the conducting plates such that the bottom, plate is at $+V_0$ and the top plate at $-V_0$. Due to their conducting surface, the balls will get charged, will become equipotential with the plate and are repelled by it. The balls will eventaully collide with the top plate, where the coefficient of restitution can be taken to be zero due to the soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collisions between the balls and the interaction between them is negligible. (Ignore gravity)

[JEE Advance-2016]



17. Which one of the following statements is correct?

(A) The balls will bounce back to the bottom plate carrying the opposite charge they went up with

(B) The balls will execute simple harmonic motion between the two plates

(C) The balls will bounce back to the bottom plate carrying the same charge they went up with

(D) The balls will stick to the top plate and remain there

18. The average current in the steady state registered by the ammeter in the circuit will be

(A) Proportional to $V_0^{1/2}$ (B) Proportional to V_0^2

(C) Proportional to the potential V_0 (D) Zero

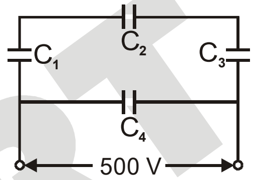
EXERCISE-IV

BOARD QUESTIONS

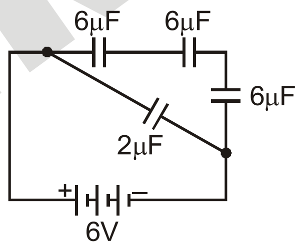
1. A parallel plate capcitor is charged by a battery. After sometime the battery is disconnected and a dielectric slab with its thickness equal to the plate separation is inserted between the plates. How will (i) the capacitance of the capacitor, (ii) potential difference between the plates and (ii) the energy stored in the capacitor be affected? **[CBSE 2010]**
Justify your answer in each case.

2. A parallel plate capacitor is charged to a potential difference V by a dc source. The capacitor is then disconnected from the source. If the distance between the plates is doubled, state with reason how the following will change; **[CBSE 2010]**
(i) electric field between the plates, (ii) capacitance, and
(iii) energy stored in the capacitor.

3. A network of four capacitors each of 12 μF capacitance is connected to a 500 V supply as shown in the figure. Determine (a) equivalent capacitance of the network and (b) charge on each capacitor. **[CBSE 2010]**



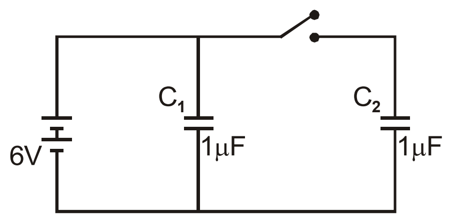
4. Four capacitors of values 6 μF, 6 μF, 6 μF and 2 μF, are connected to a 6V battery as shown in the figure. Determine the
(i) equivalent capacitance of the network.
(ii) the charge on each capacitor. **[CBSE 2010]**



5. Net capacitance of three identical capacitors in series is 1μF. What will be their net capacitance of connected in parallel? **[CBSE 2011]**

Find the ratio of energy stored in the two configurations if they are both connected to the same source.

6. Figure shows two identical capacitors, C_1 and C_2, each of 1 μF capacitance connected to a battery of 6V. Initially switch 'S' is closed. After sometime 'S' is left open and dielectric slabs of dielectric constant K = 3 are inserted to fill completely the space between the plates of the two capacitors. How will the (i) charge and (ii) potential difference between the plates of the capacitors be affected after the slabs are inserted?



[CBSE 2011]

7. Deduce the expression for the energy stored in a parallel plate capacitor C having charges +Q and –Q on its plates. **[CBSE 2011]**

8. Define the dielectric constant of a medium. What is its unit? **[CBSE 2011]**

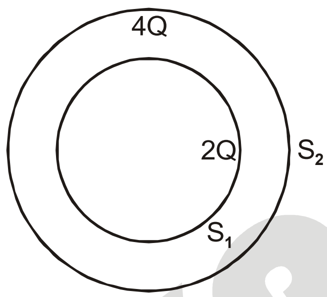
9. (a) A parallel plate capacitor is charged by a battery to a potential. The battery is disconnected and a dielectric slab is inserted to completely fill the space between the plates. How will (i) its capacitance, (ii) electric field between the plates and (iii) energy stored in the capacitor be affected? Justify your answer giving necessary mathematical expressions for each case.

(b) Sketch the pattern of electric field lines due to (i) a conducting sphere having negative charge on it, (ii) an electric dipole. **[CBSE 2011]**

10. A slab of material of dielectric constant K has the same area as that of the plates of a parallel plate capacitor but has the thickness 2d/3, where d is the separation between the plates. Find out the expression for its capacitance when the slab is inserted between theplates of the capacitor. **[CBSE 2013]**

11. A parallel plate capacitor of capacitance C is charged to a potential V. It is then connected to another uncharged capacitor having the same capacitance. Find out the ratio of the energy stored in the combined system to that stored initially in the single capacitor. **[CBSE 2014]**

12. Consider two hollow concentric spheres, S_1 and S_2, enclosing charges 2Q and 4Q respectively as shown in the figure. (i) Find out the ratio of the electric flux through them. (ii) How will the electric flux through the sphere S_1 change if a medium of dielectric constant 'ε_r' is introduced in the space inside S_1 in place of air? Deduce the necessary expression. **[CBSE 2014]**



13. Define dielectric constant of a medium. What is its S.I. unit ? **[CBSE 2015]**

14. (a) Distinguish, with the help of a suitable diagram, the difference in the behaviour of a conductor and a dielectric placed in an external electric field. How does polarised dielectric modify the original external field?
(b) A capacitor of capacitance C is charged fully by connecting it to a battery of emf E. It is then disconnected from the battery. If the separation between the plates of the capacitor is now doubled, how will the following change ? **[CBSE 2016]**
(i) charge stored by the capacitor.
(ii) field strength between the plates.
(iii) energy stored by the capacitor.
Justify your answer in each case.

ANSWER KEY

EXERCISE-I

1.	A	**2.**	C	**3.**	D	**4.**	A	**5.**	D	**6.**	A	**7.**	B
8.	B	**9.**	B	**10.**	B	**11.**	B	**12.**	B	**13.**	C	**14.**	A
15.	C	**16.**	C	**17.**	D	**18.**	D	**19.**	C				
20.	B	**21.**	B	**22.**	AC	**23.**	D	**24.**	ACD	**25.**	ABCD	**26.**	BD

27. A – S ; B – Q, R ; C – P, R ; D – Q

EXERCISE-II

1. 30 V **2.** C **3.** 3Q/2C **4.** 0.8 **5.** $C\left(\frac{E}{R_1+R_3}\right)R_3$

6. (a) I = $\frac{V_0}{R}$ $e^{-2t/Rc}$; (b) $\frac{1}{4}CV_0^2$ **7.** 12μC **8.** W = $\frac{1}{2}$ C_0 V_0^2 $\left(1-\frac{1}{K}\right)$

9. 4.425×10^{-9} Ampere **10.** 0.10 J

EXERCISE-III

OLD AIEEE QUESTIONS

1.	B	**2.**	A	**3.**	A	**4.**	D	**5.**	D	**6.**	C	**7.**	A
8.	C	**9.**	C	**10.**	D	**11.**	C	**12.**	C				

OLD IIT-JEE QUESTIONS

1. D **2.** $\frac{\beta A}{2}\Big/\ell n\left(\frac{2\in_0+\beta d}{2\in_0}\right)$ **3.** C **4.** B

5. $Q_0 = \frac{CVR_2}{R_1+R_2}$ *and a* = $\frac{R_1+R_2}{CR_1R_2}$ ***6.*** B **7.** D

8.	C	**9.**	A	**10.**	2	**11.**	D	**12.**	C
13.	B, D	**14.**	A, D	**15.**	D	**16.**	A,B,C,D	**17.**	A
18.	B								