CLASSROOM STUDY PACKAGE

MATHEMATICS

ARITHMETIC PROGRESSION

JEE EXPERT

CHAPTER -1

ARITHMETIC PROGRESSION

CONTENTS

KEY CONCEPTS

Sequence A succession of numbers $a_1, a_2, ..., a_n$ formed according to some definite rule is called a sequence.

A sequence is a function whose domain is the set N of natural numbers and range a subset of real numbers of complex numbers.

A sequence whose range is a subset of real numbers is called a real sequence. Since we shall be dealing with real sequences only, we shall use the term sequence to denote a real sequence.

Notation The different terms of a sequence are usually denoted by $a_1, a_2, a_3,……$or by $t_1, t_2, t_3, …..$ The subscript (always a natural number) denotes the position of the term in the sequence. The term at the nth place of a sequence, i.e., t_n is called the general term of the sequence.

A sequence is said to be finite or infinite according to whether it has finite or infinite number of terms.

Progression If the terms of a sequence follow certain pattern, then the sequence is called a progression.

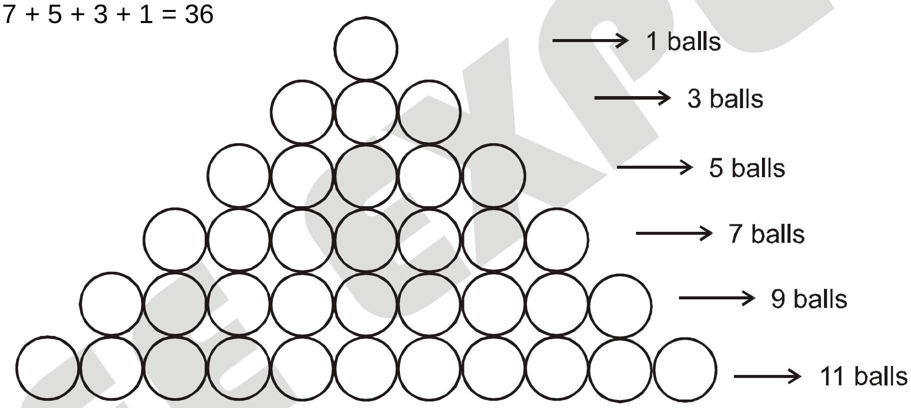
Introduction to Arithmetic Progression (A.P.)

Consider this collection of balls one row above the other.

Such that last row consists of 11 balls, last but one consists of 9 balls, second last has 7 balls and so on.

If we want to add these balls we get

11 + 9 + 7 + 5 + 3 + 1 = 36



Now let us suppose we have 99 balls in last row and with gap of 9, 18, 27 and so on as we move towards the top. i.e. 99 + 90 +81 + ... + 9

Summing them up like this will be a bit time consuming.

Arithmetic Progression (A.P.)

A sequence whose terms increase or decrease by a fixed number (common difference) is called A.P. The common difference may be +ve or –ve or zero.

In an A.P., the first term is usually denoted by 'a', the common difference by 'd' and the n^{th} term by 't_n'. Obviously

$$d = t_n - t_{n-1}$$

thus, an A.P. can be written as

$$a, a + d, a + 2d, …. a + (n – 1) d,…$$

For example,

(i) 1, 3, 5, 7, 9,….

Since, 2^{nd} term – 1^{st} term = 3^{rd} term – 2^{nd} term = 4^{th} term – 3^{rd} term

The sequence 1, 3, 5, 7, …are in A.P. whose first term is 1 and common difference is 2.

(ii) 5, 3, 1, –1, –3, – 5, – 7,….. are in A.P. whose first term is 5 and common difference is – 2.

The n^{th} term of an Arithmetic Progression

To understand the nth term of an A.P. let us take an A.P. with first term 'a' and common difference 'd'.

Now A.P. is $a_1, a_2, a_3, \ldots a_n$

Where $a_1 = a$

$a_2 = a + d$

$a_3 = (a + d) + d = a + 2d$

$a_4 = (a + 2d) + d = a + 3d$

……………………………..

…………………………….

$a_n = a_{n-1} + d = [a + (n – 2)d] + d$

$\Rightarrow \quad a_n = a + (n - 1)d$

NOTE :

(i) If an A.P. has n terms, then the nth term is called the last term of A.P. and it is denoted by l. That is $l = a + (n – 1)d$.

(ii) Three numbers a, b, c are in A.P. if and only if $b – a = c – b$, i.e., if and only if $a + c = 2b$.

(iii) If a is the first term and d the common difference of an A.P. having m terms, then nth term from the end is $(m – n + 1)^{th}$ term from the beginning. Thus nth term from the end $= a + (m – n)d$.

Sum of n terms of an A.P. The sum of n terms of an A.P. with first term 'a' and common difference 'd' is given by

$$S_n = \frac{n}{2}[2a + (n – 1)d]$$

Let, us derive the result for sum of the first 'n' terms of an A.P.

A.P. is given by

a ,a + d, a + 2d, ….

Now, nth term is $[a + (n – 1)d]$

Let, S_n be the sum of first 'n' terms of an A.P.

Then,

$S_n = a + (a + d) + (a + 2d) + \ldots + [a + (n – 1)d]$ …..(i)

Rewriting in reverse order

$S_n = [a + (n – 1)d] + [a + (n – 2)d] + \ldots + a$ …..(ii)

Adding (i) and (ii) we get

$2S_n = [2a + (n – 1)d] + [2a + (n – 1)d] + \ldots + [2a + (n – 1)d]$

$\Rightarrow \quad 2S_n = n[2a + (n – 1)d]$

$$\Rightarrow \quad S_n = \frac{n}{2}[2a + (n – 1)d]$$

Which is the sum of first 'n' terms. Now, $S_n = \frac{n}{2}[a + a + (n – 1)d]$ $S_n = \frac{n}{2}[a + a_n]$

We know, $a_n = l$ = last term

$$\Rightarrow \quad S_n = \frac{n}{2}[a + l]$$

NOTE :

(i) The nth term is given by $t_n = S_n - S_{n-1}$.

(ii) Sum of first 'n' odd natural numbers = n^2

(iii) Sum of first 'n' even natural numbers = n(n + 1)

SELECTION OF TERMS IN AN A.P. : Sometimes we require certain number of terms in A.P. The following ways of selecting terms are generally very convenient.

Number of terms	Terms	common difference
3	a – d, a, a + d	*d*
4	a – 3d, a – d, a + d, a + 3d	2*d*
5	a – 2d, a – d, a, a + d, a + 2d	*d*
6	a – 5d, a – 3d, a – d, a + d, a + 3d, a + 5d	2*d*

NOTE : We select them in this way because if we add them, 'd' gets cancelled out, so we have just one variable that can be easily eliminated.

PROPERTIES OF A.P.

(i) If $a_1, a_2, a_3, \dots a_n$ are in A.P, then

(A) $a_1 + k, a_2 + k, \dots, a_n + k$ are also in A.P.

(B) $a_1 - k, a_2 - k, \dots\dots, a_n - k$ are also in A.P.

(C) $ka_1, ka_2, \dots\dots ka_n$ are also in A.P.

(D) $a_1/k, a_2/k, \dots\dots a_n/k$, $k \neq 0$ are also in A.P.

(ii) If $a_1, a_2, a_3, \dots a_n$ and $b_1, b_2, b_3, \dots$ are two A.P.s, then

(A) $a_1 + b_1, a_2 + b_2, a_3 + b_3 \dots$, are also in A.P.

(B) $a_1 - b_1, a_2 - b_2, a_3 - b_3 \dots$, are also in A.P.

(iii) If $a_1, a_2, a_3, \dots a_n$ are in A.P, then

(A) $a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = \dots$

(B) $a_r = \dfrac{a_{r-k} + a_{r+k}}{2}$, $0 \le k \le r - 1$.

(iv) If n^{th} term of a sequence is a linear expression in 'n' then the sequence is an A.P.

(v) If the sum of first n terms of a sequence is a quadratic expression in 'n' of form $An^2 + Bn$, then the sequence is an A.P.

ARITHMETIC MEAN (A.M.)

Single Arithmetic Mean : A number 'A' is said to be the single A.M. between two given numbers 'a' and 'b' provided a, A, b are in A.P.

For example, since 2, 4, 6 are in A.P., therefore, 4 is the single A.M. between 2 and 6.

n-Arithmetic Means : The numbers $A_1, A_2, \dots\dots A_n$ are said to be 'n' arithmetic means between two given numbers 'a' and 'b' provided

a, $A_1, A_2, \dots\dots A_n$, b are in A.P.

For example, since 2, 4, 6, 8, 10, 12 are in A.P. therefore, 4,6,8,10 are the four arithmetic means between 2 and 12.

Inserting Single A.M. between Two given Numbers : Let 'a' and 'b' be two given numbers and 'A' be the A.M. between them. Then a, A, b are in A.P. Thus A =(a + b)/2

Inserting n-Arithmetic Means between Two given Numbers : Let A_1, A_2, ….. A_n be the n arithmetic means between two given numbers 'a' and 'b'. Then a, A_1, A_2, ….. A_n, b are in A.P.

Now, b = (n +2)th term of A.P.

= a + (n + 2 -1)d = a + (n +1)d

Or $d = \frac{b-a}{n+1}$, where d is common difference of A.P.

And $A_1 = a + d = a + \left(\frac{b-a}{n+1}\right)$,

$A_2 = a + 2d = a + 2\left(\frac{b-a}{n+1}\right)$,

: : :

$A_n = a + nd = a + n\left(\frac{b-a}{n+1}\right)$.

NOTE : The sum of n arithmetic means between two given numbers is n times the single A.M. between them, i.e. if a and b are two given numbers and A_1, A_2, ….. A_n are n arithmetic means between them, then

$A_1 + A_2 + \ldots\ldots + A_n = n\left(\frac{a+b}{2}\right)$.

MOREOVER :

(i) For given two real numbers 'a' & 'b' :

$\rightarrow$ A.M. $= \frac{a+b}{2}$

$\rightarrow$ G.M. $= \sqrt{ab}$ (a > 0, b > 0)

$\rightarrow$ H.M. $= \frac{2ab}{a+b}$ $(a, b \neq 0)$

where, A.M. ≡ Arithmetic Mean

G.M. ≡ Geometric Mean

H.M. ≡ Harmonic Mean

(ii) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

(iii) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

(iv) $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$

(v) a_1, a_2, a_3, ..., a_n, where $a_1 \neq 0$, are said to be in G.P. (Geometric Progression) if $\frac{a_{i+1}}{a_i} = r$ (common ratio)

$\forall \; i \in \{1, 2, ..., (n-1)\}$

eg : 2, 4, 8, 16 .... (r = 2)

$3, 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$, ... $\left(r = \frac{1}{3}\right)$

2, 6, 18, 54, ... (r = 3)

(vi) a_1, a_2, a_3, ..., a_n, where

$a_i \neq 0$, are said to be in H.P. (Harmonic Progression) if

$\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, ..., \frac{1}{a_n}$ are in A.P.

eg : $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ ....

$\frac{5}{3}, 1, \frac{5}{7}, \frac{5}{9}$ ....

SOME SOLVED ILLUSTRATIONS

ILLUSTRATION : If five times 3^{rd} term is equal to three times the 5^{th} term and A.P. is made up to all natural numbers then prove that 3^{rd} term is divisible by 3.

SOLUTION : According to question let, first term = a

And common difference = d

Since all terms are natural numbers, 'a' and 'd' are natural numbers too.

Now, $5a_3 = 3a_5$

5(a + 2d) = 3(a + 4d)

$\Rightarrow$ 5a + 10d = 3a + 12d

$\Rightarrow$ 2a = 2d

$\Rightarrow$ a = d

Now, $a_3 = a + 2d = d + 2d \Rightarrow a_3 = 3d$

ILLUSTRATION : If the mth term of an A.P. is (1/n) and the nth terms is (1/m), show that the sum of mn terms is

$\frac{1}{2}(mn+1)$.

SOLUTION : Let a be the first and d the common difference of the given A.P., then,

$a_m = a + (m-1)d = (1/n)$ ...... (i) [$\therefore$ $a_m = (1/n)$ given in this]

and $a_n = a + (n-1)d = (1/m)$

Subtracting above , we get

$a_m - a_n = (m-n)d = (1/n) - (1/m)$

$\Rightarrow$ $(m-n)d = [(m-n)/(mn)]$ $\Rightarrow$ $d = (1/mn)$

Substituting this value of d in (i), we get $a + (m-1) \times (1/mn) = (1/n)$

$\Rightarrow$ $a + (1/n) - (1/mn) = (1/n)$ $\Rightarrow$ $a = (1/mn)$

Now, $S_{mn} = (mn/2)[2a + (mn-1)d]$

$= [mn/2]\,[2 \times (1/mn) + (mn-1) \times (1/mn)]$

$= [mn/2]\,[(2/mn) + 1 - (1/mn)]$

$= (mn/2)\,[1 + (1/mn)] = (mn/2) \times (mn+1)/(mn)$

$= \frac{1}{2}(mn + 1)$

ILLUSTRATION : Fourth term of an arithmetic progression is 8. what is the sum of the first 7 terms of the arithmetic progression :

SOLUTION : Fourth term = 8 $\Rightarrow$ a + 3d = 8

sum of terms $= S_7 = (7/2)\,[2a + (7 - 1)d] = (7/2) \times 2\,(a + 3d) = 7 \times 8 = 56$

ILLUSTRATION : If the sum of the first 2n terms of 2, 5, 8…is equal to the sum of the first n terms of 57, 59, 61 …then n is equal to :

SOLUTION : Given, $(2n/2)\,\{2\times2 + (2n - 1)3\} = (n/2)\,\{2\times57 + (n - 1)2\}$

Or $2\,(6n + 1) = 112 + 2n$ or $10n = 110$, so $n = 11$

ILLUSTRATION : The sum of 24 terms of the following series : $\sqrt{2}+\sqrt{8}+\sqrt{18}+\sqrt{32}+.....$

SOLUTION : We have $\sqrt{2}+\sqrt{8}+\sqrt{18}+\sqrt{32}+.....$

$= 1\sqrt{2}+2\sqrt{2}+3\sqrt{2}+4\sqrt{2}+...... = \sqrt{2}\,[1 + 2 + 3 + 4 + \ldots$upto 24 terms]

$= [\sqrt{2} \times (24 \times 25)/2] = 300\sqrt{2}$

ILLUSTRATION : The ratio of sum of m and n terms of an AP is $m^2 : n^2$. The find the ratio of m^{th} term to n^{th} term.

SOLUTION : Given that : $\dfrac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \dfrac{m^2}{n^2}$

$$\frac{[2a+(m-1)d]}{[2a+(n-1)d]} = \frac{m}{n} \Rightarrow \frac{a+\frac{1}{2}(m-1)d}{a+\frac{1}{2}(n-1)d} = \frac{m}{n}$$

Ĝ an + ½(m-1)nd = am + ½(n –1)md

$a\,(n - m) + \dfrac{d}{2}\,[mn - n - mn + m] = 0$

$a\,(n - m) + \dfrac{d}{2}\,(m - n) = 0$

$(a - \dfrac{d}{2})\,(n - m) = 0$

Since $m \neq n$, $a = \dfrac{d}{2}$ or $d = 2a$

so required ratio, $\dfrac{T_m}{T_n} = \dfrac{a+(m-1)d}{a+(n-1)d} = \dfrac{a+(m-1)2a}{a+(n-1)2a}$

$\Rightarrow \quad \dfrac{1+2m-2}{1+2n-2} = \dfrac{2m-1}{2n-1}$

ILLUSTRATION : If S_1 , S_2 and S_3 denote the sum of first n_1, n_2 and n_3 terms respectively of A.P.,

find $(S_1/n_1)(n_2 - n_3) + (S_2/n_2)(n_3 - n_1) + (S_3/n_3)(n_1 - n_2)$

SOLUTION: We have, $S_1 = (n_1/2)\,[2a + (n_1 - 1)d]$

$\Rightarrow \quad (2S_1/n_1) = 2a + (n_1 - 1)d$

$S_2 = (n_2/2)[\,2a + (n_2 - 1)d] \quad \Rightarrow \quad (2S_2/n_2) = 2a + (n_2 - 1)d$

$S_3 = (n_3/2)[\,2a + (n_3 - 1)d] \quad \Rightarrow \quad (2S_3/n_3) = 2a + (n_3 - 1)d$

$(2S_1/n_1)(n_2 - n_3) + (2S_2/n_2)(n_3 - n_1) + (2S_3/n_3)(n_1 - n_2)$

$= [2a + (n_1 - 1)d]\,(n_2 - n_3) + [\,2a + (n_2 - 1)d]\,(n_3 - n_1) + [\,2a + (n_3 - 1)d]\,(n_1 - n_2) = 0$

EXERCISE-I

1. If p*th*, q*th* and r*th* terms of an A.P. are a, b, c respectively, then show that :
(i) $a(q - r) + b(r - p) + c(p - q) = 0$ (ii) $(a - b)r + (b - c)p + (c - a)q = 0$

2. If m times the mth term of an A.P is equal to n times its nth term, show that the (m + n)th term of the A.P is zero.

3. How many numbers of two digits are divisible by 7 ?

4. Find the sum of first 24 terms of the A.P $a_1, a_2, a_3, \ldots$, if it is known that $a_1+a_5+a_{10}+a_{15}+a_{20}+a_{24} = 225$.

5. If the first term of an A.P is 2 and the sum of first five terms is equal to one-fourth of the sum of the next five terms, find the sum of first 30 terms.

6. The sum of *n* terms of three A.P. are S_1, S_2 and S_3. The first term of each is unity and the common differences are 1, 2 and 3 respectively. Prove that $S_1 + S_3 = 2S_2$.

7. If p*th* term of an A.P is *a* and q*th* term is *b*. Prove that the sum of its (p + q) terms is $0.5 \times (p + q)\,[a + b + (a - b)/(p - q)]$.

8. If a, b, c are in A.P.,prove that the following are also in A.P.

(i) $\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ (ii) $a(1/b + 1/c), b(1/c + 1/a), c(1/a + 1/b)$

9. (i) The nth term of the series is given to be $(3 + n)/4$, find the sum of 105 terms of this series.
(ii) Find $a_1+a_6+a_{11}+a_{16}$ if it is known that $a_1, a_2, a_3, \ldots$ is an A.P and $a_1+a_4+a_7+\ldots+a_{16} = 147$.

10. A class consist of a number of boys whose ages are in A.P the common difference being 4 months. If the youngest boy is just 8 years old & if the sum of the ages is 168 years, find the no. of boys.

11. Find four numbers in A.P whose sum is 20 and the sum of whose squares is 120.

12. The ratio of the sum of n terms of 2 A.P's is (7n + 1) : (4n + 27). Find the ratio of their m*th* terms.

13. If $S_1, S_2, \ldots, S_m$ are the sums of *n* terms of *m* A.P's whose first terms are 1,2,3,…,m and common differences are 1,3,5…,(2m – 1)respectively. Show that : $S_1 + S_2 + S_3 + \ldots + S_m = \frac{mn}{2}(mn + 1)$.

14. If $(b + c - a)/a, (c + a - b)/b, (a + b - c)/c$ are in A.P., prove that : $1/a, 1/b, 1/c$ are also in A.P.

15. If $(b - c)^2, (c - a)^2, (a - b)^2$ are in A.P., prove that : $1/(b - c), 1/(c - a), 1/(a - b)$ are in A.P.

16. Two cars start together in the same direction from the same place. The 1st goes with uniform speed of 10 km/h. The 2nd goes at a speed of 8 km/h in the first hour and increases the speed by ½ km/hr each succeeding hr. After how many hrs. will the 2nd car overtake the 1st car if both cars go non-stop?

17. Show that the sum of an A.P whose first term is *a*, second term is *b* and the last term is *c* is equal to $[(a + c)(b + c - 2a)]/2(b - a)$.

18. Show that the sum of all odd numbers between 1 and 1000, which are divisible by 3 is 83667.

19. Divide 28 into four parts in A.P. so that the ratio of the product of 1st and 3rd with the product of 2nd and 4th is 8 : 15.

20. The sum of two numbers is 13/6. An even number of arithmetic means are being inserted between them and their sum exceeds their number by 1. Find the number of means inserted.

EXERCISE-II

1. If the pth term of an A.P. is q and the qth term is p, then its (p +q)th term is
(A) 0 (B) p – q (C) p + q (D) None of these

2. The number of numbers lying between 100 and 500 that are divisible by 7 but not by 21 is :
(A) 57 (B) 19 (C) 38 (D) None of these

3. In the series 3, 7, 11, 15,… and 2, 5, 8,… each continued to 100 terms, the number of terms that are identical is
(A) 21 (B) 27 (C) 25 (D) None of these

4. If m times the mth term of an A.P. is equal to n times the nth term, then its (m + n)th term is -
(A) 1 (B) –1 (C) 0 (D) None of these

5. The sum of first 24 terms of the A.P. a_1, a_2, a_3,..if it known that $a_1+a_5+a_{10}+a_{15}+a_{20}+a_{24} = 225$, is :
(A) 865 (B) 900 (C) 930 (D) None of these

6. The maximum sum of the series 20 + 19 (1/3) + 18 (2/3) + 18 +…is
(A) 310 (B) 290 (C) 320 (D) None of these

7. The sum of 11 terms of an A.P. whose middle term is 30, is
(A) 320 (B) 330 (C) 340 (D) 350

8. The sum of an A.P. is 525. If its first term is 3 and last term is 39, then the common difference is
(A) 3/2 (B) 1 (C) 1/2 (D) None of these

9. The common difference of an A.P., whose first term is 100 and the sum of whose first six terms is five times the sum of the next six terms, is
(A) 10 (B) –10 (C) 5 (D) – 5

10. The first and last terms of an A.P. are a and l respectively. If S is the sum of all the terms of the A.P. and the common difference is $(l^2 - a^2)/[k - (l + a)]$, then k is equal to
(A) S (B) 2S (C) 3S (D) None of these

11. The sum of all two digit numbers which when divided by 4, yield unity as remainder, is
(A) 1100 (B) 1200 (C) 1210 (D) None of these

12. Let S_n denotes the sum of n terms of an A.P. whose first term is a. If the common difference $d = S_n - kS_{n-1} + S_{n-2}$ then k = …
(A) 1 (B) 2 (C) 3 (D) None of these

13. If the sum of the first n even natural numbers is equal to k times the sum of first n odd natural numbers, then k is equal to
(A) 1/n (B) (n – 1)/n (C) (n + 1)/2n (D) (n + 1)/n

14. If S_1, S_2, S_3 are the sum of n, 2n, 3n terms of an A.P., then $3(S_2 - S_1) =$
(A) S_3 (B) $2S_3$ (C) $4S_3$ (D) None of these

15. If the first, second and last terms of an A.P. are a, b then 2a respectively, then its sum is
(A) [ab]/[2(b – a)] (B) [ab]/[(b – a)] (C) [3ab]/[2(b – a)] (D) None of these

16. If m arithmetic means are inserted between 1 and 31 so that the ratio of the 7^{th} and $(m - 1)^{th}$ means is 5 : 9, then the value of m is
(A) 9 (B) 11 (C) 13 (D) 14

17. If there are (2n + 1) terms in A.P., then the ratio of the sum of odd terms and the sum of even terms is

(A) n : (n + 1) (B) (n + 1) : n (C) (n – 1) : n (D) None of these

18. If the ratio of the sum of n terms of two A.P.s is (3n-13) :(5n+21), then ratio of 24^{th} terms of the two progressions is

(A) 2 : 3 (B) 2 : 1 (C) 1 : 2 (D) None of these

19. If a is the first term, d the common difference and S_k the sum to k terms of an A.P., then for S_{kx}/S_x to be independent of x.

(A) a = 2d (B) a = d (C) 2a = d (D) None of these

20. Let $a_1, a_2, a_3 \ldots\ldots a_n$ be in A.P.

If $1/(a_1a_n) + 1/(a_2\, a_{n-1}) + \ldots + 1/(a_n\, a_1) = k/(a_1 + a_n)\,[(1/a_1) + (1/a_2) + \ldots + (1/a_n)]$, then k is equal to :

(A) 1 (B) 2 (C) 3 (D) None of these

21. If x, y, z are in A.P., then(x + 2y – z) (2y + z – x) (z + x – y)

(A) 4xyz (B) 2xyz (C) xyz (D) None of these

22. The sum of n terms of m A.P.s are $S_1, S_2, S_3, \ldots, S_m$. If the first term and common difference are equal for each A.P., and are 1, 2, 3,….,m respectively, then $S_1 + S_2 + S_3 + \ldots + S_m =$

(A) (1/4)mn (m + 1)(n + 1) (B) (1/2)mn (m + 1)(n + 1)

(C) mn (m + 1)(n + 1) (D) None of these

23. The middle term in the following arithmetic progression 20, 16, 12, …, –180 is

(A) – 46 (B) – 76 (C) – 80 (D) None of these

24. Find the sum to 200 terms of the series 1 + 4 + 6 + 5 + 11 + 6 + ……….

(A) 30,400 (B) 29,800 (C) 30,200 (D) None of these

25. If $S_n = nP + n(n-1)Q$, where S_n denotes the sum of the first n term of an A.P., then the common difference is :

(A) 2P +3Q (B) P + Q (C) Q (D) 2Q

26. The sum of all terms of the arithmetic progression having ten terms except for the first term, is 99, and except for the sixth term, 89. Find the third term of the progression.

(A) 15 (B) 5 (C) 8 (D) 10

27. 8^{th} term of the series $2\sqrt{2}$, $\sqrt{2}$, 0,… will be :

(A) $-5\sqrt{2}$ (B) $5\sqrt{2}$ (C) $10\sqrt{2}$ (D) $-10\sqrt{2}$

28. If the first term of a series in AP is 17, the last term is –12(3/8) and the sum 25(7/16), then find the common difference.

(A) – 43/18 (B) – 45/17 (C) – 47/16 (D) None of these

29. If the first, second and the last terms of an AP are a, b, c respectively, then the sum is –

(A) [(a + b)(a + c – 2b)]/ [2(b – a)] (B) [(b + c)(a + b – 2c)]/ [2(b – a)]

(C) [(a + c)(b + c – 2a)]/ [2(b – a)] (D) None of these

30. If (b + c – a)/a, (c + a – b)/b, (a + b – c)/c are in AP then which of the following is in A.P. –

(A) a,b,c (B) a^2, b^2, c^2 (C) (1/a),(1/b), (1/c) (D) None of these

31. If $a_1, a_2, a_3 \ldots\ldots a_n$ are in AP where $a_i > 0\ \forall$ i, then find the value of

$$\frac{1}{\left(\sqrt{a_1}+\sqrt{a_2}\right)}+\frac{1}{\left(\sqrt{a_2}+\sqrt{a_3}\right)}+\ldots\ldots+\frac{1}{\left(\sqrt{a_{n-1}}+\sqrt{a_n}\right)}$$

(A) $\frac{1}{\left(\sqrt{a_1}+\sqrt{a_n}\right)}$ (B) $\frac{1}{\left(\sqrt{a_1}-\sqrt{a_n}\right)}$ (C) $\frac{n}{\left(\sqrt{a_1}-\sqrt{a_n}\right)}$ (D) $\frac{n-1}{\left(\sqrt{a_1}+\sqrt{a_n}\right)}$

32. If b_1, b_2, b_3,........ belongs to A.P. such that
$b_1 + b_4 + b_7 + ... + b_{28} = 220$, then the value of
$b_1 + b_2 + b_3 + ... + b_{28}$ equals
(A) 616 (B) 308 (C) 2,464 (D) 1,232

33. If $a^2(b + c)$, $b^2(c + a)$, $c^2(a + b)$ are in A.P., then, either a, b, c are in A.P. or
(A) $ab + bc + ca = 0$ (B) $a + b + c = 0$ (C) $a - b - c = 0$ (D) $a - b + c = 0$

34. The n^{th} term of the series $1^2, (1^2 + 2^2), (1^2 + 2^2 + 3^2)$, ... is

(A) n (B) $\frac{n(n+1)}{2}$ (C) $\frac{n^2(n+1)^2}{2}$ (D) $\frac{n(n+1)(2n+1)}{6}$

35. A circle with area A_1 is contained in the interior of a larger circle with area $A_1 + A_2$. If the radius of the larger circle is 3 unit and $A_1, A_2, A_1 + A_2$ are in A.P., then the radius of the smaller circle is

(A) $\frac{\sqrt{3}}{2}$ unit (B) 1 unit (C) $\frac{2}{\sqrt{3}}$ unit (D) $\sqrt{3}$ unit

36. If the p^{th}, q^{th} and r^{th} terms of an A.P. are P, Q, R respectively, then $P(q - r) + Q(r - p) + R(p - q)$ equals
(A) 0 (B) $pq + qr + rp$ (C) pqr (D) $p + q + r$

37. In an A.P., if it is given that $t_{p+1} = 2t_{q+1}$, then, t_{3p+1} is equal to
(A) $2t_{p+q+1}$ (B) $2t_{p+q-1}$ (C) $2t_{p-q+1}$ (D) $2t_{p-q-1}$

38. If the A.M. of 'a' and 'b' is $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$, then the value of n is

(A) 1 (B) $\frac{1}{2}$ (C) – 1 (D) 0

39. Let $s_1(n)$ be the sum of the first n terms of the arithmetic progression 8, 12, 16, ... and let $s_2(n)$ be the sum of the first n terms of arithmetic progression 17, 19, 21 .... If for some value of n, $s_1(n) = s_2(n)$ then this common sum is : **[KVPY 2007]**
(A) not uniquely determinable (B) 260
(C) 216 (D) 200

40. The sum of 7 consecutive positive integers is equal to the sum of the next five consecutive integers. What is the largest among the 12 numbers ? **[KVPY 2008]**

(A) 24 (B) 23 (C) 22 (D) 21

ANSWER KEY

EXERCISE-I

3.	13	**4.**	900	**5.**	–2550	**9.**	(i)1470 (ii)98	**10.**	16	**11.**	2,4,6,8
12.	(14m – 6) : (8m + 23)	**16.**	9 hours	**19.**	4,6,8,10	**20.**	6				

EXERCISE-II

1.	(A)	**2.**	(C)	**3.**	(C)	**4.**	(C)	**5.**	(B)	**6.**	(A)	**7.**	(B)
8.	(A)	**9.**	(B)	**10.**	(B)	**11.**	(C)	**12.**	(B)	**13.**	(D)	**14.**	(A)
15.	(C)	**16.**	(D)	**17.**	(B)	**18.**	(C)	**19.**	(C)	**20.**	(A)	**21.**	(A)
22.	(A)	**23.**	(C)	**24.**	(C)	**25.**	(D)	**26.**	(B)	**27.**	(A)	**28.**	(C)
29.	(C)	**30.**	(C)	**31.**	(D)	**32.**	(A)	**33.**	(A)	**34.**	(D)	**35.**	(D)
36.	(A)	**37.**	(A)	**38.**	(D)	**39.**	(B)	**40.**	(B)				