JEE EXPERT

ANSWER KEY
RANK ELEVATOR TEST SERIES
(RETS/PT-02)
12TH (Zenith X01 & X02)
Date 01.09.2019

PHYSICS

1	(D)	2	(C)	3	(D)	4	(D)	5	(A)
6	(A)	7	(C)	8	(B)	9	(C)	10	(B)
11	(D)	12	(B)	13	(C)	14	(D)	15	(C)
16	(D)	17	(A)	18	(B)	19	(C)	20	(A)
21	(B)	22	(A)	23	(A)	24	(C)	25	(B)
26	(D)	27	(C)	28	(A)	29	(C)	30	(A)

CHEMISTRY

31	(A)	32	(C)	33	(B)	34	(B)	35	(D)
36	(B)	37	(C)	38	(C)	39	(A)	40	(B)
41	(A)	42	(B)	43	(B)	44	(A)	45	(C)
46	(D)	47	(A)	48	(D)	49	(B)	50	(A)
51	(C)	52	(B)	53	(A)	54	(D)	55	(A)
56	(D)	57	(D)	58	(D)	59	(D)	60	(C)

MATHEMATICS

61	(A)	62	(B)	63	(C)	64	(A)	65	(A)
66	(A)	67	(A)	68	(C)	69	(B)	70	(A)
71	(C)	72	(D)	73	(D)	74	(C)	75	(A)
76	(C)	77	(A)	78	(A)	79	(A)	80	(C)
81	(C)	82	(D)	83	(B)	84	(C)	85	(C)
86	(B)	87	(B)	88	(C)	89	(B)	90	(C)

JEE EXPERT

SOLUTIONS

RANK ELEVATOR TEST SERIES

(RETS/PT-02)

12^{TH} (Zenith X01 & X02)

Date 01.09.2019

PART - I [PHYSICS]

1. Sol. (D)

At equilibrium final pressure and final temperature in both compartment are equal.

$$\frac{PV}{RT} = \frac{P'V'}{RT} \quad \text{...(i)}$$

$$\frac{(2P)(2V)}{RT} = \frac{P'V'}{RT} \quad \text{...(ii)}$$

$V'' = 4V'$ ...(iii)

$V' + V'' = 3V$

on solving we get the result.

2. Sol. (C)

As ; $W_{ext} = \Delta(ME)$; ME = Mechanical energy.

Mechanical energy will keep on increasing upto the instant the W_{ext} is positive, which will happen till there is no compression in the spring. First the spring gets extended to a maximum and after which the extension decreases upto the natural length. After that there is a compression in the spring, results in a –ve external work (so as to move the end of spring at constant speed u).

Hence maximum energy stored is at the natural length and $ME_{max} = \frac{1}{2}mv^2$

At the natural length v = 2u, since the block is moving at this instant at a speed u with respect to the other end of the spring.

Hence $ME_{max} = \frac{1}{2}m(2v)^2 = 2mu^2$.

3. Sol. (D)

$\omega = \frac{2a}{T} = 4$

$a_{max} = \omega^2 A = 16 \times 1$

$T_{max} - mg = m \times 16 \Rightarrow T_{max} = 26m$

To just slip

$26\ m = \mu(2mg)$

$\mu = \dfrac{13}{10}$

4. Sol. (D)

$\Delta U = \Delta Q - \Delta W = 2256 - P(\Delta V) = 2256 - 10^5\ (1670 \times 10^{-6}) = 2256 - 167$

5. Sol. (A)

$$\frac{\Delta \ell}{\ell} = \frac{\text{Stress}}{Y} = \frac{5\times10^{7}}{2\times10^{11}} = \frac{5}{2}\times10^{-4}$$

$$\Delta\ell = \left(\frac{5}{2}\ell\times10^{-4}\right)$$

$$V = \pi r^2 \ell$$

$$\frac{\Delta V}{V} = \frac{\Delta\ell}{\ell} - \frac{2\Delta r}{r}$$

$$2\times10^{-4} = \frac{5}{2}\times10^{-4} - \frac{2\Delta r}{r}$$

$$\frac{\Delta r}{r} = \frac{1}{4}\times10^{-4}$$

6. Sol. (A)

Surface energy = $(8\pi r^2)T$

$$p = \frac{d}{dt}(8\pi r^2 T) = 8\pi T\left(2r\frac{dr}{dt}\right)$$

$\Rightarrow \quad p \propto r$

7. Sol. (C)

8. Sol. (B)

9. Sol. (C)

10. Sol. (B)

11. Sol. (D)

$\dfrac{E}{A} = y\dfrac{\Delta\ell}{\ell} \Rightarrow \Delta\ell = \dfrac{F\ell}{Ay}$; $\Delta\ell_1 = \dfrac{F}{A}\left(\dfrac{1.0}{10^{11}}\right) = 1\,\text{mm}$; $\Delta\ell_2 = \dfrac{F}{A}\left(\dfrac{0.5}{2\times10^{11}}\right) = \dfrac{1}{4}\,\text{mm}$. Total extension = 1.25 mm

12. Sol. (B)

Due to induction damping will be there $\omega = \sqrt{\omega_0^2 - \gamma^2}$, where $\gamma = \dfrac{b}{2m}$, due to damping angular frequency decreases. Thesefore time period will increases.

13. Sol. (C)

$ms\left(\frac{dT}{dt}\right) = K(\Delta T)$ using approximate method.

For first case $(50 + 30)(s_w)\left(\frac{5}{2}\right) = \left(\frac{30+25}{2} - T\right)k$

$(100S + 30s_w)\left(\frac{5}{2}\right) = \left(\frac{20+25}{2} - T\right)k$

So, $s_w = 100s + 30s_w$

So, $s_w = 100s$ Þ $s = \frac{s_w}{2} = 0.5$ kcal/g

14. Sol. (D)

(1) $n\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$ Þ $nr^3 = R^3$ Þ $\frac{nr^2}{R^3} = \frac{1}{r}$

(2) $nT4pr^2 - T4pR^2 = DU = \frac{1}{2}\left(\rho\cdot\frac{4}{3}\pi R^3\right)v^2$

$$v = \sqrt{\frac{6T}{\rho}\left(\frac{nr^2 - R^2}{R^3}\right)}$$

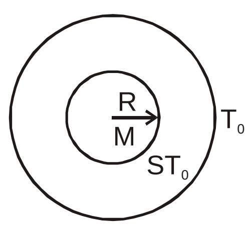
$$v = \sqrt{\frac{6T}{\rho}\left(\frac{nr^2}{R^3} - \frac{1}{R}\right)} = \sqrt{\frac{6T}{\rho}\left(\frac{1}{r} - \frac{1}{R}\right)}$$

15. Sol. (C)

$P_L = \sigma(4\pi R^2)[(T^4) - T_0^4]$

$$-m\alpha T^3\frac{dT}{dt} = \sigma 4\pi R^2(T^4 - T_0^4)$$

$$\int_{3T_0}^{2T_0}\frac{T^3 dT}{T^4 - T_0^3} = \int_0^t \frac{-\sigma 4\pi R^2}{m\alpha}\,dt$$



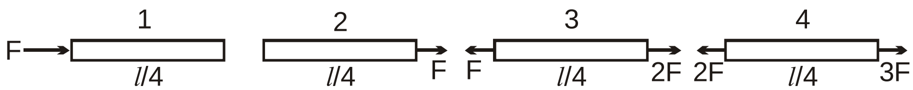
Solving get

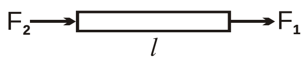
$$t = \frac{m\alpha}{16\sigma\pi R^2}\ell n\left(\frac{16}{3}\right)$$

16. Sol. (D)

r_0 = perpendicular separation of 'p' from either of the wire. In steady state, the F.B.D. of the rod can be drawn by dividing it into four parts as shown force is shown the $\gamma \dfrac{\Delta \ell}{\ell}$ = (stress)$_{\text{average}} \Rightarrow \Delta \ell = \dfrac{\ell}{\gamma} \cdot \dfrac{(F_1 + F_2)}{2A}$

Now, $\Delta \ell_{1(\text{compression})} = \dfrac{F\frac{\ell}{4}}{2A\gamma} = \dfrac{F\ell}{8A\gamma}$; $\quad \Delta \ell_{2(\text{elongation})} = \dfrac{F\ell}{8A\gamma}$



For any rod. If 

$\Delta \ell_{3(\text{elongation})} = \dfrac{3F\ell}{8A\gamma}, \Delta \ell_{4(\text{elongation})} = \dfrac{5F\ell}{8A\gamma}$

$\Delta \ell = \dfrac{8F\ell}{8A\gamma} = \dfrac{F\ell}{A\gamma}$.

17. Sol. (A)

If θ be temperature of B, then

$$\frac{2kA(\theta - 100)}{\ell} + \frac{(k/2)A(\theta - 0)}{\ell} = 200$$

Substituting value $\theta = 880^\circ C$

$\therefore \quad \phi_t = mL$

$$\frac{kA(880 - 0)}{2\ell} \times t = 440 \times 80$$

$$t = \frac{80 \times 1 \times 2 \times 440}{100 \times 10 \times 10^{-4} \times 880} = 800 \text{ sec} = \frac{40}{3} \text{min}$$

18. Sol. (B)

$eA\sigma T^4$ = Power emitted per unit time

$= e(4\pi r^2)\sigma T^4$

$r \to 2r$

Power gets 4 times

Power received by planet also gets 4 times higher, power received = Power radiated (for equilibrium of temperature of planet)

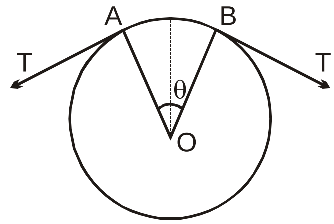
$4P = e(4\pi r^2)(T_1)^4$, intially $P = e(4\pi r^2)\,\sigma T^4$ (for planet),

$$\frac{T_1}{T} = \sqrt{2}$$

19. **Sol. (C)**

For small element AB = $Rd\theta$

Tension provides centripetal force



$$2T\sin\frac{\theta}{2} = 2T\left(\frac{\theta}{2}\right)$$

$= \rho(A)(r\theta)\omega^2 r$

$T = \rho A\omega^2 r^2$

stress = $\dfrac{T}{A} = \rho\omega^2 r^2$

20. **Sol. (A)**

$(a^3\rho + m)g = a^2 x_1 \rho_\ell g$ …(1)

$a^3\rho g = a^2 x_2 r_\ell g$ …(2)

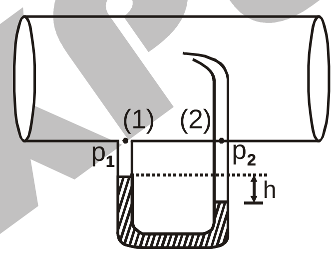
(1) – (2)

$mg = a^2 r_\ell g(x_1 - x_2)$

21. **Sol. (B)**

Applying Bernouli equation between points (1) & (2)

$$p_1 + \frac{1}{2}\rho v^2 = p_2$$

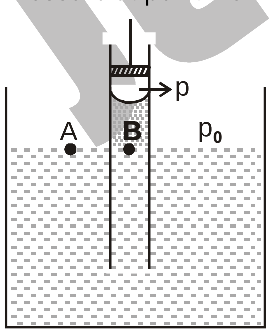


$$p_2 - p_1 = \frac{1}{2}\rho v^2 = \rho_0 g\Delta h$$

$$v = \sqrt{\frac{2\rho_0 g\Delta h}{\rho}}$$

22. **Sol. (A)**

Pressure at point A & B will be equal



$10^5 = \dfrac{1}{2}\times 10^5 - \dfrac{2T}{r} + \rho gh$ $\therefore$ h = 15 m

23. **Sol. (A)**

Relative density of cube = $\dfrac{1}{3}$ (R.D. of mercury) = $\dfrac{1}{3}$ (g) = 3

Now after pouring water.

Mass of cube = mass of mercury displaced + mass of water displaced.

$$V(3) = \frac{V}{x}(g) + V\left(1-\frac{1}{x}\right) \; ; \; 3 = \frac{g}{x} + 1 - \frac{1}{x} \; ; \; 2 = \frac{8}{x} \; ; x = 4$$

24. Sol. (C)

$$\frac{PV}{R} = nT = \text{const} \; ; nT = \text{const.} \quad \left(\frac{dn}{dt}\right)T + n\left(\frac{dT}{dt}\right) = 0$$

$$\frac{\Delta Q}{\Delta t} = \frac{T_0 - 0.8T_0}{VR} = \frac{RT_0}{5}$$

25. Sol. (B)

Let n drops coalesce to form a big drop. Then

$$N \times \frac{1}{3}\pi r^3 = \frac{4}{3}\pi r^3 \;\Rightarrow\; n = \frac{R^3}{r^3}$$

$$\text{Heat produced} = \frac{\text{Work done}}{J} = S.T \times \frac{\text{decrease in area}}{J} = mc\, d\theta \Rightarrow d\theta = \frac{3T}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$$

26. Sol. (D)

$W_{BC} = P\Delta V = nR\Delta T = -nRT_0$

$W_{CA} = +2\, nRT_0 \ln 2$

$$\Delta Q_{BC} = nC_P\Delta T = \frac{nR_\gamma T_0}{\gamma - 1}$$

Hence, efficiency $\dfrac{(2\ln 2 - 1)}{\gamma/(\gamma-1)}$

27. Sol. (C)

By definition, Bulk's modulus $K = -\dfrac{dP}{\left(\dfrac{dV}{V}\right)} - \dfrac{mg/A}{dV/V}$

$$V = \frac{4}{3}\pi R^3 \qquad \frac{dV}{V} = 3\frac{dR}{R} \qquad K = \frac{mg}{3A\left(\dfrac{dR}{R}\right)} \qquad \frac{dR}{R} = -\frac{mg}{3AK}$$

28. Sol. (A)

29. Sol. (C)

30. Sol. (A)

PART - III [MATHEMATICS]

61. **(A)**

62. **(B)** Normals at t_1 and t_2 meets at some point $(-t_5)$

$$\Rightarrow \quad -t_3 = \left(-t_1 - \frac{2}{t_1}\right) = \left(-t_2 - \frac{2}{t_2}\right)$$

$$\Rightarrow \quad (t_1 - t_2) + 2\left[\frac{1}{t_1} - \frac{1}{t_2}\right] = 0 \Rightarrow \frac{(t_1t_2 - 2)(t_1 - t_2)}{t_1t_2} = 0$$

$$\Rightarrow \quad t_1t_2 = 2$$

63. **(C)** Equation of tangent to the parabola,

$$y = mx + \frac{a}{m}$$

$$y = m(x+2) + \frac{2}{m}$$

$$\therefore \quad -4 = -2m + \frac{2}{m}$$

$$-4m = -2m^2 + 2$$

$$m^2 - 2m - 1 = 0$$

$$m_1 + m_2 = 2$$

$$m_1m_2 = -1$$

$$\Rightarrow \quad \theta = \pi/2$$

64. **(A)** The equation of the tangent to $y^2 = 8x$ at $P(2, 4)$ is

$4y = 4(x + 2)$ or $x - y + 2 = 0$ …(i)

Let (x_1, y_1) be the mid-point of chord QR. Then, equation of QR is

$$yy_1 - 4(x + x_1) - 5 = y_1^2 - 8x_1 - 5$$

$$4x - yy_1 - 4x_1 + y_1^2 = 0 \qquad \ldots\text{(ii)}$$

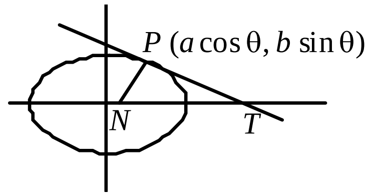
Clearly, (i) and (ii) represent the same line. So,

$$\frac{4}{1} = -\frac{y_1}{-1} = \frac{-4x_1 + y_1^2}{2}$$

$$\Rightarrow \quad y_1 = 4 \text{ and } 8 = 4x_1 + y_1^2$$

$$\Rightarrow \quad y_1 = 4 \text{ and } x_1 = 2.$$

65. **(A)** $e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{3}{5}$



$T \equiv \left(\frac{a}{\cos\theta}, 0\right)$,

$\frac{a\sec\theta - ae^2\cos\theta}{2ae} = \frac{91}{60} \Rightarrow \cos\theta = \frac{1}{2}, \ \sin\theta = \frac{\sqrt{3}}{2}$

$\Delta PSS' = \frac{1}{2} \times 6 \times 4 \times \frac{\sqrt{3}}{2} = 6\sqrt{3}$ sq. units

66. **(A)** Let an end of a latus-rectum be $(ae, b\sqrt{1-e^2})$, then the equation of the normal at this end is

$$\frac{x - ae}{ae/a^2} = \frac{y - b\sqrt{1-e^2}}{b\sqrt{1-e^2}/b^2}$$

It will pass through the end (0, –b) if

$$-a^2 = \frac{-b^2(1+\sqrt{1-e^2})}{\sqrt{1-e^2}} \quad \text{or} \quad \frac{b^2}{a^2} = \frac{\sqrt{1-e^2}}{1+\sqrt{1-e^2}}$$

or $(1-e^2)[1+\sqrt{1-e^2}] = \sqrt{1-e^2}$

or $\sqrt{1-e^2} + 1 - e^2 = 1$ or $e^4 + e^2 = 1$.

67. **(A)** For the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, we have $a^2 = 16$, $b^2 = 9$.

Let e be the eccentricity of the ellipse. Then

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$$

The coordinates of the foci are ($\pm ae$, 0) or ($\pm\sqrt{7}$, 0).

So, radius of the circle = distance between ($\pm\sqrt{7}$, 0) and (0, 3) = $\sqrt{7+9} = 4$.

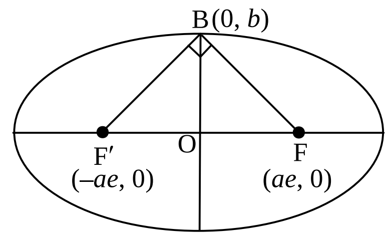
68. **(C)** Chords of contact are $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$ and $\frac{xx_2}{a^2} + \frac{yy_2}{b^2} = 1$

Product of the slopes = –1

$$\Rightarrow -\frac{x_1/a^2}{y_1/b^2} \cdot -\frac{x_2/a^2}{y_2/b^2} = -1 \Rightarrow \frac{x_1x_2}{a^4} \cdot \frac{b^4}{y_1y_2} = -1 \Rightarrow \frac{x_1x_2}{y_1y_2} = -\frac{a^4}{b^4}$$

69. **(B)** Clearly FB = F′B $\therefore$ ΔFBF′ is isosceles.

$\therefore$ $\angle OBF = 45°$



$$\tan OBF = \frac{OF}{OB} = \frac{ae}{b}$$

y $\quad \frac{ae}{b} = 1 \ (\because \ \tan 45° = 1)$

$$\Rightarrow \quad ae = b \Rightarrow a^2e^2 = b^2 = a^2(1-e^2)$$

$$\Rightarrow \quad e^2 = 1 - e^2 \Rightarrow 2e^2 = 1 \Rightarrow e^2 = \frac{1}{2} \Rightarrow e = \frac{1}{\sqrt{2}}$$

70. **(A)** Let circle is $x^2 + y^2 + 2gx + 2fy + 1 = 0$

Solve with $y = \frac{1}{x}$, we get, $x^4 + 2gx^2 + cx^2 + 2fx = 1 = 0$

$\Rightarrow \quad x_1x_2x_3x_4 = 1 \Rightarrow y_1y_2y_3y_4 = 1$

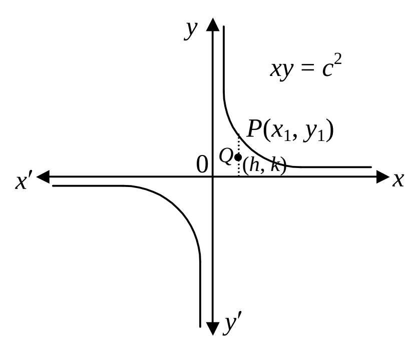
71. **(C)** We have $\frac{2b^2}{a} = 8$ and $2b = \frac{1}{2}(2ae)$ $\therefore$ $\frac{2}{a}\left(\frac{ae}{2}\right)^2 = 8 \Rightarrow ae^2 = 16$ …(i)

Now, $2\frac{b^2}{a} = 8 \Rightarrow b^2 = 4a \Rightarrow a^2(e^2 - 1) = 4a \Rightarrow ae^2 - a = 4$ …(ii)

From (i) and (ii), $16 - \frac{16}{e^2} = 4 \Rightarrow \frac{16}{e^2} = 12 \Rightarrow e = \frac{2}{\sqrt{3}}$.

72. **(D)** Let $xy = c^2$ be the rectangular hyperbola, and let $P(x_1, y_1)$ be a point on it. . Let $Q\,(h, k)$ be the mid-point of PN.

Then the coordinates of Q are $\left(x_1, \frac{y_1}{2}\right)$

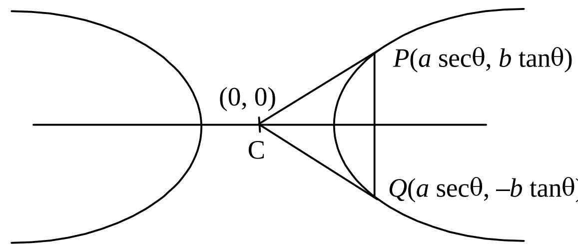


$\therefore \quad x_1 = h$ and $\frac{y_1}{2} = k \Rightarrow x_1 = h$ and $y_1 = 2k$

But (x_1, y_1) lies on $xy = c^2$

$h\,.(2k) = c^2 \Rightarrow hk = c^2/2.$

73. **(D)** Let $P(a\sec\theta,\ b\tan\theta)$, $Q(a\sec\theta,\ -b\tan\theta)$ be end points of double oridinate and C(0, 0) is the centre of the hyperbola.



Now, $PQ = 2b\tan\theta$

$$CQ = CP = \sqrt{a^2\sec^2\theta + b^2\tan^2\theta}$$

Since $CQ = CP = PQ$

$\therefore \quad 4b^2\tan^2\theta = a^2\sec^2\theta + b^2\tan^2\theta$

$\Rightarrow \quad 3b^2\tan^2\theta = a^2\sec^2\theta \ \Rightarrow\ 3b^2\sin^2\theta = a^2$

$\Rightarrow \quad 3a^2(e^2-1)\sin^2\theta = a^2 \ \Rightarrow\ 3(e^2-1)\sin^2\theta = 1$

$\Rightarrow \quad \dfrac{1}{3(e^2-1)} = \sin^2\theta < 1 \quad (\because\ \sin^2\theta < 1)$

$\Rightarrow \quad \dfrac{1}{e^2-1} < 3 \ \Rightarrow e^2 - 1 > \dfrac{1}{3} \quad \Rightarrow e^2 > \dfrac{4}{3} \ \Rightarrow\ e > \dfrac{2}{\sqrt{3}}$

74. **(C)** Adding the given two.

$\sin(B+C) = 1 \ \Rightarrow\ B + C = 90°$

Subtracting, $\sin(B-C) = \dfrac{\sqrt{3}-2}{\sqrt{3}} \neq 0$

$\therefore \quad B \neq C$

$\therefore$ Triangle is right-angled.

75. **(A)** A.M. $\geq$ G.M $\Rightarrow \dfrac{\tan A + \tan B + \tan C}{3} \geq (\tan A\ \tan B\ \tan C)^{1/3}$

Also we know that $\tan A + \tan B + \tan C = \tan A \tan B \tan C$ when $A + B + C = \pi$.

76. **(C)** $\left(1 - \dfrac{s-b}{s-a}\right)\left(1 - \dfrac{s-c}{s-a}\right) = 2$

$\Rightarrow \quad (b-a)(c-a) = 2(s-a)^2 \ \Rightarrow 2(bc - ac - ab + a^2) = (b+c-a)^2 \ \Rightarrow a^2 = b^2 + c^2$

77. **(A)** We have $a + b + c = \dfrac{6(\sin A + \sin B + \sin C)}{3}$

$\Rightarrow \quad k(\sin A + \sin B + \sin C) = 2(\sin A + \sin B + \sin C)$,

where $k = \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$\Rightarrow \quad k = 2 \quad [\because \sin A + \sin B + \sin C \neq 0]$

$$\Rightarrow \quad \frac{a}{\sin A} = 2 \quad \Rightarrow \quad \sin A = \frac{1}{2} \quad [\because a = 1]$$

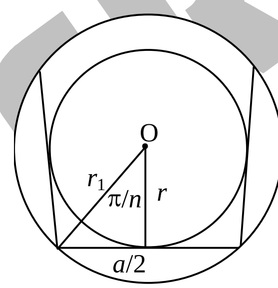
$$\Rightarrow \quad A = \frac{\pi}{6}$$

78. **(A)** $\cos A + \cos B = 4\sin^2 \frac{C}{2}$

$$\Rightarrow \quad \tan\frac{A}{2} . \tan\frac{B}{2} = \frac{1}{3} \Rightarrow \frac{\Delta^2}{s(s-a)s(s-b)} = \frac{1}{3} \Rightarrow 3(s-c) = s$$

$$\Rightarrow \quad a+b+c = 3c \Rightarrow \frac{a+b}{2} = c$$

79. **(A)** $\because \quad r = \frac{a}{2}\cot\frac{\pi}{n}$ and $r_1 = \frac{a}{2}\operatorname{cosec}\frac{\pi}{n}$



$$\Rightarrow \quad r + r_1 = \frac{a}{2} . \frac{1+\cos\frac{\pi}{n}}{\sin\frac{\pi}{n}} = \frac{a}{2} . \cot\left(\frac{\pi}{2n}\right)$$

80. **(C)** $\tan^{-1}\frac{x}{y} - \tan^{-1}\left(\frac{1-\frac{y}{x}}{1+\frac{y}{x}}\right)$

$$= \tan^{-1}\frac{x}{y} - \left(\tan^{-1}1 - \tan^{-1}\frac{y}{x}\right) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

81. **(C)** Given, $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi \Rightarrow \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right) = \pi - \cos^{-1}z$

$$\Rightarrow \quad xy - \sqrt{1-x^2}\sqrt{1-y^2} = \cos\left(\pi - \cos^{-1}z\right) = -\cos\left(\cos^{-1}z\right) = 0$$

$$\Rightarrow \quad xy + z = \sqrt{1-x^2}\sqrt{1-y^2} \Rightarrow x^2y^2 + z^2 + 2xyz = \left(1-x^2\right)\left(1-y^2\right)$$

$$= 1 - x^2 - y^2 + x^2y^2$$

$$\Rightarrow \quad x^2 + y^2 + z^2 = 1 - 2xyz$$

82. **(D)** If we denote $\cos^{-1}x$ by y, then

since $0 \le \cos^{-1}x \le \pi \quad \Rightarrow \quad 0 \le 2y \le 2\pi \qquad \ldots$(i)

Also since $-\frac{\pi}{2} \le \sin^{-1}(2x\sqrt{1-x^2}) \le \frac{\pi}{2}$

$\Rightarrow \qquad -\frac{\pi}{2} \le 2y \le \frac{\pi}{2} \qquad \ldots$(ii)

From (i) and (i) we find $0 \le 2y \le \frac{\pi}{2}$

$\Rightarrow \qquad 0 \le y \le \frac{\pi}{4}$

$\Rightarrow \qquad 0 \le \cos^{-1} x \le \frac{\pi}{4}$

which holds if $1/\sqrt{2} \le x \le 1$.

83. **(B)** The function $\sec^{-1} x$ is defined for all $x \in R-(-1, 1)$

and the function $\frac{1}{\sqrt{x-[x]}}$ is defined for all $x \in R - Z$

So the given function is defined for all $x \in R - \{(-1,1) \cup (n \mid n \in Z)\}$

84. **(C)** ${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + ... + (-1)^r . {}^nC_r$

$= {}^{n-1}C_0 - ({}^{n-1}C_0 + {}^{n-1}C_1) + ({}^{n-1}C_1 + {}^{n-1}C_2) - ({}^{n-1}C_2 + {}^{n-1}C_3) + ... + (-1)^r . ({}^{n-1}C_{r-1} + {}^{n-1}C_r)$

$= (-1)^r . {}^{n-1}C_r$

$(-1)^r . {}^{n-1}C_r = 28 \Rightarrow r$ must be even

${}^{n-1}C_r = 28 \Rightarrow {}^{n-1}C_r = 7 \times 4 = \frac{7 \times 8}{2} = {}^8C_2 \Rightarrow n-1 = 8 \Rightarrow n = 9$

85. **(C)** $x + y + 3z = 33 \quad \Rightarrow \quad x + y = 33 - 3z$

Let $z = k$. Then, $x + y = 33 - 3k$.

The number of non-negative integral solutions of $x + y = 33 - 3z$ is

${}^{33-3k+2-1}C_{2-1} = {}^{34-3k}C_1 = (34 - 3k)$

But $0 \le 33 - 3k \le 33 \quad \Rightarrow \quad 0 \le k \le 11$

Hence, total number of solutions $= \sum_{k=0}^{11}(34 - 3k) = \frac{12}{2}(34 + 1) = 210$.

86. **(B)** Select four cards in ${}^7C_4$ ways to put them in correct envelopes then remaining all will go wrong in 2 ways (since number of no matches for three objects is 2).

$\Rightarrow$ Required answer $= 2 \times {}^7C_4 = 70$

87. **(B)** Let x_4 be such that $x_4 = \frac{120}{a}$

Then the number of positive integral solutions of $x_1 x_2 x_3 = a$

$\Rightarrow \quad x_1 x_2 x_3 x_4 = 120 = 2^3 \times 3 \times 5$

We can assign 3 and 5 to unknown quantities in 4×4 ways

We can assign all 2 to one unknown in ${}^4C_1$ ways, to two unknowns in $\left({}^4C_2\right)(2)$

and to three unknown in ${}^4C_3$ ways

Hence, the number of required solutions

$$= 4 \times 4 \times [{}^4C_1 + \left({}^4C_2\right)(2) + {}^4C_3] = 4 \times 4 \times 20 = 320$$

88. **(C)** $t_{r+1} = (-1)^r.(n-r+2)\ {}^nC_r \cdot 2^{n-r+1}$

$$= (n+2)\cdot 2^{n+1}\cdot(-1)^r \cdot {}^n C_r \cdot \left(\frac{1}{2}\right)^r - 2^{n+1}\cdot(-1)^r \cdot r \cdot {}^n C_r \cdot \left(\frac{1}{2}\right)^r$$

$$= (n+2)\cdot 2^{n+1}\cdot(-1)^r \cdot {}^n C_r \cdot \left(\frac{1}{2}\right)^r - 2^n \cdot n \cdot (-1)^r \cdot {}^{n-1}C_{r-1} \cdot \left(\frac{1}{2}\right)^{r-1}$$

$$\therefore \quad \text{sum} = (n+2)\cdot 2^{n+1}\cdot\left\{{}^nC_0 - {}^nC_1 \cdot \frac{1}{2} + {}^nC_2 \cdot \left(\frac{1}{2}\right)^2 - \ldots..\right\}$$

$$- n \cdot 2^n \left\{- {}^{n-1}C_0 + {}^{n-1}C_1 \cdot \frac{1}{2} - {}^{n-1}C_2 \cdot \left(\frac{1}{2}\right)^2 + \ldots.\right\}$$

$$= (n+2)\cdot 2^{n+1}\cdot\left(1-\frac{1}{2}\right)^n + n\cdot 2^n \cdot \left(1-\frac{1}{2}\right)^{n-1} = 2(n+2) + 2n = 4n+4.$$

89. (B)

90. (C)