

# JEE EXPERT

## ANSWER KEY RANK ELEVATOR TEST SERIES

(RETS/PT-02)  
12<sup>TH</sup> (Zenith X01 & X02)  
Date 01.09.2019

### PHYSICS

1	(D)	2	(C)	3	(D)	4	(D)	5	(A)
6	(A)	7	(C)	8	(B)	9	(C)	10	(B)
11	(D)	12	(B)	13	(C)	14	(D)	15	(C)
16	(D)	17	(A)	18	(B)	19	(C)	20	(B)
21	(B)	22	(A)	23	(A)	24	(C)	25	(B)
26	(D)	27	(C)	28	(A)	29	(C)	30	(A)

### CHEMISTRY

31	(A)	32	(C)	33	(B)	34	(B)	35	(D)
36	(B)	37	(C)	38	(C)	39	(A)	40	(B)
41	(A)	42	(B)	43	(B)	44	(A)	45	(C)
46	(D)	47	(A)	48	(D)	49	(B)	50	(A)
51	(C)	52	(B)	53	(A)	54	(D)	55	(A)
56	(D)	57	(D)	58	(D)	59	(D)	60	(C)

### MATHEMATICS

61	(A)	62	(B)	63	(C)	64	(A)	65	(A)
66	(A)	67	(A)	68	(C)	69	(B)	70	(A)
71	(C)	72	(D)	73	(D)	74	(C)	75	(A)
76	(C)	77	(A)	78	(A)	79	(A)	80	(C)
81	(C)	82	(D)	83	(B)	84	(C)	85	(C)
86	(B)	87	(B)	88	(C)	89	(B)	90	(C)

# JEE EXPERT

## SOLUTIONS RANK ELEVATOR TEST SERIES

(RETS/PT-02)  
12<sup>TH</sup> (Zenith X01 & X02)  
Date 01.09.2019

### PART - I [PHYSICS]

1. **Sol. (D)**

At equilibrium final pressure and final temperature in both compartment are equal.

$$\frac{PV}{RT} = \frac{P'V'}{RT} \quad \dots(i)$$

$$\frac{(2P)(2V)}{RT} = \frac{P'V'}{RT} \quad \dots(ii)$$

$$V'' = 4V' \quad \dots(iii)$$

$$V' + V'' = 3V$$

on solving we get the result.

2. **Sol. (C)**

As ;  $W_{\text{ext}} = \Delta(\text{ME})$  ; ME = Mechanical energy.

Mechanical energy will keep on increasing upto the instant the  $W_{\text{ext}}$  is positive, which will happen till there is no compression in the spring. First the spring gets extended to a maximum and after which the extension decreases upto the natural length. After that there is a compression in the spring, results in a -ve external work (so as to move the end of spring at constant speed u).

Hence maximum energy stored is at the natural length and  $ME_{\text{max}} = \frac{1}{2}mv^2$

At the natural length  $v = 2u$ , since the block is moving at this instant at a speed u with respect to the other end of the spring.

$$\text{Hence } ME_{\text{max}} = \frac{1}{2}m(2v)^2 = 2mu^2.$$

3. **Sol. (D)**

$$\omega = \frac{2a}{T} = 4$$

$$a_{\text{max}} = \omega^2 A = 16 \times 1$$

$$T_{\text{max}} - mg = m \times 16 \Rightarrow T_{\text{max}} = 26m$$

To just slip

$$26 \text{ m} = \mu(2\text{mg})$$

$$\mu = \frac{13}{10}$$

4. **Sol. (D)**

$$\Delta U = \Delta Q - \Delta W = 2256 - P(\Delta V) = 2256 - 10^5 (1670 \times 10^{-6}) = 2256 - 167$$

5. **Sol. (A)**

$$\frac{\Delta \ell}{\ell} = \frac{\text{Stress}}{Y} = \frac{5 \times 10^7}{2 \times 10^{11}} = \frac{5}{2} \times 10^{-4}$$

$$\Delta \ell = \left( \frac{5}{2} \ell \times 10^{-4} \right)$$

$$V = \pi r^2 \ell$$

$$\frac{\Delta V}{V} = \frac{\Delta \ell}{\ell} - \frac{2\Delta r}{r}$$

$$2 \times 10^{-4} = \frac{5}{2} \times 10^{-4} - \frac{2\Delta r}{r}$$

$$\frac{\Delta r}{r} = \frac{1}{4} \times 10^{-4}$$

6. **Sol. (A)**

$$\text{Surface energy} = (8\pi r^2)T$$

$$p = \frac{d}{dt} (8\pi r^2 T) = 8\pi T \left( 2r \frac{dr}{dt} \right)$$

$$\Rightarrow p \propto r$$

7. **Sol. (C)**

8. **Sol. (B)**

9. **Sol. (C)**

10. **Sol. (B)**

11. **Sol. (D)**

$$\frac{E}{A} = y \frac{\Delta \ell}{\ell} \Rightarrow \Delta \ell = \frac{F \ell}{A y} ; \Delta \ell_1 = \frac{F}{A} \left( \frac{1.0}{10^{11}} \right) = 1 \text{ mm} ; \Delta \ell_2 = \frac{F}{A} \left( \frac{0.5}{2 \times 10^{11}} \right) = \frac{1}{4} \text{ mm} . \text{ Total extension} = 1.25 \text{ mm}$$

12. **Sol. (B)**

Due to induction damping will be there  $\omega = \sqrt{\omega_0^2 - \gamma^2}$ , where  $\gamma = \frac{b}{2m}$ , due to damping angular frequency decreases. Therefore time period will increase.

13. Sol. (C)

$$ms\left(\frac{dT}{dt}\right) = K(\Delta T) \quad \text{using approximate method.}$$

$$\text{For first case} \quad (50 + 30)(s_w) \left(\frac{5}{2}\right) = \left(\frac{30 + 25}{2} - T\right)k$$

$$(100S + 30s_w) \left(\frac{5}{2}\right) = \left(\frac{20 + 25}{2} - T\right)k$$

$$\text{So, } s_w = 100s + 30s_w$$

$$\text{So, } s_w = 100s \text{ } \therefore s = \frac{s_w}{2} = 0.5 \text{ kcal/g}$$

14. Sol. (D)

$$(1) \quad n \frac{4}{3} \pi r^3 = \frac{4}{3} \pi R^3 \text{ } \therefore nr^3 = R^3 \text{ } \therefore \frac{nr^2}{R^3} = \frac{1}{r}$$

$$(2) \quad nT_4pr^2 - T_4pR^2 = DU = \frac{1}{2} \left( \rho \cdot \frac{4}{3} \pi R^3 \right) v^2$$

$$v = \sqrt{\frac{6T}{\rho} \left( \frac{nr^2 - R^2}{R^3} \right)}$$

$$v = \sqrt{\frac{6T}{\rho} \left( \frac{nr^2}{R^3} - \frac{1}{R} \right)} = \sqrt{\frac{6T}{\rho} \left( \frac{1}{r} - \frac{1}{R} \right)}$$

15. Sol. (C)

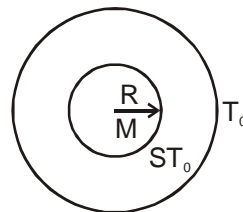
$$P_L = \sigma (4\pi R^2) [(T^4) - T_0^4]$$

$$-m\alpha T^3 \frac{dT}{dt} = \sigma 4\pi R^2 (T^4 - T_0^4)$$

$$\int_{3T_0}^{2T_0} \frac{T^3 dT}{T^4 - T_0^4} = \int_0^t \frac{-\sigma 4\pi R^2}{m\alpha} dt$$

Solving get

$$t = \frac{m\alpha}{16\sigma\pi R^2} \ln\left(\frac{16}{3}\right)$$

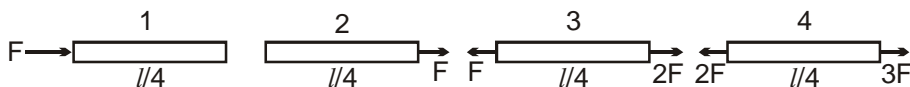


16. **Sol. (D)**

$r_0$  = perpendicular separation of 'p' from either of the wire. In steady state, the F.B.D. of the rod can be drawn

by dividing it into four parts as shown force is shown the  $\gamma \frac{\Delta \ell}{\ell} = (\text{stress})_{\text{average}} \Rightarrow \Delta \ell = \frac{\ell}{\gamma} \cdot \frac{(F_1 + F_2)}{2A}$

$$\text{Now, } \Delta \ell_{1(\text{compression})} = \frac{F \frac{\ell}{4}}{2A\gamma} = \frac{F\ell}{8A\gamma}; \quad \Delta \ell_{2(\text{elongation})} = \frac{F\ell}{8A\gamma}$$



For any rod. If  $F_2 \rightarrow \text{rod} \rightarrow F_1$

$$\Delta \ell_{3(\text{elongation})} = \frac{3F\ell}{8A\gamma}, \quad \Delta \ell_{4(\text{elongation})} = \frac{5F\ell}{8A\gamma}$$

$$\Delta \ell = \frac{8F\ell}{8A\gamma} = \frac{F\ell}{A\gamma}.$$

17. **Sol. (A)**

If  $\theta$  be temperature of B, then

$$\frac{2kA(\theta - 100)}{\ell} + \frac{(k/2)A(\theta - 0)}{\ell} = 200$$

Substituting value  $\theta = 880^\circ\text{C}$

$$\therefore \phi_t = mL$$

$$\frac{kA(880 - 0)}{2\ell} \times t = 440 \times 80$$

$$t = \frac{80 \times 1 \times 2 \times 440}{100 \times 10 \times 10^{-4} \times 880} = 800 \text{ sec} = \frac{40}{3} \text{ min}$$

18. **Sol. (B)**

$eA\sigma T^4$  = Power emitted per unit time

$$= e(4\pi r^2)\sigma T^4$$

$$r \rightarrow 2r$$

Power gets 4 times

Power received by planet also gets 4 times higher, power received = Power radiated (for equilibrium of temperature of planet)

$$4P = e(4\pi r^2)(T_1)^4, \text{ initially } P = e(4\pi r^2) \sigma T^4 \text{ (for planet),}$$

$$\frac{T_1}{T} = \sqrt{2}$$

19. **Sol. (C)**

For small element  $AB = R d\theta$

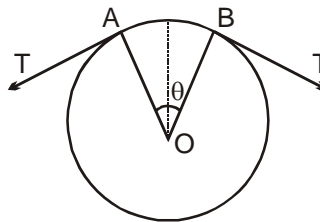
Tension provides centripetal force

$$2T \sin \frac{\theta}{2} = 2T \left( \frac{\theta}{2} \right)$$

$$= \rho(A)(r\theta)\omega^2 r$$

$$T = \rho A \omega^2 r^2$$

$$\text{stress} = \frac{T}{A} = \rho \omega^2 r^2$$



20. **Sol. (A)**

$$(a^3 \rho + m)g = a^2 x_1 \rho_\ell g \quad \dots(1)$$

$$a^3 \rho g = a^2 x_2 \rho_\ell g \quad \dots(2)$$

$$(1) - (2)$$

$$mg = a^2 r_\ell g (x_1 - x_2)$$

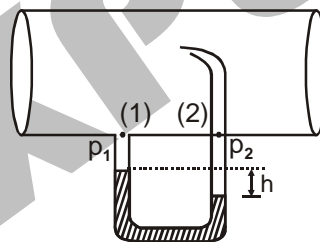
21. **Sol. (B)**

Applying Bernoulli equation between points (1) & (2)

$$p_1 + \frac{1}{2} \rho v^2 = p_2$$

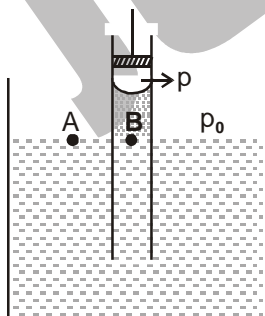
$$p_2 - p_1 = \frac{1}{2} \rho v^2 = \rho_0 g \Delta h$$

$$v = \sqrt{\frac{2\rho_0 g \Delta h}{\rho}}$$



22. **Sol. (A)**

Pressure at point A & B will be equal



$$10^5 = \frac{1}{2} \times 10^5 - \frac{2T}{r} + \rho g h \quad \therefore h = 15 \text{ m}$$

23. **Sol. (A)**

$$\text{Relative density of cube} = \frac{1}{3} \text{ (R.D. of mercury)} = \frac{1}{3} (g) = 3$$

Now after pouring water.

Mass of cube = mass of mercury displaced + mass of water displaced.

$$V(3) = \frac{V}{x}(g) + V\left(1 - \frac{1}{x}\right); 3 = \frac{g}{x} + 1 - \frac{1}{x}; 2 = \frac{8}{x}; x = 4$$

24. **Sol. (C)**

$$\frac{PV}{R} = nT = \text{const}; nT = \text{const.} \quad \left(\frac{dn}{dt}\right)T + n\left(\frac{dT}{dt}\right) = 0$$

$$\frac{\Delta Q}{\Delta t} = \frac{T_0 - 0.8T_0}{VR} = \frac{RT_0}{5}$$

25. **Sol. (B)**

Let  $n$  drops coalesce to form a big drop. Then

$$N \times \frac{1}{3} \pi r^3 = \frac{4}{3} \pi R^3 \Rightarrow n = \frac{R^3}{r^3}$$

$$\text{Heat produced} = \frac{\text{Work done}}{J} = S.T \times \frac{\text{decrease in area}}{J} = mc \, d\theta \Rightarrow d\theta = \frac{3T}{J} \left( \frac{1}{r} - \frac{1}{R} \right)$$

26. **Sol. (D)**

$$W_{BC} = P \Delta V = nR \Delta T = -nRT_0$$

$$W_{CA} = +2nRT_0 \ln 2$$

$$\Delta Q_{BC} = nC_P \Delta T = \frac{nR_\gamma T_0}{\gamma - 1}$$

$$\text{Hence, efficiency} = \frac{(2 \ln 2 - 1)}{\gamma / (\gamma - 1)}$$

27. **Sol. (C)**

$$\text{By definition, Bulk's modulus} \quad K = - \frac{dP}{\left(\frac{dV}{V}\right)} = \frac{mg/A}{dV/V}$$

$$V = \frac{4}{3} \pi R^3 \quad \frac{dV}{V} = 3 \frac{dR}{R} \quad K = \frac{mg}{3A \left(\frac{dR}{R}\right)} \quad \frac{dR}{R} = - \frac{mg}{3AK}$$

28. **Sol. (A)**

29. **Sol. (C)**

30. **Sol. (A)**

## PART - III [MATHEMATICS]

61. (A)

62. (B) Normals at  $t_1$  and  $t_2$  meet at some point  $(-t_3)$

$$\Rightarrow -t_3 = \left(-t_1 - \frac{2}{t_1}\right) = \left(-t_2 - \frac{2}{t_2}\right)$$

$$\Rightarrow (t_1 - t_2) + 2\left[\frac{1}{t_1} - \frac{1}{t_2}\right] = 0 \Rightarrow \frac{(t_1 t_2 - 2)(t_1 - t_2)}{t_1 t_2} = 0$$

$$\Rightarrow t_1 t_2 = 2$$

63. (C) Equation of tangent to the parabola,

$$y = mx + \frac{a}{m}$$

$$y = m(x + 2) + \frac{2}{m}$$

$$\therefore -4 = -2m + \frac{2}{m}$$

$$-4m = -2m^2 + 2$$

$$m^2 - 2m - 1 = 0$$

$$m_1 + m_2 = 2$$

$$m_1 m_2 = -1$$

$$\Rightarrow \theta = \pi/2$$

64. (A) The equation of the tangent to  $y^2 = 8x$  at  $P(2, 4)$  is

$$4y = 4(x + 2) \text{ or } x - y + 2 = 0 \dots(i)$$

Let  $(x_1, y_1)$  be the mid-point of chord  $QR$ . Then, equation of  $QR$  is

$$yy_1 - 4(x + x_1) - 5 = y_1^2 - 8x_1 - 5$$

$$4x - yy_1 - 4x_1 + y_1^2 = 0 \dots(ii)$$

Clearly, (i) and (ii) represent the same line. So,

$$\frac{4}{1} = \frac{-y_1}{-1} = \frac{-4x_1 + y_1^2}{2}$$

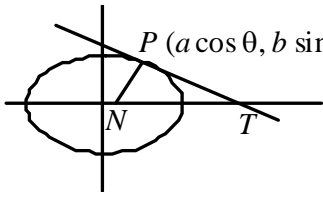
$$\Rightarrow y_1 = 4 \text{ and } 8 = 4x_1 + y_1^2$$

$$\Rightarrow y_1 = 4 \text{ and } x_1 = 2.$$



65. (A)  $e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{3}{5}$

$T \equiv \left( \frac{a}{\cos \theta}, 0 \right),$



$$\frac{a \sec \theta - ae^2 \cos \theta}{2ae} = \frac{91}{60} \Rightarrow \cos \theta = \frac{1}{2}, \sin \theta = \frac{\sqrt{3}}{2}$$

$$\Delta PSS' = \frac{1}{2} \times 6 \times 4 \times \frac{\sqrt{3}}{2} = 6\sqrt{3} \text{ sq. units}$$

66. (A) Let an end of a latus-rectum be  $(ae, b\sqrt{1-e^2})$ , then the equation of the normal at this end is

$$\frac{x - ae}{ae/a^2} = \frac{y - b\sqrt{1-e^2}}{b\sqrt{1-e^2}/b^2}$$

It will pass through the end  $(0, -b)$  if

$$-a^2 = \frac{-b^2(1 + \sqrt{1-e^2})}{\sqrt{1-e^2}} \quad \text{or} \quad \frac{b^2}{a^2} = \frac{\sqrt{1-e^2}}{1 + \sqrt{1-e^2}}$$

or  $(1-e^2)[1 + \sqrt{1-e^2}] = \sqrt{1-e^2}$

or  $\sqrt{1-e^2} + 1 - e^2 = 1$  or  $e^4 + e^2 = 1$ .

67. (A) For the ellipse  $\frac{x^2}{16} + \frac{y^2}{9} = 1$ , we have  $a^2 = 16, b^2 = 9$ .

Let  $e$  be the eccentricity of the ellipse. Then

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$$

The coordinates of the foci are  $(\pm ae, 0)$  or  $(\pm \sqrt{7}, 0)$ .

So, radius of the circle = distance between  $(\pm \sqrt{7}, 0)$  and  $(0, 3) = \sqrt{7+9} = 4$ .

68. (C) Chords of contact are  $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$  and  $\frac{xx_2}{a^2} + \frac{yy_2}{b^2} = 1$

Product of the slopes = -1

$$\Rightarrow -\frac{x_1/a^2}{y_1/b^2} \cdot -\frac{x_2/a^2}{y_2/b^2} = -1 \Rightarrow \frac{x_1 x_2}{a^4} \cdot \frac{b^4}{y_1 y_2} = -1 \Rightarrow \frac{x_1 x_2}{y_1 y_2} = -\frac{a^4}{b^4}$$

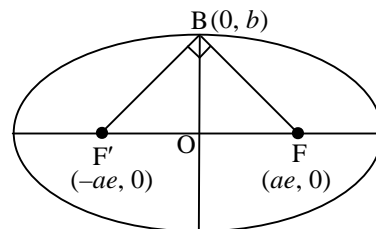
69. (B) Clearly  $FB = F'B \therefore \Delta FBF'$  is isosceles.  
 $\therefore \angle OBF = 45^\circ$

$$\tan OBF = \frac{OF}{OB} = \frac{ae}{b}$$

$$\text{m} \quad \frac{ae}{b} = 1 \quad (\because \tan 45^\circ = 1)$$

$$\Rightarrow ae = b \Rightarrow a^2 e^2 = b^2 = a^2 (1 - e^2)$$

$$\Rightarrow e^2 = 1 - e^2 \Rightarrow 2e^2 = 1 \Rightarrow e^2 = \frac{1}{2} \Rightarrow e = \frac{1}{\sqrt{2}}$$



70. (A) Let circle is  $x^2 + y^2 + 2gx + 2fy + 1 = 0$

Solve with  $y = \frac{1}{x}$ , we get,  $x^4 + 2gx^2 + cx^2 + 2fx = 1 = 0$

$$\Rightarrow x_1 x_2 x_3 x_4 = 1 \Rightarrow y_1 y_2 y_3 y_4 = 1$$

71. (C) We have  $\frac{2b^2}{a} = 8$  and  $2b = \frac{1}{2}(2ae) \therefore \frac{2}{a}\left(\frac{ae}{2}\right)^2 = 8 \Rightarrow ae^2 = 16 \dots(i)$

$$\text{Now, } 2\frac{b^2}{a} = 8 \Rightarrow b^2 = 4a \Rightarrow a^2(e^2 - 1) = 4a \Rightarrow ae^2 - a = 4 \dots(ii)$$

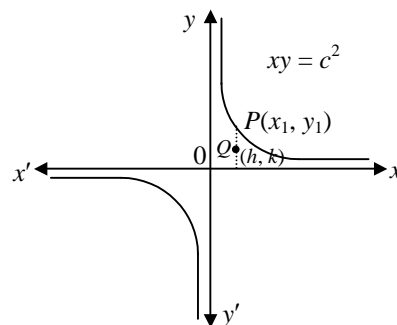
$$\text{From (i) and (ii), } 16 - \frac{16}{e^2} = 4 \Rightarrow \frac{16}{e^2} = 12 \Rightarrow e = \frac{2}{\sqrt{3}}.$$

72. (D) Let  $xy = c^2$  be the rectangular hyperbola, and let  $P(x_1, y_1)$  be a point on it. Let  $Q(h, k)$  be the mid-point of  $PN$ .

Then the coordinates of  $Q$  are  $\left(x_1, \frac{y_1}{2}\right)$

$$\therefore x_1 = h \text{ and } \frac{y_1}{2} = k \Rightarrow x_1 = h \text{ and } y_1 = 2k$$

$$\text{But } (x_1, y_1) \text{ lies on } xy = c^2 \\ h \cdot (2k) = c^2 \Rightarrow hk = c^2/2.$$



73. (D) Let  $P(a \sec \theta, b \tan \theta)$ ,  $Q(a \sec \theta, -b \tan \theta)$  be end points of double ordinate and  $C(0, 0)$  is the centre of the hyperbola.

Now,  $PQ = 2b \tan \theta$

$$CQ = CP = \sqrt{a^2 \sec^2 \theta + b^2 \tan^2 \theta}$$

Since  $CQ = CP = PQ$

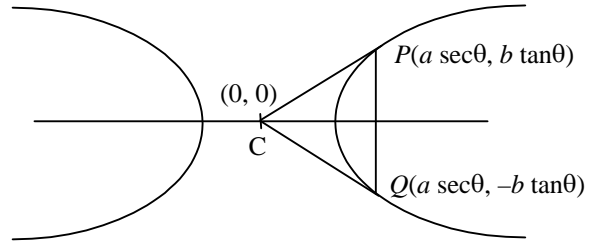
$$\therefore 4b^2 \tan^2 \theta = a^2 \sec^2 \theta + b^2 \tan^2 \theta$$

$$\Rightarrow 3b^2 \tan^2 \theta = a^2 \sec^2 \theta \Rightarrow 3b^2 \sin^2 \theta = a^2$$

$$\Rightarrow 3a^2(e^2 - 1)\sin^2 \theta = a^2 \Rightarrow 3(e^2 - 1)\sin^2 \theta = 1$$

$$\Rightarrow \frac{1}{3(e^2 - 1)} = \sin^2 \theta < 1 \quad (\because \sin^2 \theta < 1)$$

$$\Rightarrow \frac{1}{e^2 - 1} < 3 \Rightarrow e^2 - 1 > \frac{1}{3} \Rightarrow e^2 > \frac{4}{3} \Rightarrow e > \frac{2}{\sqrt{3}}$$



74. (C) Adding the given two.

$$\sin(B + C) = 1 \Rightarrow B + C = 90^\circ$$

$$\text{Subtracting, } \sin(B - C) = \frac{\sqrt{3} - 2}{\sqrt{3}} \neq 0$$

$$\therefore B \neq C$$

$\therefore$  Triangle is right-angled.

75. (A) A.M.  $\geq$  G.M  $\Rightarrow \frac{\tan A + \tan B + \tan C}{3} \geq (\tan A \tan B \tan C)^{1/3}$

Also we know that  $\tan A + \tan B + \tan C = \tan A \tan B \tan C$  when  $A + B + C = \pi$ .

76. (C)  $\left(1 - \frac{s-b}{s-a}\right)\left(1 - \frac{s-c}{s-a}\right) = 2$

$$\Rightarrow (b-a)(c-a) = 2(s-a)^2 \Rightarrow 2(bc - ac - ab + a^2) = (b+c-a)^2 \Rightarrow a^2 = b^2 + c^2$$

77. (A) We have  $a + b + c = \frac{6(\sin A + \sin B + \sin C)}{3}$

$$\Rightarrow k(\sin A + \sin B + \sin C) = 2(\sin A + \sin B + \sin C),$$

$$\text{where } k = \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow k = 2 \quad [\because \sin A + \sin B + \sin C \neq 0]$$

$$\Rightarrow \frac{a}{\sin A} = 2 \Rightarrow \sin A = \frac{1}{2} \quad [\because a = 1]$$

$$\Rightarrow A = \frac{\pi}{6}$$

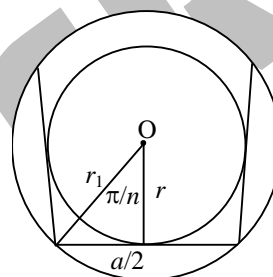
78. (A)  $\cos A + \cos B = 4 \sin^2 \frac{C}{2}$

$$\Rightarrow \tan \frac{A}{2} \cdot \tan \frac{B}{2} = \frac{1}{3} \Rightarrow \frac{\Delta^2}{s(s-a)s(s-b)} = \frac{1}{3} \Rightarrow 3(s-c) = s$$

$$\Rightarrow a + b + c = 3c \Rightarrow \frac{a+b}{2} = c$$

79. (A)  $\therefore r = \frac{a}{2} \cot \frac{\pi}{n}$  and  $r_1 = \frac{a}{2} \operatorname{cosec} \frac{\pi}{n}$

$$\Rightarrow r + r_1 = \frac{a}{2} \cdot \frac{1 + \cos \frac{\pi}{n}}{\sin \frac{\pi}{n}} = \frac{a}{2} \cdot \cot \left( \frac{\pi}{2n} \right)$$



80. (C)  $\tan^{-1} \frac{x}{y} - \tan^{-1} \left( \frac{1 - \frac{y}{x}}{1 + \frac{y}{x}} \right)$

$$= \tan^{-1} \frac{x}{y} - \left( \tan^{-1} 1 - \tan^{-1} \frac{y}{x} \right) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

81. (C) Given,  $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi \Rightarrow \cos^{-1} (xy - \sqrt{1-x^2} \sqrt{1-y^2}) = \pi - \cos^{-1} z$

$$\Rightarrow xy - \sqrt{1-x^2} \sqrt{1-y^2} = \cos(\pi - \cos^{-1} z) = -\cos(\cos^{-1} z) = 0$$

$$\Rightarrow xy + z = \sqrt{1-x^2} \sqrt{1-y^2} \Rightarrow x^2 y^2 + z^2 + 2xyz = (1-x^2)(1-y^2)$$

$$= 1 - x^2 - y^2 + x^2 y^2$$

$$\Rightarrow x^2 + y^2 + z^2 = 1 - 2xyz$$

82. (D) If we denote  $\cos^{-1}x$  by  $y$ , then  
 since  $0 \leq \cos^{-1}x \leq \pi \Rightarrow 0 \leq 2y \leq 2\pi \dots(i)$

$$\text{Also since } -\frac{\pi}{2} \leq \sin^{-1}(2x\sqrt{1-x^2}) \leq \frac{\pi}{2}$$

$$\Rightarrow -\frac{\pi}{2} \leq 2y \leq \frac{\pi}{2} \dots(ii)$$

$$\text{From (i) and (ii) we find } 0 \leq 2y \leq \frac{\pi}{2}$$

$$\Rightarrow 0 \leq y \leq \frac{\pi}{4}$$

$$\Rightarrow 0 \leq \cos^{-1}x \leq \frac{\pi}{4}$$

$$\text{which holds if } 1/\sqrt{2} \leq x \leq 1.$$

83. (B) The function  $\sec^{-1}x$  is defined for all  $x \in R - (-1, 1)$

$$\text{and the function } \frac{1}{\sqrt{x-[x]}} \text{ is defined for all } x \in R - Z$$

$$\text{So the given function is defined for all } x \in R - \{(-1, 1) \cup (n | n \in Z)\}$$

84. (C)  ${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \dots + (-1)^r \cdot {}^nC_r$   
 $= {}^{n-1}C_0 - ({}^{n-1}C_0 + {}^{n-1}C_1) + ({}^{n-1}C_1 + {}^{n-1}C_2) - ({}^{n-1}C_2 + {}^{n-1}C_3) + \dots + (-1)^r \cdot ({}^{n-1}C_{r-1} + {}^{n-1}C_r)$   
 $= (-1)^r \cdot {}^{n-1}C_r$   
 $(-1)^r \cdot {}^{n-1}C_r = 28 \Rightarrow r \text{ must be even}$   
 ${}^{n-1}C_r = 28 \Rightarrow {}^{n-1}C_r = 7 \times 4 = \frac{7 \times 8}{2} = {}^8C_2 \Rightarrow n-1 = 8 \Rightarrow n = 9$

85. (C)  $x + y + 3z = 33 \Rightarrow x + y = 33 - 3z$   
 Let  $z = k$ . Then,  $x + y = 33 - 3k$ .  
 The number of non-negative integral solutions of  $x + y = 33 - 3z$  is  
 ${}^{33-3k+2-1}C_{2-1} = {}^{34-3k}C_1 = (34 - 3k)$   
 But  $0 \leq 33 - 3k \leq 33 \Rightarrow 0 \leq k \leq 11$

$$\text{Hence, total number of solutions} = \sum_{k=0}^{11} (34 - 3k) = \frac{12}{2} (34 + 1) = 210.$$

86. (B) Select four cards in  ${}^7C_4$  ways to put them in correct envelopes then remaining all will go wrong in 2 ways (since number of no matches for three objects is 2).  
 $\Rightarrow \text{Required answer} = 2 \times {}^7C_4 = 70$

87. (B) Let  $x_4$  be such that  $x_4 = \frac{120}{a}$

Then the number of positive integral solutions of  $x_1 x_2 x_3 = a$

$$\Rightarrow x_1 x_2 x_3 x_4 = 120 = 2^3 \times 3 \times 5$$

We can assign 3 and 5 to unknown quantities in  $4 \times 4$  ways

We can assign all 2 to one unknown in  ${}^4C_1$  ways, to two unknowns in  $({}^4C_2)(2)$

and to three unknown in  ${}^4C_3$  ways

Hence, the number of required solutions

$$= 4 \times 4 \times [{}^4C_1 + ({}^4C_2)(2) + {}^4C_3] = 4 \times 4 \times 20 = 320$$

88. (C)  $t_{r+1} = (-1)^r \cdot (n-r+2) {}^nC_r \cdot 2^{n-r+1}$

$$= (n+2) \cdot 2^{n+1} \cdot (-1)^r \cdot {}^nC_r \cdot \left(\frac{1}{2}\right)^r - 2^{n+1} \cdot (-1)^r \cdot r \cdot {}^nC_r \cdot \left(\frac{1}{2}\right)^r$$

$$= (n+2) \cdot 2^{n+1} \cdot (-1)^r \cdot {}^nC_r \cdot \left(\frac{1}{2}\right)^r - 2^n \cdot n \cdot (-1)^r \cdot {}^{n-1}C_{r-1} \cdot \left(\frac{1}{2}\right)^{r-1}$$

$$\begin{aligned} \therefore \text{sum} &= (n+2) \cdot 2^{n+1} \cdot \left\{ {}^nC_0 - {}^nC_1 \cdot \frac{1}{2} + {}^nC_2 \cdot \left(\frac{1}{2}\right)^2 - \dots \right\} \\ &\quad - n \cdot 2^n \cdot \left\{ -{}^{n-1}C_0 + {}^{n-1}C_1 \cdot \frac{1}{2} - {}^{n-1}C_2 \cdot \left(\frac{1}{2}\right)^2 + \dots \right\} \\ &= (n+2) \cdot 2^{n+1} \cdot \left(1 - \frac{1}{2}\right)^n + n \cdot 2^n \cdot \left(1 - \frac{1}{2}\right)^{n-1} = 2(n+2) + 2n = 4n + 4. \end{aligned}$$

89. (B)

90. (C)