ROTATIONAL MECHANICS SOLUTIONS

1. $|\vec{L}| = |\vec{r} \times \vec{p}|$ = mv_0a

2. From COE, mgh $= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$\Rightarrow$ mgh $= \frac{1}{2}mv^2 + \frac{1}{2}.\ (\frac{1}{2}mR^2) \times \frac{v^2}{R^2} = \frac{3}{4}mv^2$

$\Rightarrow v = \sqrt{\frac{4gh}{3}}$

3. $I_1\ \omega_1 = I_2\ \omega_2$

$\Rightarrow \frac{2}{5}mR^2.\frac{2\pi}{24} = \frac{2}{5}m\left(\frac{R}{2}\right)^2 \times \frac{2\pi}{T_2}$

$\Rightarrow T_2 = 6$ hrs.

4. Speed at P = $\sqrt{v_0^2 + v_0^2}$ $= v_0\sqrt{2}$.

Speed at Q = 0

5. (a) Upward along inclined surface.

(b) $\frac{E_{Translational}}{E_{Total}} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2} = \frac{2}{3}$.

(c) In case of rolling, there is no slipping so work done by the friction force is zero.

6. (a) Upward along inclined surface.

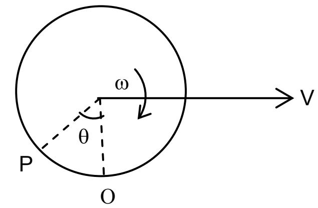
(b) $\frac{E_{Translational}}{E_{Total}} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2} = \frac{2}{3}$.

(c) In case of rolling, there is no slipping so work done by the friction force is zero.

7. $|\vec{v}_p| = (OP)\omega$

$OP = 2R \sin \theta/2$

$|v_P| = 2R \sin \theta/2.\ \omega = 2V \sin \theta/2$



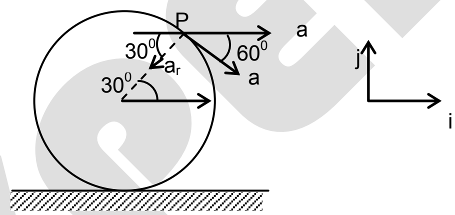
8. $|\vec{L}| = |\vec{r} \times \vec{p}| = mv_0a$

9. From COE, $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}.\ (\frac{1}{2}mR^2) \times \frac{v^2}{R^2} = \frac{3}{4}mv^2$

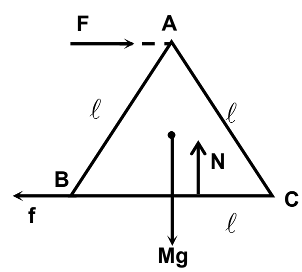
$\Rightarrow v = \sqrt{\frac{4gh}{3}}$

10. $a_P = \left(\frac{3a}{2} - \frac{v^2\sqrt{3}}{2R}\right)\hat{i} - \left(\frac{a\sqrt{3}}{2} + \frac{v^2}{2R}\right)\hat{j}$



11. Torque of normal reaction about B on the wedge is τ_N = Torque due to frictional force (f) + torque due to applied force (F) + torque due to weight of the wedge (Mg)

$\tau_N = 0 + F\left(\frac{\sqrt{3}}{2}\ell\right) + Mg\left(\frac{\ell}{2}\right) = \frac{(Mg + \sqrt{3}F)\ell}{2}$

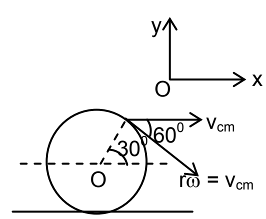


12. As disc is in pure rolling state

$v_{cm} = r\omega = 10$ m/s.

$\bar{v}_A = 10\hat{i} + 10\cos 60\hat{i} - 10\sin 60\hat{j}$`

$\bar{v}_A = 15\hat{i} - 5\sqrt{3}\,\hat{j}$.



13. $I_{x_1x_2} = \frac{2}{3}MR^2$

Coordinate of C. M. = (0, R/2)

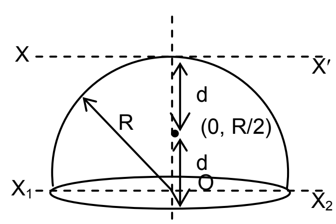
Treating O as origin

d = R – R/2 = R/2

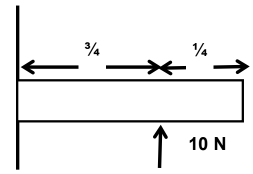
$I_{XX'} = I_{cm} + Md^2$

$I_{XX'} = I_{x_1x_2} - Md^2 + Md^2$

$I_{XX'} = \frac{2}{3}MR^2$.



14. $\tau = I\,\alpha$



$10\times\frac{3}{4}=\frac{(2)(1)^2}{3}\times\alpha$

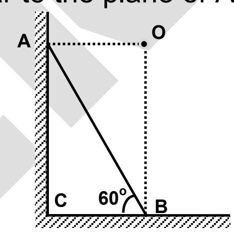
$\alpha = \frac{45}{4}$ rad/sec^2

$\theta=\frac{1}{2}\alpha t^2$

$\theta = \frac{1}{2}\,\frac{45}{4}\,(1)^2 = 5.62$ radian.

15. Consider the ends A and B of the ladder. Velocity of end A is vertically downward and velocity of end B is along the horizontal floor. As perpendiculars to the velocities meet at point O. Therefore, axis of rotation will pass through this point and will be perpendicular to the plane of ABC.

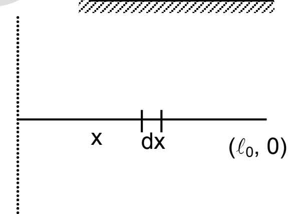
Taking C as origin co-ordinates of the point O are 10 cos60^o m and 10sin60^o m. Hence radius vector of the point O w.r.t. point C is $\vec{R}=5\hat{i}+5\sqrt{3}\hat{j}$ where $\hat{i}$ and $\hat{j}$ are the unit vectors along horizontal and vertical.



16. Take a small element at a distance x

$dI = (\lambda_0\, x dx)\, x^2 \qquad = \lambda_0\, x^3\, dx$

$I = \int_0^{\ell_0}\lambda_0\, x^3 dx = \frac{\lambda_0\ell_0^4}{4}$



17. $\frac{PL}{2}=\frac{ML^2\omega}{12}$

$\omega = \frac{6P}{mL}$

$\theta = \omega t = \frac{6P}{mL}\times\frac{\pi mL}{24P}=\frac{\pi}{4}$

18. (a) Instantaneous angular velocity, $\omega=\frac{d\theta}{dt}=\frac{d}{dt}(at^2)$

$\Rightarrow\ \omega = 2at = 0.4t$

Angular velocity at t = 2.5 sec. is ω = 0.4(2.5)=1.0 rad/sec.

Instantaneous angular acceleration , $\alpha=\frac{d\omega}{dt}=\frac{d}{dt}(0.4t)$

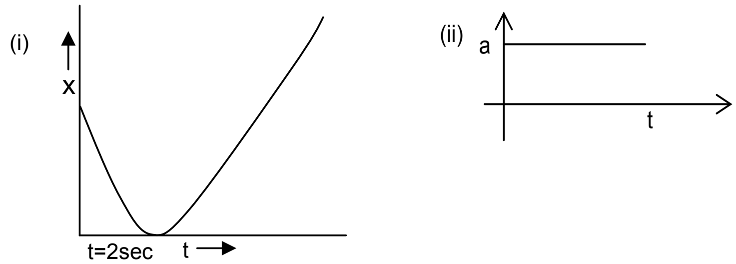
Angular acceleration at 2.5 sec is 0.4 rad/sec^2

$\Rightarrow\ a_n = R\omega^2 = (0.65)(1.0)^2 = 0.65 m/sec^2$

$a_t = R\alpha = (0.65)(0.4) = 0.26\ m/sec^2$

Magnitude of total acceleration $a=\sqrt{a_n^2+a_T^2}=\sqrt{(0.65)^2+(0.26)^2}=0.7 m/s^2$

(b)



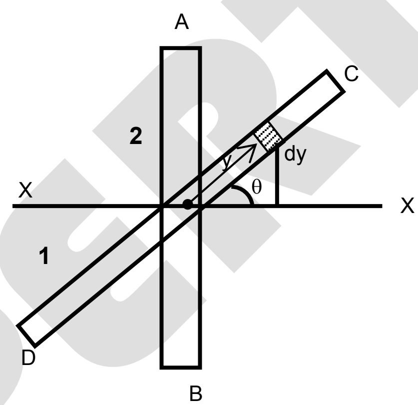
19.

$$I_1 = \int_{-L/2}^{+L/2} \frac{M}{L} dy (y \sin\theta)^2$$

$$I_1 = \frac{ML^2}{12}\sin^2\theta$$

$$I_2 = \frac{ML^2}{12}$$

$$\therefore I = I_1 + I_2 = \frac{ML^2}{12}(1+\sin^2\theta)$$



20.

$$\Delta PE = \tfrac{1}{2} I\omega^2$$

$$mg\left(\frac{2R}{\pi}\right) = \frac{1}{2}\left(\frac{m}{2}\right)R^2\omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{8g}{\pi R}}$$

$$\omega = \sqrt{\frac{8\times 10\times 2}{\pi}}$$

$$\omega = 4\sqrt{\frac{10}{\pi}}\ \text{rad/sec}$$

21.

$$I_1 + I_2 = I_{total}$$

$$2\times\frac{M}{12}\left[\left(\frac{\ell}{2}\right)^2 + \left(\frac{\ell}{2}\right)^2\right]$$

$$\frac{ML^2}{12} = I_{Total}$$

$$\tau = \frac{ML^2}{12}.\alpha$$

$$\alpha = \frac{10\times 12}{2\times(1)^2} = 60\ \text{rad/sec}^2$$

$$\omega_t = \omega_0 + 60\times 2 = 120\ \text{rad/sec}$$

22. (i) $\alpha = \frac{d\omega}{dt} \quad \Rightarrow \quad d\omega = \alpha\, dt$

$$\Rightarrow \quad \int_{\omega_o}^{\omega} d\omega = \int_0^t \alpha\, dt = \int_0^t (4at^3 - 3bt^2)\, dt$$

$$\Rightarrow \quad \omega = \omega_0 + at^4 - bt^3$$

(ii) Further,

$$\omega = \frac{d\theta}{dt} \Rightarrow d\theta = \omega\, dt$$

$$\Rightarrow \int_0^{\theta} d\theta = \int_0^t \omega\, dt = \int_o^t (\omega_0 + at^4 - bt^3)\, dt$$

$$\Rightarrow \theta = \omega_0 t + \frac{at^5}{5} - \frac{bt^4}{4}$$

23. $\frac{MR^2}{2}\omega$

24. Mass per unit area $\sigma = \frac{M}{\pi(R_2^2 - R_1^2)}$

$I_0 = (I_{complete})_0 - (I_{removed})_0$

$= \frac{1}{2}(\sigma\pi R_2^2)R_2^2 - \frac{1}{2}(\sigma\pi R_1^2)R_1^2$

$= \frac{1}{2}\sigma\pi(R_2^4 - R_1^4)$

$= \frac{1}{2}M(R_2^2 + R_1^2)$

25. Let σ = mass per unit square unit of square mass of full square (M) = $\sigma(2\ell)^2 = 4\sigma\ell^2$

mass of cut off square $(M') = \sigma\ell^2$

$$x_{cm} = \frac{M\ell + (-M')\frac{3\ell}{2}}{M + (-M')} = \frac{4\sigma\ell^3 + (-\sigma\ell^2)(3\ell/2)}{\sigma\ell(4-1)} = \frac{5\ell}{6}$$

$$y_{cm} = \frac{M\ell + (-M')(\ell/2)}{M + (-M')} = \frac{7\ell}{6}$$

26. (a) About AA′

due to rod MI = $\frac{M\ell^2}{3}$

due to small masses = $\left(\frac{M}{2}\right)\left(\frac{\ell}{2}\right)^2 + \frac{M}{2}(\ell)^2$

Total MI of system about AA′ = $M\ell^2\left[\frac{1}{3} + \frac{1}{2} + \frac{1}{8}\right] = \frac{23M\ell^2}{24}$

(b) About BB′

due to rod, MI = $\frac{M\ell^2}{12}$

Due to small masses $= 0 + \frac{M}{2}\left(\frac{\ell}{2}\right)^2$

Total MI of system about BB′ $= M\ell^2 \left[\frac{1}{8} + \frac{1}{12}\right] = \frac{5}{24}M\ell^2$

27. Newton's second law in the horizontal direction——

$F + F_f = ma_G$ ..(1)

For angular acceleration α

$FR - F_f R = I_G\alpha$

$\therefore F - F_f = \frac{mR\alpha}{2} = \frac{ma_G}{2}$ ...(2)

Solving for F_f from equation (1) and (2)

$F_f = \frac{F}{3}$ $\therefore F_f = \frac{F}{3}$ rightward $\Rightarrow$ The force of friction exerted on the surface $= \frac{F}{3}$ left ward.

28. As the net torque about A during the collision is zero, the angular momentum of the system about A is conserved.

$\therefore mv_0 \left(\frac{L}{2}\right) = \left(m\left(\frac{L}{2}\right)^2 + \frac{mL^2}{3}\right)\omega$

$\omega = \frac{6V_0}{7L}$ anticlockwise

29. $\frac{E_R}{E_T} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2} = \frac{1}{3}$ (where $I = MR^2/2$)

30. At the highest point it has only horizontal velocity $v_x = v\cos\theta$

$H_{max} = \frac{v^2 \sin^2\theta}{2g}$

$\Rightarrow$ Angular momentum

$L = \frac{mv^3 \sin^2\theta\cos\theta}{2g}\ (-\hat{k})$.

31. $kx = m\omega^2 (\ell_0 + x)$

$\Rightarrow x \approx 1$ cm.

32. $2I_d = I_G$

$I_G = 2MK^2$

33. $I\omega = 2I\omega'$ $\Rightarrow \omega' = \omega/2$

loss in K.E. $= \frac{1}{2}I\omega^2 - \frac{1}{2}(2I)(\omega')^2$

$= \frac{1}{4}I\omega^2$.

34. Let the thickness of the plate be t

$$x_{cm} = \frac{\frac{A}{2}t\rho_1 \times \frac{\ell}{4} + \frac{A}{2}t\rho_2 \times \frac{3\ell}{4}}{\frac{A}{2}t\rho_1 + \frac{A}{2}t\rho_2} = \frac{(\rho_1 + 3\rho_2)\ell}{4(\rho_1 + \rho_2)}$$

35. $v_{cm} = \frac{mv_0 + m(0)}{2m} = \frac{v_0}{2}$

so $x(t) = \frac{\ell}{2} + \frac{v_0 t}{2}$

36. $2\left[\frac{mr^2}{4} + mr^2\right] = \frac{5}{2}mr^2$

37. $a_{cm} = \frac{F}{m}$

$\tau_{\text{about COM}} = \frac{F\ell}{2}$

$I_{cm} = \frac{m\ell^2}{12}$ $\therefore$ $\alpha = \frac{6F}{m\ell}$

$\therefore$ $a_{B,\,g} = a_{cm,\,g} + a_{B,\,cm}$

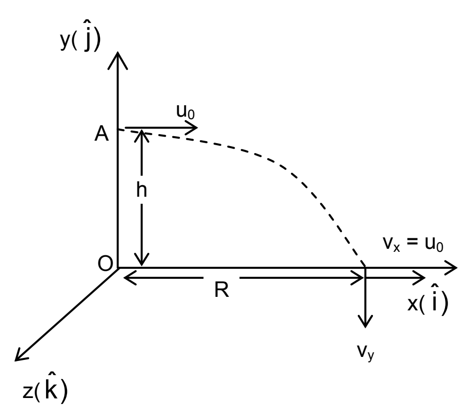
$= \frac{F}{m} + \frac{6F}{m\ell} \times \frac{\ell}{2} = \frac{4F}{m}$

38. $I = ML^2 / 12$ axis passing through CG.

$I = Mk^2$

$\frac{1}{12}ML^2 = Mk^2 \Rightarrow$ $k = 0.289$ m

39. If t is the time taken by the particle to reach the ground: vertical motion: $h = 0 + \frac{1}{2}gt^2$



$\Rightarrow t = \frac{2h}{g}$ and $v_y^2 = 0^2 + 2gh$

$\Rightarrow v_y = \sqrt{2gh}$

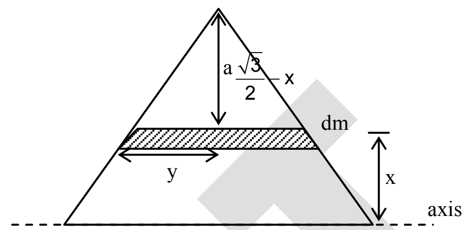
Horizontal range: $R = u_0 t = \frac{2u_0 h}{g}$

$\therefore$ Angular momentum about O:

$\vec{L}_0 = (R\hat{i}) \times [m(v_x\hat{i} - v_y\hat{j})]$

$$= - mRv_y\hat{k} = -\left(2mu_0\sqrt{\frac{2h^3}{g}}\right)\hat{k}.$$

40. Mass per unit area = $\dfrac{dm}{2y(dx)} = \dfrac{dm}{2\left(\dfrac{a/2}{\frac{a\sqrt{3}}{2}}\left(a\dfrac{\sqrt{3}}{2}-x\right)\right)}dx$



$$\Rightarrow \frac{4m}{a^2\sqrt{3}} = \frac{dm}{\left(a-\frac{2x}{\sqrt{3}}\right)dx}$$

$$\Rightarrow \frac{4m}{a^2\sqrt{3}}\left(a-\frac{2x}{3}\right)dx = dm$$

$$\Rightarrow I = \int x^2 dm = \int_0^{\frac{\sqrt{3}}{2}} x^2\,\frac{2m}{a^2}\left(\frac{\sqrt{3}a}{2}-x\right)dx$$

$$= \frac{2m}{a^2}\left[\frac{\sqrt{3}a}{2}\frac{x^3}{3}-\frac{x^4}{4}\right]_0^{\frac{\sqrt{3}}{2}a} = \frac{2m}{a^2}\left[\frac{1}{12}\left(\frac{9a^4}{16}\right)\right] = \frac{3}{32}ma^2.$$

41. (a) Instantaneous angular velocity, $\omega = \dfrac{d\theta}{dt} = \dfrac{d}{dt}(at^2)$

$\Rightarrow \quad \omega = 2at = 0.4t$

Angular velocity at t = 2.5 sec. is ω = 0.4(2.5)=1.0 rad/sec.

Instantaneous angular acceleration , $\alpha = \dfrac{d\omega}{dt} = \dfrac{d}{dt}(0.4t)$

Angular acceleration at 2.5 sec is 0.4 rad/sec^2

$\Rightarrow \quad a_n = R\omega^2 = (0.65)(1.0)^2 = 0.65\,m/sec^2$

$a_t = R\alpha = (0.65)(0.4) = 0.26\ m/sec^2$

Magnitude of total acceleration $a = \sqrt{a_n^2 + a_T^2} = \sqrt{(0.65)^2 + (0.26)^2} = 0.7\,m/s^2$

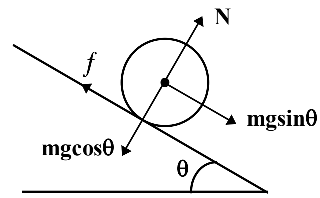
42. If you consider plank and cylinder as a system acceleration is g sin θ. Since there is no tendency of relative motion between cylinder and plank, acceleration of both are same i.e. g sin θ.

43. $a = g\sin\theta - \dfrac{f}{M}$

Angular acceleration α is given by

$$\alpha = \frac{fR}{I_c} = \frac{5f}{2MR} \qquad \text{and} \qquad a = \frac{5f}{2M}$$

Solving these $f = \frac{2}{7}Mg\sin\theta$



44. Angular momentum about any point,

$\vec{L}_P = \vec{L}_{cm} + M\vec{r}_{cm,P} \times \vec{v}_{cm}$

(a) On smooth horizontal plane, only disc is rotating,

so, $L_{cm} = I_{cm}\,\omega = \frac{1}{2}MR^2\omega$ clockwise sense.

(b) same as in part (a)

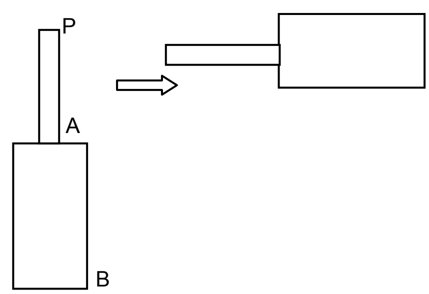
(c) $L_{cm} = I_{cm}\,\omega = \frac{1}{2}MR^2\omega$ clockwise sense

$L_0 = L_{cm} + MRv_{cm} = \frac{1}{2}MR^2\omega + MR\omega R = \frac{3}{2}MR^2\omega$ clockwise sense.

45. $L = L_f$

$mv(2\ell) = I\alpha$,

Moment of inertia of system about P = $m(2\ell)^2 + m_A(\ell/3)^2 + m_B\left[\frac{\ell^2}{12} + \left(\frac{\ell}{2} + \ell\right)^2\right]$



$I = 0.09$ kg – m^2

$\omega = \frac{2mv\ell}{I} = \frac{2(0.05)v(0.6)}{0.09}$

$\omega = 0.67$ v.

decrease in rotational K.E. = increase in gravitational P.E.

$\frac{1}{2}I\omega^2 = mg(2\ell) + m_A g(\ell/2) + m_B(\ell + \ell/2)$

$\omega^2 = \frac{g\ell(4m + m_A + 3m_B)}{I}$

$= \frac{9.8 \times 0.6 \times (4 \times 0.05 + 0.01 + 3 \times 0.02)}{0.09}$

$= 17.69$ rad/s^2

$\omega = 4.2$ rad/s … (ii)

from the above equation we get

$v = \frac{4.2}{0.67} = 6.8$ m/s

46. $I = \frac{1}{4}Mr^2 = 8 \times 10^{-5}$ kg-m^2

Therefore KE = $\frac{1}{2}I\omega^2$ = 4×10^{-3} J

Angular momentum = $I\omega$ = 8×10^{-4} J-s

47. For no slipping $v_{cm} = \omega R$ ... (i)

$v_B = \omega r + v_{cm} = v_{cm}\left(1+\frac{r}{R}\right)$ .... (ii)

$\frac{I_{cm}}{I} = \frac{v_{cm}}{v} = \left(1+\frac{r}{R}\right) \Rightarrow I_B = I_{cm}\left(1+\frac{r}{R}\right)$.

48. $\tau = rT$ ... (1)

$\alpha = \tau/I$ ... (2)

$a = r\alpha$ ... (3)

From (1), (2) and (3)

$T = \frac{I\alpha}{r}$ ... (4)

Since the YO - YO starts from rest and descents a vertical height say H

$v^2 = 2aH$ or $a = \frac{v^2}{2H}$

By conservation of energy , gain in KE = Loss in PE

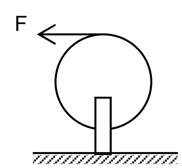
$$\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = MgH$$

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\frac{v^2}{r^2} = MgH$$

$$\frac{v^2}{2} = \frac{Mgr^2H}{Mr^2 + I}$$

$a = \frac{v^2}{2H}$, $T = \frac{I\alpha}{r} = \frac{Ia}{r^2}$ or $T = \frac{MgI}{Mr^2 + I}$

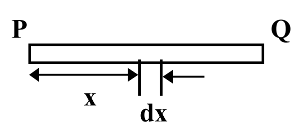
49. $\tau = Fr\sin\theta$



$\Rightarrow \alpha = \tau / I = \frac{20 \times 0.2}{0.2} = 20$ rad/sec^2

$\omega = \omega_0 + \alpha t = 20 \times 5 = 100$ rad/s.

50. $dm = \lambda dx$



$m = \int_0^l \frac{m}{2l}(1 + ax)dx \Rightarrow a = 2/l$

From P

$$x_{cm} = \frac{\int x\,dm}{m} = \frac{\int_0^l x\frac{m}{2l}\left(1+\frac{2x}{l}\right)dx}{m} = \frac{7l}{12}$$

51. At break off

$mg\cos\theta - m\omega_0 \sin\theta - N = \dfrac{mv^2}{R}$

when the body break off N = 0

$v^2 = gR\cos\theta - \omega_0 R\sin\theta$ …(i)

From work energy theorem

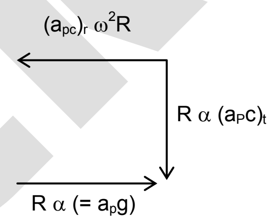
$\dfrac{1}{2}mv^2 - mg(R - R\cos\theta) = m\,\omega_0 R\sin\theta$ …(ii)

from (i) and (ii)

$v = \sqrt{(2/3)gR}$

52. $\vec{a}_{pg} = \vec{a}_{pc} + \vec{a}_{cg}$

$\vec{a}_{pg} = (R\alpha - \omega^2 R)\hat{i} - R\alpha\hat{j}$



53. $2mg\sin\theta - f = ma_{cm}$ … (i)

taking torque about centre of mass

$fr = \dfrac{mr^2}{2}\cdot\dfrac{a_{cm}}{r}$ … (ii)

from (i) and (ii)

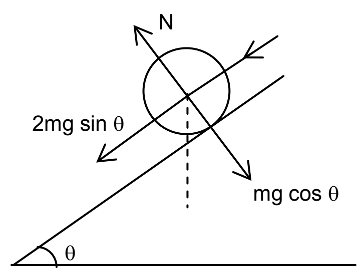
$2mg\sin\theta - f = 2f$

$\therefore\ 3f = 2mg\sin\theta \quad \Rightarrow f = \dfrac{2}{3}mg\sin\theta$

At critical stage

$f = \mu_{min}\, mg\cos\theta$

$\therefore\ \mu_{min} = \dfrac{2}{3}\tan\theta$



54. Before cutting $2T_1\cos 60 = mg$

$\Rightarrow T_1 = mg$

after cutting $T_2 = mg\cos 60 = \dfrac{mg}{2}$

Now, $\dfrac{T_1}{T_2} = \dfrac{2mg}{mg} = 2$

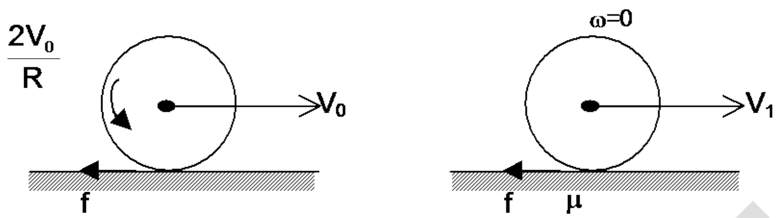
55. $m_1 = m,\ m_2 = -\dfrac{m}{4}$, $x_1 = a/2,\ x_2 = 3a/4$

$x_{cm} = \dfrac{m(a/2) - (m/4).(3a/4)}{m - (m/4)} = \dfrac{7a}{12}$ $y_1 = a/2,\ y_2 = a/4$

$$y_{cm} = \frac{m(a/2)-(m/4)(a/4)}{m-(m/4)} = \frac{a}{3}$$

56. Friction force will act towards left. As a result after sometime t_1, $\omega = 0$ and V_0 will reduce to V_1. Further due to friction the sphere will start rotating in clockwise sense and V_1 will decrease. If after time t_2, $V_2 = \omega r$ is satisfied, then disc will start rolling.

Calculation of t_1 :



Rotation

$\omega_2 = \omega_1 - \alpha t$

$$\Rightarrow 0 = \frac{2V_0}{r} - \frac{(\mu mg)\times r}{\frac{1}{2}mr^2} . t_1$$

$$\Rightarrow t_1 = \frac{V_0}{\mu g}$$

Translation

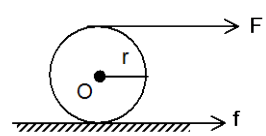
$V_1 = V_0 - at_1$

$$= V_0 - \mu g \times \frac{V_0}{\mu g}$$

$= 0$

Thus, we find that after time $\frac{V_0}{\mu g}$, the linear velocity and angular velocity both become 0 simultaneously. Hence, there will be no further motion and disc will not achieve rolling.

57. The situation is shown in the figure. As force F rotates the sphere point of the contact has a tendency to slip towards left so that static friction of sphere will act towards right.



Let a = linear acceleration, r = radius of the sphere, then

angular acceleration $\alpha = \frac{a}{r}$

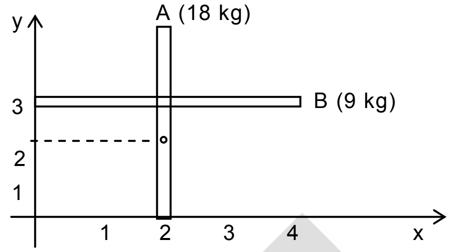
Thus, $F + f = ma$ . . . (i)

And $Fr - fr = I\alpha = \left[\frac{2}{5}mr^2\right]\left(\frac{a}{r}\right)$ . . . (ii)

From (i) and (ii)

$a = 10F/7m$

58. $X_{cm} = \dfrac{18(2)+9(2)}{18+9} = 2$



$Y_{cm} = \dfrac{18(2)+9(3)}{18+9} = \dfrac{7}{3}$

$(X_{cm},\ Y_{cm}) \Rightarrow (2, \dfrac{7}{3})$

$I_{cm} = \dfrac{18(4)^2}{12} + 18\left(\dfrac{1}{3}\right)^2 + \dfrac{9(4)^2}{12} + 9(2/3)^2 = 42 kgm^2.$

59. $mg\sin\theta - f = ma$ ... (1)

$fR = I\alpha$ ... (2)

$\alpha = \dfrac{a}{R}$ ... (3)

$\Rightarrow$ from equations (1) (2) and (3)

$a = \dfrac{mg\sin\theta}{m+\dfrac{I}{R^2}} = \dfrac{mg\sin\theta}{m+\dfrac{2}{5}m} = \dfrac{5}{7}g\sin\theta$

$\Rightarrow f = \left(\dfrac{2}{5}m\right)\dfrac{5}{7}g\sin\theta$

$f = \dfrac{2}{7}mg\sin\theta$ (up the incline)

work done by the f is zero since it is static frictional force.

$v(t) = at = \dfrac{5}{7}g(\sin\theta)t$

$a(t) = \dfrac{5}{7}g\sin\theta$

60. Particle $\vec{v}_m = 2\vec{v}_{cm} = 2v_0$

It will fall a vertical distance in time t,

$2R = \dfrac{1}{2}gt^2 \qquad \Rightarrow t = \sqrt{\dfrac{4R}{g}}$

horizontal distance $= 2v_0 \times t$

$= 4v_0\sqrt{\dfrac{R}{g}}$

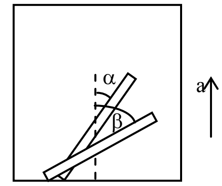
Co-ordinate of the particle $\left(4v_0\sqrt{\dfrac{R}{g}}, 0\right)$

Co-ordinate of the disc CM $\left(2v_0\sqrt{\dfrac{R}{g}}, R\right)$

61. Using mechanical energy conservation principle in the reference frame •

of lift.

$$m\,(g + a)\frac{\ell}{2}\cos\alpha = m\,(g + a)\frac{\ell}{2}\cos\beta + \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$



Here $I = \frac{1}{12}M\ell^2$ & $v = \frac{\omega\ell}{2}\sin\beta$

$$m\,(g + a)\frac{\ell}{2}\cos\alpha$$

$$= m\,(g + a)\frac{\ell}{2}\cos\beta + \frac{1}{2}\left(\frac{1}{12}m\ell^2\right)\omega^2 + \frac{1}{2}m\omega^2\frac{\ell^2}{4}\sin^2\beta$$

$$\Rightarrow \omega = \sqrt{\frac{12(g+a)(\cos\alpha-\cos\beta)}{\ell(1+3\sin^2\beta)}}.$$

62. From COE

$$mgH = \frac{1}{2}mv_c^2 + \frac{1}{2}.\frac{2}{5}mr^2\frac{v_c^2}{r^2} \Rightarrow mgH = \frac{7}{10}m\,v_c^2$$

From B onward

$$\frac{1}{2}I_c\omega^2 + mgh = \frac{1}{2}I_c\omega^2 + \frac{1}{2}mv_c^2$$

$$\Rightarrow h = \frac{v_c^2}{2g} = \frac{10}{14}H = \frac{5H}{7}$$

63. $mg + ma_0 - 2T = ma$ … (i)

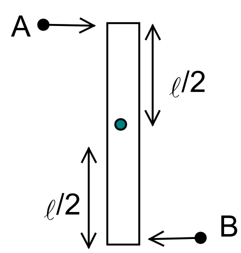
$2T.\,R = I\alpha$ … (ii)

$a = R\alpha$ … (iii)

from the above equations

$a = \frac{2}{3}(g + a_0)$ and $2T = \frac{2m}{3}(g - a_0)$

64. Conservation of linear momentum :



$P_i = mv - mv = 0$

$\Rightarrow \quad v_{cm} = 0$

Conservation of angular momentum

$\Rightarrow \quad L_{initial} = L_{final}$

where $L_{initial} = \left|\frac{m\ell}{2}v + \frac{m\ell}{2}v\right| = mv\ell$

Let the system rotate about its c.m. 'O' with an angular speed ω

$\Rightarrow \quad L_{final} = (I_{system})\,\omega$

where $I_{system} = \frac{M\ell^2}{12} + m\left(\frac{\ell}{2}\right)^2 + m\left(\frac{\ell}{2}\right)^2 = \left(\frac{M+6m}{12}\right)\ell^2$

$\therefore \quad \left(\frac{M+6m}{12}\right)\omega\ell^2 = mv\ell$

$\Rightarrow \quad \omega = \frac{12mv}{(M+6m)\ell}$.

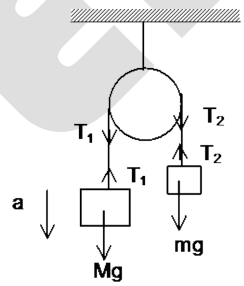
65. Decrease in P.E. = increase in kE

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \qquad [\omega = v/r]$

$\Rightarrow \quad v = \left[\frac{2mgh}{m + I/r^2}\right]^{1/2}$.

66. $Mg - T_1 = Ma$ for block M
$T_2 - mg = ma$ for block m
$T_1R - T_2R = I\alpha$ for pulley
$a = \alpha R$ constraint

Solving $a = \frac{(M-m)gR^2}{I + (M+m)R^2}$



67. The total kinetic energy of the sphere $= E = E_{tran} + E_{rot} = \frac{1}{2}(1 + k^2/r^2)mv^2$.

Putting $k^2/r^2 = 2/5$ for the sphere, we obtain

$E = \frac{1}{2}(1 + 2/5)mv^2 = (7/10)mv^2$

$\Rightarrow \quad v = \sqrt{\frac{10E}{7m}}$.

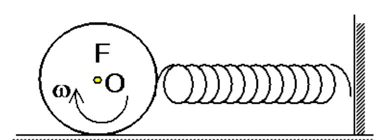
Since the spring force F passes through the centre O of the sphere, it causes no torque about O. Therefore, the angular momentum and hence angular velocity of the sphere remains constant.

Since the surface AB is smooth no frictional loss take place. Therefore, conserving the energy of the system (sphere-spring) between the given (initial) position and the final position (maximum compression of the spring),

we obtain,

$\Delta KE_{sphere} + \Delta PE_{spring} = 0$

$\Rightarrow \quad \Delta KE_{tran.} + \Delta KE_{rot} + \Delta PE_{spring} = 0$



Since the initial angular velocity of the sphere remains constant due to the absence of friction.

$\Delta KE_{rotation} = 0 \Rightarrow \quad \Delta KE_{tans} + \Delta PE_{spring} = 0$

$\Rightarrow \quad \left(0-\frac{1}{2}mv^2\right)+\left(\frac{1}{2}kx^2-0\right) = 0$

where v = $\sqrt{\frac{10E}{7m}}$ and x = maximum compression of the spring

$\Rightarrow \quad \frac{1}{2}Kx^2 = \frac{1}{2}m\left(\sqrt{\frac{10E}{7m}}\right)^2$

$\Rightarrow \; x^2 = \frac{10E}{7k}$.

$\Rightarrow \quad x = \sqrt{\frac{10E}{7k}}$.

68. (a) From the conservation of angular momentum about the centre of mass of the rod,

$mv_0\, L/2 = \frac{ML^2}{12}\,\omega$

$\Rightarrow \quad \omega = \left(\frac{6m}{M}\right)\frac{v_0}{L}$

From conservation of linear momentum $mv_0 = MV_{cm}$

$\Rightarrow \quad V_{cm} = \left(\frac{m}{M}\right)v_0$

$\Rightarrow$ Just after collision ; $V_A = V_{cm} + \omega(L/2) = 4v_0\,(m/M)$

For perfectly elastic collision $V_A = v_0 \Rightarrow \quad \frac{m}{M} = \frac{1}{4}$

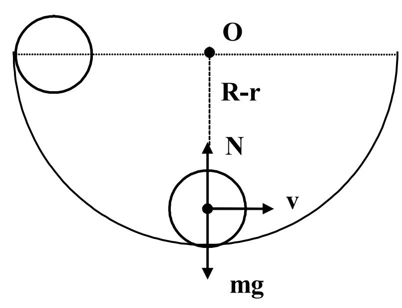
(b) If P remains instantaneously at rest

$\vec{V}_{PC} + \vec{V}_C = 0$

$\Rightarrow \quad V_{CM} = \omega\,[x - L/2]$ Where x = AP

$\Rightarrow \quad x = 2L/3$

69. $K_{trans} = \frac{1}{2}mv^2$



$K_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{5}mv^2$

$K = K_{trans} + K_{rot} = \frac{7}{10}mv^2$

$\therefore \quad \frac{K_{trans}}{K} = \frac{5}{7}$

$\frac{K_{rot}}{K} = \frac{2}{7}$

$mg(R - r) = \frac{7}{10}mv^2$

$\left(\frac{mv^2}{R-r}\right) = \frac{10}{7}mg$

$$N = mg + \left(\frac{mv^2}{(R-r)}\right) = \frac{17}{7}mg$$

70. (a) $mg \sin\theta - f = ma_c$ … (i)

about cm, $fR = I\alpha$ … (ii)

for pure rolling, $a_c = \alpha R$ … (iii)

$f = \frac{2}{7} mg \sin\theta$

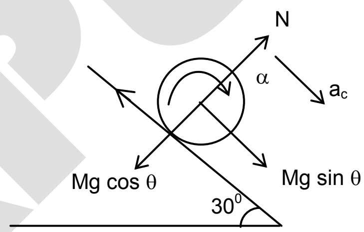
maximum friction, $f_{r(max)} = \mu_s N$

$s = \mu_s\, mg \cos\theta$

$fr(max) \geq f$

so, $\mu \geq \frac{2}{7}\tan\theta$

$\mu_{min} = \frac{2}{7}\tan\theta$



(b) From (i) and (ii)

$a_c = \frac{5}{7} g \sin\theta$

$\alpha = \frac{5}{7}\frac{g\sin\theta}{R}.$

71. Frictional forces are shown in diagram

$F - f_1 = ma_P$ …(1)

$f_1 + f_2 = 2ma_C$ …(2)

$(f_1 - f_2)\, r = \frac{mr^2}{2} \times \alpha$ …(3)

since there is no slipping

$\therefore\ a_C = r\alpha$ …(4)

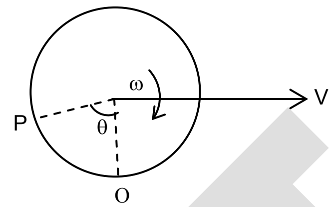
and $a_P = 2a_C$ …(5)

by solving the above equations and putting F = 2mg

we get $a_C = \frac{8}{13} g = \frac{80}{13}$ m/sec^2

$a_P = \frac{16}{13} g = \frac{160}{13} m/sec^2$

72. (a) $|\vec{v}_p| = (OP)\omega$



OP = 2R sin θ/2

$|v_P|$ = 2R sin θ/2. ω = 2V sin θ/2

(b) From above distance moved by the point in time dt is

$ds = v_A\, dt = 2R\omega \sin\left(\frac{\omega t}{2}\right) dt$

$s = \int_0^{2\pi/\omega} 2R\omega \sin(\omega t/2) dt = 8R.$

73. M = 5 kg, r = 1m, ω = 1 rad/sec., m = 0.05 kg

(a) initial velocity of the mas (=0.05 kg) is v which equal to Rω = 1 × 1 = 1 m/s

$h = \frac{v^2}{2g} = \frac{1 \times 1}{2 \times 10} = \frac{1}{20}$ = 0.05 m.

(b) Angular momentum of the sphere before mass (= 0.05 kg) breaks off

$L = I\omega = \frac{2}{5}(MR^2)\omega$ (M = mass of sphere)

Angular momentum of the smaller mass that about the point it breaks off.

$L_m = mvR \sin 90^0 = mvR = m\omega R^2$

$L_{M-m} = \frac{2}{5} MR^2\omega - m\omega R^2$

$= \frac{2}{5} \times 5 \times 1 \times 1 - \frac{1}{20} \times 1 \times 1$

$= 2 - 0.05 = 1.95$ kg m^2/s.

74. At equilibrium, let T be the tension in each string.

∴ The extension x == T/k

2T = mg

2kx = mg

x = mg/2k

so the string is extended by a distance mg/2k. Let the pulley be displaced by x. Then extension in the string will be 2x.

Energy of the system

$U = \frac{1}{2} I\omega^2 + \frac{1}{2} m\omega^2 - mgx + \frac{1}{2} k\left(\frac{mg}{2k} + 2x\right)^2$

$= \frac{1}{2}\left[\frac{I}{r^2} + m\right] v^2 + \frac{m^2g^2}{8k} + 2kx^2$

$\frac{dU}{dt} = 0$ [$\because$ energy is conserved]

$\therefore$ $0 = \left(\frac{I}{r^2} + m\right)v\frac{dv}{dt} + 4kxv$

or $\frac{dv}{dt} = -\frac{4kx}{\frac{I}{r^2} + m}$

or $a = -\omega^2 x$ where $\omega^2 = \frac{4k}{(I/r^2) + m}$

$\therefore$ $T = 2\pi\sqrt{\frac{(I/r^2) + m}{4k}}$.

75. (i) $F_{ext} = 0$, by applying COM

$-2m \times v + m \times 2v + 0 = (2m + m + 8m)v$

$v = 0$

(ii) $\tau_{ext} = 0$, by applying COAM

$2mva + m(2v)(2a) = [2m(a)^2 + m(2a)^2 + 8m \times (6a)^2/12]\omega$

$\omega = (v/5a)$

(iii) from part(i) and (ii), the system has no translating but only rotating motion

$E = \frac{1}{2}I\omega^2 = \frac{1}{2}(30ma^2)\left[\frac{v}{5a}\right] = \frac{3}{5}mv^2$

76. For translational motion

$f = \mu R$ and $R = mg$

$a = f/m = \mu g$ ...(1)

For rotational motion

$\tau = I\alpha$

$fr = I\alpha$ [$I = \frac{2}{5}mr^2$]

$v = v_0 - \mu gt$ [as $u = v_0$ and $a = -\mu g$] ...(2)

$\omega = \omega_0 + \alpha t$

$\omega = \left(\frac{5\mu g}{2r}\right)t$ [as $\omega_0 = 0$ and $\alpha = \frac{5\mu g}{2r}$ ...(3)

rolling without sliding $v = r\omega$

$(v_0 - \mu gt) = r(\left(\frac{5\mu g}{2r}\right)t$ i.e $t = \frac{2v_0}{7\mu g}$ ...(4)

Substituting the value of (t) in equation (2)

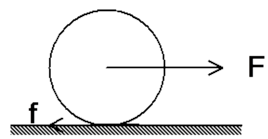
$v = v_0 - \mu g \times \frac{2v_0}{7\mu g} = \frac{5}{7}v_0$

from equation of motions $v^2 = u^2 + 2as$ we get

$(5v_0/7)^2 = v_0^2 - 2\mu gs \Rightarrow s = \frac{12v_0^2}{49\mu g}$

and $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ and $n = \left(\frac{\alpha t^2}{4\pi}\right)[as\,\omega_0 = 0]$, $n \Rightarrow \frac{5v_0^{\,2}}{98\pi\mu gr}$

77. By Newton's law



$F - f = m.\,a_C$

torque about centre of mass

$\tau = f.\,r$

$I\,\alpha = f.r$

$\frac{mr^2}{2}\alpha = f.\,r \Rightarrow \alpha = \frac{2f}{mr}$

Since pure rolling takes place

$\therefore a_c = \alpha.r$

$\therefore \frac{a_c}{r} = \frac{2f}{mr} \Rightarrow f = \frac{ma_c}{2}$

$\therefore F - f = m.\,\frac{2f}{m}$

$f = \frac{F}{3}$

$\therefore$ direction of frictional force is opposite to the F.

without causing slip $f_{max} = \mu mg$

$\therefore F = 3\mu mg$

$\therefore F - f = ma_{cm}$

$3\mu mg - \mu mg = ma_{cm}$

$a_{cm} = 2\mu g$

78. (a) $\alpha = 4t - t^2$

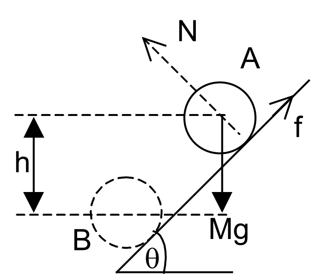
$\therefore \frac{dw}{dt} = 4t - t^2$

or $\int_0^w dw = \int_{t=0}^{3}(4t - t^2)dt$

$\therefore$ w = 9 rad/sec

For t > 3 sec, w will not change as $\alpha = 0$ so at t = 5 sec $\omega = 9$ rad /sec

(b) Applying COE for A and B,



$Mgh = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$

Where v = velocity of sphere at B

ω = angular velocity at B

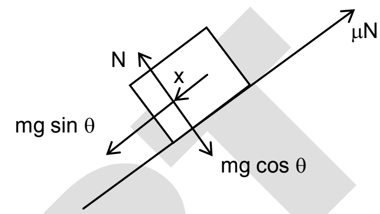
$\Rightarrow Mgh = \frac{1}{2}MV^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\frac{V^2}{R^2}$

$\therefore V^2 = \frac{4gh}{3}$

For AB; u = 0, $V = \sqrt{\frac{4gh}{3}}$, a = ?, $s = \frac{h}{\sin\theta}$

Applying $V^2 = u^2 + 2as$, we get $a = \frac{2g\sin\theta}{3}$

79. (a) FBD of block



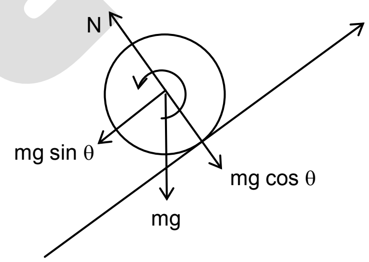
As the block slides down with uniform speed ; net force on it along the inclined plane = 0

$mg\sin\theta = \mu N = \mu mg\cos\theta$ as $N = mg\cos\theta$

Now; distance line of action of normal force has by a distance x from the centre line of the block to counter the torque of friction about the centre. As the block does not topple ;

(N) (x) = torque of normal force about the centre = μN (a/2) = ($\mu mg\cos\theta$) (a/2) = ($mg\sin\theta$) (a/2)

(b) FBD of the shell'



f is friction whose maximum value is μN. If a_{cm} = acceleration of centre of mass and α = angular acceleration, then

$mg\sin\theta - f = ma_{cm}$ …(i)

$(fR) = \frac{2}{3}mR^2\alpha$ …(ii)

and $a_{cm} = R\alpha$ for pure rolling …(iii)

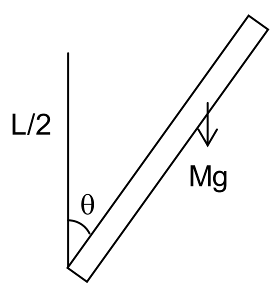
from (i), (ii) and (iii) ;

$a_{cm} = \frac{3g\sin\theta}{5}$

$\alpha = \frac{3g\sin\theta}{5R}$ and $f = \frac{2}{5}mg\sin\theta$

As $f \le \mu mg\cos\theta \Rightarrow \mu \ge \frac{2}{5}\tan\theta$

80. L/2 Mg sin θ = Iα



$I = ML^2/3$

$\alpha = 3g\sin\theta/2L$

(i) $a_T = \alpha L = \frac{3g}{2L}\sin\theta \times L = \frac{3g}{2}\sin\theta$

(ii) $a_R = \frac{V^2}{R} = \omega^2 R$

$\frac{d\omega}{dt} = \frac{3g}{2L}\sin\theta$

$$\frac{d\omega}{d\theta}.\frac{d\theta}{dt} = \frac{3g}{2L}\sin\theta$$

$$\int_0^{\omega}\omega d\omega = \int_0^{\theta}\frac{3g}{2L}\sin\theta d\theta$$

$$\frac{\omega^2}{2} = -\frac{3g}{2L}(\cos\theta - 1) \qquad \Rightarrow \quad \omega^2 = \frac{3g}{L}(1 - \cos\theta)$$

81. Apply conservation of linear momentum

$$mv_0 = \frac{mv_0}{2} + 2mv_{cm} \quad \Rightarrow \quad v_{cm} = \frac{v_0}{4}$$

Apply conservation of angular momentum about C.O.M. of rod

$$mv_0(\ell/2) = (2m\ell^2)\omega/12 + \frac{mv_0}{2}\frac{\ell}{2}$$

$$\Rightarrow \qquad \frac{mv_0\ell}{4} = \frac{m\ell^2}{12}\omega \qquad \Rightarrow \qquad \omega = \frac{3v_0}{2\ell}$$

velocity of point B

$$v_B = v_{cm} + \omega(\ell/2)$$

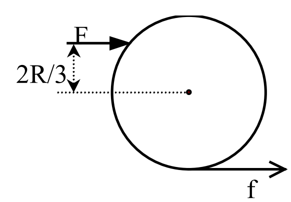
$= v_0$

$$e = -\frac{\text{vel. of separation}}{\text{vel. of approach}}$$

$$= -\frac{v_0 - v_0/2}{0 - v_0}$$

$$e = \frac{1}{2}$$

82. • Let f be the friction force acting on the cylinder as shown in the F.B.D of the cylinder



Torque acting on the cylinder

$$\tau = F \times \frac{2R}{3} - fR = \frac{(2F - 3f)R}{3}$$

Angular acceleration of the cylinder

$$\alpha = \frac{\tau}{I} = \frac{(2F - 3f)R}{3MR^2/2} = \frac{(4F - 6f)}{3MR}$$

• Acceleration of the centre of mass of the cylinder

$$a_{cm} = \frac{F + f}{M}$$

• Since cylinder rolls without slipping

$a_{cm} = \alpha R$

$$\Rightarrow \frac{F + f}{M} = \frac{(4F - 6f)}{3MR} \times R$$

$$\Rightarrow F + f = \frac{4F - 6f}{3} \qquad \Rightarrow f = \frac{F}{9}$$

• Now, the friction force f acting on the cylinder static because cylinder rolls without slipping

Also we know that static friction $f \le \mu N$

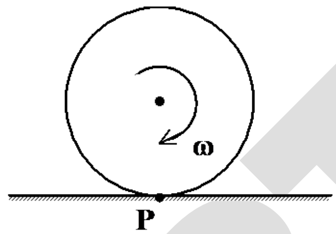
$\Rightarrow f \le \mu mg$

$\Rightarrow F \le 9\mu mg$

$\Rightarrow F_{max} = 9\mu mg = 9 \times 0.2 \times 6 \times 10 = 108N.$

83. $v_1^2 = 2gh \quad \Rightarrow \quad v_1 = \sqrt{2gh} = v(\text{say})$

$v_2^2 = \sqrt{2gh/2} \quad \Rightarrow \quad v_2 = \sqrt{gh} = v/\sqrt{2}$



$$\int N dt = m\left(\frac{v}{\sqrt{2}} + v\right) \qquad \ldots \text{(i)}$$

Where N = impact force on the ball during collision.

$$\int fR dt = I_{c.m.}(-\omega' + \omega)$$

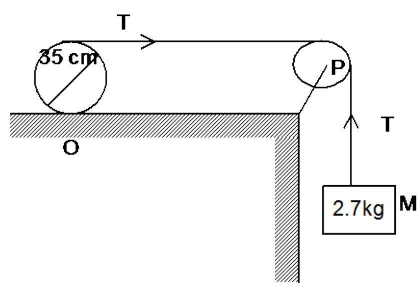
$$\Rightarrow \int \mu N R dt = I_{c.m.}(\omega - \omega') \qquad \ldots \text{(ii)}$$

From (i) and (ii), we get

$$\mu R.\frac{1}{\sqrt{2}} mv\,(1 + \sqrt{2}) = \frac{2}{5} mR^2(\omega - \omega')$$

$$\Rightarrow \omega' = \omega - \frac{5\mu}{2R}(1+\sqrt{2})\sqrt{gh}.$$

84. If the acceleration of center of mass of cylinder is 'a' then the acceleration of 2.7 kg mass is 2a.



$2.7\,g - T = 2.7 \times 2a \qquad \ldots \text{(i)}$

$T \times R = (MR^2/2) \times a/R$

$T = Ma/2 \qquad \ldots \text{(ii)}$

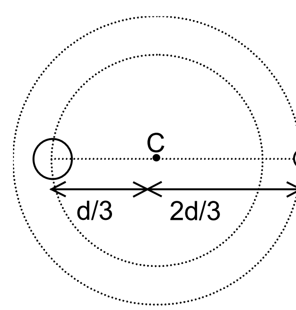
From eq. (i) and (ii)

$2.7\,g - Ma/2 = 2.7 \times 2a$

$2.7\,g = a\left(5.4 + \frac{8.1}{2}\right)$

$\therefore$ acceleration of mass 2.7 kg = 2a = 108/18.9 = 5.71 m/s^2

85. The centre of mass C will be at a distance d/3 and 2d/3 from the masses 2m and m respectively.



Both the stars rotate with same angular velocity 'ω' around C in their individual orbits.

Ratio of angular momentum $= \dfrac{m(2d/3)^2\omega}{2m(d/3)^2\omega} = \dfrac{2}{1}$

Ratio of kinetic energies $= \dfrac{\frac{1}{2}m(2d/3)^2\omega^2}{\frac{1}{2}2m(d/3)^2\omega^2} = \dfrac{2}{1}$

86. Let mass of remaining part of sphere is M_1 then mass of smaller sphere scooped out of bigger sphere of radius R is

$$M_2 = \frac{M}{8} \qquad \therefore M_1 + M/8 = M \qquad \Rightarrow \qquad M_1 = \frac{7M}{8}$$

Gravitational field inside cavity will be uniform

$$g' = \frac{GM(R/2)}{R^3} = \frac{GM}{2R^2} \text{ and it points towards O.} \qquad \therefore R = \frac{1}{2}g't^2$$

$$\Rightarrow t = \sqrt{\frac{2R}{g'}} = \sqrt{\frac{2R}{GM/2R^2}} = 2R\sqrt{\frac{R}{GM}}.$$

87. The acceleration of the whole system, $a_1 = \dfrac{F}{m_1 + m_2}$

The acceleration of the of the point K w.r.t. the axis of the cylinder

$$a_2 = \alpha R$$

where, α is given by

$$FR = I\alpha \qquad \Rightarrow \qquad \alpha = \frac{FR}{m_1R^2/2} = \frac{2F}{m_1R} \Rightarrow a_2 = \frac{2F}{m_1}$$

$\therefore$ The acceleration of the point K w.r.t. ground

$$= a_1 + a_2 = \frac{F}{m_1 + m_2} + \frac{2\ F}{m_1} = F\frac{3m_1 + 2m_2}{m_1(m_1 + m_2)}$$

88. For rolling motion

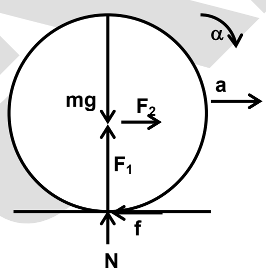
$a = r\alpha$

$fr = \left(\dfrac{mr^2}{2}\right)\alpha$

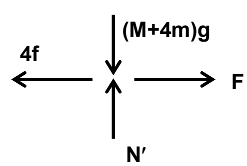
$f = \dfrac{mr\alpha}{2} = \dfrac{ma}{2}$

$F - 4f = (M + 4m)a$

$\therefore\ a = \dfrac{F}{M + 6m}$



FBD of the wheel (F_1 and F_2 are the reactions of the axle)



FBD of the cart (with wheels)

89. (a) For the position of the rod, shown in figure

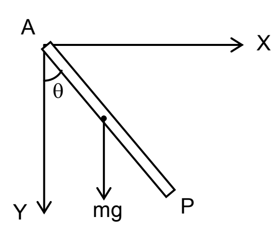
total external torque $= Mg(\ell/2)\sin\theta$

as $\tau = I\alpha$, about A,

we get $Mg(\ell/2)\sin\theta = \dfrac{M\ell^2}{3}\alpha$

$\alpha = \dfrac{3g}{2\ell}\sin\theta$

As $\theta = 30^0$, $\alpha = \dfrac{3g}{2\ell} \times \dfrac{1}{2} = \dfrac{30}{4} = 7.5$ rad/sec^2



(b) Apply energy conservation, $Mg\frac{\ell}{2}\cos\theta = \frac{1}{2}\frac{M\ell^2}{3}\omega^2$

$\omega^2 = \frac{3g\cos\theta}{\ell} = 15\sqrt{3}$ rad/sec.

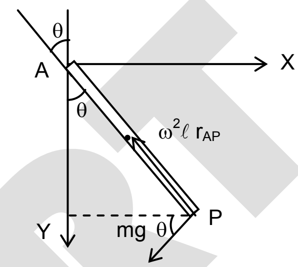
$\Rightarrow$ $\omega \approx 5.1$ rad/sec.

(c) $\vec{a}_P = \omega^2\ell\hat{r}_{AP} + \alpha\ell\hat{n}$

$= \omega^2\ell\,[-\sin\theta\ \hat{i} + (-\cos\theta\,\hat{j})] + \alpha\ell\,[-\cos\theta\ \hat{i} - \sin\theta\ \hat{j}]$

$\vec{a}_P = [\omega^2\ell\,[\sin\theta] + \alpha\ell\cos\theta\,](-\hat{i})$

$+ [\omega^2\ell\cos\theta + \alpha\ell\sin\theta]\,(-\hat{j})$



90. (a) From the conservation of angular momentum about the centre of mass of the rod,

$mv_0\,L/2 = \frac{ML^2}{12}\omega$

$\Rightarrow \omega = \left(\frac{6m}{M}\right)\frac{v_0}{L}$

From conservation of linear momentum $mv_0 = MV_{cm}$

$\Rightarrow V_{cm} = \left(\frac{m}{M}\right)v_0$

$\Rightarrow$ Just after collision ; $V_A = V_{cm} + \omega(L/2) = 4v_0\,(m/M)$

For perfectly elastic collision $V_A = v_0 \Rightarrow \frac{m}{M} = \frac{1}{4}$

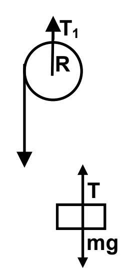
(b) If P remains instantaneously at rest

$\vec{V}_{PC} + \vec{V}_C = 0$

$\Rightarrow V_{CM} = \omega\,[x - L/2]$ Where $x = AP$

$\Rightarrow x = 2L/3$

91. $\tau = I\alpha$



$TR = \frac{MR^2}{2}\left(\frac{a}{R}\right) = \frac{Ma}{2}$

$mg - T = ma$

$a = \frac{2mg}{M+2m}$; $h = \frac{1}{2}at^2$

$t = \sqrt{\frac{2h}{a}} = \sqrt{\frac{2h}{2mg}(M+2m)} = \sqrt{\frac{h(M+2m)}{mg}}$

$v = \sqrt{2ha} = \sqrt{\frac{4mgh}{M+2m}}$

92. For the cylinder

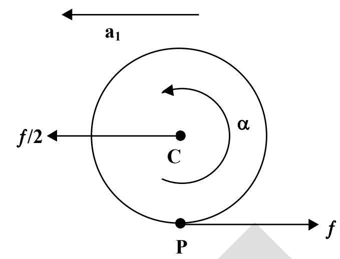
$\frac{F}{2} - f = ma_1$ (1)

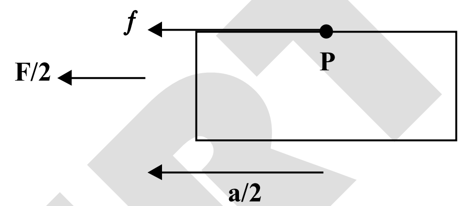
Taking moment of forces about e

$fR = I\alpha$

$\Rightarrow \quad f = \frac{mR\alpha}{2}$ (2)

$\because \; I = mR^2/2$





For the plank

$\frac{F}{2} + f = (2m)a_2$ (3)

And the point of contact P, the acceleration of the two bodies must be same

$a_1 - R\alpha = a_2$ (4)

From (1) and (3) , (2) and (4)

$F = (ma_1 + 2ma_2)$

and $\frac{F}{2} + \frac{mR\alpha}{2} = 2ma_2$

Substituting value of α and solving for a_1 and a_2 gives,

Acceleration of plank $a_2 = \frac{2F}{7m}$

Acceleration of cylinder $a_1 = \frac{3F}{7m}$

93. Suppose after time t cylinder will start pure rolling on horizontal plane,

$v = v_0 - \mu gt$

$\omega = o + \frac{2\mu g}{R}t$

for pure rolling : $v = R\omega$

$\Rightarrow v_0 - \mu gt = 2\mu gt \Rightarrow t = \frac{v_0}{3\mu g}$ . . . (1)

velocity of center of mass

$= v_0 - \mu gt = v_0 - \frac{v_0}{3} = \frac{2v_0}{3}$ also $\omega = \frac{2v_0}{3R}$ . . . (2)

Suppose velocity of center of mass of cylinder be v' when it reaches the edge of the top horizontal .

By COE : $\frac{1}{2}\frac{mR^2}{2}\omega^2 + \frac{1}{2}m(\frac{2v_0}{3})^2 = mgh + \frac{1}{2}\frac{mR^2}{2}\omega^2 + \frac{1}{2}mv^2$

$m\frac{v_0^2}{9} + \frac{m}{2}\frac{4v_0^2}{9} = mgh + \frac{m}{4}v'^2 + \frac{mv^2}{2}$

$v'^2 = \frac{4}{3}(\frac{v_0^2}{3} - gh)$ $\cdots$(1)

If the velocity of the cylinder becomes v'' on top horizontal then by COE:

$$\frac{1}{2}I\omega'^2 + \frac{1}{2}mv'^2 = mgR(1-\cos\alpha) + \frac{1}{2}I\omega''^2 + \frac{1}{2}mv''^2$$

$$\frac{mv_0^2}{3} = \frac{3mv''^2}{4} + mgR(1-\cos\alpha)$$

At the edge: $N + \frac{mv''^2}{R} = mg\cos\alpha$

For cylinder not to loose contact :

$N >= 0 \Rightarrow mg\cos\alpha - \frac{mv''^2}{R} >= 0$

$\Rightarrow mg\cos\alpha - \frac{4}{3}m[g(1-\cos\alpha) - \frac{v_0^2}{3R}] >= 0$

$\frac{4mv_0^2}{9R} >= \frac{4}{3}mg(1-\cos\alpha) - mg\cos\alpha$

$v_0^2 >= \frac{3R}{4}g[4-7\cos\alpha]$

$v_0 >= \sqrt{\frac{3Rg}{4}(4-7\cos\alpha)}$

94. Conservation of linear momentum

$m \times 20 + 3m \times 10 = 4m \times v$

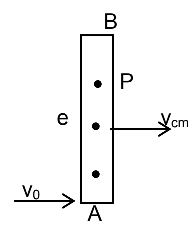
$\therefore \quad v = 12.5$ m/s

Conservation of angular momentum

$m \times 20 \times \frac{2}{3} \times 1 - 3m \times 10 \times \frac{1}{3} = 4m \times (2^2/12) \times \omega$

$\therefore \quad \omega = 2.5$ rad /s.

95. (i) From the conservation of angular momentum about the centre of mass of the rod,



$mv_0 L/2 = \frac{ML^2}{12}\omega$

$\Rightarrow \omega = \left(\frac{6m}{M}\right)\frac{v_0}{L}$

From conservation of linear momentum $mv_0 = MV_{cm}$

$\Rightarrow V_{cm} = \left(\frac{m}{M}\right)v_0$

$\Rightarrow$ Just after collision ; $V_A = V_{cm} + \omega(L/2) = 4v_0\,(m/M)$

For perfectly elastic collision $V_A = V_0 \Rightarrow \frac{m}{M} = \frac{1}{4}$ { Since e = 1, at the point of contact}

(ii) If P remains instantaneously at rest

$\vec{V}_{PC} + \vec{V}_C = 0$

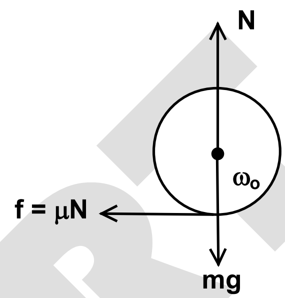
$\Rightarrow V_{CM} = \omega\,[x - L/2]$ Where x = AP

$\Rightarrow x = 2L/3$

(iii) Required linear speed of P = $\sqrt{V_{PC}^2 + V_C^2 + 2V_{PC}V_C\cos(\pi - \omega t)}$

Putting the values we get, V_P (at t = $\pi L/3v_0$) = $\sqrt{2}V/A = v_0/2\sqrt{2}$.

96. F.B.D of the disc



$\because$ N = mg . . . (i)

$\Rightarrow$ f = μmg . . . (ii)

hence, $|\vec{a}_{cm}| = \mu g$

$v_{cm}(t) = v_0 - \mu gt$

let $v_{cm} = 0$ at $t = t_1$

$\Rightarrow$ $t_1 = \dfrac{v_o}{\mu g}$

$\Rightarrow$ $t_1 = \dfrac{\omega_o R}{4\mu g}$ . . . (iii)

from rotation

$|\vec{\alpha}| = \dfrac{\mu mgR}{I_{cm}} = \dfrac{2\mu g}{R}$

$\omega(t) = \omega_0 - \alpha t$

$= \omega_0 - \dfrac{2\mu g}{R}t$

Let $\omega = 0$ at $t = t_2$

$\Rightarrow$ $t_2 = \dfrac{\omega_o R}{2\mu g}$ . . . (iv)

as $t_1 < t_2$

Hence disc will return to its starting point. Let the speed of the centre of mass of the disc when it starts pure rotation be v (in the reverse direction). Applying the principle of conservation of angular momentum about a point on the surface,

$$\frac{mR^2}{2}\omega_o - mv_oR = \frac{mR^2}{2}\omega + mvR$$

Here, $\omega = \dfrac{v}{R}$

$\Rightarrow$ $v = \dfrac{\omega_o R}{6}$ . . . (v)

Suppose it starts rolling after a time t.

Then,

$-v = v_0 - \mu gt$

$\Rightarrow$ $t = \dfrac{5\omega_o R}{12\mu g}$ . . . (vi)

Time interval between the instant when v_{cm} becomes zero and when the disc starts pure rolling $\Delta t = t - t_1$

$\Rightarrow$ $\Delta t = \dfrac{\omega_o R}{6\mu g}$ . . . (vii)

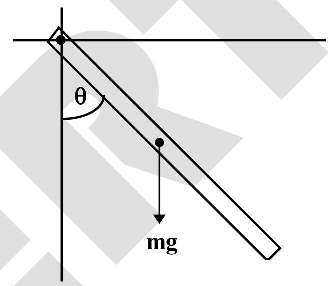
If the total time taken to return to the initial point be T, then

$$T = t + \frac{d}{v}$$

where $d = v_0 t - \frac{1}{2}\mu g t^2$ and $t = \frac{5\omega_o R}{12\mu g}$ & $v = \frac{\omega_o R}{6}$

$$T = \frac{25}{48\mu g}\omega_o R$$

97. (a) For the position of the rod, shown in the figure.



Total external torque = Mg[(L/2) sinθ]

as $\tau = I$□□

We get, $Mg[(L/2)\sin\theta] = \left(\frac{ML^2}{3}\right)\frac{d\omega}{dt}$ or $\alpha = \frac{d\omega}{dt} = \frac{3g}{2L}\sin\theta$

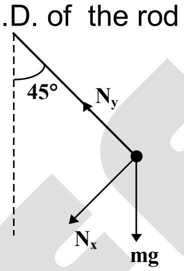
As $\theta = 45°$, $\alpha = \frac{3g}{2\sqrt{2}\ L}$

(b) $\frac{d\omega}{d\theta}\omega = \frac{3}{2}\frac{g}{L}\sin\theta \Rightarrow \int_0^{\omega}\omega d\omega = \frac{3g}{2L}\int_{\pi/2}^{\theta}\sin\theta d\theta$

$\frac{\omega^2}{2} = \frac{3g}{2L}\cos\theta \Rightarrow \omega = \sqrt{\frac{3g\cos\theta}{L}}$

for $\theta = 45°$, $\omega = \sqrt{\frac{3g}{\sqrt{2}L}} = 4.6$ rad/s

(c) F.B.D. of the rod



Where N_x and N_y are tangential and normal components of the force exerted the pivot on the rod.

As the centre of mass of the rod moves along a circle of radius (L/2) with angular velocity ω and angular acceleration α.

For the rod, a_r (centripetal acceleration) $= \omega^2 \frac{L}{2}$

and $a_{tangent} = \alpha \frac{L}{2}$

Now $\Sigma F_{tangent} = Mg\sin 45° + N_x = M\alpha\frac{L}{2} \Rightarrow N_x = M\alpha\frac{L}{2} - \frac{Mg}{\sqrt{2}} = \frac{3Mg}{4\sqrt{2}} - \frac{Mg}{\sqrt{2}} = -\frac{Mg}{4\sqrt{2}}$

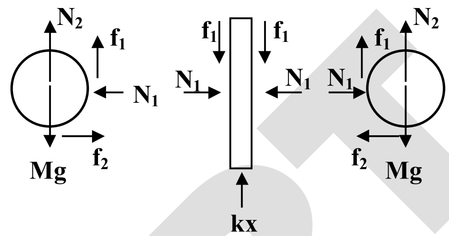
Similarly

$N_y - Mg\cos 45° = M\omega^2\frac{L}{2} = \frac{3Mg}{2\sqrt{2}} \Rightarrow N_y = \frac{3Mg}{2\sqrt{2}} + \frac{Mg}{\sqrt{2}} = \frac{5Mg}{2\sqrt{2}}$

$\Rightarrow$ Force exerted by the pivot on rod = $\sqrt{N_x^2 + N_y^2} = \left(\sqrt{\dfrac{101}{32}}\right)Mg$

Hence the magnitude of force exerted by rod at the pivot, according to Newton's third law equals.

98. For sphere, along y axis



$N_2 + f_1 - Mg = 0$

or $N_2 = Mg - f_1$

along x axis

$N_1 - f_2 = 0$

or $N_1 = f_2$

$f_2 = \mu N_2 = \mu(Mg - f_1)$

$f_1 = \mu N_1 = \mu f_2 = \mu^2(Mg - f_1)$

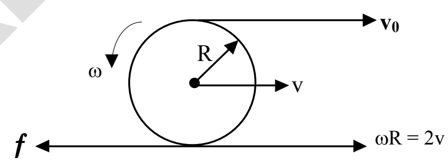
$f_1 = \dfrac{\mu^2 Mg}{1+\mu^2}$

$\therefore$ $f_2 = \mu[Mg - \dfrac{\mu^2}{1+\mu^2}Mg]$; $f_2 = \dfrac{\mu Mg}{1+\mu^2}$

If x be the compression of the spring

$kx = 2f_1 = \dfrac{2\mu^2 Mg}{1+\mu^2}$ or $x = \left[\dfrac{2\mu^2}{1+\mu^2}\right]\dfrac{Mg}{k}$

99. (i) Let v_0 be the velocity when $\omega = 0$



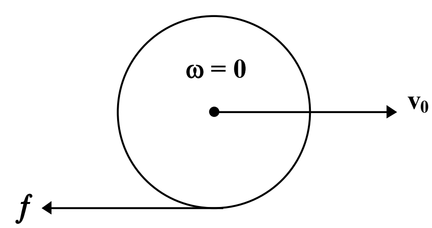
$-\int f dt = (mv_0 - mv)$

$-\int f R dt = 0 - (I\omega)$

$\therefore$ $R = \dfrac{-I\omega}{m(v_0 - v)} = \dfrac{-\frac{2}{5}mR^2\left(\frac{2v}{R}\right)}{m(v_0 - v)}$

$v_0 - v = -\dfrac{4}{5}v$ or $v_0 = \dfrac{v}{5}$

(ii) Let v_f and ω_f be the final velocities



$-\int f dt = (mv_f - mv_0)$

$\int f R dt = I\omega_f - 0$

$v_f = R\omega_f$

$\therefore$ $(-R) = \dfrac{I\omega_f}{m(v_f - v_0)}$

$\Rightarrow$ $(v_f - v_0) = \dfrac{2}{5}v_f \Rightarrow$ $v_f = \dfrac{5}{7}v_0 = \dfrac{v}{7}$

100. $T - F_1 - F_2 = M_2 a$ . . . (i)

$M_1 g - T = M_1(a/2)$ . . . (ii)

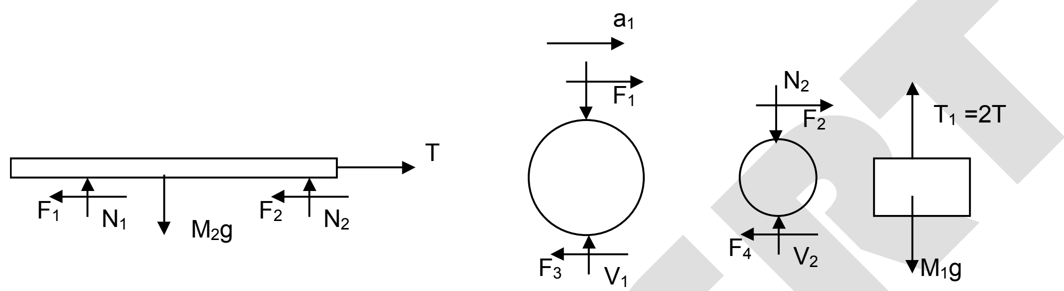
$\alpha_1 = a/2R$, $\alpha_2 = a/2r$ . . . (iii)

from (I) & (ii) $a = \dfrac{M_1 g - F_1 - F_2}{((M_1/2) + M_2)}$ . . . (iv)

For bigger roller $F_1.2R = I_1\,\alpha_1$ . . . (v)

$F_2.\,2r = I_2\,\alpha_2$ . . . (vi)

and for smaller roller



from (v) , (vi) and (iii), $F_1 + F_2 = \dfrac{a}{2Rr}(I_1 + I_2)$ . . . (vii)

$I_1 = \dfrac{2}{5} \times 5 \times (.20)^2 = 8 \times 10^{-2}$ kg m^2, $I_2 = 2/5 \times 2.5 \times (-0.1)^2 = 10^{-2}$ kgm^2

from (vii) $F_1 + F_2 = (9/4)a$

hence from (iv) $a = \dfrac{8}{3}$ ms^{-2}

Hence acceleration of the block $M_1 = \dfrac{4}{3}$ ms^{-2}

$\alpha_1 = \dfrac{8}{3 \times 2 \times 0.2} = \dfrac{20}{3}$ rad/sec^2, $\alpha_2 = \dfrac{8}{3 \times 2 \times 0.1} = \dfrac{40}{3}$ rad/sec^2

Hence acceleration of smaller roller $= \dfrac{4}{3}$ ms^{-2}

From (v) $F_1 = \dfrac{I_1\alpha_1}{2R} = \dfrac{2}{3}$N, $F_2 = \dfrac{16}{3}$N (right ward)

For bigger roller : $F_1 - F_3 = Ma_1$

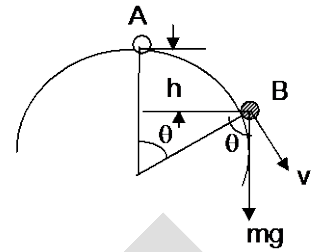
$\dfrac{2}{3} - F_3 = Ma_1$

$\dfrac{2}{3} - F_3 = 5 \times \dfrac{4}{3}$

$F_3 = \dfrac{2}{3} - \dfrac{20}{3} = -6$N

hence $F_3 = 6$N (right ward)

101. Let the ball escapes at B. Conservation of energy between A and B yields



$\Delta PE + \Delta KE = 0$

$\Rightarrow$ $-mgh + (1/2)\,mv^2 + (1/2)\,I\omega^2 = 0$

$\Rightarrow$ $-mg\,[(R + r) - (R + r)\cos\theta] + (1/2)\,mv^2 + \frac{1}{2}\times\frac{2}{5}mr^2\,\omega^2$

Putting $v = r\,\omega$ for rolling we obtain,

$$v = \sqrt{\frac{10g(R+r)(1-\cos\theta)}{7}} \qquad \text{. . . (i)}$$

At the points of escape, normal contact force = 0

$\Rightarrow$ The centripetal force $= mg\cos\theta$

$\Rightarrow$ $F = mg\cos\theta = \frac{mv^2}{R+r}$

$\Rightarrow$ $v = \sqrt{(R+r)g\cos\theta}$ . . . . (ii)

solving (i) and (ii) we obtain

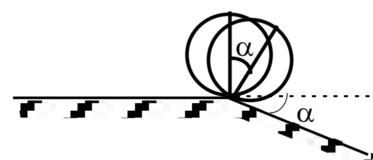
$\cos\theta = \frac{10}{7}(1 - \cos\theta)$

$\Rightarrow$ $\frac{17}{7}\cos\theta = \frac{10}{17}$ $\Rightarrow$ $\theta = \cos^{-1}\frac{10}{17}$

Putting $\cos\theta = \frac{10}{17}$ in eq. (ii) we obtain

$$v = \sqrt{g(R+r)\left(1-\frac{10}{17}\right)} \Rightarrow v = \sqrt{\frac{7g(R+r)}{17}}.$$

102. Initial energy $E_1 = \frac{1}{2}mv_0^2 + \frac{1}{2}I_{c.m.}\omega^2 + mgR$



For rolling $\frac{v_o}{R} = \omega$

$\Rightarrow$ $E_1 = \frac{1}{2}mv_0^2 + \frac{1}{2}.\frac{1}{2}mR^2\frac{v_0^2}{R^2} + mgR$

$= \frac{3}{4}mv_0^2 + mgR$

$E_2 = \frac{1}{2}mv^2 + \frac{1}{2}I_{c.m.}\omega'^2 + mgR\cos\alpha$

$= \frac{3}{4}mv^2 + mgR\cos\alpha$

From COE

$$\frac{3}{4}mv^2 + mgR\cos\alpha = \frac{3}{4}mv_0^2 + mgR$$

$\Rightarrow \quad mv^2 = mv_0^2 + \frac{4}{3}mgR(1-\cos\alpha) \qquad$ (i)

F.B.D. of the cylinder when it is at the edge.

Centre of mass of the cylinder describes circular motion about P.

Hence mg cos□- N = mv^2/R

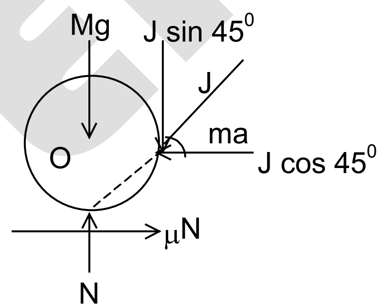
$\Rightarrow$ N = mg cos□- mv^2/R = mg cos□- $\frac{mv_0^2}{R} - \frac{4}{3}mg + \frac{4}{3}mg\cos\alpha$

For no jumping, $N \geq 0$

$\Rightarrow \quad \frac{7}{3}mg\cos\alpha - \frac{4}{3}mg - \frac{mv_0^2}{R} \geq 0$

$\Rightarrow \quad v_o \leq \sqrt{\frac{7gR}{3}\cos\alpha - \frac{4}{3}g}$

103. Since impulse applied is sharp and its line of action does not pass through the centre of mass of the sphere, therefore (just after the impulse) the sphere starts to move translatery as well as rotationally. Translational component is provided by moment of the impulse. Then its horizontal component



$J\cos 45^0 = mv_0 \quad \Rightarrow \quad J = 4\sqrt{2}$ kg ms^{-1}.

$R.\ J\sin 45^0 = I\omega_0 \quad \Rightarrow \quad \omega_0 = 250$ rad/s (clockwise)

as $I = \frac{2}{5}MR^2$

Retardation $a = \frac{\mu N}{m} = \mu g = 1$ ms^{-2}.

Taking torque about O,

$\mu NR = I\alpha \quad \Rightarrow \quad \alpha = 25$ rad/sec^2 (anticlockwise)

$v = v_0 - at$

$= (10 - t)$ms^{-1} (towards left)

and angular velocity

$\omega = (-\omega_0) + \alpha t$

$\omega = 25t - 250$

when sliding stops $v = \omega R$

$(10 - t) = (25t - 250) \times 0.1$

Hence t = 10 sec.

at that instant v = 10 - 10 = 0

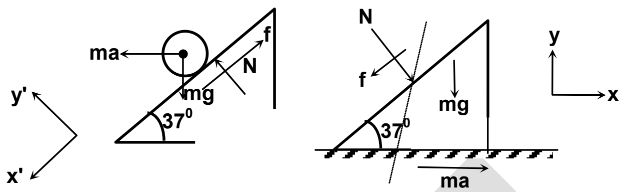
Distance covered $s = v_0 t - \frac{1}{2}at^2$

$S = 10 \times 10 - \frac{1}{2} \times 1 \times 10^2$

S = 50 m .

Energy lost against friction = $\frac{1}{2}mv_0^2 + \frac{1}{2}I\omega^2 = 70$ Joule.

104. for sphere $\Sigma F_{x'} = max'$



$mg\sin37° - f + ma\cos37° = mR\alpha$ (i)

$\Sigma F_{y'} = 0$

$mg\cos37° - N = ma\sin37°$ (ii)

for wedge

$N\sin37° - f\cos37° = ma$ (iii)

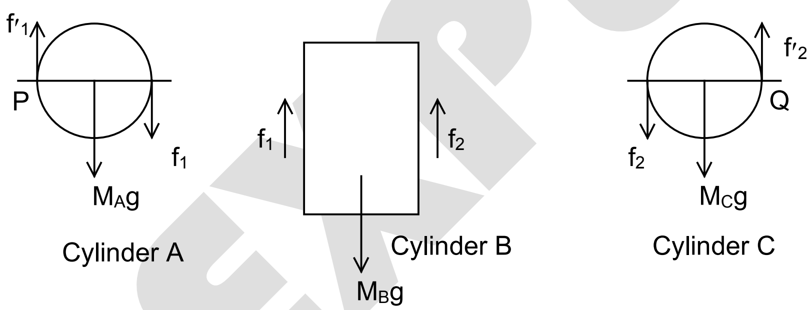
$fR = I\alpha$

$f = 2/5\ MR\alpha$ (iv)

from (i) , (ii) (iii) and (iv)

$f = 2/9\ Mg$

105. Free body diagrams :



Equation of motion $\tau_P = I_P\alpha_A = -f_1 \times 2r_1 - M_Ag \times r_1$

$\tau_P = I_P\ \alpha_A = -f_1 \times 0.4 - 2 \times 10 \times 0.2$

$[\frac{1}{2} \times 2 \times (0.2)^2 + 2 \times (0.2)^2\ ]\ \alpha_A = -0.4\ f_1 - 4$

$\frac{3}{25}\alpha_A = -\ 0.4f_1 - 4$ . . . (i)

$\tau_Q = I_Q\alpha_C = f_2 \times 0.2 + 10 \times 0.1$

$[\frac{1}{2} \times 1 \times (0.1)^2 + 1 \times (0.1)^2]\ \alpha_C = 0.2f_2 + 1$

$\frac{3}{20}\alpha_C = 0.2\ f_2 + 1$ . . . (ii)

$-M_Bg + f_1 + f_2 = M_Ba_B$

$-\ 2.5 \times 10 + f_1 + f_2 = 2.5\ a_B$

$-25 + f_1 + f_2 = 2.5\ a_B$ . . . .(iii)

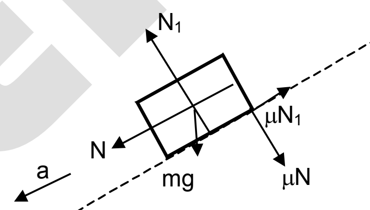
As cylinders A and B are rolling without sliding hence, $a_B = 0.2\,\alpha_A$, or $\alpha_A = 5\,a_B$

and $-a_B = 0.1\,\alpha_C$, or $\alpha_C = -10\,a_B$

Substituting these values in (i) and (ii) and solving, $a_B = \frac{-8}{2.3} = 3.478$ m/s^2

Hence cylinder B will go down with acceleration 3.478 m/s^2.

106. If the block and the cylinder move independently on the incline their accelerations will be $\frac{g}{2\sqrt{2}}$ and $\frac{\sqrt{2}g}{3}$ respectively i.e. $a_{block} < a_{cylinder}$.
(Here friction is found to be sufficient to sustain pure rolling of the cylinder)
Hence in the given configuration they will move with a common acceleration (say a)



From F.B.D. of the block

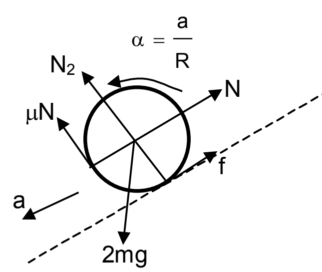
$$\frac{mg}{\sqrt{2}} + N - \mu N_1 = ma \qquad \ldots(1)$$

Here N is normal reaction between block and cylinder

$$N_1 = \frac{mg}{\sqrt{2}} + \mu N = 0 \qquad \ldots(2)$$

Eliminating N_1 between (1) and (2)

$$\frac{mg}{\sqrt{2}} + N - \mu^2 N - \frac{\mu mg}{\sqrt{2}} = ma \qquad \ldots(3)$$



For translation of cylinder

$$\frac{2mg}{\sqrt{2}} - N - f = 2ma \qquad \ldots(4)$$

For rotation of cylinder (assuming pure rolling)

$$(f - \mu N)R = \frac{1}{2}(2m)R^2\left(\frac{a}{R}\right)$$

or $f - \mu N = ma \qquad \ldots(5)$

Adding (4) and (5)

$$\sqrt{2}\,mg - N - \mu N = 3ma \qquad \ldots(6)$$

$$N = \frac{m\left(\sqrt{2}g - 3a\right)}{1+\mu} \qquad \ldots(7)$$

Putting in (3), we get

$$a = \frac{3(1-\mu)g}{\sqrt{2}(4-3\mu)} = \frac{3g}{5\sqrt{2}}$$

Putting in (7), we get $N = \frac{\sqrt{2}mg}{15}$

Putting the values of N and a in (5), we get

$$f = \frac{\sqrt{2}mg}{3}$$

Also from the F.B.D. of the cylinder

$$N_2 = \frac{2mg}{\sqrt{2}} - \mu N = \sqrt{2}\ mg\ [1 - \mu/15) = \frac{29\sqrt{2}mg}{30}$$

$\therefore\ \mu N_2 > f$

Hence pure rolling will takeplace and the value of acceleration will be $\frac{3g}{5\sqrt{2}}$.

107. (a) $m(3\hat{i} + 2\hat{j}) + 0 = m(-2\hat{i} + \hat{j}) + 13\ m(v_x\hat{i} + v_y\hat{j})$

$$v = \frac{5}{13}\hat{i} + \frac{\hat{j}}{13}$$

(b) Impulse on small mass (m) = $m(\vec{v}_f - \vec{v}_i) = m(-2\hat{i} + \hat{j} - 3\hat{i} - 2\hat{j}) = m(-5\hat{i} - \hat{j})$

Impulse on big mass = $m\ (5\hat{i} + \hat{j})$

(c) $e = \left(\frac{\text{velocity of separation along common normal}}{\text{velocity of approach along common normal}}\right)$

(common normal is in the direction of impulse)

$$= -\left(\frac{(-2\hat{i} + \hat{j})\left(\frac{5\hat{i} + \hat{j}}{\sqrt{26}}\right) - \left(\frac{5}{13}\hat{i} + \frac{\hat{j}}{13}\right).\left(\frac{5\hat{i} + \hat{j}}{\sqrt{26}}\right)}{(3\hat{i} + 2\hat{j})\left(\frac{5\hat{i} + \hat{j}}{\sqrt{26}}\right) - 0}\right) = \left(\frac{11}{17}\right).$$