JEE EXPERT

PRACTICE TEST – 03 (31 MARCH 2020)

ANSWER KEY & SOLUTION

Physics [PART-I]

1.	**A**	2.	**A**	3.	**B**	4.	**B**
5.	**D**	6.	**A**	7.	**B**	8.	**D**
9.	**A, B, C, D**	10.	**B, C**	11.	**A, B**	12.	**A, C**
13.	**(A)→ (p, r)**		**(B)→ (q, s)**		**(C)→ (q, r)**		**(D)→ (p, s)**
14.	**(A)→ (s)**		**(B)→ (s)**		**(C)→ (p, s)**		**(D)→ (q, r, t)**
15.	**2**	16.	**1**	17.	**8**	18.	**0**
19.	**3**	20.	**6**				

Chemistry [PART-II]

1.	**A**	2.	**A**	3.	**C**	4.	**B**
5.	**D**	6.	**C**	7.	**A**	8.	**B**
9.	**B, C**	10.	**A, B, C, D**	11.	**A, B, C, D**	12.	**A, B, D**
13.	**(A) → (r); (B) → (s); (C) → (q, t); (D)→ (p)**						
14.	**(A) → (p, r); (B) → (q); (C) → (s); (D)→ (t)**						
15.	**2.**	16.	**4**	17.	**6**	18.	**5**
19.	**6**	20.	**4**				

Mathematics [PART-III]

1.	**B**	2.	**D**	3.	**A**	4.	**C**
5.	**A**	6.	**A**	7.	**D**	8.	**D**
9.	**A, B**	10.	**A, B**	11.	**A, B**	12.	**A, B, C**
13.	**(A) → (q)**		**(B) → (s)**		**(C) → (r)**		**(D) → (p)**
14.	**(A) → (r)**		**(B) → (p)**		**(C) → (s)**		**(D) → (q)**
15.	**1**	16.	**4**	17.	**4**	18.	**8**
19.	**4**	20.	**1**				

HINTS AND SOLUTIONS

PHYSICS

2. $\frac{dQ}{dt} = k\frac{AdT}{dt}$

$\int_0^L dx = -\left(\frac{\alpha}{H}\right)^{A\ T_2}\left(\frac{dT}{T}\right)$

$L = -\frac{\alpha A}{H}\left(T_1 \ell_n \frac{T_2}{T_1}\right)$

$\int_0^x dx = \left(\frac{dA}{H}\right)\left(d\frac{T}{T}\right)$

$x = -\frac{\alpha A}{H}\ell_n\left(\frac{T}{T_1}\right)$

4. $F = \pi R^3\rho g - \frac{1}{3}\pi R^3\rho g = \frac{2}{3}\pi R^3\rho g$

6. $W = FS\cos\theta = m\ (g + a) \times \frac{1}{2}at^2\cos\theta = \frac{m(g+a)}{2}at^2$

7. $\tan\theta = \frac{a}{g}$ $\qquad \frac{5}{2.5} = \frac{a}{g} \Rightarrow a = 2g$

8. Work done $= -\frac{Gmm}{3R} + \frac{Gmm}{R} = \frac{20}{3}mgR$

9. $P = (P_0 + mg)$

$W = P.\ \Delta S$

$= (P_0 + mg) \times \Delta S = 20$ J

$\Delta U = nc_v\Delta T = \frac{3}{2}nR\Delta T = \frac{3}{2}P\Delta v = 30\,J$

$\Delta Q = \Delta U + \Delta w = 50\,J$

11. $\Rightarrow |a_T| = |a_c|$

$-\frac{dv}{dt} = \frac{v^2}{r}$

$-\int\frac{dv}{v^2} = \int\frac{dt}{r} + \frac{1}{v}\Big|_{v_0}^{v} = \frac{1}{r}t$

$\frac{1}{v} - \frac{1}{v_0} = \frac{t}{r}$

$v = \frac{v_0}{1 + v_0 t / R}$

$v\frac{dv}{ds} = \frac{v^2}{r}$

$\Rightarrow v_0 = v_0 e^{-s/R}$

12. $a = -\alpha V$

$V\frac{dv}{d.s} = -\alpha v$

$$\int_{v_0}^{0} dv = -\alpha\int_0^s ds$$

$$\frac{dv}{dt} = -\alpha v$$

$$|\,nv\,|_{v_0}^{v} \;\; -\alpha t\,|_0^t \Rightarrow v = v_0 e^{-\alpha t}$$

$t \to \infty$
$v \to 0$

16. $\mu = \frac{2}{4} = \frac{1}{20}$

$t = 01 \qquad v = \frac{4}{1} = 40$

$$v = \sqrt{\frac{T}{\mu}}$$

$1600 = T \times 20$
$T = 80$
$K = 1$

17. $t_{min} = \frac{330}{165} + \frac{1320}{\sqrt{1320 \times 33 \times 10^5 \times 0.001}} + \frac{1650 - 330}{330} \quad t_{min} = 8\,\text{sec}.$

20. $I_{zz} = \frac{ML^2}{6}$

CHEMISTRY

1.

$$2SO_2 + O_2 \rightleftharpoons 2SO_3$$

	$2SO_2$	O_2	$2SO_3$
Initial moles	2	1	0
	$2-x$	$1-x/2$	x

$$K_c = \frac{x^2}{1-x/2 \;\; 2-x^{\;2}}$$

Mili equivalent of $KMnO_4^-$ = mili equivalent of SO_2
$0.4 \times 5 = 2 \times (2 - x)$
$2 = 2 \times (2 - x)$
$1 = 2 - x$
$x = 1$

$$K_c = \frac{1}{\left(1-\frac{1}{2}\right)\; 2-1^{\;2}} = 2$$

2.

$$2NH_3 \longrightarrow N_2 + 3H_2$$

	$2NH_3$	N_2	$3H_2$
Initial moles	2	0	0
	0	1	3

Before starting the reaction:

$$D = \frac{w}{2 \times 2} \qquad \text{... 1}$$

After the reaction:

$$d = \frac{w}{2 \times 4} \qquad \text{... 2}$$

$$\frac{D}{d} = 2$$

$$d = \frac{D}{2} = \frac{8.5}{2} = 4.25$$

9. P^H of acidic buffer $P^H = P^{K_a} + \log\frac{\text{conjugate base}}{[\text{acid}]}$

10. (A) For same central atom, if the electronegativity of surrounding atoms increases, then bond angle decreases.
(B) ClF_2^- has sp^3d hybridization with 3 l.P. and 2 b.p. Hence it is linear, but ClF_2^+ has sp^3 hybridization with 2 l.P and 2 b.p. hence it is bent
(C) Dipole moment $\propto$ E.N
Dipole moment is zero for symmetrical molecule.
(D) O–hydroxy benzaldehyde molecule has intra molecular hydrogen bondings but inter molecular hydrogen bonds present between molecules of p –hydroxy benzaldehyde.

15. $V_0 = \frac{2.18\times10^8\times Z}{n}\text{cm / sec}$

Velocity of electron in certain Bohr's orbit $V_x = \frac{1}{275}\times\text{velocity of light}$

$= \frac{1}{275}\times3\times10^{10} = 1.09\times10^8\ \text{cms}^{-1}$

and $= \frac{1}{275}\times3\times10^{10} = 1.09\times10^8\ \text{cms}^{-1}$

So, n = 2

16. Molecular weight of gas = 54
So $\therefore$ 12n + 2n – 2 = 54
$\Rightarrow$ n = 4

19. The number of α hydrogen in the carbocation are 6.

20. $-NH_2, -OH, -NR_2, -OR$ are o, p directing and activating groups.

MATHEMATICS

1. Equation of tangent $y = mx - am^2$ …(1)
Let co-ordinate of mid-point of PQ is (h, k)
then equation of PQ = $hx - ky = h^2 - k^2$ …(2)
from (1) and (2), we get
$k^3 = h^2(k - a)$
hence curve is $y^3 = x^2(y - a)$.

2. Here $\frac{1}{t_1} = \tan\left(\frac{\pi}{4}+\theta\right)$ and $\frac{1}{t_2} = \tan\left(\frac{\pi}{4}-\theta\right)$
so, $t_1t_2 = 1 \Rightarrow$ the x-coordinate of P = $at_1t_2 = a$.

3. $\tan\alpha \tan2\alpha \tan3\alpha \ldots \tan(2n-1)\alpha$
$= \{\tan\alpha \tan(2n-1)\alpha\}\{\tan2\alpha \tan(2n-2)\alpha\} \ldots\ldots \{\tan(n-1)\alpha \tan(n+1)\alpha\} \tan n\alpha$,
$= \{\tan\alpha \tan(\pi/2 - \alpha)\}\{\tan2\alpha \tan(\pi/2 - 2\alpha)\} \ldots \tan\frac{\pi}{4} = 1.1.1. \ldots.1 = 1$

4. $\tan^{-1}(\cot\alpha) - \cot^{-1}(\tan\alpha)$
$= \tan^{-1}\left(\frac{1}{\tan\alpha}\right) - \left(\frac{\pi}{2} - \tan^{-1}(\tan\alpha)\right) = -\pi$ $\left(\text{As } -\frac{\pi}{2} < \alpha < 0\right)$.

5. In ΔABC,
$a = 2R\sin A \Rightarrow k\left(\sqrt{\cot\frac{B}{2}\cot\frac{C}{2} - 1}\right)\cos\frac{A}{2} = 4R\sin\frac{A}{2}\cos\frac{A}{2}$

$\Rightarrow k = 4R\sqrt{\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}$

$\Rightarrow k = 2\sqrt{Rr}$.

6. (1 + (a + 3) + a) (9 + (a + 3)3 + a) $\le$ 0

$\Rightarrow -\frac{9}{2} \le a \le -2$

Now, as $\frac{1-a^2}{a} = \frac{1}{a} - a$ is decreasing in a, its minimum value = $\frac{1}{(-2)} - (-2) = \frac{3}{2}$.

7. $S_n - S_{n-2} = 2$.

$\Rightarrow T_n + T_{n-1} = 2$

Also $T_n + T_{n-1} = \left(\frac{1}{n^2}+1\right)T_{n-1} = 2$

$\Rightarrow T_{n-1} = \frac{2}{1+\frac{1}{n^2}} = \frac{2n^2}{1+n^2}$.

So, $T_m = \frac{2(m+1)^2}{1+(m+1)^2}$.

8. $x_1x_2x_3 = 2\cdot35\cdot7 = 2\cdot49\cdot5 = 10\cdot7\cdot7 = 14\cdot7\cdot5$.

So total number of solution set = $3\cdot3! + \frac{3!}{2!}$ = 21.

9. $169\, e^{i\left(\pi+\cos^{-1}\frac{5}{13}+\sin^{-1}\frac{12}{13}\right)}$

$= -169\left[\cos\left(\cos^{-1}\frac{5}{13}\right)+i\sin\left(\cos^{-1}\frac{5}{13}\right)\right]\left[\cos\left(\sin^{-1}\frac{12}{13}\right)+i\sin\left(\sin^{-1}\frac{12}{13}\right)\right]$

$= -169\left[\frac{5}{13}+i\frac{12}{13}\right]\left[\frac{5}{13}+i\frac{12}{13}\right]$

$= [119 - 120\, i] = -i[\,120 + 119i]$.

10. $\frac{z-4}{z-2i} + \frac{\bar{z}-4}{\bar{z}+2i} = 0 \Rightarrow z\bar{z} + (-2+i)z + (-2-i)\bar{z} = 0$ which is a circle $x^2 + y^2 - 4x - 2y = 0$.

Let (x_1, y_1) be the mid-point of a chord, then its equation is

$xx_1 + yy_1 - 2(x + x_1) - (y + y_1) = x_1^2 + y_1^2 - 4x_1 - 2y_1$.

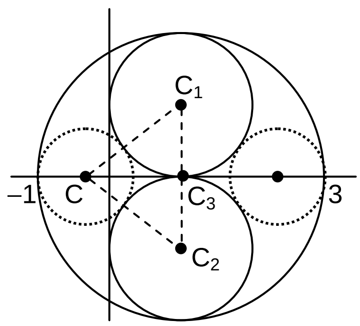
It passes through (0, 0), so, the locus of (x_1, y_1) is $x^2 + y^2 - 2x - y = 0$. So, $z_1 = x_1 + iy_1$, lies on this circle for which the points (2, 0) and (0, 1) are extremities of a diameter. Also (0, 0) and (2, 1) represent extremities of another diameter.

11. Given circles touch externally at real axis. So, the centre C of the desired circle lies on real axis, which has radius r. Thus $CC_1 = CC_2 = 1 + r$

$C_1C_3 = C_2C_3 = 1$

$CC_3 = 2 - r$

$\therefore (1 + r)^2 = 1^2 + (2 - r)^2 \Rightarrow r = \frac{2}{3}$.



So, the centre of desired circle is at

$-1+\frac{2}{3} = -\frac{1}{3}$ or $3-\frac{2}{3} = \frac{7}{3}$.

So the equation of the circles are $\left|z+\frac{1}{3}\right| = \frac{2}{3}$ and $\left|z-\frac{7}{3}\right| = \frac{2}{3}$.

13. (A). $x^2 - y^2 = 10$

Equation of asymptotes are $y = \pm x$

Let P_1 and P_2 be length of perpendicular from any point on the asymptotes

$$P_1 = \left|\frac{\sqrt{10}\tan\theta - \sqrt{10}\sec\theta}{\sqrt{2}}\right|$$

$$P_2 = \left|\frac{\sqrt{10}\tan\theta + \sqrt{10}\sec\theta}{\sqrt{2}}\right|$$

$$P_1P_2 = \frac{10}{2}\left(\sec^2\theta - \tan^2\theta\right) = 5$$

(B) Director circle of hyperbola $\frac{x^2}{4} - \frac{y^2}{1} = 1$ is $x^2 + y^2 = 3$

Solving this with $\frac{x^2}{4} + \frac{y^2}{3} = 1$ gives

$3x^2 + 4(3 - x^2) = 12$

$\Rightarrow x = 0 \;\; \therefore \;\; y = \pm\sqrt{3}$

$\therefore$ number of points are 2.

(C) $(4x - 8)^2 + 16y^2 = \left(x + \sqrt{3}y + 10\right)^2$

$$(x - 2)^2 + y^2 = \frac{1}{4}\left(\frac{x + \sqrt{3}y + 10}{2}\right)^2 \Rightarrow e = \frac{1}{2}$$

one of the focus is (2, 0) and directrix is $x + \sqrt{3}y + 10 = 0$

Distance between one of the focus and its corresponding directrix is

$$\frac{a}{e} - ae = a\left(2 - \frac{1}{2}\right) = \frac{12}{2} = 6$$

$$a = \frac{6 \times 2}{4} = 4$$

Distance between directrices is $\frac{2a}{e} = \frac{2 \times 4}{1/2} = 16$.

(D) Any point on line $y - x + 2 = 0$ is $(\lambda, \lambda - 2)$

equation of chord of contact to $y^2 = 4x$ is

$y(\lambda - 2) = 2(x + \lambda) = (x + y) - \lambda(y - 2) = 0 \Rightarrow y = 2$ and $x + y = 0$.

15. Let the equation of chord be $y = mx + c$. Combined equation of lines joining the point of intersection with origin is $3x^2 - y^2 - 2(x - 2y)\left(\frac{y - mx}{c}\right) = 0$

i.e., $x^2.(3c + 2m) - y^2.(c - 4) - 2xy.(1 + 2m) = 0$

these lines will be mutually perpendicular if $3c + 2m - c + 4 = 0$

$\Rightarrow 2m + 2c = -4 \Rightarrow m + c = -2$ that means the chord $y = mx + c$ is always pass through the point (1, –2)

16. $S_1 - S_2 = 0 \Rightarrow 3x - y = 2$ which is directrix of parabola whose vertex is (0, 0).

The axis of the parabola is $x + 3y = 0$.

Point of intersection of axis and directrix $A \equiv \left(\frac{3}{5}, -\frac{1}{5}\right)$.

Let the focus be (x_1, y_1) then

$$\frac{x_1 + \frac{3}{5}}{2} = 0, \quad \frac{y_1 - \frac{1}{5}}{2} = 0$$

$x_1 = -\frac{3}{5}$, $y = \frac{1}{5}$.

17. Equation of tangent is $y = 2x \pm \sqrt{4a^2 + b^2}$
$\Rightarrow$ this is normal to the circle $x^2 + y^2 + 4x + 1 = 0$
$\Rightarrow$ this tangent passes through (–2, 0).
$\Rightarrow 0 = -4 \pm \sqrt{4a^2 + b^2} \Rightarrow 4a^2 + b^2 = 16$
$\Rightarrow$ using A.M $\geq$ G.M, we get
$\frac{4a^2 + b^2}{2} \geq \sqrt{4a^2 + b^2} \Rightarrow ab \leq 4$.

18. $3^{n_1} = (4-1)^{n_1} = 4\lambda_1 + (-1)^{n_1}$
$5^{n_2} = (4+1)^{n_2} = 4\lambda_2 + 1$
$7^{n_3} = (8-1)^{n_3} = 4\lambda_3 + (-1)^{n_3}$
Hence, any positive integer power of 5 will be in the form of $4\lambda_2 + 1$. Even power of 3 and 7 will be in the form of $4\lambda + 1$ and odd power of 3 and 7 will be in the form of $4\lambda - 1$ thus required no. of divisors = 8 (3.5 + 3.5) = 240

19. Given that cos A cos C = $\frac{ac}{b^2} \Rightarrow \frac{b^2 + c^2 - a^2}{2bc} . \frac{a^2 + b^2 - c^2}{2ba} = \frac{ac}{b^2} \Rightarrow b^2 = a^2 + c^2 \Rightarrow \angle B = \frac{\pi}{2}$.

20. x = tan 1
$\sqrt{3} > \tan 1 > 1$