CLASSROOM STUDY PACKAGE

MATHEMATICS

GEOMETRY - 1

JEE EXPERT

CHAPTER -1

CONTENTS

EUCLID'S GEOMETRY

COORDINATE GEOMETRY

EUCLID'S GEOMETRY

Key Concepts

"Around 300 B.C., Greek mathematician Euclid, a teacher of mathematics at Alexandria in Egypt collected all known work in the field of Geometry and arranged it in his famous treatise called 'Elements. "

Euclid's axioms and postulates

The Greek mathematicians of Euclid's time expressed some basic terms in geometry such as point, line, plane, solid, etc. According to what they observed in the world around them. From their observation, a solid is an object in space which has three dimensions called length, breadth and thickness. It has shape, size, position and place. It can be moved from one place to another. The boundaries of a solid are called surface. Surface has two dimensions which are length and breadth. It has no thickness. The boundaries of a surface are lines or curves. Lines and curves has no breadth and no thickness. The ends of a line or a curve are points. A point has no dimension. Euclid presented his work in the form of definitions , axioms, postulates and theorems. Some terms defined by Euclid and other mathematicians of that time may not be fully explained but still the observations were very strong and hence, form the basis for the further development of the subject.

Euclid defined some terms precisely as below

Point : A point is that which has no part.

Line : A line is breadthless length. The end of a line are points.

Surface : A surface is that which has length and breadth only.

Ends of a line: The ends of a line are points.

Edges of a surface: The edges of a surface are lines.

Straight line: A straight line is a line which lies evenly with the point on itself.

Plane surface : A plane surface is a surface which lies evenly with the straight lines on itself.

If you carefully study these definitions you find that some of the terms like part, breadth, length, evenly, etc. need to be further explained clearly. For example, consider his definition of a point. In this definition, 'a part' needs to be defined. Suppose if you define 'a part' to be that which occupies 'area', again 'an area' needs to be defined. So, to define one thing, you need to define many other things, and you may get a long chain of definition without an end. For such reasons, mathematicians agree to leave some geometric terms undefined.

The basic undefined terms in geometry are

(i) point (ii) Line (iii) Plane

(i) Point: A point has position only. It has no length, no width and no thickness.

(ii) Line: A line has length but no width and no thickness.

(iii) Plane: If any two points are taken anywhere on a surface and joined by a straight line, then if each and every point of this line lies in the surface, the surface is called a plane or a plane surface.

Though Euclid defined a point, a line and a plane, the definitions are not accepted by today's mathematicians. They take terms as undefined terms. Because these terms can be represented intuitively or explain with the help of 'Physical Models'.

For example : The tip of a fine sharp pencil, or the tip of a needle represent a point. A thread hold tightly by two hands represent a line. the top of a table represent a plane etc.

Undefined properties (axioms and postulates)

On the basis of above definitions and some more observations, Euclid started some obvious universal truth as axioms and postulates. Euclid assumed these universal truths as such, which were not to be proved. In present time, the terms axioms and postulates can be used interchangeably but Euclid made a fine distinction between the two terms.

Axioms

The assumptions, which are granted without proof and are used throughout in mathematics which are obvious universal truths, and not specifical linked to geometry are termed as axioms.

Some of the Euclid's Axioms are given below:

(i) Things which are equal to the same thing are equal to one another.

i.e. if $x = y$ and $y = z$, then $x = z$.

(ii) If equals are added to equals, the wholes are equal.

i.e. if $a = b$ and $c = d$, then $a + c = b + d$

Also $a = b$ $\Rightarrow a + c = b + c$

E.g. If $5 = 5$, $\Rightarrow 5 + 2 = 5 + 2$

or $7 = 7$

(iii) If equals are subtracted from equals, the remainders are equal.

i.e. if $a = b$ and $c = d$, then $a – c = b – d$.

E.g $3 = 3$

$\Rightarrow$ $3 – 1 = 3 – 1$

or $2 = 2$

Here magnitudes of same kinds can be compare and subtract. We cannot subtract a line from a triangle similarly we cannot subtract kg from litres.

(iv) The things which coincide with one another are equal to one another.

In fig. 1, two line segment AB and CD coincide with each other

$\therefore$ AB = CD = 2 cm

(v) The whole is greater than the part.

i.e. if $a > b$, then there exists c such that $a = b + c$.

Here b is a part of a and therefore, a is greater than b.

In Fig. 2, AB = AC + BC

= 1 + 2

= 3cm

$\therefore$ AB > AC and AB > BC

(vi) Things which are halves of the same things are equal to one another.

in fig. 4, CD and EF are halves of AB

$\therefore$ CD = EF

Postulates

The assumptions, which are specifically linked to geometry and are obvious universal truths, are termed as postulates.

Euclid gave five postulates as stated below:

(i) A straight line may be drawn from any one point to any other point.

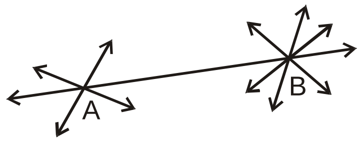


Fig. (1)

(ii) A terminated line (i.e., a line segment) can be produced indefinitely one either side.

A terminated line is called line segmetn these days.

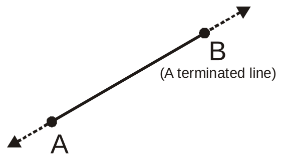


Fig. (2)

The Fig. 2 shows a terminated line (line segment) AB with two end points A and B.

(iii) A circle can be drawn with any centre and any radius.

(iv) All right angles are equal to one another.

(v) If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then two straight lines, if produced indefinitely, meet on that side on which the angles are less than two right angles.

3.3 A Statement as a theorem

Propositions or theorems are statements which are proved, using definition, axioms, previously proved statements and by deductive reasoning.

e.g., (i) "The sum of all angles of a triangles is equal to 180º" is a theorem.

(ii) "The sum of all angles of a quadrilateral is 360º" is a theorem.

Euclid deduced 465 propositions (theorem) in a logical chain using his axioms, postulates, definitions and already proved theorems.

When we proved a theorem, then it becomes a general statements for other theorem. A theorem and assumption (postulate) are both statement. The difference is that an assumption (postulate) is a accepted to be true only when it has been proved.

Some axioms of points and lines

We shall assume some properties about lines and points without any proof but these properties are obvious universal truths. These properties are taken as axioms.

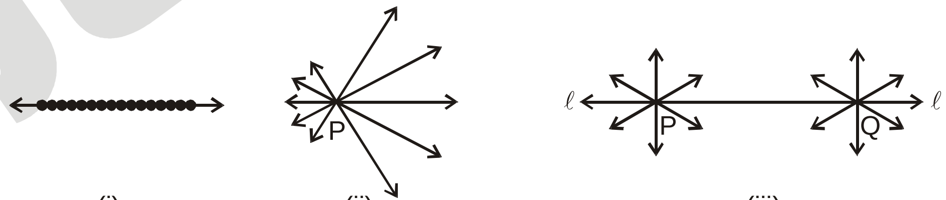


Fig. (3)

(i) A line contains infinitely many points. (see fig. 3(i))

(ii) Through a given point, infinitely many lines can be drawn. in Fig.3(ii), infinitely many lines pass through the point P.

(iii) Given two distinct points, there exists one and only one line through them.

In fig. 3(iii), we observe that, out of all lines passing through the point P there is exactly one line 'ℓ' which also passes through Q. Similarly, out of all lines passing through the point Q there is exactly one line 'ℓ' which also passes through P. Hence we find exactly one line 'ℓ' which also passes through P. Hence, we find exactly one line 'ℓ' which can be drawn through two points P and Q.

Theorem 1 : Two distinct lines cannot have more than one point in common.

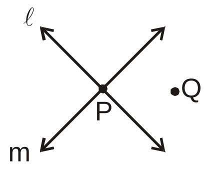


Fig. (4)

Given : Two distinct line ℓ and m. (see fig. 4)

To prove : Lines ℓ and m have at most one point in common.

Proof : Two distinct lines ℓ and m intersect at a point P.

Let us suppose they will intersect at another point, say Q (different from P). It means two lines ℓ and m passing through two distinct point P and Q. But it is contrary to the axiom which states that "Given two distinct points, there exists one and only one line pass throgh them".

So our supposition is wrong.

hence, two distinct lines cannot have more than one point in common.

Theorem 2 : Two lines which are both parallel to the same line, are parallel to each other.

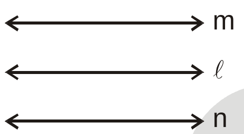


Fig.(5)

Given: Three lines ℓ, m, n in a plane such that m || ℓ and n || ℓ . (See Fig. 5)

To prove : m || n

Proof : If possible, let m be not parallel to n. Then, m and n intersect in a unique point, say P.

Thus, through a point P outside ℓ, there are two lines m and n both parallel to ℓ. This is a contradiction to the parallel axiom. So, our supposition is wrong. Hence m||n.

Theorem 3 : If ℓ, m, n, are lines in the same plane such that ℓ intersects m and n || m, then ℓ intersect n also.

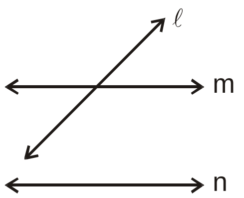


Fig. (6)

Given : Three lines ℓ, m, n in the same plane such that ℓ intersect m and n||m.

To prove : Lines ℓ and n are intersecting lines.

Proof : Let ℓ and n be non intersecting lines. Then ℓ||n (see Fig. 6)

But, n||m

$\therefore$ ℓ || n and n ||m $\Rightarrow$ ℓ||m $\Rightarrow$ ℓ and m are non intersecting lines.

This is a contradiction to the hypothesis that ℓ and m are intersecting lines.

So, our supposition is wrong.

Hence, line ℓ intersects line n.

Exercise # 1

1. A proof is required for :

(A) Postulate (B) Axiom (C) Theorem (D) Definition

2. How many number of lines does pass through two distinct points.

(A) 1 (B) 2 (C) 3 (D) 4

3. Which of the following is an example of a geometrical line.

(A) Black Board (B) Sheet of paper

(C) Meeting place of two walls (D) Tip of the sharp pencil

4. Given four points such that no three of them are collinear, then the number of lines that can be drawn through any of the two points is :

(A) 2 lines (B) 4 lines (C) 6 lines (D) 8 lines

5. Two planes intersect each other to form a :

(A) Plane (B) Point (C) Straight line (D) Angle

6. 'Lines are parallel if they do not intersect' is stated in the form of :

(A) An axiom (B) A definition (C) A postulate (D) A proof

7. Select the wrong statement :

(A) Only one line can pass through a single point.

(B) Only one line can pass through two distinct points.

(C) A line consists of infinite number of points.

(D) If two circles are equal, then their radii are equal.

8. Number of dimension(s) a surface has :

(A) 0 (B) 1 (C) 2 (D) 3

9. The number of line segments determined by three collinear points is :

(A) Two (B) Three (C) Only one (D) Four

10. If the point P lies in between M and N and C is midpoint of MP, then :

(A) MC + PN = MN (B) MP + CP = MN (C) MC + CN = MN (D) CP + CN = MN

11. Euclid's second axiom is

(A) The things which are equal to the same thing are equal to one another

(B) If equals be added to equals, the wholes are equal

(C) If equals be subtracted from equals, the remainders are equals

(D) Things which coincide with one another are equal to one another

12. Euclid's fifth postulate is

(A) The whole is greater than the part.

(B) A circle may be described with any centre and any radius.

(C) All right angles are equal to one another.

(D) If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, tehn the two straight lines if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

13. The things which are double of the same thing are

(A) equal (B) unequal

(C) halves of the same thing (D) double of the same thing

14. Axioms are assumed

(A) universal truths in all branches of mathematics

(B) universal truths specific to geometry

(C) theorems

(D) definitions

15. John is of the same age as Mohan. Ram is also of the same age as Mohan. State the Euclid's axiom that illustrates the relative ages of John and Ram

(A) First Axiom (B) Second Axiom (C) Third Axiom (D) Fourth Axiom

16. If a straight line falling on two straight lines makes the interior angles on the same side of it, whose sum is 120º, then the two straight lines, if produced indefinitely, meet on the side on which the sum of angles is

(A) less than 120º (B) greater than 120º (C) is equal to 120º (D) greater than 180º

17. The three steps from solids to points are :

(A) Solids -surfaces - lines - points (B) solids - lines - surfaces- points

(C) Lines - points - surfaces - solids (D) Lines - surface - points - solids

18. The number of dimensions, a solid has :

(A) 1 (B) 2 (C) 3 (D) 0

19. The number of dimensions, a surface has :

(A) 1 (B) 2 (C) 3 (D) 0

20. The number of dimensions, a surface has :

(A) 0 (B) 1 (C) 2 (D) 3

21. Euclid divided his famous treatise "The Elements" into

(A) 13 chapters (B) 12 chapters (C) 11 chapters (D) 9 chapters

22. The total number of propositions in the Elements are :

(A) 465 (B) 460 (C) 13 (D) 55

23. Boundaries of solids are :

(A) Surfaces (B) curves (C) lines (D) points

24. Boundaries of surfaces are :

(A) Surfaces (B) curves (C) lines (D) points

25. In indus Valley Civilisation (about 300 B.C.), the bricks used for construction work were having dimensions in the ratio

(A) 1 : 3 : 4 (B) 4 : 2 : 1 (C) 4 : 4 : 1 (D) 4 : 3 : 2

26. A pyramid is a solid figure, the base of which is

(A) only a triangle (B) only a square (C) only a rectangle (D) any polygon

27. The side faces of a pyramid are :

(A) Triangles (B) Squares (C) Polygons (D) Trapeziums

28. it is known that if $x + y = 10$ then $x + y + z = 10 + z$. The Euclid's axiom that illustrates this statement is :

(A) First Axiom (B) Second Axiom (C) Third Axiom (D) Fourth Axiom

29. In ancient India, the shapes of altars used for house hold rituals were :

(A) Squares and circles (B) Triangles and rectangles

(C) Trapeziums and pyramids (D) Rectangles and squares

30. The number of interwoven isosceles triangles in Sriyantra (in the Atharvaveda) is :

(A) Seven (B) Eight (C) Nine (D) Eleven

31. Greek's emphasised on :

(A) Inductive reasoning (B) Deductive reasoning

(C) Both A and B (D) practical use of geometry

32. In Ancient India, altars with combination of shapes like rectangles, triangles and trapeziums were used for:

(A) Public worship (B) Household rituals (C) Both A and B (D) None of A, B, C

33. Euclid belongs to the country :

(A) Babylonia (B) Egypt (C) Greece (D) India

34. Thales belongs to the country :

(A) Babylonia (B) Egypt (C) Greece (D) Rome

35. Pythagoras was a student of :

(A) Thales (B) Euclid (C) Both A and B (D) Archimedes

36. Which of the followings needs a proof ?

(A) Theorem (B) Axiom (C) Definition (D) Postulate

37. Euclid stated that all right angles are equal to each other in the form of

(A) an axiom (B) a definition (C) a postulate (D) a proof

38. Lines are parallel if they do not intersect ' is stated in the form of

(A) an axiom (B) a definition (C) a postulate (D) a proof

39. According to Euclid's definition, the ends of a line are

(A) Breadthless (B) Points (C) Lengthless (D) None of these

40. According to Euclid, the first axiom is

(A) Things which are equal to the same things, are equal to each other

(B) If equals are added to equals, the results are equal

(C) If equals are subtracted from equals, the results are equal

(D) The whole is greater than its part

41. Things which are three times of the same thing are

(A) Equal to each other (B) Not equal to each other

(C) Half of the same (D) Double of the same thing

42. If a point C lies between two points A and B such that AC = BC, then

(A) AC = AB (B) AC = $\frac{1}{2}$ AB (C) AB = $\frac{1}{2}$ AC (D) AC = $\frac{1}{3}$ AB

43. $\angle A = \angle B$ and $\angle B = \angle C$, According to which axiom of Euclid the relation between $\angle A$ and $\angle C$ is established?

(A) I (B) II (C) III (D) IV

44. Two distinct lines ℓ and m can not have

(A) Any point in common (B) One point in common

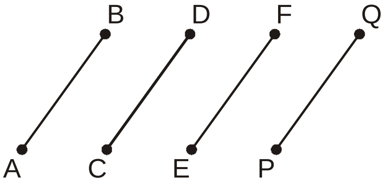
(C) Two points in common (D) None of the above

45. Through two points

(A) No line can be drawn (B) A unique line can be drawn

(C) More than one line can be drawn (D) None of these

46. If AB = CD, CD = EF and EF = PQ, then which one of the following is not true



(A) AB = PQ (B) CD = PQ (C) AB = EF (D) AB $\neq$ CD

47. For every line ℓ and for every point P (not on ℓ), there exists a unique line passing through P.

(A) which is || to ℓ (B) which is $\perp$ to ℓ

(C) which is coincident with ℓ (D) None of these

True or false

1. Pyramid is a solid figure, the base of which is a triangle of square or some other polygon and its side faces are equilateral triangles that converges to a point at the top.

2. In Vedic period, squares and circular shapes altars were used for household rituals, while altars whose shapes were combination or rectangles, triangles and trapeziums were used for public worship.

3. In geometry, we take a point, a line and a plane as undefined terms.

4. If the area of a triangle equals the area of a rectangle and the area of the rectangle equals that of a square, then the area of the triangle also equals the area of the squares.

5. Euclid's fourth axiom says that everything equals itself.

6. The Euclidean geometry is valid only for figures in the plane.

7. Euclidean geometry is valid only for curved surfaces.

8. The boundaries of the solids are curves.

9. The edges of a surface are curves.

10. The things which are double of the same thing are equal to one another.

Exercise # 2

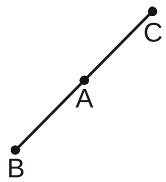
(Subjective)

1. How many lines can pass through :

(i) One point (ii) two distinct points

2. Write the largest number of points in which two distinct straight lines may intersect.

3. A, B and C are three collinear points such that point A lies between B and C. Name all the line segments determined by these points and write the relation between them.



4. State, true or false :

(i) A point is a undefined term.

(ii) A line is a defined term.

(iii) Two distinct lines always intersect at one point.

(iv) Two distinct point always determine a line.

(v) A ray can be extended infinitely on both the sides of it.

(vi) A line segment has both of its end-points fixed and so it has a definite length.

5. Name three undefined terms.

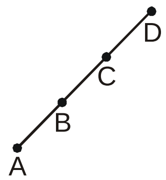
6. If AB is a line and P is a fixed point, outside AB, how many lines can be drawn through P which are :

(i) Parallel to AB (ii) Not parallel to AB

7. Out of the three lines AB, CD and EF, if AB is parallel to EF and CD is also parallel to EF, then what is the relation between AB and CD.

8. If A, B and C three points on a line, and B lies between A and C, then prove that AB + BC = AC.

9. In the given figure, if AB = CD ; prove that AC = BD.



10. (i) How many lines can be drawn to pass through three given point if they are not collinear ?

(ii) How many line segments can be drawn to pass through two given points if they are collinear ?

Answer key

Exercise # 1

1.	C	**2.**	A	**3.**	C	**4.**	C	**5.**	C	**6.**	B	**7.**	A
8.	C	**9.**	B	**10.**	C	**11.**	B	**12.**	D	**13.**	A	**14.**	A
15.	A	**16.**	A	**17.**	A	**18.**	C	**19.**	B	**20.**	A	**21.**	A
22.	A	**23.**	A	**24.**	B	**25.**	B	**26.**	A	**27.**	A	**28.**	B
29.	A	**30.**	C	**31.**	B	**32.**	C	**33.**	C	**34.**	C	**35.**	A
36.	A	**37.**	C	**38.**	B	**39.**	D	**40.**	A	**41.**	A	**42.**	B
43.	A	**44.**	C	**45.**	B	**46.**	D	**47.**	A				

True of False

1. False **2.** True **3.** True **4.** True **5.** True **6.** True **7.** False
8. False **9.** False **10.** True

Exercise # 2

1. (i) Infinite (ii) only one **2.** one **3.** Ba, AC & BC ; BA + AC = BC

4. (i) True (ii) False (iii) False (iv) True (v) False (vi) True

5. Point, line and plane **6.** (i) onlye one (ii) infinite **7.** AB || CD **10.** (i) Three lines (ii) One

COORDINATE GEOMETRY

Key Concepts

Co-ordinate System

In two dimensional coordinate geometry, we use generally two types of co-ordinate system.

(i) Cartesian or Rectangular co-ordinate system.

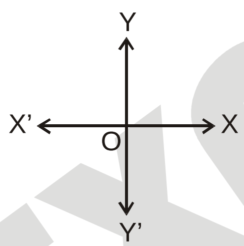
(ii) Polar co-ordinate system.

In Cartesian co-ordinate system we represent any point by ordered pair (x,y), where x and y are called x and Y co-ordinate of that point respectively.

In polar co-ordinate system we represent any point by ordered pair (r, θ) where 'r' is called radius vector and 'θ' is called vectorial angle of the point.

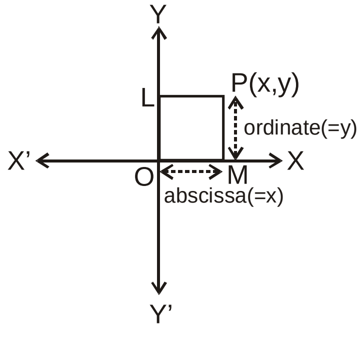
Rectangular Co-ordinates

Take two perpendicular lines X' OX and Y'OY intersecting at the point O. X'OX and Y'OY are called the co-ordinate axes. X'OX is called the X-axis, Y'OY is called the Y-axis and O is is called the origin. Lines X'OX and Y'OY are sometimes also called rectangular axes.



(a) Co-ordinates of a Point:

Let P be any point as shown in figure. Draw PL and PM perpendiculars on Y-axis and X-axis, respectively. The length LP (or OM) is called the x-coordinate or the abscissa of point P and MP is called the y-coordinate or the ordinate of point P. A point whose abscissa is x and ordinate is y named as the point (x, y) or P(x,y).



The two lines **X' OX** and **Y'OY** divide the plane into four parts called quadrants. XOY, YOX', X'OY' and Y'OX are, respectively, called the first, second, third and fourth quadrants. The following table shows the signs of the coordinates of points situated in different quadrants:

Quadrant	X-coordinate	Y-coordinate	Point
First quadrant	+	+	(+,+)
Second quadrant	–	+	(–,+)
Third quadrant	–	–	(–, –)
Fourth quadrant	+	–	(+,–)

Remarks:

(i) Abscissa is the perpendicular distance of a point from y-axis. (i.e., positive to the right of y-axis or negative to the left of y-axis)

(ii) Ordinate is positive above x-axis or negative below x-axis.

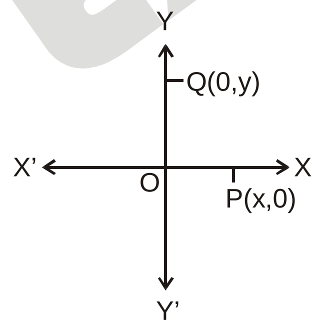
(iii) Abscissa of any point on y-axis is zero.

(iv) ordinate of any point on x-axis is zero.

(v) Co-ordinates of the origin are (0, 0).

(b) Points on Axes:

If point P lies on X-axis then clearly its distance from X-axis will be zero, therefore we can say that its Y-coordinate will be zero. Similarly if any point Q lies on Y-axis, then its distance from Y-axis will be zero therefore we can say its X-coordinate will be zero.



(c) Plotting the Points:

In order to plot the points in a plane, we may use the following algorithm.

Step I : Draw two mutually perpendicular lines on the graph paper, one horizontal and other vertical.

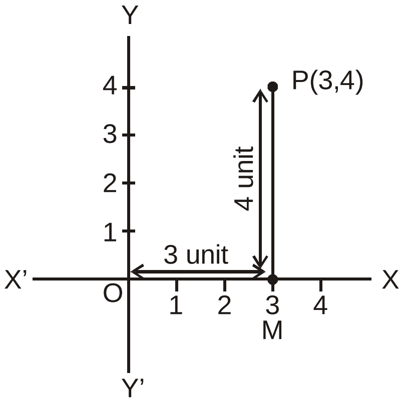
Step II : Mark their intersection point as O (origin).

Step III: Choose a suitable scale on X-axis and Y-axis and mark the points on both the axis.

Step IV: Obtain the coordinates of the point which is to be plotted. Let the point be P(a, b). To plot this point start from the origin and |a| units move along OX, OX' according as 'a' is positive or negative respectively. Suppose we arrive at point M from point M move vertically upward or downward |b| through units according as 'b' is positive or negative. The point where we arrive finally is the required point P(a, b).

Ex.1 Plot the point (3, 4) on a graph paper.

Sol. Let X'OX and Y'OY be the coordinate axis. Here given point is P(3,4), first we move 3 units along OX as 3 is positive then we arrive a point M. Now from M we move vertically upward as 4 is positive. Then we arrive at P(3, 4).



Ex.2 Write the quadrants for the following points.

(i) A(3,4) (ii) B(–2, 3) (iii) C(–5, –2) (iv) D(4, –3) (v) E(–5, –5)

Sol. (i) Here both coordinates are positive therefore point A lies in Ist quadrant.

(ii) Here x is negative and y is positive therefore point B lies in IInd quadrant

(iii) Here both coordinates are negative therefore point C lies in IIIrd quadrant.

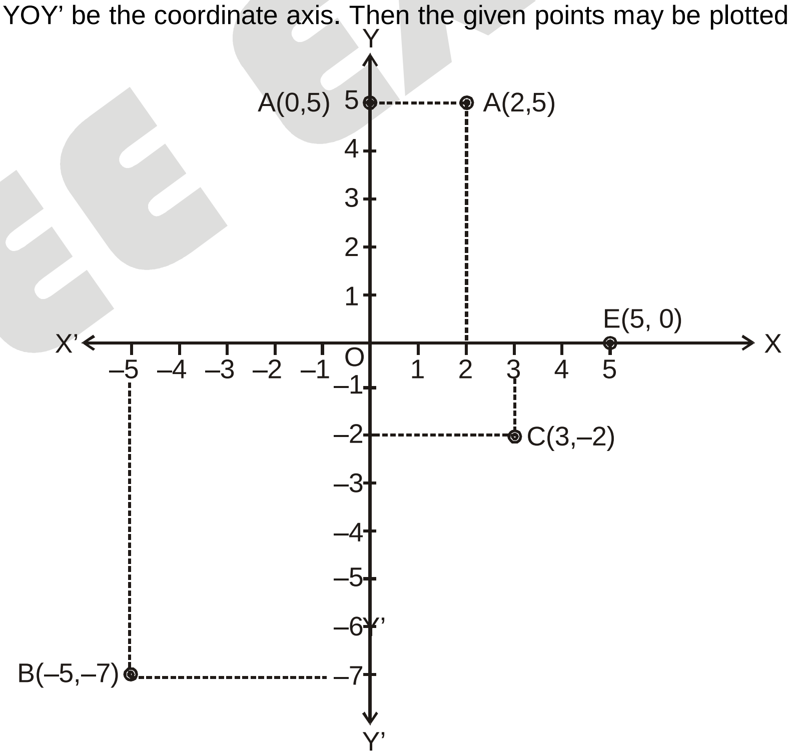
(iv) Here x is positive and y is negative therefore point D lies in IVth quadrant.

(v) Here both coordinates are negative therefore point E lies in IIIrd quadrant

Ex.3 Plot the following points on the graph paper and also specify the quadrants or co-ordinate axes in which they lies:

(i) A(2, 5) (ii) B(–5, –7) (iii) C(3, –2) (iv) D(0, 5) (v) E(5, 0)

Sol. Let XOX' and YOY' be the coordinate axis. Then the given points may be plotted as given below:



(i) Here, both coordinates are positive therefore point A lies in Ist quadrant.

(ii) Here both coordinates are negative therefore point B lies in IIIrd quadrant.

(iii) Here x coordinate is positive and y coordinate is negative therefore point C lies in IVth quadrant.

(iv) Here x coordinate is zero and y coordinate is positive therefore point D lies on positive y-axis.

(v) Here x is positive and y is zero therefore point E lies on positive x-axis.

Exercise # 1

1. The abscissa of a point is distance of that point from :

(A) X-axis (B) Y-axis (C) Origin (D) None of these

2. The y coordinate of a point is distance of that point from :

(A) X-axis (B) Y-axis (C) Origin (D) None of these

3. If both co-ordinates of any point are negative then that point will lie in :

(A) First quadrant (B) Second quadrant (C) Third quadrant (D) Fourth quadrant

4. If the abscissa of any point is zero then that point will lie :

(A) On X-axis (B) On Y-axis (C) At origin (D) None of these

5. The distance of the point (3, 5) from x-axis is :

(A) $\sqrt{34}$ (B) 3 (C) 5 (D) None of these

6. Position of point (–6, 0) on graph paper is :

(A) OY' (B) OX'

(C) In the second quadrant (D) In the fourth quadrant

7. Point (0, 4) lies :

(A) In I quadrant (B) On x-axis (C) On y-axis (D) In IV quadrant

8. The points (–5, 2) and (2, –5) lie in the :

(A) Same quadrants (B) II and III quadrants respectively

(C) II and IV quadrants respectively (D) IV and III quadrants respectively

9. Which of the following points lie on the negative side of x-axis ?

(A) (–4, 0) (B) (–3, 2) (C) (0, –4) (D) (5, –7)

10. Ordinate of a point is negative in :

(A) III and IV quadrant (B) III quadrant only

(C) II and III quadrant (D) IV quadrant only

Exercise # 2

(Subjective)

1. Plot the points in the plane if its co-ordinates are given as A(5, 0) B(0, 3), C(7, 2), D(–4, 3), E(–3, –2) and F(3, –2). Also find the quadrant or coordinate axes on which they lies.

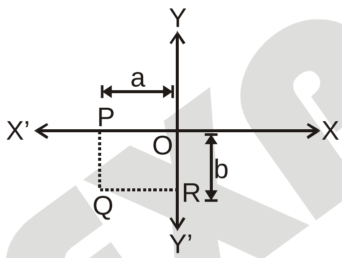
2. Plot the following pairs of numbers as points in the Cartesian plane.

x	–3	–2.5	8	4	0	–5
y	5	0	3.5	8	–2	–1

3. With rectangular axes, plot the points O(0, 0), A(4, 0) and C(0, 6). Find the coordinates of the fourth point B such that OABC forms a rectangle.

4. Plot the points P(–3, 1) and Q (2, 1) in a reactangular coordinate system and find all possible coordinates of other two vertices of a square having P and Q as two adjacent vertices.

5. In the figure given below, determine :



(i) Abscissa of point Q

(ii) Ordinate of point Q

(iii) Coordinate of point Q

6. Plot the points and find out the quadrants or axis on which the following point lies :

(i) P (–7, 6) (ii) Q(0, 5.5) (iii) $R\left(-\frac{3}{2}, -2.5\right)$ (iv) S(6, –9)

7. Write the coordinates of a point :

(a) Above x-axis, lying on y-axis and at a distance of 6 units.

(b) lying on x-axis to the left of origin and at distance of 3 units.

Answer key

Exercise # 1

1.	B	**2.**	A	**3.**	C	**4.**	B	**5.**	C	**6.**	B	**7.**	C
8.	C	**9.**	A	**10.**	A								

Exercise # 2

1. A (On positive x-axis), B (On positive y-axis), C (First quadrant), D (Second quadrant)
E (Third quadrant), F (Fourth quadrant)

3. Fourth Point B (4, 6) **4.** Other two vertices of square : (2, 6) and (–3, 6) or (2, –4) and (–3, –4)

5. (i) Abscissa of point Q = –a (ii) Ordinate of point Q = –b (iii) Co-ordinate of point Q = (–a, –b)

6. P (Ii quadrant), Q (on positive y -axis), R (III quadrant), S (IV quadrant)

7. (a) (0, 6) (b) (–3, 0)

<u>*CHAPTER -2*</u>

LINES, ANGLES AND TRIANGLES RELATIONSHIP

<u>*CONTENTS*</u>

KEY CONCEPTS

ANGLES AT A POINT :

Angle : An angle is a figure formed by two rays with the same initial point. The common initial point is called the vertex of the angle and the rays forming the angle are called its arms or sides.

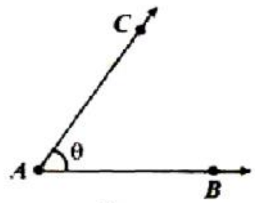


Fig.

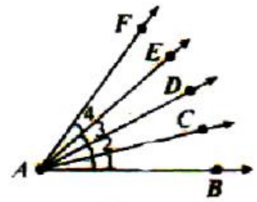


Fig.

Thus, in Fig., A is the vertex, AB and AC are arms. The angle formed by two rays AB and AC is denoted by the symbol ∠BAC or ∠CAB. Sometimes, ∠A, but if there are more than one angle with the same vertex A, then we cannot denote any angle by ∠A.

INTERIOR OF AN ANGLE : The interior of an angle BAC is the region such that every point in the region lies on the same side of the ray AB and C, and also on the same side of the ray AC and B.

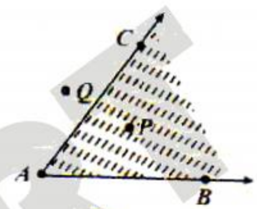


Fig.

THE EXTERIOR OF AN ANGLE

The region outside the interior of ∠BAC is called the exterior of ∠BAC. The point Q (Fig.) lies in the exterior of ∠BAC.

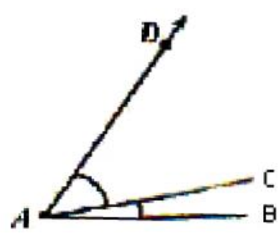
Congruent angles : Two angles are said to be congruent if one angle can superpose the other completely and exactly. If ∠BAC is congruent to an angle, say ∠XYZ, then we write it symbolically as ∠BAC as]ꞏĎ XYZ.

MEASURE OF AN ANGLE : The amount of turning from a ray AB to a ray is called the measure of ∠BAC. It is generally denoted by m ∠BAC or simply as ∠BAC. In this module we shall use ∠BAC to indicate its measure only. There is a standard angle called a degree which is the unit of angle measure. We denote x degrees by the symbol x°.

Note : if 1° is divided into 60 equals parts, then each part is called 1 minute and is written as 1°. If 1' is divided into 60 equals parts, then each part is called 1 second and is written as 1". Thus, 60' = 1° and 60" = 1'.

AXIOMS ON ANGLE MEASURE :

Axiom : Two congruent angles have the same measure and conversely, two angles of equal measure are congruent.



Axiom : If C is a point in the interior of ∠BAD [Fig.], the ∠BAD = ∠BAC + m∠CAD

DIFFERENT TYPES OF ANGLES :

Right angle : An angle whose measure is 90° is called as right angles.

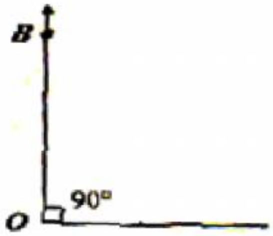
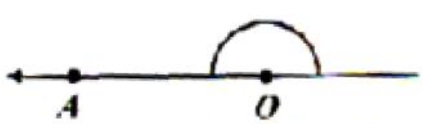


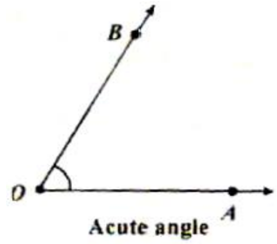
Fig.



A straight angle
Fig.

Straight angle: An angle whose measure is 180°, is called a straight angles.

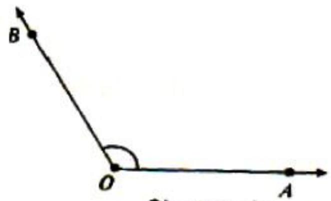
Acute angle: An angle whose measure is less than 90° is called an acute angles.



Acute angle

Fig.

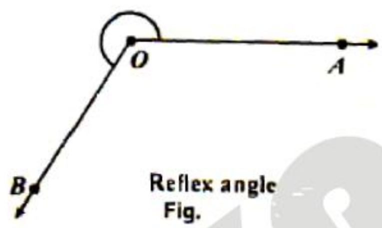
Obtuse angle: An angle whose measure in more than 90º and less than 180º is called an obtuse angle.



Obruse angle

Fig.

Reflex angle: An angle whose measure is more than 180º, but less than 360º is called a reflex angle.

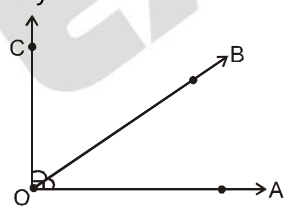


Reflex angle

Fig.

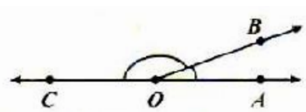
Perpendicular lines : Two lines AB and CD lying in the same plane are said to be perpendicular, if they form a right angle.

Complementary angles : Two angles are said to be complementary, if their sum is 90º [Fig.]. For examples, angles of 30º and 60º are complementary.



∠AOB and ∠BOC are complementary Angles

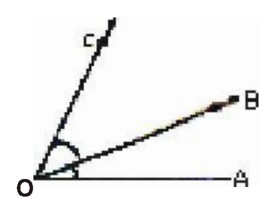
Supplementary angles: Two angles are said to be supplementary, if their sum is 180º* [Fig]. For examples, angles of 160º and 20º are supplementary.



∠*AOB* and ∠*BOC* are supplementary angles

Fig.

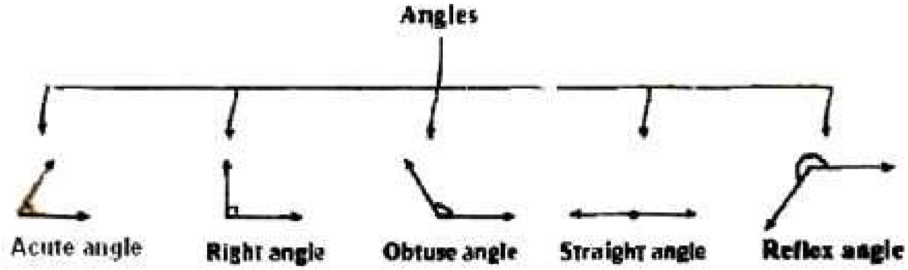
Adjacent angles: Two angles are said to be adjacent, if they have (i) a common vertex, (ii) a common arm and (iii) non-common arms lying on either side of the common arm [Fig.]. In Fig., ∠AOB and ∠BOC are adjacent angles, since these two angles have a common vertex O, a common arm OB and the two non-common arms OA and OC lie on either side of the common arm OB.

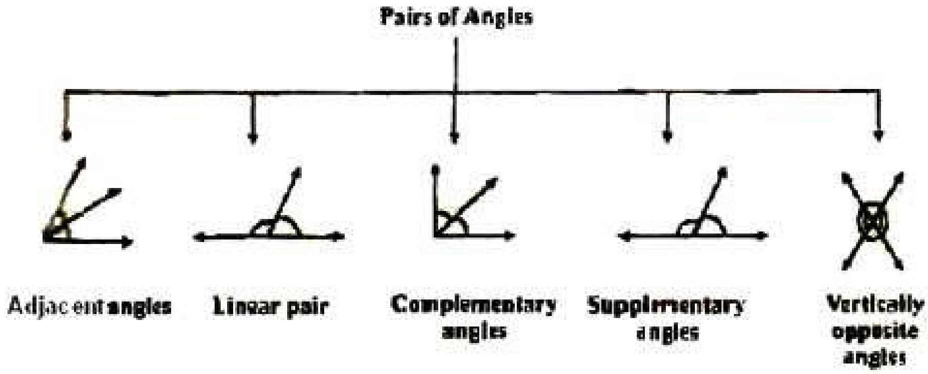


Adjacent Angles

Linear pair of angles: Two adjacent angles are said to form a linear pair, if their non-common arms are opposite rays

Vertically opposite angles: A pair of angles is said to be vertically opposite, if the arms of the two angles form two pair of opposite rays.





DIFFERENT TYPES OF ANGLES

LINEAR PAIR AXIOM :

Axiom : If a ray stands on a line, then the sum of two adjacent angles so formed is 180º. Conversely, if the sum of two adjacent angles is 180º, then the non-common arms of the angles are two opposite rays.

Angle Bisector : A ray OB is said to be the bisector of $\angle$AOC if B is a point in the interior of $\angle$AOC and $\angle$AOB = $\angle$BOC.

Clearly, $\angle$AOB = $\angle$BOC = ½ $\angle$AOC.

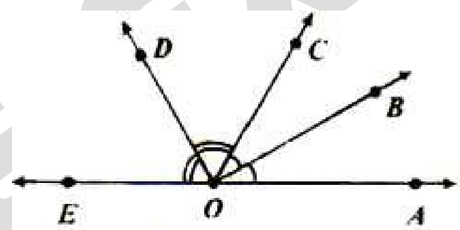
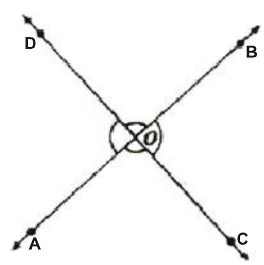


Fig.

Internal And External Bisectors : The rays which bisect an angles and the adjacent angle made by producing one of its arms are called the internal and external bisectors of the given angle respectively.

A theorem on vertically opposite angles : Theorem : If two lines intersect, then the vertically opposite angles are equal.



Given that two lines AB and CD intersect at O.

To prove that

(i) $\angle$AOC = $\angle$BOD and (ii) $\angle$AOD = $\angle$COB.

Proof: Since ray OC stands on the line AB,

y $\angle$AOC + $\angle$COB = 180º [Linear pair axiom] ….(1)

Again, since ray OB stands on the line CD,

y $\angle$DOB + $\angle$COB = 180º [Linear pair axiom] ….(2)

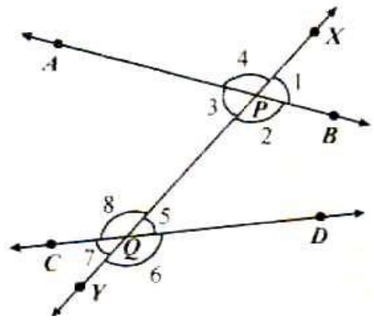
From (1) and (2), we have

y $\quad \angle AOC + \angle COB = \angle DOB + \angle COB$

= $\quad \angle AOC = \angle DOB$ [Cancelling $\angle COB$ from both sides]

Similarly, $\angle AOD = \angle COB$

Transversal : A transversal of two or more given lines (line-segments) is a line (a line segment or a ray) which intersects the lines (lines segments or rays) at distinct points.



Corresponding angles: A pair of angles on the same side of a transversal of two given lines is said to be corresponding angles if the two angles lie either above the two given lines or below them.

Interior angles: The angles lying between two given lines are called interior angles.

Exterior angles: The angles not lying between two given lines are called exterior angles.

Consecutive interior angles: The pairs of interior angles on the same side the transversal of the given lines are called the pairs of consecutive interior angles.

Alternate interior angles: The two interior angles lying on opposite sides of a transversal to two given lines are known as alternate interior angles.

In Fig., the pairs (i) $\angle 3$ and $\angle 5$ and (ii) $\angle 2$ and $\angle 8$ are two pairs of alternate interior angles (or, simply alternate angles).

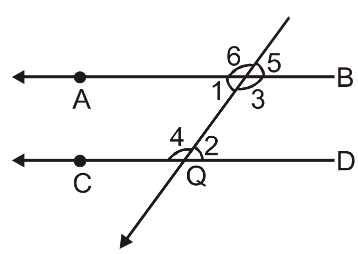
Axiom (Corresponding Angles Axiom) : If a transversal intersects two parallel lines, then each pair of corresponding angles is equal. Conversely, if a transversal intersects two line making a pair of corresponding angles equal, then the two lines are parallel.

We now state and prove the theorems concerning other pairs of angles.

Theorem : If a transversal intersect two parallel lines, then the alternate interior angles are equal to one another.

Given that AB and CD are lines such that such that AB || CD. A transversal l intersects these two parallel lines at P and Q respectively making alternate interior angles $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$ [Fig.]

To prove that $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$.



Proof: We have

	$\angle 2 = \angle 5$	[Corresponding angles axiom]
Also,	$\angle 5 = \angle 1$	[Vertically opposite angles]
Hence,	$\angle 1 = \angle 2$	
Again,	$\angle 4 = \angle 6$	[Corresponding angles axiom]]
Also,	$\angle 6 = \angle 3$	[Vertically opposite angles]
Hence,	$\angle 3 = \angle 4$	

Theorem (Converse of Theorem): If a transversal intersects two lines in such a way that a pair of alternate interior angles is equal, the lines are parallel.

Given that a transversal l intersect two lines AB and CD at two point P and Q respectively such that a pair of alternate interior angles $\angle 1$ and $\angle 2$ is equal i.e. $\angle 1 = \angle 2$ [Fig]

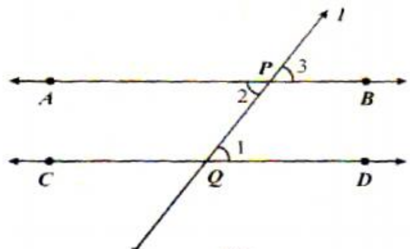


Fig.

To prove that AB || CD.

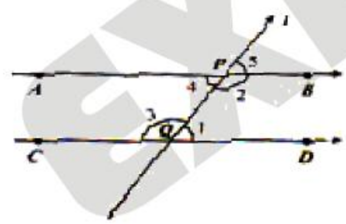
Proof: We have

	$\angle 1 = \angle 2$	[Given]
Also,	$\angle 2 = \angle 3$	[Vertically opposite angles]]
y	Ď1 = $\angle 3$	

But $\angle 1$ and $\angle 3$ are corresponding angles. Hence, a pair of corresponding angles is equal. Therefore, by corresponding angles axiom, AB || CD

Theorem : If a transversal intersects two parallel lines, then each pair of consecutive angles is supplementary.

Given that AB and CD are lines such that AB || CD. A transversal l intersects these two parallel lines at P and Q respectively making two pairs of consecutive interior angles $\angle 1$, $\angle 2$ and $\angle 3$, $\angle 4$ [Fig.].



To prove that

$\angle 1 + \angle 2 = 180^o$ and $\angle 3$ +Ď4 = 180^o

Proof: The ray PB stands on the line ℓ.

y	$\angle 5 + \angle 2 = 180^o$	[Linear pair]	
also	$\angle 5 = \angle 1$	[Corresponding angles axiom]	
y	$\angle 1 + \angle 2 = 180^o$		…(i)

Again, the ray PQ stands on the line CD.

y	Ď1 + $\angle 3 = 180^o$	[Linear pair]	…(ii)

Also, the ray QP stands on the lie AB.

y	Ď2 + $\angle 4 = 180^o$	[Linear pair]	…(iii)

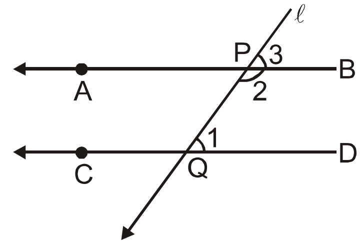
Adding (ii) and (iii), we get

	$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^o + 180^o$		
=	$180^o + \angle 3 + \angle 4 = 180^o + 180^o$	[Form (i)]	
Ĝ	Ď3 + $\angle 4 = 180^o$		…(iv)

Hence, from (i) and (iv), it is proved that each pair of consecutive interior angles is supplementary.

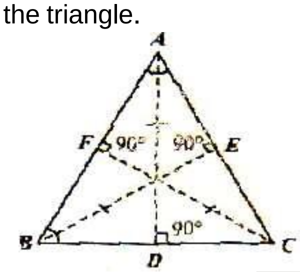
Theorem (Converse of Theorem): if a transversal intersects two lines in such a way that a pair of consecutive interior angles is supplementary, then the lines are parallel.

Given that a transversal l intersects two lines AB and CD at P and Q respectively such that , when the $\angle 1$ and $\angle 2$ are a pair of consecutive interior angles and $\angle 1 + \angle 2 = 180^0$ [Fig.].
To prove that AB || CD.
Proof: Since the ray PB, stands on l,



y Ď2 + $\angle 3 = 180^o$ [Linear pair]
Also, $\angle 1 + \angle 2 = 180^o$ [Given]
y $\angle 1 + \angle 2 = \angle 2 + \angle 3$
= $\angle 1 = \angle 3$
But these two are corresponding angles.
Hence, AB || CD [Corresponding angles axiom]

A TRIANGLE AND ITS PARTS : A triangle is a plane figure bounded by three line segments. We denote a triangle by the symbol a. ABC [Fig] is written as aABC. A triangle has six elements or, parts viz., three angles $\angle$BAC, $\angle$ABC and $\angle$BCA or, $\angle$A, $\angle$B, and $\angle$C and three sides viz., BC, CA and AB. The three points A, B, C are called the vertices of a triangle. Note that the vertices of triangle are always non-collinear. The side BC is called the base of the triangle with respect to the vertex A. If we draw AD { BC, then the line segment AD is called the altitude (or, height) of the triangle.



DIFFERENT KINDS OF TRIANGLES : Triangles can be classified into different types on the basis of the lengths of their sides and the measure of their angles.

(a) Types of triangles on the basis of sides: There are three kinds of triangles on the basis of the lengths of their sides. These are called an equilateral triangle, an isosceles triangle and a scalene triangles.

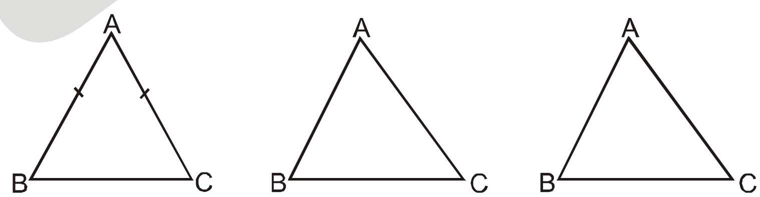
Equilateral triangle: A triangle is said to be an equilateral triangle when the lengths of its three sides are equal.

Isosceles triangle: A triangle is said to be an isosceles triangle when the lengths of any two of its sides are equal

Scalene triangle: A triangle is said to be a scalene triangle when the lengths of its sides are all different

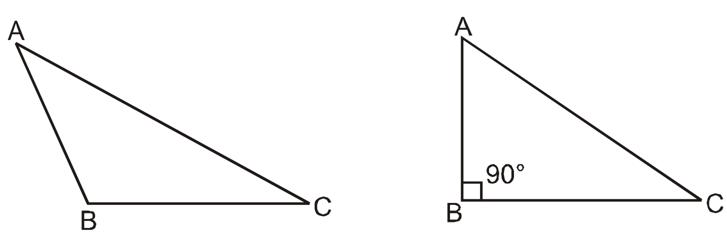
TYPES OF TRIANGLE ON THE BASIS OF ANGLES : There **ARE THREE KINDS OF TRIANGLES** on the basis of the measures of their angles. These are called an acute-angled triangle, an obtuse-angle triangle and a right-angled triangle.

Acute-angle triangle: A triangle is said to be an acute-angled triangle when each of its three angles is acute (i.e., less than 90º) [Fig.]



An isosceles triangle A scalene triangle An acute angled triangle

Obtuse-angled triangle : A triangle is said to be an obtuse-angled triangle when any one of its angle is obtuse (i.e., greater than 90º and clearly less than 180º). [Figure.]

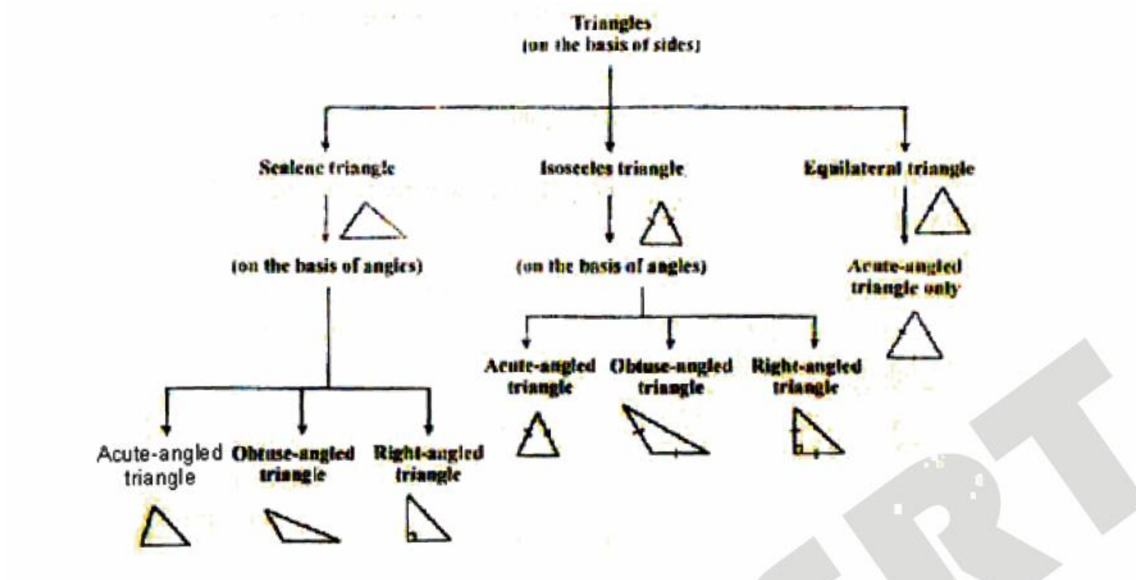


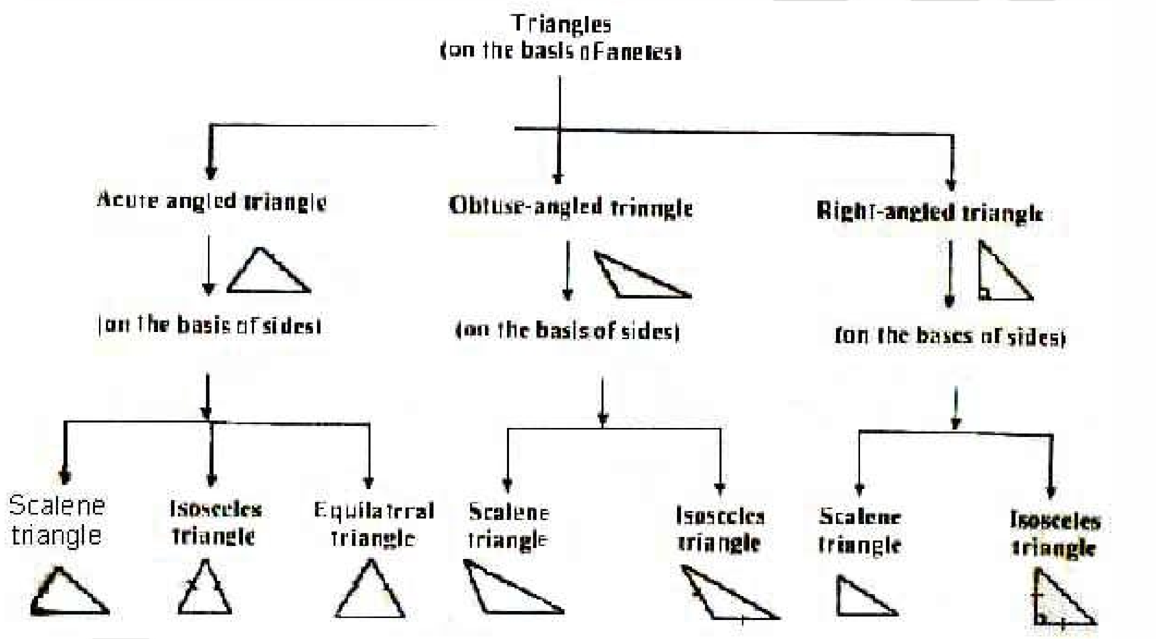
An obtuse angled triangle **A right - angled triangle**

Right-angled triangle : A triangle is said to be a right-angled triangle when any one of its angles is a right angle (i.e., 90°)

The side opposite to the right angle is called the hypotenuse of the right-angled triangle.

We can now show these different kinds of triangles in tabular form as follows :





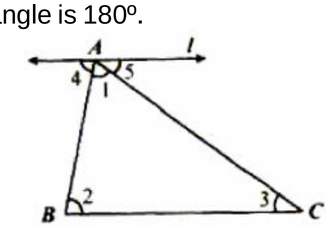
ANGLES SUM PROPERTY OF A TRIANGLE : We shall now deduce an important property about the angles of a triangle viz., the sum of three angles of any triangle is 180°.

Theorem (angles-sum property): The sum of three angles of any triangle is 180°.

Given that ABC is a triangle.

To prove that $\angle A + \angle B + \angle C = 180^\circ$

i.e., $\angle 1 + \angle 2 + \angle 3 = 180^\circ$



Construction: We draw a line l though the vertex A parallel to the base BC of ΔABC.

Proof: We have

	$\angle 2 = \angle 4$	[Alternate angles]	…(i)
And	$\angle 3 = \angle 5$	[Alternate angles]	…(ii)

Adding (i) and (ii), we get

$\angle 2 + \angle 3 = \angle 4 + \angle 5$

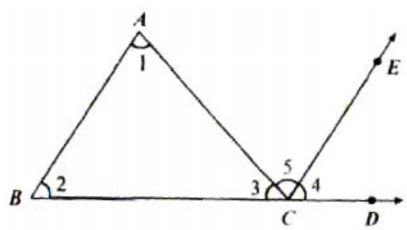
$\Rightarrow \quad \angle 1 + \angle 2 + \angle 3 = \angle 1 + \angle 4 + \angle 5$ [Alternate $\angle 1$ on both sides]

$= 180^o$ [$\because$ Sum of angles at a point on a line is 180^o]

Hence, the sum of three angles of a triangle of a triangle is 180^o, proving the theorem.

The above theorem can also be proved by a slightly different construction as follows:

Construction: Produce the side BC to a point D to form a ray BD and draw a ray CE through C parallel to the side AB of $\triangle$ ABC [Fig.]



Proof : We have

$\angle 2 = \angle 4$ [Corresponding angles on the same side the transversal BD] ....(1)

$\angle 1 = \angle 5$ [Alternate angles] ....(2)

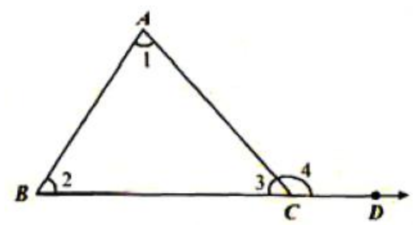
Adding (1) and (2) $\angle 1 + \angle 2 = \angle 4 + \angle 5$

$\Rightarrow \quad \angle 1 + \angle 2 + \angle 3 = \angle 3 + \angle 5 + \angle 4 = 180^0$

[So sum of angles at a point on a line is 180^0.]

Theorem (Exterior Angle Theorem) : If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of two interior opposite angles.

Given that ABC is a triangle, D is a point on BC produced so that $\angle$ACD is an exterior angle (i.e., $\angle 4$) $\angle 1$ and $\angle 2$ are interior opposite angles.



To prove that $\angle 4 = \angle 1 + \angle 2$

Proof: We have

$\angle 1 + \angle 2 + \angle 3 = 180^o$ [Angles-sum property of a triangle] …(1)

Also, $\angle 3 + \angle 4 = 180^o$ [Linear part] …(2)

From (1) and (2), we have

$\angle 1 + \angle 2 + \angle 3 = \angle 3 + \angle 4$

$\Rightarrow \quad \angle 1 + \angle 2 = \angle 4$

i.e., $\angle 4 = \angle 1 + \angle 2$

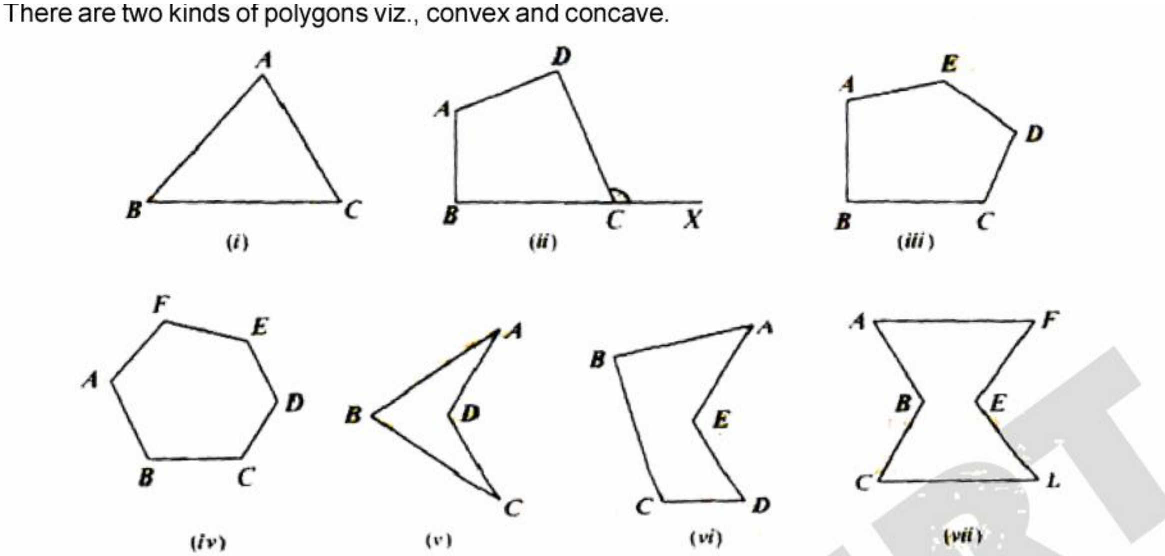
Hence, the exterior angle is equal to the sum of the interior opposite angles.

POLYGONS AND THEIR ANGLES-SUM PROPERTY :

Definition: A Polygon is a closed plane figure bounded by three or more line-segments that terminate in pairs at the same number of points called vertices and do not intersect other than at their vertices.

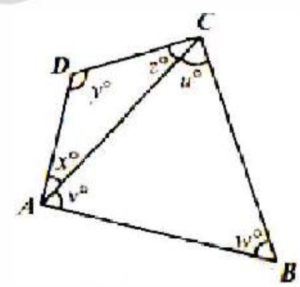
A polygon is said to be a regular polygon if the lengths of all its sides are equal and its angles are also equal.

There are two kinds of polygons viz., convex and concave.



A polygon is said to be convex if each of its interior angles is less than 180º so that all lines joining any pair of points on the boundary of the figure lie wholly inside it. The polygon which is not convex is called concave.

Different kinds of polygons : A polygon has different names on the number of its sides. Thus, polygons with three, four, five, six, seven, eight, nine and ten sides are respectively called a triangle, a quadrilateral, a pentagon, a hexagon, a heptagon, an octagon, a nonagon and a decagon.



(i) Theorem : The sum of four angles of a quadrilateral is 360º.

Given that ABCD is a quadrilateral [Fig.].

To prove that $\angle A + \angle B + \angle C + \angle D = 360^o$

Construction: We join AC.

Proof: Let $\angle ADC = y^o$, $\angle DAC = x^o$, $\angle DCA = z^o$, $\angle ACB = u^o$, $\angle CAB = v^o$, and $\angle ABC = w^o$

In aABC, we have

$x^o + y^o + z^o = 180^o$ [Angles-sum property for a triangle] …(1)

In aABC, we have

$u^o + v^o + w^o = 180^o$ [Angles-sum property for a triangle] …(2)

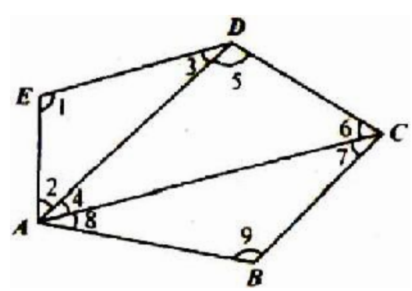
Adding (1) and (2), we have

$$x^o + y^o + z^o + u^o + v^o + w^o = 180^o + 180^o = 360^o.$$

= $y^o + (x^o + y^o) + (z^o + u^o) + w^o = 360^o$

= $\angle D + \angle A + \angle C + \angle B = 360^o$.

= $\angle A + \angle B + \angle C + \angle D = 360^o$.



(ii) Theorem : The sum of five angles of a pentagon is 540º.

Given that ABCDE is a pentagon.

To prove that $\angle A + \angle B + \angle C + \angle D + \angle E = 540^0$.

Construction : We join AD and AC [Fig.]

Proof : In aAED, we have

$\angle 1 + \angle 2 + \angle 3 = 180^o$ [Angles-sum property in aAED] …(1)

In aADC, we have

$\angle 4 + \angle 5 + \angle 6 = 180^o$ [Angles-sum property in aADC] ...(2)

In ΔABC, we have

$\angle 7 + \angle 8 + \angle 9 = 180^o$ [Angles-sum property in ΔABC] …(3)

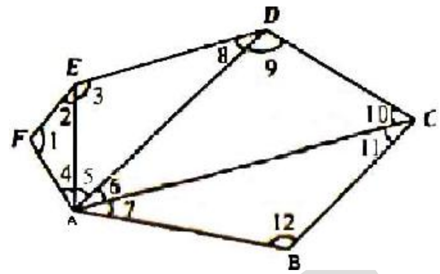
Adding (1), (2) and (3) we get

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 + \angle 9 = 180^o + 180^o + 180^o$

$\angle 1 + (\angle 3 + \angle 5) + (\angle 2 + \angle 4 + \angle 8) + (\angle 6 + \angle 7) + \angle 9 = 540^o$

$\angle E + \angle D + \angle A + \angle C + \angle B = 540^o$

$\angle A + \angle B + \angle C + \angle D + \angle E = 540^o$



(iii) Theorem : The sum of six angles of hexagon is 720^o.

Given that ABCDEF is a hexagon.

To prove that $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 720^o$.

Proof : In ΔAFE, we have

$\angle 1 + \angle 2 + \angle 4 = 180^o$ [Angles-sum property in ΔAFE] …(1)

In ΔAED, we have

$\angle 3 + \angle 5 + \angle 8 = 180^o$ [Angles-sum property in ΔAED] ...(2)

In ΔADC, we have

$\angle 6 + \angle 9 + \angle 10 = 180^o$ [Angles-sum property in ΔADC] …(3)

In ΔABC, we have

$\angle 7 + \angle 11 + \angle 12 = 180^o$ [Angles-sum property in ΔABC] …(4)

Adding (1),(2), (3), (4), we get

$\angle 1 + (\angle 2 + \angle 3) + (\angle 8 + \angle 9) + (\angle 10 + \angle 11) + \angle 12 + (\angle 4 + \angle 5 + \angle 6 + \angle 7) = 180^o \times 4 = 720^o$

$\Rightarrow \quad \angle F + \angle E + \angle D + \angle C + \angle B + \angle A = 720^o$

$\Rightarrow \quad \angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 720^o$

Note: We observe from the three theorems above that the sum of angles of quadrilateral with 4 sides is $180^o \times 2 = 360^o$, that for a pentagon with 5 sides is $180^o \times 3 = 540^o$, and that for a hexagon with 6 sides is $180^o \times 4 = 720^o$. Similarly, we can prove that the sum of all the angles of heptagon with 7 sides is $180^o \times 5 = 900^o$, that of an octagon with 8 sides = $180^o \times 6 = 1080^o$ and so on. We can thus generalize this result as follows:

The sum of the interior angles of a polygon of n sides

$= (n - 2) \times 180^0 (n \geq 3)$

If the polygon is regular, then each interior angle

$= \frac{(n-2) \times 180^o}{n}$, since all angle of a regular polygon are equal.

Although it should be noted that the sum of all exterior angles of a polygon always is a constant=360^o irrespective of its number of sides.

SOME SOLVED ILLUSTRATIONS

Illustration: In Fig., OA, OB, OC, OD, OE and OF are rays such that ∠AOB = 45º, ∠BOC = 85º, ∠BOC = 50º, ∠EOF = 85º, and ∠FOA = 50º. Prove that AD, BE and CF are three lines. Hence, find ∠DOE.

Solution: We have, ∠AOB +∠BOC + ∠COD = 45º + 85º + 50º = 180º

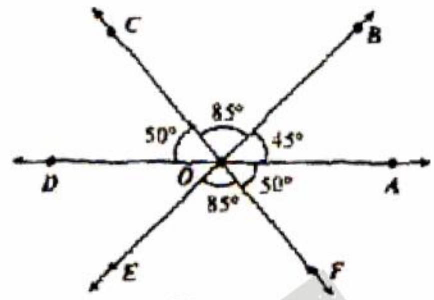


Fig.

Ĝ ĎDOA = 180^0

Hence, OA and OD are opposite rays i.e., AD is a line.

Again, ∠FOA +∠AOB +∠BOC = $50^0 + 45^0 + 85^0 = 180^0$

= ∠FOC = 180

Hence, OF and OC are opposite rays i.e., CF is a line.

Finally, ∠BOA + ∠AOF + ∠FOE = $45^0 + 50^0 + 85^0 = 180^0$

= ∠BOE = 180^0

Hence, OB and OE are opposite rays i.e., BE is a line.

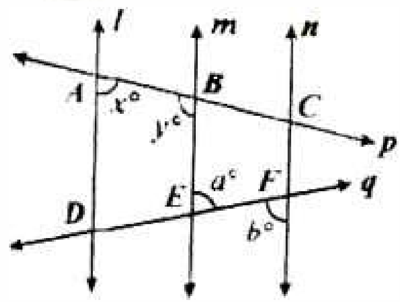
Now, since AD is a line,

Hence, ∠DOE + ∠EOF +∠FOA= 180^0

= ∠DOE + $85^0 + 50^0 = 180^0$

∠DOE = $180^0 - 85^0 - 50^0 = 45^0$.

Illustration: In Fig., $x^\circ + y^\circ = 180^\circ$ and $a^\circ = b^\circ$. show that l || n.



Solution: We see from the figure that the transversal p intersects the lines l and m at A and B respectively such that the sum of the consecutive interior angles x° and y° is 180°.

i.e., $x^\circ + y^\circ = 180^\circ$.

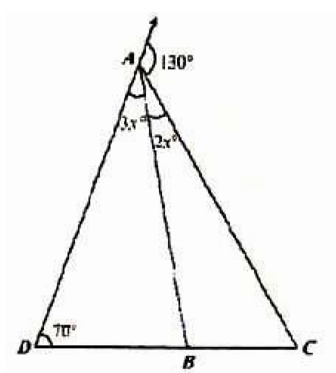
y l || m …(1)

Again the transversal q intersects the lines m and n at E and F respectively such that the alternate angles a° and b° are equal i.e., $a^\circ = b^\circ$ …(2)

From (1) and (2), it follows that l || n.

Illustration: In Fig., AB divides ∠DAC in the ratio∠DAB; ∠BAC = 3 : 2. Find ∠ACB and ∠ABC.

Solution : Let the angles ∠DAB = 3xº and ∠BAC = 2xº.



Now, ∠130º + ∠DAC = 180º [Linear pair]

= ∠DAC = 180º – 130º = 50º

y 3xº + 2xº = 50º

= 5xº = 50º, x = 10º

Now, by angles sum property, we have

∠ADB + ∠DAC + ∠ACB = 180º

= 70º + 50º + ∠ACB = 180º

= ∠ACB = 180º – 120º = 60º

Now, ∠ABC = 70º + 3xº = 70º + 3 x 10º = 70º + 30º = 100º.

Illustration: Two regular polygons are such that the ratio between their number of sides is 1 : 2 and the ratio of the measures of their angles is 4 : 5. Find the number of sides of each polygon.

Solution: Let the number of sides of the two polygons be n and 2n. then the measures of their interior angles are

$$\frac{(n-2)\times 180^\circ}{n} \text{ and } \frac{(2n-2)\times 180^\circ}{2n} \text{ respectively.}$$

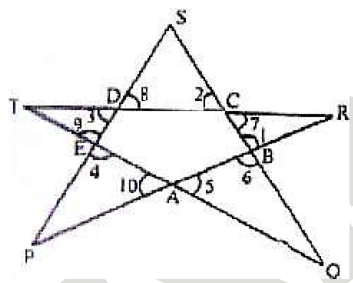
It is given that

$$\frac{(n-2)\times 180^\circ}{n} : \frac{(2n-2)\times 180^\circ}{2n} = 4 : 5$$

$$\Rightarrow \quad \frac{\left\{\frac{(n-2)\times 180^\circ}{n}\right\}}{\frac{(2n-2)\times 180^\circ}{2n}} = \frac{4}{5} \Rightarrow \frac{(n-2)}{(n-1)} = \left(\frac{4}{5}\right) \Rightarrow \quad 5n - 10 = 4n - 4 \Rightarrow n = 6.$$

$\therefore$ The numbers of sides of the two polygons are 6 and 12.

Illustration: In Fig., Find the value of $\angle P + \angle Q + \angle R + \angle S + \angle T$



Solution : As ABCDE is a pentagon, sum of the exterior angles = 360º

i.e. $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 = 360^\circ$ ....(i)

but : $\angle 1 = \angle 6$, $\angle 2 = \angle 7$

$\angle 3 = \angle 8$, $\angle 4 = \angle 9$

And : $\angle 5 = \angle 10$ [Vertically opposite angles]

Thus: $\angle 6 + \angle 7 + \angle 8 + \angle 9 + \angle 10 = 360^\circ$ ....(ii)

From ΔPAE, ΔQAB, ΔSDC and ΔTDE

$(\angle P + \angle 4 + \angle 10) + (\angle Q + \angle 5 + \angle 6) + (\angle R + \angle 1 + \angle 7) +$

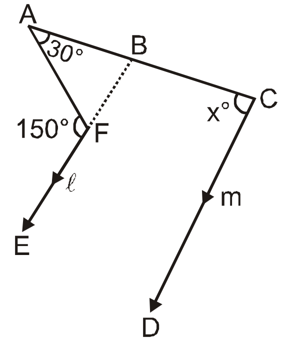
$(\angle S + \angle 2 + \angle 8) + (\angle T + \angle 9 + \angle 3) = 5 \times 180^\circ = 900^\circ$ ....(iii)

From : (i), (ii) & (iii), we get :

$\angle P + \angle Q + \angle R + \angle S + \angle T + 360^\circ = 900^\circ$

$= \angle P + \angle Q + \angle R + \angle S + \angle T = 900^\circ - 2 \times 360^\circ = 180^\circ$

Illustration: In the given figure, find the value of x°, if $\ell \parallel m$.



(A) 30° (B) 80° (C) 120° (D) 60°

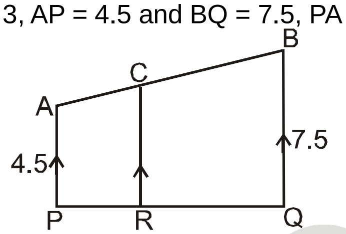
Solution : $\angle ABF = \angle AFE - \angle BAF$

$= 150° - 30° = 120°$

Now $\angle DCB = \angle ABF$ (Corresponding angles of parallel sides)

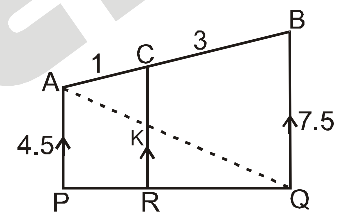
$=$ x = 120°

Illustration: In the figure, if AC : CB = 1 : 3, AP = 4.5 and BQ = 7.5, PA || RC ||QB, then find CR.



Solution : Join AQ cutting CR at K.

Consider as ACK and ABQ.



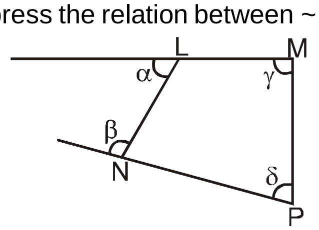
$\frac{CK}{7.5} Z \frac{1}{4}$ $=$ CK = $\frac{7.5}{4}$

Consider aKRQ and APQ.

$=$ $\frac{KR}{4.5} Z \frac{3}{4}$ $=$ KR = $\frac{13.5}{4}$

y CR = CK + KR = $\frac{7.5}{4} H \frac{13.5}{4} Z \frac{21}{4} Z 5\frac{1}{4}$

Illustration: From the adjoining figure, express the relation between ~, , ¥, ¢.

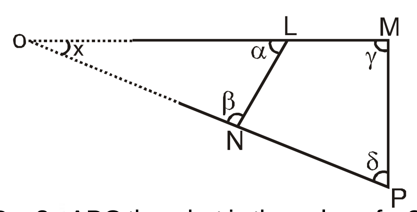


(A) ~ – = ¥ + ¢ (B) ~ + = ¥ – ¢ (C) ~ + = ¥ + ¢ (D) ~ – = ¥ – ¢

Solution : Since lines LM and NP are not parallel,

and lie in the same plane, so they meet say at a point O.

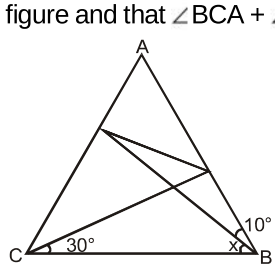
Let $\angle$LON and $\angle$MOP each be X.



y ~ + + X = ¥+ ¢ + X.

Ĝ ~+ = ¥ + ¢.

Illustration: Given the angle measures is the figure and that $\angle BCA + \angle CAB = 2\angle ABC$ the what is the value of x ?



(A) 50° (B) 40° (C) 75° (D) 72°

Solution : ∠BCA + ∠CAB = 2∠ABC = 180°

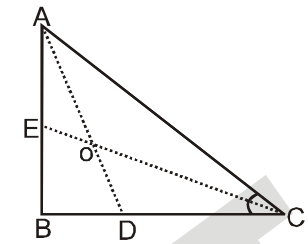
y   2∠ABC + ∠ABC = 180°

Ĝ   ĎABC = 60° = 10° + x°   y   x = 50°.

Illustration: Consider the angle bisectors of the acute angle in a right triangle. Find the measure of the obtuse angle of their intersection

(A) 109°   (B) 120°   (C) 150°   (D) 135°

Solution : In a right triangle the sum of the acute angle = 90°



∠BAC + ∠BCA = 90°

y $\frac{1}{2}$(∠BAC H ∠BCA) Z 45î

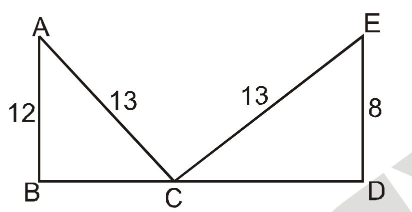
or   ∠OAC + ∠OCA = 45°

y   ĎAOC = 180° – 45° = 135°

Illustration: A ladder reaches a window that is 8 m above the ground on one side of the street. Keeping its foot on the same point, the ladder is turned to the other side of the street to reach a window 12 m high. Find the width of the street if the ladder is 13 m long.

(A) 15.2 m   (B) 14 m   (C) 14.6 m   (D) 12 m

Solution :



CE = AC = length of ladder, width of street = BC + CD

BC = $\sqrt{13^2 - 12^2}$ Z 5m.

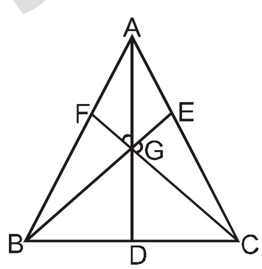
CD = $\sqrt{13^2 - 8^2}$ Z $\sqrt{105}$ Z 10.2m (approximately)

y   BD = 15.2 m

Illustration: In a triangle ABC median from the vertex A intersects BC at D, median from B intersects AC at E and median from C intersects AB at F. If G is the centroid, what is ∠AGF +ĎCGE + ∠BGD ?

(A) 120 °   (B) 180°   (C) 150°   (D) cannot be determined

Solution :



∠AGF = ∠DGC (Vertically opposite angles)

∠BGD + ∠DGC + ∠CGE = 180° (angles on a straight line)

y   ĎAGF + ∠CGE + ∠BGD = 180°

EXERCISE-I

Fill in the blanks :

1. When two lines meet at a point forming right angles they are said to be ………. to each other.
2. If one angles of a linear pair is double the other one, then their measure are…..&…… .
3. If one angle of lines are intersected by a transversal, then each pair of corresponding angles are ……… .
4. Two lines perpendicular to the same line are ….. to each other.
5. If the sum of two adjacent angles is 180º, then the ……..arms of the two angles are opposite rays.
6. If one angle of a linear pair is acute, then its other angle will be…….. .

Assertion & Reason :

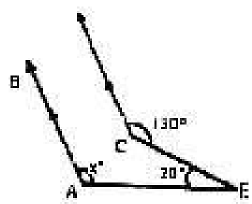
Instructions: In the following questions a Assertain (A) is given followed by a Reason (R). Mark you response from the following options.
(A) Both Assertion and Reason are true and Reason is the correct explanation of 'Assertion'.
(B) Both Assertion and Reason are true and Reason is not the correct explanation of 'Assertion'
(C) Assertion is true but Reason is false
(D) Assertion is false but Reason is true.

Assertion: If one angle of a linear pair is acute, then its other angle will be obtuse.
Reason: The sum of two angles of a linear pair is always 180º.

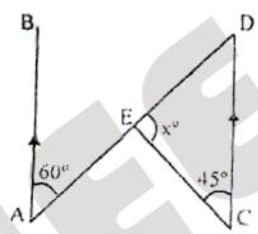
Assertion: Line L_1,is parallel to line L_2 and L_2 is parallel to line L_3, then L_1, parallel to L_3.
Reason: Two lines perpendicular to the same line are parallel to each other.

Match of the following (one to one)

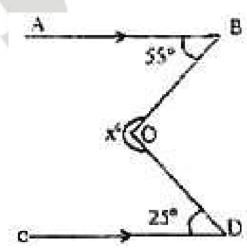
1. **Column - I** **Column - II**

(A) 

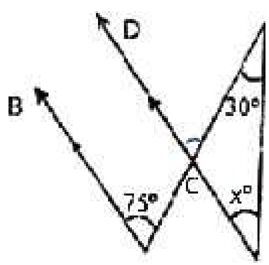
(P) 75º

(B) 

(Q) 110º

(C) 

(R) 45º

(D) 

(S) 280º

2. **Column I** **(Regular plane figure)**

(A) Triangle

(B) Square

(C) Pentagon

(D) Hexagon

Column II **(Measure of interior angles)**

(P) 108^0

(Q) 120^0

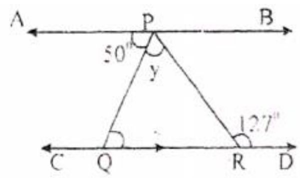
(R) 90^0

(S) 60^0

3. **Column I**

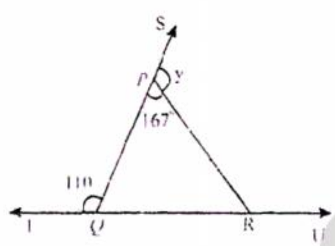
(A) Complement of 13^0

(B)



given AB || CD

(C)



(D) Supplement of 13º

Column II

(P) $\frac{1}{2}$ [right angle] – 4 × 8º

(Q) 167º

(R) 77º

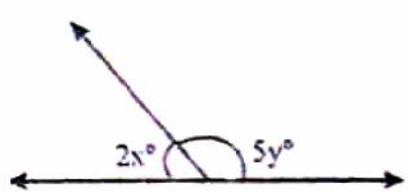
(S) Right angle + 11 × 7º

OBJECTIVE TYPE :

1. The difference of two complementary angles is 40º. Then the angles are :

(A) 62º, 25º (B) 70º, 20º (C) 70º, 30º (D) 60º, 30º

2. From the adjoining figure x = 30^0, the value of y is :



(A) 25º (B) 24º (C) 36º (D) 45º

3. If B lies between A and C, AC = 15 cm and BC = 9 cm, then $(2AB)^2$, is :

(A) 306 (B) 144 (C) 36 (D) 24

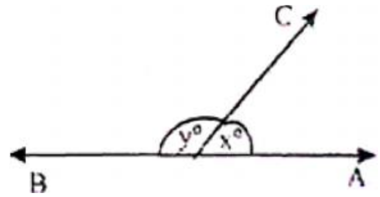
4. All linear pairs are :

(A) Supplementary (B) Vertically opposite (C) Right angles (D) None of these

5. At 3 O' clock, the angle formed between the hands of a clock is :

(A) Reflex angle (B) Straight angle (C) Acute angle (D) Right angle

6. In the given Figure, $\angle x$ is greater than one fifth of a right angle, then :



(A) y >162º (B) y ñ 162º (C) y á 162º (D) y < 162º

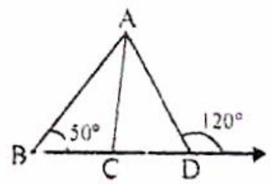
7. In figure AC bisects $\angle$BAD. What type of triangle is aACB ?

(A) Acute angle triangle

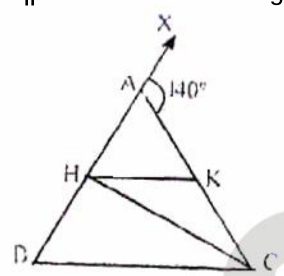
(B) Right angle triangle

(C) Obtuse angle triangle

(D) Reflex Triangle



8. In the figure AB = AC, CH = CB and HK || BC. If the exterior angle CAX is 140^0 then the angle HCK is

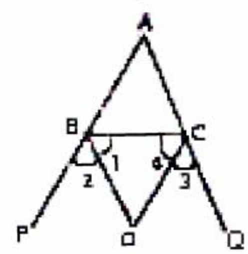


(A) 45º (B) 30º (C) 50º (D) 40º

9. In the adjoining figure BO, CO are angle bisectors of external angles of $\angle$ABC. Then $\angle$BOC is :

(A) $90 - \frac{1}{2}\angle A$ (B) $90 + \frac{1}{2}\angle A$

(C) $180 - \frac{1}{2}\angle A$ (D) $180 + \frac{1}{2}\angle A$



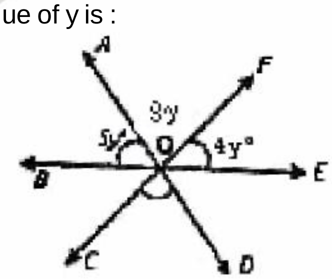
10. From the adjoining figure the value of y is :

(A) 24º

(B) 22º

(C) 20º

(D) 10º



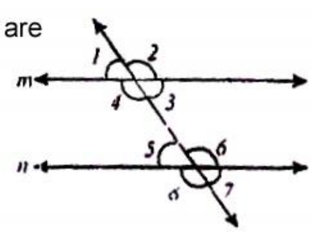
11. From the adjoining figure, if $\angle 2 = 155º$ And $\angle 5 = 60º$ then the lines m and n are

(A) parallel

(B) not parallel

(C) cannot say

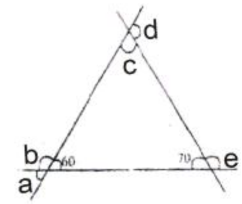
(D) none of these



12. If X is a point on the line AB and Y, Z are points outside such that $\angle AXY = 45º$ and $\angle YXZ = 60º$ then $\angle AXZ$ is equal to:

(A) 120^0 (B) 135^0 (C) 150^0 (D) 105^0

13. In the figure above, which of the following is the correct inequality ?



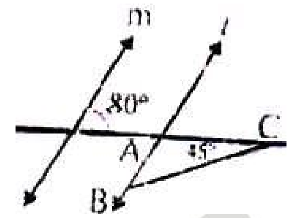
(A) $c^o < a^o < e^o < b^o < d^o$

(B) $c^o < a^o < e^o < d^o < e^o$

(C) $a^o < c^o < e^o < b^o < d^o$

(D) $c^o < a^o < d^o < b^o < e^o$

14. In figure l || m, then $\angle$ABC will be



(A) 45^o

(B) 30^o

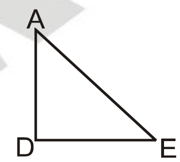
(C) 35^o

(D) 125^o

15. What is the measure of each angle of a regular hexagon?

(A) 120^o (B) 60^o (C) 80^o (D) 160^o

16. For triangle ADE as shown in figure, which of the following angle relation is/are correct:



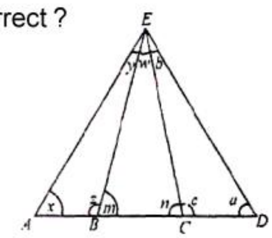
(A) Of the twelve angle formed at the three vertices by adjoining lines, four are acute, four Obtuse and four right angles.

(B) The mean measure of the above twelve angles is 90^o

(C) The measure of the above twelve angle will generally have three or five distinct value.

(D) Of the twelve angles formed, the greatest angle formed is right angle.

17. For triangle ADE as shown in figure, which of the following angle relation is/are correct ?



(A) $x + y + n = a + b + m$

(B) $x + z + n = w + c + m$

(C) $x + y = m$

(D) $y + z = a + b$

18. Which of the following statements relating to 3 lines L_1, L_2 and L_3 in the same plane is 'correct

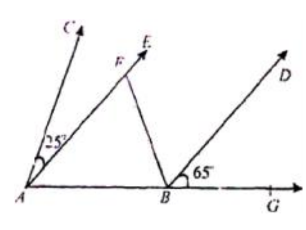
(A) If L_2 and L_3 are both parallel to L_1, then they are parallel to each other.

(B) If acute angle between L_1 and L_2 is equal to the acute angle between L_1 & L_3, then L_2 is parallel to L_1.

(C) If L_2 and L_3 are both perpendicular to L_1 then they are parallel to each other.

(D) If perpendicular distance between L_1 and L_3 is equal to perpendicular distance between L_1 and L_2 , then L_1, L_2 and L_3 are parallel to each other.

19. In the given figure AC || BD, $\angle CAF = 25^0$, $\angle DBG = 65^0$ and BF =BA. Then which of the following statements is correct :-



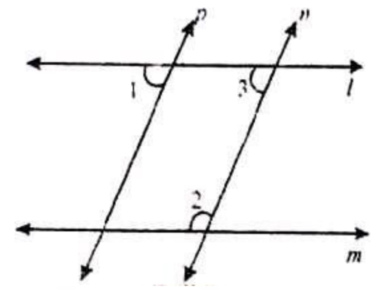
(A) $\angle FBG = 150^o$

(B) $\angle FAB = \angle BCA$

(C) $\angle BFE = 140^o$

(D) $\angle FBG = 80^o$

20. In the given figure, l || m and p || n, if ∠1 = 75º, then.



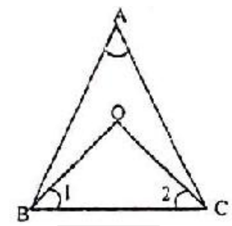
(A) ∠2 = ∠1 + 1 / 3 (of a right angle)

(B) ∠1 and ∠3 form a pair of corresponding angles.

(C) ∠2 and ∠3 form a pair of alternate angle.

(D) ∠2 and ∠3 form a pair of cointerior angles.

21. In the given figure, BO and CO are respectively the bisector of ∠ABC and ∠ACB, then∠BOC is equal to :



(A) $90 - \frac{1}{2}$ ∠A

(B) 180 – ∠ 1 + ∠2

(C) $90 + \frac{1}{2}$∠A

(D) 180 – (∠1 + ∠2)

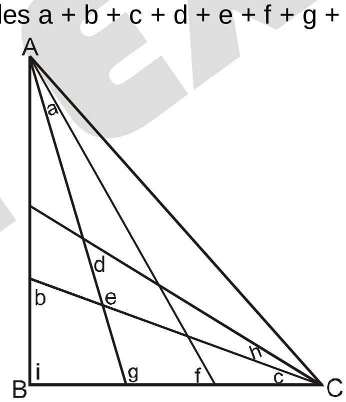
22. A triangle can have at most ______obtuse angles.

(A) one (B) two (C) three (D) none of these

23. If the angles of a triangle are $(x-40)^0$, $(x-20)^0$ and $(½x - 10^0)$, then the smallest angle is :

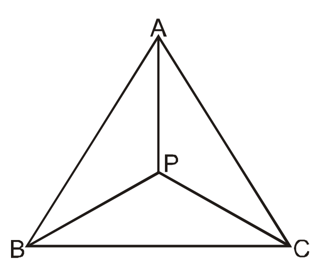
(A) (x – 40º) (B) $\left(\frac{1}{2}x - 10^{o}\right)$ (C) (x – 40º) (D) (180 – xº)

24. In aABC, what is sum of the angles a + b + c + d + e + f + g + h + i ?



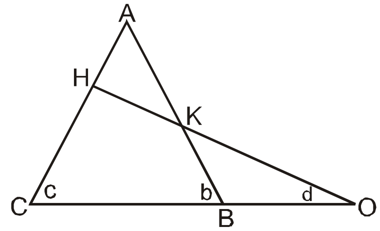
(A) 360° (B) 540° (C) 600° (D) Cannot be determined

25. P is a point inside aABC. If ∠PBA = 20°, ∠BAC = 50° and, ∠PCA = 35°, then the measure of ∠BPC is :



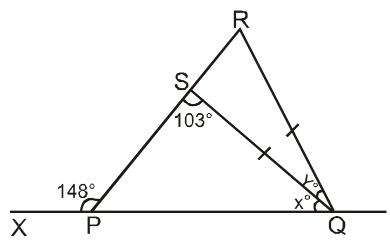
(A) 65° (B) 75° (C) 90° (D) 105°

26. In the given figure OBC and OKH are straight lines. If AH = AK, b = 80° and c = 30° then the value of d is :



(A) 20° (B) 25° (C) 30° (D) 45°

27. In given figure find the values of x and y, if QS = RQ.

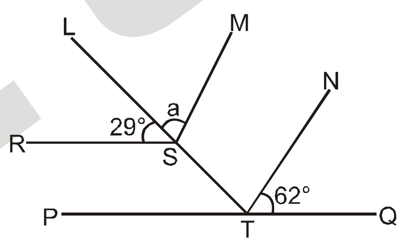


(A) x = 36°, y = 32° (B) x = 45°, y = 32° (C) x = 32°, y = 45° (D) x = 45°, y = 26°

28. Triangles of perimeter 9 units are drawn with their sides having integral and unequal lengths. How many such triangles are possible ?

(A) 1 (B) 2 (C) 3 (D) 4

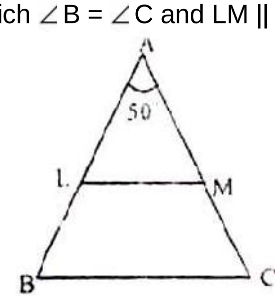
29. In the figure shown PQ || RS and SM || TN. then measure of angle ~ is :



(A) 58° (B) 118° (C) 89° (D) 91°

Passage –I

aABC is an isosceles triangle in which ∠B = ∠C and LM || BC. If ∠A =50º.



1. The quadrilateral LMCB is

(A) Trapezium (B) Square (C) rectangle (D) rhombus

2. The value of $\angle LMC$ is :

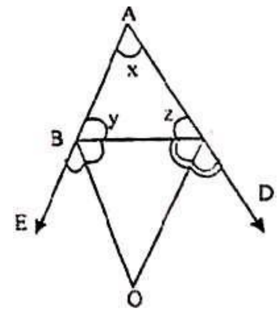
(A) 65^o (B) 115^o (C) 130^o (D) 100^o

3. The value of $\angle ALM$ is:

(A) 130^o (B) 80^o (C) 65^o (D) 100^o

Passage –II

The sides AB and AC of is $\triangle ABC$ are produced to points E and D respectively. If bisectors BO and CO of $\angle CBE$ and $\angle BCD$ respectively meet at point O.



1. The value of $\angle CBO$ is :

(A) $\frac{180-y}{2}$ (B) $\frac{90}{2}-y$ (C) $\frac{1}{2}(90-y)$ (D) $90^o-\frac{y}{z}$

2. The value of $\angle BOC$ is :

(A) $\frac{1}{2}(y+z)$ (B) $\frac{1}{2}\left[\frac{y}{2}+\frac{z}{2}\right]$ (C) 180^o-x (D) $180^o-\frac{x}{2}$

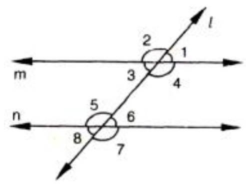
3. Which of the following relations is correct :-

(A) $x + y = 180 - z$ (B) $\angle BOC = 90 - \frac{1}{2}\angle BAC$

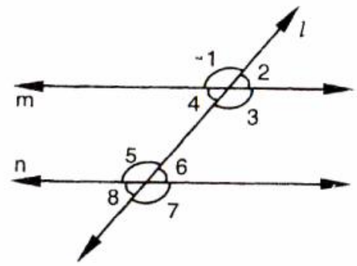
(C) $\angle BOC = 90 + ½\angle BAC$ (D) $x + z = 180 - y$

EXERCISE-II

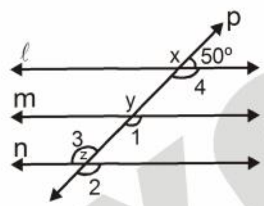
1. In Fig., m||n and $\angle 1 = 65^o$. Find $\angle 5$ and $\angle 8$.



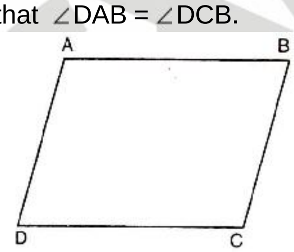
2. In Fig. , m||n and angles 1 and 2 are in the ratio 3 : 2. Determine all the angles from 1 to 8.



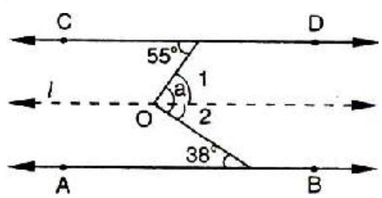
3. In Fig. l,m and n are parallel lines intersected by a transversal p at X, Y and Z respectively. Find $\angle 1$, $\angle 2$ and $\angle 3$. Give reasons.



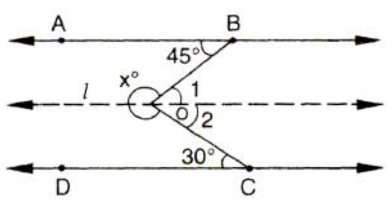
4. In Fig. , AB||DC and AD||BC. Prove that $\angle DAB = \angle DCB$.



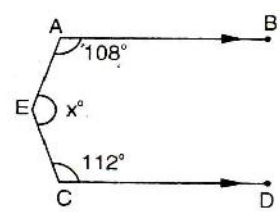
5. In Fig., AB||CD. Determine $\angle a$.



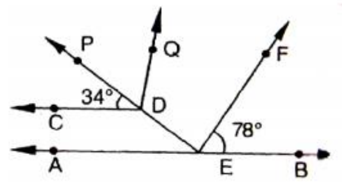
6. In Fig., AB||CD. Determine X.



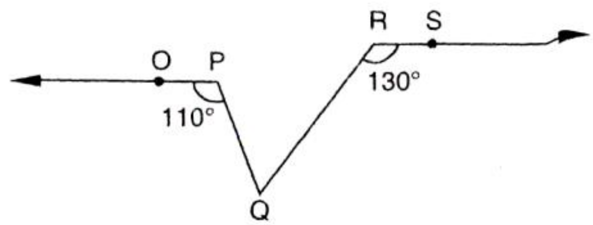
7. In Fig. , AB||CD. Find the value of X.



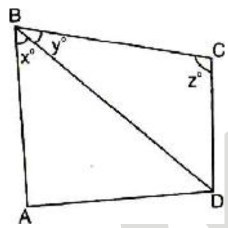
8. In Fig., AB||CD and EF||DQ. Determine $\angle$PDQ, $\angle$AED and $\angle$DEF.



9. In Fig., OP||RS. Determine $\angle$PQR.

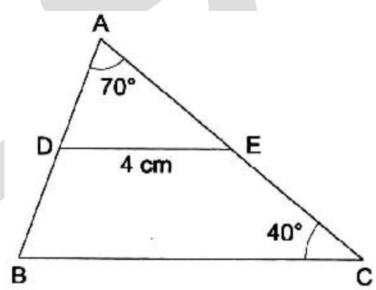


10. In Fig., AB||CD . If $x = \frac{4y}{3}$ and $y = \frac{3z}{8}$, find the values of x,y and z.



11. In Fig., D and E are the mid points of sides AB and AC respectively of aABC. If $\angle$A=70º,$\angle$C=40º and DE=4cm. Find $\angle$EDB and BC.

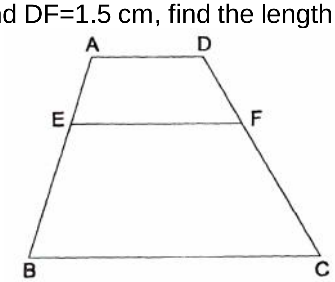
12. In aABC, AD is the median through A and E is the mid-point of AD. BE produced meets AC in F. Prove that $AF = \frac{1}{3}$ AC.



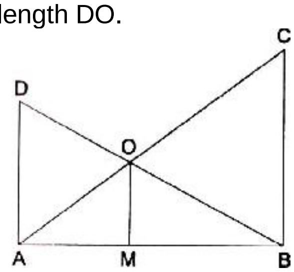
13. Prove that the Figure formed by joining the mid points of the pairs of consecutive sides of a quadrilateral is a parallelogram.

14. In a trapezium ABCD, AD and BC are non parallel sides. E is the mid point of AD. The line segment EF||AB meets BC in F. Prove that F is the mid point of BC and $EF = \frac{1}{2}$(AB + DC).

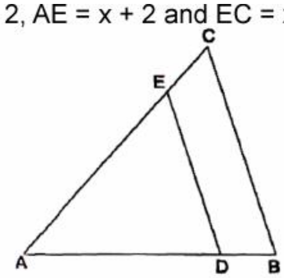
15. In Fig., AD||EF||BC. If EB=2 AE and DF=1.5 cm, find the length of FC.



16. In Fig. , DA { AB, CB { AB, AC and BD intersect at O, and OM { AB meets AB in M. If AO=2.4 cm, OC=3.6 cm and BO=3 cm, find the length DO.

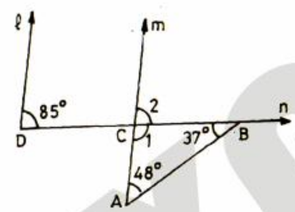


17. In Fig. , DE||BC. If AD = x, DB = x – 2, AE = x + 2 and EC = x – 1, find the value of x.

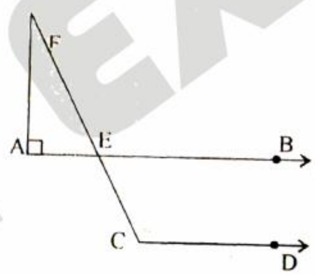


18. D and E are respectively the points on the sides AB and AC of a aABC such that AB = 5.6cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm, show that DE || BC

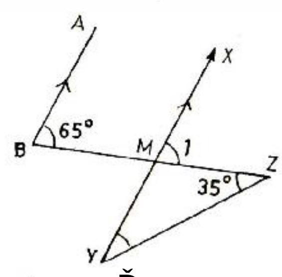
19. In Fig. , prove that ℓ ||m.



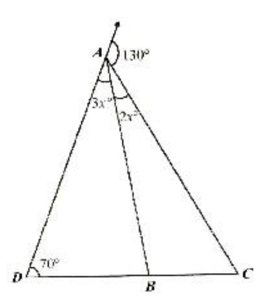
20. In Fig., AB||CD and ∠F=30º. Find ∠FCD.



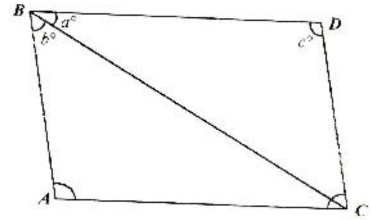
21. In Fig., AB||XY, ∠ABZ=65º, ∠BZY=35º. Calculate ∠XYZ.



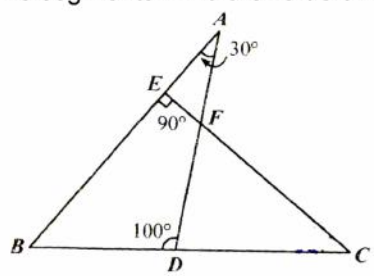
22. In Fig. , AB divides ∠DAC in the ratio ∠DAB :-ĎBAC = 3 : 2. Find ∠ACB and ∠ABC.



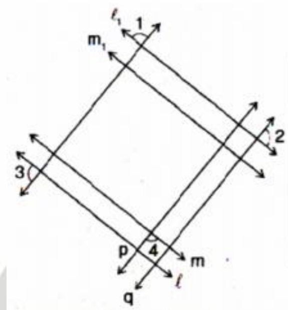
23. In Fig., AB||CD. If $b=\dfrac{5a}{3}$ and $c=\dfrac{10a}{3}$, find the values of a,b and c.



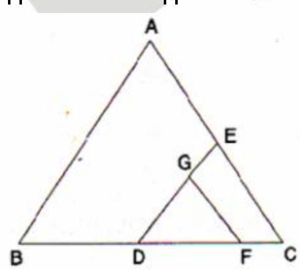
24. In Fig., AB,BC,CE and AD are line segments. Find the value of x.



25. In the figure above (not to scale), l || m, l_1 || m_1 and p || q such that $\angle 1 = 90^o$, $\angle 2 = 130^o$ and $\angle 3 = 70^o$. Find $\angle 4$.

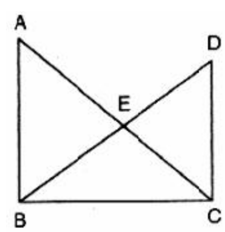


26. In the figure above (not to scale), AB || DE and EC || GF. If $\angle$EGF = 100º and $\angle$ECF = 40º, find the following.

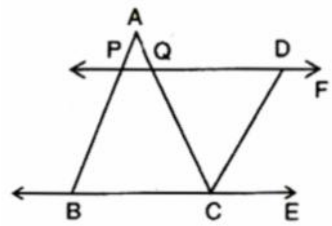


(i) $\angle$ABC (ii) $\angle$GFD (iii) $\angle$GDF

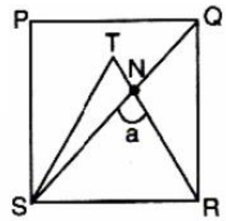
27. In the above figure, $\overline{AC}$ and $\overline{BD}$ intersect at E such that BE = EC, $\angle ABE = 70^0$ and $\angle DCE = 80^0$. If $\angle BAC = 3/2 \angle CDE$, then find $\angle BEC$.



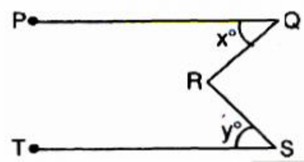
28. In the figure above (not to scale), $\overline{PF} \parallel \overline{BE}$ and $\overline{AB} \parallel \overline{CD}$. If $\angle FDC = 130^0$ and $\angle ACD = 20^0$, find $\angle ACB$ and $\angle ABC$.



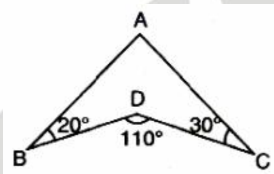
29. In the figure below, PQRS is a square and STR is an equilateral triangle. Find the value of a.



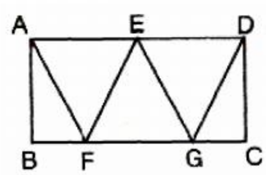
30. In the figure below (not to scale), $\overline{PQ} \parallel \overline{TS}$, reflex $\angle QRS = 300^o$ and $x - y = 30^o$. The measure of y will be



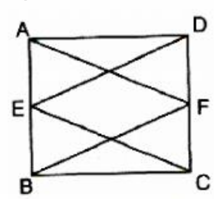
31. In the figure above, $\angle ABD = 20^o$, $\angle BDC = 110^o$ and $\angle DCA = 30^o$. What is the value of $\angle BAC$?



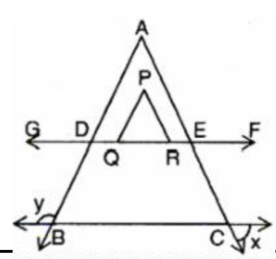
32. In the figure above (not to scale), $\overline{EF} // \overline{GD}$, $\overline{AF} // \overline{EG}$, $\overline{AD} // \overline{BC}$ and $\angle DCG = 100^o$. If $\angle CDG = 40^o$, then find $\angle AEF$.



33. In the above figure (not to scale), E and F are the mid points of AB and CD respectively. $\overline{AB} \parallel \overline{CD}$, $\overline{BC} \parallel \overline{AD}$, $\angle ADE = 70^o$, and $\angle BCE = 40^o$. $\angle DEC$ is

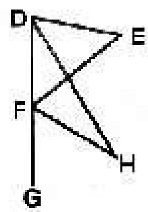


34. In the above figure (not to scale), $\overline{GF} \parallel \overline{BC}$, $\overline{AB} \parallel \overline{PQ}$ and $\overline{AC} \parallel \overline{PR}$. If $\angle x = 40^o$ and $\angle y = 110^o$, then find $\angle QPR$.

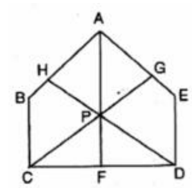


35. In ΔPQR, PD $\perp$ QR and PO is the bisector of $\angle$QPR. If $\angle$PQR = 65º and PRQ = $23\,\frac{1}{2}^{\circ}$ then $\angle$DPO in degrees =

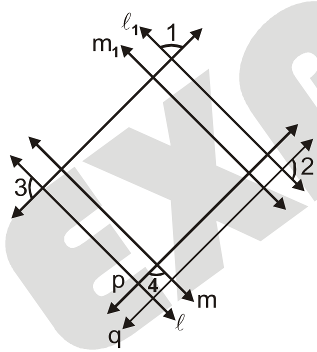
36. In the above figure, DEF is a triangle whose side DF is produced to G. HF and HD are the bisectors of $\angle$EFG and $\angle$EDG respectively. If $\angle$DEF = $23\left(\frac{1}{2}\right)^{\circ}$, then $\angle$DHF (in degrees) =



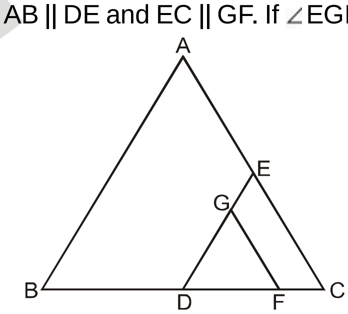
37. In the above figure, $\overline{AF} \parallel \overline{ED}$, $\overline{CG} \parallel \overline{AB}$ and $\overline{AE} \parallel \overline{HD}$. If $\angle$FPD = 40^0, then $\angle$AED =



38. In the figure above (not to scale), $\ell \parallel m$, $\ell_1 \parallel m_1$ and p || q such that $\angle$1 = 90º, $\angle$2 = 130º and $\angle$3 = 70º. Find $\angle$4.



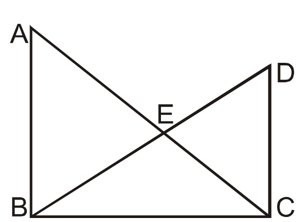
39. In the figure above (not to scale), AB || DE and EC || GF. If $\angle$EGF = 100º and $\angle$ECF = 40º, find the following



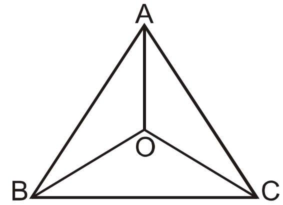
(i) $\angle$ABC (ii) $\angle$GFD (iii) $\angle$GDF

40. In the above figure, $\overline{AC}$ and $\overline{BD}$ intersect at E such that BE = EC, $\angle$ABE = 70º and $\angle$DCE = 80º.

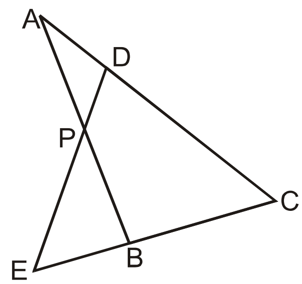
If $\angle$BAC = $\frac{3}{2}$ $\angle$CDE, then find $\angle$BEC.



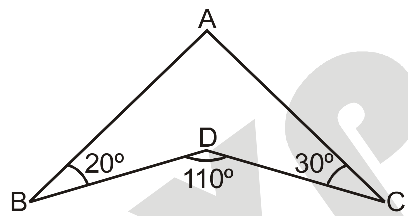
41. In the above aABC (not to scale), OA is the angle bisector of $\angle$BAC. If OB = OC, $\angle$OAC = 40º and $\angle$ABO = 20º. If $\angle OCB = \frac{1}{2}\angle ACO$, then find $\angle$BOC.



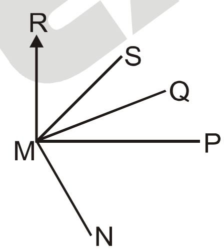
42. In the above figure, AB and DE are straight lines. $\angle$BAC = 40º, $\angle$BPD = 110º and $\angle$DEC = 40º. Find $\angle$ACE.



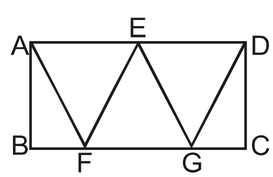
43. In the figure above, $\angle$ABD = 20º, $\angle$BDC = 110º and $\angle$DCA = 30º. What is the value of $\angle$BAC ?



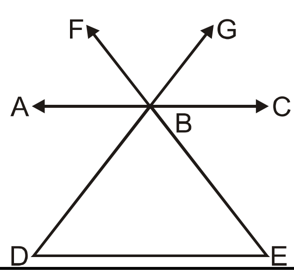
44. In the figure below (not to scale), $\overline{MR}$ { $\overline{MP}$, $\overline{MQ}$ { $\overline{MN}$, and $\overline{MS}$ is bisector of $\angle$RMQ. If $\angle$PMN = 50º, then find the measure of $\angle$RMS



45. In the figure above (not to scale), $\overline{EF}$ // $\overline{GD}$, $\overline{AF}$ // $\overline{EG}$, $\overline{AD}$ // $\overline{BC}$ and $\angle$DCG = 100º. If $\angle$CDG = 40º, then find $\angle$AEF

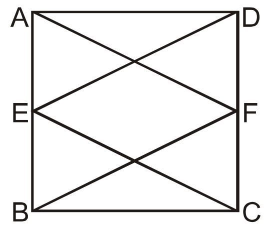


46. In the above figure, BDE is a triangle in which EB is produced to F and DB is produced to G. If $\angle$BDE = xº, $\angle$FBG = (x + 2)º and $\angle$BED = (x + 7)º, then the value of x is



47. In the above figure (not to scale), E and F are the mid points of AB and CD respectively. $\overline{AB} \parallel \overline{CD}$, $\overline{BC} \parallel \overline{AD}$,

$\angle ADE = 70^{\circ}$, and $\angle BCE = 40^{\circ}$. $\angle DEC$ is



48. The number of diagonals of a regular polygon is 27. Then, each of the interior angles of the polygon is__________ .

SOME EXAMPLE WITH PROPER HINTS

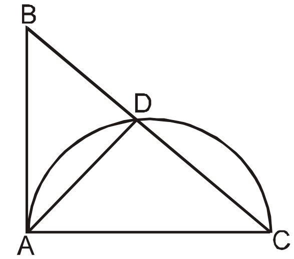
1. On the larger leg of a right angled triangle a semicircle is described. Find the semicircumference if the smaller leg is equal to 30 cm and the chord joining the vertex of right angle with the point of intersection of the hypotenuse and the semi circle is equal to 24 cm.

(A) $30\pi + 60$ cm (B) $22\pi + 10$ cm (C) $20\pi + 40$cm (D) None of these

Sol. ΔADC is right angled triangle [$\angle$ in semicircle]

Hence, ΔADB is right angled triangle

From the triangle ABD, we have



$BD = \sqrt{BA^2 - AD^2} = 18$ cm

Since, $BC.BD = BA^2$

$\therefore \quad BC = \dfrac{BA^2}{BD} = 50$ cm

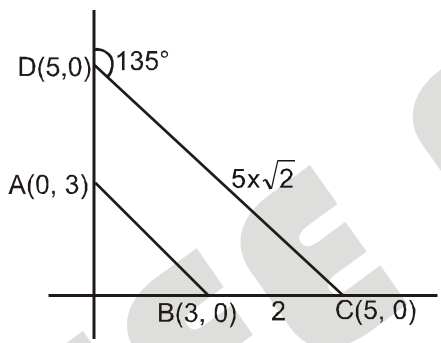
Consequently, $AC = \sqrt{BC^2 - BA^2} = 40$ cm

$\therefore$ The semicircumference is equal to $20\pi + 40$.

2. An ant was running on a graph paper towards North from where she has started. Then she has turned right on her right side through 135° and walked straight for $5\sqrt{2}$ m. Again turned right through 135° and covered 2 metres. Again when she turned towards the initial point she was facing North-West direction. Find the total distance covered by the ant when the ant reaches initial point.

(A) 4 m (B) $8\sqrt{2}$ m (C) $8 + 4\sqrt{2}$ m (D) $4 + 8\sqrt{2}$ m

Sol. A is the initial position of the ant and she ends her journey at B passing through D and C.

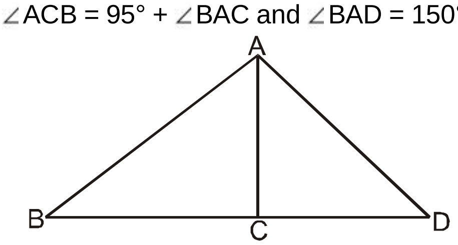


$AB = 3\sqrt{2}$ m $BC = 2$ m

$AD = 2$m, $DC = 5\sqrt{2}$ m

Total distance covered = 2 + 2 + $3\sqrt{2} + 5\sqrt{2} = 4 + 8\sqrt{2}$ m

3. In the given figure AB = AD, $\angle ACB = 95° + \angle BAC$ and $\angle BAD = 150°$. What is the measure of $\angle ACB$?



(A) 110° (B) 120° (C) 130° (D) 140°

Sol. $\because \quad AB = AD$

$\therefore \quad \angle ABD = \angle ADB = 15°$ ($\because \angle BAD = 150°$)

Now $\angle ABD + \angle BAC + \angle ACB = 180°$

$\Rightarrow \quad 2\angle ACB = 260°$

$\therefore \quad \angle ACB = 130°$

4. A man, 5 ft. high standing at a certain distance from a lamp post, finds that the length of his shadow is 8 ft. On moving in the direction of the shadow through 3 ft, he finds the length of his shadow is now 11 ft. Find the difference of the height of the post and distance of the man from the post originally.

(A) 2 ft (B) 3 ft (C) 6 ft (D) 8 ft

Sol. Let 'x' be the height of the post and y the distance of the man from the post originally.

Let AB be the post, ab the original position of the man, a 'b' the position to which he moves.

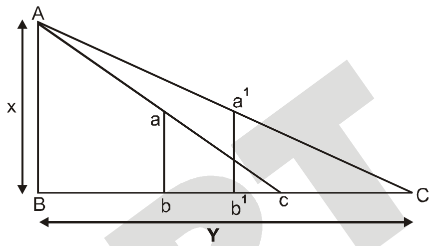
Let bc the length of the shadow initially b 'c' the length of the shadow latter.

Then, ab = 5 ft. bc = 8 ft.

a 'b' = 5ft, bbà = 3 ft, bàcà = 11 ft.

Then AB = x ft, Bb = y ft and Bc = Bb + bc = y + 8 ft and Bbà = y + 3 ft.

Now, $\frac{AB}{Bc}$ Z $\frac{ab}{bc}$ and $\frac{AB}{Bcà}$ Z $\frac{aàbà}{bàcà}$



y $\frac{x}{y H 8}$ Z $\frac{5}{8}$ and $\frac{x}{y H 3 H 11}$ Z $\frac{5}{11}$

Or $8x - 5y = 40$ .... (1)

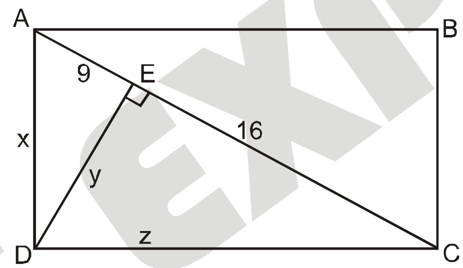
and $11x - 5y = 70$ .... (2)

Solving, we have $3x = 30$;

y $x = 10$ ft and $y = 8$ ft.

So the difference = 10 – 8 = 2 ft.

5. What are the respective values of x, y and z in the given rectangle ABCD ?



(A) 15, 12, 20 (B) 12, 15, 20 (C) 8, 10, 12 (D) None of these

Sol. From the figure given in the question, we get

$x^2 - y^2 = 81$

$x^2 + y^2 = 625$

and $y^2 + 256 = z^2$.

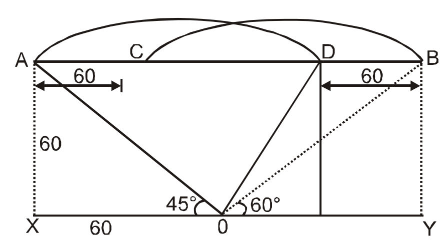
From the option, the only triplet satisfying the three equations is 15, 12, 20.

6. A boy observes a point at the circumference of a disc at an angle of 45° when the disc is flying horizontally at the rate of 30 m/min and 60 m above the ground. After 2 minutes he turns and observes another point diametrically opposite to previous at an angle of 60°. Find the radius of the disc (in m).

(A) $20\sqrt{3}$ (B) $60 H 20\sqrt{3}$ (C) $60 H 10\sqrt{3}$ (D) $10\sqrt{3}$

Sol. Let the observer be at point O : He sees two points A and B. OX Z $\frac{60}{\tan 45î}$ Z 60

OY Z $\frac{60}{\tan 60}$; ======= y AB = XY = 60 + $20\sqrt{3}$



Distance covered by disc in 2 min is given by AC = 30 × 2 = 60 m

Diameter of disc will be given by (AB – DB) or (AB – AC)

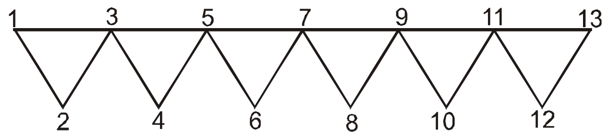
= $60 H 20\sqrt{3} - 60$ Z $20\sqrt{3}$ m

y Radius = $10\sqrt{3}$ Ans.

7. There is a chain of triangles whose vertices are metal beads. If the sides of the triangles are formed by joining a single thread to the metal beads, what will be the length of the thread required to form a chain of 6 triangle of unit sides using 13 beads ?

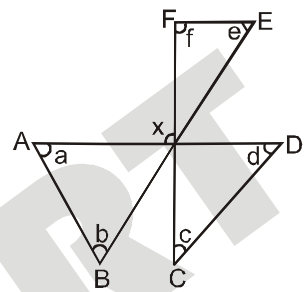
(A) 12 (B) 13 (C) 15 (D) 18

Sol.



The chain will be as shown in figure. First join all the beads from 1 to 13 and then join again, skipping one bead i.e., 13 to 11 to 9 to ..... to 1.

8. In the given figure, APD, BPE and CPF are straight lines, find the measure of



(a) the angle x, if x = c + d.

(b) the angle e, if b = c = d = e = f = x.

(c) the values of a + b + c + d + e + f.

Sol. (a) x = ∠CPD (vertically opposite ∠s).

c + d + ∠CPD = 180°(∠sum of a).

y c + d + x = 180°.

y x + x = 180.

y x + x = 180° (given x = c + d)

y 2x = 180° = x = 90°.

(b) x = ∠CPD (vertically opposite ∠s)

x + c + d = 180° (∠sum of a).

= 3x = 180° (∵ c = d = x).

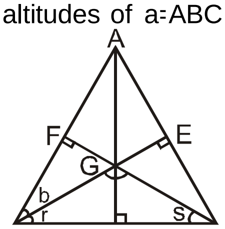
= x = 60°

y e = f = 60° (given e = f = x)

(c) a + b + c + d + e + f

$= 3 \times 180° - \left(\frac{360°}{2}\right) = 540° - 180° = 360°$

9. In the figure; AD, Be and CF are the altitudes of aABC intersecting at G. Find the sum of ∠BAC and ∠BGC.



Sol. Consider aBGC

∠BGC + r + s = 180°

y ∠BGC = 180° – r – s ..... (i)

Consider aBAE.

∠BAE + b + 90° = 180°.

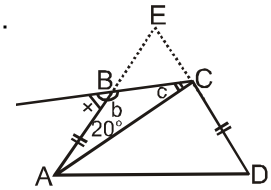
⇒ ∠BAE = 90° – b ..... (ii)

Adding equation (1) and 92),

∠BGC + ∠BAC = 270° – (b + r + s).

= 270° – 90° = 180°

10. In the figure, AB, BC, CD are three sides of a regular polygon and ∠ BAC is 20°. Calculate
(a) the exterior angle of the polygon,
(b) the number of sides of the polygon,
(c) ∠ADC



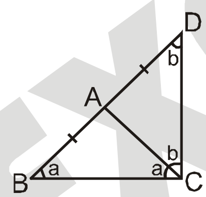
Sol. (a) c = 20° (base ∠s, isosceles a).
b = 180° – 20° – 20° (∠sum of a) = 140°.
y The exterior angle = 180° – 140°
(adjacent angles on line).
i.e. x = 40°.
(b) Let n be the number of sides.

$\frac{(n-2) ò 180î}{n}$ Z 140î or 180 n – 360 = 140 n.

40n = 360 = n = 9.
(c) Produce AB and DC to meet at E.
∠ EBC = ∠ECB = 180° – 140° = 40°.
y EB = EC (sides opposite equal angles).
∠ BEC = 180° – 40° – 40° = 100°.
EB + BA = EC + CD y EA = ED.
y ĎEDA = ∠EAD (base angles, isosceles triangle).

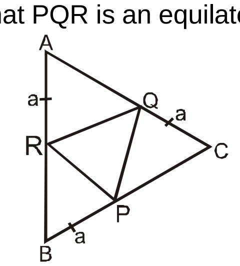
y ĎEAD = $\frac{180î - 100î}{2}$ Z 40î i.e. ∠ADC = 40°.

11. ABC is an isosceles triangle such that AB = AC. Side BA is produced to a point D such that BA = AD. What is the measure of ∠BCD ?



Sol. If AB = AC, then ∠ABC = ∠ACB = a°(say).
Also if AC = AD, then ∠ACD = ∠ADC = b°(say).
Now in aBCD, a + a + b + b = 180°
= a + b = ∠BCD = 180/2 = 90°.

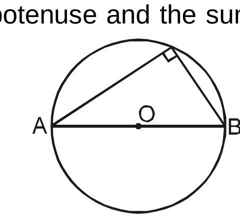
12. In the figure, ABC is an equilateral triangle of side 3a. P, Q and R are points on BC, CA and AB respectively such that AR = BP = CQ = a. Prove that PQR is an equilateral triangle



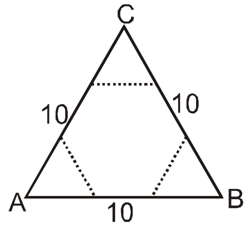
Sol. In as ARQ, BPR and CQP,
AR = BP = CQ = a (given).
Also ∠RAQ = ∠PBR = ∠QCP = 60° (given)
AQ = BR = CP = 3a – a = 2a.
y aARQ ≅ aBPR ≅ aCQP (S.A.S.). i.e. aPQR is equilateral.

13. One side of a right triangle is 3/5 times of the hypotenuse and the sum of that side and hypotenuse is 16 cm. What is the circum radius of the triangle ?

Sol. Forming the equations we have a = 3/5 h.
AC = (3/5) h, BC = h.
Given a + h = 16.
n So 3/5 h + h = 16 or h = (5/8) × 16 = 10



14. The corners of an equilateral triangle of side 10 cm each are cut to form a regular hexagon. What is the area of the hexagon ?

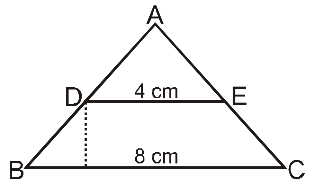


Sol. Since one side of the triangle is 10 cm, when we cut it, the side of the hexagon becomes 10/3 = 3.33 cm.

$\Rightarrow$ Area of the hexagon = 6 × area of an equilateral triangle

= $6 ò \sqrt{3}/4 ò 3.33^2$ = 28.8 sq. cm.

15. In the given figure, DE||BC. If DE = 4 cm, BC = 8 cm and area of aADE = 25 sq. cm, then the area of aABC is

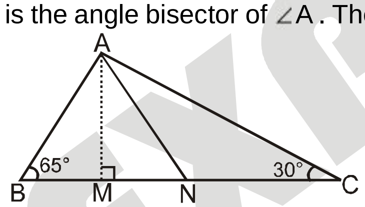


(A) 70 sq cm (B) 100 sq cm (C) 150 sq cm (D) None of these

Sol. $\frac{aADE}{aABC} Z \left(\frac{DE}{BC}\right)^2 \Rightarrow \frac{25}{aABC} Z \left(\frac{4}{8}\right)^2 Z \frac{1}{4}.$

y aABC = 100 sq. cm.

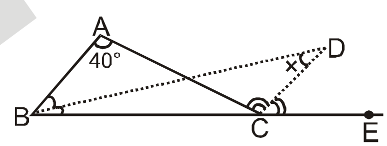
16. In the given figure, AM { BC, AN is the angle bisector of $\angle A$. Then $\angle MAN$ is



(A) 15° (B) 32.5° (C) 17.5° (D) None of these

Sol. By rule, $\angle MAN = \frac{1}{2}(\angle B - \angle C) Z \frac{1}{2}$ (65° – 30) = $17\frac{1}{2}$î .

17. The measure of angle x in the given figure is

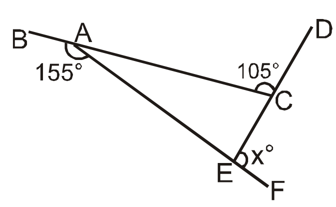


(A) 20° (B) 30° (C) 40° (D) None of these

Sol. Here BD and CD are the respective bisectors of $\angle ABC$ and $\angle ACE$.

Thus, by rule $\angle x = \frac{1}{2}\angle A = \frac{1}{2} ò 40î Z 20î$

18. Find the value of x in the given figure.



(A) 160° (B) 130° (C) 100° (D) None of these

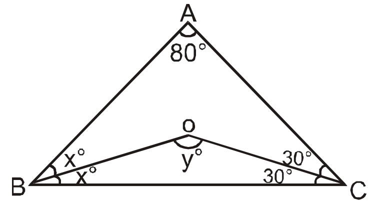
Sol. Since $\angle ACD$ = 105°

y ĎACE = 75°. Also since $\angle BAE$ = 155°,

y ĎEAC = 25°.

y x = $\angle ACE + \angle EAC$ $\Rightarrow$ x = 75 + 25 = 100°.

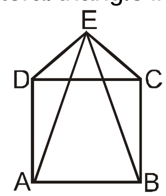
19. The values of x° and y° in the given figure are respectively



(A) 20° and 130° (B) 30° and 120° (C) 70° and 80° (D) None of these

Sol. In ΔABC, $80° + 30° + x + 30° + x = 180°$ $\Rightarrow$ $x = 20°$
Also, In ΔBOC, $x + y + 30° = 180°$ $\Rightarrow$ $y = 130°$.

20. If ABCD is a square and EDC is an equilateral triangle in the given figure, then the measure of $\angle DAE$ is

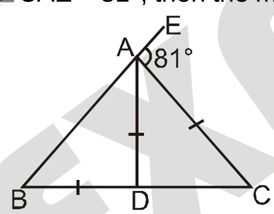


(A) 15° (B) 22½ (C) 45° (D) 30°

Sol. Here, $\angle ADE = \angle ADC + \angle EDC = 90° + 60° = 150°$.
Also, since AD = DE.
$\therefore$ $\angle DAE = \angle DEA$ ..... (i)
But $\angle DAE + \angle DEA = 180° - 150° = 30°$

$\therefore$ $\angle DAE = \frac{1}{2}(30°) = 15°$ [due to (i)].

21. In the given figure, AD = BD = AC. If $\angle CAE = 81°$, then the measure of $\angle ACD$ is



(A) 40.5° (B) 81° (C) 54° (D) None of these

Sol. In ΔABD, let $\angle BAD = x°$. Also let $\angle ADC = y°$.
Since BD = AD = AC,
$\angle BAD = x° = \angle ABD$
$\angle ADC = y° = x° + x°$ $\Rightarrow$ $y° = 2x°$.

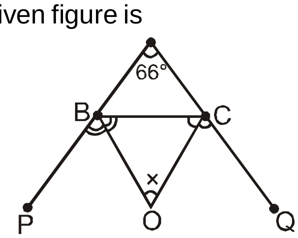
$\therefore$ $x° = \frac{1}{2}(y°)$ ..... (i)

Now consider the straight line BAE,
$\angle EAC + \angle DAC + \angle BAD = 180°$.
$\Rightarrow$ $81° + 180° - y° - y° + x° = 180°$

$\Rightarrow$ $2y° - \frac{1}{2}(y°) = 81°$

$\therefore$ $\angle ACD = y° = 54°$

22. The measure of the angle x in the given figure is



(A) 66° (B) 33° (C) 57° (D) 133°

Sol. By rule, $\angle x = \angle BOC = 90° - \frac{1}{2}\angle BAC$

$= 90° - \frac{1}{2}(66î)$ Z 57î

23. P and Q are the points on the sides AB and AC respectively of a aABC. If AP = 2 cm, PB = 4 cm, AQ = 3 cm, QC = 6 cm. Then $\frac{BC}{PQ}$ Z ?

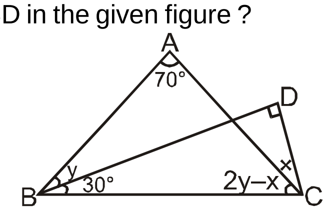
(A) 4 (B) 2 (C) 3 (D) 5

Sol. The two trianlges ABC and APQ are similar.
Thus BC : PQ = AB : AP.

On substituting the values, we get BC = PQ × $\frac{(2 \text{ H } 4)}{2}$ Z 3PQ.

= $\frac{BC}{PQ}$ Z 3 **Ans.**

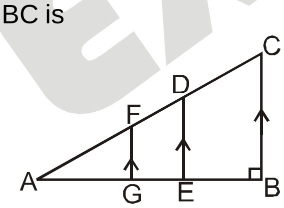
24. What is the value of $\angle DCA + \angle ABD$ in the given figure ?



(A) 100° (B) 70° (C) 50° (D) 40°

Sol. In aDBC, 2y – x + x + 90 + 30 = 180.
y y = 30°
In aABC, 70 + 30 + 30 + 60 – x = 180.
y x + y = 30 + 10 = 40°. **Ans.**

25. In the given figure, ABC is a right angled triangle. Also FG || DE || BC and AG = GE = EB.
If DE = 12 cm, then the measure of BC is



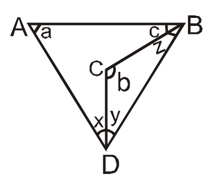
(A) 12 cm (B) 18 cm (C) 24 cm (D) 30 cm

Sol. Since AG = GE = EB y $\frac{AE}{AB}$ Z $\frac{2}{3}$
Since aAED is similar to aABC,

y $\frac{AE}{AB}$ Z $\frac{DE}{BC}$ = $\frac{2}{3}$ Z $\frac{12}{BC}$

y BC Z $\frac{12 \text{ ò } 3}{2}$ Z 18 cm **Ans. (2)**

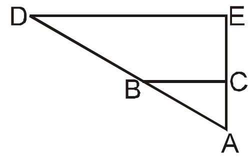
26. Find the value of x in the following figure



(A) a + b + c (B) b – a – c (C) 180 – c (D) None of these

Sol. In aCDB, b + y + z = 180°.
y y + z = 180° – b.
in aADB = a + x + y + c + z = 180°
y a + c + x + 180° – b = 180°
y x = b – a – c.

27. Line BC divides aADE into 2 sections, one of them an isosceles triangle (AB = AC). Angle DBC is equal to 105°. What is the sum of the measures of angles D and E ?



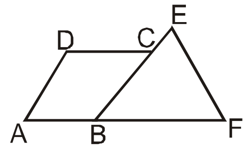
(A) 100° (B) 125° (C) 150° (D) 175°

Sol. AB = AC = $\angle$ABC = $\angle$ACB = 75°

y Ď A = 30°, $\angle$A + $\angle$D + $\angle$E = 180°

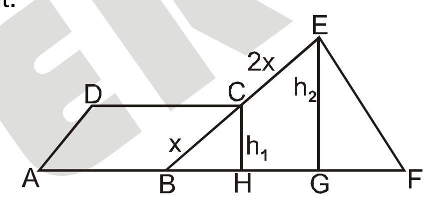
Ĝ Ď D + $\angle$E = 150°.

28. In the figure, the area of parallelogram ABCD is 24. If CE = 2BC and AB = BF, then what is the area of aBFE ?



(A) 24 (B) 72 (C) 48 (D) 36

Sol. AB = BF and CE = 2BC. Area of parallelogram ABCD = base × height.



= AB × h_1 = 24.

y AB = $\frac{24}{h_1}$

Area of aBFE = $\frac{1}{2}$ ò h_2 ò BF Z $\frac{1}{2}$ ò h_2 ò AB Z $\frac{1}{2}$ ò h_2 ò $\frac{24}{h_1}$ Z 12 $\frac{h_2}{h_1}$

aBHC and aBGE are similar triangles.

= $\frac{x}{3x}$ Z $\frac{h_1}{h_2}$ = $\frac{h_1}{h_2}$ Z $\frac{1}{3}$

= $\frac{h_2}{h_1}$ Z 3 .

y Area of aBFE = 12 × 3 = 36

29. The sides of a triangle are in the ratio of 6 : 8 : 9. Thus

(A) the triangle is acute (B) the triangle is right-angled

(C) the triangle is obtuse (D) None of these

Sol. In a aABC where a, b and c are the three sides and a is the largest side,

if $a^2 < b^2 + c^2$, then the triangle is acute,

if $a^2 = b^2 + c^2$, then it is a right-angled triangle,

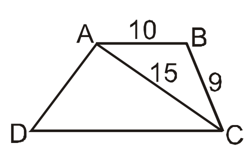
if $a^2 > b^2 + c^2$, then the triangle is obtuse.

Let sides of the triangle be 6k, 8k and 9k.

We have $(9k)^2 < (6k)^2 + (8k)^2$.

y It is an acute triangle

30. In the given figure, BA is parallel to CD and AB = 10 cm, BC = 9 cm and AC = 15 cm. Also $\angle$CBA = $\angle$DAC. Find the length of AD.



(A) 13 cm (B) $\frac{27}{2}$ cm (C) $16\frac{2}{3}$ cm (D) None of these

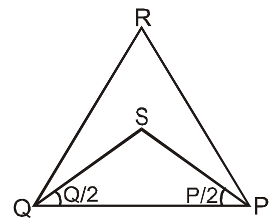
Sol. It is clear that ABC and CAD are similar triangles.

So $\frac{AB}{BC} = \frac{CA}{AD}$.

$\Rightarrow \quad \frac{10}{9} = \frac{15}{AD}$

$= \quad AD = \frac{15 \times 9}{10} = \frac{27}{2}$ cm.

31. In the given triangle PQR, angle P is greater than angle Q and the bisector of angle P and Q meet in S. Then



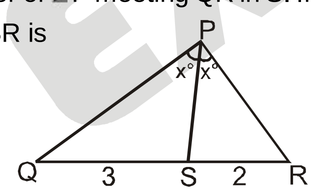
(A) SQ = SP (B) SQ ≠ SP (C) SQ < SP (D) SQ > SP

Sol. Since it is given that $\angle P > \angle Q$, so $\angle \frac{P}{2} > \angle \frac{Q}{2}$.

So, the side opposite to the greater angle $\left(\frac{P}{2}\right)$ is greater than the side opposite to the smaller angle $\left(\frac{Q}{2}\right)$.

$\Rightarrow \quad SQ > SP$.

32. In the given figure, PS is the bisector of $\angle P$ meeting QR in S. If area of $\Delta PQS = 9$ sq. units, QS = 3 units and SR = 2 units, then the area of ΔPSR is



(A) 6 sq. units (B) 7.5 sq. units (C) 9 sq. units (D) None of these

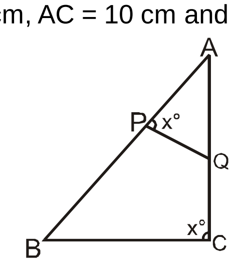
Sol. Given that PS is the bisector of $\angle P$.

$\Rightarrow \quad \frac{\text{Area } (\Delta PQS)}{\text{Area} (\Delta PSR)} = \frac{QS}{SR}$

$= \quad \frac{9}{\text{Area}(\Delta PSR)} = \frac{3}{2}$

$\Rightarrow \quad$ Area of $\Delta PSR = \frac{9 \times 2}{3} = 6$ sq. units.

33. In the figure, ABC is triangle, AP = 4 cm, AC = 10 cm and QC = 2 cm. The length of PB is



(A) 12 cm (B) 14 cm (C) 16 cm (D) 13.5 cm

Sol. In aAPQ and aACB,

(a) $\angle$PAQ = $\angle$CAB (common).

(b) $\angle$APQ = $\angle$ACB = x° (given).

(c) y $\angle$AQP = $\angle$ABC (3rd angle of a).

y aAPQ ~ aACB (A.A.A).

$\frac{AB}{AC}$ Z $\frac{AQ}{AP}$ (corresponding sides ~ as).

$\frac{AB}{AC}$ Z $\frac{AC - QC}{AP}$ = $\frac{AB}{10}$ Z $\frac{10 - 2}{4}$ = AB Z $\frac{10 ò 8}{4}$ Z 20 cm .

y PB = AB – AP = 20 – 4 = 16 cm.

34. If the angles of a quadrilateral are in the ratio of 1 : 4 : 2 : 3, what kind of quadrilateral is it ?

(A) Rectangle (B) Square (C) Trapezium (D) Rhombus

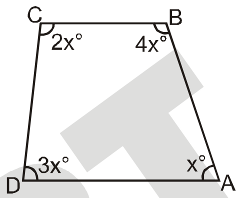
Sol. x + 4x + 2x + 3x = 360°

= 10x = 360°

= x = 36°

Angles are 36°, 144°, 72°, 108°.

So the quadrilateral is a trapezium.



35. How many sides does a regular polygon have whose exterior angle is 1/11 of its interior angle ?

(A) 22 (B) 24 (C) 20 (D) 26

Sol. If x be exterior angle, then according to the question x = $\frac{1}{11}$(180î – x) .

= 11x = 180° – x = 12x = 180° = x = 15°

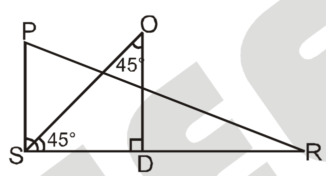
y Number of sides = $\frac{360î}{15î}$ Z 24

41. PSR is a triangle right angled at S. D is the mid-point of SR. If the bisector of $\angle$PSR and perpendicular bisector of SR meet at O, then triangle OSD is :

(A) scalene (B) equilateral

(C) isosceles right angled (D) acute angled

Sol.

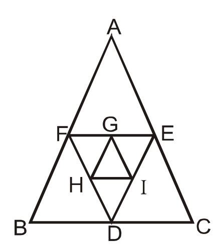


Clearly $\angle$OSD = $\angle$SOD

= SD = OD. Further $\angle$SDO = 90°.

y aOSD is isosceles right angled.

49. D, E, F are midpoints of BC, CA and AB respectively. G, H, f are midpoints of FE, FD, DE respectively. Areas of aDHf and aAFE are in the ratio

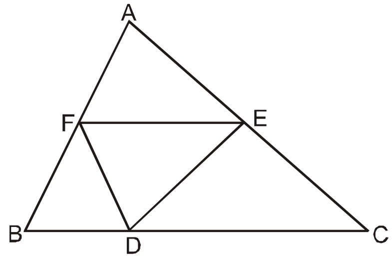


(A) 1 : 3 (B) 1 : 4 (C) 1 : 9 (D) 1 : 16

Sol. We have area of triangle AFE = A/4. (If A = area of triangl ABC) and area of triangle DHI = (A/4)/4 = A/16.

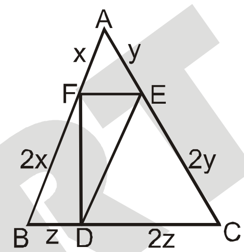
Hence, ratio = 1 : 4.

50. In triangle ABC, D, E, F are points of trisection of BC (BD : DC = 1 : 2), AC (CE : EA = 2 : 1) and AB(AF : FB = 1 : 2) respectively. Which of the following statement is not true ?



(A) Area aEDC = 4/9 area aABC
(B) Area aFBD = 2/7 area AFDC
(C) Area aDEF = 2/9 area aABC
(D) Area (aEDC + aDBF + aAFE = 2 area aDEF

Sol. Area of aAFE = $\frac{1}{2}(x\,\grave{o}\,y)\sin A$



Area of aABC = $\frac{1}{2}(3x\,\grave{o}\,3y)\sin A$

Area of aFBD = Area of aDEF = $\frac{1}{2}(2x\,\grave{o}\,z)\sin B$

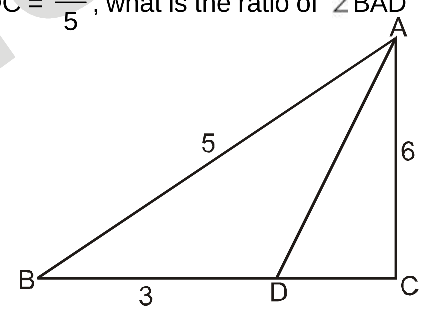
Area of aABC = $\frac{1}{2}(3x\,\grave{o}\,3z)\sin B$

Area of aEDC = $\frac{1}{2}(2y\,\grave{o}\,2z)$ sin C and Area of aABC = $\frac{1}{2}(3y\,\grave{o}\,3z)$ sin C

51. Two sides of a triangle measure 7 and 12. What is the possible measure of the third side ?
(A) 4
(B) 14
(C) 24
(D) All of the above

Sol. Option (1) is not possible as 7 + 4 < 12.
Option (3) is not possible as 24 – 7 > 12.
therefore option (B) is correct

52. In the given triangle if DC = $\frac{18}{5}$, what is the ratio of $\angle BAD$ to $\angle CAD$?



(A) 5 : 6
(B) 3 : 5
(C) 1 : 1
(D) None of these

Sol. $\frac{AB}{AC}\,Z\,\frac{5}{6},\ \frac{BD}{DC}\,Z\,\frac{3}{18}.5\,Z\,\frac{5}{6}$

y $\frac{AB}{AC}\,Z\,\frac{BD}{DC}$.

y AD is angle bisector

y $\frac{m\angle BAD}{m\angle CAD}\,Z\,\frac{1}{1}$

ANSWER KEY

Fill in the blanks :

1. Perpendicular **2.** 60º, 120º **3.** equal **4.** || **5.** uncommon

6. obtuse

Assertion & Reason :

1. (A) **2.** (A)

Match of the following :

1. A (q), B (p), C (s), D (r) **2.** A(s), B(r), C(p), D(q) **3.** A(r), B(r), C(p), D(q, s)

Objective type :

1. (A) **2.** (B) **3.** (B) **4.** (A) **5.** (D) **6.** (B) **7.** (C)

8. (B) **9.** (A) **10.** (D) **11.** (B) **12.** (D) **13.** (A) **14.** (C)

15. (A) **16.** (A, B, C) **17.** (A, C) **18.** (A, C, D) **19.** (C, D)

20. (A, B, D) **21.** (C, D) **22.** (A) **23.** (B) **24.** B **25.** D

26. B **27.** D **28.** A **29.** C

Passage–I

1. (A) **2.** (B) **3.** (C)

Passage–II

1. (A) **2.** (A) **3.** (A, B, D)

EXERCISE

1. $\angle 5 = 115^\circ, \angle 8 = 65^\circ$ **2.** 108º, 72º, 108º, 72º, 108º, 72º, 108º, 72º

3. $\angle 1 = \angle 2 = \angle 3 = 130^\circ$ **5.** $\angle a = 93^\circ$ **6.** X = 285º **7.** X=140º

8. $\angle PDQ = \angle DEF = 68^\circ$ and $\angle AED = 34^\circ$ **9.** 60º **10.** 48º,36º, 96º

11. 110º, 8 cm **15.** 3 cm **16.** 2 cm **17.** x = 4 **20.** 120º

21. 30º **22.** $\angle ACB = 60^\circ, \angle ABC = 100^\circ$ **23.** 30º, 50º, 100º **24.** 40

25. 70º **26.** (i) 60, (ii) 40º, (iii) 60º **27.** 100º **28.** 110º, 50º **29.** 75º

30. 15º **31.** 60º **32.** 40º **33.** 110º **34.** 70º

35. $20\left(\frac{3}{4}\right)$ **36.** $11\left(\frac{3}{4}\right)$ **37.** 140º **38.** 70º

39. (i) 60º ; (ii) 40º ; (iii) 60º **40.** 100º **41.** 140º **42.** 30º **43.** 60º

44. 25º **45.** 40º **46.** 57 **47.** 110º **48.** 140°

CHAPTER -3

CONGRUENT TRIANGLES

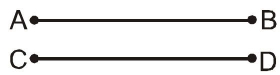
CONTENTS

CONGRUENT TRIANGLES

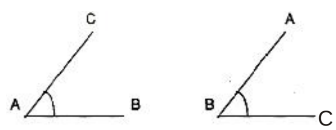
KEY CONCEPTS

CONGRUENCE OF TWO FIGURES : Two geometrical figures in a plane are congruent if, without bending or twisting, we can superimpose one figure on the other so that the two can be brought into coincidence.

(i) Two line segments are congruent if and only if they have the same length.



(ii) Two angles are congruent if and only if they have the same angle measure.

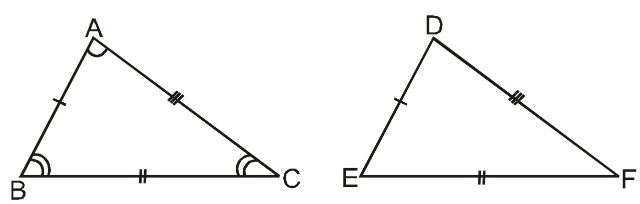


(iii) Two circles are congruent if and only if they have the same radius.

Congruence of Two Triangles

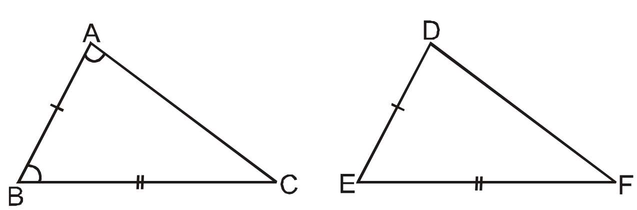
Two triangles ABC and DEF are congruent if and only if all the sides and angles of one triangle are equal to the corresponding sides and angles of the other. We write $\Delta ABC \cong \Delta DEF$.

AB = DE
BC = EF
CA = FD
$\angle A = \angle D$
$\angle B = \angle E$
$\angle C = \angle F$



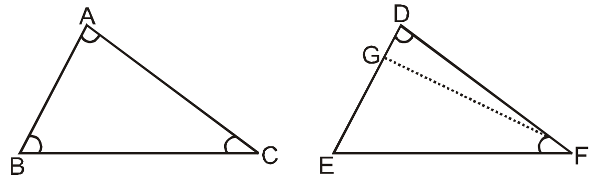
Corresponding Parts of Congruent Triangles are Equal [cpct]
If two triangles are congruent then their corresponding parts are equal.

CRITERIA FOR CONGRUENCE OF TRIANGLES



We being with the following axiom regarding congruence of two triangles :

1. **SAS (Side-Angle-Sides-Axiom of Congruency) :** Two triangles are congruent, if and only if two sides and the include angle of one triangle are equal to any to sides and the included angle of the other triangle.



In SAS criterion the equality of the included angle is essential.

2. **ASA (Angle Side Angle) Congruence Rule :**

Theorem 1: Two triangles are congruent if two angles and the Included side of one triangle are equal to two angles and the included sides of the other triangle.

Given: Two triangles ABC and DEF such that $\angle B = \angle E$, $\angle C = \angle F$, and BC = EF.
To prove: $\Delta ABC \cong \Delta DEF$
Proof: There are three possibilities:
(i) AB = DE (ii) AB < DE (iii) AB > DE

Case (i) When AB = DE
In this case, we have:

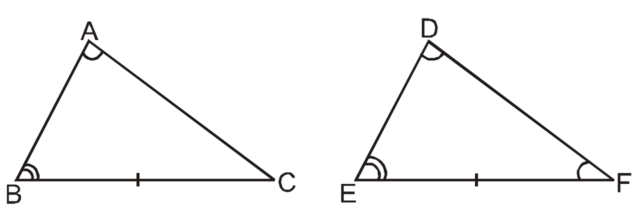
	AB = DE	[by construction]
	$\angle B = \angle E$	[given]
And,	BC = EF	[given]
Thus,	$\Delta ABC = \Delta DEF$	[SAS axiom of congruence]

Case(ii) When AB < DE
In this case take a point G on ED such that EG = AB. Join GF
Now, in triangles ABC and GET, we have

	AB = GE	[by construction]
	$\angle B = \angle E$	[given]
And,	BC = EF	[given]
Thus,	$\Delta ABC = \Delta GEF$	[SAS axiom of congruence]
$\Rightarrow$	$\angle ACB = \angle GEF$ [cpct]	
But	$\angle ACB = \angle DFE$ [given]	
	$\angle GFE = \angle DEF$	

This is possible only when ray FG coincides with ray FD, or G coincides with D.
Therefore, AB must be equal to DE.
Thus, $\Delta ABC \cong \Delta DEF$ [SAS axiom of congruence]
Case (iii) When AB > DE.
In this case we take a point G on ED produced in such a way that GE = AB and repeating the argument as in case (ii), we can conclude that AB = DE and hence
$\Delta ABC \cong \Delta DEF$.
Thus, two triangle are congruent if two angles and the included side of one triangle are equal to two angles and the included side of the other triangle.

Corollary: AAS (Angle-Angle-Side) Congruence Rule



Two triangles are congruent if any two angles and a sick of one triangle are equal to two angles and the corresponding of the other triangle.

GIVEN: $\angle A = \angle D$, $\angle B = \angle E$ AND BC = EF
To prove: $\Delta ABC \cong \Delta DEF$

Proof:	$\angle A = \angle D$	[given]	…(i)
	$\angle B = \angle E$	[given]	…(ii)

Now,

$\angle C = 180^0 - (\angle A + \angle B)$ [Sum of the angles of a triangle is 180^0]

$= 180^0 - (\angle D + \angle E)$ [using (i) and (ii)]

$\angle C = \angle F$

In ΔABC and ΔDEF

$\angle B = \angle E$ [given]

$\angle C = \angle F$ [proved]

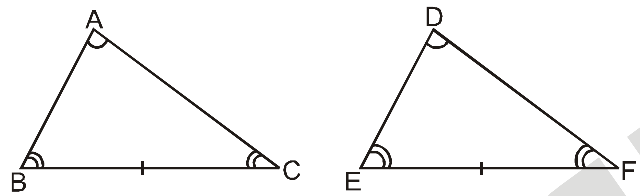
And $BC = EF$ [given]

Thus,

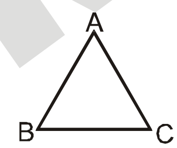
$\Delta ABC \cong \Delta DEF$ [ASA congruence rule]

Thus, two triangles are congruent if any two angles and a side of one triangle are equal to two angles and the corresponding side of the other triangle.

Important: If all the three angles of one triangle are equal to all the three angles of another triangle. Then the two triangles need not be congruent.



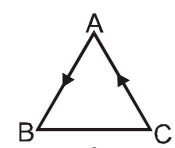
PROPERTIES OF AN ISOSCELES TRIANGLE



An isosceles is a triangle whose two sides are equal.

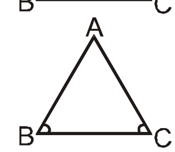
Property chart of Isosceles Triangle :

A triangle ABC is an isosceles triangle if and only if



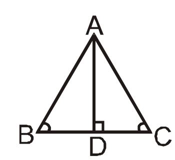
Any one of the following conditions is satisfied :

1. Sides



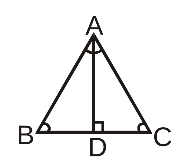
AB = AC

2. Angles



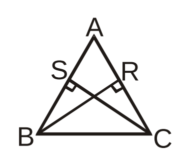
$\angle B = \angle C$

3. Altitude



Altitude AD bisects the side BC.

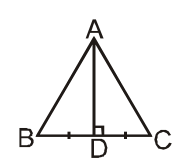
4. Altitude



Altitude AM bisects $\angle BAC$.

i.e. $\angle 1 = \angle 2$

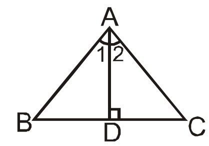
5. Altitudes



Altitudes BR and CS of the triangle are

Equal, i.e. BR = CS.

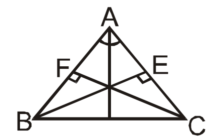
6. Median



Median AD is perpendicular to BC.

i.e. AD $\perp$ BC.

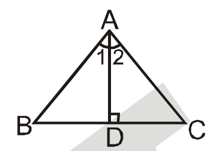
7. Median



Median AD bisects the $\angle$BAC

i.e. $\angle 1 = \angle 2$

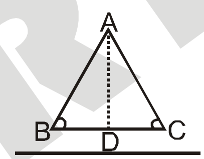
8. Medians



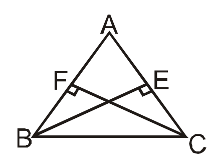
Medians BE and CF are equal. i.e.
BE = CF.

9. Bisector

Bisector AD of $\angle$BAC bisects the side
BC, i.e. BD = CD

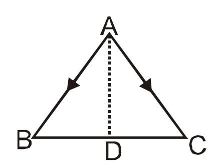


10. Bisector



Bisector AD of $\angle$BAC is perpendicular to the side BC i.e. AD $\perp$ BC

11. Bisectors



Bisectors of $\angle$B and $\angle$C are equal in length i.e. BE = CF.

Theorem 2: The angles opposite to equal sides of a triangle are equal.

Given: ABC is an isosceles triangle in which:
AB = AC

To prove: $\angle B = \angle C$

Construction: Take a point D on BC such that AD bisects $\angle$BAC.
Proof : In triangles ABD and ACD.

	AB = AC	[given]
	$\angle BAD = \angle CAD$	[by construction]
And	AD = AD	[common]

Therefore, ΔABD and ΔACD [by SAS axiom of congruence]
Thus, angles opposite to equal sides of a triangle are equal.

Theorem 3: If two angles of a triangles are equal, then the sides opposite to them are also equal.
Given: ABC is a triangle in which $\angle B = \angle C$.
To prove: AC = AB
Construction: From A, draw AD $\perp$ BC
Proof : In triangles ABD and ACD

$\angle B = \angle C$ [given]

$\angle ADB = \angle ADC$

[each = 90^0, since $AD \perp BC$]

$AD = AD$ [common]

Therefore,

$\Delta ABD \cong \Delta ACD$ [AAS congruence rule]

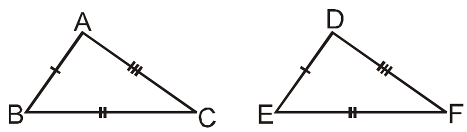
Hence : $AC = AB$

Thus if two angles of a triangle are equal then the sides opposite to them are also equal.

SOME MORE CRITERIA FOR CONGRUENCE OF TRIANGLES

Theroem4: (SSS Congruence Rule)

Two triangles are congruent if three sides of triangle are equal to the three sides of the other triangle.



If $AB = DE$, $BC = EF$, and $CA = FD$, then $\Delta ABC \cong \Delta DEF$

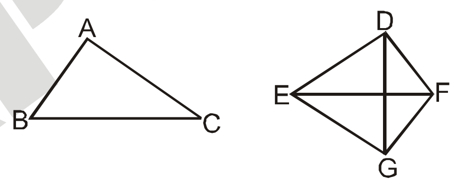
Side-Side-Rule (SSS Rule): Two triangles are congruent if the three sides of one triangle are equal to the three sides of the other triangle.

Given: In ΔABC and ΔDEF

$AB = DE$, $BC = EF$ and $AC = DF$

To prove $\Delta ABC \cong \Delta DEF$

Construction : Draw a line segment EG at the other side of ΔDEF such that $EG = AB$ and $\angle ABC = \angle FEG$ then join EG, GF and DG



Proof : In ΔABC and ΔGEF

$AB = GE$ (By construction)

$\angle ABC = \angle GEF$ (By construction)

$BC = EF$ (Given)

Hence by SAS rule

$\Delta ABC \cong \Delta GEF$

Hence the corresponding sides and angles are equal

$\Rightarrow$ $\angle A = \angle G$, $AB = GF$ …(1)

Now $AB = EG$ (By construction)

$AB = DE$ (Given)

$\Rightarrow$ $AB = DE$ …(2)

Similarly $AC = GF$ and $AC = DF$ (Given)

$\Rightarrow$ $GF = DF$ …(3)

Now in ΔEDG the opposite angle of equal sides EG and DE are equal

$\Rightarrow$ $\angle EDG = \angle EGD$ …(4)

Similarly in ΔFDG the opposite angle of equal, sides GF and DF are equal

$\angle GDF = \angle DGF$ …(5)

$\angle EDG + \angle GDF = \angle EGD + \angle DGF$

$\Rightarrow$ $\angle D = \angle G$ …(6)

From eq. (1) $\angle A = \angle G$ …(7)

Hence, from (6) and (7), we have

$\angle A = \angle D$

In ΔABC and ΔDEF

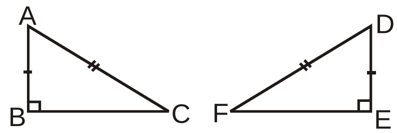
$AB = DE$ (Given)

$\angle A = \angle D$ [By (8)]

$AC = DF$ (Given)

By SAS rule $\Delta ABC \cong \Delta DEF$.

Theorem 5: RHS (Right Angles Hypotenuse-Side) : Two right triangles are congruent if the hypotenuse and a side of one triangle are respectively equal to the hypotenuse and the corresponding side of the other triangle.



$AC = PR$, $AB = PQ$ and $\angle ABC = \angle PQR$, then

$\Delta ABC \cong \Delta PQR$

Right-Hypotenuse-Side Rule (RHS Rule)

Two right triangles are congruent if the hypotenuse and a side of one triangle are equal to hypotenuse and a side of the other triangle respectively.

Given: In two right triangle ΔABC and ΔDEF

$\angle B = \angle E = 90^0$

Hypotenuse AC = Hypotenuse DF

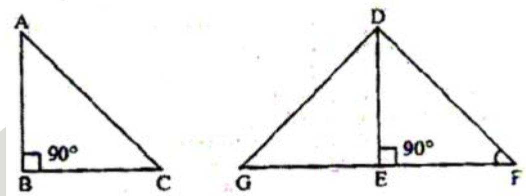
And side AB = Side DE

To prove: $\Delta ABC \cong \Delta DEF$

Construction : Produce FE to G such that GE = BC and join G and D.

Proof : In ΔDEF

$\angle DEF = 90^0$ …(1)



Now in ΔABC and ΔDEG

AB = DE (Given)

BC = GE (By construction)

$\angle ABC = \angle DEG = 90^0$ [By (1)]

Therefore, by SAS rule, $\Delta ABC \cong \Delta DEG$

Hence the corresponding sides and angles are equal

$AC = DG$ and $\angle C = \angle G$ …(2)

But given that AC = DF …(3)

From (2) and (3) DG = DF …(4)

So angles opposite to the equal sides (DG = DF) of ΔDGF are equal

Hence $\angle G = \angle F$ …(5)

Therefore, from (2) and (5)

$\angle C = \angle F$ …(6)

Now in ΔABC and ΔDEF

AB = DE (Given)

$\angle C = \angle F$ [from (6)]

And $\angle ABC = \angle DEF = 90^0$ (Given)

Hence by ASA rule $\Delta ABC \cong \Delta DEF$.

Theorem 6 :

If two sides of a triangle are unequal, the large side has the greater angle opposite to it.

Given: In triangle ABC, AB > AC

To prove: $\angle C > \angle B$

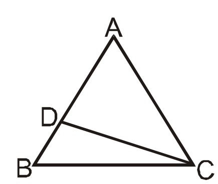
Construction: Draw a line segment CD from vertex C such that AC = AD.

Proof: In ΔACD, AC = AD

Therefore, $\angle ACD = \angle ADC$ ...(1)

But $\angle ADC$ is an exterior angle of ΔBDC

$\therefore$ $\angle ADC > \angle B$ …(2)



From (1) and (2), we have

$\angle ACD > \angle B$ …(3)

By figure, $\angle ACB > \angle ACD$ …(4)

From (3) and (4), we have

$\angle ACB > \angle ACD > \angle B$

$\Rightarrow$ $\angle ACB > \angle B$

$\Rightarrow$ $\angle C > \angle B$

Theorem 7:

In a triangle, the greater angle has a large side opposite to it.

Given: A triangle ABC in which $\angle B > \angle C$

To prove: AC > AB

Proof : We have the following three possibilities for sides AB and AC of ΔABC.

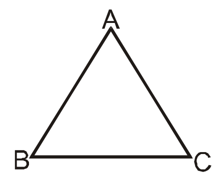
(i) AC = AB (ii) AC < AB and (iii) AC > AB

Case (i) : If AC = AB :

If AC = AB, then opposite angles of equal sides are equal then $\angle B > \angle C$.

Hence $AC \neq AB$.

Case(ii) : If AC < AB :



We know that the larger side has greater angle opposite to it

$\therefore$ AC < AB $\Rightarrow$ $\angle B < \angle C$

Which is also contrary to given ($\angle B > \angle C$)

Hence $AC \leq AB$

Case(iii) : If AC > AB :

We are left only this possibility which must be true.

Hence AC > AB.

SUM AND DIFFERENCE OF TWO SIDES OF TRIANGLE

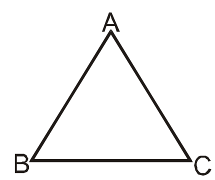
Theorem 6: The sum of two sides of a triangle is always greater than the third.

Thus,

AB + AC > CB

AC + BC > AB

BC + AB > AC



Deduction:

The difference of two sides is always less than the third side.

AB > AC – BC and BC – AC

AC > AB – BC and BC – AB

BC > AC – AB and AB – AC

Theorem 8 :

The sum of any two sides of a triangle is greater than its third side.

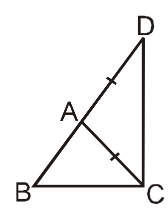
Given: A triangle ABC.

To prove:

(i) AB + BC > AC

BC + AC > AB

AC + AB > BC



Construction: produce BA to D, such that AD = AC and join DC.

Proof : In ΔACD, by construction AD = AC, then opposite angles are equal.

$\therefore$ $\angle ACD = \angle ADC$ …(1)

And $\angle BCD > \angle ACD$ …(2)

From (1) and (2)

$\angle BCD > \angle ADC = \angle BDC$

Therefore, BD > AC [greater angle has a larger opposite side]

$\Rightarrow$ BA + AD > BC [so BD = BA + AD]

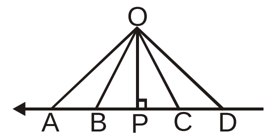
$\Rightarrow$ BA + AC > BC [By construction AD = AC]

Similarly, we may show that

AB + BC > AC

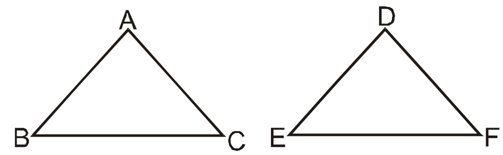
BC + AC > AB

Perpendicular Line Segment is the Shortest : Of all the segments that can be drawn to a given line l from a point O outside the line l, the perpendicular line segment is the shortest. That is :
OP < OA, OB, OC OD … etc.



Proof: Suppose OP $\perp$ l.
Let OC by any line other than OP.
In ΔOPC, $\angle$OPC = 90^0 > $\angle$OCP
OP < OC

Similar Triangles

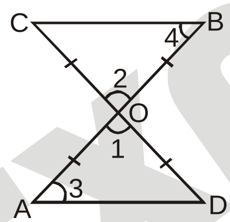


Two triangles are side to be similar. If either of their
(i) Corresponding angles are equal and
(ii) Corresponding sides are proportional
ΔABC ~ ΔDEF, if
$\angle$A = $\angle$D
$\angle$B = $\angle$E and AB / DE = BC / EF = AC / DF
$\angle$C = $\angle$F

Basic Proportionality Theorem : If a line is drawn parallel to one side of a triangle intersecting the other two side. Then it divides the two sides in the same ratio.

SOME SOLVED ILLUSTRATION

Illustration: In figure, O is the mid-point of both AB and CD. Prove that (i) ΔAOD $\cong$ ΔBOC and (ii) AD || BC.

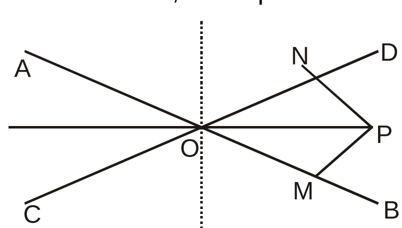


Solution. (i) In ΔAOD $\cong$ ΔBOC

	OA = OB	[O is the midpoint of AB]
	OD = OC	[O is the midpoint of DC]
And	$\angle$1 = $\angle$2	[vertically opposite angles]
Thus ΔAOD $\cong$ ΔBOC		[SAS axiom of congruence]
(ii) from (i), we have		
$\angle$3 = $\angle$4		[cpct]
AD || BC		[a pair of alternate angles are equal]

Illustration: Prove that any point on bisector of an angle is equidistant from the arms of the angles.

Solution: Two straight lines AB and CD intersect at O, P is a point on the bisector of $\angle$BOD.

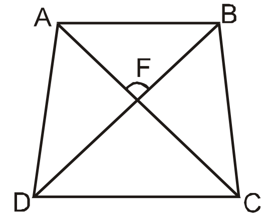


Construction : Draw MP $\perp$ OB and NP $\perp$ OD
In ΔOMP and ΔONP

$\angle$MOP = $\angle$NOP	[P lies on the bisector $\angle$BOD]
$\angle$OPM = $\angle$OPN	[each is 90^0]
OP = OP	[Common]
$\therefore$ ΔOMP $\cong$ ΔONP	[by ASA congruence rule]
MP = NP	[cpct]

Illustration: In fig. AD = BC and BD = CA, Prove that $\angle ADB = \angle BCA$ and $\angle DAB = \angle CBA$.

Solution: In triangles ABD and ABC.



We have :

AD = BC [given]

BD = CA [given]

AB = AB [common]

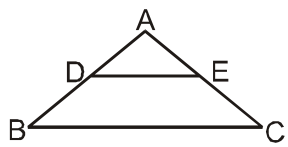
So, by SSS congruence criterion, we have:

$\Delta ABD \cong \Delta CBA$

$\angle ADB = \angle BCA$ [Since, corresponding parts of congruent triangles are equal]

$\angle DAB = \angle CBA$

Illustration: D and E are respectively the points on the sides AB and AC of a ΔABC such that AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm. show that DE || BC.



Solution: We have

AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm

$\therefore$ BD = AB – AD

= (5.6 – 1.4) = 4.2 cm

And EC = AC – AE = (7.2 – 1.8) cm = 5.4 cm

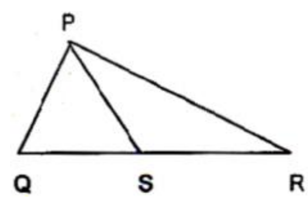
Now, AD / DB = 1.4 / 4.2 = 1 / 3 and AE / EC = 1.8 / 5.4 = 1 / 3

$\Rightarrow$ AD / DB = AE / EC

Thus, DE divides side AB and AC of ΔABC in the same ratio. Therefore, by the converse of basic proportionality. Theorem, we have DE || BC

Illustration: In figure S is any point on the side QR, of ΔPQR. Prove that PQ + QR + RP > 2PS.

Solution: In ΔPQS.



PQ + QS > PS …(i)

Since the sum of any two sides of a Δ is greater than the third side

In ΔPRS. PR + RS > PS …(ii)

Since, The sum of any two sides of a triangle is greater than the third side.

On adding, (i) and (ii),

(PQ + QS) + (PR + RS) > 2PS

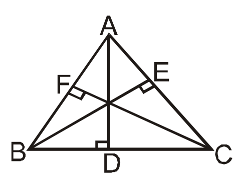
PQ + (QS + RS) + PR > 2PS

PQ + (QS + RS) + PR > 2 PS

PQ + QR + RP > 2PS hence proved.

Illustration: Prove that the sum of three altitudes of a triangle is less than the sum of the three sides of the triangle.

Solution: In ΔABD, $\angle D = 90^0$ and $\angle B$ is acute.



y $\angle D > \angle B$

y AB > AD …(i) [Side opposite to greater angle is longer]

In ΔACD, $\angle D = 90^0$ and $\angle C$ is acute.

$\angle D > \angle C$

$\therefore$ AC > AD …(ii) [Side opposite to greater angel is longer]

Adding (i) and (ii), we have

AB + AC > 2 AD …(iii)

Similarly, we can prove that,

BC + BA > 2BE …(iv) (Since BE $\perp$ AC)

And CA + CB > 2 CF …(v) (Since CF $\perp$ AB)

Adding (iii), (iv) and (v), we get

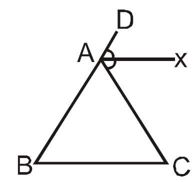
$\Rightarrow$ 2 (AB + BC + CA) > 2(AD + BE + CF)

$\Rightarrow$ AB + BC + CA > AD + BE + CF

$\Rightarrow$ AD + BE + CF < AB + BC + CA

Illustration: If the bisector of the exterior vertical angle of a triangle is parallel to the base, then show that the triangle is isosceles.

Solution.



Given that AX is the bisector of the exterior angle CAD of the vertical angle BAC of ΔABC so that

$\angle$CAX = $\angle$XAD [fig.] …(1)

Also, AX || BC.

To prove that ΔABC is isosceles i.e., AB = AC.

Proof : We have $\angle$BCA = alternate $\angle$CAX …(2)

[so AX || BC and CA is a transversal]

Also, $\angle$XAD = $\angle$CBA …(3)

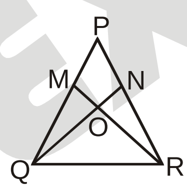
[Corresponding angles with respect to the transversal BA]

From (1), (2) and (3), it follows that

$\angle$BCA = $\angle$CBA

$\therefore$ AB = AC [So Sides opposite to equal angles of a triangle are equal]

Illustration: In fig., it is given that PQ = PR and QN and RM meet at o such that $\angle$MOQ = 2 $\angle$NQR and $\angle$NOR = 2 $\angle$MRQ. Prove that ΔPQN $\cong$ ΔPRM.



Solution: We have

$\angle$MOQ = $\angle$NOR [Vertically opposite angles]

$\Rightarrow$ 2$\angle$NQR = 2$\angle$MRQ [given]

$\Rightarrow$ $\angle$NQR = $\angle$ORQ

$\therefore$ In ΔOQR, we have

$\angle$OQR = $\angle$ORQ …(1)

Now, in ΔPQR, since PQ = PR,

Hence, $\angle$PQR = $\angle$PRQ …(2)

[Angles opposite to equal sides of a triangle are equal]

subtracting (1) and (2), we get

$\angle$PQR - $\angle$OQR = $\angle$PRQ – $\angle$ORQ

$\Rightarrow$ $\angle$PQN = $\angle$PRM

Now, in ΔPQN and ΔPRM, we have

PQ = PR [given]

$\angle$PQN = $\angle$PRM [from (3)]

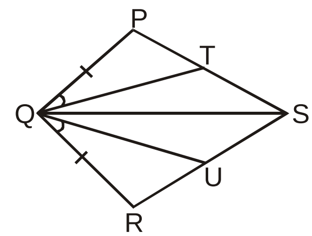
And $\angle$QPR is common to both the triangles.

Hence, by AAS criterion of congruence.

ΔPQN $\cong$ ΔRPM.

Illustration: In Fig., PQRS is a quadrilateral and T and U are respectively points on PS and RS such that PQ = RQ, $\angle PQT = \angle RQU$ and $\angle TQS = \angle UQS$. Prove that QT = QU

Solution: In ΔPQS and ΔRQS, we have



QP = QR.

QS is common to both the triangles and

$\angle PQS = \angle PQT + TQS$

$= \angle RQU + \angle UQS$ [Given]

$= \angle RQS$

Hence, by SAS criterion of congruence, we have

$\Delta PQS \cong \Delta RQS$

$\therefore$ $\angle PSQ = \angle RSQ$ [c.p.c.t.]

$\Rightarrow$ $\angle TSQ = \angle USQ$ …(1)

Now, in ΔQTS and ΔQUS, we have

$\angle TQS = \angle UQS$, [Given]

$\angle TSQ = \angle USQ$ [From (1)]

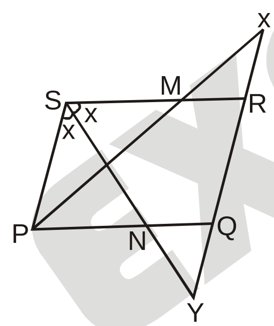
And QS is common to both the triangles.

Hence, by ASA criterion of congruence, we have

$\Delta QTS \cong \Delta QUS$

$\therefore$ QT = QU [c.p.c.t.]

Illustration: In Fig. PQRS is a quadrilateral with its opposite sides parallel and equal. The internal bisectors of $\angle SPQ$ and $\angle PSR$ meet QR (produced if necessary) at X and Y respectively. Prove that ΔPQX is isosceles and that QY = RX.



Solution: Given that PQ || RS, PQ = SR, SP || RQ and SP = RQ,

$\angle SPX = \angle QPX$

[so PX is the internal bisector of $\angle SPQ$]

$\angle RSY = \angle PSY$ [so SY is the internal bisector of $\angle PSR$]

Let PX and SY intersect SR and PQ at M and N respectively.

Now, $\angle SPM = \angle MPN$

= alternate $\angle PMS$ [so SR || PQ and PM is a transversal]

$\therefore$ In ΔPSM, we have

SM = SP [so Sides opposite to equal angles of a triangle are equal]

= RQ [so PQRS is a ||gm] …(1)

Again, $\angle PSN = \angle NSM$ [so SN is the bisector of $\angle PSR$]

= alternate $\angle SNP$ [so PS || QR and SN is a transversal]

$\therefore$ In ΔSPN, we have

PN = SP = QR …(2)

From (1) and (2), we have

SM = PN ...(3)

$\therefore$ MR = SR – SM = PQ – PN = NQ [so SR = PQ and SM = PN] …(4)

Now, $\angle QPM = \angle SMP$ [Alternate angles]

$= \angle XMR$ [Vertically opposite angles]

Also, $\angle SPM = \angle MXR$ [Alternate angles]

But $\angle QPM = \angle SPM$

$\therefore$ $\angle XMR = \angle MXR$

$\therefore$ $MR = RX$ [so sides opposite to equal angles of a Triangle are equal]

similarly, $NQ = QY$

but $MR = NQ$ [From (4)]

$\therefore$ $QY = RX = NQ$ …(5)

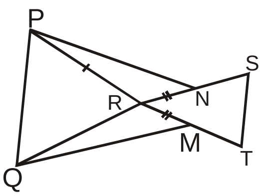
Again, $QX = QR + RX = PN + NQ$ [so from (2), QR = PN and from (5), RX = NQ]

$\therefore$ In ΔPQX, we have

$QX = PQ$.

Hence, ΔPQX is an isosceles triangle with $QX = PQ$ and from (5), we get $QY = RX$.

Illustration: In Fig., PR = QR, SR = TR and M and N are the mid-points of TR and SR respectively of ΔRST. If $\angle PRT = \angle QRS$, prove that QM = PN.



Solution: In ΔPRN and ΔQRM, we have

$PR = QR$ [Given]

$RN = \frac{1}{2} SR$ [so N is the mid-point of SR]

$= \frac{1}{2} TR = RM$ [so SR = TR and M is the mid-point of TR]

AND $\angle PRN = \angle PRT - \angle NRM$

$= \angle QRS - \angle NRM$ [so $\angle PRT = \angle QRS$]

$= \angle QRM$.

hence, by SAS criterion of congruence,

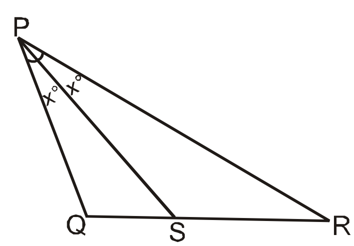
$\Delta PRN \cong \Delta QRM$.

$\therefore$ $PN = QM$ [c.p.c.t.]

Illustration: In Fig., PR > PQ and PS is the bisector of $\angle QPR$. Show that $\angle PSR > \angle PSQ$.

Solution: Since PS is the bisector of $\angle QPR$,

Hence we have $\angle QPS = \angle SPR = x^0$ say.



Now, in ΔPQR, since PR > PQ.

$\therefore$ $\angle Q > \angle R$ …(1)

[so Angle opposite to longer side of a triangle is greater]

Now, in ΔPQS,

$\angle PSR = \angle Q + x^0$ [By exterior angle theorem]

$\Rightarrow$ $\angle Q = \angle PSR - x^0$

Also, in ΔPSR,

$\angle PSQ = \angle R + x^0$ [By exterior angle theorem]

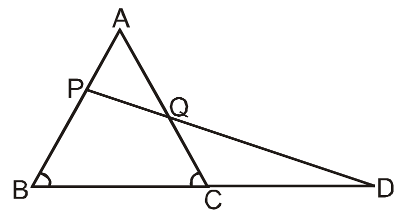
$\Rightarrow$ $\angle R = \angle PSQ - x^0$

Hence, putting the values of $\angle Q$ and $\angle R$ in (1), we get

$\angle PSR - x^0 > \angle PSQ - x^0$

$\Rightarrow$ $\angle PSR > \angle PSQ$.

Illustration: In Fig., if AB = AC, prove that AP < AQ, where PD is a line-segment intersecting AC at Q and BC produced at D.



Solution:

$\therefore$ AB = AC, [given] …(1)

$\therefore$ $\angle$ACB = $\angle$ABC [$\therefore$ angles opposite to equal sides of a triangle are equal]

Now, in ΔQCD, $\angle$QCB in an exterior angle.

$\therefore$ $\angle$QCB > $\angle$CQD …(2)

[An exterior angle of a triangle is greater than either of the interior opposite angles]

But $\angle$CQD = vertically opposite $\angle$AQP …(3)

$\therefore$ $\angle$QCB > $\angle$AQP [Using (2) and (3)]

$\Rightarrow$ $\angle$AQP < $\angle$ACB …(4)

[$\therefore\angle$QCB is a part of $\angle$ACB]

Again, in ΔBPD, $\angle$APQ is an exterior angle.

$\therefore$ $\angle$APQ > $\angle$ABC [An exterior angle of a triangle is greater than either of the interior opposite angles]

$\therefore$ $\angle$APQ > $\angle$ACB [From (1)] …(5)

From (4) and (5), it follows that

$\angle$APQ > $\angle$ABC > $\angle$AQP [$\therefore\angle$ABC = $\angle$ACB]

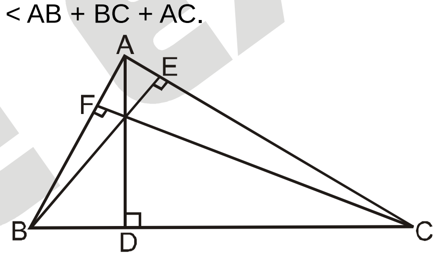
$\Rightarrow$ AQ > AP [$\therefore$ Side opposite to greater angle of a triangle is longer]

Illustration: Show that the sum of the three altitudes of a triangle is less than the sum of the three sides of the triangle [or, prove that the perimeter of a triangle is greater than the sum of is altitudes].

Solution:

Given that AD , BE and CF are the altitudes of ΔABC so that AD $\perp$ BC. BE $\perp$ CA and CF $\perp$ AB [fig.]

To prove that AD + BE + CF < AB + BC + AC.



Proof : In ΔABD, we have

$\angle$ADB = 90^0

$\therefore$ $\angle$AB < AD [$\therefore$side opposite to greater angle of a triangle is longer]

Similarly, AC > AD from ΔACD.

$\therefore$ AB + AC > AD + AD = 2AD. …(1)

Similarly, from ΔACF and ΔBCF,

AC > CF AND BC > CF

$\therefore$ AC + BC > CF + CE = 2CF …(2)

and from ΔABE and ΔBCE, we have

AB > BE AND BC > BE

$\therefore$ AB + BC > BE + BE = 2BE …(3)

Adding (1), (2) and (3), we have

2(AB + AC + BC) > 2(AD + CF + BE)

$\Rightarrow$ AB + BC + AC > AD + CF + BE

$\Rightarrow$ AD + BE + CF < AB + BC + AC

i.e., the sum of three altitudes of a triangles is less than the sum of the three sides of the triangle i.e., the perimeter of the triangle.

Illustration: O is any point in the exterior of ΔPQR, Prove that $OP + OQ + OR > \frac{1}{2}(PQ + QR + RP)$.

Solution: From ΔOPQ [fig.], we have

$OP + OQ > PQ$ …(1)

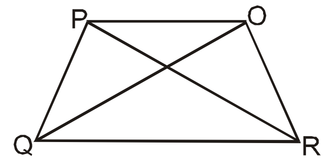
[$\therefore$ Sum of any two sides of a triangle is greater than the third side]

Again, from ΔOQR, $OQ + OR > QR$ …(2)

[For the same reason as stated above]

And from ΔOPR, $OP + OR > RP$

[For the same reason as stated above]



Adding (1), (2) and (3), we get

$$2(OP + OQ + OR) > PQ + QR + RP$$

$$\Rightarrow \quad OP + OQ + OR > \frac{1}{2}(PQ + QR + RP).$$

QUESTION - BANK

FILL IN THE BLANKS

1. If two angles of a triangle are equal and complementary. Then the triangle is _______ and triangle.

2. Difference of any two sides of a triangle is ______________ than the third side.

3. The perimeter of a triangle is ___________ than the sum of its medians.

4. If AB = 9.3 cm and BC = 7.8 cm, then AC must be between ________ and ________.

5. In ΔABC, $\angle A = 60^0$, $\angle B = 42^0$, then greatest side of the ΔABC is __________.

6. In an obtuse angled triangle, the greatest side is always opposite the _______ angles.

7. In ΔABC, AB = 10 cm, BC = 8 cm and CA = 6 cm then the greatest angle of ΔABC is _________ and the smallest angle of ΔABC is __________.

8. Each equal side of an isosceles triangle is _____________ than half the base.

9. Of all the line segments drawn from a point P to a line l not passing through P, the _________ line segment is shortest.

10. In ΔABC, $\angle A > \angle B$ and $\angle B > \angle C$, then smallest side is __________________.

11. If $\angle C$ is right angle in ΔABC, then largest side is ______________.

12. The angles opposite of equal sides of a triangle are ___________.

13. Sum of the angles of a quadrilateral is equal to __________ right angles.

14. Two circles of the same radii are _____________.

15. If two angles and the included side of one triangle are equal to two angles and the included side of the other triangle, the then two triangles are _____________.

16. Angles opposite to equal sides of a triangle are _________.

17. In a triangle, angle opposite to the longer side is _____________.

18. Sum of any two sides of a triangle is greater than the __________ side.

EXERCISE - I

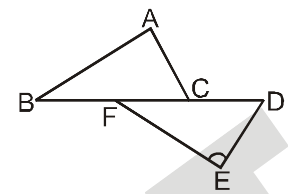
1. If X is a point on the line AB and Y, Z are points outside on the same side of the line AB such that $\angle AXY = 45^0$ and $\angle YXZ = 60^0$ then $\angle BXZ$ is equal to

(A) 120^0 (B) 75^0 (C) 150^0 (D) 105^0

2. In a triangle ABC, $\angle A + \angle B = 144°$ and $\angle A + \angle C = 124°$ $\angle B$ = ?

(A) 56° (B) 60° (C) 65° (D) 45°

3. In the figure, $BA \perp AC$ and $DE \perp EF$ such that BA = DE and BF = DC. Then triangle ΔABC is congruent to ΔEDF by



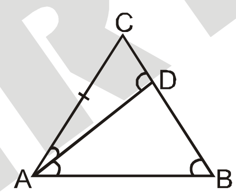
(A) SSS

(B) RHS

(C) ASA

(D) AAS

4. In triangle ABC, AC = CD and $\angle CAB - \angle ABC = 30^0$. Then $\angle BAD$ is



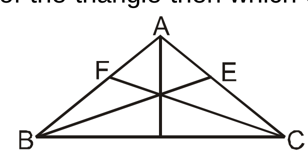
(A) 30^0

(B) 20^0

(C) 15^0

(D) 10^0

5. ABC is an isosceles triangle with AB = AC BE and CF are two medians of the triangle then which of the following is / are true ?



(A) BE = CF (B) AB = AC

(C) $\Delta ABE \cong \Delta ACF$ (D) AE = ½ AC

6. If the perpendicular drawn from the midpoint on one side of a triangle to its other two sides are equal , then triangle is :

(A) Equilateral (B) Isosceles (C) Equiangular (D) Scalene

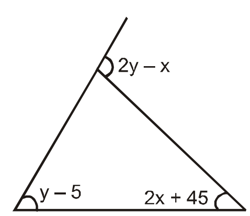
7. In a isosceles triangle AB = AC and BA is produced to D, such that AB = AD then $\angle BCD$ is :

(A) 70^0 (B) 90^0 (C) 60^0 (D) 45^0

8. In ABC, $BD \perp AC$ and $CE \perp AB$. If BD and CE intersect at O, then $\angle BOC$ =

(A) $\angle A$ (B) $90 + \angle A$ (C) $180 + \angle A$ (D) $180 - \angle A$

9. In figure value y, if x = 5° is :



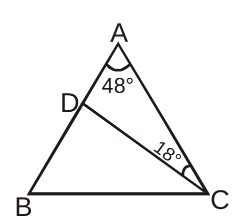
(A) 55°

(B) 60°

(C) 65°

(D) 45°

10. In the figure alongside, AB = AC, $\angle A$ = 48 and $\angle ACD = 18^0$. BC equal to :



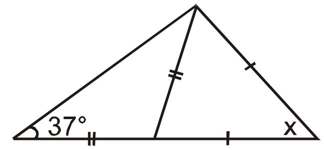
(A) AC

(B) CD

(C) BD

(D) AB

11. x, in figure shown is



(A) 25°

(B) 45°

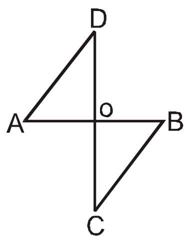
(C) 32°

(D) 38°

MATCH OF THE FOLLOWING

1. **Column - I** **Column - II**

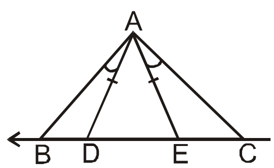
(A)



(p) AB = AC

Given : DO = CO ; AO = BO

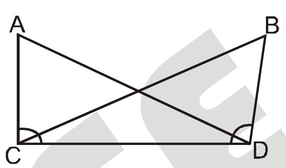
(B)



(q) $\angle C = \angle D$

Given : AD = AE; $\angle BAD = \angle CAE$

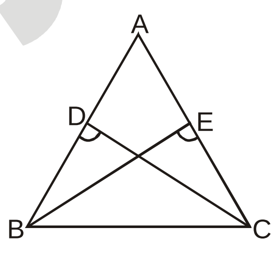
(C)



(r) AD = BC

Given : $\angle BCD = \angle ADC$; $\angle ACB = \angle BDA$

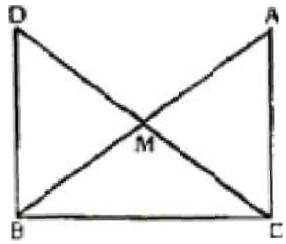
(D)



(s) $\angle B = \angle C$

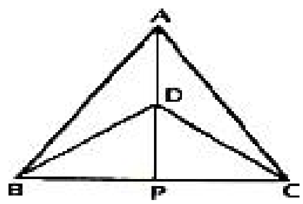
Given : BD = CE; $\angle BDC = \angle CEB = 90^0$

2. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such to DM = CM. Point D is joined to point B (fig.), match them correctly :



Column - I		**Column - II**	
(A)	ΔAMC	(p)	congruent ΔBMD
(B)	$\angle$DBC	(q)	a right angle
(C)	ΔDBC	(r)	congruent ΔACB
(D)	CM	(s)	(1/2) AB

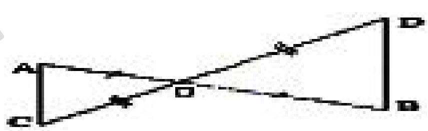
3. ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (fig.) If AD is extended to intersect BC at P. Match them correctly.



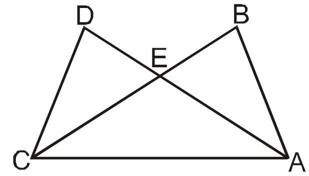
Column - I	**Column - II**
(A) ΔABD	(p) congruent ΔACD
(B) ΔAPB	(q) congruent ΔACP
(C) AP bisects	(r) $\angle$A
(D) AP is the perpendicular bisector	(s) BC

EXERCISE-II

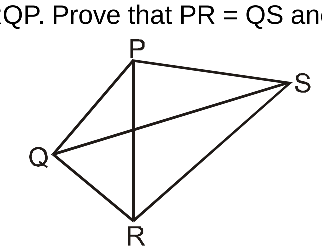
1. In the Fig., O is the mid-point of AB and CD. Prove that AC = BD and AC||BD.



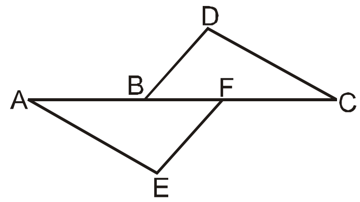
2. In Fig., it is given that AB = CD and AD = BC. Prove that $\Delta ADC \cong \Delta CBA$.



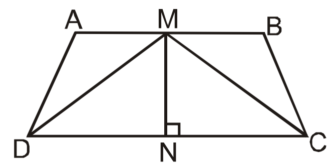
3. In Fig., PS = QR and $\angle SPQ = \angle RQP$. Prove that PR = QS and $\angle QPR = \angle PQS$



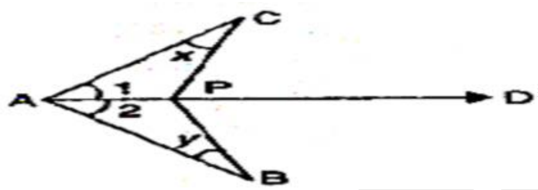
4. In Fig., it is given that AB = CF, EF = BD and $\angle$AFE = $\angle$DBC.
Prove that ΔAFE $\cong$ ΔCBD.



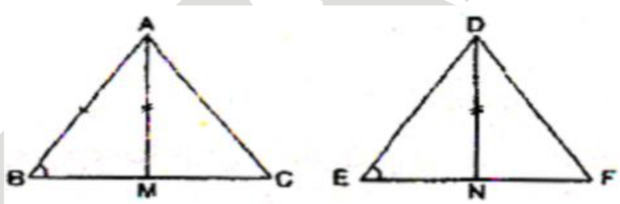
5. In Fig., the line segment joining the mid-points M and N of opposite sides AB and DC of quadrilateral ABCD is perpendicular to both these sides. Prove that the other sides of the quadrilateral are equal.



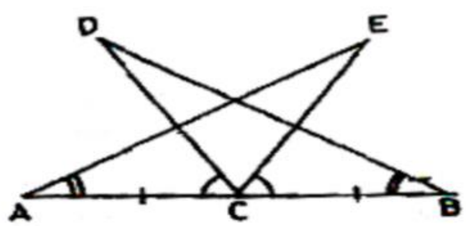
6. In Fig., $\angle$CPD =$\angle$BPD and AD is the bisector of $\angle$BAC. Prove that ΔCAP $\cong$ ΔBAP and hence CP = BP.



7. In Fig., two sides AB and BC and the median AM of ΔABC are respectively equal to sides DE and EF and the median DN of ΔDEF. Prove that ΔABC $\cong$ ΔDEF.

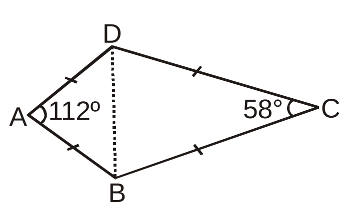


8. In Fig., AC = BC, $\angle$DCA = $\angle$ECB and $\angle$DBC = $\angle$EAC. Prove that ΔDBC and ΔEAC are congruent and hence DC = EC.

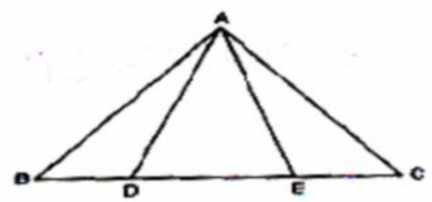


9. M is the mid-point of the hypotenuse AB a right triangle ABC. Prove that
CM = ½ AB.

10. Find $\angle$ADC in Fig.

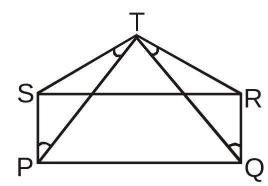


11. In Fig., AB = AC and BD = CE. Prove that AD = AE.



12. In isosceles ΔABC, $AB = AC$. Prove that the perpendicular from the vertices B and C to their opposite sides are equal.

13. In Fig., PQRS is a square and SRT is an equilateral triangle. Prove that

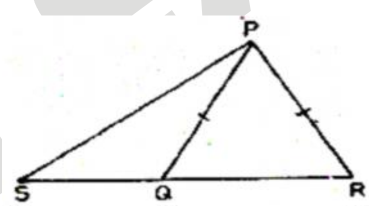


(i) PT = QT (ii) $\angle TQR = 15^0$.

14. E is a point on the side BC of a ΔABC, such that the perpendiculars from E on the sides AB and AC are equal. Show that (i) AE bisects $\angle BAC$ (ii) AC = AB.

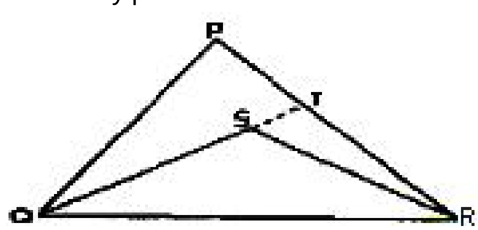
15. ΔABC is an isosceles triangle with AB = AC. Side BA is produced to D such that AB = AD. Prove that $\angle BCD$ is a right angle.

16. In the Given Fig. PQ = PR. Show that PS > PQ.

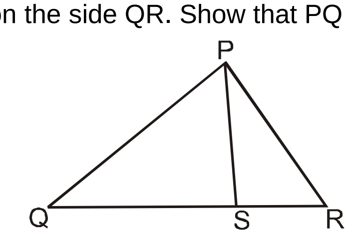


17. ABCD is a parallelogram. If the two diagonals are equal, find the measure of $\angle ABC$.

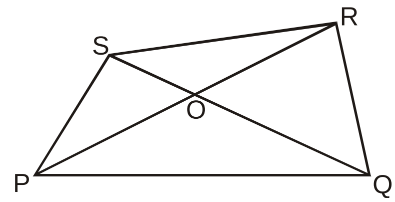
18. In Fig., PQR is a triangle and S is any point in its interior. Show that SQ + SR < PQ + PR.



19. In ΔPQR in Fig., S is any point on the side QR. Show that PQ + QR + RP > 2PS.

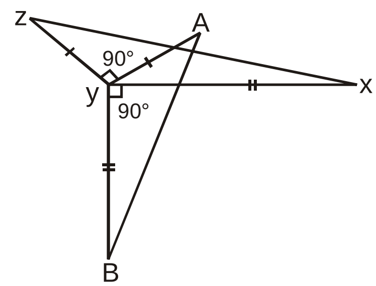


20. In Fig., PQRS is a quadrilateral in which diagonal PR and QS intersect in O.

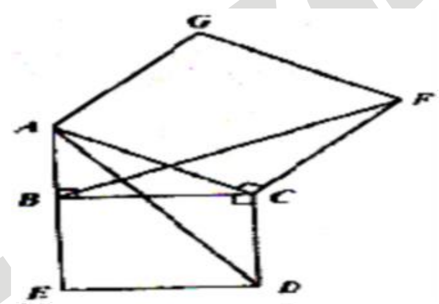


Show that : (i) PQ + QR + RS + SP > PR + QS (ii) PQ + QR + RS + SP < 2(PR + QS).

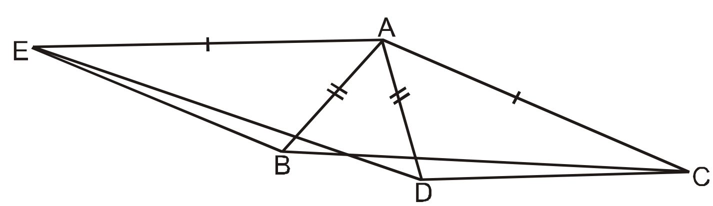
21. In Fig., AY ⊥ ZY and BY ⊥ XY such that AY=ZY, BY = XY, then prove that ΔABY ≅ ΔZXY.



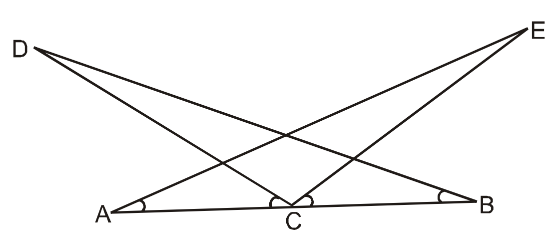
22. In fig., ABC is a triangle, right-angled at B. If BCDE is a square on side BC and ACFG is a square on AC, prove that AD = BF.



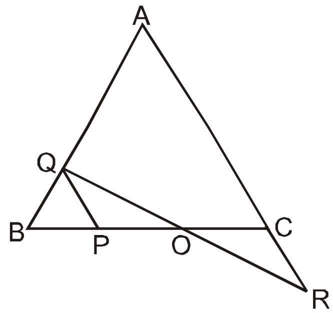
23. In Fig., AC = AE, AB = AD and ∠BAE = ∠DAC. Prove that BC = DE.



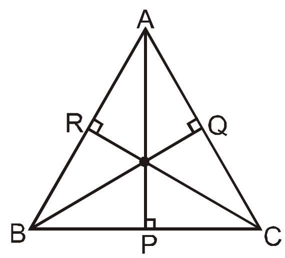
24. In Fig., AC = BC, ∠DCA = ∠ECB and ∠DBC = ∠EAC. Prove that ΔDBC ≅ ΔEAC and hence DC = EC.



25. In Fig., ΔABC is an equilateral triangle, PQ||AC and AC is produced to R such that CR = PQ. Prove that QR bisects PC.



26. The altitude of the given triangle are AP = 3, BQ = 4 and CR = 5. What is the ratio of BC : AC : AB ?



(A) 20 : 15 : 12 (B) 20 : 12 : 15 (C) 15 : 12 : 20 (D) 3 : 4 : 5

27. In a ΔABC, AB = AC = 2.5 cm, BC = 4 cm. Find its height from A to the opposite base :
(A) 1.5 cm (B) 1 cm (C) 2 cm (D) 3 cm

28. The number of isosceles triangles with integer sides such that no side is greater than 4 units is :
(A) 8 (B) 9 (C) 16 (D) 12

29. Three line segments in a plane have lengths a, b and c. No two of these are parallel. If $\sqrt{a}+\sqrt{b}=\sqrt{c}$, then these three line segments.
(A) can form an acute angled triangle (B) can form a right angled triangle.
(C) can form an isosceles triangle (D) cannot form a traingle.

30. In ΔABC, D is a point on side AC such that $\angle ABD = 1/2\ \angle ABC$. If AB = 36, BC = 48, CD = 28, then the length DA will be :
(A) 20 (B) 21 (C) 22 (D) 24

31. In a triangle ABC, AB = AC = 37. Let D be a point on BC such that BD = 7, AD = 33. The length of CD is :
(A) 7 (B) 11 (C) 40 (D) not determinate

32. Let ABC be a triangle with $\angle B = 90°$. Let AD be the bisector of $\angle A$ with D on BC. Suppose AC = 6 cm and the area of the triangle ADC is 10 cm^2. Then the length of BD in cm is equal to :

(A) $\frac{3}{5}$ (B) $\frac{3}{10}$ (C) $\frac{5}{3}$ (D) $\frac{10}{3}$

ANSWER KEY

QUESTION BANK

Fill in the blanks

1.	isosceles, right	**2.**	less	**3.**	greater	**4.**	1.5 cm, 17.1 cm		
5.	AB	**6.**	Obtuse	**7.**	$\angle C$, $\angle B$	**8.**	greater	**9.**	$\perp$
10.	AB	**11.**	AB	**12.**	Equal	**13.**	4	**14.**	Congruent
15.	Congruent	**16.**	equal	**17.**	larger	**18.**	third		

EXERCISE-I

1.	B	**2.**	A	**3.**	B	**4.**	C		
5.	A, B, C, D	**6.**	B	**7.**	B	**8.**	D	**9.**	A
10.	B	**11.**	C						

MATCH THE FOLLOWING

1. A – (q, r), B – (p, s), C – (q, r), D – (p, s)

2. A – (p), B – (q), C – (r), D – (s)

3. A – (p), B – (q), C – (r), D – (s)

EXERCISE-II

10.	95°	**17.**	90°	**26.**	A	**27.**	A	**28.**	D	**29.**	D		
30.	B	**31.**	C	**32.**	D								