NUMBER SYSTEM

CLASSIFICATION OF NUMBERS

(I) Natural numbers:

Set of all non-fractional number from 1 to + ∞ , N = {1,2,3,4,....}.

(II) Whole numbers :

Set of numbers from 0 to + ∞ , W = {0,1,2,3,4,.....}.

(III) Integers :

Set of all-non fractional numbers from ∞ to + ∞ , I or Z = (...., -3,-2,-1,0,1,2,3,....}.

(IV) Rational numbers :

These are real numbers which can be expressed in the form of p/q, where **p** and **q** are integers and q $\neq$ 0.

e.g. 2/3, 37/15, -17/19.

- ❖ All natural numbers, whole numbers and integers are rational.
- ❖ Rational numbers include all Integers (without any decimal part to it), terminating fractions (fractions in which the decimal parts terminating e.g. 0.75, - 0.02 etc.) and also non-terminating but recurring decimals e.g. 0.666...., -2.333....., etc.

Fractions :

(a) Common fraction : Fractions whose denominator is not 10.

(b) Decimal fraction : Fractions whose denominator is 10 or any power of 10.

(c) Proper fraction : Numerator < Denominator i.e. $\frac{3}{5}$.

(d) Improper fraction : Numerator > Denominator i.e. $\frac{5}{3}$.

(e) Mixed fraction : Consists of integral as well as fractional part i.e. $3\frac{2}{7}$.

(f) Compound fraction : Fraction whose numerator and denominator themselves are fractions. i.e. $\frac{2/3}{5/7}$.

- ❖ Improper fraction can be written in the form of mixed fractions.

(v) Irrational Numbers :

All real number which are not rational are irrational numbers. These are non-recurring as well as non-terminating type of decimal numbers e.g. $\sqrt{2}, \sqrt[3]{4}, 2+\sqrt{3}, \sqrt{2+\sqrt{3}}, \sqrt[4]{\sqrt[7]{3}}$ etc.

(vi) Real numbers : Number which can represent actual physical quantities in a meaningful way are known as **real numbers.** These can be represented on the number line. Number line in geometrical straight line with arbitrarily defined zero (origin).

(vii) Prime number : All natural numbers that have one and itself only as their factors are called prime numbers i.e. prime numbers are exactly divisible by 1 and themselves. e.g. 2,3,5,7,11,13,17,19,23....etc. If P is the set of prime number then **P** = {2,3,5,7....}.

(viii) Composite numbers : All natural number, which are not prime are composite numbers. If C is the set of composite number then C = {4,6,8,9,10,12,.....}.

❖ 1 is neither prime nor composite number.

(ix) Co-prime numbers : If the H.C.F. of the given numbers (not necessarily prime) is 1 then they are known as co-prime numbers. e.g. 4, 9, are co-prime as H.C.F. of (4, 9) = 1.

❖ Any two consecutive numbers will always be co-prime.

(x) Even Numbers : All integers which are divisible by 2 are called even numbers. Even numbers are denoted by the expression 2n, where n is any integer. So, if E is a set even numbers, then E = {...., -4, -2, 0, 2, 4,....}.

(xi) Odd Numbers: All integers which are not divisible by 2 are called odd numbers. Odd numbers are denoted by the general expression 2n - 1 where n is any integer. If O is a set of odd numbers, then O = {...., -5, -3, -1, 1, 3, 5,....}.

(xii) Imaginary Numbers: All the numbers whose square is negative are called imaginary numbers. e.g. 3i, 4i, i, ... where i = $\sqrt{-1}$.

(xiii) Complex Numbers : The combined form of real and imaginary numbers is known as complex numbers. It is denoted by Z = A + iB where A is real part and B is imaginary part of Z and A, B $\in$ R.

❖ The set of complex number is the super set of all the sets of numbers.

IDENTIFICATION PRIME NUMBER

Step 1 : Find approximate square root of given number.

Step 2 : Divide the given number by prime numbers less than approximate square root of number. If given number is not divisible by any of this prime number then the number is prime otherwise not.

Ex.1 571, is it a prime ?

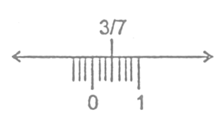
Sol. Approximate square root of 571 = 24.

Prime number < 24 are 2, 3, 5, 7, 11, 13, 17, 19, & 23. But 571 is not divisible by any of these prime numbers so 571 is a prime number.

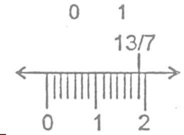
Ex.2 Is 1 prime or composite number ?

Sol. 1 is neither prime nor composite number.

REPRESENTATIO FO RATIONAL NUMBER OF A REAL NUMBER LINE

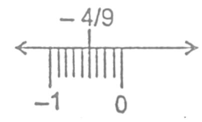


(i) 3/7 Divide a unit into 7 equal parts.



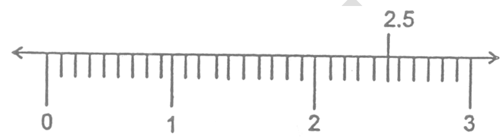
(ii) $\frac{13}{7}$

(iii) $-\frac{4}{9}$

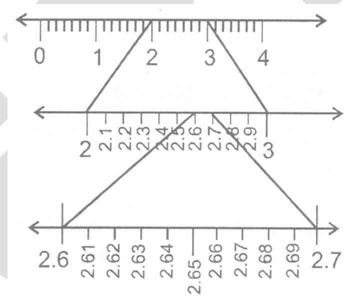


(a) Decimal Number (Terminating) :

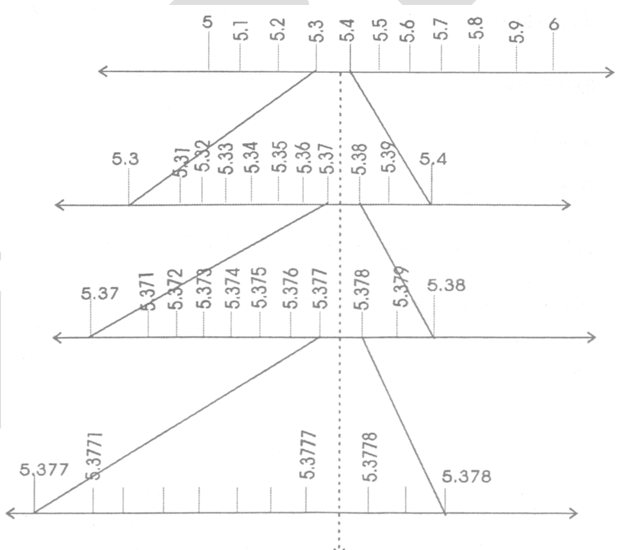
(i) 2.5



(ii) 2.65 (process of magnification)



Ex.3 Visualize the representation of $5.3\overline{7}$ on the number line upto 5 decimal place. i.e. 5.37777.



(b) Find Rational Numbers Between Two Integral Numbers :

Ex.4 Find 4 rational numbers between 2 and 3.

Sol. **Steps :**

(i) Write 2 and 3 multipling in N^r and D^r with (4+1).

(ii) i.e. $2\ \dfrac{2\times(4+1)}{(4+1)}=\dfrac{1}{5}\&3=\dfrac{3\times(4+1)}{(4+1)}=\dfrac{15}{5}$

(iii) So, the four required numbers are $\dfrac{11}{5},\dfrac{12}{5},\dfrac{13}{5},\dfrac{14}{5}$.

Ex.5 Find three rational no's between a and b (a < b).

Sol. $a < b$

$\Rightarrow a + a < b + a$

$\Rightarrow 2a < a + b$

$\Rightarrow a < \frac{a+b}{2}$

Again, $a < b$

$\Rightarrow a + b < b + b.$

$\Rightarrow a + b < 2b$

$\Rightarrow \frac{a+b}{2} < b.$

$\therefore a < \frac{a+b}{2} < b.$

i.e. $\frac{a+b}{2}$ lies between a and b.

Hence 1st rational number between a and b is $\frac{a+b}{2}$.

For next rational number

$$\frac{a+\frac{a+b}{2}}{2} = \frac{\frac{2a+a+b}{2}}{2} = \frac{3a+b}{4} \quad \therefore \quad a < \frac{3a+b}{4} < \frac{a+b}{2} < b.$$

Next, $$\frac{\frac{a+b}{2}+b}{2} = \frac{a+b+2b}{2\times 2} = \frac{a+3b}{4}$$

$$\therefore \quad a < \frac{3a+b}{4} < \frac{a+b}{2} < \frac{a+3b}{4} < b$$, and continues like this.

Ex.6 Find 3 rational numbers between $\frac{1}{3}$ & $\frac{1}{2}$.

Sol. 1st Method $$\frac{\frac{1}{3}+\frac{1}{2}}{2} = \frac{\frac{2+3}{6}}{2} = \frac{5}{12} \quad \therefore \quad \frac{1}{3}, \frac{5}{12}, \frac{1}{2}$$

$$= \frac{\frac{1}{3}+\frac{5}{12}}{2} = \frac{\frac{4+5}{12}}{2} = \frac{9}{24} \quad \therefore \quad \frac{1}{3}, \frac{9}{24}, \frac{5}{12}, \frac{1}{2}$$

$$= \frac{\frac{5}{12}+\frac{1}{2}}{2} = \frac{\frac{5}{12}+\frac{6}{12}}{2} = \frac{11}{24} \quad \therefore \quad \frac{1}{3}, \frac{9}{24}, \frac{5}{12}, \frac{11}{24}, \frac{1}{2}.$$

Verify : $\frac{8}{24} < \frac{9}{24} < \frac{10}{24} < \frac{11}{24} < \frac{12}{24}.\left(\text{as} \frac{8}{24} = \frac{1}{3} \ \& \ \frac{1}{2}\right)$

2nd Method : Find n rational numbers between a and b (a < b).

(i) Find d = $\frac{b-a}{n+1}$.

(ii) 1st rational number will be a + d.

2nd rational number will be a + 2d.
3rd rational number will be a + 3d and so on....
nth rational number is a + nd.

Ex.7 Find 5 rational number between $\frac{3}{5}$ and $\frac{4}{5}$

Here, $a = \frac{3}{5}, b = \frac{4}{5} d = \frac{b-a}{n+1} = \frac{\frac{4}{5} - \frac{3}{5}}{5+1} = \frac{1}{5} \times \frac{1}{6} = \frac{1}{30}$.

$1^{st} = a + b = \frac{3}{5} + \frac{1}{30} = \frac{19}{20}$, $2^{nd} = a + 2d = \frac{3}{5} + \frac{2}{30}$,

$3^{rd} = a + 3d = \frac{3}{5} + \frac{3}{30} = \frac{21}{30}$, $4^{th} = a + 4d = \frac{3}{5} + \frac{4}{30} = \frac{22}{30}$,

$5^{th} = a + 5d = \frac{3}{5} + \frac{5}{30} = \frac{23}{30}$.

RATIONAL NUMBER IN DECIMAL REPRESENTATION

(a) Terminating Decimal :

In this a finite number of digit occurs after decimal i.e. $\frac{1}{2}$ = 0.5, 0.6875, 0.15 etc.

(b) Non-Terminating and Repeating (Recurring Decimal) :

In this a set of digits or a digit is repeated continuously.

Ex.8 $\frac{2}{3} = 0.6666 ------- = 0.\overline{6}$.

Ex.9 $\frac{5}{11} = 0.454545 ------- = 0.\overline{45}$.

PROPERTIES OF RATIONAL NUMBER

If a,b,c are three rational numbers.

(i) Commutative property of addition. a + b = b + a

(ii) Associative property of addition (a+b)+c = a+(b+c)

(iii) Additive inverse a + (-a) = 0

0 is identity element, -a is called additive inverse of a.

(iv) Commutative property of multiplications a.b. = b.a.

(v) Associative property of multiplication (a.b).c = a.(b.c)

(vi) Multiplicative inverse $(a) \times \left(\frac{1}{a}\right) = 1$

1 is called multiplicative identity and $\frac{1}{a}$ is called multiplicative inverse of a or reciprocal of a.

(vii) Distributive property a.(b+c) = a.b + a.c

$\therefore$ L.H.S. $\neq$ R.H.S.

Hence in contradicts our assumption that $2 + \sqrt{3}$ rational.

$\therefore$ $2 + \sqrt{3}$ is irrational.

Ex.9 Prove that $\sqrt{3} - \sqrt{2}$ is an irrational number

Sol. Let $\sqrt{3} - \sqrt{2} = r$ where r be a rational number

Squaring both sides

$\Rightarrow$ $\left(\sqrt{3} - \sqrt{2}\right)^2 = r^2$

$\Rightarrow$ $3 + 2 - 2\sqrt{6} = r^2$

$\Rightarrow$ $5 - 2\sqrt{6} = r^2$

Here, $5 - 2\sqrt{6}$ is an irrational number but r^2 is a rational number

$\therefore$ L.H.S. $\neq$ R.H.S.

Hence it contradicts our assumption that $\sqrt{3} - \sqrt{2}$ is a rational number.

(b) Irrational Number in Decimal Form :

$\sqrt{2} = 1.414213$ ...... i.e. it is not-recurring as well as non-terminating.

$\sqrt{3} = 1.732050807$ ...... i.e. it is non-recurring as well as non-terminating.

Ex.10 Insert an irrational number between 2 and 3.

Sol. $\sqrt{2 \times 3} = \sqrt{6}$

Ex.11 Find two irrational number between 2 and 2.5.

Sol. 1st **Method :** $\sqrt{2 \times 2.5} = \sqrt{5}$

Since there is no rational number whose square is 5. So $\sqrt{5}$ is irrational..

Also $\sqrt{2 \times \sqrt{5}}$ is a irrational number.

2nd Method : 2.101001000100001.... is between 2 and 5 and it is non-recurring as well as non-terminating.

Also, 2.201001000100001......... and so on.

Ex.12 Find two irrational number between $\sqrt{2}$ and $\sqrt{3}$.

Sol. **1st Method :** $\sqrt{\sqrt{2} \times \sqrt{3}} = \sqrt{\sqrt{6}} = \sqrt[4]{6}$

Irrational number between $\sqrt{2}$ and $\sqrt[4]{6}$

$$\sqrt{\sqrt{2}\times\sqrt[4]{6}} = \sqrt[4]{2}\times\sqrt[8]{6}$$

2nd Method : As $\sqrt{2}$ = 1.414213562 ...... and $\sqrt{3}$ = 1.732050808......

As , $\sqrt{3} > \sqrt{2}$ and $\sqrt{2}$ has 4 in the 1st place of decimal while $\sqrt{3}$ has 7 is the 1st place of decimal.

$\therefore$ 1.501001000100001......., 1.601001000100001...... etc. are in between $\sqrt{2}$ and $\sqrt{3}$

Ex.13 Find two irrational number between 0.12 and 0.13.

Sol. 0.1201001000100001......., 0.12101001000100001 ....etc.

Ex.14 Find two irrational number between 0.3030030003..... and 0.3010010001 ....

Sol. 0.302020020002...... 0.302030030003.... etc.

Ex.15 Find two rational number between 0.2323323332..... and 0.252552555255552.......

Sol. 1st place is same 2.

2nd place is 3 & 5.

3rd place is 2 in both.

4th place is 3 & 5.

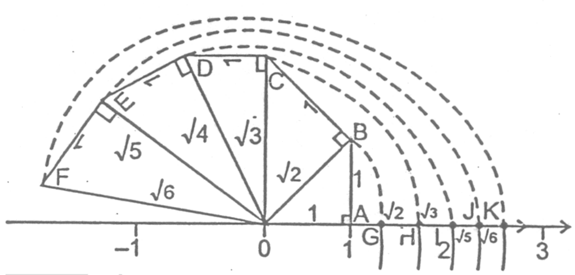
Let a number = 0.25, it falls between the two irrational number.

Also a number = 0.2525 an so on.

(c) Irrational Number on a Number Line :

Ex.16 Plot $\sqrt{2},\sqrt{3},\sqrt{5}\sqrt{6}$ on a number line.

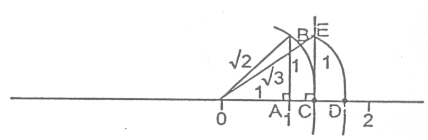
Sol.



Another Method for :

(i) Plot $\sqrt{2},\sqrt{3}$

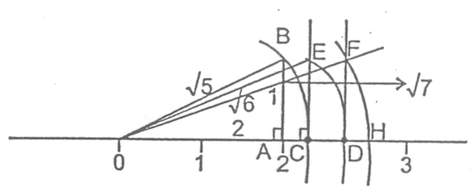
So, OC = $\sqrt{2}$ and OD = $\sqrt{3}$



(ii) Plot $\sqrt{5},\sqrt{6},\sqrt{7}\sqrt{8}$

OC = $\sqrt{5}$

OD = $\sqrt{6}$ OH = $\sqrt{7}$ ....



(d) Properties of Irrational Number :

(i) Negative of an irrational number is an irrational number e.g. $-\sqrt{3}-\sqrt[4]{5}$ are irrational.

(ii) Sum and difference of a rational and an irrational number is always an irrational number.

(iii) Sum and difference of two irrational numbers is either rational or irrational number.

(iv) Product of a non-zero rational number with an irrational number is either rational or irrationals

(v) Product of an irrational with a irrational is not always irrational.

Ex.17 Two number's are 2 and $\sqrt{3}$, then

Sum = 2 + $\sqrt{3}$, is an irrational number.

Difference = 2 - $\sqrt{3}$, is an irrational number.

Also $\sqrt{3}-2$ is an irrational number.

Ex.18 Two number's are 4 and $\sqrt[3]{3}$, then

Sum = 4 + $\sqrt[3]{3}$, is an irrational number.

Difference = 4 - $\sqrt[3]{3}$, is an irrational number.

Ex.19 Two irrational numbers are $\sqrt{3},-\sqrt{3}$, then

Sum = $\sqrt{3}+\left(-\sqrt{3}\right)=0$ which is rational.

Difference = $\sqrt{3}-\left(-\sqrt{3}\right)=2\sqrt{3}$, which is irrational.

Ex.20 Two irrational numbers are 2 + $\sqrt{3}$ and 2 - $\sqrt{3}$, then

Sum = $\left(2+\sqrt{3}\right)+\left(2-\sqrt{3}\right)=4$, a rational number

Two irrational numbers are $\sqrt{3}+3m\sqrt{3}-3$

Difference = $\sqrt{3}+3-\sqrt{3}+3=6$, a rational number

Ex.21 Two irrational numbers are $\sqrt{3}-\sqrt{2},\sqrt{3}+\sqrt{2}$, then

Sum = $\sqrt{3}-\sqrt{2}+\sqrt{3}+\sqrt{2}=2\sqrt{3}$, an irrational

Ex.22 2 is a rational number and $\sqrt{3}$ is an irrational.

2 × $\sqrt{3}=2\sqrt{3}$, an irrational.

Ex.23 0 a rational and $\sqrt{3}$ an irrational.

0 × $\sqrt{3}$ = 0, a rational.

Ex.24 $\frac{4}{3}\times\sqrt{3}=\frac{4}{3}\sqrt{3}=\frac{4}{\sqrt{3}}$ is an irrational.

Ex.25 $\sqrt{3}\times\sqrt{3}=\sqrt{3\times3}=\sqrt{9}=3$ a rational number.

Ex.26 $2\sqrt{3}\times3\sqrt{2}=2\times3\sqrt{3\times2}=6\sqrt{6}$ and irrational number.

Ex.27 $\sqrt[3]{3}\times\sqrt[3]{3^2}=\sqrt[3]{3\times 3^2}=\sqrt[3]{3^3}=3$ a rational number.

Ex.28 $\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)=(2)^2-\left(\sqrt{3}\right)^2=4-3=1$ a rational number.

Ex.29 $\left(2+\sqrt{3}\right)\left(2+\sqrt{3}\right)=\left(2+\sqrt{3}\right)^2$

$$=(2)^2+\left(\sqrt{3}\right)^2+2(2)\times\left(\sqrt{3}\right)$$

$$=4+3+4\sqrt{3}$$

$= 7 + 4\sqrt{3}$ an irrational number

NOTE :

(i) $\sqrt{-2}\neq-\sqrt{2}$, it is not a irrational number.

(ii) $\sqrt{-2}\times\sqrt{-3}\neq\left(\sqrt{-2\times-3}=\sqrt{6}\right)$

Instead $\sqrt{-2},\sqrt{-3}$ are called Imaginary numbers.

$\sqrt{-2}=i\sqrt{2}$, where i (= iota) = $\sqrt{-1}$

$\therefore$ (A) $i^2 = -1$

(B) $i^3 = i^2 \times i = (-1) \times i = -i$

(C) $i^4 = i^2 \times i^2 = (-1) \times (-1) = 1$

(iii) Numbers of the type (a + ib) are called complex numbers where (a, b) $\in$ R.e.g. 2 + 3i, -2 + 4i, -3i, 11 - 4i, are complex numbers.

GEOMETRICAL REPRESENTATION OF REAL NUMBERS

To represent any real number of number line we follows the following steps :

STEP I : Obtain the positive real number x (say).

STEP II : Draw a line and mark a point A on it.

STEP III : Mark a point B on the line such that AB = x units.

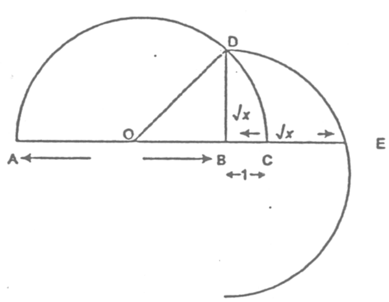
STEP IV : From point B mark a distance of 1 unit and mark the new point as C.

STEP V : Find the mid - point of AC and mark the point as O.

STEP VI : Draw a circle with centre O and radius OC.

STEP VII : Draw a line perpendicular to AC passing through B and intersecting the semi circle at D.

Length BD is equal to $\sqrt{x}$.



4. Examine whether the following numbers are rational or irrational :

(i) $\left(2-\sqrt{3}\right)^2$ (ii) $\left(\sqrt{2}+\sqrt{3}\right)^2$ (iii) $\left(3+\sqrt{2}\right)\left(3-\sqrt{2}\right)$ (iv) $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

5. Represent $\sqrt{8.3}$ on the number line.

6. Represent $(2+\sqrt{3})$ on the number line.

7. Prove that $\left(\sqrt{2}+\sqrt{5}\right)$ is an irrational number.

8. Prove that $\sqrt{7}$ is not a rational number.

9. Prove that $(2+\sqrt{2})$ is an irrational number.

10. Multiply $\sqrt{27a^3b^2c^4} \times \sqrt[3]{128a^7b^9c^2} \times \sqrt[6]{729ab^{12}c^2}$.

11. Express the following in the form of p/q.

(i) $0.\overline{3}$ (ii) $0.\overline{37}$ (iii) $0.\overline{54}$ (iv) $0.\overline{05}$ (v) $1.\overline{3}$ (vi) $0.\overline{621}$

12. Simplify : $0.\overline{4}+.01\overline{8}$

NUMBER SYSTEM

SURDS

Any irrational number of the form $\sqrt[n]{a}$ is given a special name surd. Where 'a' is called radicand, it should always be a rational number. Also the symbol $\sqrt[n]{\ }$ is called the radical sign and the index n is called order of the surd.

$\sqrt[n]{a}$ is read as 'nth **root a'** and can also be written as $a^{\frac{1}{n}}$.

(a) Some Identical Surds :

(i) $\sqrt[3]{4}$ is a surd as radicand is a rational number.

Similar examples $\sqrt[3]{5}, \sqrt[4]{12}, \sqrt[5]{7}, \sqrt{12}$,.........

(i) $2\sqrt{3}$ is a surd (as surd + rational number will give a surd)

Similar examples $\sqrt{3}+1, \sqrt[3]{3}+1$,....

(iii) $\sqrt{7-4\sqrt{3}}$ is a surd as 7 - $4\sqrt{3}$ is a perfect square of $\left(2-\sqrt{3}\right)$

Similar examples $\sqrt{7+4\sqrt{3}}, \sqrt{9-4\sqrt{5}}, \sqrt{9+4\sqrt{5}}$,.........

(i) $\sqrt[3]{\sqrt{3}}$ is a surd as $\sqrt[3]{\sqrt{3}} = \left(3^{\frac{1}{2}}\right)^{\frac{1}{3}} = 3^{\frac{1}{6}} = \sqrt[6]{3}$

Similar examples $\sqrt[3]{\sqrt[3]{5}}, \sqrt[4]{\sqrt[5]{6}}$,....... ...

(b) Some Expression are not Surds :

(i) $\sqrt[3]{\ }8$ because $\sqrt[3]{8} = \sqrt[3]{2^3} = 2$, which is a rational number.

(ii) $\sqrt{2+\sqrt{3}}$ because $2+\sqrt{3}$ is not a perfect square.

(iii) $\sqrt[3]{1+\sqrt{3}}$ because radicand is an irrational number.

LAWS OF SURDS

(i) $\left(\sqrt[n]{a}\right)^n = \sqrt[n]{a^n} = a$

e.g. (A) $\sqrt[3]{8} = \sqrt[3]{2^3} = 2$ (B) $\sqrt[4]{81} = \sqrt[4]{3^4} = 3$

(ii) $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$ [Here order should be same]

e.g. (A) $\sqrt[3]{2} \times \sqrt[3]{6} = \sqrt[3]{2\times 6} = \sqrt[3]{12}$

but, $\sqrt[3]{3} \times \sqrt[4]{6} \neq \sqrt{3\times 6}$ [Because order is not same]

1st make their order same and then you can multiply.

(iii) $\sqrt[n]{a} + \sqrt[n]{b} = \sqrt[n]{\dfrac{a}{b}}$

(iv) $\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a} = \sqrt[m]{\sqrt[n]{a}}$ e.g. = $\sqrt{\sqrt{\sqrt{2}}} = \sqrt[8]{8}$

(v) $\sqrt[n]{a} = \sqrt[n\times p]{a^p}$ [Important for changing order of surds]

or, $\sqrt[n]{a^m} = \sqrt[n\times p]{a^{m\times p}}$

e.g. $\sqrt[3]{6^2}$ make its order 6, then $\sqrt[3]{6^2} = \sqrt[3\times 2]{6^{2\times 2}} = \sqrt[6]{6^4}$.

e.g. $\sqrt[3]{6}$ make its order 15, then $\sqrt[3]{6} = \sqrt[3\times 5]{6^{1\times 5}} = \sqrt[15]{6^5}$.

OPERATION OF SURDS

(a) Addition and Subtraction of Surds :

Addition and subtraction of surds are possible only when order and radicand are same i.e. only for surds.

Ex.1 Simplify

(i) $\sqrt{6} - \sqrt{216} + \sqrt{96} = 15\sqrt{6} - \sqrt{6^2} \times 6 + \sqrt{16\times 6}$ [Bring surd in simples form]

$= 15\sqrt{6} - 6\sqrt{6} + 4\sqrt{6}$

$= (15 - 6 + 4)\sqrt{6}$

$= 13\sqrt{6}$ **Ans.**

(ii) $5\sqrt[3]{250} + 7\sqrt[3]{16} - 14\sqrt[3]{54}$ $= 5\sqrt[3]{125\times 2} + 7\sqrt[3]{8\times 2} - 14\sqrt[3]{27\times 2}$

$= 5\times 5\sqrt[3]{2} + 7\times 2\sqrt[3]{2} - 14\times 3\times \sqrt[3]{2}$

$= (25 + 14 - 42)\sqrt[3]{2}$

$= -3\sqrt[3]{2}$ **Ans.**

(ii) $5\sqrt[3]{250} + 7\sqrt[3]{16} - 14\sqrt[3]{54}$ $= 5\sqrt[3]{125\times 2} + 7\sqrt[3]{8\times 2} - 14\sqrt[3]{27\times 2}$

$= 5\times 5\sqrt[3]{2} + 7\times 2\sqrt[3]{2} - 14\times 3\times\sqrt[3]{2}$

$= (25+14-42)\sqrt[3]{2}$

$= -3\sqrt[3]{2}$ **Ans.**

(iii) $4\sqrt{3} + 3\sqrt{48} - \frac{5}{2}\sqrt{\frac{1}{3}} = 4\sqrt{3} + 3\sqrt{16\times 3} - \frac{5}{2}\sqrt{\frac{1\times 3}{3\times 3}}$

$= 4\sqrt{3} + 3\times 4\sqrt{3} - \frac{5}{2}\times\frac{1}{3}\sqrt{3}$

$= 4\sqrt{3} + 12\sqrt{3} - \frac{5}{6}\sqrt{3}$

$= \left(4+12-\frac{5}{6}\right)\sqrt{3}$

$= \frac{91}{6}\sqrt{3}$ **Ans.**

(b) Multiplication and Division of Surds :

Ex.2 (i) $\sqrt[3]{4}\times\sqrt[3]{22} = \sqrt[3]{4\times 22} = \sqrt[3]{2^3\times 11} = 2\sqrt[3]{11}$

(i) $\sqrt[3]{2}\times\sqrt[4]{3} = \sqrt[12]{2^4}\times\sqrt[12]{3^3} = \sqrt[12]{2^4\times 3^3} = \sqrt[12]{16\times 27} = \sqrt[12]{432}$

Ex.3 Simplify $\sqrt{8a^5b}\times\sqrt[3]{4a^2b^2}$

Hint : $\sqrt[6]{8^3a^{15}b^3}\times\sqrt[6]{4^2a^4b^4} = \sqrt[6]{2^{13}a^{19}b^7} = \sqrt[6]{2ab}$. **Ans.**

Ex.4 Divide $\sqrt{24}\div\sqrt[3]{200} = \frac{\sqrt{24}}{\sqrt[3]{200}} = \frac{\sqrt[6]{(24)^3}}{\sqrt[6]{(200)^2}} = \sqrt[6]{\frac{216}{625}}$ **Ans..**

(c) Comparison of Surds :

It is clear that if x > y > 0 and n > 1 is a positive integer then $\sqrt[n]{x} > \sqrt[n]{y}$.

Ex.5 $\sqrt[3]{16} > \sqrt[3]{12}, \sqrt[5]{35} > \sqrt[5]{25}$ and so on.

Ex.6 Which is greater is each of the following :

(i) $\sqrt[3]{16}$ and $\sqrt[5]{8}$ (ii) $\sqrt{\frac{1}{2}}$ and $\sqrt[3]{\frac{1}{3}}$

L.C.M. of 3 and 5 15. L.C.M. of 2 and 3 is 6.

$$\sqrt[3]{6} = \sqrt[3\times5]{6^5} = \sqrt[15]{7776}$$

$$\sqrt[5]{8} = \sqrt[3\times5]{8^5} = \sqrt[15]{512}$$

$$\therefore \quad \sqrt[75]{7776} > \sqrt[15]{512}$$

$$\Rightarrow \quad \sqrt[3]{6} > \sqrt[5]{8}$$

$$\sqrt[6]{\left(\frac{1}{2}\right)^3} \text{ and } \sqrt[3]{\left(\frac{1}{3}\right)^2}$$

$$\sqrt[6]{\frac{1}{8}} \text{ and } \sqrt[6]{\frac{1}{9}} \qquad \left[\text{As } 8 < 9 \therefore \frac{1}{8} > \frac{1}{9}\right]$$

so, $$\sqrt[6]{\frac{1}{8}} > \sqrt[6]{\frac{1}{9}}$$

$$\Rightarrow \quad \sqrt{\frac{1}{2}} > \sqrt[3]{\frac{1}{3}}$$

Ex.7 Arrange $\sqrt{2}, \sqrt[3]{3}$ and $\sqrt[4]{5}$ is ascending order.

Sol. L.C.M. of 2, 3, 4 is 12.

$\therefore \quad \sqrt{2} = \sqrt[2\times6]{2^6} = \sqrt[12]{64}$

$\sqrt[3]{3} = \sqrt[3\times4]{3^4} = \sqrt[12]{81}$

$\sqrt[4]{5} = \sqrt[4\times3]{5^3} = \sqrt[12]{125}$

As, $64 < 81 < 125$.

$\therefore \quad \sqrt[12]{64} < \sqrt[12]{81} < \sqrt[12]{125}$

$\Rightarrow \quad \sqrt{2} < \sqrt[3]{3} < \sqrt[4]{5}$

Ex.8 Which is greater $\sqrt{7}-\sqrt{3}$ or $\sqrt{5}-1$?

Sol. $$\sqrt{7}-\sqrt{3} = \frac{(\sqrt{7}-\sqrt{3})(\sqrt{7}+\sqrt{3})}{(\sqrt{7}+\sqrt{3})} = \frac{7-3}{\sqrt{7}+\sqrt{3}} = \frac{4}{\sqrt{7}+\sqrt{3}}$$

And, $$\sqrt{5}-1 = \frac{(\sqrt{5}-1)(\sqrt{5}+1)}{(\sqrt{5}+1)} = \frac{5-1}{\sqrt{5}+1} = \frac{4}{\sqrt{5}+1}$$

Now, we know that $\sqrt{7} > \sqrt{5}$ and $\sqrt{3} > 1$, add

So, $\sqrt{7}+\sqrt{3} > \sqrt{5}+1$

$$\Rightarrow \quad \frac{1}{\sqrt{7}+\sqrt{3}} < \frac{1}{\sqrt{5}+1}$$

$$\Rightarrow \quad \frac{4}{\sqrt{7}+\sqrt{3}} < \frac{4}{\sqrt{5}+1}$$

$\Rightarrow \quad \sqrt{7}-\sqrt{3} < \sqrt{5}-1$

So, $\sqrt{5}-1 > \sqrt{7}-\sqrt{3}$

RATIONALIZATION OF SURDS

Rationalizing factor product of two surds is a rational number then each of them is called the rationalizing factor (R.F.) of the other. The process of converting a surd to a rational number by using an appropriate multiplier is known as **rationalization.**

Some examples :

(i) R.F. of $\sqrt{a}$ is $\sqrt{a}$ $\left(\because \sqrt{a}\times\sqrt{a} = a\right)$.

(ii) R.F. of $\sqrt[3]{a}$ is $\sqrt[3]{a^2}$ $\left(\because \sqrt[3]{a} \times \sqrt[3]{a^2} = \sqrt[3]{a^3} = a\right)$.

(iii) R.F. of $\sqrt{a}+\sqrt{b}$ is $\sqrt{a}-\sqrt{b}$ & vice versa $\left[\therefore \left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)=a-b\right]$.

(iv) R.F. of a $+\sqrt{b}$ is $a-\sqrt{b}$ & vice versa $\left[\therefore \left(a+\sqrt{b}\right)\left(a-\sqrt{b}\right)=a^2-b\right]$

(v) R.F. of $\sqrt[3]{a}+\sqrt[3]{b}$ is $\left(\sqrt[3]{a^2}-\sqrt[3]{ab}+\sqrt[3]{b^2}\right)\left[\therefore \left(\sqrt[3]{a}+\sqrt[3]{b}\right)\left(\sqrt[3]{a^2}-\sqrt[3]{ab}+\sqrt[3]{b^2}\right)\right]$

$\left[\therefore \left(\sqrt[3]{a}\right)^3+\left(\sqrt[3]{b}\right)^3=a+b\right]$ which is rational.

(vi) R.F. of $\left(\sqrt{a}+\sqrt{b}+\sqrt{c}\right)is\left(\sqrt{a}+\sqrt{b}-\sqrt{c}\right)nd\left(a+b-c+2\sqrt{ab}\right)$.

Ex.9 Find the R.G. (rationalizing factor) of the following :

(i) $\sqrt{10}$ (ii) $\sqrt{12}$ (iii) $\sqrt{162}$ (iv) $\sqrt[3]{4}$ (v) $\sqrt[3]{16}$ (vi) $\sqrt[4]{162}$ (vii) $2+\sqrt{3}$

(viii) $7-4\sqrt{3}$ (ix) $3\sqrt{3}+2\sqrt{2}$ (x) $\sqrt[3]{3}+\sqrt[3]{2}$ (xi) $1+\sqrt{2}+\sqrt{3}$

(i) $\sqrt{10}$

Sol. $[\therefore \sqrt{10}\times\sqrt{10}=\sqrt{10\times10}=10]$ as 10 is rational number.

$\therefore$ R.F. of $\sqrt{10}$ is $\sqrt{10}$ **Ans.**

(ii). $\sqrt{12}$

Sol. First write it's simplest from i.e. $2\sqrt{3}$.

Now find R.F. (i.e. R.F. of $\sqrt{3}$ is $\sqrt{3}$)

$\therefore$ R.F. of $\sqrt{12}$ is $\sqrt{3}$ **Ans.**

(iii) $\sqrt{162}$

Sol. Simplest from of $\sqrt{162}$ is $9\sqrt{2}$.

R.F. of $\sqrt{2}$ is $\sqrt{2}$.

$\therefore$ R.F. of $\sqrt{162}$ is $\sqrt{2}$ **Ans.**

(iv) $\sqrt[3]{4}$

Sol. $\sqrt[3]{4}\times\sqrt[3]{4^2}=\sqrt[3]{4^3}=4$

$\therefore$ **R.F.** of $\sqrt[3]{4}$ is $\sqrt[3]{4^2}$ **Ans.**

(v). $\sqrt[3]{16}$

Sol. Simplest from of $\sqrt[3]{16}$ is $2\sqrt[3]{2}$

Now R.F. of $\sqrt[3]{2}$ is $\sqrt[3]{2^2}$

$\therefore$ R.F. of $\sqrt[3]{16}$ is $\sqrt[3]{2^2}$ **Ans.**

(vi) $\sqrt[4]{162}$

Sol. Simplest form of $\sqrt[4]{162}$ is $3\sqrt[4]{2}$

Now R.F. of $\sqrt[4]{2}$ is $\sqrt[4]{2^3}$

R.F. of $(\sqrt[4]{162})$ is $\sqrt[4]{2^3}$ **Ans.**

(vii) $2+\sqrt{3}$

Sol. As $\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)=(2)^2-\left(\sqrt{3}\right)^2=4-3=1,$, which is rational.

$\therefore$ R.F. of $(2+\sqrt{3})$ is $(2-\sqrt{3})$ **Ans.**

(viii) $7-4\sqrt{3}$

Sol. As $\left(7-4\sqrt{3}\right)\left(7+4\sqrt{3}\right)=(7)^2-\left(4-\sqrt{3}\right)^2$ = 49 - 48 = 1, which is rational

$\therefore$ R.F. of $(7-4\sqrt{3})$ is $\left(7+4\sqrt{3}\right)$ **Ans.**

(ix). $3\sqrt{3}+2\sqrt{2}$

Sol. As $\left(3\sqrt{3}+2\sqrt{2}\right)\left(3\sqrt{3}-2\sqrt{2}\right)=\left(3\sqrt{3}\right)^2-\left(2\sqrt{2}\right)^2=27-8=19$, which is rational.

$\therefore$ R.F. of $(3\sqrt{3}+2\sqrt{2})$ is $(3\sqrt{3}-2\sqrt{2})$ **Ans.**

(x) $\sqrt[3]{3}+\sqrt[3]{2}$

Sol. As $\left(\sqrt[3]{3}+\sqrt[3]{2}\right)\left(\sqrt[3]{3^2}-\sqrt[3]{3}\times\sqrt[3]{2}+\sqrt[3]{2^2}\right)=\left(\sqrt[3]{3^3}+\sqrt[3]{2^3}\right)$ = 3 + 2 = 5, which is rational.

$\therefore$ R.F. of $(\sqrt[3]{3}+\sqrt[3]{2})$ is $\left(\sqrt[3]{3^2}-\sqrt[3]{3}\times\sqrt[3]{2}+\sqrt[3]{2^2}\right)$ **Ans.**

(xi) $1+\sqrt{2}+\sqrt{3}$

Sol. $\left(1+\sqrt{2}+\sqrt{3}\right)\left(1+\sqrt{2}-\sqrt{3}\right)=\left(1+\sqrt{2}\right)^2-\left(\sqrt{3}\right)^2$

$=1)^2+\left(\sqrt{2}\right)^2+2(1)\left(\sqrt{2}\right)-3$

$=1+2+2\sqrt{2}-3$

$=3+2\sqrt{2}-3$

$=2\sqrt{2}$

$2\sqrt{2}\times\sqrt{2}=2\times2=4$

$\therefore$ R.F. of $1+\sqrt{2}+\sqrt{3}$ is $\left(1+\sqrt{2}-\sqrt{3}\right)$ and $\sqrt{2}$. **Ans.**

NOTE : R.F. of $\sqrt{a}+\sqrt{b}$ or $\sqrt{a}-\sqrt{b}$ type surds are also called conjugate surds & vice versa.

Ex.10 (i) $2-\sqrt{3}$ is conjugate of $2+\sqrt{3}$

(ii) $\sqrt{5}+1$ is conjugate of $\sqrt{5}-1$

NOTE : Sometimes conjugate surds and reciprocals are same.

Ex.11 (i) $2+\sqrt{3}$, it's conjugate is $2-\sqrt{3}$, its reciprocal is $2-\sqrt{3}$ & vice versa.

(ii) $5-2\sqrt{6}$, it's conjugate is $5+2\sqrt{6}$, its reciprocal is $5-2\sqrt{6}$ & vice versa.

(iii) $6-\sqrt{35}, 6+\sqrt{35}$

(iv) $7-4\sqrt{3}, 7+4\sqrt{3}$

(v) $8+3\sqrt{7}, 8-3\sqrt{7}$ .... and so on.

Ex.12 Express the following surd with a rational denominator.

Sol.
$$\frac{8}{\sqrt{15}+1-\sqrt{5}-\sqrt{3}} = \frac{8}{\left[\left(\sqrt{15}+1\right)-\left(\sqrt{15}+\sqrt{3}\right)\right]} \times \left[\frac{\left(\sqrt{15}+1\right)+\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{15}+1\right)+\left(\sqrt{5}+\sqrt{3}\right)}\right]$$

$$= \frac{8\left(\sqrt{15}+1+\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{15}+1\right)^2-\left(\sqrt{5}+\sqrt{3}\right)^2}$$

$$= \frac{8\left(\sqrt{15}+1+\sqrt{5}+\sqrt{3}\right)}{15+1+2\sqrt{15}-\left(5+3+2\sqrt{15}\right)}$$

$$= \frac{8\left(\sqrt{15}+1+\sqrt{5}+\sqrt{3}\right)}{8}$$

$$= (\sqrt{15}+1+\sqrt{5}+\sqrt{3})$$ **Ans.**

Ex.13 Rationalize the denominator of $\dfrac{a^2}{\sqrt{a^2+b^2}+b}$

Sol.
$$\frac{a^2}{\sqrt{a^2+b^2}+b} = \frac{a^2}{\sqrt{a^2+b^2}+b} \times \frac{\sqrt{a^2+b^2}-b}{a^2+b^2-b}$$

$$= \frac{a^2\left(\sqrt{a^2+b^2}-b\right)}{\left(\sqrt{a^2+b^2}\right)^2-(b)^2}$$

$$=\frac{a^2\left(\sqrt{a^2+b^2}-b\right)}{a^2+b^2-b^2}=\left(\sqrt{a^2+b^2}-b\right)$$ **Ans.**

Ex.14 If $\frac{3+2\sqrt{2}}{3-\sqrt{2}}=a+b\sqrt{2}$, where a and b are rational then find the values of a and b.

Sol. L.H.S. $\frac{3+2\sqrt{2}}{3-\sqrt{2}}=\frac{(3+2\sqrt{2})(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})}$

$$=\frac{9+3\sqrt{2}+6\sqrt{2}+4}{9-2}$$

$$=\frac{13+9\sqrt{2}}{7}$$

$$=\frac{13}{7}+\frac{9}{7}\sqrt{2}$$

$$\therefore \quad \frac{13}{7}+\frac{9}{7}\sqrt{2}=a+b\sqrt{2}$$

Equating the rational and irrational parts

We get $a=\frac{13}{7}, b=\frac{9}{7}$ **Ans.**

Ex.15 If $\sqrt{3}=1.732$, find the value of $\frac{1}{\sqrt{3}-1}$

Sol. $\frac{1}{\sqrt{3}-1}=\frac{1}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$

$$=\frac{\sqrt{3}+1}{3-1}$$

$$=\frac{\sqrt{3}+1}{2}$$

$$=\frac{1.732+1}{2}$$

$$=\frac{2.732}{2}$$

$$=1.366$$ **Ans.**

Ex.16 If $\sqrt{5}$ = 2.236 and $\sqrt{2}$ = 1.414, then

Evaluate : $\frac{3}{\sqrt{5}+\sqrt{2}}+\frac{4}{\sqrt{5}-\sqrt{2}}$

Sol. $\frac{3}{\sqrt{5}+\sqrt{2}}+\frac{4}{\sqrt{5}-\sqrt{2}}=\frac{3\sqrt{5}-\sqrt{2}+4(\sqrt{5}+\sqrt{2})}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})}$

$=\frac{3\sqrt{5}-3\sqrt{2}+4\sqrt{5}+4\sqrt{2}}{5-2}$

$=\frac{7\sqrt{5}+\sqrt{2}}{5-2}$

$=\frac{7\sqrt{5}+\sqrt{2}}{3}$

$=\frac{7\times 2.236+1.414}{3}$

$=\frac{15.652+1.414}{3}$

$=\frac{17.066}{3}$

= 5.689 (approximate)

Ex.17 If $=\frac{1}{2+\sqrt{3}}$ find the value of $x^3 - x^2 - 11x + 3$.

Sol. As, $x=\frac{1}{2+\sqrt{3}}=2-\sqrt{3}$

$\Rightarrow$ $x - 2 = -\sqrt{3}$

$\Rightarrow$ $(x - 2)^2 = \left(-\sqrt{3}\right)^2$ [By squaring both sides]

$\Rightarrow$ $x^2 + 4 - 4x = 3$

$\Rightarrow$ $x^2 - 4x + 1 = 0$

Now, $x^3 - x^2 - 11x + 3 = x^3 - 4x^2 + x + 3x^2 - 12x + 3$

$= x(x^2 - 4x + 1) + 3(x^2 - 4x + 1)$

$= x(0) + 3(0)$

$= 0 + 0 = 0$ **Ans.**

Ex.18 If $x = 3 - \sqrt{8}$, find the value of $x^3+\frac{1}{x^3}$.

Sol. $x = 3 - \sqrt{8}$

$\therefore$ $\frac{1}{x}=\frac{1}{3-\sqrt{8}}$

$\Rightarrow$ $\frac{1}{x}=3+\sqrt{8}$

Now, $x+\frac{1}{x}=3-\sqrt{8}+3+\sqrt{8}=6$

$\Rightarrow \quad x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3x\frac{1}{x}\left(x+\frac{1}{x}\right)$

$\Rightarrow \quad x^3+\frac{1}{x^3}=(6)^3-3(6)$

$\Rightarrow \quad x^3+\frac{1}{x^3}=216-18$

$\Rightarrow \quad x^3+\frac{1}{x^3}=198$ **Ans.**

Ex.19 If $x = 1 + 2^{1/3} + 2^{2/3}$, show that $x^3 - 3x^2 - 3x - 1 = 0$

Sol. $x = 1 + 2^{1/3} + 2^{2/3}$

$\Rightarrow$ $x - 1 (2^{1/3} + 2^{2/3})$

$\Rightarrow$ $(x - 1)^3 = (2^{1/3} + 2^{2/3})^3$

$\Rightarrow$ $(x - 1)^3 = (2^{1/3}) + (2^{2/3})^3 + 3.2^{1/3}.2^{2/3}(2^{1/3} + 2^{1/3-})$

$\Rightarrow$ $(x - 1)^3 = 2 + 2^2 + 3.2^1 (x - 1)$

$\Rightarrow$ $(x - 1)^3 = 6 + 6 (x - 1)$

$\Rightarrow$ $x^3 - 3x^2 + 3x - 1 = 6x$

$\Rightarrow$ $x^3 - 3x^2 - 3x - 1 = 0$ **Ans.**

Ex.20 Solve : $\sqrt{x+3}+\sqrt{x-2}=5$.

Sol. $\sqrt{x+3}=5-\sqrt{x-2}$

$\Rightarrow$ $\left(\sqrt{x+3}\right)^2=\left(5-\sqrt{x-2}\right)^2$ [By squaring both sides]

$\Rightarrow$ $x + 3 = 25 + (x - 2) - 10\sqrt{x-2}$

$\Rightarrow$ $x + 3 = 25 + x - 2 - 10\sqrt{x-2}$

$\Rightarrow$ $3 - 23 = - 10\sqrt{x-2}$

$\Rightarrow$ $-20 = -10\sqrt{x-2}$

$\Rightarrow$ $2 = \sqrt{x-2}$

$\Rightarrow$ $x - 2 = 4$ [By squaring both sides]

$\Rightarrow$ $x = 6$ **Ans.**

Ex.21 If $x = 1 + \sqrt{2}+\sqrt{3}$, prove that $x^4 - 4x^3 - 4x^2 + 16 - 8 = 0$.

Hint : $x = 1 + \sqrt{2}+\sqrt{3}$

$\Rightarrow$ $x - 1 = \sqrt{2}+\sqrt{3}$

$\Rightarrow$ $(x-1)^2=\left(\sqrt{2}+\sqrt{3}\right)^2$ [By squaring both sides]

$\Rightarrow$ $x^2 + 1 - 2x = 2 + 3 + 2\sqrt{6}$

$\Rightarrow$ $x^2 - 2x - 4 = 2\sqrt{6}$

$\Rightarrow$ $(x^2 - 2x - 4)^2 = (2\sqrt{6})^2$

$\Rightarrow$ $x^4 + 4x^2 + 16 - 4x^3 + 16x - 8x^2 = 24$

$\Rightarrow$ $x^4 - 4x^3 - 4x^2 + 16x + 16 - 24 = 0$

$\Rightarrow$ $x^4 - 4x^3 - 4x^2 + 16x - 8 = 0$ **Ans.**

EXPONENTS OF REAL NUMBER

(a) Positive Integral Power :

For any real number a and a positive integer 'n' we define a^n as :

$a^n = a \times a \times a \times$ .... x a (n times)

a^n is called then n^{th} power of a. The real number 'a' is called the base and 'n' is called the exponent of the n^{th} power of a.

e.g. $2^3 = 2 \times 2 \times 2 = 8$

NOTE : For any non-zero real number 'a' we define $a^0 = 1$.

e.g. thus, $3^0 = 1$, 5^0, $\left(\frac{3}{4}\right)^0 = 1$ and so on.

(b) Negative Integral Power :

For any non-zero real number 'a' and a positive integer 'n' we define $a^{-n} = \frac{1}{a^n}$

Thus we have defined a^n find all integral values of n, positive, zero or negative. a^n is called the n^{th} power of a.

RATIONAL EXPONENTS OR A REAL NUMBER

(a) Principal of n^{th} Root of a Positive Real Numbers :

If 'a' is a positive real number and 'n' is a positive integer, then the principal n^{th} root of a is the unique positive real number x such that $x^n = a$.

The principal n^{th} root of a positive real number a is denoted by $a^{1/n}$ or $\sqrt[n]{a}$.

(b) Principal of n^{th} Root of a Negative Real Numbers :

If 'a' is a negative real number and 'n' is an odd positive integer, then the principle n^{th} root of a is define as $-|a|^{1/n}$ i.e. the principal n^{th} root of -a is negative of the principal n^{th} root of $|a|$.

Remark :

It 'a' is negative real number and 'n' is an even positive integer, then the principle n^{th} root of a is not defined, because an even power of real number is always positive. Therefore $(-9)^{1/2}$ is a meaningless quantity, if we confine ourselves to the set of real number, only.

(c) Rational Power (Exponents) :

For any positive real number 'a' and a rational number $\frac{p}{q} \neq$ where $q \neq 0$, we define $a^{p/q} = (a^p)^{1/q}$ i.e. $a^{p/q}$ is the principle q^{th} root of a^p.

LAWS OF RATIONAL EXPONETNS

The following laws hold the rational exponents

(i) $a^m \times a^n = a^{m+1}$

(ii) $a^m \div a^n = a^{m-n}$

(ii) $(a^m)^n = a^{mn}$

(iv) $a^{-n} = \frac{1}{a^n}$

(v) $a^{m/n} = (a^m)^{1/n} = (a^{1/n})^m$ i.e. $a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

(vi) $(ab)^m = a^m b^m$

(vii) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

(viii) $a^{bn} = a^{b+b+b \ldots . n \text{ tmes}}$

Where a,b are positive real number and m,n are relational numbers.

ILLUSTRATIONS :

Ex.22 Evaluate each of the following:

(i) $5^2 \times 5^4$ (ii) $5^8 \div 5^3$ (iii) $\left(3^2\right)^2$ (iv) $\left(\frac{11}{12}\right)^3$ (v) $\left(\frac{3}{4}\right)^{-3}$

Sol. Using the laws of indices, we have

(i) $5^2.5^4 = 5^{2+4} = 5^6 = 15625$ $\qquad \because a^m \times a^n = a^{m+n}$

(ii) $5^8 \div 5^3 = \dfrac{5^8}{5^3} = 5^{8-3} = 5^5 = 3125$ $\qquad \because a^m + a^n = a^{m-n}$

(iii) $\left(3^2\right)^3 = 3^{2\times 3} = 3^6 = 729$ $\qquad \because (a^m)^n = a^{m\times n}$

(iv) $\left(\dfrac{11}{12}\right)^3 = \dfrac{11^3}{12^3} = \dfrac{1331}{1728}$ $\qquad \because \left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

(v) $\left(\dfrac{3}{4}\right)^{-3} = \dfrac{1}{\left(\dfrac{3}{4}\right)^3} = \dfrac{1}{\dfrac{3^3}{4^3}} = \dfrac{1}{\dfrac{27}{64}} = \dfrac{64}{27}$ $\qquad \because a^{-n} = \dfrac{1}{a^n}$

Ex.23 Evaluate each of the following :

(i) $\left(\dfrac{2}{11}\right)^4 \times \left(\dfrac{11}{3}\right)^2 \times \left(\dfrac{3}{2}\right)^3$ (ii) $\left(\dfrac{1}{2}\right)^5 \times \left(\dfrac{-2}{3}\right)^4 \times \left(\dfrac{3}{5}\right)^{-1}$

(iii) $2^{55} \times 2^{60} - 2^{97} \times 2^{18}$ (iv) $\left(\dfrac{2}{3}\right)^3 \times \left(\dfrac{2}{5}\right)^{-3} \times \left(\dfrac{3}{5}\right)^2$

Sol. We have.

(i) $\left(\dfrac{2}{11}\right)^4 \times \left(\dfrac{11}{3}\right)^2 \times \left(\dfrac{3}{2}\right)^3 = \dfrac{2^4}{11^4} \times \dfrac{11^2}{3^2} \times \dfrac{3^3}{2^3}$

$= \dfrac{2\times 3}{11^2}$

$= \dfrac{6}{121}$ **Ans.**

(ii) We have,

$$\left(\frac{1}{2}\right)^5 \times \left(\frac{-2}{3}\right)^4 \times \left(\frac{3}{5}\right)^{-1} = \left(\frac{1}{2}\right)^5 \times \left(\frac{-2}{3}\right)^4 \times \left(\frac{1}{\frac{3}{5}}\right)$$

$$= \frac{1^5}{2^5} \times \frac{(-2)^2}{3^4} \times \frac{5}{3}$$

$$= \frac{1\times 16\times 5}{32\times 81\times 3}$$

$$= \frac{5}{2\times 81\times 3}$$

$= \frac{5}{486}$ **Ans.**

(iii) We have,

$$2^{55}\times 2^{60} - 2^{97}\times 2^{18} =^{55+60} -2^{97+18}$$

$$= 2^{15} - 2^{115}$$

$$= 0$$ **Ans.**

(iv) We have,

$$\left(\frac{2}{3}\right)^3 \times \left(\frac{2}{5}\right)^{-3} \times \left(\frac{3}{5}\right)^{-2} = \frac{2^3}{3^3}\times\frac{1}{(2/5)}\times\frac{3^2}{5^2}$$

$$= \frac{2^3}{3^3}\times\frac{1}{2^3/5^3}\times\frac{3^2}{5^2}$$

$$= \frac{2^3\times 5^3\times 3^2}{3^3\times 2^3\times 5^2}$$

$$= \frac{5}{3}$$ **Ans.**

Ex.24 Simplify :

(i) $\dfrac{(25)^{3/2}\times(243)^{3/5}}{(16)^{5/4}\times(8)^{4/3}}$ (ii) $\dfrac{16\times 2^{n+1} - 4\times 2^n}{16\times 2^{n+2} - 2\times 2^{n+2}}$

Sol. We have,

(i) $$\frac{(25)^{3/2}\times(243)^{3/5}}{(16)^{5/4}\times(8)^{4/3}} = \frac{\left(5^2\right)^{3/2}\times\left(3^5\right)^{3/5}}{\left(2^4\right)^{5/4}\times\left(2^3\right)^{4/3}}$$

$$= \frac{5^{2\times 3/2}\times 3^{5\times 3/5}}{2^{4\times 5/4}\times 2^{3\times 4/3}}$$

$$= \frac{5^3\times 3^3}{2^5\times 2^4}$$

$$= \frac{125\times 27}{32\times 16}$$

$$= \frac{3375}{512}$$ **Ans.**

(ii) $$\frac{16\times 2^{n+1} - 4\times 2^n}{16\times 2^{n+2} - 2\times 2^{n+2}} = \frac{2^4\times 2^{n+1} - 2^2\times 2^n}{2^4\times 2^{n+2} - 2\times 2^{n+2}}$$

$$= \frac{2^{n+5} - 2^{n+2}}{2^{n+6} - 2^{n+3}}$$

$$= \frac{2^{n+5} - 2^{n+2}}{2.2^{n+5} - 2.2^{n+2}}$$

$$= \frac{2^{n+5} - 2^{n+2}}{2(2^{n+5} - 2^{n+2})} = \frac{1}{2}$$ **Ans.**

Ex.25 Simplify $\left(\frac{81}{16}\right)^{-3/4} \times \left[\left(\frac{25}{9}\right)^{-3/2} \div \left(\frac{5}{2}\right)^{-3}\right]$

Sol. We have

$$\left(\frac{81}{16}\right)^{-3/4} \times \left[\left(\frac{25}{9}\right)^{-3/2} \div \left(\frac{5}{2}\right)^{-3}\right] = \left(\frac{3^4}{2^4}\right)^{-3/4} \times \left[\left(\frac{5^2}{3^2}\right)^{-3/2} \div \left(\frac{5}{2}\right)^{-3}\right]$$

$$= \left[\left(\frac{3}{2}\right)^4\right]^{-3/4} \times \left[\left(\frac{5}{3}\right)^2\right]^{-3/2} \div \left[\left(\frac{5}{2}\right)^{-3}\right]$$

$$= \left(\frac{3}{2}\right)^{4x-3/4} \times \left[\left(\frac{5}{3}\right)^{2x-3/2} \div \left(\frac{5}{2}\right)^{-3}\right]$$

$$= \left(\frac{3}{2}\right)^{-3} \times \left[\left(\frac{5}{3}\right)^{-3} \div \left(\frac{5}{2}\right)^{-3}\right]$$

$$= \left(\frac{2}{3}\right)^{3} \times \left[\left(\frac{5}{3}\right)^{-3} \times \left(\frac{5}{2}\right)^{-3}\right]$$

$$= \frac{2^3}{3^3} \times \left[\frac{3^3}{5^3} \div \frac{2^3}{5^3}\right]$$

$$= \frac{2^3}{3^3} \times \left[\frac{3^3}{5^3} \times \frac{5^3}{2^3}\right]$$

= 1 **Ans.**

EXERCISE

OBJECTIVE DPP - 3.1

1. If x= 3 + $\sqrt{8}$ and y = 3 - $\sqrt{8}$ then $\frac{1}{x^2}+\frac{1}{y^2}=$

(A) -34 (B) 34 (C) 12 $\sqrt{8}$ (D) -12 $\sqrt{8}$

2. If $\frac{3+\sqrt{7}}{3-\sqrt{7}}$ = a + b $\sqrt{7}$ then (a,b) =

(A) (8, -3) (B) (-8, -3) (C) (-8, 3) (D) (8, 3)

3. $\frac{\sqrt{5}-2}{\sqrt{5}+2}-\frac{\sqrt{5}+2}{\sqrt{5}-2}=$

(A) $8\sqrt{5}$ (B) $-8\sqrt{5}$ (C) $6\sqrt{5}$ (D) $-6\sqrt{5}$

4. If x = $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and y = 1, the value of $\frac{x-y}{x-3y}$ is :

(A) $\frac{5}{\sqrt{5}-4}$ (B) $\frac{5}{\sqrt{6}+4}$ (C) $\frac{\sqrt{6}-4}{5}$ (D) $\frac{\sqrt{6}+4}{5}$

5. Which one is greatest in the following :

(A) $\sqrt{2}$ (B) $\sqrt[3]{3}$ (C) $\sqrt[3]{4}$ (D) $\sqrt[3]{2}$

6. The value of $\sqrt[5]{(32)^{-3}}$ is :

(A) 1/8 (B) 1/16 (C) 1/32 (D) None

7. If $x=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $y=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ the value of $x^2 + xy + y^2$ is :

(A) 99 (B) 100 (C) 1 (D) 0

8. Simplify : $\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}$

(A) 1 (B) 0 (C) 10 (D) 100

9. Which of the following is smallest ?

(A) $\sqrt[4]{5}$ (B) $\sqrt[5]{4}$ (C) $\sqrt{4}$ (D) $\sqrt{3}$

10 The product of $\sqrt{3}$ and $\sqrt[3]{5}$ is :

(A) $\sqrt[6]{375}$ (B) $\sqrt[6]{675}$ (C) $\sqrt[6]{575}$ (D) $\sqrt[6]{475}$

11. The exponential from of $\sqrt{\sqrt{2}\times\sqrt{2}\times\sqrt{2}}$ is :

(A) $2^{1/16}$ (B) $8^{3/4}$ (C) $2^{3/4}$ (D) $8^{1/2}$

12. The value of x, if $5^{x-3} . 3^{2x-8} = 225$, is :

(A) 1 (B) 2 (C) 3 (D) 5

13. If $2^{5x} \div 2^{x} = \sqrt[5]{2^{20}}$ then x =

(A) 0 (B) - 1 (C) $\frac{1}{2}$ (D) 1

14. $\sqrt[3]{(729)^{2.5}} =$

(A) $\frac{1}{81}$ (B) 81 (C) 243 (D) 729

15. $\sqrt[4]{\sqrt[3]{x^2}} =$

(A) x (B) $x^{\frac{1}{2}}$ (C) $x^{\frac{1}{3}}$ (D) $x^{\frac{1}{6}}$

SUBJECTIVE DPP - 3.2

1. Arrange the following surds in ascending order of magnitude :

(i) $4\sqrt{10}, 3\sqrt{6}, \sqrt{3}$ (ii) $3\sqrt{4}, 4\sqrt{5}, \sqrt{3}$

2. Whish is greater :

$\sqrt{17}-\sqrt{12}$ or $\sqrt{11}-\sqrt{6}$.

3. Simplify : $\frac{8}{\sqrt{15}+1-\sqrt{5}-\sqrt{3}}$.

4. If p and q are rational number and $p-\sqrt{q} = \frac{4+\sqrt{2}}{3+\sqrt{2}}$ find p and q.

5. Find the simplest R.F. of :

(i) $\sqrt[3]{32}$ (ii) $\sqrt[3]{36}$ (iii) $2^{3/5}$

6. Retionalise the denominator :

(i) $\frac{3}{\sqrt{5}}$ (ii) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$

7. Retionalise the denominator and simplify :

(i) $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ (ii) $\dfrac{1+\sqrt{2}}{3+2\sqrt{2}}$ (iii) $\dfrac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

8. Simplify :

(i) $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5-\sqrt{3}}}+\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ (ii) $\dfrac{7+3\sqrt{5}}{3+\sqrt{5}}-\dfrac{7-3\sqrt{5}}{3-\sqrt{5}}$

9. Find the value of a and b

(i) $\dfrac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}}=a-b\sqrt{77}$ (ii) $\dfrac{5+\sqrt{6}}{5-\sqrt{6}}=a+b\sqrt{6}$

10. If x = $\dfrac{\sqrt{3}+1}{2}$ find the value of $4x^3 + 2x^2 - 8x + 7$.

11. If x = $\dfrac{5-\sqrt{21}}{2}$ show that $\left(x^3+\dfrac{1}{x^3}\right)-5\left(x^2+\dfrac{1}{x^2}\right)+\left(x+\dfrac{1}{x}\right)=0.$

12. Show that a = x + 1/x, where x = $\dfrac{\sqrt{a+2}+\sqrt{a-2}}{\sqrt{a+2}-\sqrt{a-2}}$.

13. Prove that : $\dfrac{1}{3-\sqrt{8}}-\dfrac{1}{\sqrt{8}-\sqrt{7}}+\dfrac{1}{\sqrt{7}-\sqrt{6}}-\dfrac{1}{\sqrt{6}-\sqrt{5}}+\dfrac{1}{\sqrt{5}-2}=5.$

14. If x = $\dfrac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ and y = $\dfrac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$ find the value of $3x^2 + 4xy - 3y^2$.

15. Evaluate:

$$\frac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}}-\sqrt{3-2\sqrt{2}}.$$

16. If x= 3, b = 4 then find the values of :

(i) $a^b + b^a$ (ii) $a^a + b^b$ (iii) $a^b - b^a$

17. Simplify :

$$\left(\sqrt{x}\right)^{-2/3}\sqrt{y^4}\div\sqrt{xy^{-1/2}}.$$

18. Simplify :

(i) $\left[16^{-1/5}\right]^{5/2}$ (ii) $[0.001]^{\frac{1}{3}}$

19. If $\dfrac{9^n\times 3^2\times\left[3^{-n/2}\right]^{-2}-(27)^n}{3^{3m}\times 2^3}=\dfrac{1}{27}$, then prove than m - n = 1.

20. Find the value of x, if $5^{x-3(2x-3)} = 625$.

ANSWER KEY

(Objective DPP # 1.1)

Qus.	1	2	3	4	5	6
Ans.	A	B	C	B	D	C

(Subjective DPP # 1.2)

1. (i) Non-terminating and repeating (ii) Non-terminating and non-repeating
(iii) Non-terminating and repeating (iv) Terminating

2. $\frac{-7}{6}, \frac{-4}{3}, \frac{-3}{2}, \frac{-5}{3}, \frac{-11}{6}$

3. -4, -3, -2, -1

4. $\frac{-5}{24}$

5. $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$

6. $\frac{9}{24}, \frac{10}{24}, \frac{11}{24}$

7. $\frac{-5}{14}, \frac{-4}{14}, \frac{-3}{14}$

(Objective DPP # 2.1)

Qus.	1	2	3	4	5	6	7	8
Ans.	A	B	A	A	C	D	C	C

(Subjective DPP # 2.2)

3. 0.110101001000100001

4. (i) irrational (ii) irrational (iii) rational (iv) irrational

10. $36A^4B^6C^3\sqrt[6]{108}$

11. (i) 1/3 (ii) $\frac{37}{99}$ (iii) $\frac{6}{11}$ (iv) $\frac{5}{99}$ (v) $\frac{4}{3}$ (vi) $\frac{4}{3}$ (v) $\frac{23}{37}$

12. $\frac{19}{30}$

(Objective DPP # 3.1)

Qus.	1	2	3	4	5	6	7	8	9	10
Ans.	B	D	B	D	C	A	A	B	B	B
Qus.	11	12	13	14	15					
Ans.	C	D	D	C	D					

(Subjective DPP # 3.2)

1. (i) $4\sqrt{10} > 3\sqrt{6} > \sqrt{3}$ (ii) $4\sqrt{5} > 3\sqrt{4} > \sqrt{3}$

2. $\sqrt{11}-\sqrt{6}$

3. $\sqrt{16}+1+\sqrt{5}+\sqrt{5}$

4. $P=\frac{10}{7}, Q=\frac{2}{49}$

5. (i) $\sqrt[3]{2}$ (ii) $\sqrt[3]{6}$ (iii) $2^{2/5}$

6. (i) $\frac{3}{2}\sqrt{5}$ (ii) $\frac{\sqrt{6}+\sqrt{15}}{3}$

7. (i) $5-2\sqrt{6}$ (ii) $7+5\sqrt{2}$ (iii) $\frac{9+4\sqrt{6}}{15}$

8. (i) 8 (ii) $\sqrt{5}$

9. (i) $a = 9/2, b = 1/2$ (ii) $a=\frac{31}{19}, b=\frac{10}{19}$

10. 10

14. $\frac{12+56\sqrt{10}}{3}$

15. 1

16. (i) 145 (ii) 283 (iii) 17

17. $\frac{y^{9/4}}{x^{5/6}}$

18. (i) 1/4 (ii) 0.1

20. 1