1. $\frac{dQ}{dt}\frac{1}{k}\frac{(20/100)}{A}$ = (100- 0) … (i)

$\frac{dQ}{dt}\frac{1}{k}\frac{(9/100)}{A} = (100-\theta)$ … (ii)

from (i) and (ii), $\theta = 55^0C$

2. $\frac{1}{2}nmv^2 = \frac{f}{2}nR\Delta T$ … (i)

$f = \frac{2}{\gamma - 1}$ … (ii)

from (i) and (ii)

$$\Delta T = \frac{mv^2}{2R}(\gamma - 1)$$

3. From B to A,
$0 = \Delta U_{BA} + \Delta W_{BA}$
$\Delta U_{BA} = +30$
From A to B
$20 = \Delta U_{AB} + \Delta W_{AB}$
$20 = -30 + \Delta W_{AB}$
$\Delta W_{AB} = 50$

4. $\theta = \theta_2 - \theta_1$
$= 78.3 - 40.6 = 37.7°C$
$\Delta\theta = (\Delta\theta_1 + \Delta\theta_2)$
$= \pm(0.2 + 0.3) = \pm 0.5$ °C
$= (37.7 \pm 0.5)$°C.

5. $\Delta W_{AB}$ = Area under A $\rightarrow$ B bounded by volume axis.

$= 10 \times 10^3\,(25 - 10) \times 10^{-6} + \frac{1}{2}(25 - 10) \times 10^{-6} \times 20 \times 10^3$

= 0.15 + 0.15 = 3.0 J.

In path A $\rightarrow$ C $\rightarrow$ B $= 30 \times 10^3\,(25 - 10) \times 10^{-6} = 0.45$ J.

6. $\frac{dQ}{dt}\frac{1}{k}\frac{(20/100)}{A}$ = (100- 0) … (i)

$\frac{dQ}{dt}\frac{1}{k}\frac{(9/100)}{A} = (100-\theta)$ … (ii)

from (i) and (ii), $\theta = 55^0C$

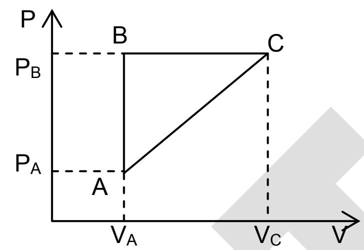
7. $P^2V$ = constant

also $\frac{PV}{T} = \text{constant} \Rightarrow \frac{T^2}{V^2}V = \text{constant}$

$\therefore$ $T^2V^{-1}$ = constant

$\therefore\ T_f^2 = T_0^2\left(\dfrac{3V_0}{V_0}\right) \quad \Rightarrow T_f = \sqrt{3}\,T_0.$

8. $W_{AB} = 0$




$\Delta Q_{AB} = \Delta v_{AB} + \Delta W_{AB}$

$600 = \Delta U_{AB}$

$\Delta Q_{BC} = \Delta U_{BC} + \Delta W_{BC}$

$200 = \Delta U_{BC} + 8 \times 10^4 \times 3 \times 10^{-3}$

$\Delta U_{BC} = -40$

$\Delta U_{AC} = 560$

9. $\dfrac{dQ}{ndt} = \dfrac{dU}{ndT} + \dfrac{dW}{ndT}$

$C = C_v + \left(\dfrac{Pdv}{ndT}\right)$

$P = \alpha v^2$

$PV = nRT$

$PdV + VdP = nRdT$

also from $P = \alpha v^2 \qquad dP = 2\alpha V\, dV$

$PdV + 2\alpha V^2\, dV = nRdT$

$3P\, dV = nRdT$

$\dfrac{PdV}{ndT} = \dfrac{R}{3}$

$\therefore\ C = \dfrac{3}{2}R + \dfrac{R}{3} = \dfrac{11}{6}R$

10. Process: $dQ = -\dfrac{1}{2}dU + \dfrac{1}{2}dW$

Ist law: $dQ = dU + dW$

$\therefore\ dU + dW = -\dfrac{1}{2}dU + \dfrac{1}{2}dW \Rightarrow dW = -3\,dU$

$dQ = dU - 3dU = -2dU$

$\therefore\ C = \dfrac{dQ}{ndT} = -2\dfrac{dU}{ndT} = -2C_V = -5R$

11. Let $\ell$ and a be the length and cross-section area of each rod.

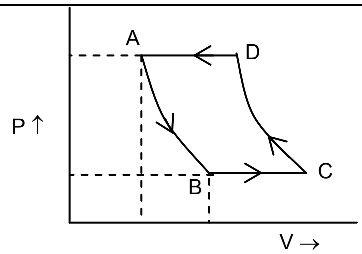
$\therefore Q_{AB} = Q_{BC} \quad \therefore \dfrac{kA(100-T)}{\ell} = \dfrac{ka(T-50)}{\ell}$

$\therefore\ T = 75^0C$

also if $Q_{AB} = Q_{AC} \qquad \therefore \dfrac{ka(100-75)}{\ell} = \dfrac{k'a(100-50)}{\ell}$

$\therefore\ 25\,k = 50\,k' \qquad \therefore k' = k/2$

12.




13. Average rotational K.E. = $\frac{nRTf_r}{2}$

where $f_r$ = rotational degree of freedom )

Avg. rotational KE of $CO_2$ = $\frac{1 \times R \times T \times 2}{2} = RT$

Avg. rotational KE of $N_2$ = $\frac{2 \times R \times T \times 2}{2} = 2RT$

Ratio = $\frac{1}{2}$.

14. Applying COE

$\frac{1}{2}mv_0^2 = nC_v\Delta T$ = $\frac{m}{M}\frac{3}{2}R\Delta T$

$\therefore$ $\Delta T$ = $\frac{Mv_0^2}{3R}$

15. Using Newton's law of cooling

$\frac{\Delta\theta}{\Delta t} = -k(\theta_{avg} - \theta_{surrounding})$

for 1, 2

$\therefore \quad \frac{50-48}{5} = -k\left(\frac{50+40}{2} - 27\right)$

$\frac{2}{5} = -k(49-27)$ … (i)

for 3, 4

$\frac{40-38}{\Delta t'} = -k\left(\frac{40+38}{2} - 27\right)$

$\frac{2}{\Delta t'} = -k(39-27)$ … (ii)

on solving (i) and (ii) we get

$\Delta t'$ = 9.1 min.

16. $R_1$ = $\frac{\ell}{3kA}$, $R_2$ = $\frac{\ell}{2kA}$

$R_3 = \frac{\ell}{kA}$

$100 - \theta = I_1R_1$ , $\theta - 50 = R_2I_2, \theta - 0 - R_3 (I_1 - I_2)$

$\theta = \frac{200}{3}\,^0C$

17. Loss in K.E. of the gas $\Delta E = \frac{1}{2}(nm)\,v_0^2$, where n = number of moles.

If its temperature change by $\Delta T$.

Then $n\frac{3}{2}R\Delta T = \frac{1}{2}(nm)\,v_0^2$

$\Rightarrow \Delta T = \frac{mv_0^2}{3R}$.

18. (a) $\Delta Q = u_1 - U_2 = -(u_2 - u_1) = -\Delta u$

$dQ = -du = -n\,C_v dT = -\frac{nR}{\gamma - 1}dt$

$C = \frac{dQ}{\nu dT} = -\frac{R}{\gamma - 1}$

(b) **not available**

19. For first ten minutes

$\frac{dT}{dt} = -\left[\frac{62 - 50}{10}\right] = -1.2\,^0C/min$

$\Delta T = \left[\frac{62 + 50}{10}\right] - T_0 = (56 - T_0)^0C$

$-kA(56 - T_0)^0 = -1.2\,^0C / min.$ …(i)

Similarly for next 10 minutes

$\frac{dT}{dt} = \left[\frac{42^0 - 50^0}{10}\right] = -0.8\,^0C/min$

$\Delta T = \left(\frac{42 + 50}{2}\right) - T_0 = (46 - T_0)^0C$

$-0.8^0C / min = -kA\,(46 - T_0)^0C$ …(ii)

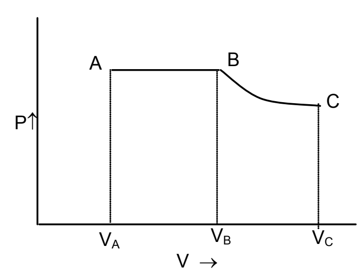
dividing (i) and (ii)

$T_0 = 26^0$

20. (a) For an ideal gas

$P = \frac{nRT}{V} = \frac{2 \times 8.3 \times 300}{20 \times 10^5}\,N/m^2$

$= 2.5 \times 10^5\,N/m^2$




(b) $T_A = T$, $T_B = 2T$

at B, $P'_B = P'_A = 2.49 \times 10^5\,N/m^2$

$v_B = 2v_A = 40 \times 10^{-3}\,m^3$, $T_B = 600\,k$

from $TV^{r-1}$ = constant at B and C

$$\frac{v_c}{v_B} = 2^{1/\gamma-1} = 2^{3/2}$$

$v_c = 2\sqrt{2}\ v_B = 2 \times 1.414 \times 40 = 113\ \ell$

$$P_c = \frac{NRTC}{VC} = \frac{2 \times 8.3 \times 300}{113.13 \times 10^{-3}}$$

$= 0.44 \times 10^5\ N/m^2$

(c) $W_{AB} = 2.49 \times 10^5 (40 - 20) \times 10^{-3} = 4980\ J$

$$W_2 = \frac{nR}{r-1}[T_2 - T_1] = \frac{2 \times 8.3}{1-(5/3)}[300 - 600]$$

$= 7470J$

$W_{net} = 4980 + 7470 = 12450\ J$

21. Molecular weight of the mixture is given by $\frac{\Sigma m}{M} = \Sigma(m/M)$

$$\therefore \quad M = \frac{75+25}{\frac{75}{28}+\frac{25}{32}} = 28.9$$

$\gamma$ of the mixture given by

$$\frac{n_1+n_2}{\gamma_m - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$$

$\gamma_m = 1.4$

$$\therefore \quad \text{velocity of sound } v = \sqrt{\frac{\gamma RT}{M}} = 331.3\ m/s$$

22. $H = \frac{v^2}{R}.t$

$R = \frac{v^2}{H}.t = k.t$

$$\frac{R_1}{R_2} = \frac{t_1}{t_2}$$

$$\frac{R_{eq}}{R_1} = \frac{t_{eq}}{t_1}$$

$$t_{eq} = \frac{R_1 + R_2}{R_1}.t_1 = \left[1 + \frac{t_2}{t_1}\right]t_1 = t_1 + t_2 = 10\ \text{mins}$$

23. $R_{100} = R_0 (1 + \alpha\Delta T)$

$= (2\,cm)\left[1 + (11x10^{-6}\ /^\circ C)(100^\circ C)\right]$

$= (2\,cm)(1 + 11x10^{-4})$

$= 2.0022$ cm.

24. $A \to B$ represents an isobaric process,

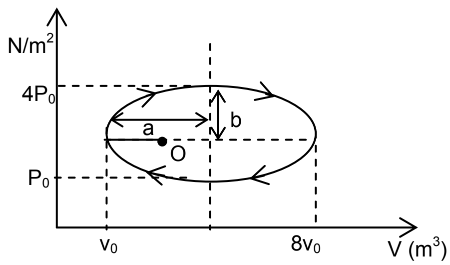
$$\therefore \quad \Delta Q_{AB} = 1 \times \frac{5}{2}R(2T_0 - T_0) = \frac{5}{2}RT_0$$

$B \rightarrow C$ represents an isothermal expansion $\therefore$ $\Delta U_{BC} = 0$

$\Rightarrow$ $\Delta Q_{BC} = 1.R.2T_0 \ln\left(\frac{3P_0}{P_0}\right) = 2RT_0 \ln 3$

$\therefore$ $\Delta Q = RT_0[2.5 + 2\ln 3]$

25. W = area enclosed by the ellipse = $\pi ab$
$= \pi\ (3P_0)\ (7v_0)$ Nm
$= 21\pi P_0 v_0$ Nm




26. (a) At constant volume $Q = nC_v\ \Delta T$
$= 2\ (3R/2)100$
$= 300\ R$
$= 300 \times 8.31 = 2493$ Joules

(b) At constant pressure $Q = nC_p\ \Delta T = 2\ (5R/2)\ (100)$
$= 500 \times 8.31 = 4155$ Joules

27. The heat required to melt the ice at $0^0$C $= 100 \times 80 = 8000$ cal.
The heat given by water when it cools down from $25^0$C to $0^0$C $= 200 \times 1 \times 25 = 5000$ cal.
Clearly, the whole of the ice can not be melted, as the required amount of heat is not provided by the water. Therefore, the final temperature of the mixture is $0^0$C.

28. $\frac{dQ}{dt}\frac{1}{k}\frac{(20/100)}{A} = (100-0)$ … (i)

$\frac{dQ}{dt}\frac{1}{k}\frac{(9/100)}{A} = (100-\theta)$ … (ii)

from (i) and (ii), $\theta = 55^0$C

29. Let $A_0$ be the cross-section of cube and $\rho_0$ the density of liquid before temperature rise.
After $\Delta t^0$C increase in temperature, the density of liquid becomes

$$\rho = \frac{\rho_0}{(1+\gamma\Delta t)}$$

while new cross-sectional area of cube is, $A = A_0\ (1 + 2\alpha\Delta t)$
Since $Mg = A_0 x_0\ \rho_0 g$ . . . (i)
where $x_0$ is the length of cube in liquid and M is the mass of cube.
Also $Mg = Ax_0\ \rho g$ . . . (ii)

$\Rightarrow A_0 x_0\rho_0 g = A_0(1 + 2\alpha\Delta t)\dfrac{x_0\rho_0}{(1+\gamma\Delta t)}g$

$\Rightarrow \gamma = 2\alpha$.

30. $\left(\dfrac{dQ}{dt}\right)\dfrac{1}{k}\dfrac{dr}{4\pi r^2} = -dT$

$\dfrac{dT}{dr} = -\dfrac{C_1}{4\pi k}\dfrac{1}{r^2}$

Integrating,

$T = \dfrac{C_1}{4\pi k}\dfrac{1}{r} + C_2$

At $r = a$, $T = 2T_0$ and at $r = 2a$, $T = T_0$

$\Rightarrow C_2 = 0, C_1 = 8\pi akT_0 \qquad \therefore T = \dfrac{2a}{r}T_0$

(i) $\dfrac{dQ}{dt} = 8\pi akT_0$

(ii) $T(r = \dfrac{3a}{2}) = \dfrac{4T_0}{3}$

31. (a) $e_A \sigma A_A T_A^4 = \rho_B\sigma A_B T_B^4$

$0.01 \times (5802)^2 = 0.81\ (T_B)^4$

$T_B = 1934$ k.

(b) $\lambda_A T_A = \lambda_B T_B$

$\lambda_B - \lambda_A = 1\ \mu m$

$\lambda_B = 1.5\ \mu m$.

32. $\dfrac{dQ}{dt} = L\left(\dfrac{dm}{dt}\right)$, $\dfrac{kA[0-(-20)]}{y} = LA\ \rho\ \dfrac{dy}{dt}$

$\int_5^{10} y dy = \dfrac{20k}{\rho L}\int_0^t dt$, $T = \left(\dfrac{10^2}{2} - \dfrac{5^2}{2}\right)$

t = 27600 sec. = 7 hrs. 40 minutes.

33. We have heat supplied by heater = heat lost by tank by radiation under steady state.

$\therefore$ 2 = k (65 – 15) where k is a constant

$\therefore$ K = 2/50 = 2(240)/50 = 48/5 Cal/s-$^0$C

At any instant if the temperature of the tank be ‘T’ then

we have $\dfrac{dT}{dt} = -\dfrac{K}{m.s}(T-15)$

or $-\dfrac{dT}{T-15} = \dfrac{K}{m.s}.dt$

or $-\int_{65}^{50}\dfrac{dT}{T-15} = \dfrac{K}{m.s}\int_0^t dt$

$\Rightarrow -\ln\ (T-15)\Big|_{65}^{50} = \dfrac{K}{m.s}.t$

$\Rightarrow \quad \ln\left[\frac{65-15}{50-15}\right] = \frac{K}{m.s}.t$

or $\quad t = \frac{m.s}{K}\ln\left(\frac{50}{35}\right) = \frac{10^3 \times 2 \times 10^3}{48/5}\ln\left(\frac{10}{7}\right)$

$= 20$ hrs. (Approximately)

34. (i) work done = area under the curve.

$= \frac{1}{2}[2v_0 - v_0][3P_0 - P_0] = P_0v_0$

(ii) heat rejected in the path CA

$= nC_P.dT$ (constant pressure process)

$= 1 \times \frac{5}{2} R(T_C - T_A)$

$= \frac{5R}{2}\left[\frac{P_0(2v_0)}{R} - \frac{P_0v_0}{R}\right] = \frac{5P_0v_0}{2}$

heat absorbed in path AB = $nC_V.dT$(constant vol process.

$= 1 \times \frac{3}{2} \times R[T_B - T_A] = \frac{3R}{2}\left[\frac{3P_0v_0}{R} - \frac{P_0v_0}{R}\right]$

$= 3P_0v_0$

(iii) for ABC, Q = W (cyclic process)

$\therefore \quad \frac{-5}{2}P_0v_0 + 3P_0v_0 + Q_{BC} = P_0v_0$

$\therefore \quad Q_{BC} = P_0v_0 - 3P_0v_0 + \frac{5}{2}P_0v_0$

$= \frac{P_0v_0}{2}$

(iv) $\frac{Pv}{T}$ = constant so, when Pv is maximum, T also is maximum.

Pv is maximum for process BC.

Hence temperature will be maximum between B & C.

Let equation for BC be $P = Kv + K_1$ satisfying both points B and C

For B, $3P_0 = Kv_0 + K_1$

For C, $P_0 = K(2v_0) + K_1$

So $\quad K = \frac{-2P_0}{v_0}$ and $K_1 = 5P_0$

$\therefore$ equation for BC becomes $P = \frac{-2P_0}{v_0}v + 5P_0$

or $\quad \frac{RT}{v} = -\frac{2P_0v_0}{v_0} + 5P_0 \quad$ or $\quad T = \frac{P_0}{R}\left[5v - 2\frac{v^2}{v_0}\right]$

for maximum $\frac{dT}{dv} = 0$

$\therefore \quad \frac{P_0}{R}\left[5 - \frac{4v}{v_0}\right] = 0$

or $v = \frac{5v_0}{4}$

$\therefore \quad T_{max} = \frac{P_0}{R}\left[5\left(\frac{5v_0}{4}\right) - 2\left(\frac{5v_0}{4}\right)^2 \frac{1}{v_0}\right] = \frac{25P_0v_0}{8R}$

35. $\eta = \frac{W_{net}}{(\Delta Q)_{supplied}}$

$\eta = \frac{Q_{BC}+Q_{DA}}{Q_{BC}} = 1 - \frac{T_D - T_A}{T_C - T_B}$

Let $\frac{v_A}{v_B} = \frac{v_P}{v_C} = k$

$\frac{T_A}{T_B} = \frac{T_D}{T_C} = \left(\frac{1}{k}\right)^{\gamma-1}$

$\eta = 1 - \frac{1}{(k)^{\gamma-1}}$

$= 1 - \frac{1}{(8)^{2/3}} = 75\ \%.$

36. Table of determination

| Constituent | Molar fraction | Molecular weight | Mass/mole of mixture | $C_v$ | Specific heat per mole of mixture |
|---|---|---|---|---|---|
| He | 0.2 | 4 | 0.8 | $\frac{3}{2}R$ | 0.3 R |
| $H_2$ | 0.1 | 2 | 0.2 | $\frac{5}{2}R$ | 0.25 R |
| $O_2$ | 0.3 | 32 | 9.6 | $\frac{5}{2}R$ | 0.75 R |
| $N_2$ | 0.4 | 28 | 11.2 | $\frac{5}{2}R$ | R |

(a) For this mixture mass per mole = 21.8

(b) Specific heat at constant volume for the mixture

$= (0.3+0.25+0.75+1)R = 2.3\ R = 19.09\ J\ mole^{-1}k^{-1}$

and $c_p = c_v + R = 27.39\ J\ mol^{-1}\ k^{-1}$

(c) gas constant per kg $= \frac{R}{M} = \frac{8.3}{21.8} = 0.4\ Jkg^{-1}\ k^{-1}$

37. (i) $T_I\ V_i^{y-1} = T_f\ V_f^{y-1} \Rightarrow \quad T_2 = 189\ k$

(ii) $\Delta U = nC_v\,\Delta T$

where $C_v = \frac{3}{2}R$ and $\Delta T = -111$ k

$\Rightarrow \quad \Delta V = 2 \times \frac{3}{2} \times (-111) = -2767.2$ (J)

(iii) In adiabatic proces

$\Delta U = -\Delta W$

$\Rightarrow \Delta W = 2767.2$ J

38. (i) we have velocity of sound in a medium given by $\sqrt{\frac{\gamma P}{\rho}}$

at N.T.P. $\frac{v_H}{v_a} = \sqrt{\frac{\rho_a}{\rho}}$

at N.T.P. $\frac{v_H}{v_a} = \sqrt{\frac{\rho_a}{\rho_H}} = (16)^{1/2} = 4$

$\therefore \quad v_H = 4v_a = 4\,(332) = 1328$ m/s.

(ii) we also have $v \propto \sqrt{T}$

$\therefore \quad \frac{v_{819}}{v_0} = \sqrt{\frac{273+819}{273}} = 2$

$\therefore \quad v_{819} = 2v_0 = 2\,(1328) = 2656$ m/s

39. (a) $V_2 = V_1\frac{T_2}{T_1} = (1000)\frac{(375)}{300} = 1250\text{cm}^3$

(b) $p_2 = \frac{p_1V_2}{V_1} = \frac{(1.1\times10^5)(1250)}{1000} = 1.375\times10^5\text{Pa}$

(c) $W_{net} = W_{AB} + W_{BC} + W_{CA}$

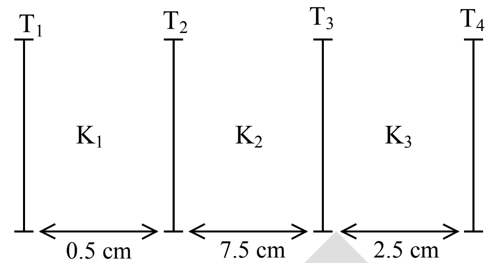
$W_{AB} = p_1(V_2 - V_1) = 1.1 \times 10^5\,(1250 - 1000) \times 10^{-6} = 27.5$ J

$W_{BC} = p_2V_1 \ln\left|\frac{V_1}{V_2}\right| = (1.375 \times 10^5)(1000 \times 10^{-6}) \ln\left|\frac{1000}{1250}\right| = -30.7$ J

$W_{CA} = 0$

$W_{net} = 27.5 - 30.7 = -3.2$ J, Work done on the gas = 3.2 J

40. $\frac{dQ}{dt} = \frac{(T_1 - T_4)}{\left[L_1/K_1 + L_2/K_2 + L_3/K_3\right]}$




q = $\frac{150 - 38}{\left[0.5/52.3 + 7.5/0.075 + 2.5/0.081\right] 10^{-2}}$ = 85.58 w/m$^2$

and $T_2 = T_1 - q\left(\frac{L_1}{K_1}\right) = 150 - 85.85\left(\frac{0.5}{52.3}\right)10^{-2}$

= 149.99$^0$C

$T_3 = T_4 + q\left(\frac{L_3}{K_3}\right) = 64.4^0C$

41. In the steady state, the net outward thermal current is constant, and does not depend on the radial position.

Thermal current, $C_1 = \left(\frac{dQ}{dt}\right) = -K.(4\pi r^2)\frac{dT}{dr}$

$\frac{dT}{dr} = -\frac{C_1}{4\pi K}\frac{1}{r^2}$

Integrating, T = $\frac{C_1}{4\pi K}\frac{1}{r} + C_2$

At r = a, T = 2$T_0$ and at r = 2a, T = $T_0$

$\Rightarrow$ $C_2 = 0$, $C_1 = 8\pi aKT_0$ $\therefore$ T = $\frac{2a}{r}T_0$

(i) $\frac{dQ}{dt} = 8\pi aKT_0$ (ii) T(r = 3a/2) = 4$T_0$/3

42. $\frac{50 - 45}{5} = k[47.5 - \theta_0]$ Where $\theta_0$ is the temperature of the surrounding.

$\frac{45 - 40}{8} = k[42.5 - \theta_0]$

$\frac{8}{5} = \frac{k[47.5 - \theta_0]}{k[42.5 - \theta_0]}$

$\Rightarrow$ 1.6 [ 42.5 - $\theta_0$ ] = 47.5 - $\theta_0$

$\Rightarrow$ 68 - 1.6 $\theta_0$ = 47.5 - $\theta_0$

$\Rightarrow$ 0.6 $\theta_0$ = 20.5

$\Rightarrow$ $\theta_0$ = 34$^0$.

43. (i) $T_i\ V_i^{\gamma-1} = T_f\ V_f^{\gamma-1} \Rightarrow T_2 = 189$ k

(ii) $\Delta U = nC_v\,\Delta T$

where $C_v = \frac{3}{2}R$ and $\Delta T$ = -111 k

$\Rightarrow \Delta U = 2 \times \frac{3}{2}R \times (-111) = -2767.2$ (J)

44. Under equilibrium condition
temperature of the body = $T_0$
Heat lost = Decrease in internal energy
= ms ($T_i$ - $T_0$)

Newton's law of cooling $\frac{dT}{dt} = -k(T - T_0)$ where k is a constant

$\therefore\ \ln \frac{T - T_0}{T_i - T_0} = -kt$

when 60 % of the total heat is lost
ms ($T_I$ - T) = 0.6 ms ($T_i$ - $T_0$)
substituting for T

$t = \frac{1}{k}\ln 2.5.$

45. $\frac{d\theta}{dt} = -k(\theta - \theta_0)$

$\int_{40}^{36} \frac{d\theta}{\theta - \theta_0} = -k$ (5 min.)

$k = -\frac{\ln(5/6)}{5\text{min.}}$

$k = \int_{36}^{32} \frac{d\theta}{\theta - \theta_0} = -kt$

$t = \frac{\ln(4/5)}{\ln(5/6)} \times 5$ min.

46. $\sigma\, 4\pi r^2 T^4 = \left(\frac{4}{3}\pi r^3 \rho C\right)\left(-\frac{dT}{dt}\right)$

$dt = -\frac{\rho rc}{3\sigma}\frac{dT}{T^4}$

$t = -\frac{\rho rc}{3\sigma}\int_{200}^{100}\frac{dT}{T^4} = \frac{7\rho rc}{72\sigma}\times 10^{-6}\text{s}$

47. (a) $\frac{dQ}{dt} = \sigma\varepsilon A[(T_\ell)^4 - (T_a)^4]$,

Rate of heat loss per unit area = 595 watt / $m^2$.

(b) Let $T_o$ be the temperature of the hot oil

$\Rightarrow \dfrac{KA(T_o - T_\ell)}{t} = 595\,A$

$\Rightarrow T_o \approx 420$ K or 147 $^0$C

48. (a) Isobaric process

(b) $\Delta W = P(v_2 - v_1) = (P_0 + \dfrac{mg}{A})\ \ell_0 A$

(c) $\dfrac{v_1}{T_1} = \dfrac{v_2}{T_2} \quad \Rightarrow \quad \dfrac{\ell_0 A}{T_0} = \dfrac{2\ell_0 A}{T_2} \quad \Rightarrow \quad T_2 = 2T_0$

(d) $\Delta Q = nC_p\,\Delta T = \dfrac{\left(P_0 + \dfrac{mg}{A}\right)A\ell_0}{RT_0}\ \dfrac{5RT_0}{2} = \dfrac{5}{2}\left(\rho_0 + \dfrac{Mg}{A}\right)A\ell_0$

49. For adiabatic process $PV^\gamma$ = constant.
In the equilibrium state, total pressure,

$P = P_o + \dfrac{Mg}{A}$, and initial volume $= V_o$

Thus $P V_o^\gamma = (P + \Delta P)(V_o + \Delta V)^\gamma = (P + \Delta P)\, V_o^\gamma \left(1 + \gamma \dfrac{\Delta V}{V_o}\right)$

$\therefore \quad \Delta P = -\gamma\, P \dfrac{\Delta V}{V_o}$

Restoring force $F = \Delta P \times A = -\gamma\, PA\ \dfrac{\Delta V}{V_o} = -\dfrac{\gamma A^2}{V_o^2}\left(P_o + \dfrac{Mg}{A}\right)x$

$\therefore \quad$ Acceleration $a = \dfrac{F}{m} = -\dfrac{\gamma A^2}{MV_o}\left(P_o + \dfrac{Mg}{A}\right)x$

Hence the motion is SHM with angular frequency $\omega = \sqrt{\gamma A^2\left(P_o + \dfrac{Mg}{A}\right)x \Big/ MV_o}$

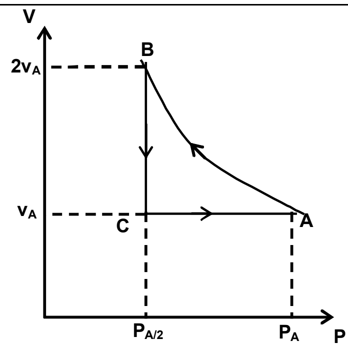
50. $\ell_C = \ell_0\,(1 + \alpha_C\,\Delta\theta)$

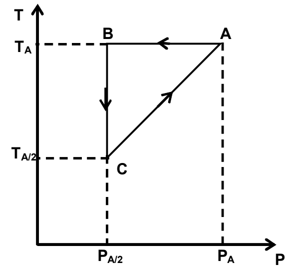
$\ell_t = \ell_0\,(1 + \alpha_T\,\Delta\theta)$

$\dfrac{(R + d/2)\theta}{(R - d/2)\theta} = \dfrac{1 + \alpha_C \Delta T}{1 + \alpha_T \Delta T}$

R = 0.77 m

51. (a) Using PV = nRT




(b) $W_{AB} = \int PdV = \int \frac{nRT}{V} dv = 17.26T_A$

$W_{BC} = \int PdV = \frac{P_A}{2}(v_A - 2v_A) = -12.45T_A$

$W_{CA} = 0$

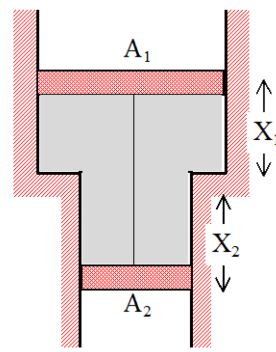
Net work done $= 17.26\ T_A - 12.45\ T_A = 4.81\ T_A$

As initial and final states of the gas are same $\Delta U = 0$

$\Delta Q = \Delta U + \Delta W$

$\Delta Q = \Delta W$.

52. $T = \frac{PV}{nR}$




$V_1 = A_1X_1 + A_2X_2$

$V_2 = A_1(X_1 + \ell) + A_2(X_2 - \ell)$

$= V_1 + \ell(A_1 - A_2)$

$= V_1 + \ell\Delta s$

Net upward force due to inner and outer pressure

$(P_1 - P_0)A_1 - (P_1 - P_0)A_2 = mg$

$\Rightarrow \quad (P_1 - P_0)\Delta s = mg$

$\Rightarrow P_1 = \frac{mg}{\Delta s} + P_0 = P_2$ (Pressure remains same for equilibrium)

$\Delta T = T_2 - T_1 = \frac{P_2V_2 - P_1V_1}{nR} = \left(\frac{mg}{\Delta s} + P_0\right)\frac{\Delta s\ell}{nR}$

$= \frac{mg\ell + \Delta sP_0\ell}{nR} = 0.91$ K.

53. Process ABCA is clockwise while process ADEA is anticlockwise in P-V diagram

Net work done

Area ABCA - area ADEA $= \frac{1}{2}\times 2\times 1 - \frac{1}{2}\times 1\times 1 = \frac{1}{2}$ (J)

During process ABCDEA

$\Delta W = 1/2$ J

$\Delta U = 0$ (cyclic process)

$\Delta Q = \Delta U + \Delta W = 1/2$ J.

54. (a) $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$\therefore \quad TV^{\gamma-1} = \frac{T}{2}(5.66\ V)^{\gamma-1}$

$\gamma - 1 = \frac{\ln 2}{\ln 5.66} = \frac{0.693}{1.7334} = 0.4$

$\gamma = 1.4$

$\therefore \quad \gamma = 1 + \frac{2}{F} \quad \Rightarrow \quad F = 5$

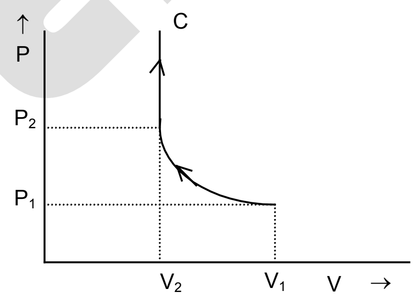
(b) $W_A = \frac{nR(T_F - T_i)}{1-\gamma} = \frac{P_FV_F - P_iV_i}{1-\gamma}$

from PV = nRT $\quad P_F V_F = \mu R\frac{T}{2} = \frac{PV}{2}$

$W_A = \frac{1}{0.4}\left[PV - \frac{PV}{2}\right] = 1.25\ PV.$

55. (a) $A \longrightarrow B$ adiabatic compression

$B \longrightarrow C$ Heating at constant volume




(b) $W_{AB} = -\frac{nR}{\gamma-1}(P_1V_1 - P_2V_2)$

$n = 2,\ \ \gamma = 5/3, \qquad P_2 = P_1\left(\frac{V_1}{V_2}\right)^{5/3}$

$\therefore \quad W_{AB} = -\frac{2R}{(5/3)-1}\left[P_1V_1 - P_1(V_1/V_2)^{5/3}V_2\right]$

$= -\frac{3R}{2}P_1V_1\left[1-\left(\frac{V_1}{V_2}\right)^{2/3}\right]$

$\Delta U = \Delta U_{AB} + \Delta U_{BC}$

$= Q - \frac{3R}{2}P_1V_1\left[1-\left(\frac{V_1}{V_2}\right)^{2/3}\right]$

For BC $Q = nC_v\,\Delta T$

$\Delta T = \frac{Q}{2\times\frac{3R}{2}} = \frac{Q}{3R}$

For point A : $P_1V_1 = 2RT_A$

For Point B : $P_2V_2 = 2RT_B$ .

For adiabatic change

$P_1V_1^{\gamma} = P_2V_2^{\gamma} \quad \Rightarrow \quad P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma}$

Further $T_B = \frac{P_2V_2}{nR} = \frac{P_2V_2}{2R}$

$= \frac{V}{2R}\left[P_1\left(\frac{V_1}{V_2}\right)^{\gamma}\right]$

$\therefore$ Final temperature $T_C = T_B + \Delta T$

$= \frac{V_2}{2R}\left[P_1\left(\frac{V_1}{V_2}\right)^{5/3}\right] + \frac{Q}{3R}$

$= \left[\frac{P_1V_1^{5/3}V_2^{-2/3}}{2R} + \frac{Q}{3R}\right]$.

56. (a) 1→2 isochoric process $\Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2}$

$1 \times \frac{600}{300} = P_2 = 2$ atm

2→ 3 adiabatic process $\Rightarrow$ $P^{1-\gamma}\,T^{\gamma}$ = constant

$\Rightarrow$ $P_3 = \left(\frac{T_2}{T_3}\right)^{\gamma/1-\gamma} P_2 = \left(\frac{600}{300}\right)^{\frac{5/3}{1-(5/3)}}$ 2 atm.

$\Rightarrow$ $P_3 = 2^{-3/2}$ atm $= \frac{1}{2\sqrt{2}}$.

(b) 1→ 2 (isochoric process)

$\Delta W_{12} = 0$

$\Delta Q_{12} = n\,C_v\,\Delta T$

$= \frac{nR}{r-1}(T_2 - T_1)$

$\frac{R}{(5/3)-1}(600 - 300)$

= 450R units.

$\Delta Q = \Delta U + \Delta W$

$\Delta Q_{12} = \Delta U_{12} = 450$ R units

2→ 3 (adiabatic process)

$\Delta W_{23} = \frac{nR}{\gamma-1}(T_1 - T_2)$

$= \frac{1\times R(600-300)}{5/3-1}$

= 450 R units

$\Delta Q = 0$

$\Delta U = -\Delta W$

$\Delta U = -450$ R units

3→ 1 (isothermal process)

$\Delta W_{31} = nRT \ln (P_3/P_1)$

$= 1 \times R \times 300 \ln 2^{-3/2}$

$\approx -312$ R units

$\Delta U = 0$

$\Delta Q = \Delta W$

$\Delta Q_{31} = -312$ R units.

Hence total work done in the cycle

$\Delta W = \Delta W_{12} + \Delta W_{23} + \Delta W_{31} = 138$ R

(c) Efficiency of the cycle

$\eta = \frac{\text{total work done}}{\text{Heat absorbed}} = \frac{138R}{450R} = \frac{138}{450} \approx 0.31$

57. Rate of heat flow through a concentric shell of radius x and thickness dx is

$Q = -k4\pi x^2 \frac{d\theta}{dx}$

Or $\frac{dx}{x^2} = -\left(\frac{4\pi k}{Q}\right)d\theta$

Integrating

$$\int_{R_1}^{R_2} \frac{dx}{x^2} = -\left(\frac{4\pi k}{Q}\right)\int_{\theta_1}^{\theta_2} d\theta$$

or $Q = \dfrac{4\pi k(\theta_1 - \theta_2)R_1R_2}{R_2 - R_1}$ . . . (1)

Integrating equation (1) from $R_1$ to r

$$\frac{r - R_1}{rR_1} = \frac{4\pi k(\theta_1 - \theta)}{Q}$$

$Q = \dfrac{4\pi k(\theta_1 - \theta)R_1 r}{r - R_1}$ . . . (2)

Equating (1) and (2) and substituting $\theta = (\theta_1+\theta_2)/2$,
We get $r = (2R_1R_2)/(R_1+R_2)$

58. From the figure

$$\frac{v_4 - v_3}{10} = \frac{4\times10^5 - 3\times10^5}{3\times10^5 - 10^5}$$

$\Rightarrow v_4 - v_3 = 5$ litres

Now work done, $W = \left(\frac{1}{2}\times10\times2\times10^5 - \frac{1}{2}\times\times5\times10^5\right)\times10^{-3} = 750$ J.

59. Process 1 - 2

${}_1W_2 = 0$, ${}_1Q_2 = U_2 - U_1 = nC_v(T_2 - T_1)$

$= n\left(\frac{3R}{2}\right)\left(\frac{v_0}{nR}\right)(P_2 - P_1)$

$= 3P_0v_0$

Process 2 - 3

${}_2W_3 = 3P_0(2v_0 - v_0) = 3P_0v_0$

${}_2Q_3 = nC_p\,(T_3 - T_2) = n\left(\frac{5R}{2}\right)\left(\frac{3P_0}{nR}\right)(V_3 - V_2) = \frac{15}{2}P_0v_0$

Process 3 - 4

${}_3W_4 = 0$, ${}_3Q_4 = nC_v\,(T_4 - T_3) < 0$ as $T_4 < T_3$

Process 4 - 1

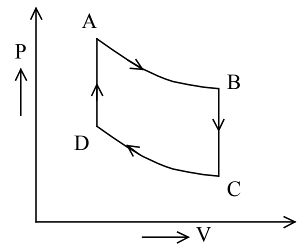
${}_4W_1 = P_0\,(v_0 - 2v_0) = -P_0v_0$

${}_4Q_1 = nC_p\,(T_1 - T_4) < 0$ as $T_1 < T_4$

$\therefore$ efficiency $= \dfrac{\text{Net work}}{\text{Heat added}} = \dfrac{2P_0v_0}{15\frac{P_0v_0}{2} + 3P_0v_0}$

$= \dfrac{4}{21} = 19.04\,\%$.

60. • (a) The P-V diagram will be as shown,




- For isothermal expansion $A \longrightarrow B$

$\Delta Q_1 = nRT \ln\left(\frac{V_2}{V_1}\right)$

$= 2 \times 8.314 \times 360 \ln\left(\frac{0.10}{0.04}\right)$

= 5483.25J

For isochoric process $B \longrightarrow C$

$\Delta Q_2 = nC_v\Delta T = nC_v (T_c - T_B) = 2 \times \frac{3}{2} R \times (300 - 360)$

$= -3 \times 8.314 \times 60 = -1496.5$ J

For isothermal compression $C \longrightarrow D$

$\Delta Q_3 = nRT\ln\left(\frac{V_D}{V_C}\right) = 2 \times 8.314 \times 300 \ln\left(\frac{0.04}{0.10}\right)$

= -4569.4J

For isochoric process $D \longrightarrow A$

$\Delta Q_4 = nC_v\Delta T = 2 \times \frac{3}{2} \times 8.314 \times (360 - 300)$

= 1496.5 J

(b) Heat absorbed by the gas during the cycle

= 5483.25 + 1496.5 = 6979.75J

(c) • Work done by the gas during the cycle

$= \Delta Q_1 + \Delta Q_2 + \Delta Q_3 + \Delta Q_4$

= 5483.25 - 1496.5 – 4569.4 + 1496.5

= 913.85 J

(d) • Efficiency of the cycle $= \frac{\text{Net work done}}{\text{Heat supplied}}$

$= \frac{913.1}{6979.75} = 0.131.$

(e) • Change in internal energy of the gas during the complete cycle = 0

61. For cooling, $\frac{dQ}{dt} = -ms\frac{d\theta}{dt} = -C\frac{d\theta}{dt}$ ($\because$ heat capacity ms = C)

and from Newton's law of cooling

$\frac{d\theta}{dt} = a(\theta - \theta_0)$

$\therefore -C\frac{d\theta}{dt} = a(\theta - 300)$ (a – constant , $\theta_0$ - temperature of surrounding)

$\therefore \int_{400}^{\theta} \frac{d\theta}{(\theta - 300)} = -\frac{a}{C}\int_0^t dt$

$\ln\frac{(\theta - 300)}{(400 - 300)} = -\frac{a}{C}.t$

$\theta = 300 + 100\ e^{-\frac{a}{c}.t}$

At time $t = t_1$ ; $\theta = 350$

Hence, $a = C \ln (2/t_1)$

Now, when the body X is connected to body Y

$$\frac{d\theta}{dt} = \left(\frac{d\theta}{dt}\right)_{conduction} + \left(\frac{dQ}{dt}\right)_{radiation}$$

$$-C\frac{d\theta}{dt} = -\frac{kA(\theta-\theta_0)}{L} + a(\theta-\theta_0)$$

$$\frac{d\theta}{dt} = -(\theta-\theta_0)\left[\frac{kA}{LC}+\frac{Q}{C}\right]$$

$$\int_{350}^{\theta_F}\frac{d\theta}{(\theta-\theta_0)} = -\left[\frac{kA}{LC}+\frac{\ln 2}{t_1}\right]\int_{t_1}^{3t_1}dt$$

or, $$\ln\frac{\theta_F-300}{350-300} = -\left[\frac{kA}{LC}+\frac{\ln 2}{t_1}\right]2t_1$$

$$\therefore\ \theta_F = 300 + 50\exp\left[-2t_1\left(\frac{kA}{LC}+\frac{\ln 2}{t_1}\right)\right]$$

62. Taking the temperature, pressure and volume at D to be $T_0$ , $P_0$ and $V_0$ using the relations.

$T_A = T_D$, $P_A v_A = P_D v_B$ for path DA

$v_A = v_B$, $\frac{P_B}{T_B} = \frac{P_A}{T_A}$ for path AB

$P_B = P_c$ $\frac{v_B}{T_B} = \frac{v_c}{T_c}$ for path BC

$T_D v_D^{\gamma-1} = T_C v_c^{\gamma-1}$ for path CD

With the given relations, we can complete the table.

| | P | V | T |
|---|---|---|---|
| A | 16 $P_0$ | $V_0/16$ | $T_0$ |
| B | $32P_0$ | $V_0/16$ | $2T_0$ |
| C | $32P_0$ | $V_0/8$ | $4T_0$ |
| D | $P_0$ | $V_0$ | $T_0$ |

Now efficiency

$$\eta = \frac{W_{cycle}}{Q_{AB}+Q_{BC}} = \frac{Q_{AB}+Q_{BC}+Q_{CD}+Q_{DA}}{Q_{AB}-Q_{BC}}$$

Here, $Q_{AB} = C_v\,(2T_0 - T_0) = \frac{3}{2}RT_0$

$Q_{BC} = C_P\,(4T_0 - 2T_0) = 5RT_0$

$Q_{CD} = 0$ (adiabatic)

$Q_{DA} = -\,RT_0 \ln 16 = -\,4RT_0 \ln 2$

putting the values

$$\eta = \frac{(3/2)+5-4\ln 2}{(3/2)+5} = 0.573.$$

63. (i) The process in B is adiabatic ($\gamma$ = 5/3)

$P_0 V_0^{5/3} = \frac{243}{32} P_0 (V_B^f)^{5/3}$,

Final volume, $V_B^f = \frac{8}{27} V_0$

$T_B^f = \frac{P_B^r V_B^f}{R} = \frac{9}{4} T_0 \qquad (P_B^f \equiv \frac{243}{32} P_0)$

(ii) $\Delta W$ = work done on B by A = $\frac{3}{2} R\ (T_B^f - T_0) = \frac{15}{8} RT_0$

$V_A^f = 2V_0 - \frac{8}{27} V_0 = \frac{46}{27} V_0$

$P_A^f = \frac{243}{32} P_0$

$T_{fA} = \frac{207}{16} T_0$

$\Delta U = \frac{573}{32} RT_0$.

The heat supplied by the heater = $\frac{633}{32} RT_0$.

64. (i) Process : $dQ = -\frac{1}{2} dU + \frac{1}{2} dW$

ist law : $dQ = dU + dW$

$\therefore \quad dU + dW = -\frac{1}{2} dU + \frac{1}{2} dW$

$\Rightarrow \quad dW = -3\,dU$

$dQ = dU - 3dU = -2dU$

$\therefore \quad C = \frac{dQ}{dT} = -2\frac{dU}{dT} = -2C_V = -5R$

(ii) $PV^2T$ = constant, for the process

or $(PV / T)\, VT^2$ = const.

or $VT^2$ = const. = A (say)

$\ln V + 2\ln T = \ln A$

$dV/V + 2dT/T = 0$

$dV/dT = -2V/T$

Now $dQ = dU + PdV$

$\therefore\ C = dQ/dT = C_V + P(dV/dT) = C_V + P(-2V/T)$

$= C_V - 2R = R/2$.

65. In case of isothermal expansion, work done by one mole of an ideal gas

$= 2.303\ RT \log (2v/v)$

$= 2.303\ RT \log (2)$

$= 1747$ J

for adiabatic compression, $dQ = 0$

$\therefore \quad du = -dw$

$du = nC_v\,dT = C_v\,dT \quad (n = 1)$

and $T_1 v_1^{\gamma-1} = T_2 v_2^{\gamma-1}$

$300\,(2v)^{0.4} = T_2\,(v)^{0.4}$

$T_2 = 300\,(2)^{0.4} = 395.85^0$k

$\therefore \quad du = (5/2)R\,[\,395.85 - 300\,]$

$= 2012.85$ J

and $dw = -\,2012.85$ J

$\therefore \quad$ total work done $= 1747 - 2012.85 = -\,265.85$ J

66. (a) $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$\therefore \quad TV^{\gamma-1} = \dfrac{T}{2}\,(5.66\,V)^{\gamma-1}$

$\gamma - 1 = \dfrac{\ln 2}{\ln 5.66} = \dfrac{0.693}{1.7334} = 0.4$

$\gamma = 1.4$

$\therefore \quad \gamma = 1 + \dfrac{2}{F} \quad \Rightarrow \quad F = 5$

(b) $W_A = \dfrac{nR(T_F - T_i)}{1-\gamma} = \dfrac{P_FV_F - P_iV_i}{1-\gamma}$

from $PV = nRT \qquad P_F\,V_F = \mu R\dfrac{T}{2} = \dfrac{PV}{2}$

$W_A = \dfrac{1}{0.4}\left[PV - \dfrac{PV}{2}\right] = 1.25\ PV.$

67. Energy supplied by the heater to the system in 10 minutes:

$Q_1 = P \times t = (90\ J/s) \times (10 \times 60\ s) = 54$ kJ

i.e., $Q_1 = (54/4.2)$kcal $= 12857$ cal.

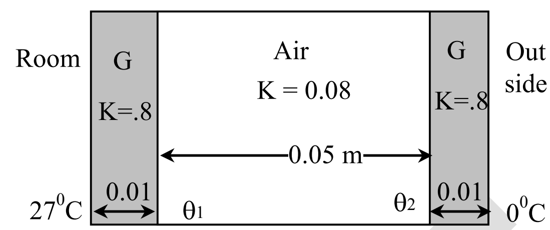
Now if T is the final temperature of the system energy absorbed by it to change its temperature from $10^0$ C to $T^0$C

$Q_2 = (m + W)c\Delta T = (360 + 40) \times 1 \times (T - 10)$ cal

According to given problem $Q_1 = Q_2$ i.e.,

$400\,(T - 10)\ 12857$ or $T = 42.14^0$C

68. In case of thermal conduction as




$$\frac{dQ}{dt} = KA\frac{\Delta\theta}{L} = \frac{\Delta\theta}{R}$$

with R = $\frac{L}{KA}$

$$R_{eq} = \Sigma\frac{L}{KA} = \frac{1}{A}\left[\frac{0.01}{0.80}\times 2 + \frac{0.05}{0.08}\right]$$

and as here A = 1 m$^2$ , $R_{eq} = \frac{1}{40} + \frac{5}{8} = \frac{26}{40}$

and hence $\frac{dQ}{dt} = \frac{\Delta\theta}{R} = \frac{(27-0)\times 40}{26}$ = 41.5 W

Now if $\theta_1$ and $\theta_2$ are the temperatures of air in contact with glass in the room and outside as shown in figure

$41.5 = 0.08 \times\ 1\frac{(27-\theta_1)}{0.01}$

and $41.5 = 0.80 \times\ 1\ \frac{(\theta_2 - 0)}{0.01}$

solving these for $\theta_1$ and $\theta_2$ we get

$\theta_1 = 26.48^0$ C and $\theta_2 = 0.52^0$ C.

69. Treating the given network of rods in terms of thermal resistance $R_x$ and $R_y$ with

$R_x = \frac{L}{A\times 0.92}$ and $R_y = \frac{L}{A\times 0.46}$ $\left[\text{as } R = \frac{L}{AK}\right]$

so that if $R_x = R$, $R_y = 2R_x = 2R$

Now as in this bridge [(P/Q) = (R/S)], so the bridge is balanced, i.e., the temperature of junctions C and D is equal and the rod CD becomes ineffective as no heat will flow through it.

Now as the thermal resistance of the bridge between junction B and E is

$\frac{1}{R_{BE}} = \frac{1}{(R+R)} + \frac{1}{(2R+2R)}$ i.e., $R_{BE} = \frac{4}{3}R$

The total resistance of bridge between A and E will be

$R_{eq} = R_{AB} + R_{BE}$

$= 2R + (4/3)R = (10/3)R$

So the net rate of flow of heat through the bridge will be

$$\frac{dQ}{dt} = \frac{\Delta\theta}{R_{eq}} = \frac{(60-10)}{(10/3)R} = \frac{15}{R}$$

Now if $T_B$ is the temperature at B,

$$\left[\frac{dQ}{dt}\right]_{AB} = \frac{\Delta Q}{R_{AB}} = \frac{60 - T_B}{2R}$$

But $\left[\frac{dQ}{dt}\right]_{AB} = \frac{dQ}{dt}$, i.e., $\frac{60 - T_B}{2R} = \frac{15}{R}$, i.e., $T_B = 30^0$ C

Also at B

$\left[\frac{dQ}{dt}\right]_{AB} = \left[\frac{dQ}{dt}\right]_{BC} + \left[\frac{dQ}{dt}\right]_{BD}$, i.e., $\frac{15}{R} = \frac{30 - T_C}{R} + \frac{30 - T_D}{2R}$

and as $T_C = T_D = T$, $30 = 3(30 - T)$, i.e., $T_C = T_D = 20^0C$

70. $\Delta U = \frac{3}{2}nR(T_2 - T_1)$

$\Delta W = \frac{k}{2}(x_2^2 - x_1^2)$

$P = \frac{kx}{S}$ or, $x = \frac{PS}{k}$ & $P = \frac{nRT}{Sx}$

or, $x^2 = \frac{nRT}{k}$

$\Delta W = \frac{nR}{2}(T_2 - T_1)$

$\Delta Q = \Delta U + \Delta W$

$= n2R(T_2 - T_1)$

$C = \frac{\Delta Q}{n\Delta T} = 2R$

71. Let $v_1$ be the total volume of iron at $0^0$C and let $V_1$ be the volume submerged in mercury

$\therefore\ k_1 = \frac{v_1}{V_1}$

at $80^0$C

$k_2 = v_2 / V_2$

also $v_2 = v_1\{1 + 80\,\gamma_{fe}\}$, $V_2 = V_1\{1 + 80\,\gamma_{Hg}\}$

$\therefore\ \frac{k_1}{k_2} = \frac{1 + 80\gamma_{Hg}}{1 + 80\gamma_{Fe}}$.

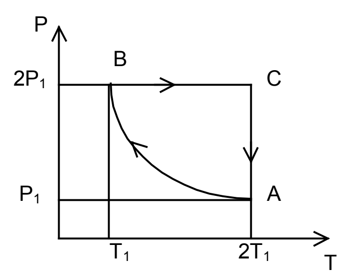
72. n = 2 moles, monatomic

$T_1 = 300$ K

PT = constant for path AB

Since PV = nRT

(a) PT = c (say)




$\Rightarrow P = \left(\frac{nRc}{V}\right)^{1/2}$

Work done on the gas $= -\int_{V_A}^{V_B}\left(\frac{nRc}{V}\right)^{1/2} dV$

$= -2\sqrt{nRc}\left[\sqrt{V_B} - \sqrt{V_A}\right] = -2nR\,[T_B - T_A]$

$= 1200$ R units

(b) Process $A \to B$

work done by the gas = -1200 R units

$\Delta U = 2 \times \dfrac{3R}{2} \times (-300) = -900R$ units

$\therefore\ \Delta Q_{AB} = \{-900\text{ R} + (-1200\text{R})\} = -2100$ R units(heat is released)

Process $B \to C$ (P = constant)

$\Delta Q_{BC} = 2 \times (5R/2) \times 300 = 1500$ R units (heat is absorbed)

Process $C \to A$

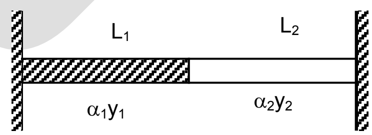
Since $\Delta U_{CA} = 0$ (T = constant)

$\therefore\ \Delta Q_{CA} = W_{CA} = nR\,(2T_1)\ \ln\,(V_A/V_C)$

$= 1200\ R\ \ln\,(P_C/P_A)$

= 831.77 R units. (heat is absorbed)

73. Increase in length of composite rod due to heating




$(\Delta L)_{increase} = (L_1\alpha_1 + L_2\alpha_2)T$

Due to compressive forces from walls, decrease in length

$(\Delta L)_{decrease} = \left[\dfrac{L_1}{Y_1} + \dfrac{L_2}{Y_2}\right]\dfrac{F}{A}$

As the length of the composite rod remains unchanged, here

$$\frac{F}{A}\left[\frac{L_1}{Y_1} + \frac{L_2}{Y_2}\right] = [L_1\alpha_1 + L_2\alpha_2]T$$

$$F = \frac{A(L_1\alpha_1 + L_2\alpha_2)T}{\left[\dfrac{L_1}{Y_1} + \dfrac{L_2}{Y_2}\right]}$$

74. First process is constant volume. $\therefore$ w = 0

and $P \propto T$

$\therefore$ If pressure is reduced to $\eta$ times then temperature is also reduced to $\eta$ times.

New temperature will be $T_0/\eta$

But $Q = n.C_V.dT = n\left(R/_{\gamma-1}\right)dT$

$$= \frac{nR}{\gamma-1}\left(\frac{T_0}{\eta} - T_0\right) = \frac{nR\,T_0(1-\eta)}{\eta(\gamma-1)}$$

…(i)

During second process pressure is constant,

$\therefore$ P.dv = n.R.dT

and $Q = \Delta U + W = \dfrac{nR.dT}{\gamma-1} + n.R.dT$

$= n.R.dT\left[\dfrac{1}{\gamma-1} + 1\right] = n.R.dT\left(\dfrac{\gamma}{\gamma-1}\right)$

$= \dfrac{nR\gamma}{\gamma-1}\left(T_0 - \dfrac{T_0}{\eta}\right) = \dfrac{nR\gamma(\eta-1)T_0}{\eta(\gamma-1)}$ … (ii)

$Q = Q_1 + Q_2$ (from (i) and (ii))

$= \dfrac{nR\,T_0(1-\eta)}{\eta(\gamma-1)} + \dfrac{nR\gamma(\eta-1)T_0}{\eta(\gamma-1)}$

$= nR.T_0\left(1-\dfrac{1}{\eta}\right)$

here n =2 R = 8.3 $T_0$

= 300 K & $\eta$ = 2

$\therefore$ Q = 2.5 KJ

75. For if at any moment temperature of spheres be $\theta_1$ and $\theta_2$ respectively, $\theta_1 > \theta_2$ and specifice heat for spheres be C

For first sphere, rate of loss of heat

$-C\dfrac{d\theta_1}{dt} = 4A\sigma e T_0^3\,(\theta_1 - T_0) + \dfrac{Ka}{\ell}(\theta_1-\theta_2)$

$\downarrow$ $\downarrow$

(Heat loss through radiation) (heat loss through conduction)

For IInd sphere

$-C\dfrac{d\theta_2}{dt} = 4A\sigma e T_0^3(\theta_2 - T_0) - \dfrac{k}{\ell}a(\theta_1-\theta_2)$ (heat gain through conduction) . . (ii)

substrating equation (ii) from (I)

$-C\dfrac{d\theta_1-\theta_2)}{dt} = 4A\sigma\, e T_0^3\,(\theta_1 - \theta_2) + \dfrac{2ka}{\ell}(\theta_1-\theta_2)$

$= (4A\sigma T_0^3\,(\theta_1 - \theta_2) + \dfrac{2kg}{\ell}(\theta_1 - \theta_2)$

$= (4A\sigma\, e T_0^3 + \dfrac{2ka}{\ell})\,(\theta_1 - \theta_2)$

let $\theta_1 - \theta_2 = \phi$ and $H = (4A\sigma e T_0^3 + 2kg/\ell)$

$\Rightarrow \quad -C\dfrac{d\phi}{dt} = H\phi$

$\Rightarrow \quad \displaystyle\int_{T_1-T_2}^{\phi}\frac{d\phi}{\phi} = -\frac{H}{C}\int_0^t dt$

$\Rightarrow \quad t = \dfrac{C}{H}\log\left[\dfrac{T_1-T_2}{\phi}\right]$

$\Rightarrow \quad \phi = e^{-tH/C}\,(T_1 - T_2)$ .

76. Efficiency $\eta = \frac{W}{Q_1}$

Where W = work done during the complete cycle and $Q_1$ is the heat input

$W_{BC} = W_{DA} = 0$

$W = W_{AB} + W_{CD}$

$= \frac{nR}{\gamma - 1}[T_0 - T_1] + \frac{nR}{\gamma - 1}[T_2 - T_3]$

$= \frac{nR}{\gamma - 1}[T_0 - T_1 + T_2 - T_3]$

And $Q_1 = n\,C_v\,(T_2 - T_1) = \frac{nR}{\gamma - 1}[T_2 - T_1]$ Since $C_v = \frac{R}{\gamma - 1}$

$\eta = \frac{W}{Q_1} = \frac{T_0 - T_1 + T_2 - T_3}{T_2 - T_1} = 1 - \frac{T_3 - T_0}{T_2 - T_1}$

$T_0 V_0^{\gamma-1} = T_0 V_2^{\gamma-1} \Rightarrow T_1 = T_0\,(v_1/v_2)^{\gamma-1}$ . . . (1)

$T_2 V_2^{\gamma-1} = T_3 V_1^{\gamma-1} \Rightarrow T_2 = T_3\,(v_1/v_2)^{\gamma-1}$ . . . (2)

$T_1 - T_2 = (v_1/v_2)^{\gamma-1}(T_0 - T_3)$

$\Rightarrow \frac{T_1 - T_2}{T_0 - T_3} = (v_1/v_2)^{\gamma-1}$

$\therefore \quad \eta = 1 - \frac{1}{\left(\frac{V_1}{V_2}\right)^{\gamma-1}} = 0.412$

77. (a) As for adiabatic change $PV^{\gamma}$ = constant

i.e. $P\left(\frac{\mu RT}{P}\right)^{\gamma} = \text{constant}$ $[\text{as} PV = \mu RT]$

i.e. $\frac{T^{\gamma}}{P^{\gamma-1}} = \text{constant, so} \left(\frac{T_B}{T_A}\right)^{\gamma} = \left(\frac{P_B}{P_A}\right)^{\gamma-1}$ with $\gamma = \frac{5}{3}$

i.e. $T_B = T_A\left(\frac{2}{3}\right)^{1-\frac{1}{\gamma}} = 1000\left(\frac{2}{3}\right)^{2/5} = 850\text{K}$

so $W_A = \frac{\mu R[T_F - T_i]}{[1-\gamma]} = \frac{1 \times 8.31[1000 - 850]}{[(5/3) - 1]}$

i.e. $W_A = (3/2) \times 8.31 \times 150 = 1869.75$ J

(b) For $B \to C$, V= constant so $\Delta W = 0$

So from first law of thermodynamics

$\Delta Q = \Delta U + \Delta W = \mu C v \Delta T + 0$

or $\Delta Q = 1 \times \left(\frac{3}{2}R\right)(T_C - 850)$ as $Cv = \frac{3}{2}R$

Now along path BC, V = constant ; $P \propto T$

i.e. $\frac{P_C}{P_B} = \frac{T_C}{T_B}$, $T_C = \frac{(1/3)P_A}{(2/3)P_A} \times T_B = \frac{T_B}{2} = \frac{850}{2} = 425$ (2)

So $\Delta Q = 1 \times \frac{3}{2} \times 8.31(425 - 850) = -5297.625J$

[Negative heat means, heat is lost by the system]

(c) As A and D are on the same isochor

$\frac{P_D}{P_A} = \frac{T_D}{T_A}$, i.e., $P_D = P_A \frac{T_D}{T_A}$

But C and D are on the same adiabatic

$$\left(\frac{T_D}{T_C}\right)^{\gamma} = \left(\frac{P_D}{P_C}\right)^{\gamma-1} = \left(\frac{P_A T_D}{P_C T_A}\right)^{\gamma-1}$$

or $(T_D)^{1/\gamma} = T_C \left[\frac{P_A}{P_C T_A}\right]^{1-\frac{1}{\gamma}}$, i.e. $T_D^{3/5} = \left(\frac{T_B}{2}\right)\left[\frac{P_A}{(1/3)P_A 1000}\right]^{2/5}$

i.e. $T_D^{3/5} = \left[\frac{1}{2}\left(\frac{2}{3}\right)^{2./5} \times 1000\right]\left[\frac{3}{1000}\right]^{2/5}$ i.e., $T_D = 500K$

78. (a) For adiabatic process BC

$P_B V_B^{\gamma} = P_C V_C^{\gamma}$ . . . (1)

For isothermal process CA

$P_A V_A = P_C V_C$ . . . (2)

From (1) and (2)

$$V_c = \left[\frac{V_B^{\gamma}}{V_A}\right]^{\frac{1}{\gamma-1}} = 64 \text{ m}^3$$

$$P_C = \frac{P_A V_A}{V_C} = \frac{10^5}{64} \text{ N/m}^2$$

(b) Work done, $W = W_{AB} + W_{BC} + W_{CA}$

$$= P(V_B - V_A) + \frac{1}{\gamma - 1}[PV_B - P_C V_C] + PV_A \, \ell n \frac{V_A}{V_C}$$

Putting the values

$W = 4.9 \times 10^5$ J

$T_i$ = initial temperature $= \frac{P_1 V_1}{R} = \frac{\alpha V^2}{R}$

And heat capacity $= \frac{Q}{T_f - T_i} = \frac{\frac{\alpha v^2}{2}(\eta^2 - 1)\left[\frac{\gamma+1}{\gamma-1}\right]}{(\alpha v^2/R)[\eta^2 - 1]}$

$$C = \frac{R}{2}\left[\frac{\gamma+1}{\gamma-1}\right]$$

79. Say v is the initial volume of the gas.

Final volume $= \eta v$

Work done $= \int_v^{\eta v} P.dv = \int_v^{\eta v} \alpha v dv = \alpha(v^2/2)_v^{\eta v}$

$= \frac{\alpha v^2}{2}(\eta^2 - 1)$.

As pr. varies with volume as $P = \alpha . v$

Initial and final pressure are $\alpha v$ and $\alpha\eta v$.

Change in internal energy ; $du = nC_v\, dT = C_v\, dT$ for $(n = 1)$

And also $du = \frac{P_1 v_1 - P_2 v_2}{\gamma - 1} = \frac{\alpha v^2 \eta^2 - \alpha v^2}{\gamma - 1} = \frac{\alpha v^2(\eta^2 - 1)}{(\gamma - 1)}$

We have $Q - w = u$

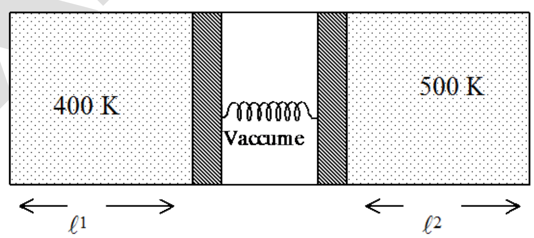
$\therefore \quad Q = u + w = \frac{\alpha v^2}{\gamma - 1}(\eta^2 - 1) + \frac{\alpha v^2}{2}(\eta^2 - 1)$

$= \frac{\alpha v^2}{2}(\eta^2 - 1)\left[\frac{2}{\gamma - 1} + 1\right]$

$= \frac{\alpha v^2}{2}(\eta^2 - 1)\left[\frac{\gamma + 1}{\gamma - 1}\right]$

hence $T_f$ = final temperature $= \frac{P_2 v_2}{R} = \alpha\eta^2 v^2/R$

80. Let $\ell_1$ and $\ell_2$ be the final length of the two parts, then from gas equation




$\frac{P_0 A \ell_0}{T_0} = \frac{PA\ell_1}{T_1} = \frac{PA\ell_2}{T_2}$ . . .. (i)

Considering the equilibrium of the piston in initial and final states, we get

$P_0 A = k\, x_0$, $PA = kx$.

$\Rightarrow \frac{P}{P_0} = \frac{x}{x_0}$ . . . .(ii)

decrease in the length of spring = total increase in the length of the two chambers

$x - x_0 = \ell_1 + \ell_2 - 2\ell_0$ . . .. (iii)

from relation (I)

$\ell_1 = \frac{P_0 T_1 \ell_0}{PT_0}$, $\ell_2 = \frac{P_0 T_2 \ell_0}{PT_0}$

using (ii)

$\ell_1 = \frac{x_0 T_1 \ell_0}{xT_0}$, $\ell_2 = \frac{x_0 T_2 \ell_0}{xT_0}$

putting these in (iii)

$x - x_0 = \frac{x_0 \ell_0}{x T_0}(T_1 + T_2) - 2\ell_0$

putting values and solving, we get $x = \frac{\sqrt{13}-1}{2} = 1.3$ m.

81. Initial charge on the capacitor $q_0 = CV_0 = 75 \times 10^{-3} \times 213\frac{1}{3} = 16C$

The charge on the capacitor decays as

$q = q_0 e^{-t/RC}$

At $t = 2.5 \ln(4)$ minutes $= 150 \ln(4)$ sec.

$q = 16 \times e^{-\ln(4)}$ $\because RC = 150s$

$= 4$ C

Total heat dissipated in the resistor in the given time $= \frac{q_0^2 - q^2}{2C} = 1.6$kJ

= heat imparted to the gas

(a) Work done by the gas at constant pressure $= P\Delta V = \nu R\Delta T \approx 0.182$ kJ

(b) Increment in the internal energy $\Delta U = Q - W = 1.6 - 0.182 = 1.418$ kJ

(c) $\gamma = 1 + \frac{\nu R\Delta T}{\Delta U} = 1.12$

82. $P_0V_0 = RT_0$, $T_0 = 200$ K

(i) $P_A V_A = RT_A$, $T_A = \frac{P_A V_A}{R} = \frac{2P_0V_0}{R} = 400$ K

Similarly, $T_B = 1200$ K, $T_C = 600$ K

(ii) $Q_{DA} = C_v \Delta T_{DA} = C_v (T_A - T_D)$

$= \frac{3}{2}R \times 200 = 300$ R

$Q_{AB} = C_P \Delta T_{AB} = C_v (T_B - T_A) = 2000$ R

$Q_{BC} = -900R$,

$Q_{CD} = -1000$ R.

(iii) $\Delta W = -400$ R, $\eta = \frac{\Delta W}{Q_+} = \frac{4}{23}$.

83. The expansion is isothermal, hence PV = constant

also P.dV + V. dp = 0

or $\frac{dP}{P} = -\frac{dv}{V}$

Let pressure decrease by $\Delta P$ and volume increase by $v_l$

During one cycle.

$$\int_{P_1}^{P_1-\Delta P}\left(\frac{dP}{P}\right) = -\int_{v}^{v+v_1}\left(\frac{dv}{v}\right)$$

$$\ln\left(\frac{P_1-\Delta P}{P_1}\right)=-\ln\left(\frac{v+v_1}{v}\right) \qquad \ldots (i)$$

at the beginning of the second cycle, pressure becomes
$P_1$ - $\Delta P$ and volume becomes V again.
At the end of second cycle, pressure becomes $P_1$ - $2\Delta P$

$$\therefore \qquad \ln\left(\frac{P_1-2\Delta P}{P_1-\Delta P}\right) = -\ln\left(\frac{v+v_1}{v}\right) \qquad \ldots (ii)$$

(ii) can also written as

$$\ln\left[\frac{P_1-2\Delta P}{P_1}.\frac{P_1}{P_1-\Delta P}\right]=-\ln\left(\frac{v+v_1}{v}\right)$$

$$\text{or } \ln\left(\frac{P_1-2\Delta P}{P_1}\right)-\ln\left(\frac{P_1-\Delta P}{P_1}\right)=-\ln\left(\frac{v+v_1}{v}\right)$$

$$\text{or } \ln\left(\frac{P_1-2\Delta P}{P_1}\right)=\ln\left(\frac{P-\Delta P}{P_1}\right)-\ln\left(\frac{v+v_1}{v}\right) \qquad \ldots (iii)$$

From (i) and (iii)

$$\ln\left(\frac{P_1-2\Delta P}{P_1}\right)=-2\ln\left(\frac{v+v_1}{v}\right)$$

When Process is repeated 'n' times,

$$\ln\left(\frac{P_1-n\Delta P}{P_1}\right)=-n\ln\left(\frac{v+v_1}{v}\right)$$

But $P_1$ - n. $\Delta P$ = $P_2$ = final pressure

$$\therefore \qquad \ln\left(\frac{P_2}{P_1}\right)=-n\ln\left(\frac{v+v_1}{v}\right)$$

and $n = \dfrac{\ln(P_1/P_2)}{\ln(v+v_1)/v}$.

84. Time lost $\Delta t = 43200 \;\propto T$
$= 43200 \times 1.2 \times 10^{-5} \times 20$
= 10.4 sec.

85. $P_{O_2} = \dfrac{\eta_{O_2}RT}{V} = 0.1\left[\dfrac{8.31\times 300}{2\times 10^{-3}}\right]$
$= 1.25 \times 10^5$ Pa
$P_{CO2} = \dfrac{\eta_{Co_2}RT}{V} = 0.2\left[\dfrac{8.31\times 300}{2\times 10^{-3}}\right]$
$= 2.5 \times 10^5$ Pa
$\therefore$ Total pressure $= P_{O_2} + P_{CO_2}$
$= (1.25 \times 10^5 + 2.5 \times 10^5)$ Pa
$= 3.75 \times 10^5$ Pa.

86. $PV = nRT = \frac{m}{M}RT$

$$\frac{P}{\rho T} = \frac{R}{M}$$

$$\therefore \quad \left(\frac{P}{\rho T}\right)_{Top} = \left(\frac{P}{\rho T}\right)_{bottom}$$

$$\frac{\rho_T}{\rho_B} = \frac{P_T}{P_B} \times \frac{T_B}{T_T} = \frac{70}{76} \times \frac{300}{280} = \frac{75}{76} = 0.9868.$$

87. $f = \mu N$ (Kinetic Friction) = $\mu$ mg

$\therefore$ Work done against friction = $\mu$ mg(s)

's' is the distance moved by body.

$\therefore$ $\mu$ mg(s) = 0.5(25) 10 (20) ($10^3$) = (2.5) $10^6$J

$\therefore$ Heat generated in calories = $\frac{2.5 \times 10^6}{4.2}$ = 595 x $10^3$ calories.

Heat absorbed = 50% of (595) $10^3$ = (297.5) $10^3$ calories

But $Q = m.s.\Delta t$

$297.5 \times 10^3 = 25 \times 0.1 \times 10^3 \times \Delta t$

$\Delta t = 118.8^0C$

88. Say v is the initial volume of the gas.

Final volume = $\eta v$

Work done = $\int_v^{\eta v} P.dv = \int_v^{\eta v} \alpha v dv = \alpha(v^2/2)_v^{\eta v}$

$$= \frac{\alpha v^2}{2}(\eta^2 - 1).$$

As pr. varies with volume as $P = \alpha . v$

Initial and final pressure are $\alpha v$ and $\alpha \eta v$.

Change in internal energy ; $du = nC_v\, dT = C_v\, dT$ for (n = 1)

And also $du = \frac{P_1v_1 - P_2v_2}{\gamma - 1} = \frac{\alpha v^2\eta^2 - \alpha v^2}{\gamma - 1} = \frac{\alpha v^2(\eta^2 - 1)}{(\gamma - 1)}$

$$\therefore \quad Q = u + w = \frac{\alpha v^2}{\gamma - 1}(\eta^2 - 1) + \frac{\alpha v^2}{2}(\eta^2 - 1)$$

$$= \frac{\alpha v^2}{2}(\eta^2 - 1)\left[\frac{2}{\gamma - 1} + 1\right]$$

$$= \frac{\alpha v^2}{2}(\eta^2 - 1)\left[\frac{\gamma + 1}{\gamma - 1}\right].$$

89. The heat lost by body $\Delta Q = ms(\theta_1 - \theta_0)$

Let $\theta_2$ be the temperature after loosing 90 % of $\Delta Q$

$$\frac{90}{100}\Delta Q = ms(\theta_1 - \theta_2)$$

$0.9(\theta_1 - \theta_0) = (\theta_1 - \theta_2)$

$\theta_2 = \theta_0 = 0.1\ (\theta_1 - \theta_0)$

$$\frac{\theta_2 - \theta_0}{\theta_1 - \theta_0} = 0.1$$

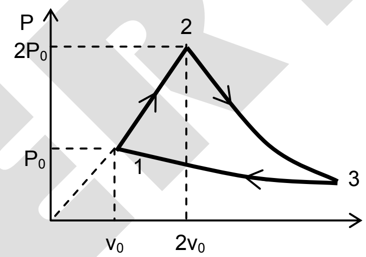
According to Newton's law of cooling

$$-\frac{d\theta_2}{dt} = k(\theta - \theta_0)$$

$$\ln\frac{(\theta_2 - \theta_0)}{(\theta_1 - \theta_0)} = -kt$$

$$t = -\frac{1}{k}\ln(1/10) = \frac{\ln 10}{k}.$$

90. For process $1 \to 2$




$P = V$

Since $PV = nRT$

$\Rightarrow V^2 = RT \qquad \Rightarrow T_2 = 4T_0$

$$\Delta W_{12} = \int_{v_0}^{2v_0} Pdv = \frac{4v_0^2 - v_0^2}{2} = \frac{3}{2}v_0^2 = \frac{3}{2}RT_0$$

$$\Delta U_{12} = C_v\ \Delta T = \frac{5}{2}R(3T_0) = \frac{15RT_0}{2}$$

$\Rightarrow \Delta Q_{12} = 9RT_0$

for process $2 \to 3$ $\quad (TV^{\gamma-1} = \text{constant} \Rightarrow v_3 = 64\ v_0)$

$\Delta Q_{23} = 0 \qquad \Delta U_{23} = C_v\ \Delta T = -\ C_v(3T_0)$

$$\Rightarrow \Delta U_{23} = -\frac{15}{2}RT_0 \qquad \Rightarrow \Delta W_{23} = -\ \Delta U_{23} = \frac{15}{2}RT_0$$

for process $3 \to 1$

$$\Delta U_{31} = 0, \qquad \Delta Q_{31} = \Delta W_{31} = -\ RT_0 \ln\frac{v_3}{v_1}$$

$\Rightarrow \Delta Q_{31} = -\ RT_0 \ln 64$

$$\eta = \frac{\text{work done}}{\text{heat input}} \times 100 = \frac{\left(\frac{3}{2}RT_0 + \frac{15}{2}RT_0 - RT_0 \ln 64\right)}{9RT_0} \times 100$$

$$= \left(\frac{9 - 6\ln 2}{9}\right) \times 100 = \left(\frac{3 - 2\ln 2}{3}\right) \times 100 = 53.8\ \%.$$