St. Francis Institute of Technology

Complex Variable (Practice Questions)

(i) Determine the constant a, b, c, d, e if
$f(z) = (ax^3 + b\,xy^2 + 3x^2 + cy^2 + x) + i(dx^2y - 2y^3 + exy + y)$ is analytic.

(ii) Determine the constant a, b, c, d, e if
$f(z) = (ax^4 + b\,x^2y^2 + cy^4 + dx^2 - 2y^2) + i(4x^3y - exy^3 + 4xy)$ is analytic.

(iii) Check whether the following function are analytic or not.
1. $f(z) = (x^3 - 3xy^2 + 3x) + i\,(3x^2y - y^3 + 3y)$
2. $f(z) = z^2 - \bar{z}$.
3. $f(z) = \cosh z$ and hence find $f'(z)$

(iv) Show that $u = y^3 - 3x^2y$ is a harmonic function.

(v) Check whether $v = e^x \cos y + x^3 - 3xy^2$ is a harmonic function

(vi) Construct an analytic function whose real part is
$$\frac{x}{2}\log(x^2+y^2) - y\tan^{-1}\left(\frac{y}{x}\right) + \sin x \cosh y$$
OR If $u = \frac{x}{2}\log(x^2+y^2) - y\tan^{-1}\left(\frac{y}{x}\right) + \sin x \cosh y$, find its harmonic conjugate and the corresponding analytic function

OR Find the orthogonal trajectories of the family of curve
$$\frac{x}{2}\log(x^2+y^2) - y\tan^{-1}\left(\frac{y}{x}\right) + \sin x \cosh y = c$$

(vii) Find the imaginary part of the analytic function whose real part is
$e^x(x\cos y - y\sin y)$

(viii) Find the analytic function whose real part is $x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$

(ix) Construct an analytic function whose Imaginary part is $x^2 - y^2 + \frac{x}{x^2+y^2}$

OR If $v = x^2 - y^2 + \frac{x}{x^2+y^2}$, find its harmonic conjugate and the corresponding analytic function

OR Find the orthogonal trajectories of the family of curve
$$x^2 - y^2 + \frac{x}{x^2+y^2} = c$$

(x) Find the analytic function whose imaginary part is
$$\frac{x}{x^2+y^2} + \cosh x \cos y$$

(xi) Find the analytic function $f(z) = u + iv$ such that
$$u + v = e^x(\cos y + \sin y)\ .$$

(xii) Find the analytic function $f(z) = u + iv$ such that
$$u - v = (x-y)(x^2 + 4xy + y^2)$$