St. Francis Institute of Technology, Mumbai-400 103

Department of Information Technology

A.Y. 2020-2021

Class: SE-ITA/B, Semester: III

Subject: DATA STRUCTURE LAB

Experiment – 11 Case Study on Tower of Hanoi

1. **Aim:** Case Study on Tower of Hanoi to calculate the runtime complexity.

2. **Objectives:** After study of this experiment, the student will be able to
   - To learn the applications of stacks
   - To understand the Tower of Hanoi algorithm

3. **Outcomes:** After study of this experiment, the student will be able to • Identify the complexity of Tower of Hanoi algorithm
   - Understand the applications of stacks.
   - Illustrate and examine the methods of stacks

4. **Prerequisite:** Stack, operations of stack

5. **Requirements:** PC and Turbo C compiler version 3.0

6. **Laboratory Exercise:** (Prepare a word document and print it) **Brief Theory:**

A. Tower of Hanoi

a. Explain the concept

The **Tower of Hanoi,** is a mathematical problem which consists of three rods and multiple disks. Initially, all the disks are placed on one rod, one over the other in ascending order of size similar to a cone-shaped tower.

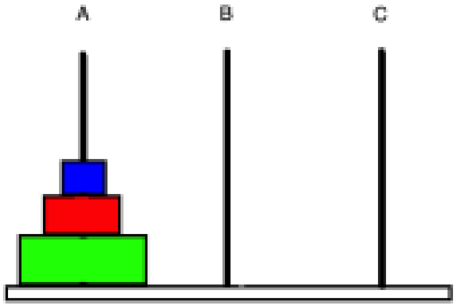
The objective of this problem is to move the stack of disks from the initial rod to another rod, following these rules:

- A disk cannot be placed on top of a smaller disk
- No disk can be placed on top of the smaller disk.

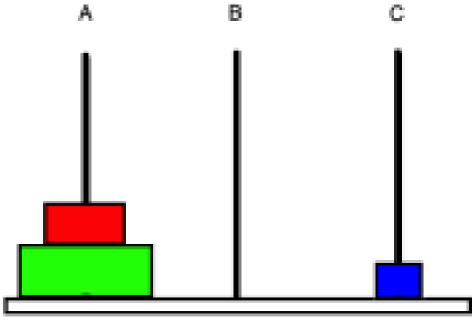
The goal is to move all the disks from the leftmost rod to the rightmost rod. To move N disks from one rod to another, 2^��−1 steps are required. So, to move 3 disks from starting the rod to the ending rod, a total of 7 steps are required.

b. Example with suitable diagrams

Let's see an example of the puzzle in action when there are a total of three disks. We denote the rods by A, B, and C. We start with all three disks stacked on one of the three rods (rod A in this case):

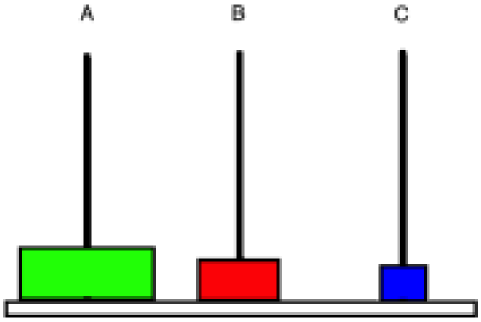


Let’s say the goal is to move all three disks to rod C (which rod we choose as our destination doesn’t matter). Our first move can only involve the smallest disk since there is only one stack and this disk is at the top of it. So let’s move the smallest disk to rod C:

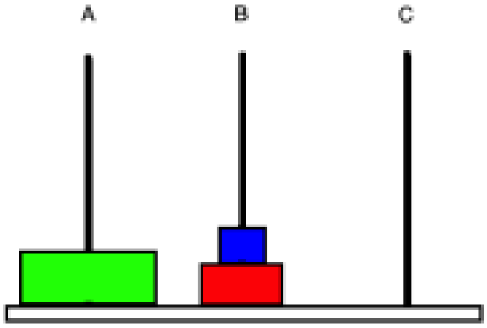


Now we have two disks that we can move, namely the smallest and the second smallest. Moving the smallest disk will not accomplish much since moving it to rod A gets us back to the original state, and moving it to rod B is redundant since we could have just moved the smallest disk to rod B in the first place.

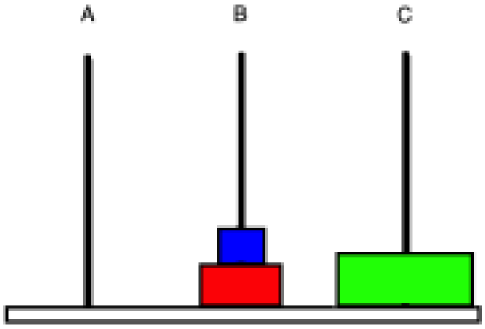
So, it’s clear that we should move the second smallest disk. Let’s move it to rod B:



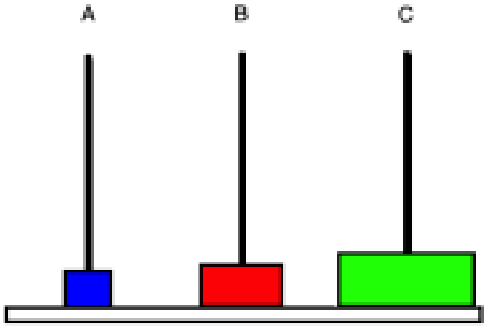
Now let’s move the smallest disk to rod B as well:



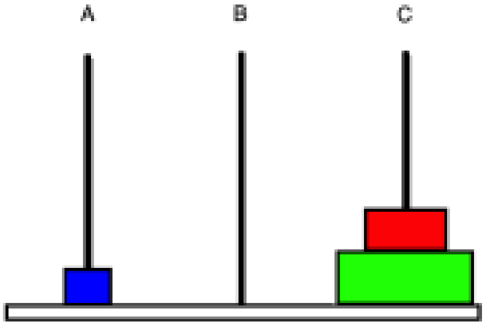
We know that we want the largest disk at the bottom of rod C so our next move is to move it there:



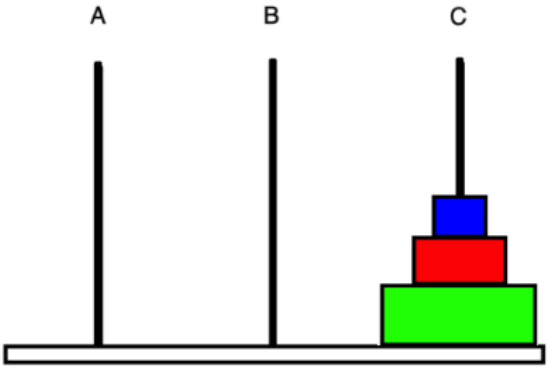
Now we need to move the stack on rod B to rod C. To do this we can first move the smallest disk to rod A:



Then we can move the second smallest disk to rod C:



Finally, we move the smallest disk to rod C as well and we are done!



c. Discuss and derive the time complexity

Recursive Equation : T(n) = 2T(n-1) + 1 ——-equation-1

Solving it by BackSubstitution :

T(n-1) = 2T(n-2) + 1 ————equation-2
T(n-2) = 2T(n-3) + 1 ————equation-3
Put value of T(n-2) in equation–2 with help of equation-3
T(n-1)= 2(2T(n-3) + 1) + 1 ——equation-4

Put value of T(n-1) in equation-1 with help of equation-4
T(n)= 2(2(2T(n-3) + 1) + 1) + 1
T(n) = 2^3 T(n-3) + 2^2 + + 2^1 + 1

After Generalization :
T(n)= 2^k T(n-k) + 2^{(k-1)} + + 2^{(k-2)} + ... +2^2 + + 2^1 + 1

Base condition T(0) == 1
n – k = 0
n = k;
put, k = n
T(n) = 2^n T(0) + 2^{(n-1)} + + 2^{(n-2)} + ... +2^2 + + 2^1 + 1

It is GP series, and sum is 2^{(n+1)} - 1
T(n)= O(2^{(n+1)} - 1), or you can say O(2^n) which is exponential.
7. Post-Experiments Exercise: (handwritten)

A. Questions:

1. Show the steps in finding solution for Tower of Hanoi puzzle for moving 3 discs from pole A to pole B using pole C.
2. Implement Tower of Hanoi using ‘C’
3. Explain Tail and Non-tail recursion.
4. Define winding and unwinding phase of recursion.

B. **Conclusion:**

1. Summary of Experiment
2. Importance of Experiment

C. **References:**

1. S. K Srivastava, Deepali Srivastava; Data Structures through C in Depth; BPB Publications; 2011.
2. Reema Thareja; Data Structures using C; Oxford.
3. Data Structures A Pseudocode Approach with C, Richard F. Gilberg & Behrouz A. Forouzan, second edition, CENGAGE Learning.
4. https://www.educba.com/stack-in-data-structure/
5. https://www.javatpoint.com/data-structure-stack