PULSE MODULATION TECHNIQUES

[redacted]

→ Here, modulating signal → sine wave (analog)

→ Carrier → is a periodic rectangular pulses

→ Depending upon what will be the output — Pulse mod. tech can be analog or digital.

Pulse Modulation Techniques

- Analog
  - PAM
  - PWM/PDM/PLM
  - PPM
- Digital — Dig O/P in terms of 0's & 1's.
  - PCM
  - DM
  - ADM

→ PAM — Pulse amplitude modulation

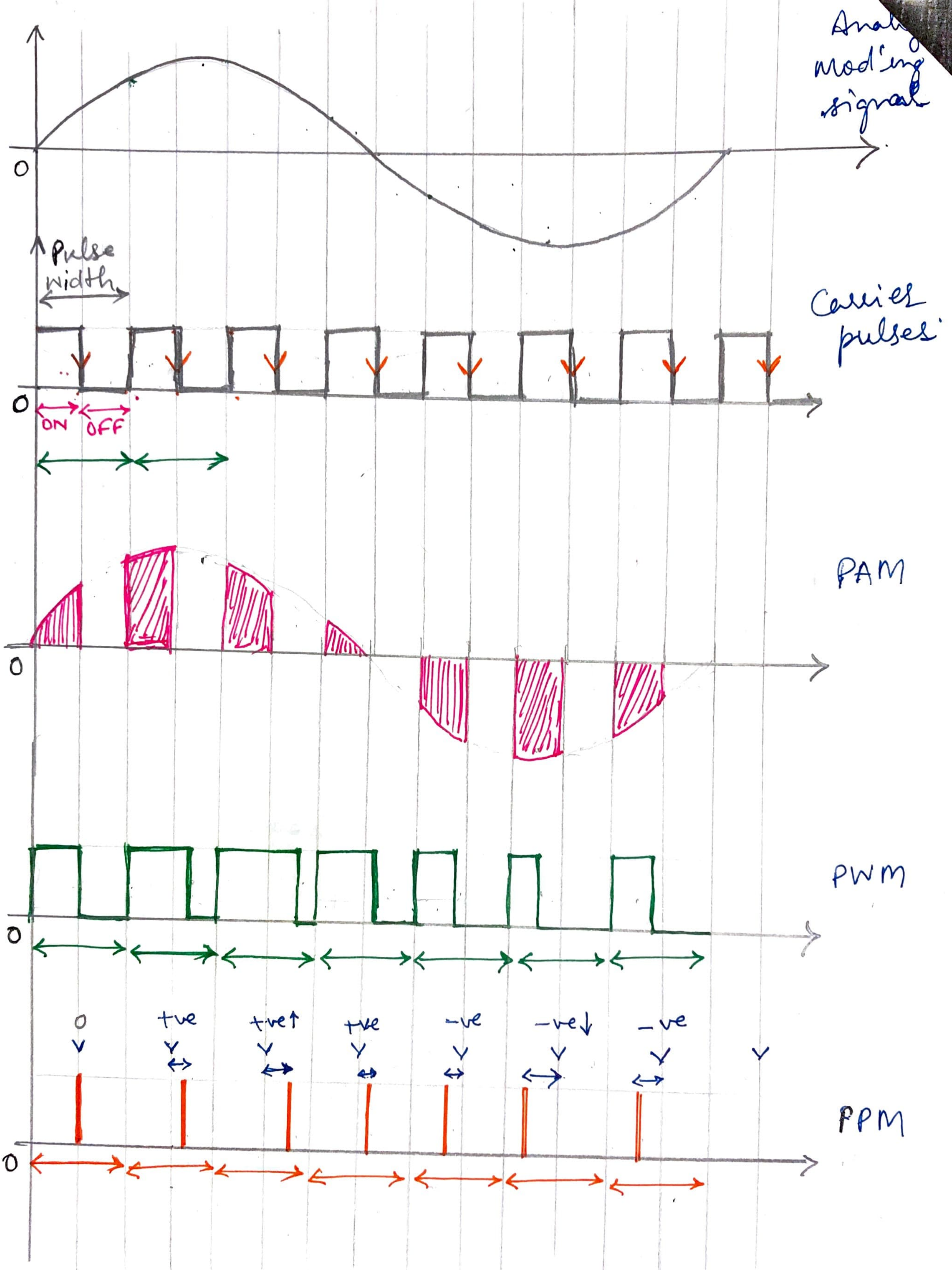
→ PWM/PDM/PLM — Pulse width modulation/ Pulse distance modulation/ Pulse length modulation

→ PPM — Pulse position modulation

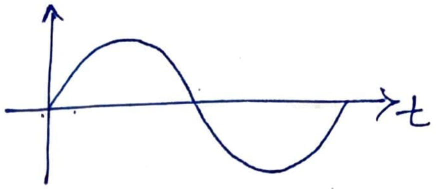
→ PCM — Pulse code modulation

→ DM — Delta modulation

→ ADM — Adaptive delta modulation


Anal
mod'ing
signal
0
Pulse width
Carrier pulses
0
ON
OFF
PAM
0
PWM
0
0
+ve
+ve↑
+ve
-ve
-ve↓
-ve
PPM
0

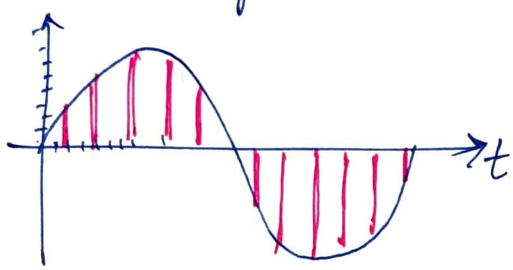
Analog → continuous signal



Digital → ✗ NOT
in terms of 0's & 1's.

→ Then... what did I get...
DISCRETE SIGNAL

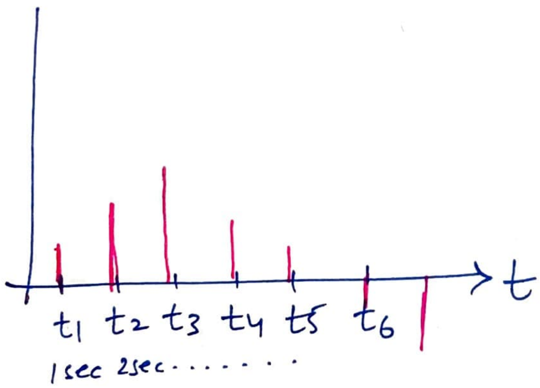
Analog



x-axis → continuous
y-axis → continuous.

Digital → 2 values (0 & 1)

Discrete

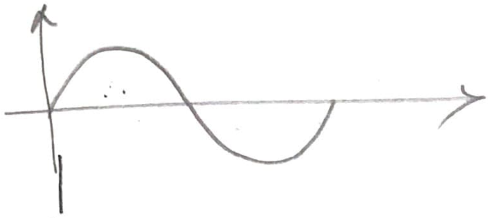


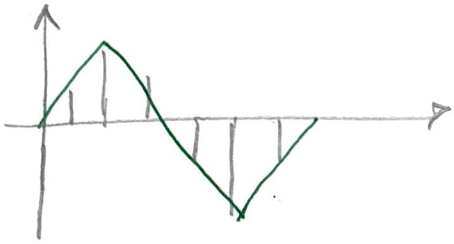
3·946459
5·004987

→ SAMPLING PROCESS –

→ Sampling is the process of converting a continuous analog signal to a discrete analog signal and the sampled signal is the discrete time representation of original modulating signal.

→ NOTE – Sampled signal should represent the original signal faithfully and we should be able to reconstruct the original signal from its sampled version.





→ We require to take as many samples of original signal as possible.

→ Number of samples depends on "Sampling Rate" and max. frequency of the signal to be sampled.

[redacted]

→ NYQUIST CRITERION :-

→ A continuous time signal $x(t)$ can be completely represented in its sampled form and recovered back from the sampled form if the sampling frequency, $f_s \geq 2f_m$, where, ~~fm~~ f_m is the maximum frequency of continuous time signal, $x(t)$.

→ This minimum sampling rate of $2f_m$ samples / sec for $x(t)$ having max. freq of f_m Hz is called Nyquist Rate.

→ The reciprocal of Nyquist Rate (ie. $\frac{1}{2f_m}$) is called Nyquist Interval.

Eg. If max. modulating frequency,

$$f_m = 4000 \text{ Hz}$$

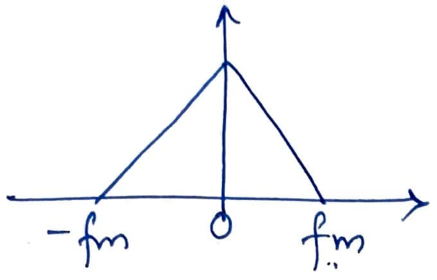
$$\Rightarrow \text{Sampling frequency} = f_s = 2f_m$$

$$= 2 \times 4000 \text{ Hz}$$

$$= 8 \text{ kHz}$$

$$\therefore \text{Sampling rate} = \frac{1}{8000} = 125 \ \mu\text{sec}.$$

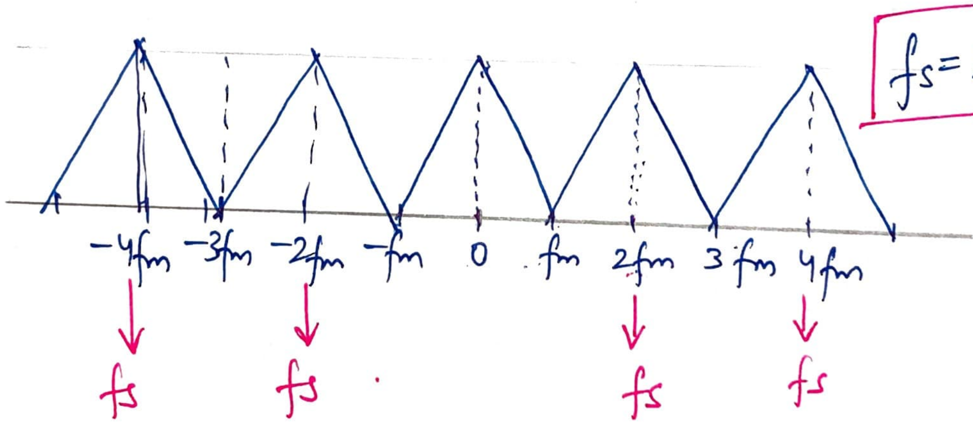
→ Frequency Spectrum –



Frequency spectrum of original signal, $x(t)$

Sampling results in a periodic spectrum of periodicity equal to sampling rate.

∴ freq. spectrum of sampled signal is –



$\boxed{f_s = 2f_m}$

→ Aliasing or foldover Error –

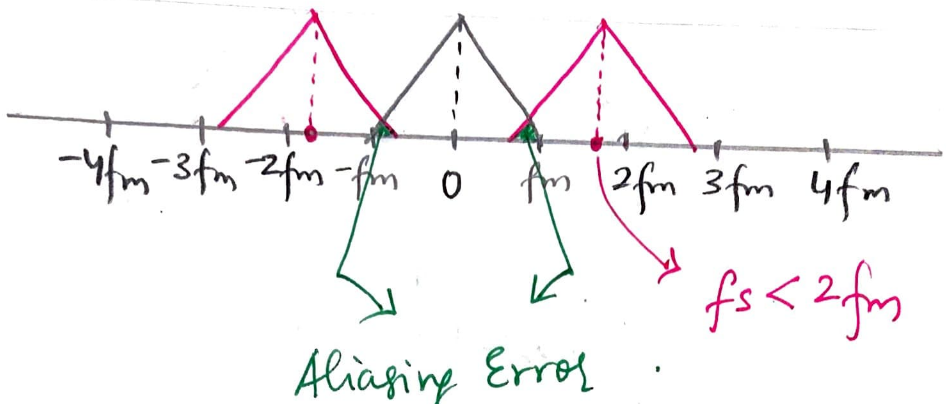
→ If signal $x(t)$ is not strictly bandlimited.

and/or

If sampling freq, $f_s < 2f_m$

then, an error called aliasing or foldover error is observed.

$\boxed{f_s < 2f_m}$



→ Higher frequency in $x(t)$ takes on the identity of lower frequency due to aliasing

or

Some information ~~is~~ contained in $x(t)$ is lost in the process of sampling.

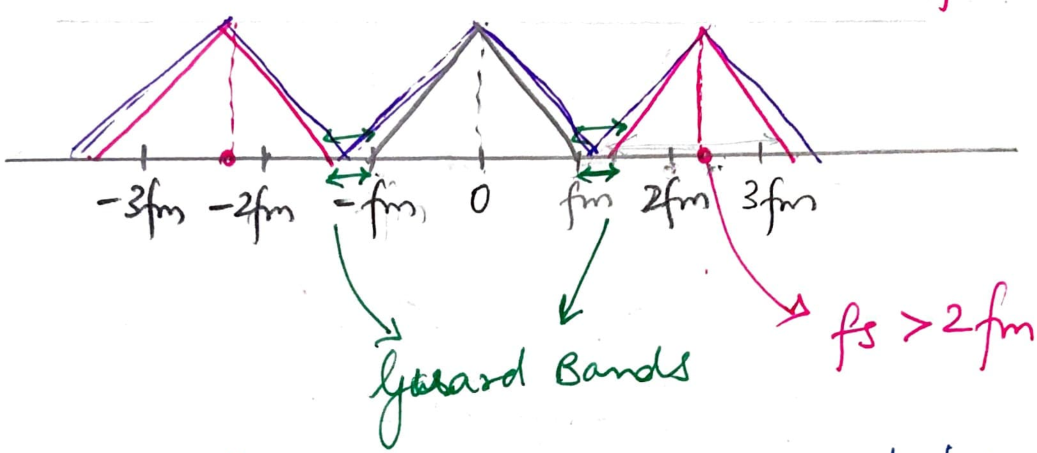
→ Due to overlapping, portions of frequency shifted replicas are folded over inside the desired spectrum.

→ <u>How to eliminate Aliasing —</u>

→ Use bandlimited LPF (Low Pass filter) before sampling (Filter called Anti-aliasing filter or pre-alias filter)

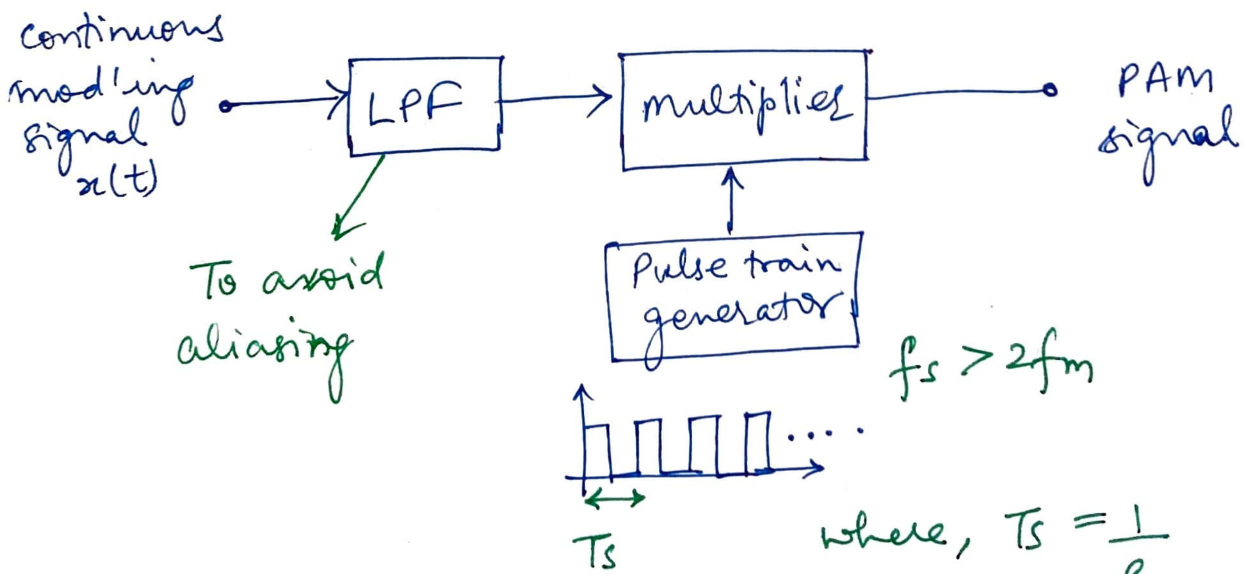
→ Increase f_s such that $f_s >> 2f_m$.
(Due to this, even though $x(t)$ not strictly bandlimited, spectrum will not overlap.

$f_s >> 2f_m$



Increasing $f_s >> 2f_m$, creates guard bands between adjacent spectrums.

→ PAM Generation Techniques –

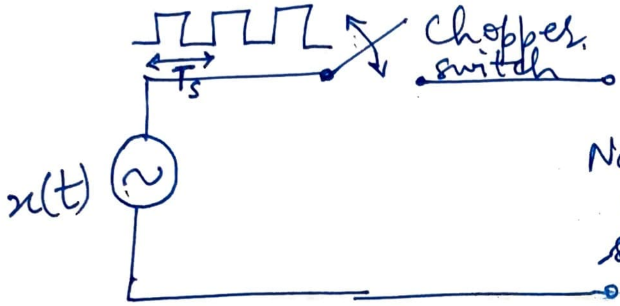


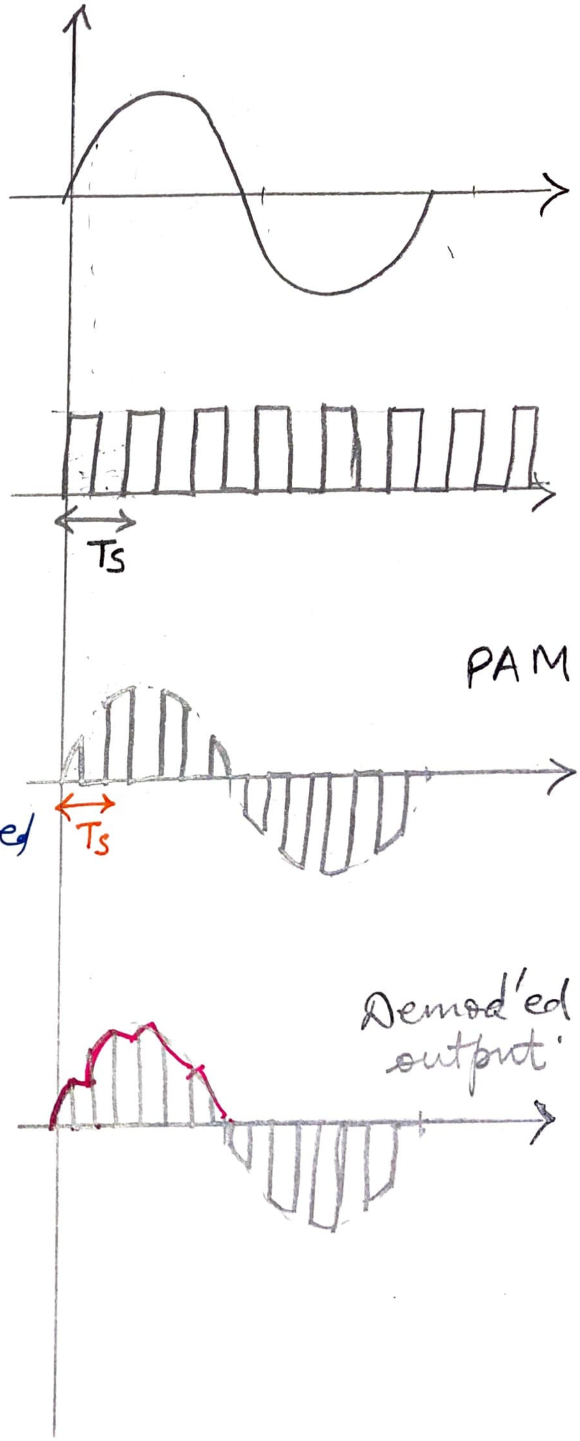
where, $T_s = \frac{1}{f_s}$

→ PAM can be classified into 3 Types.

I. Natural Sampling

Generation —



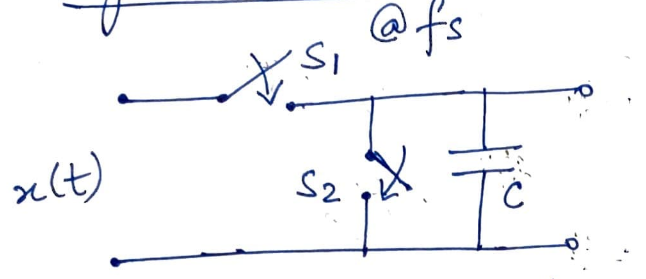


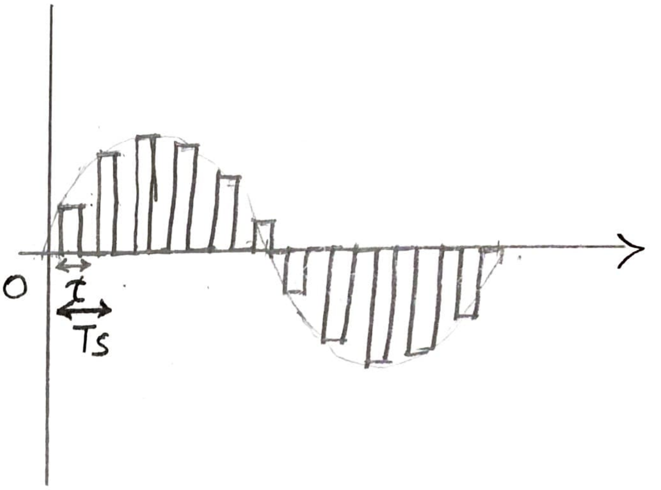
Degeneration —

→ LPF cut off adjusted to fm so that all high frequency ripples is removed and original modulating signal is recovered back.

II. Flat-top Sampling –

Generation





$S_1 \rightarrow$ Sampling switch
$S_2 \rightarrow$ Discharge switch.

→ The circuit consists of two switches S_1 & S_2 and a capacitor, C.

→ S_1 is closed at the instant of sampling for a very short time.

→ C charges to the sampled value, $x(nT_s)$

→ S_1 is opened.

→ Both S_1 & S_2 kept open for a duration of "τ" sec. & C holds voltage across it constant for this period

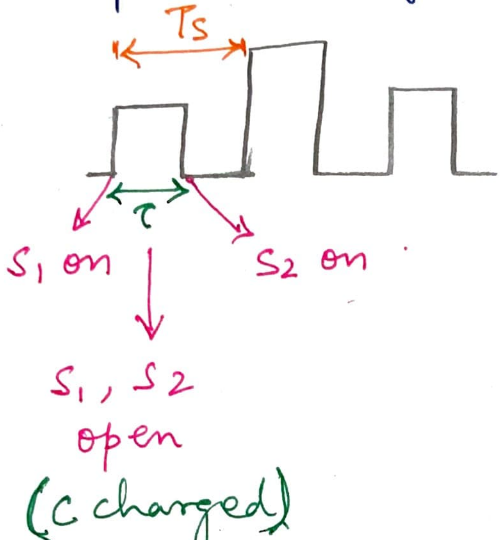
⇒ Pulse is stretched for "τ" sec.

→ At the end of time, τ, switch S_2 is closed.

→ This will short circuit the C and output reduces to zero.

→ S_1 is controlled by clock with time period of T_s $(= 1/f_s)$

→ Analog signal $x(t)$ is sampled instantaneously @ $f_s = \frac{1}{T_s}$ and the duration of each sample is lengthened ~~to~~ to a duration, τ.

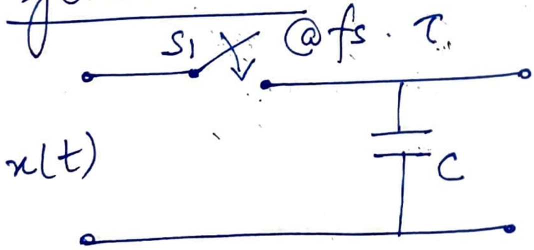


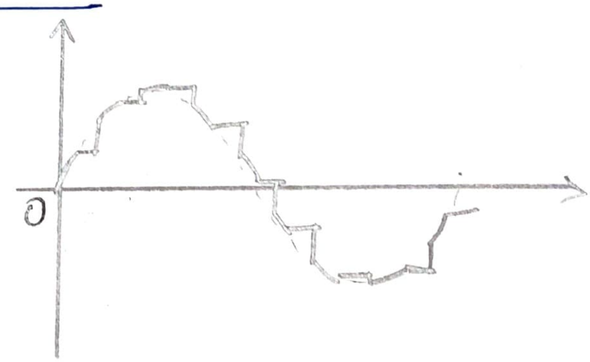
→ Degeneration

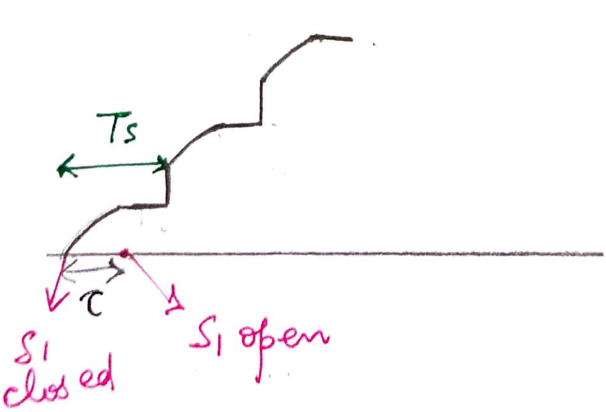
PAM signal —— LPF —— Demodulated signal.

III. Sample & Hold Circuit –

Generation –







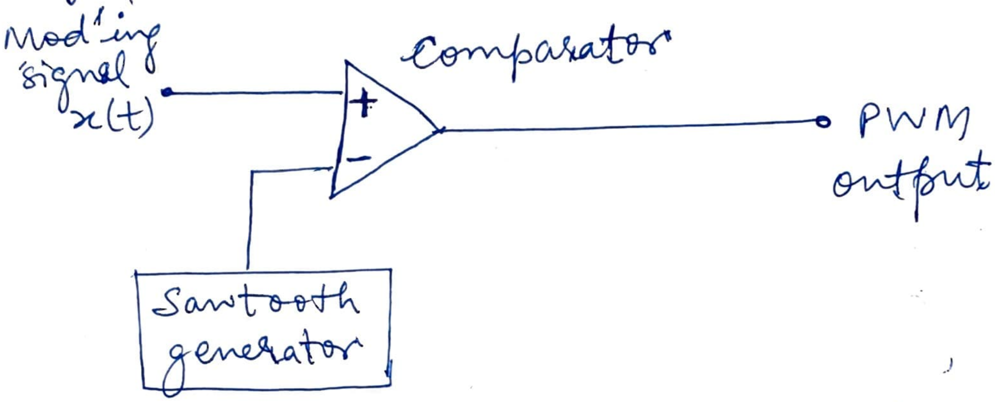
Degeneration –

PAM signal —— LPF —— Demod signal.

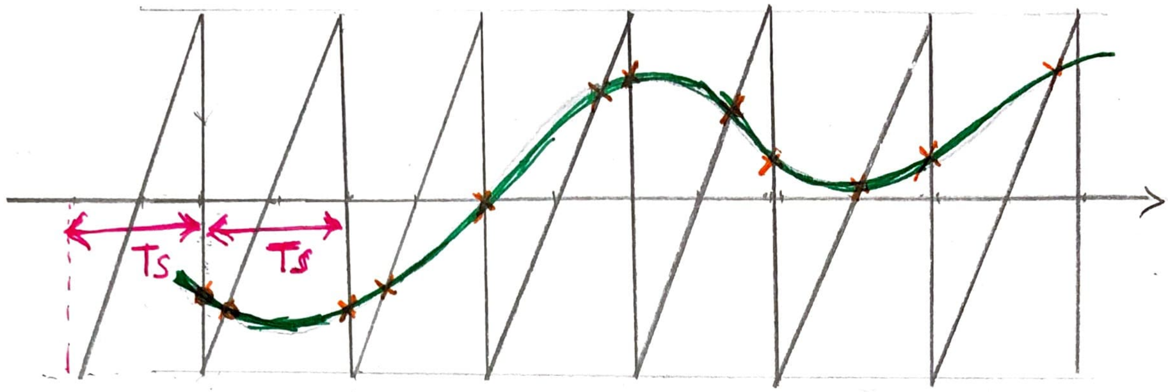
→ Pulse Width Modulation (PWM) –

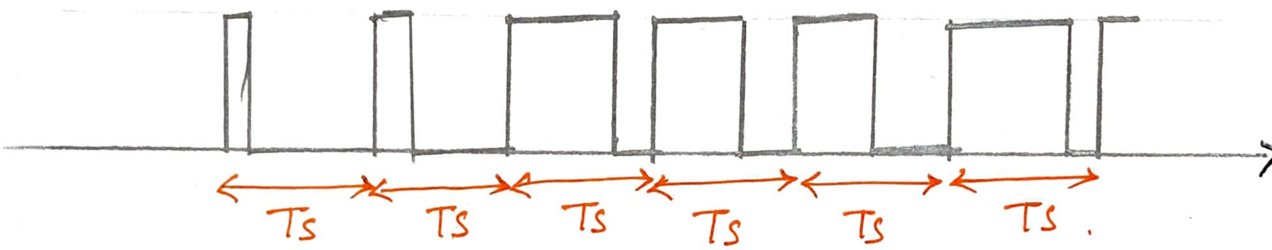
→ Here, amplitude and frequency of pulses remain constant.
→ Info is contained in width variation.
→ PWM offers better noise immunity.

Generation –



→ Sawtooth signal is the sampling signal (f_s).
→ It is applied to the inverting terminal of the comparator.
→ The modulating signal, x(t) is applied to the non-inverting terminal of the same comparator.
→ The comparator output will remain high as long as the instantaneous amplitude of x(t) is ~~greater~~ higher than that of the ramp signal.
→ This gives rise to PWM signal at comparator output.

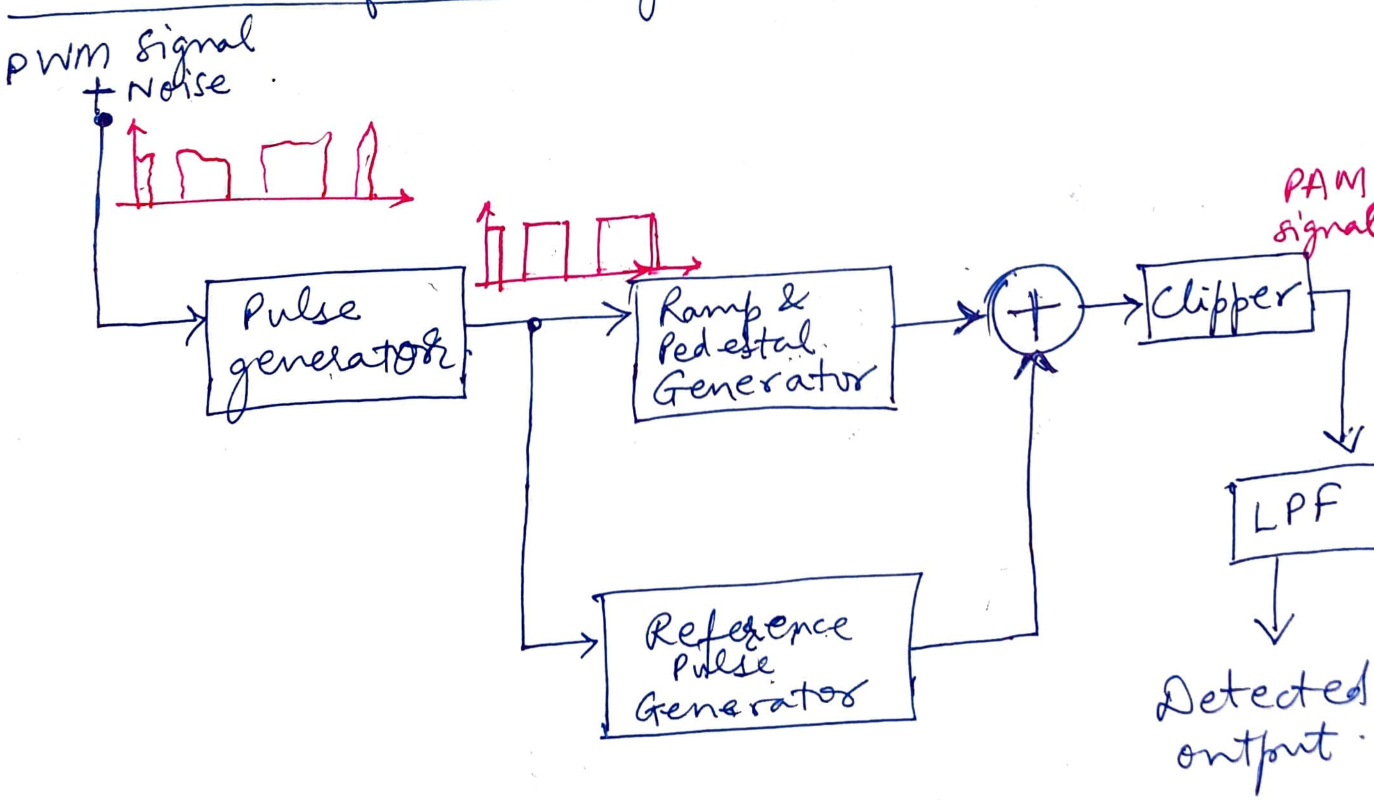


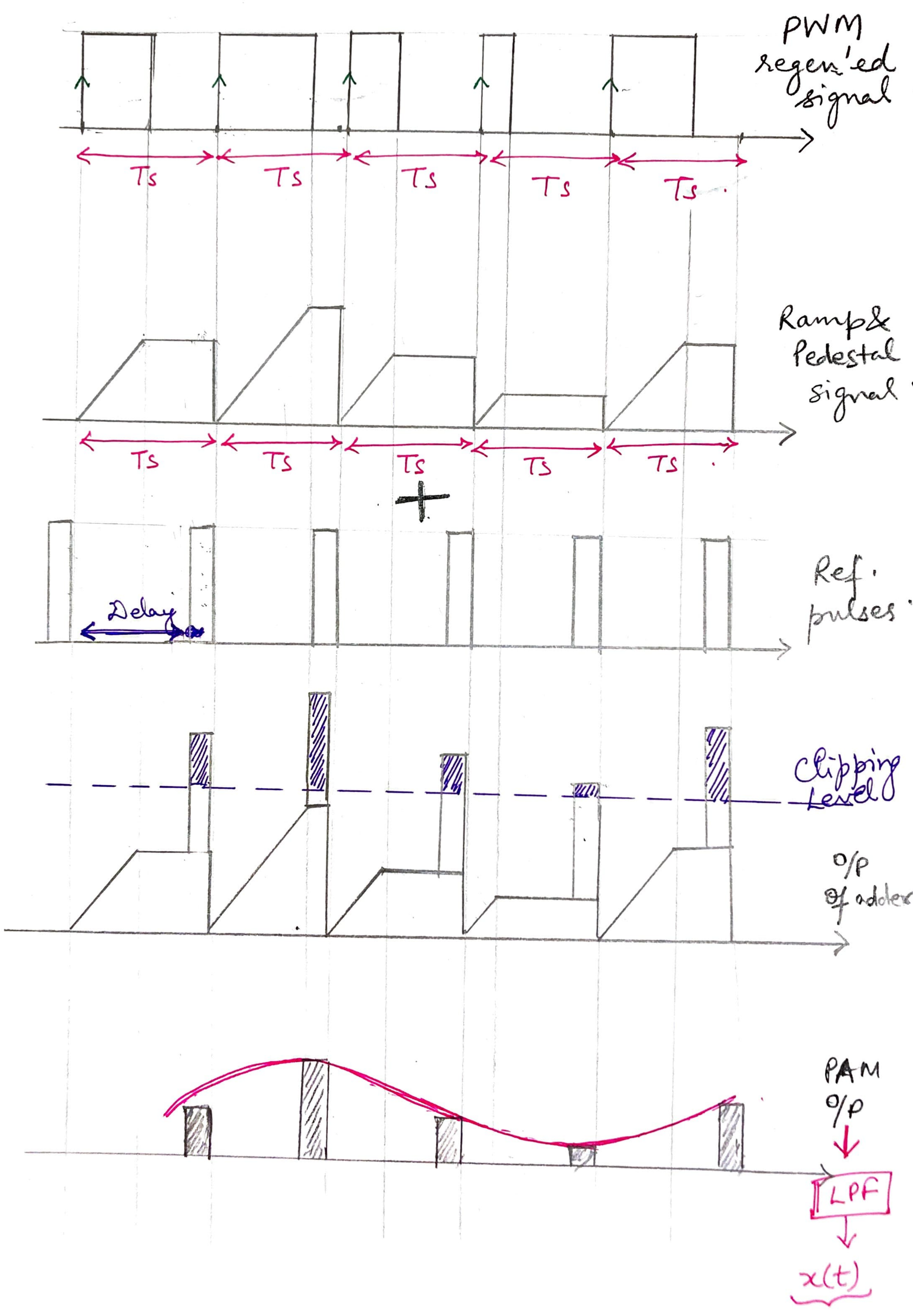


PWM

Mod IV - Lec 4

Detection of PWM Signals -





PWM regen'ed signal
Ts
Ts
Ts
Ts
Ts
Ramp & Pedestal signal
Ts
Ts
Ts
Ts
Ts
+
Delay
Ref. pulses
Clipping Level
O/P of adder
PAM O/P
LPF
x(t)

→ PWM signal received at the input of detector circuit is contaminated with noise.

→ This signal is applied to pulse generator circuit which regenerates PWM signal, i.e. pulses are squared up.

→ The regenerated pulses are applied to a reference pulse generator. It produces a train of constant amplitude, constant width pulses. These pulses are synchronized to the leading edges of regenerated PWM pulses but delayed by a fixed interval.

→ Regenerated PWM pulses are also applied to ramp generator.

→ At the output of it, we get a constant slope ramp for the duration of the pulse. Height of the ramp $\propto$ Width of PWM pulses. At the end of the pulse, the pedestal is formed of height equal to final ramp voltage until it is reset at the end of the pulse.

→ Constant amplitude pulses at output of reference pulse generator are then added to the ramp signal.

→ Output of the adder is then clipped off at a threshold level to generate a PAM signal at the output of the clipper.

→ LPF is used to recover original modulating signal from PAM signal.

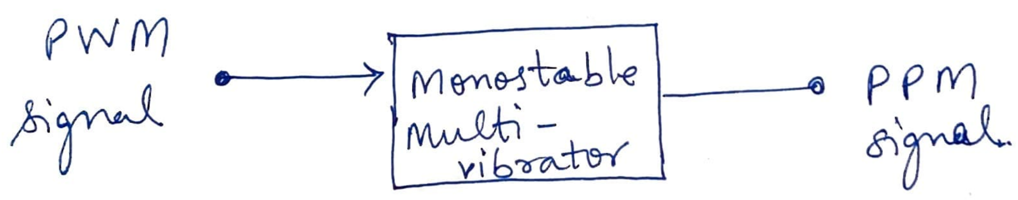
Mod IV Lec 5

→ <u>Pulse Position Modulation (PPM)</u> –

→ Position of pulses is varied in accordance with the amplitude of the modulating signal.

→ Derived from PWM.

→ <u>Generation</u>

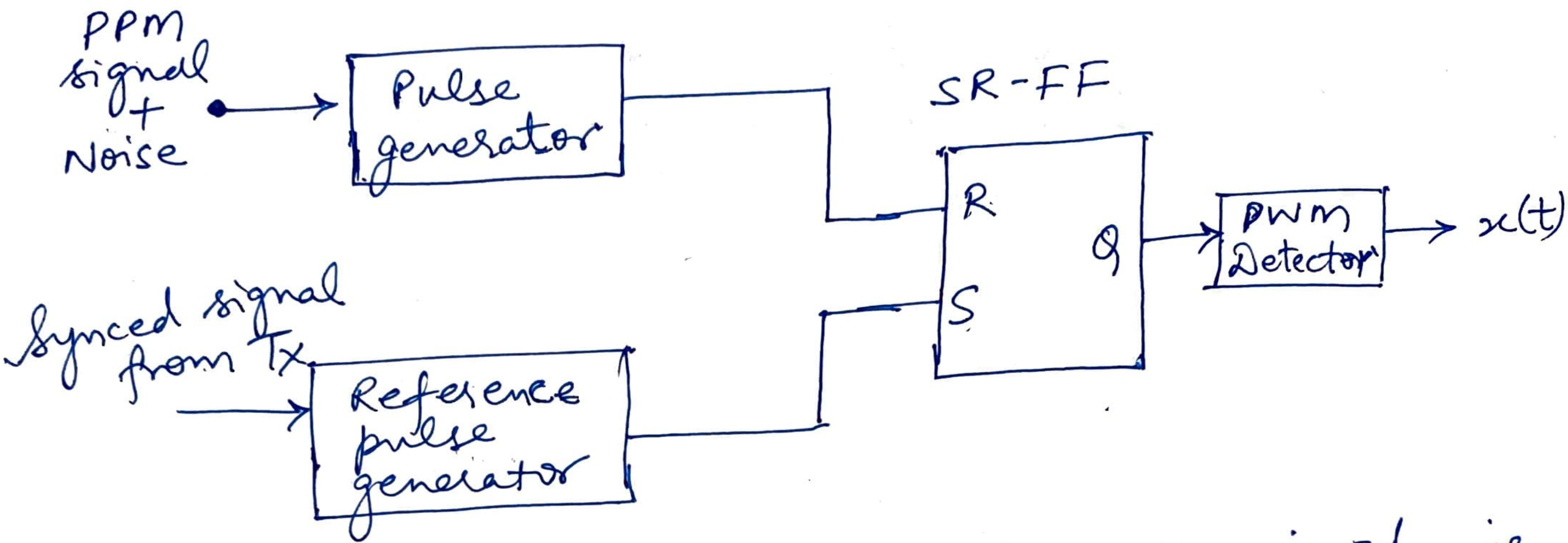


→ The PWM pulses obtained are applied to a monostable multivibrator as a trigger input. (Monostable multivibrator is negative edge triggered).

∴ Corresponding to each trailing edge of PWM signal, the monostable MV output goes high. It remains high for a fixed time decided by its own RC components

∴ As trailing edges of PWM signal keeps shifting in proportion to modulating signal, $x(t)$, PPM pulses also keeps shifting.

→ Detection of PPM signal –



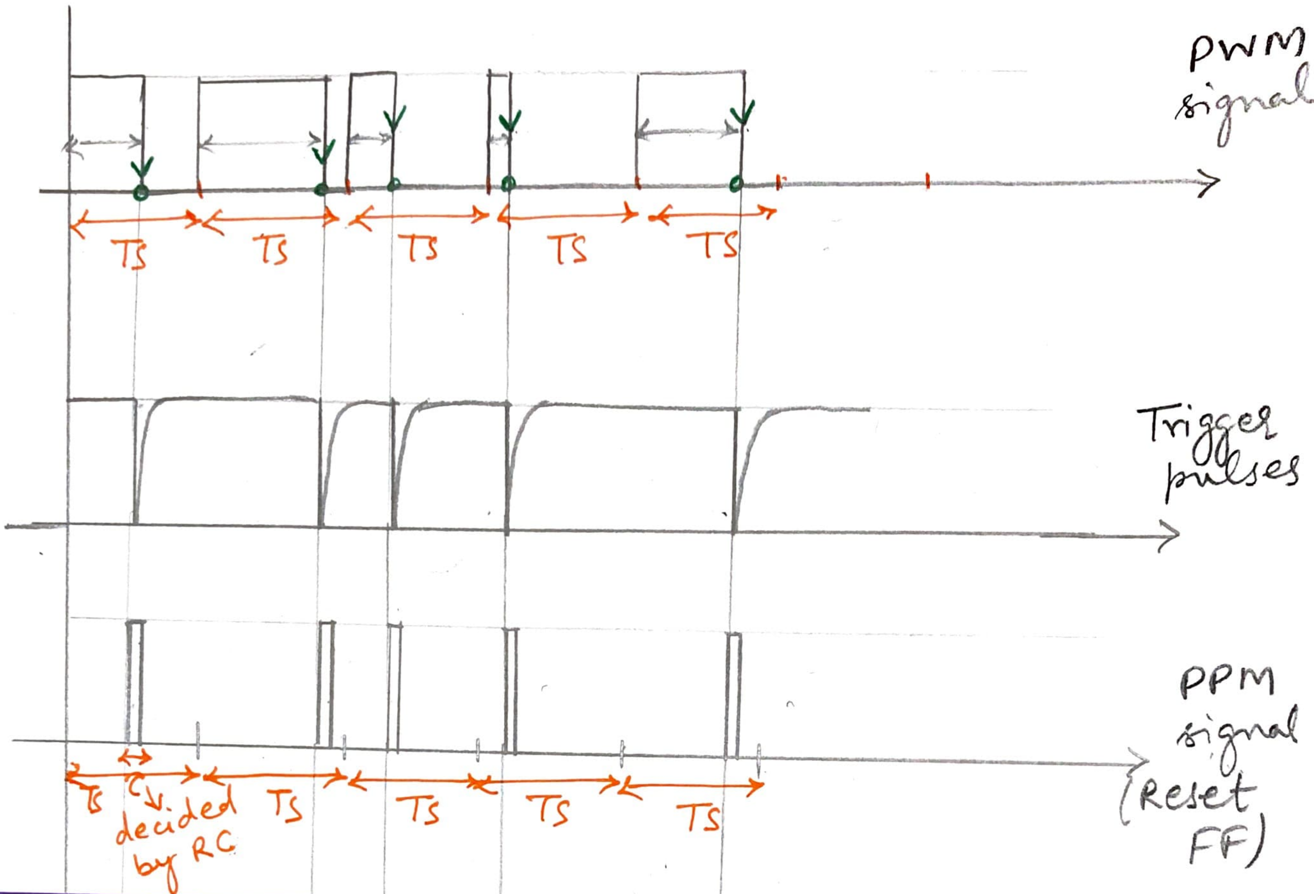
→ The noise corrupted PPM waveform received is given to a pulse generator which develops or regenerates pulsed waveforms at its output. These are applied to the reset pin of SR-FF [Set Reset - Flip flop]

→ A fixed period reference pulses are generated from the synchronized signal from Tx and SR-FF is set by these reference pulses.

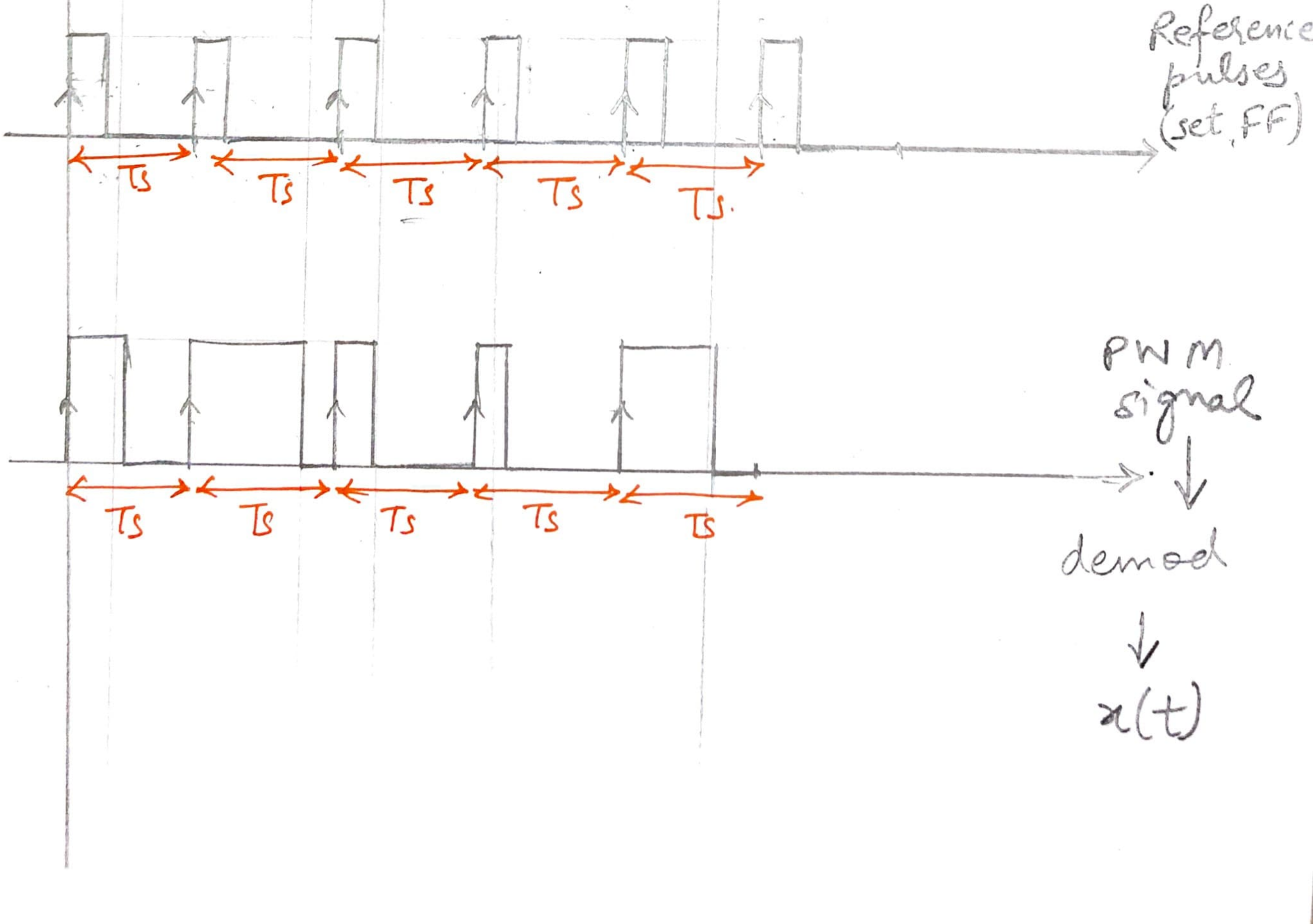
→ The output of SR-FF goes high at the start of pulse (reference) every T_s sec. However, the output of SR-FF goes low (resets) when PPM pulse appears.
Due to this set and reset signals of FF, we get PWM signal at its output.

→ PWM signal is then demodulated using PWM demodulator.

GENERATION



DEGENERATION



[redacted]

DIGITAL MODULATION

→ To generate digital signal → in terms of 0's and 1's.

→ 3 Types – Pulse Code Modulation (PCM)
  – Delta Modulation (DM)
  – Adaptive Delta Modulation (ADM)

→ Pulse Code Modulation –

→ Message is sampled.

→ Amplitude of each sample is rounded-off (approximated) to nearest one of the finite set of discrete levels (QUANTIZATION)

→ We then use A/D converter to encode these levels into "code words".

→ Finally, parallel to serial converter used to transmit the pulses:

→ QUANTIZATION PROCESS –

→ Process of approximation or rounding off

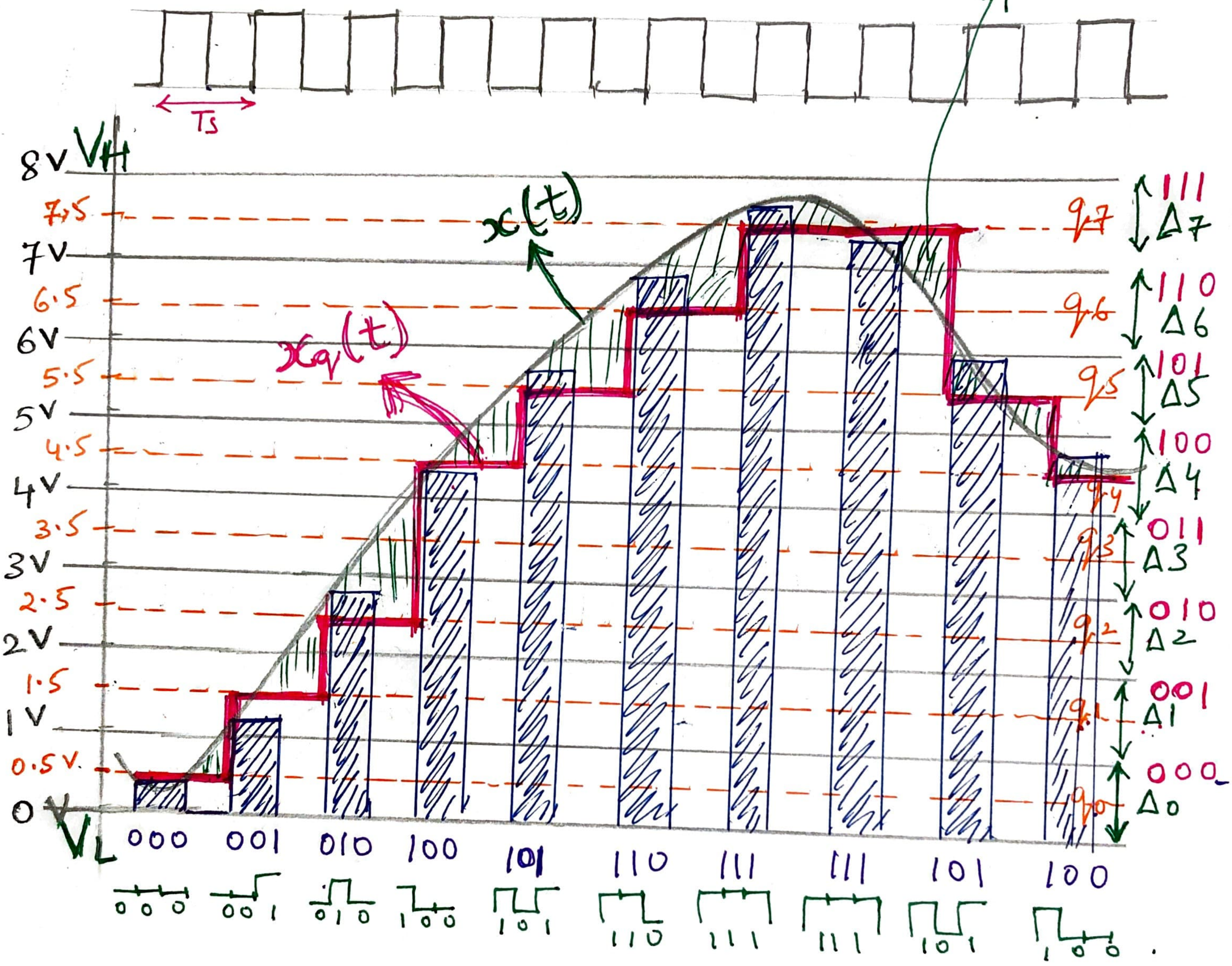
→ The sampled signal is given to a quantizer which converts sampled signal into approximate quantized signal which consists of only a finite number of pre-decided voltage levels.

→ These standard levels are known as Quantization levels.

$\epsilon = x_q(t) - x(t)$

$\downarrow$ minimum

quantization error (ϵ)



V_L to V_H

$V\ swing = V_H - V_L = 8 - 0 = 8$ volts.

Eg. $Q = 8$ = 8 levels.

$S = \dfrac{V_H - V_L}{Q} = \dfrac{8-0}{8} = 1$ volt

$2^N = Q$

$N = 3$

$Q = 2^3 = 8$ levels

N → no. of bits

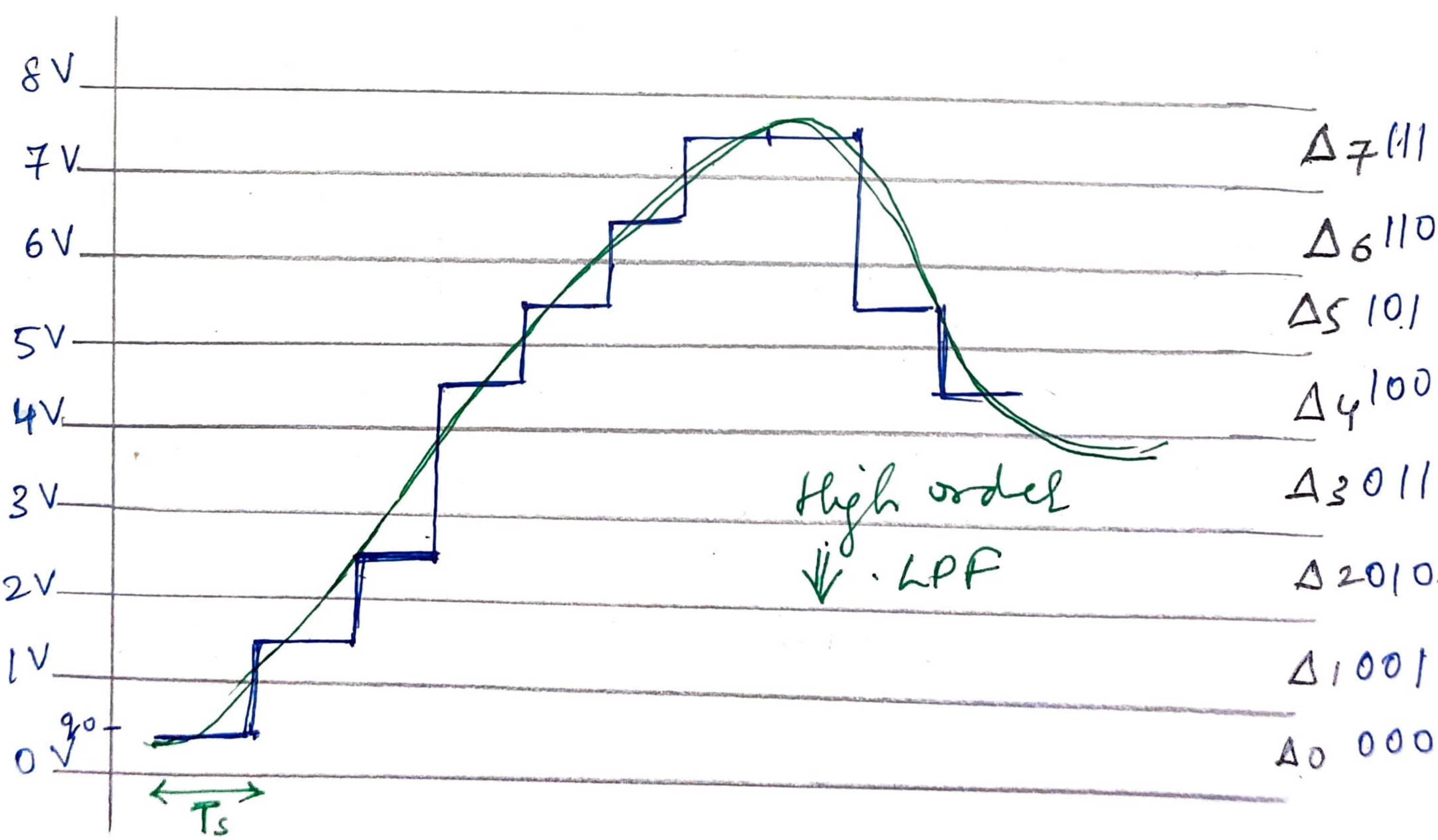
000	110
001	111
010	
011	
100	
101	

0000
0001
⋮
1110
1111

000 → combination
001 → parallel
010
100 ⇓
101 parallel to serial converter.
110
111 000001 010100101 110111111 101100
111 ← Train of pulses
101 ⇓
100 Transmitted..
⇓
Rx

000 001 010 100 101 110 111 111 101 100

⇓
serial to parallel



10 samples → 3 bits / sample → 8 levels

$3 \times 10 = 30$ bits

10 samples → 4 bits / sample → 16 levels

$4 \times 10 = 40$ bits

⇓

BW requirement ↑ → ϵ ⇓

10 samples → 5 bit / sample

$5 \times 10 = 50$ bits

$Q \uparrow \Rightarrow \epsilon \downarrow \Rightarrow$ BW ↑

TRADE OFF

Sampling rates ↑ → more samples.

12 samples → 3 bits / sample.

$12 \times 3 = 36$ bits ↑

$f_s \uparrow \Rightarrow \epsilon \downarrow \Rightarrow$ BW ↑

TRADE OFF

[redacted]

→ $x(t)$ is assumed to have a peak-to-peak voltage ~~level~~ swing from V_L to V_H volts.

→ This entire voltage range is divided into Q equal intervals each of size, S.

$S \rightarrow$ Step size.

$$S = \frac{V_H - V_L}{Q}, \quad \text{such that } Q = 2^N$$

Q can be 2, 4, 8, 16, 32 ....

→ At the centre of these steps, quantization levels; q_0, q_1, q_2 .... are located.

→ When $x(t)$ is in the range of Δ_0, then corresponding to value of $x(t)$, quantizer output will be q_0 and so on.

→ $x_q(t)$ is approximation of $x(t)$.

→ The difference between $x(t)$ and $x_q(t)$ is called quantization error/noise.

→ This error should be as small as possible.

→ To minimize this error —

Reduce step size, S, ⇒ By increasing the number of levels, Q.

→ But if we increase Q, more number of bits (N) will be required to form a codeword.

$\Rightarrow$ inc. in bit rate $\Rightarrow$ inc. in BW.

$\therefore$ Compromise / Tradeoff needs to be done.

→ Quantization Error –

$$\epsilon = x_q(t) - x(t)$$

$$\text{Max. } \epsilon = \pm \frac{S}{2}$$

$$\text{Mean square value of } \epsilon = \frac{S^2}{12}$$

→ Signal to Quantization noise ratio –

For sinusoidal signals, $N \rightarrow$ no. of bits.

$$\frac{S_i}{N_q} = (1.8 + 6N)$$

$\Rightarrow$ Better SNR_q by inc. N

Again $\Rightarrow$ inc. in bit rate

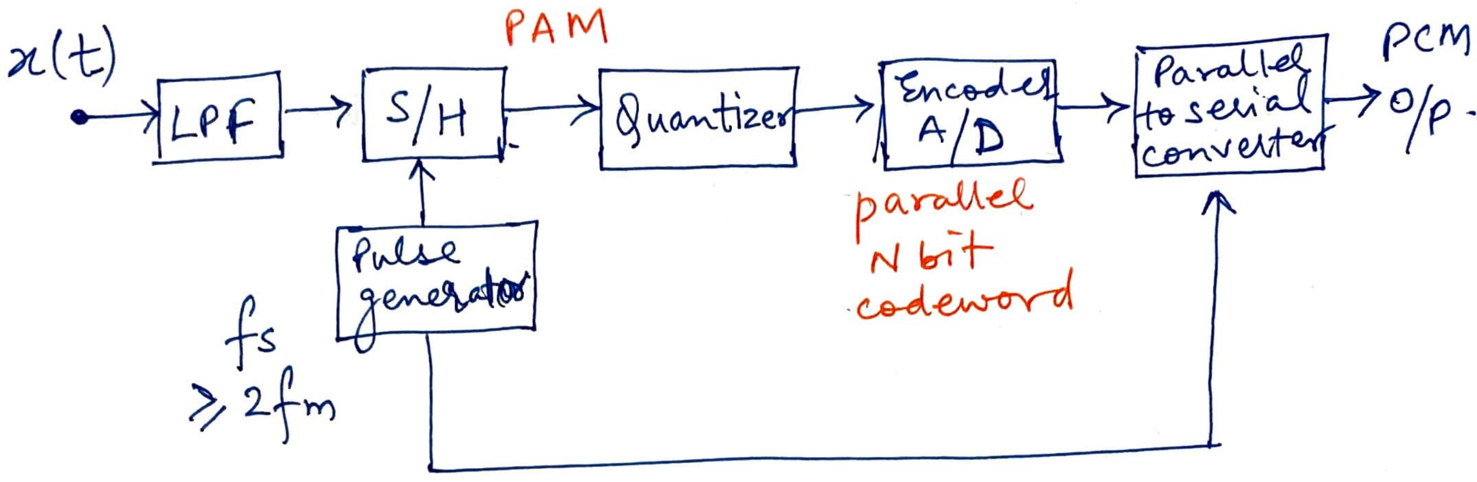
$\Rightarrow$ inc. in BW.

→ PCM system consists of –

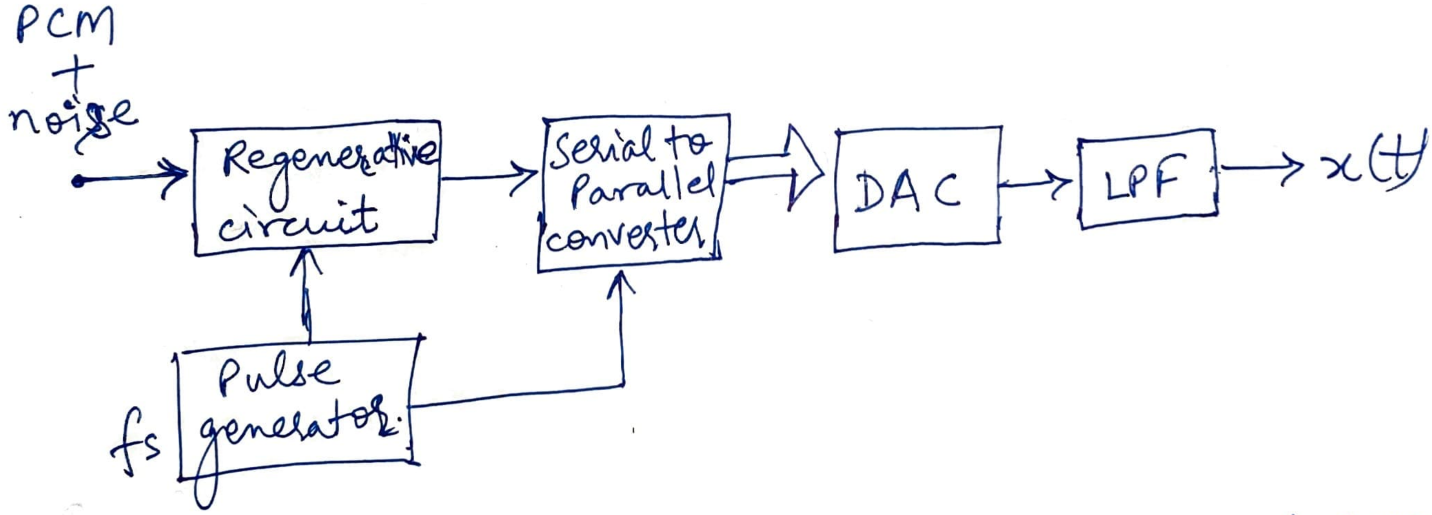
– PCM Encoder (Tx)

– PCM Decoder (Rx)

→ PCM Encoder or PCM Transmitter :-



→ PCM Decoder or PCM Receiver –



→ Signalling rate and Transmission BW of PCM–

$$Q = 2^N$$

$x(t)$ sampled at f_s

Each sample converted into a N-bit codeword.

→ Signalling Rate (r) is number of bits/sec.

r = No. of samples/sec × No. of bits/sample

$$\boxed{r = f_s \times N}$$

→ $$\boxed{\begin{aligned}\text{Transmission BW} &= \frac{1}{2} \times \text{Signalling rate} \\ &= \frac{N f_s}{2}\end{aligned}}$$

NUMERICALS

Q1. A voice signal bandlimited to 3.4 KHz is to be transmitted using PCM system. The signalling rate of PCM is not to exceed 36000 bits/sec. Find – approximate value of f_s, number of quantization levels, no. of bits per word.

Soln:

$r = N f_s$ $\qquad\qquad$ $f_m = 3.4\,\text{kHz}$

$r \leq 36000$

$\Rightarrow N f_s \leq 36000$

$f_{s_{min}} = 2 f_m = 2 \times 3.4\,\text{KHz} = 6.8\,\text{kHz}$

$N \leq \frac{36000}{6800} \leq 5.29$

Say, select $N = 5$

$$Q = 2^N = 2^5 = 32 \text{ levels}.$$

→ Now, with $N=5$, calculate max. allowable $f_{s\,max}$:

$$N f_s = 8$$

$$f_{s\,max} = \frac{36000}{5} = 7.2 \text{ KHz}$$

⇒ f_s should be in the range 6.8 KHz – 7.2 KHz

Q2: In a binary PCM system, the output signal to quantization noise ratio is to be held to a minimum of 40 dB. Calculate no. of bits per word necessary to meet this requirement and then find the actual value of output signal to quantization noise ratio.

Soln:

$$SNR_q = 1.8 + 6N \geq 40$$

$$N \geq 6.3667$$

Select, $N = 7$ $\quad [Q = 2^N = 2^7 = 128 \text{ levels}]$

$$\text{Actual } SNR_q = 1.8 + 6N$$

$$= 1.8 + (6 \times 7)$$

$$= 43.8 \text{ dB}$$

Q3. An audio signal has spectral components present in the range of 300Hz to 3300Hz. A PCM signal is generated by sampling this audio signal at $f_s = 8$ kHz. The minimum value of SNR_q is 30 dB. Calculate — (a) min. no. of quan levels & no. of bits per word

(b) Signalling rate & Transmission BW.

Soln: $SNR_q = 1.8 + 6N \geq 30$

$\Rightarrow$ ~~[illegible]~~ $N \geq 4.7$

(a) Select, $N = 5$

$Q = 2^N = 2^5 = 32$ levels.

(b) $r = Nf_s = 5 \times 8\,KHz = 40000$ bits/sec

$BW = \frac{r}{2} = \frac{40000}{2} = 20\,KHz$

Q4. BW of a video signal is 4.5 MHz. This signal is to be transmitted using PCM with $Q = 1024$. Sampling rate should be 20% higher than Nyquist rate. Calculate system bit rate.

Soln: $f_m = 4.5$ MHz $\quad f_{s\,min} = 2f_m = 2 \times 4.5$ MHz $= 9$ MHz

f_s to be 20% higher $= 9 + (20\% \text{ of } 9) = 10.8$ MHz

$Q = 2^N \Rightarrow 1024 = 2^N \Rightarrow N = 10$

$r = Nf_s = 10 \times 10.8 = 108$ MHz

[redacted]

→ Delta Modulation –

→ Disadv. of PCM – N no. of bits Xmitted per sample.

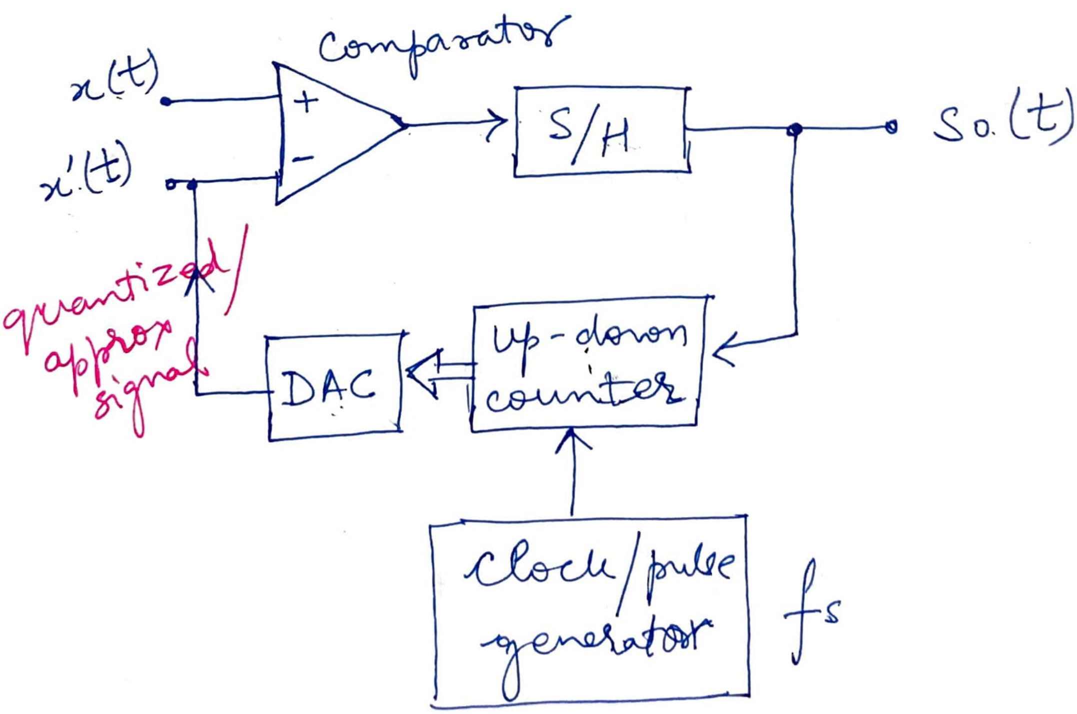
⇒ Signalling rate & transmission BW becomes v. large.

→ To overcome this limitation –
We adopt Delta Modulation.

→ Concept of DM → Transmit Only 1-bit per sample

→ Present sample value $x(t)$ is compared with previous sample value $x'(t)$ and the result of this comparison is transmitted.

→ DM Transmitter –



$x(t) \rightarrow$ analog input signal.

$x'(t) \rightarrow$ Quantized (approx.) version of $x(t)$

→ Both applied to a comparator

→ When $x(t) > x'(t)$, output is high

When $x(t) < x'(t)$, output is low.

→ S/H circuit will hold this value (0 or 1) for the entire pulse duration or clock cycle.

→ Output of S/H circuit is transmitted as output of DM system.

→ ∴ Now, only 1 bit/pulse (sample) is sent.

⇒ BW is reduced.

→ This transmitted signal is also used to decide mode of operation of up/down counter.

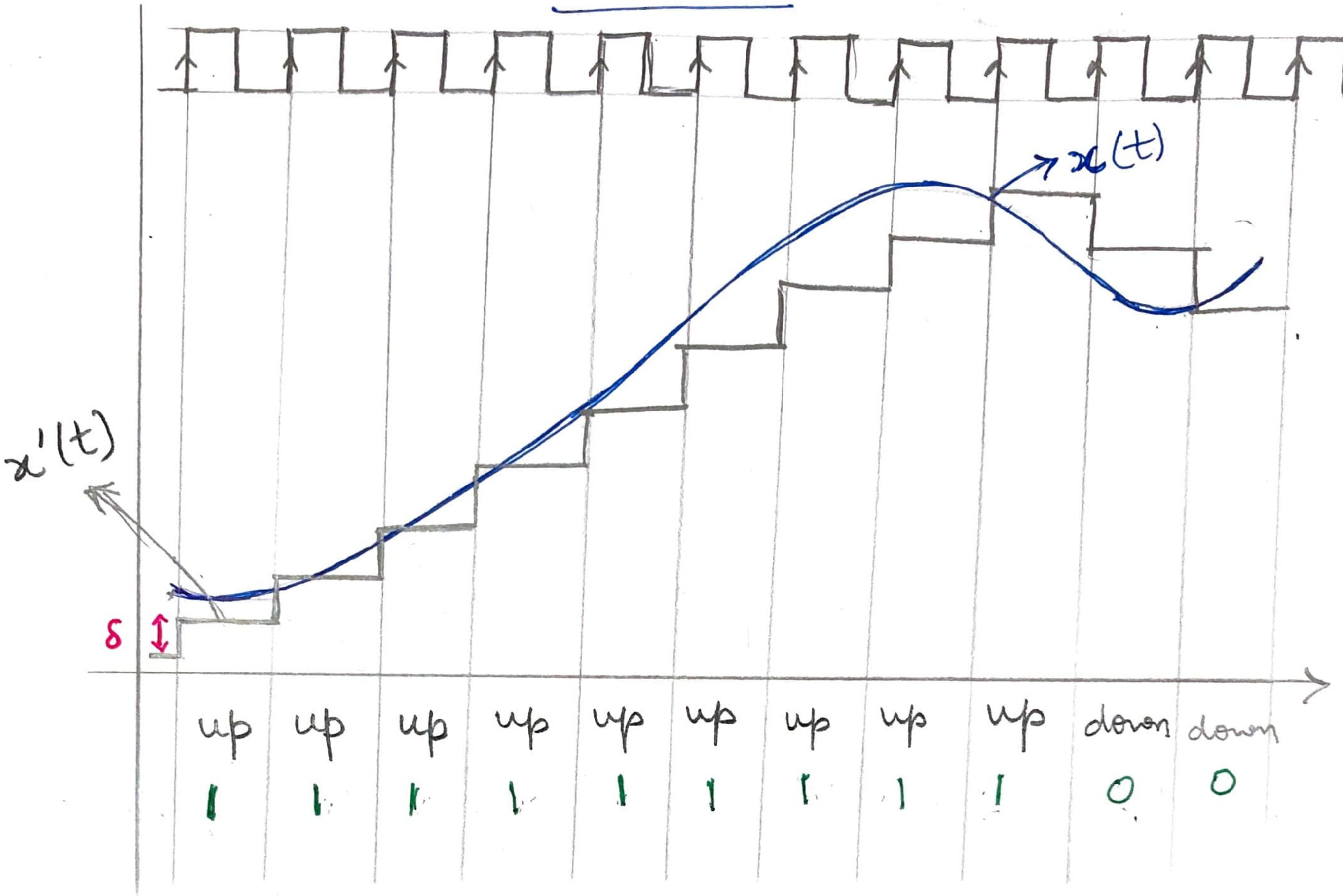
→ Counter output increments by 1 if $S_o(t) = 1$ and decrements by 1 if $S_o(t) = 0$.

→ The counter output is converted into analog signal by DAC.

∴ We get approximated signal, $x'(t)$ at output of DAC.

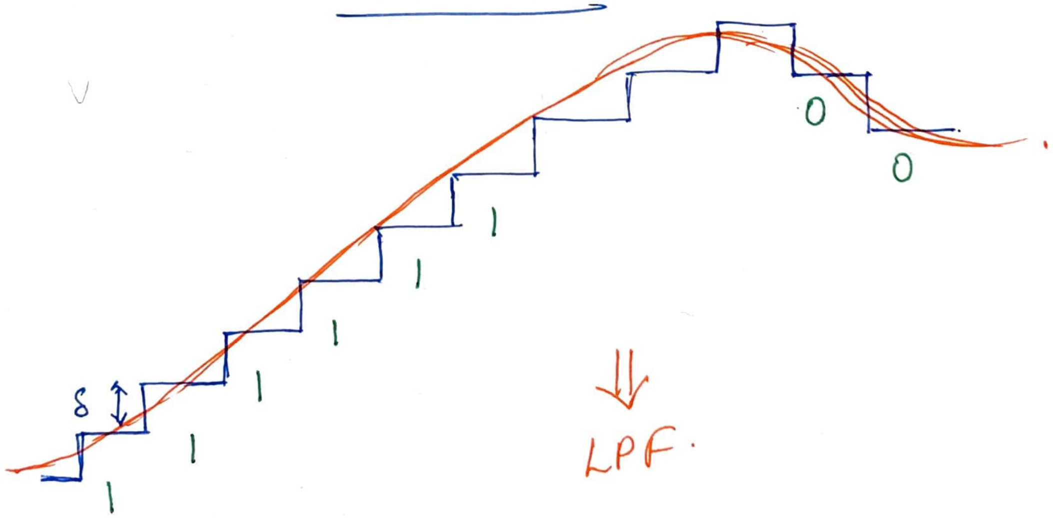
→ Step size, δ is constant.

DM - Tx



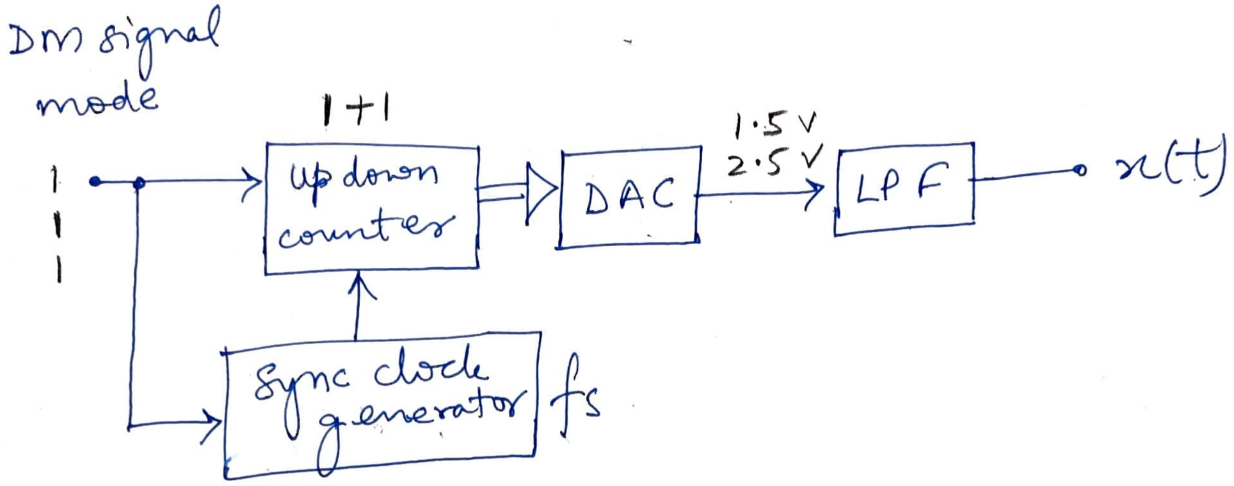
x(t)
x'(t)
δ
up up up up up up up up up down down
1 1 1 1 1 1 1 1 1 0 0

DM - Rx



δ
1 1 1 1 1 1 1 0 0
LPF.

DM – Receiver –

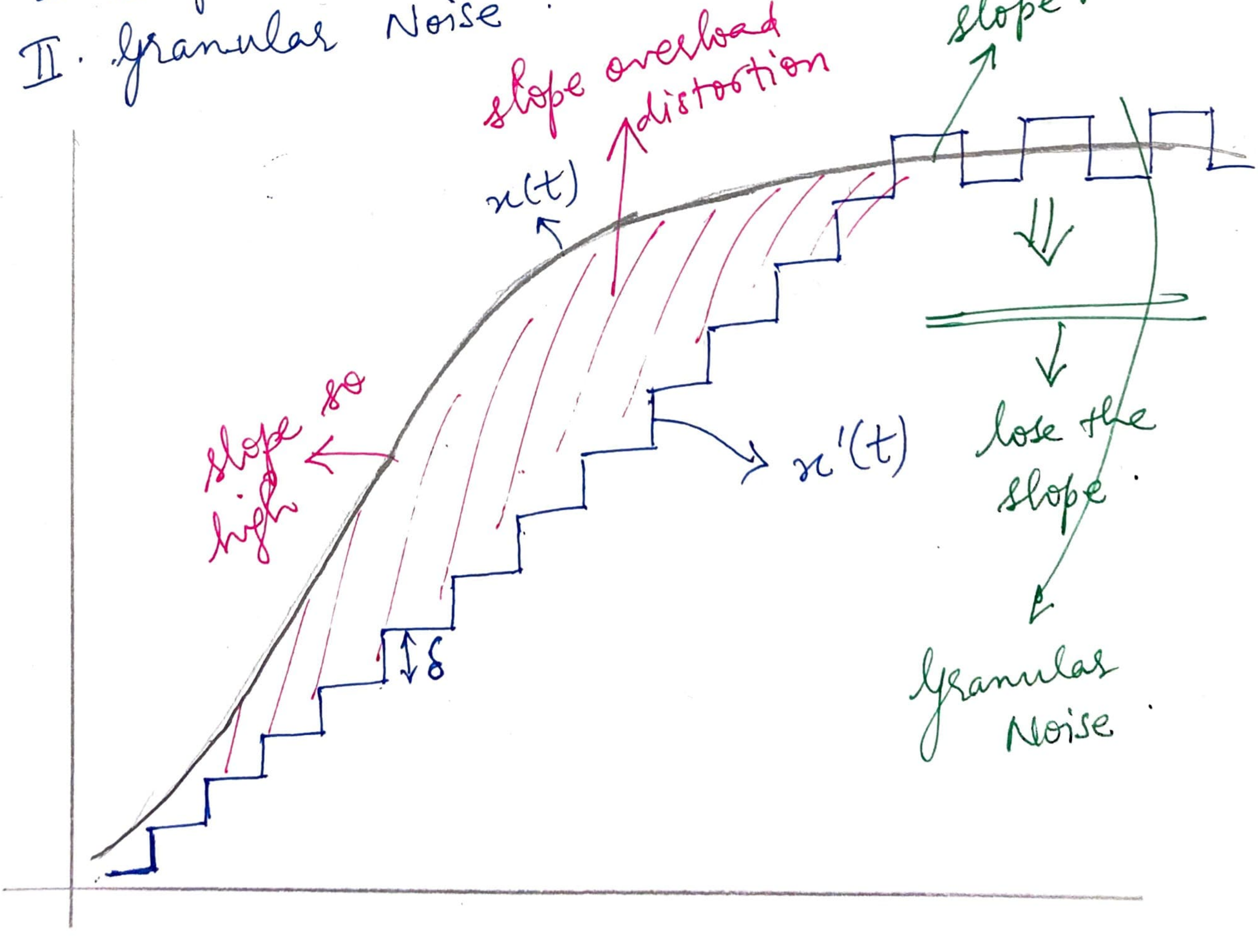


→ Limitations / Distortion in DM system –

→ Two types of Errors :

I. Slope Overload distortion

II. Granular Noise .



1. Slope Overload Error –

→ Because of ~~small~~ step size that remains constant.

→ If slope of $x(t) >> x'(t)$ over a long

[slope of $x'(t) = \delta f_s$]

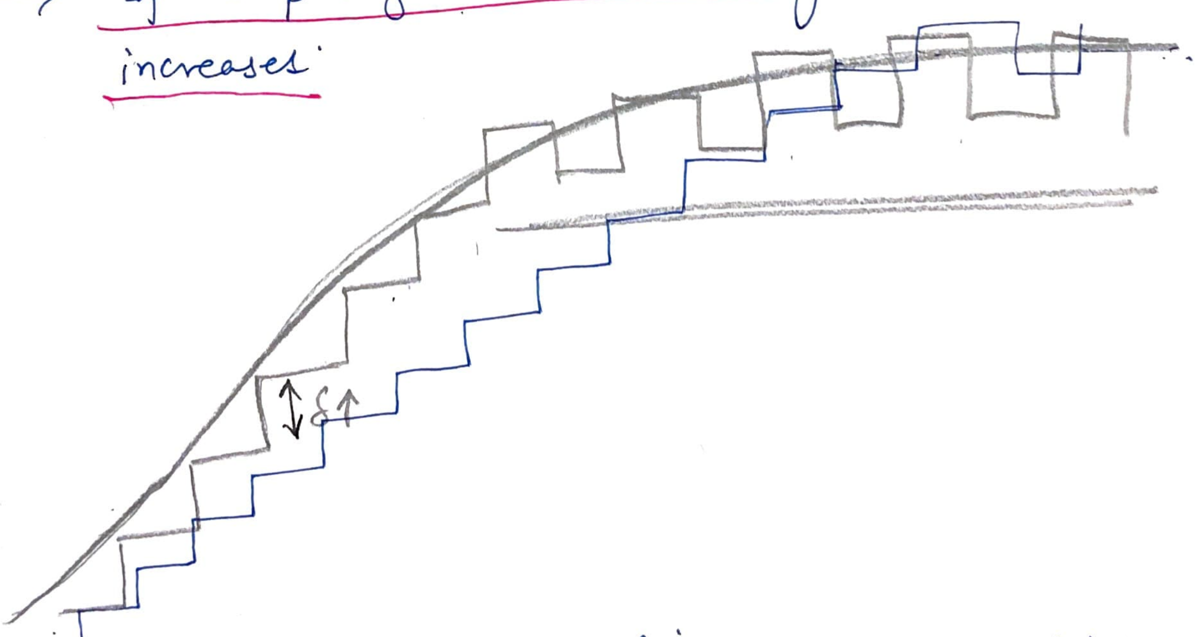
duration, then $x'(t)$ will not be ~~not~~ able to follow $x(t)$ at all.

→ This difference between $x(t)$ and $x'(t)$ is called slope overload distortion.

→ It can be reduced by increasing slope of $x'(t)$

→ Slope of $x'(t)\uparrow$ → inc. step size (δ)
→ inc. $f_s \uparrow$

→ If step size increases, granular noise increases.



→ If f_s increases, signaling rate & BW will inc.

II. Granular Noise –

→ When $x(t)$ is relatively constant in amplitude (slope is very small), $x'(t)$ hunts below and above $x(t)$

∴ Also called as Hunting Error.

→ Increases with increase in step size.

→ Can be reduced by decreasing δ to as small as possible.

→ However, this will increase slope overload error.

→ In delta modulation ... it is step size (δ) is not variable ... it is constant.

→ If we can make step size variable, such that both the types of errors can be controlled.

→ A system with variable step size is called ...

Adaptive Delta Modulation.

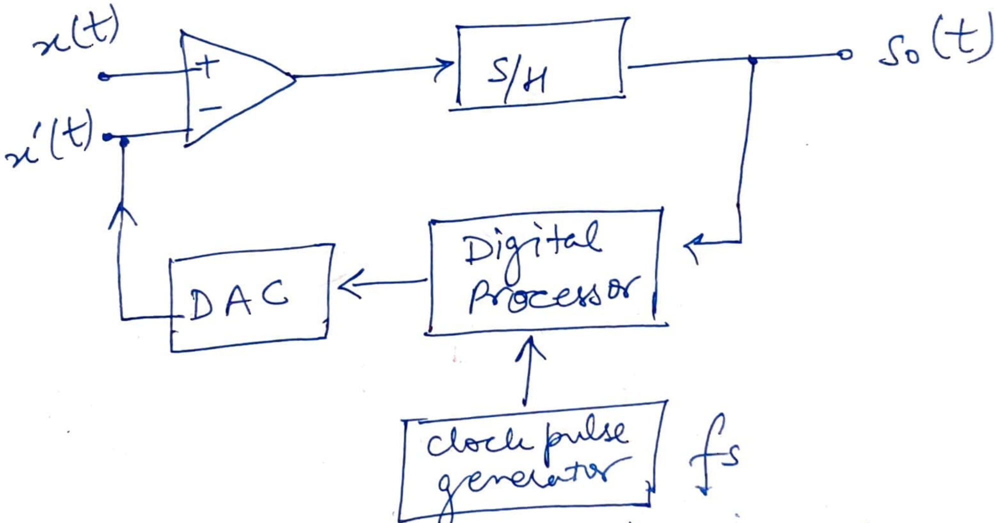
[redacted]

→ **Adaptive Delta Modulation** –

→ Here, step size is not constant

→ When slope overload occurs, step size becomes progressively larger & ∴ $x'(t)$ will catch up with $x(t)$ more rapidly.

→ **ADM – Tx** –



→ Here, the counter is replaced by digital processor.

→ In response to k^{th} pulse edge, the processor generates a step which is equal in magnitude to the step generated in response to $(k-1)^{th}$ pulse.

→ If direction of both the steps is the same, the processor will increase the magnitude of present step by δ.

→ If the direction is opposite, processor will decrease magnitude of present step by δ.

→ Output of ADM is —

$$S_o(t) = +1, \text{ if } x(t) > x'(t) \text{ just before } k^{th} \text{ clock edge}$$

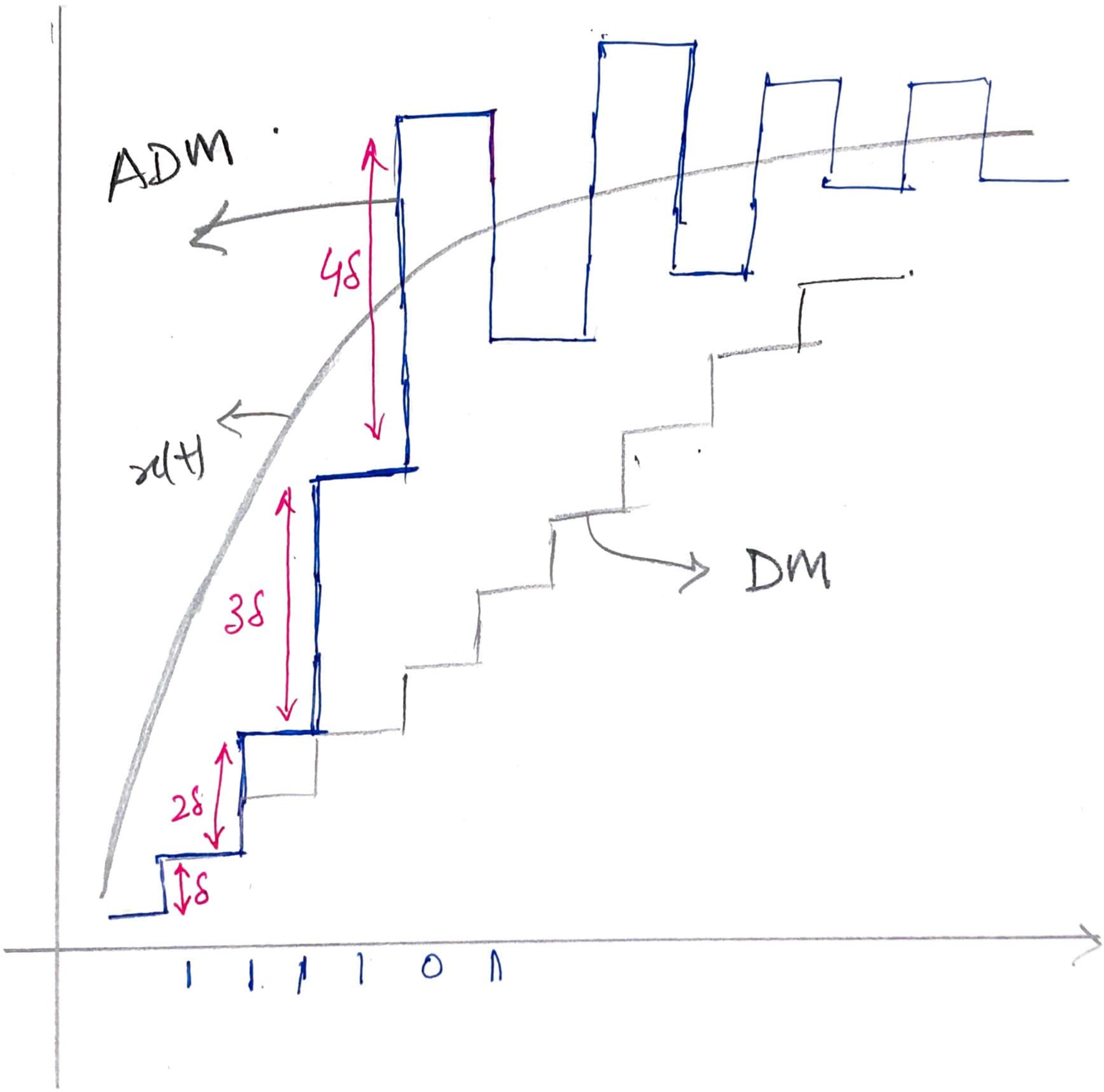
$$= -1, \text{ if } x(t) < x'(t) \text{ just before } k^{th} \text{ clock edge.}$$

→ Then, step size at sampling instant k is —

$$\underbrace{\delta(k)}_{\text{Step size at } k^{th} \text{ clock edge.}} = \underbrace{\delta(k-1)}_{\text{Step size at } (k-1)^{th} \text{ clock edge}} \underbrace{S_o(k)}_{\text{Output at } k^{th} \text{ clock edge}} + \underbrace{\delta}_{\text{Basic step size}} \underbrace{S_o(k-1)}_{\text{Output at } (k-1)^{th} \text{ clock edge.}}$$

→ Due to variable step sizes, slope overload error is reduced

But, quantization error increases.



ADM
4δ
x(t)
3δ
DM
2δ
δ
1 1 1 1 0 1

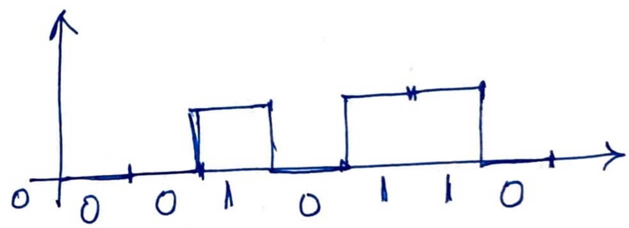
LINE CODES

→ Definition –

It is the process of converting binary data (sequence of bits) into a digital signal.

0	0	1	0	1	1	0

sequence of Bits → Line Coding



Digital signal.

→ CLASSIFICATION OF LINE CODES –

Line Codes

- Unipolar
  - RZ
  - NRZ
- Polar
  - RZ
  - NRZ
    - NRZ-I
    - NRZ-L
  - Manchester
  - Differential Manchester
- Bipolar
  - RZ
  - AMI
  - Pseudo-Ternary

→ Properties of Line Coding –

→ The digital data can be transmitted by various transmission or line codes. Each type has its advantages and disadvantages –

→ A line code must possess certain desirable properties –

1) Transmission Bandwidth –
should be as small as possible

2) Power Efficiency –
For a given BW and a specified detection error probability, transmitted power should be as small as possible

3) Error detection and correction capability

4) Favourable power spectral density –
It is desirable to have zero power spectral density at $w=0$

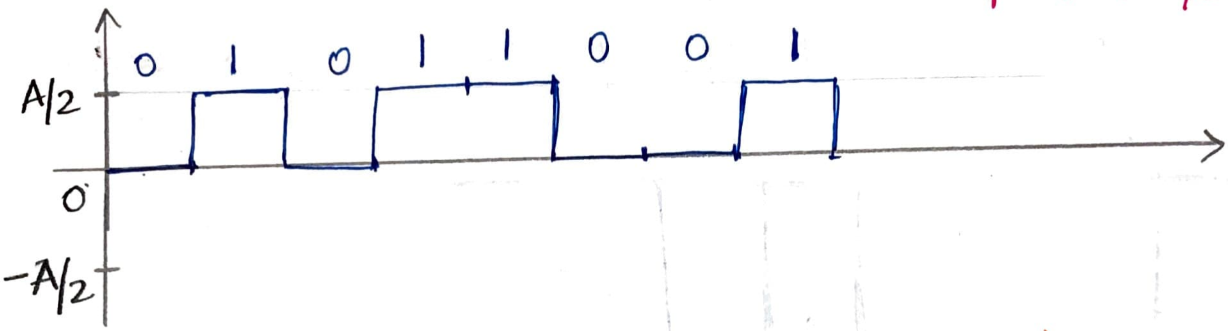
5) Adequate timing content –
It must be possible to extract timing and clock information from the signal.

6) Transparency –
It must be possible to transmit a digital signal correctly regardless of patterns of ones and zeros.

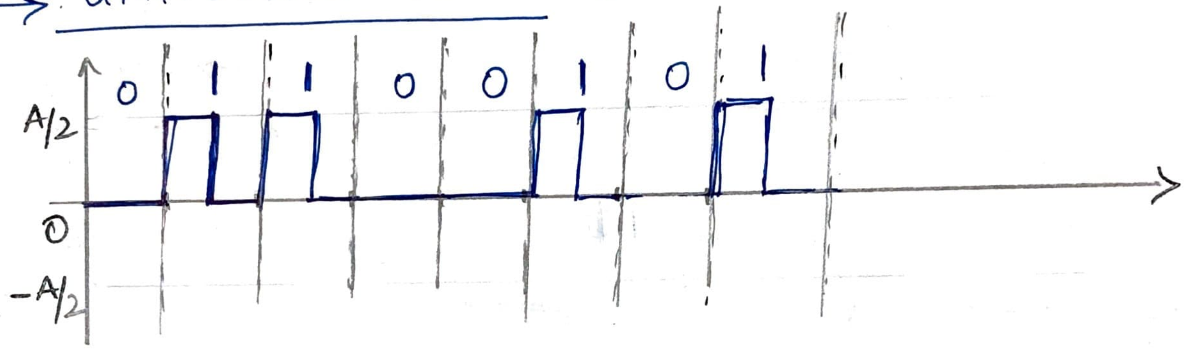
→ UNIPOLAR NRZ (Non Return to zero)

0 → 0V

1 → +A/2



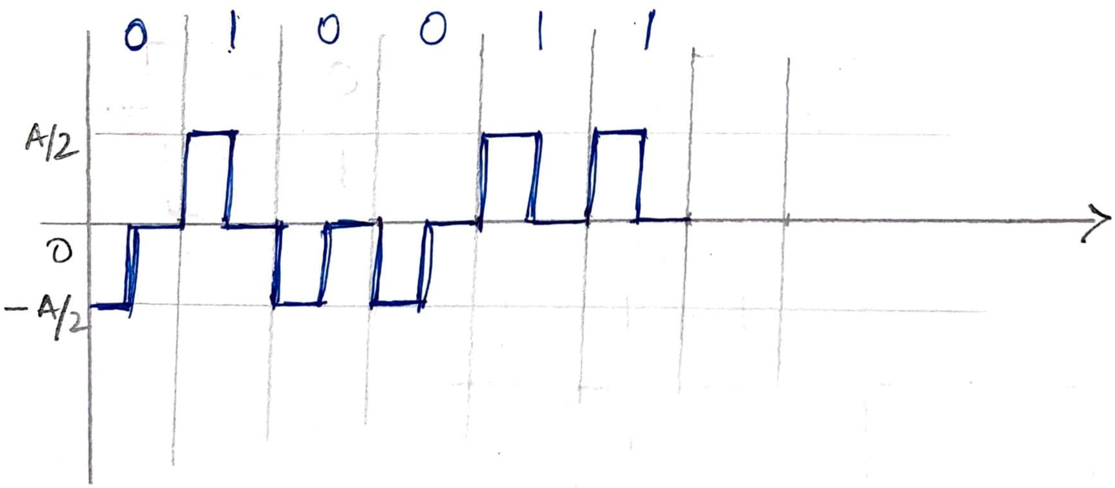
→ UNIPOLAR RZ – (Return to zero)



→ POLAR RZ –

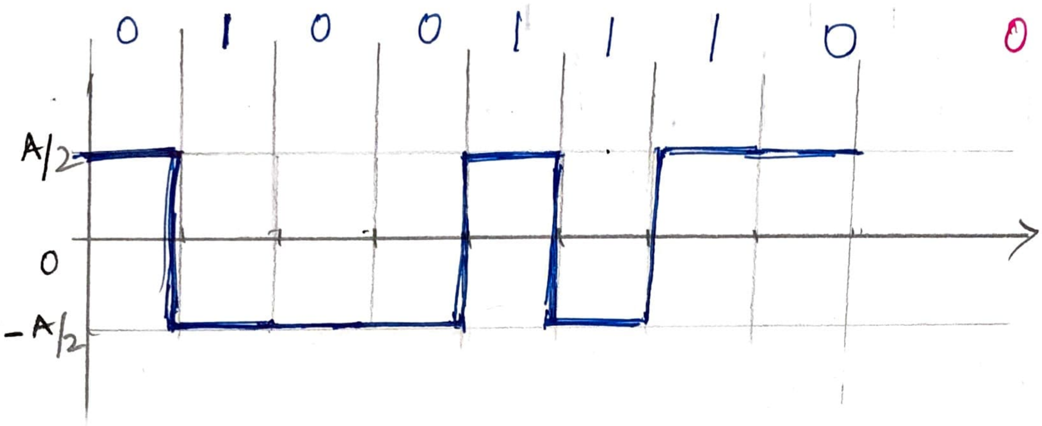
1 → +ve to zero

0 → –ve to zero.



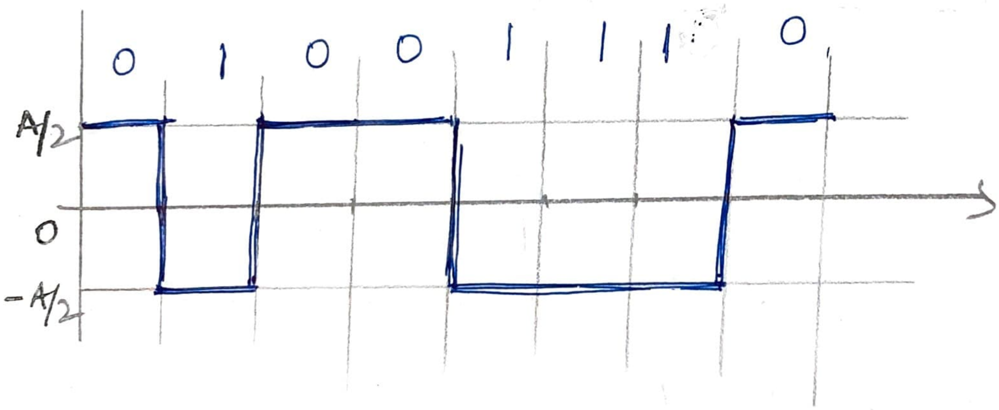
→ POLAR - NRZ - I

1 → Signal change
0 → No change



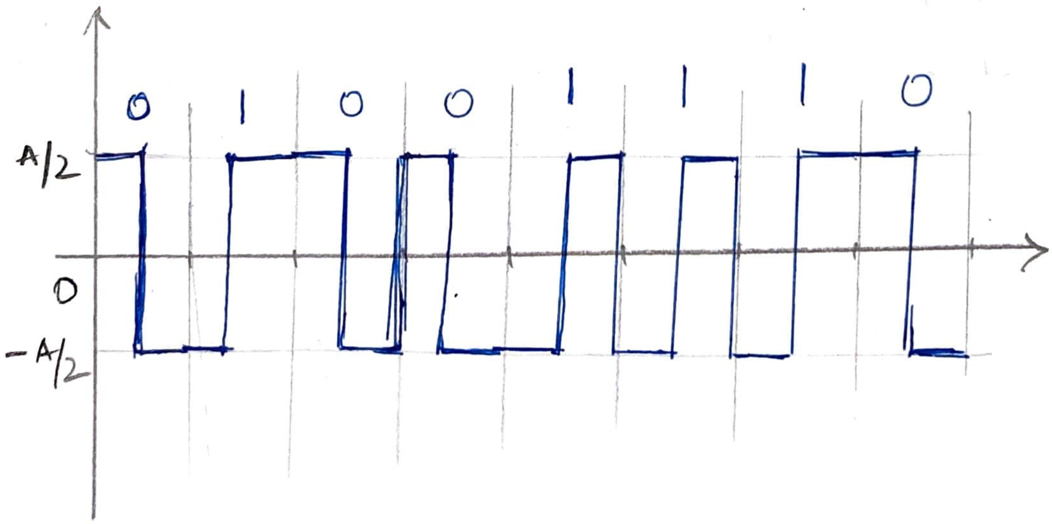
→ POLAR - NRZ-L

1 → -ve voltage
0 → +ve voltage



→ MANCHESTER

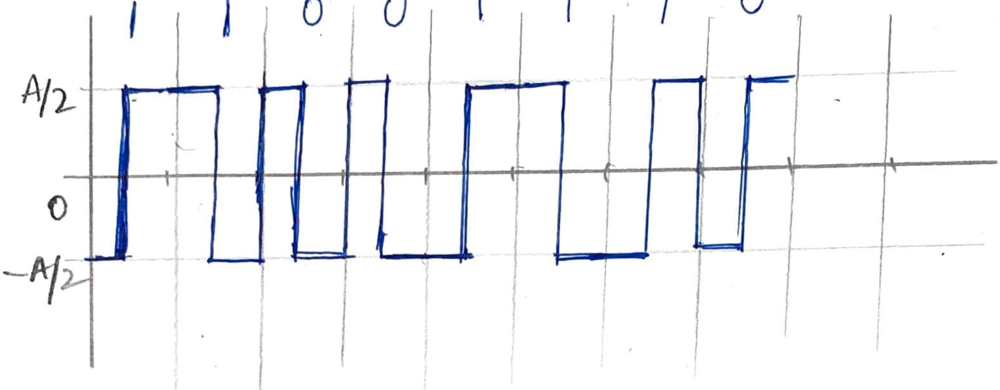
0 → 
1. → 



→ DIFFERENTIAL MANCHESTER

1 → Absence of transition at beginning of bit interval.

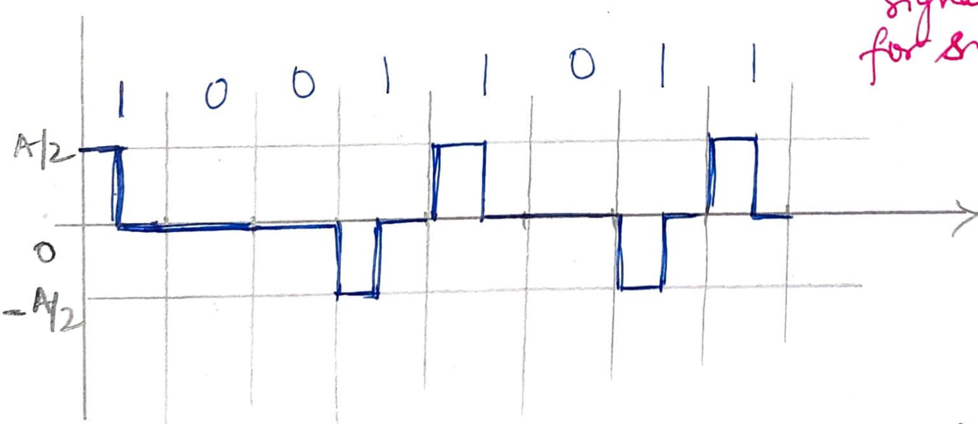
0 → presence of transition at beginning of bit interval.



→ BIPOLAR RZ

0 → no signal

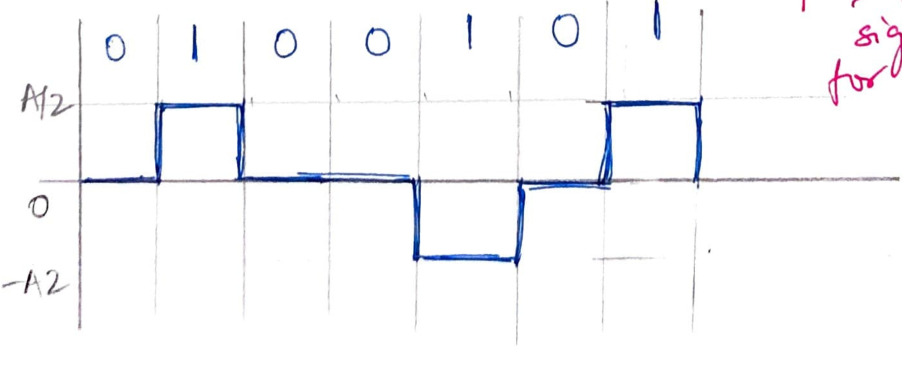
1 → +ve or -ve RZ signal, alternating for successive ones.



→ BIPOLAR AMI (Alternate Mark Inversion)

0 → no signal

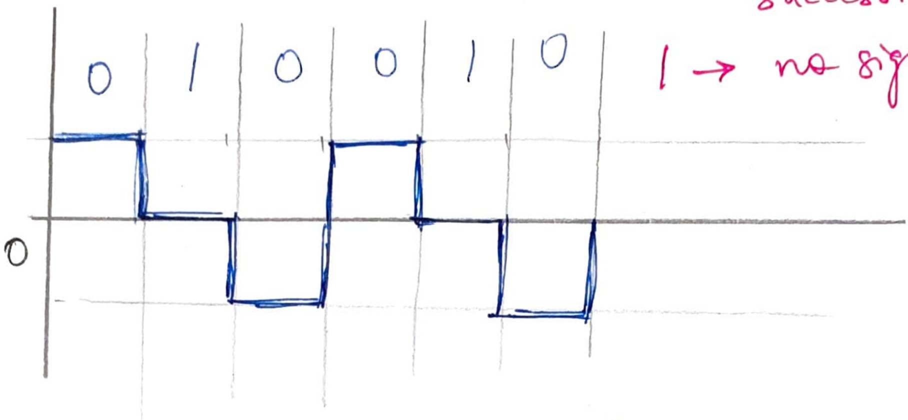
1 → +ve or -ve NRZ signal alternating for successive ones.



→ PSEUDO-TERNARY

0 → +ve or -ve level alternating for successive 0's

1 → no signal



HOME-WORK

Q1. Given the bit sequence 1100 0010, line code these using the following –

(a) Polar RZ (b) NRZ-I (c) NRZ-L

(d) AMI (e) Manchester (f) Diff. Manchester

Q2. Given the bit sequence 1100110, line code using the following –

(a) NRZ-I (b) Unipolar RZ (c) AMI

(d) Manchester (e) Diff. Manchester (f) NRZ-L

Q3. Given the bit sequence 10011011, line code using the following –

(a) Unipolar NRZ (b) Bipolar RZ

(c) Unipolar RZ (d) Pseudo ternary

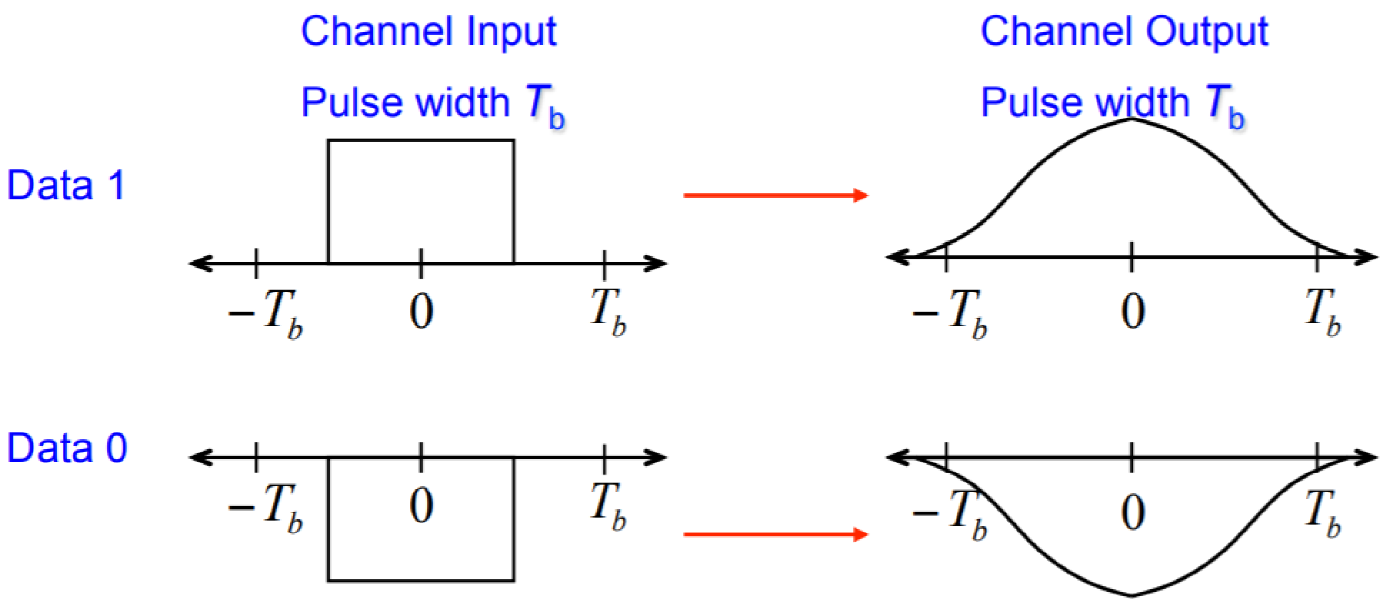
Module IV

Lecture

- Intersymbol Interference

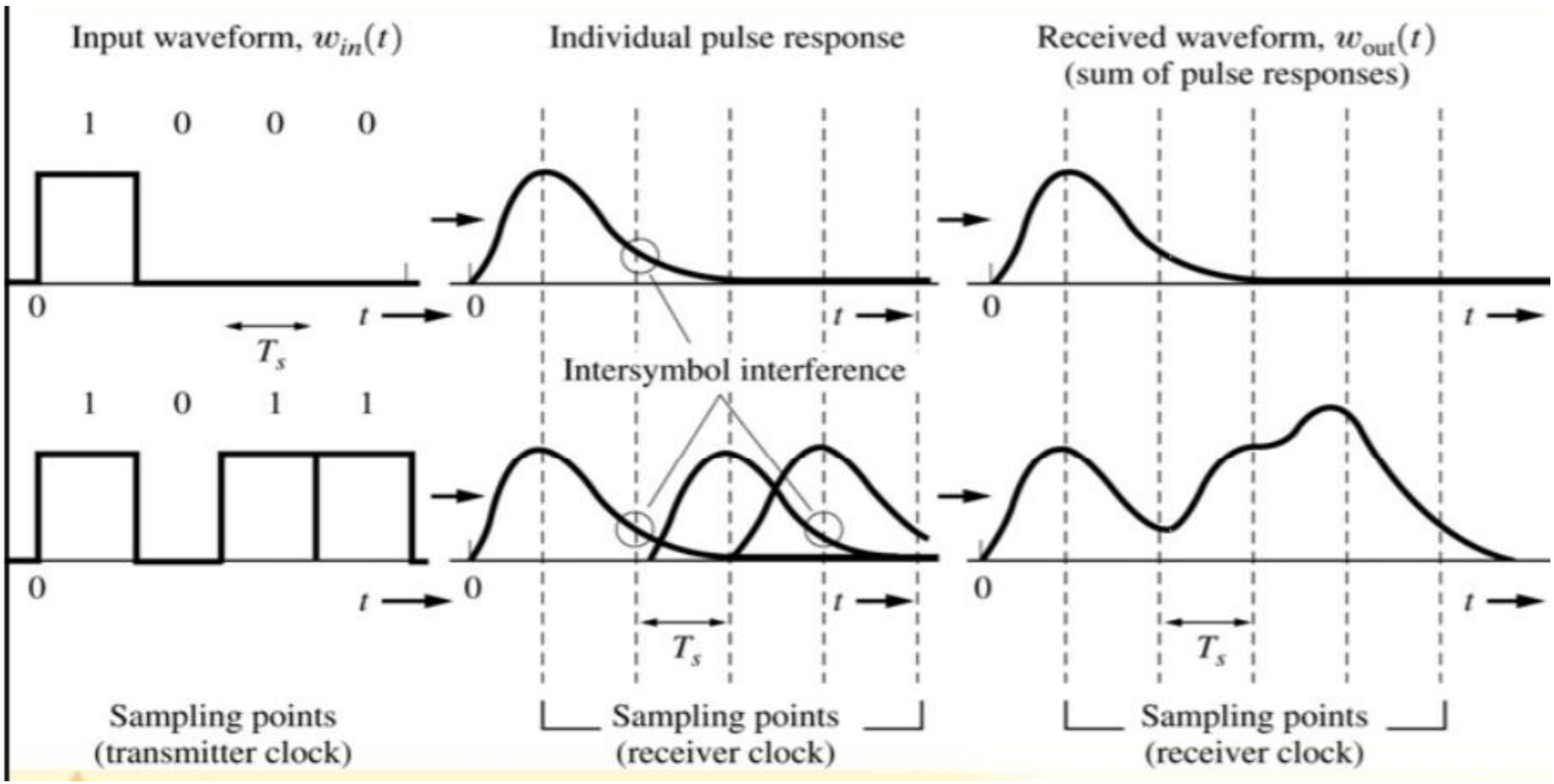
Intersymbol Interference (ISI)

- ISI occurs when a pulse spreads out in such a way that it interferes with adjacent at the sample instant, ie.
  - Extension beyond T_b is ISI
- Example: Assume polar NRZ line code. The channel outputs are "smeared" (width T_b becomes $2T_b$) pulses (spreading due to bandlimited channel)



https://slideplayer.com/slide/14784558/

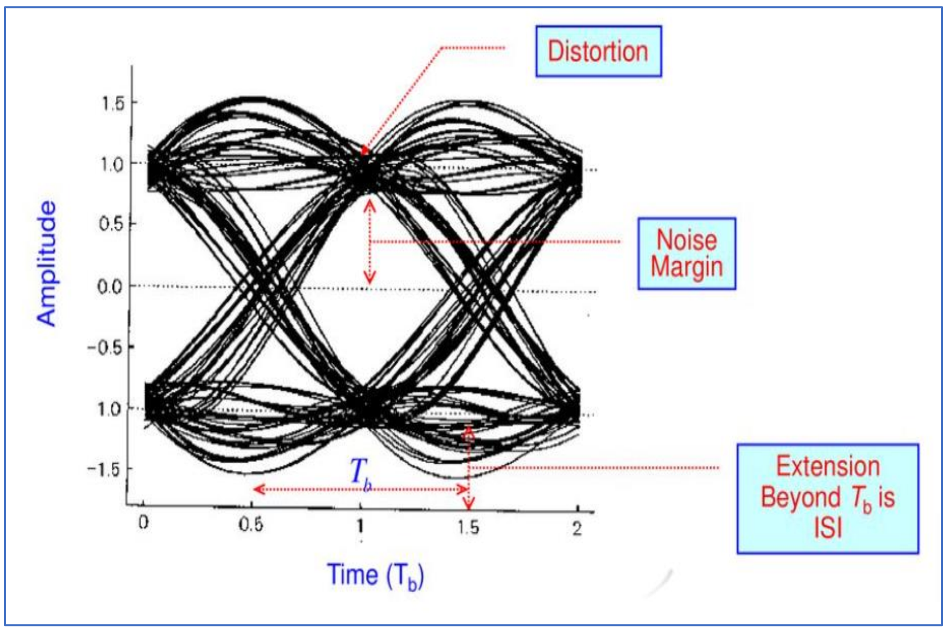
- Example of ISI on received pulses in a binary communication system

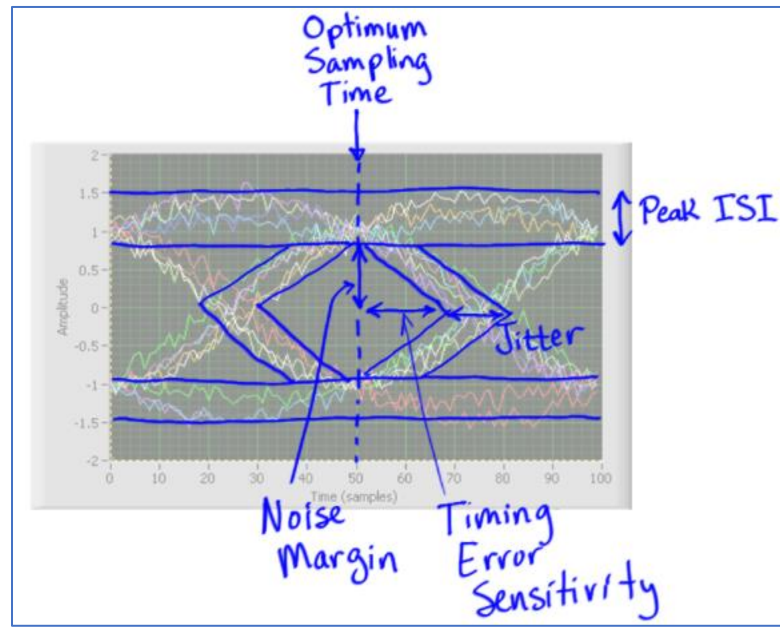


Couch, Digital and Analog Communication Systems, Seventh Edition

ISI on Eye Pattern (Eye Diagram)

- If rectangular pulses are filtered improperly as they pass through a communications system, they will spread in time, and the pulse for each symbol may be smeared into adjacent time slots and cause Intersymbol Interference
- The amount of ISI can be seen on an oscilloscope using an Eye Diagram or Eye pattern





https://slideplayer.com/slide/14784558/ https://cnx.org/contents/KiOnofD4@2/Intersymbol-Interference-ISI-and-the-Eye-Diagram

Eye Pattern (Eye Diagram)

- In telecommunication, an eye pattern, also known as an eye diagram, is an oscilloscope display in which a digital signal from a receiver is repetitively sampled and applied to the vertical input, while the data rate is used to trigger the horizontal sweep
- It is so called because pattern looks like a series of eyes between a pair of rails
- It is a tool for evaluation of combined effects of channel noise and intersymbol interference on performance of a baseband pulse-transmission system
- Several system performance measures can be derived by analyzing the display
    - An open eye pattern corresponds to minimal signal distortion
    - Distortion of the signal waveform due to intersymbol interference and noise appears as closure of the eye pattern

Combating ISI

- **Three strategies** for eliminating ISI:
- Use a line code that is absolutely bandlimited
  - Would require sinc pulse shape
  - Can't actually do this (but can approximate)
- Use a line code that is zero during adjacent sample instants
  - It's okay for pulses to overlap somewhat, as long as there is no overlap at the sample instants
  - Can come up with pulse shapes that don't overlap during adjacent sample instants
    - Raised-Cosine Rolloff pulse shaping
- Use a filter at the receiver to "undo" the distortion introduced by the channel
  - Equalizer

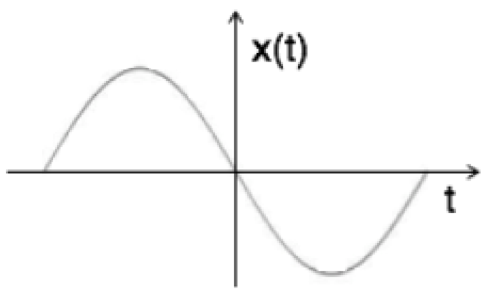
SAMPLING THEOREM WITH PROOF -

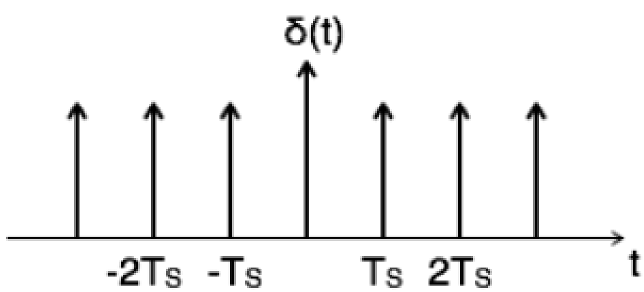
Statement: A continuous time signal can be represented in its samples and can be recovered back when sampling frequency f_s is greater than or equal to the twice the highest frequency component of message signal. i. e.

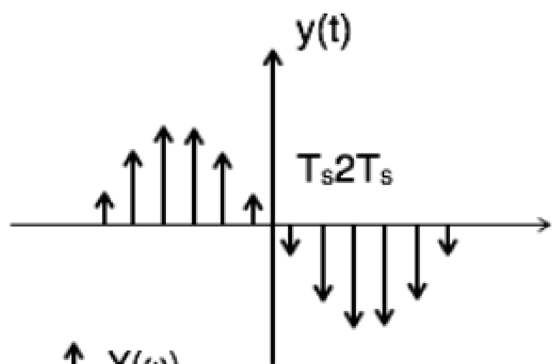
$$f_s \geq 2f_m.$$

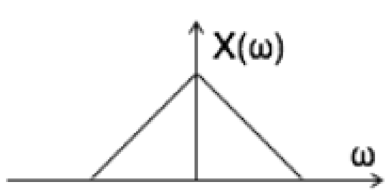
Proof: Consider a continuous time signal x(t). The spectrum of x(t) is a band limited to f_m Hz i.e. the spectrum of x(t) is zero for $|\omega|>\omega_m$.

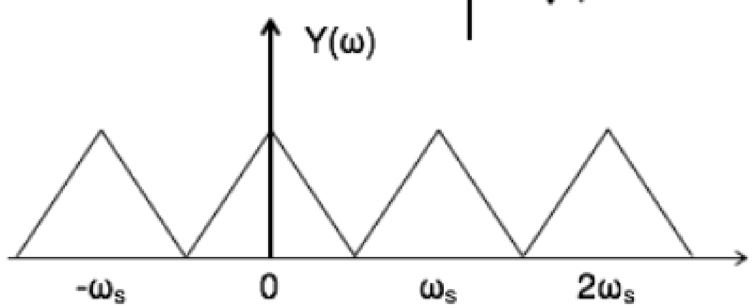
Sampling of input signal x(t) can be obtained by multiplying x(t) with an impulse train δ(t) of period T_s. The output of multiplier is a discrete signal called sampled signal which is represented with y(t) in the following diagrams:











Here, you can observe that the sampled signal takes the period of impulse. The process of sampling can be explained by the following mathematical expression:

$$\text{Sampled signal } y(t) = x(t).\, \delta(t) \; \ldots \ldots (1)$$

The trigonometric Fourier series representation of δ (t) is given by

$$\delta(t) = a_0 + \Sigma_{n=1}^{\infty}(a_n \cos n\omega_s t + b_n \sin n\omega_s t) \; \ldots \ldots (2)$$

Where $a_0 = \frac{1}{T_s} \int_{\frac{-T}{2}}^{\frac{T}{2}} \delta(t)dt = \frac{1}{T_s}\delta(0) = \frac{1}{T_s}$

$$a_n = \frac{2}{T_s} \int_{\frac{-T}{2}}^{\frac{T}{2}} \delta(t) \cos n\omega_s \, dt = \frac{2}{T_2}\delta(0) \cos n\omega_s 0 = \frac{2}{T}$$

$$b_n = \frac{2}{T_s} \int_{\frac{-T}{2}}^{\frac{T}{2}} \delta(t) \sin n\omega_s t \, dt = \frac{2}{T_s}\delta(0) \sin n\omega_s 0 = 0$$

Substitute above values in equation 2.

$$\therefore \; \delta(t) = \frac{1}{T_s} + \Sigma_{n=1}^{\infty}(\frac{2}{T_s} \cos n\omega_s t + 0)$$

Substitute δ(t) in equation 1.

$$\rightarrow y(t) = x(t).\, \delta(t)$$

$$= x(t)[\frac{1}{T_s} + \Sigma_{n=1}^{\infty}(\frac{2}{T_s} \cos n\omega_s t)]$$

$$= \frac{1}{T_s}[x(t) + 2\Sigma_{n=1}^{\infty}(\cos n\omega_s t) x(t)]$$

$$y(t) = \frac{1}{T_s}[x(t) + 2\cos \omega_s t.\, x(t) + 2\cos 2\omega_s t.\, x(t) + 2\cos 3\omega_s t.\, x(t) \ldots \ldots]$$

Take Fourier transform on both sides.

$$Y(\omega) = \frac{1}{T_s}[X(\omega) + X(\omega - \omega_s) + X(\omega + \omega_s) + X(\omega - 2\omega_s) + X(\omega + 2\omega_s) + \dots]$$

$$\therefore\ Y(\omega) = \frac{1}{T_s}\Sigma_{n=-\infty}^{\infty} X(\omega - n\omega_s) \qquad where\ n = 0, \pm1, \pm2, \dots$$

To reconstruct x(t), you must recover input signal spectrum X(ω) from sampled signal spectrum Y(ω), which is possible when there is no overlapping between the cycles of Y(ω).

Possibility of sampled frequency spectrum with different conditions is given by the following diagrams: