

Module II

Lecture 1

- Basics of signal representation and analyses
- Mathematical representation of signals (TBS)
- Concept of Time shifting, Time scaling, Time reversal (TBS)

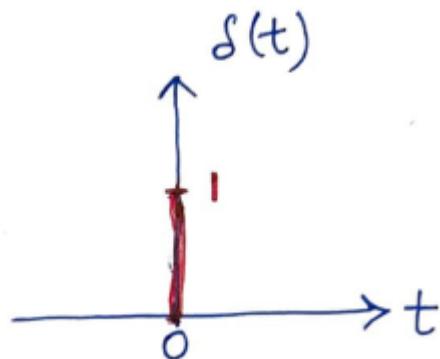


Representation of Standard Continuous Time Signals

Impulse (Delta) signal -

Denoted by $\delta(t)$

$$\begin{aligned}\delta(t) &= 1 ; t=0 \\ &= 0 ; t \neq 0\end{aligned}$$



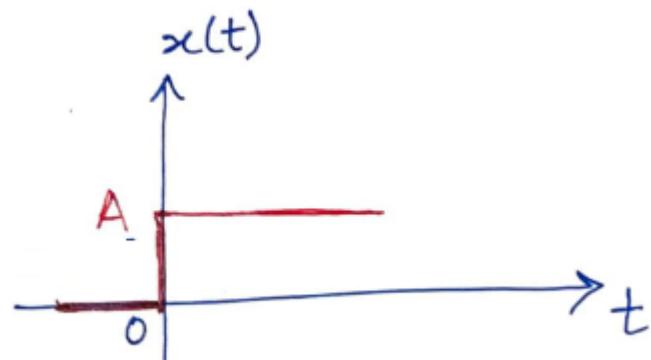
Unit impulse signal is a signal with infinite magnitude and zero duration but with unit area,

$$\int_{-\infty}^{\infty} \delta(t) \cdot dt = 1$$



Step signal -

$$x(t) = A ; t \geq 0 \\ = 0 ; t < 0$$

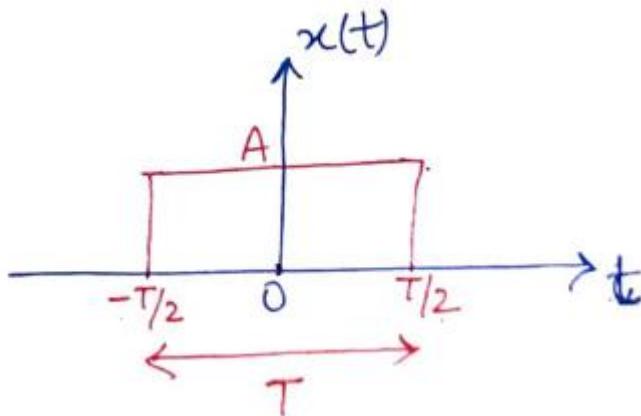


When $A=1$, unit step signal ($u(t)$)

Gate/Rectangular/Pulse signal -

$$x(t) = A ; -\frac{T}{2} \leq t \leq \frac{T}{2} \\ = 0 ; \text{otherwise}$$

when $A=1$, unit gate signal

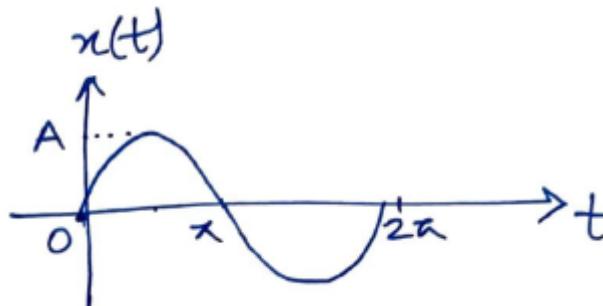


Sine signal -

$$x(t) = A \sin(wt + \phi)$$

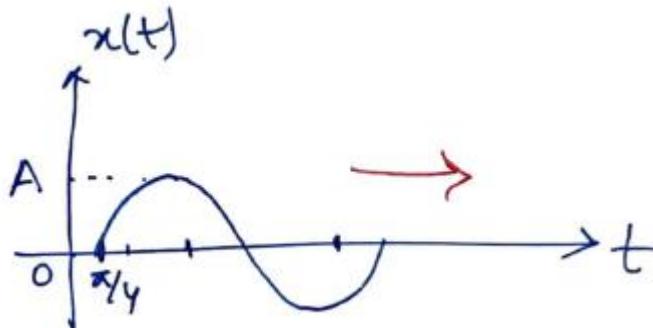
→ When $\phi = 0$

$$x(t) = A \sin wt$$



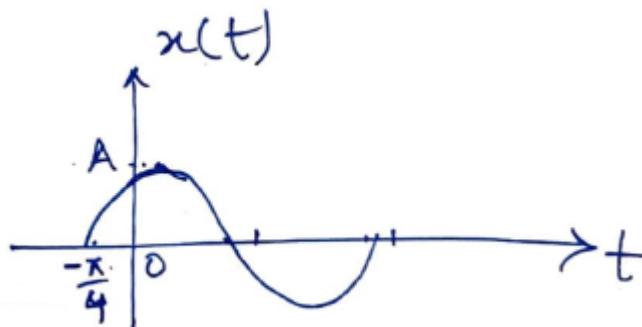
→ When $\phi = 45^\circ$ or $\frac{\pi}{4}$ but lagging

$$x(t) = A \sin \left(wt - \frac{\pi}{4} \right)$$



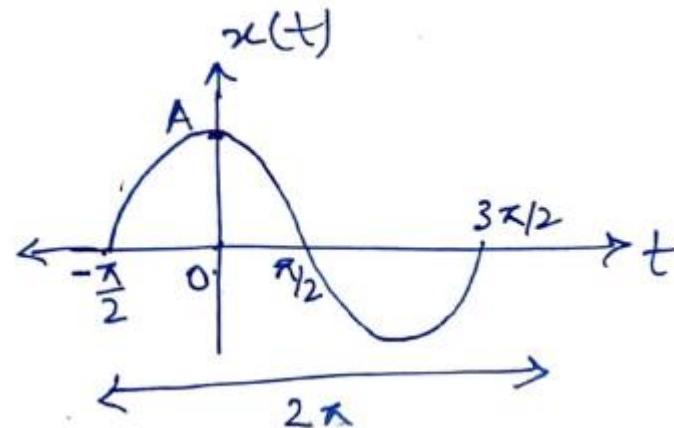
→ When $\phi = 45^\circ$ but leading

$$x(t) = A \sin \left(wt + \frac{\pi}{4} \right)$$



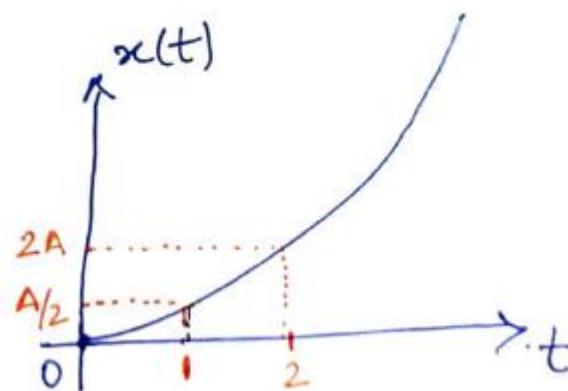
Cosine signal -

$$x(t) = A \cos(\omega t + \phi)$$



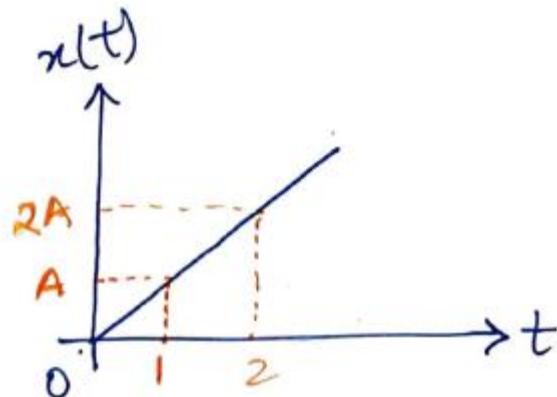
Parabolic signal -

$$\begin{aligned} x(t) &= \frac{At^2}{2} ; \quad t \geq 0 \\ &= 0 \quad ; \quad t < 0 \end{aligned}$$



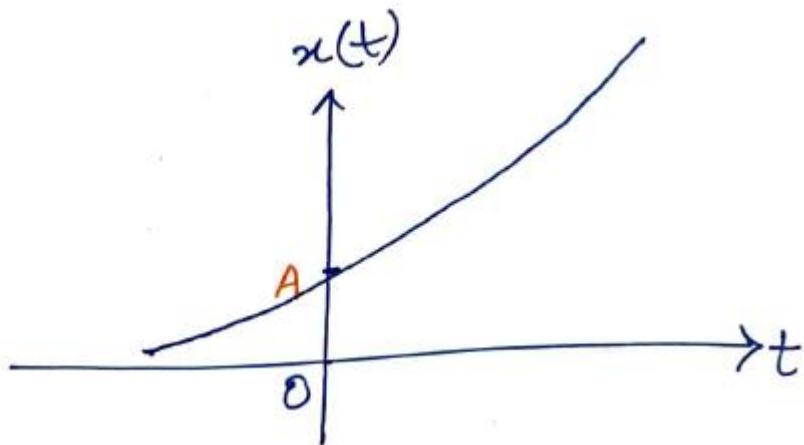
Ramp signal -

$$x(t) = At ; t \geq 0$$
$$= 0 ; t < 0$$

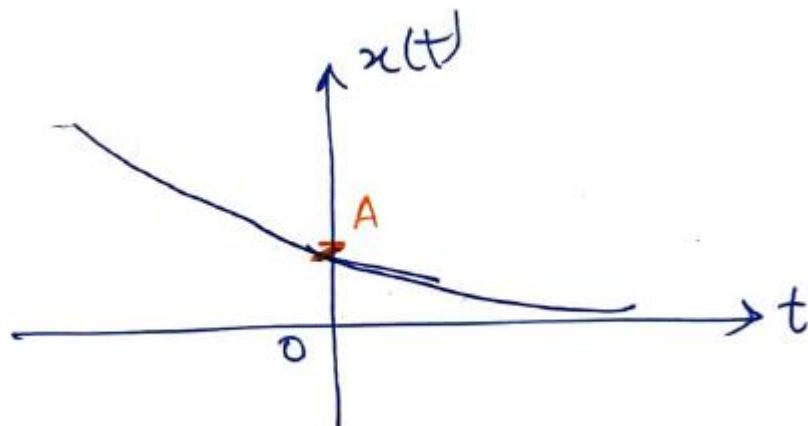


Exponential signal -

$$x(t) = A e^{bt}$$

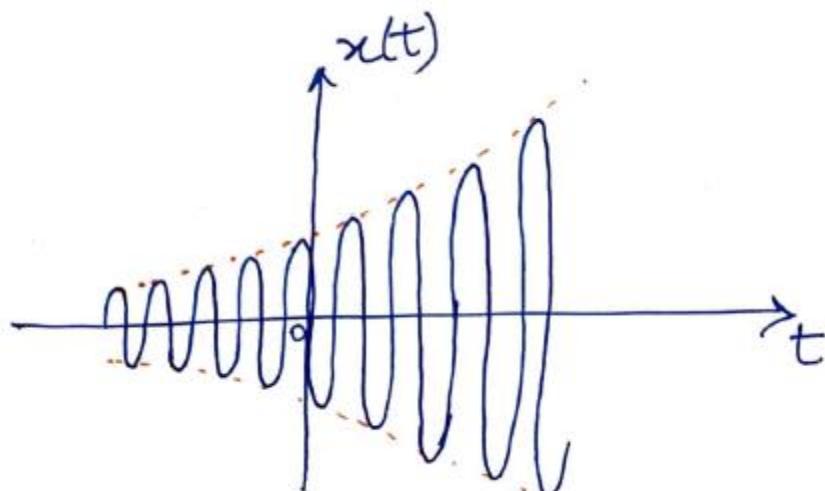


$$x(t) = A e^{-bt}$$

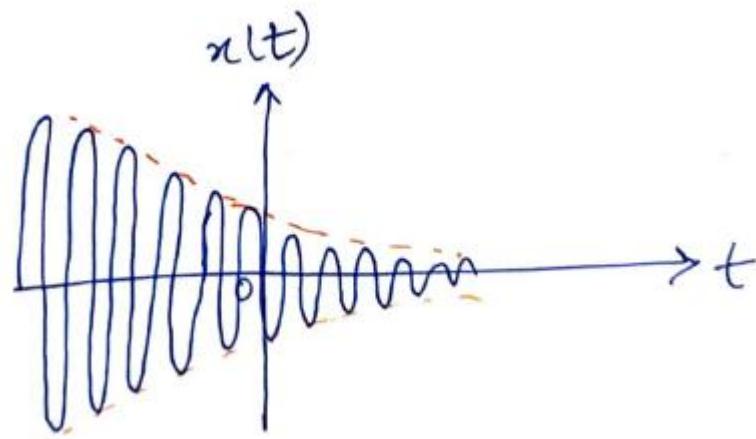


Exponentially rising & Exponentially damping signal -

$$x(t) = A \cdot e^{bt} \cdot \sin \omega t$$

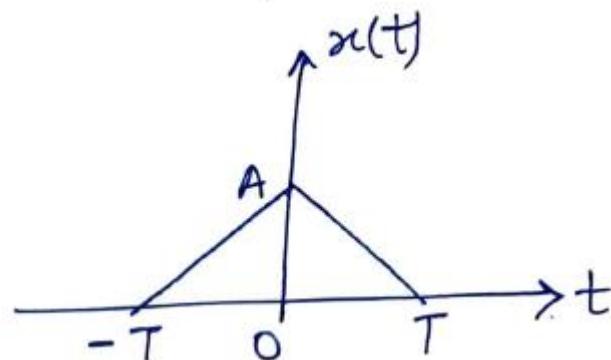


$$x(t) = A \cdot e^{-bt} \cdot \sin \omega t$$



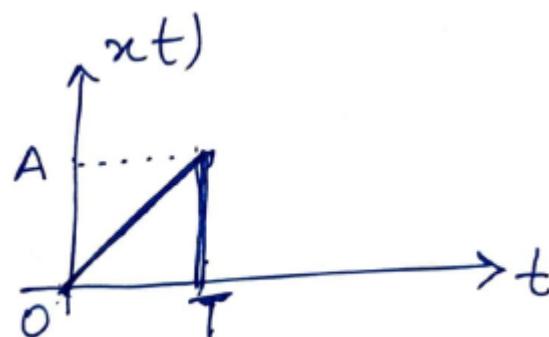
Triangular signal -

$$x(t) = A \left(1 - \frac{|t|}{T} \right)$$



Sawtooth signal -

$$\begin{aligned} x(t) &= \frac{A}{T} t ; \quad 0 \leq t \leq T \\ &= 0 ; \text{ otherwise} \end{aligned}$$



Gaussian signal -

$$x(t) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(t-\mu)^2}{2\sigma^2}}$$

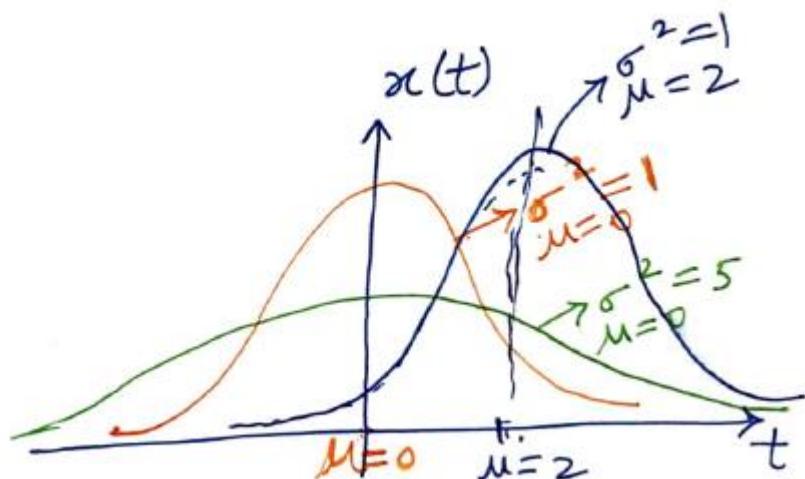
$\mu \rightarrow$ mean (w/F centred around mean)

$\sigma \rightarrow$ std. dev. (spread)

$$\rightarrow \mu = 0, \sigma^2 = 1$$

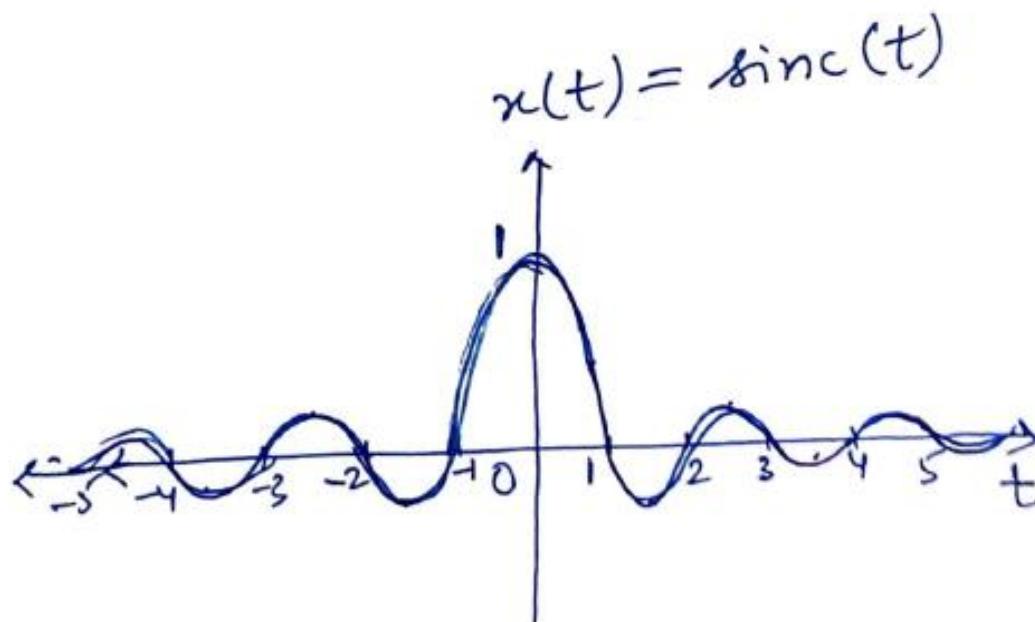
$$\rightarrow \mu = 0, \sigma^2 = 5$$

$$\rightarrow \mu = 2, \sigma^2 = 1$$



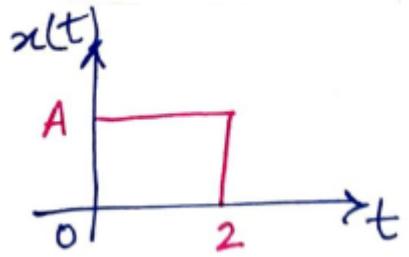
Sinc signal -

$$x(t) = \text{sinc}(t) = \frac{\sin t}{t}$$

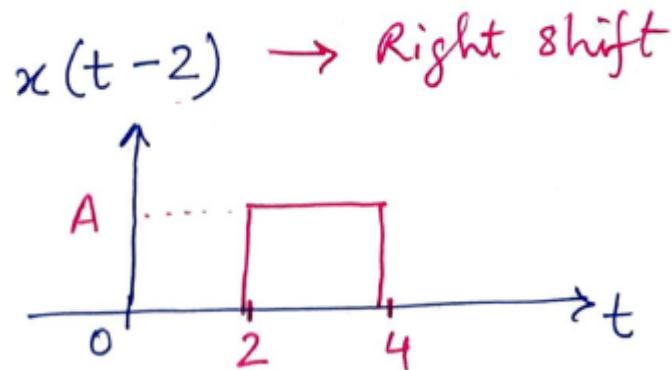


Basic Operations

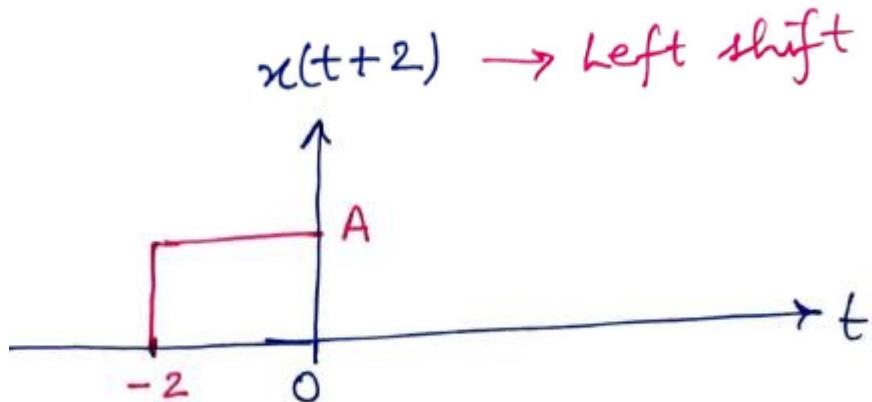
Time Shifting-



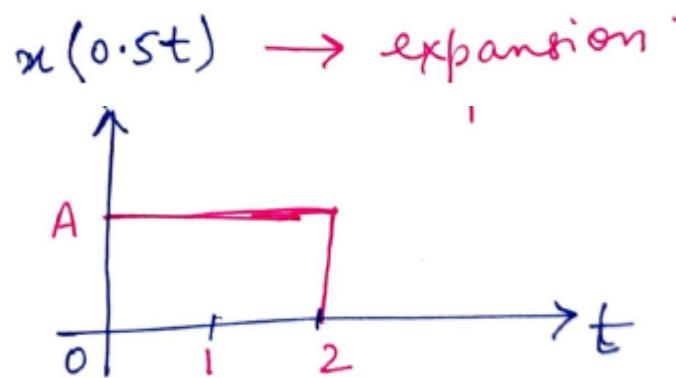
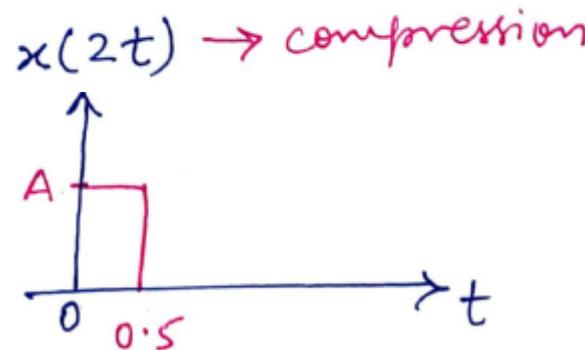
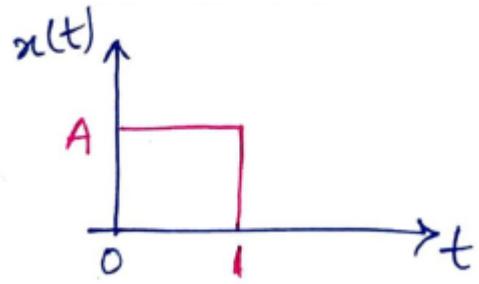
$$\begin{aligned}t &\rightarrow \text{New} \\0 &\rightarrow 2 \\2 &\rightarrow 4\end{aligned}$$



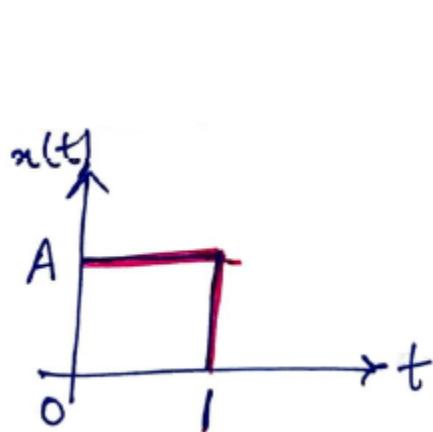
$$\begin{aligned}\underline{t} &\rightarrow \text{New} \\0 &\rightarrow -2 \\2 &\rightarrow 0\end{aligned}$$



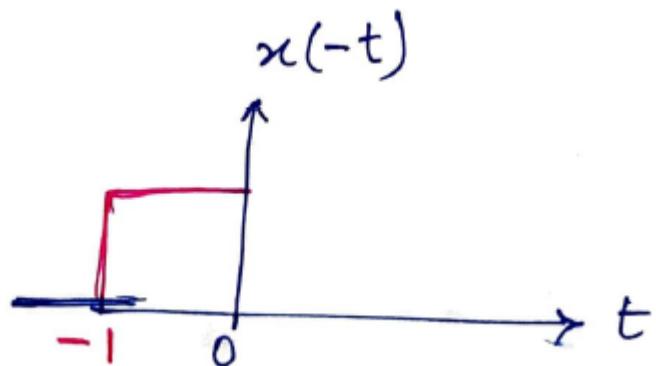
Time Scaling -



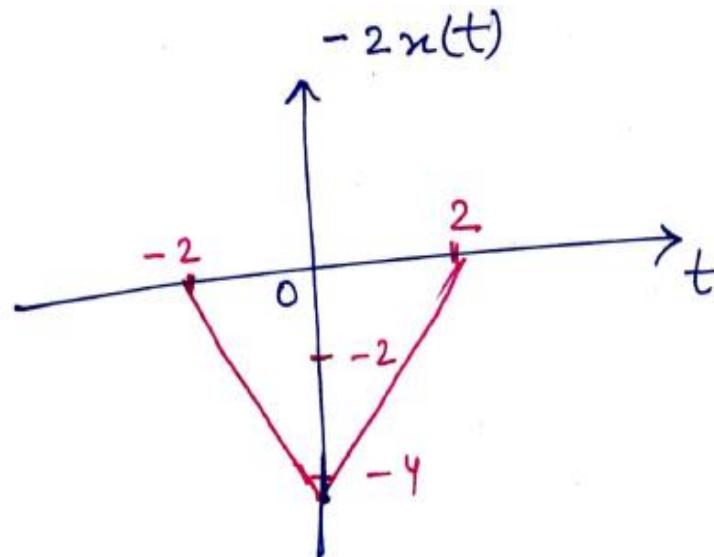
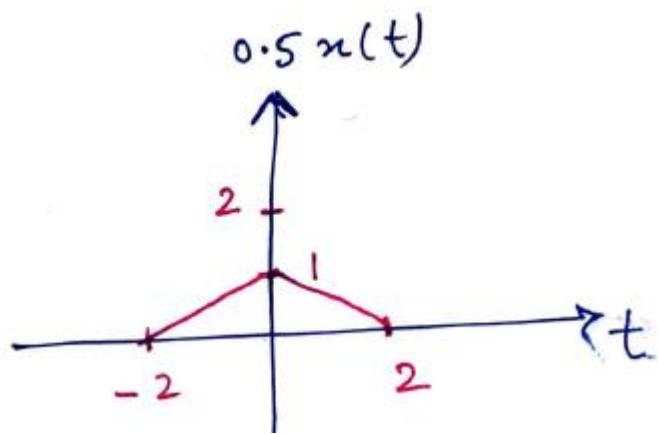
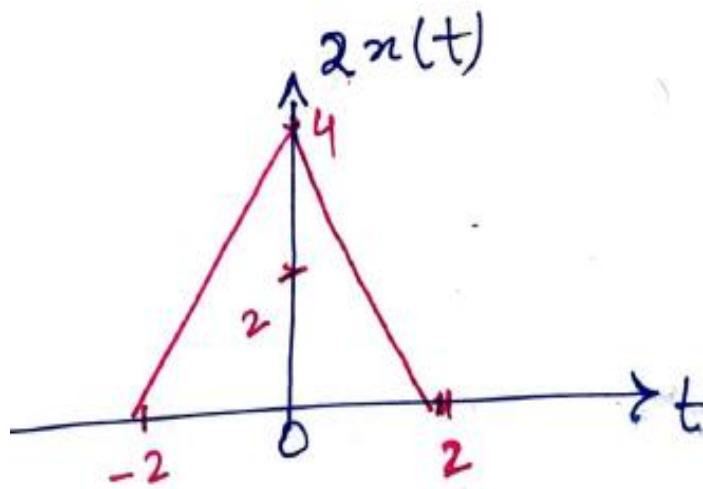
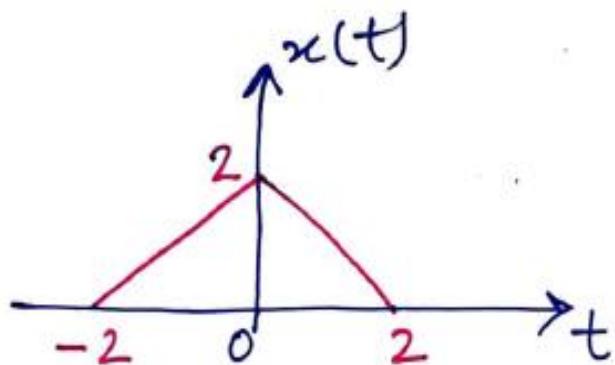
Time Reversal -



$$\begin{aligned} \frac{t}{1} &\rightarrow -t \\ 1 &\rightarrow -1 \end{aligned}$$



Amplitude Scaling -



Module II

Lecture 2

- Fourier Transform
 - Definition of Fourier Transform
 - Definition of Inverse Fourier Transform
 - Conditions for existence of Fourier Transform
 - Properties of Fourier Transform



Fourier Transform

- **Recap** - Already studied signals and its representation in time domain
- In communication systems, we need to analyze signals in frequency domain as well
- Fourier transform is used for transformation of a signal represented in time domain into a signal represented in frequency domain

Definition of Fourier Transform

- If $x(t)$ represents a continuous signal in time domain, its Fourier transform is given as –

$$F[x(t)] = X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft} dt$$

- We can get back the signal from frequency domain to time domain by carrying out Inverse Fourier Transform

Definition of Inverse Fourier Transform

- If $X(f)$ represents the signal in frequency domain, its inverse Fourier transform is given as –

$$F^{-1}[X(f)] = x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft} df$$

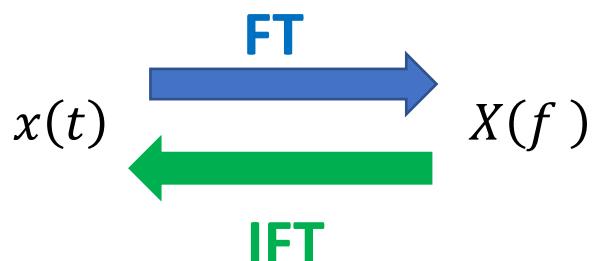
- Therefore, $x(t)$ and $X(f)$ form a Fourier transform pair

Definition of Inverse Fourier Transform

- If $X(f)$ represents the signal in frequency domain, its inverse Fourier transform is given as –

$$F^{-1}[X(f)] = x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft} df$$

- Therefore, $x(t)$ and $X(f)$ form a Fourier transform pair



Conditions for existence of Fourier Transform -

- $x(t)$ should be singled valued in any finite time interval, T
- $x(t)$ should have finite number of discontinuities in any finite time interval, T
- $x(t)$ should have a finite number of maxima and minima in any finite time interval, T
- $x(t)$ should be an absolutely integrable function

Properties of Fourier Transform

I. Linearity or Superposition -

$$a_1x_1(t) + a_2x_2(t) \xrightarrow{F} a_1X_1(f) + a_2X_2(f)$$

where, a_1 and a_2 are constants

II. Time Scaling-

$$x(at) \xrightarrow{F} \frac{1}{|\alpha|} X\left(\frac{f}{\alpha}\right)$$

where, α is a constant



III. Duality or Symmetry -

If $x(t) \xleftrightarrow{F} X(f)$, then
 $X(t) \xleftrightarrow{F} x(-f)$

IV. Time Shifting-

$$x(t - d) \xleftrightarrow{F} e^{-j2\pi f d} X(f)$$

V. Area under $x(t)$ -

$$\int_{-\infty}^{\infty} x(t) dt \xleftrightarrow{F} X(0)$$



VI. Area under X(f)-

$$\int_{-\infty}^{\infty} X(f) df \xrightarrow{F} x(0)$$

VII. Frequency Shifting-

$$e^{-j2\pi f_c t} x(t) \xrightarrow{F} X(f - f_c)$$



VIII. Differentiation in time domain -

$$\frac{d}{dt} x(t) \xleftrightarrow{F} j\pi 2f \cdot X(f)$$

IX. Integration in time domain -

$$\int_{-\infty}^{\infty} x(t) dt \xleftrightarrow{F} \frac{1}{2\pi f} X(f)$$

X. Multiplication in time domain -

$$x_1(t) \cdot x_2(t) \xleftrightarrow{F} \int_{-\infty}^{\infty} X_1(\nu) \cdot X_2(f - \nu) d\nu$$

OR

$$x_1(t) \cdot x_2(t) \xleftrightarrow{F} X_1(f) * X_2(f)$$

XI. Convolution in time domain -

$$x_1(t) * x_2(t) \xleftrightarrow{F} X_1(f) \cdot X_2(f)$$

XII. Conjugate Functions

If $x(t) \xleftrightarrow{F} X(f)$, then

$$x^*(t) \xleftrightarrow{F} X(-f)$$



In Syllabus...

Time Scaling	$x(\alpha t) \xrightarrow{F} \frac{1}{ \alpha } X\left(\frac{f}{\alpha}\right)$ where, α is a constant
Time Shifting	$x(t - d) \xrightarrow{F} e^{-j2\pi f d} X(f)$
Frequency Shifting	$e^{-j2\pi f_c t} x(t) \xrightarrow{F} X(f - f_c)$
Differentiation in time domain	$\frac{d}{dt} x(t) \xrightarrow{F} j\pi 2f \cdot X(f)$
Convolution in time domain	$x_1(t) * x_2(t) \xrightarrow{F} X_1(f) \cdot X_2(f)$



→ TIME SCALING -

The time scaling property states that if $x(t)$ and $X(f)$ form a Fourier transform pair, then -

$$x(\alpha t) \xleftrightarrow{F} \frac{1}{|\alpha|} \cdot X\left(\frac{f}{\alpha}\right)$$

where, α is a constant.

Proof - We know that -

$$F[x(t)] = X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi ft} \cdot dt$$

$$\therefore F[x(\alpha t)] = \int_{-\infty}^{\infty} x(\alpha t) \cdot e^{-j2\pi f\alpha t} \cdot dt$$

$$\text{Let, } \alpha t = y$$

$$\Rightarrow t = \frac{y}{\alpha} \Rightarrow dt = \frac{1}{\alpha} \cdot dy$$

$$\therefore F[x(\alpha t)] = \int_{-\infty}^{\infty} x(y) \cdot e^{-j2\pi f \frac{y}{\alpha}} \cdot \frac{1}{\alpha} \cdot dy$$

$$= \frac{1}{|\alpha|} \int_{-\infty}^{\infty} x(y) \cdot e^{-j2\pi f \frac{y}{\alpha}} \cdot dy$$

$$= \frac{1}{|\alpha|} \cdot X\left(\frac{f}{\alpha}\right)$$

when α is +ve
Limits $-\infty$ to $+\infty$

when α is -ve
Limits $+\infty$ to $-\infty$
To change limits $-\infty$ to $+\infty$
introduce -ve sign

Hence Proved .

→ TIME SHIFTING -

The time shifting property states that if $x(t)$ and $X(f)$ form a Fourier transform pair, then -

$$x(t - t_d) \xleftarrow{F} e^{-j2\pi f t_d} \cdot X(f)$$

Time shifted signal

Proof - We know -

$$F[x(t)] = X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi f t} \cdot dt$$

$$\therefore F[x(t - t_d)] = \int_{-\infty}^{\infty} x(t - t_d) \cdot e^{-j2\pi f t} \cdot dt$$

$$\text{Let, } (t - t_d) = \tau$$

$$\Rightarrow t = \tau + t_d \Rightarrow dt = d\tau$$

$$\therefore F[x(t - t_d)] = \int_{-\infty}^{\infty} x(\tau) \cdot e^{-j2\pi f(\tau + t_d)} \cdot d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau) \cdot e^{-j2\pi f \tau} \cdot e^{-j2\pi f t_d} \cdot d\tau$$

$$= e^{-j2\pi f t_d} \int_{-\infty}^{\infty} x(\tau) \cdot e^{-j2\pi f \tau} \cdot d\tau$$

$$= e^{-j2\pi f t_d} \cdot X(f)$$

Hence proved.

→ FREQUENCY SHIFTING -

It states that if $x(t)$ and $X(f)$ form a Fourier transform pair, then -

$$e^{j2\pi f_c t} \cdot x(t) \xleftarrow{F} X(f - f_c)$$

where, f_c is a real constant.

Proof - We know -

$$F[x(t)] = X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi ft} \cdot dt$$

$$\begin{aligned} \therefore F[e^{j2\pi f_c t} \cdot x(t)] &= \int_{-\infty}^{\infty} e^{j2\pi f_c t} \cdot x(t) \cdot e^{-j2\pi ft} \cdot dt \\ &= \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi(f-f_c)t} \cdot dt \\ &= X(f - f_c) \end{aligned}$$

shifted frequency spectrum

Hence proved.

→ DIFFERENTIATION IN TIME DOMAIN -

Let $x(t) \xleftrightarrow{F} X(f)$ and let derivative of $x(t)$ be Fourier transformable, then -

$$\frac{d}{dt} x(t) \xleftrightarrow{F} j2\pi f \cdot X(f)$$

Proof - By definition of IFT, we know -

$$x(t) = F^{-1}[X(f)] = \int_{-\infty}^{\infty} X(f) \cdot e^{j2\pi ft} \cdot df$$

$$\therefore \frac{d}{dt} x(t) = \frac{d}{dt} \int_{-\infty}^{\infty} X(f) \cdot e^{j2\pi ft} \cdot df$$

$$\frac{d}{dt} x(t) = \int_{-\infty}^{\infty} X(f) \cdot \left[\frac{d}{dt} e^{j2\pi ft} \right] \cdot df$$

$$\frac{d}{dt} x(t) = \int_{-\infty}^{\infty} X(f) \cdot e^{j2\pi ft} \cdot j2\pi f \cdot df$$

$$\frac{d}{dt} x(t) = \int_{-\infty}^{\infty} \underbrace{j2\pi f X(f)}_{\text{signal}} \cdot e^{j2\pi ft} \cdot df$$

$$\frac{d}{dt} x(t) = F^{-1}[j2\pi f \cdot X(f)]$$

Taking Fourier transform on both sides -

$$F\left[\frac{d}{dt} x(t)\right] = j2\pi f \cdot X(f)$$

Hence proved.

→ CONVOLUTIONS IN TIME DOMAIN -

This property states that the convolution of signals in time domain will be transformed into multiplication of their Fourier transform in frequency domain.

This means that -

$$[x_1(t) * x_2(t)] \xleftarrow{F} x_1(f) \cdot x_2(f)$$

Proof - The convolution of two signals in time domain is given by -

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\lambda) \cdot x_2(t - \lambda) \cdot d\lambda$$

Taking Fourier Transform, we get

$$F[x_1(t) * x_2(t)] = \int_{-\infty}^{\infty} [x_1(t) * x_2(t)] \cdot e^{-j2\pi ft} \cdot dt$$

$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x_1(\lambda) \cdot x_2(t - \lambda) \cdot d\lambda \right] \cdot e^{-j2\pi ft} \cdot dt$$

multiplying and dividing RHS by $e^{-j2\pi f t}$, we get -

$$= \int_{-\infty}^{\infty} x_1(\lambda) \cdot e^{-j2\pi f \lambda} \cdot dt \cdot \int_{-\infty}^{\infty} x_2(t - \lambda) \cdot e^{-j2\pi f (t - \lambda)} \cdot dt$$

Let $t - \lambda = m$
 $dt = dm$

$$\Rightarrow x_2(f)$$

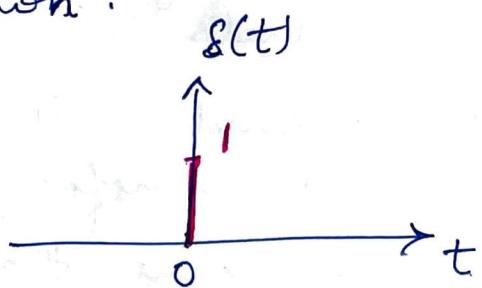
$$= x_1(f) \cdot x_2(f)$$

Hence Proved

EXAMPLES

Q1. Obtain Fourier Transform of a unit Delta (Impulse) function.

$$\delta(t) = 1 \quad ; \quad t = 0 \\ = 0 \quad ; \quad \text{otherwise}$$



Soln: we know -

$$F[x(t)] = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi ft} \cdot dt \\ = \int_{-\infty}^{\infty} \delta(t) \cdot e^{-j2\pi ft} \cdot dt \\ = \delta(t) \cdot e^{-j2\pi ft} \Big|_{t=0} \\ = \delta(0) \cdot e^{-j2\pi f \times 0}$$

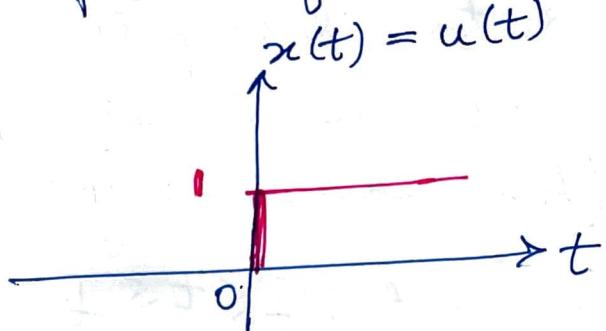
$$= 1 \cdot 1$$

$$= 1$$

Ans.

Q2. Obtain Fourier Transform of a unit step signal -

$$u(t) = x(t) = 1; t \geq 0 \\ = 0; t < 0$$



Soln. We know -

$$F[x(t)] = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi ft} \cdot dt$$

$$F[u(t)] = \int_{-\infty}^{\infty} u(t) \cdot e^{-j2\pi ft} \cdot dt$$

$$= \int_0^{\infty} 1 \cdot e^{-j2\pi ft} \cdot dt$$

$$= \frac{1}{-j2\pi f} [e^{-j2\pi ft}]_0^{\infty}$$

$$= \frac{-1}{j2\pi f} [e^{-\infty} - e^0]$$

$$= \frac{-1}{j2\pi f} [0 - 1]$$

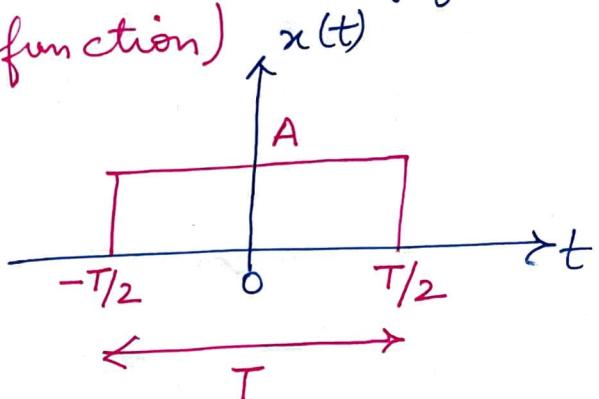
$$= \frac{1}{j2\pi f}$$

Ans

Q3. Obtain the Fourier Transform of a rectangular pulse of duration, T and amplitude, A as shown in the figure
(Also called as gate function)

$$x(t) = A ; -\frac{T}{2} \leq t \leq \frac{T}{2}$$

$$= 0 ; \text{ otherwise}$$



Soln: We know -

$$\begin{aligned} F[x(t)] &= \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi ft} \cdot dt \\ &= \int_{-T/2}^{T/2} A \cdot e^{-j2\pi ft} \cdot dt \\ &= \frac{A}{-j2\pi f} \left[e^{-j2\pi ft} \right]_{-T/2}^{T/2} \\ &= \frac{-A}{j2\pi f} \left[e^{-j2\pi f \frac{T}{2}} - e^{j2\pi f \frac{T}{2}} \right] \\ &= \frac{-A}{j2\pi f} \left[e^{-j\pi f T} - e^{j\pi f T} \right] \\ &= \frac{A}{j2\pi f} \left[e^{j\cancel{\pi f T}} - e^{-j\cancel{\pi f T}} \right] \end{aligned}$$

$$= \frac{A}{\pi f} \left[\frac{e^{j\pi f T} - e^{-j\pi f T}}{2j} \right]$$

According to Euler's theorem -

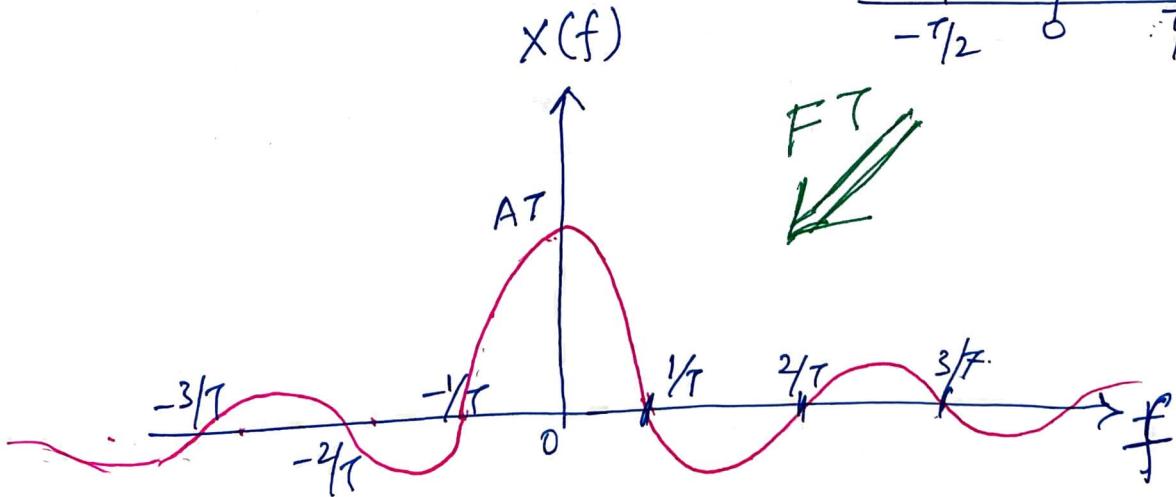
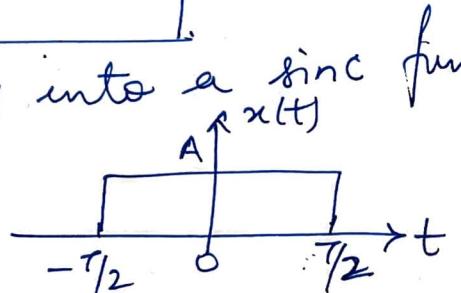
$$\sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$\begin{aligned}\therefore F[x(t)] &= \frac{A}{\pi f} \left[\sin(\pi f T) \right] \\ &= \frac{AT}{\pi f T} \left[\sin(\pi f T) \right] \\ &= AT \left[\frac{\sin(\pi f T)}{\pi f T} \right]\end{aligned}$$

We know -
 $\text{sinc } \theta = \frac{\sin \theta}{\theta}$

$$\boxed{\therefore F[x(t)] = AT \text{ sinc}(\pi f T)}$$

\Rightarrow Rectangular pulse transforms into a sinc func.



Module II

Lecture 4

- Noise in Communication Systems
- Types of Noise



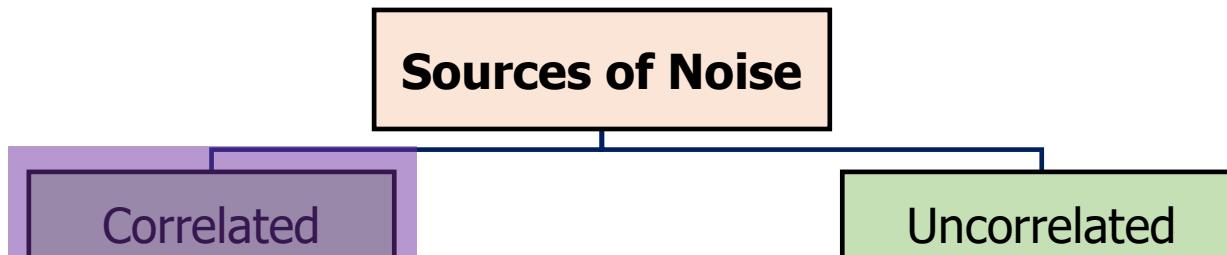
Noise

- Important parameter in communication design
- Affects receiver more as signal weak there
- Unwanted, undesired signal that interferes with the desired signal
- Gets superimposed on signal and impossible to separate signal from noise
- Random in nature
- Examples: hiss, crackle, snow

Objective: To produce highest possible SNR

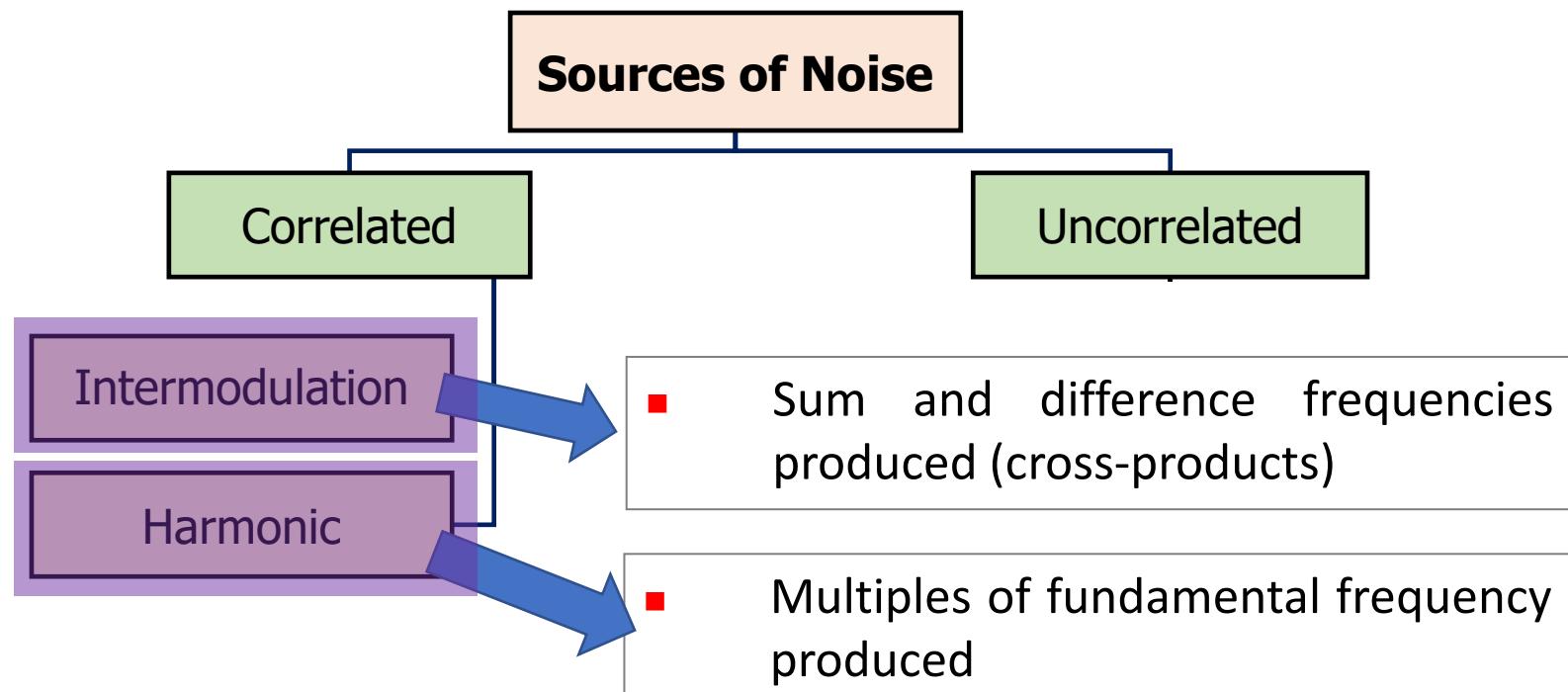


Classification of Noise



- Exists a relation between signal and noise
- Exists only when signal is present

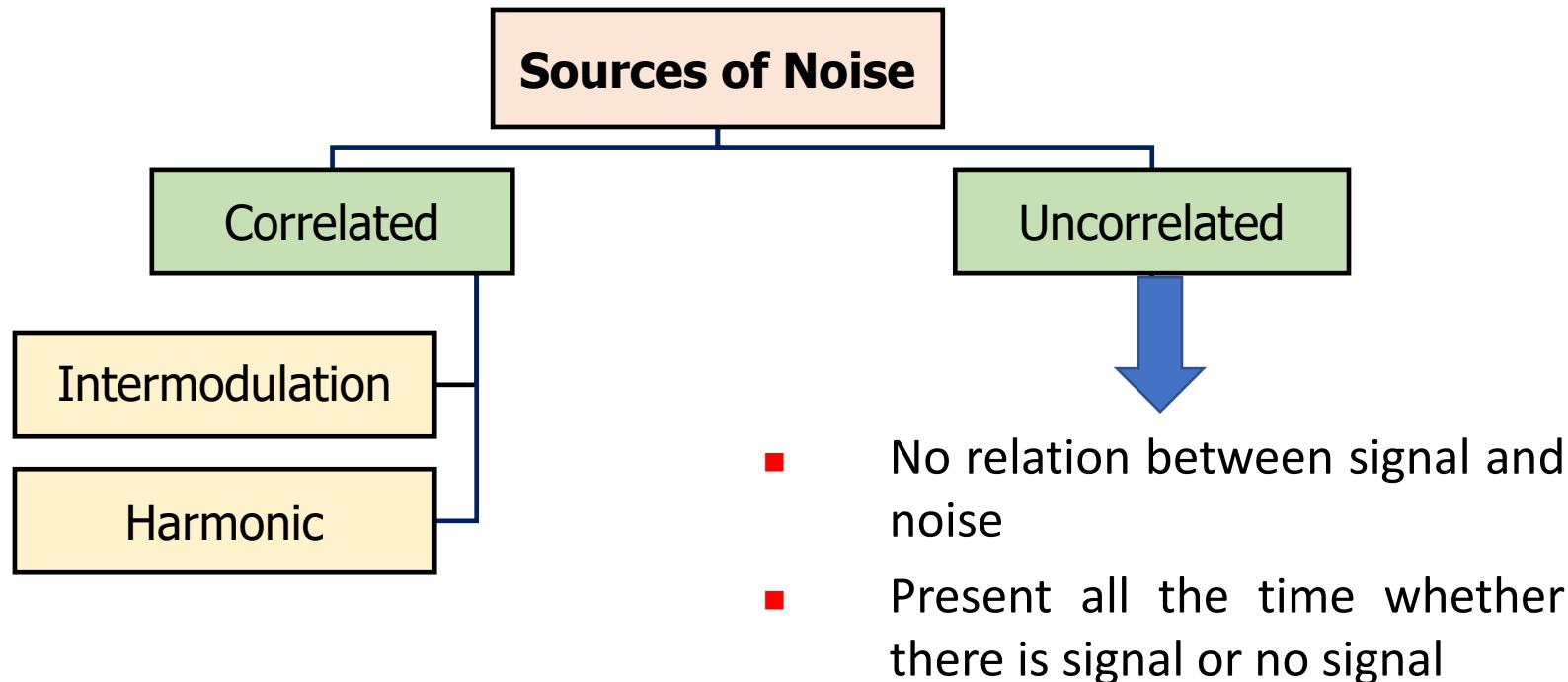
Classification of Noise



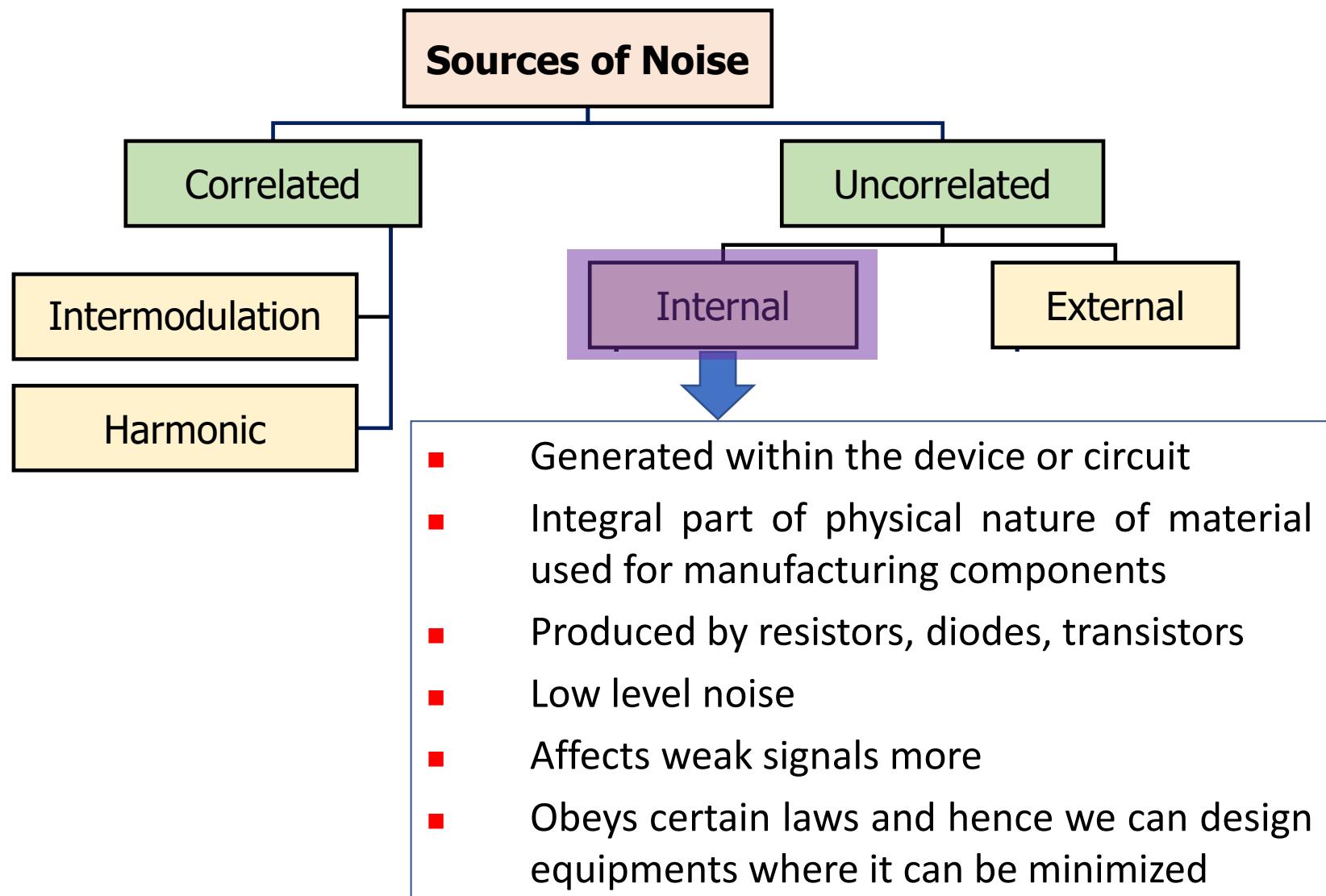
- Produced when signal passes through nonlinear devices like diodes, FET
- Hence, both together also called as nonlinear distortion



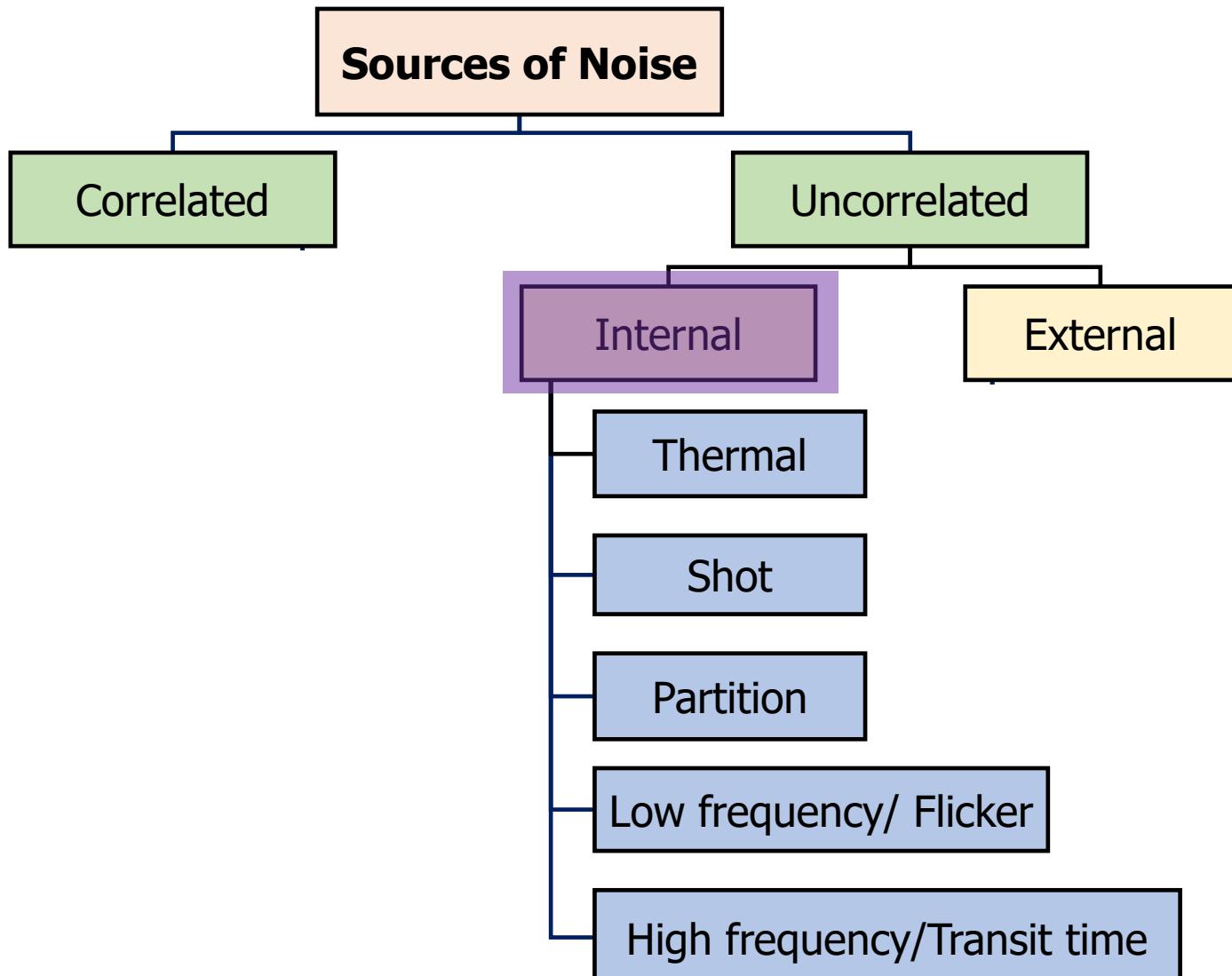
Classification of Noise



Classification of Noise



Classification of Noise



Thermal noise/ White noise/ Johnson noise

- Due to **rapid and random movement of free electrons** within a conductor due to thermal agitation
- Thermal noise power, $P \propto \text{BW} (B)$ and Temp (T)
- $N = KTB$
- Power spectral density, $S_n = N/B = KT \text{ W/Hz}$
- Since equally distributed throughout the frequency spectrum, also called White noise
- To minimize the effect...
 - Noise depends on temperature... So keep temperature low
 - Noise contains a large no. of random frequency components and noise level dependent on BW of circuit...So, use BPF
 - Noise depends on amount of current flowing in a component... So, keep I low



- Consider an equivalent circuit for a thermal noise source where internal resistance R_1 is in series with rms noise voltage, V_N
- For maximum transfer of noise power

- $R_L = R_1$

- $V_{R_L} = \frac{V_N}{2}$

- Hence, noise power developed across $R_L = KTB$

- $N = KTB = \frac{\left[\frac{V_N}{2}\right]^2}{R_L} = \frac{V_N^2}{4R_L}$

- $V_N^2 = 4KTBR_L$

where, Boltzmann constant,
 $K = 1.38064852 \times 10^{-23} \text{ m}^2 \text{ kg s}^{-2} \text{ K}^{-1}$



Shot Noise

- We assume that current in electronic devices under dc condition is constant
- However, its only time average flow of electrons which is constant
- Caused by random arrival of carriers at output element
- Sounds like a shower of lead shots on a metal sheet
- Inversely proportional to g_m of device
- Mean square shot noise current is given by –

$$I_n^2 = 2I_{dc}q_eB \text{ Amp}^2$$

where, I_{dc} is direct current in amperes

q_e is the magnitude of the electron charge ($1.6 \times 10^{-19} \text{ C}$)

B is the equivalent noise bandwidth in Hz



Partition Noise

- Occurs when current has to divide between two or more paths and results due to random fluctuations in division of current
- eg. BJT : random motion of carriers crossing junction and random recomb. in base (random division of current between C and B)
- Higher in BJT than diode
- Noise generated depends on Q-pt and source R

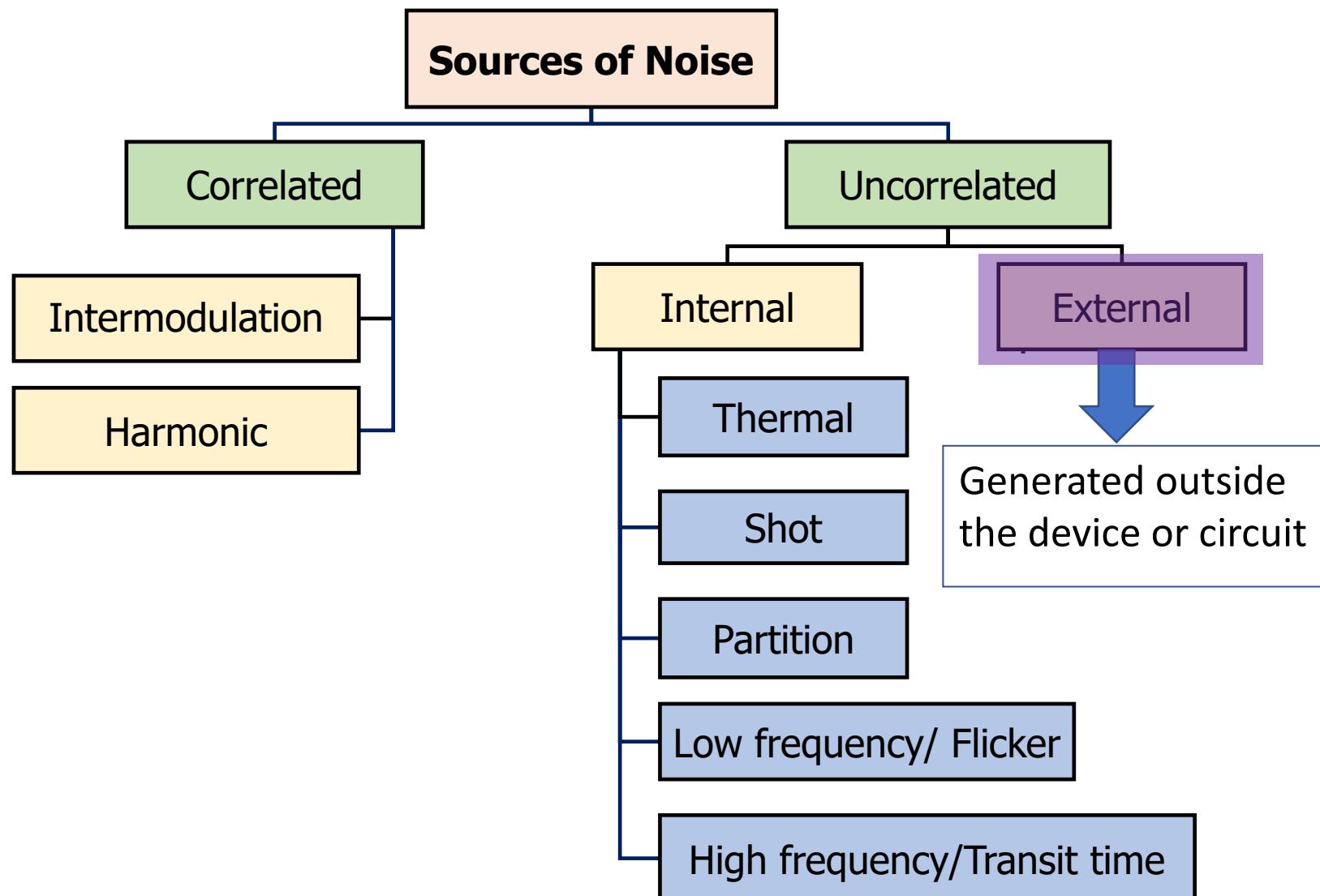
Low frequency/ Flicker Noise

- Appears for frequency < few kHz
- Inversely proportional to frequency
- It is due to fluctuations in carrier density which in turn affects the conductivity... Hence.... fluctuating voltage drop produced
- Proportional to square of direct current flowing through the device

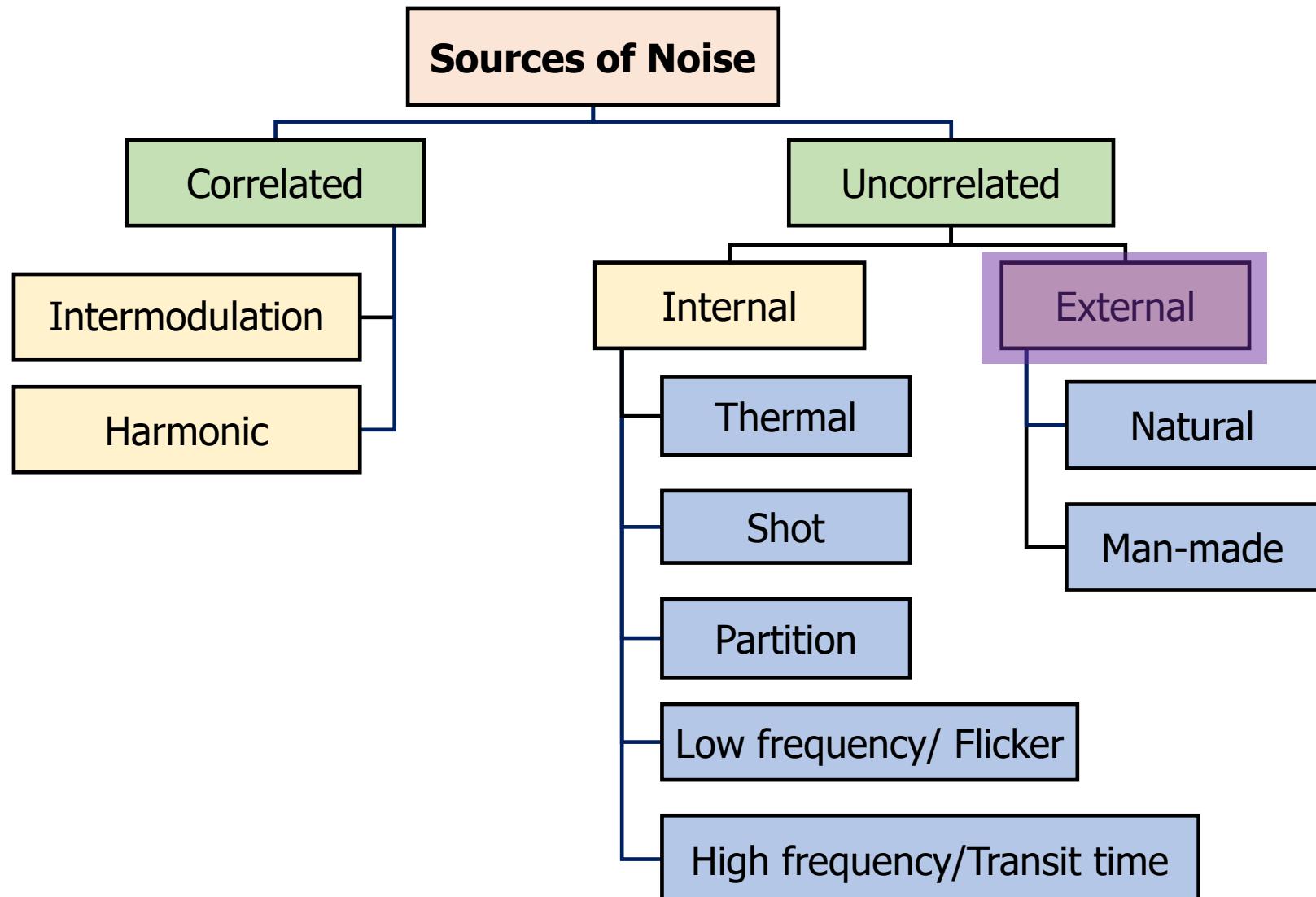
High frequency/ Transit time

- Transit time defined as time taken by carriers to cross junction
- When signal frequency high, periodic time small and maybe comparable to transit time
- Hence, some carriers may diffuse back to the source
- This results in a kind of random noise
- Determined by carrier mobility, bias voltage and transistor construction
- This noise is proportional to frequency of operation

Classification of Noise



Classification of Noise



Types of External Noise

■ **Natural**

- **Atmospheric** – produced within Earth's atmosphere
 - Lightning (electrical disturbance)
 - Electric discharge between clouds and between clouds and earth
- **Extra-terrestrial** (Source - space) - Two types
 - Solar (Source - Sun)
 - Wide range of signals in a broad noise spectrum
 - Vary with time
- **Cosmic** (Source - Stars)
 - Impact less because of distance
 - Greatest impact in 15-150MHz



■ **Man-made Noise**

- Industrial – generated due to make and break process in a current carrying circuit
- Automotive ignition systems
- Electrical motors
- Fluorescent lights
- Welding
- Switching gears
- Gas filled tubes
- Can be minimized by controlling it



NOISE

→ We know -

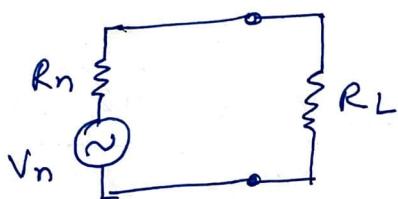
$$\text{Noise Power} = P_n = N \propto \text{BW} \& \text{Temp}$$

$$\Rightarrow N = KTB \quad \text{--- } ①$$

$$K \rightarrow \text{Boltzmann's constant} = 1.38 \times 10^{-23} \text{ J}/\text{°K}$$

→ EQUIVALENT CIRCUIT FOR THERMAL NOISE

Consider an equivalent circuit for thermal noise source where internal resistance, R_n is in series with rms noise voltage, V_n .



→ We know -

for max power transfer -

$$R_n = R_L$$

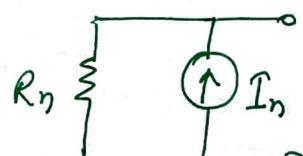
$$\therefore P_{\max} = N = \frac{V_n^2}{4R_L} \quad \text{--- } ②$$

From ① & ②,

$$\frac{V_n^2}{4R_L} = KTB$$

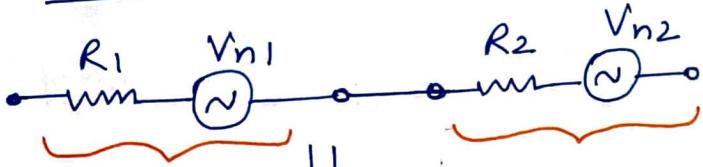
$$V_n^2 = 4 KTB R_L$$

$$\therefore \boxed{V_n = \sqrt{4 KTB R_L}}$$



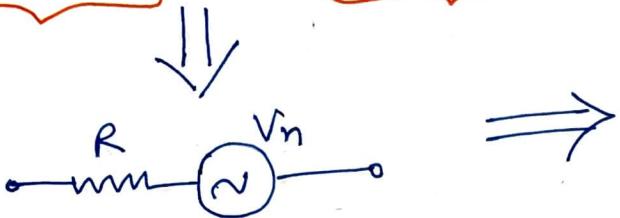
Source Transformation

→ IN SERIES -



$$V_{n1} = \sqrt{4KTB R_1}$$

$$V_{n2} = \sqrt{4KTB R_2}$$



$$R = R_1 + R_2$$

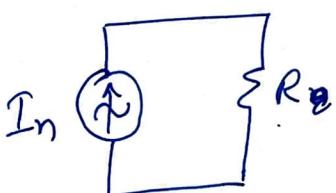
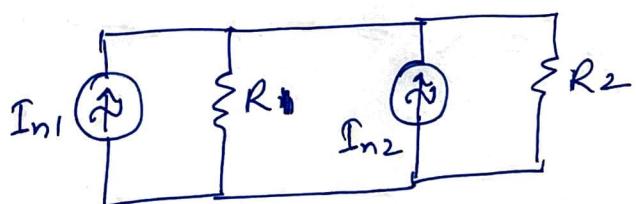
$$V_n = \sqrt{4KTB R}$$

$$\Rightarrow V_n = \sqrt{4KTB(R_1 + R_2)}$$

$$V_n = \sqrt{4KTB R_1 + 4KTB R_2}$$

$$V_n = \sqrt{V_{n1}^2 + V_{n2}^2}$$

→ IN PARALLEL -



$$I_{n1} = \frac{V_{n1}}{R_1} = \frac{\sqrt{4KTB R_1}}{R_1}$$

$$= \sqrt{\frac{4KTB}{R_1}} = \sqrt{4KTB G_1}$$

$$I_{n2} = \sqrt{4KTB G_2}$$

$$\frac{1}{R_B} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\Rightarrow G = G_1 + G_2$$

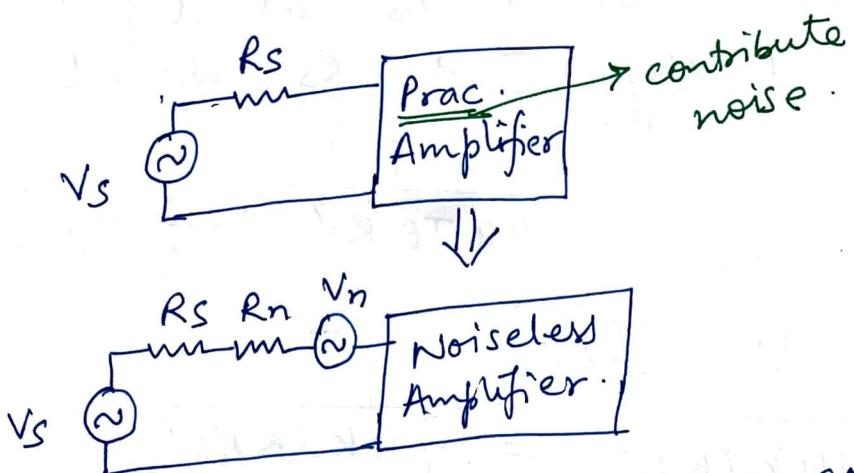
$$I_n = \sqrt{4KTB G}$$

$$\Rightarrow I_n = \sqrt{4KTB(G_1 + G_2)}$$

$$I_n = \sqrt{4KTB G_1 + 4KTB G_2}$$

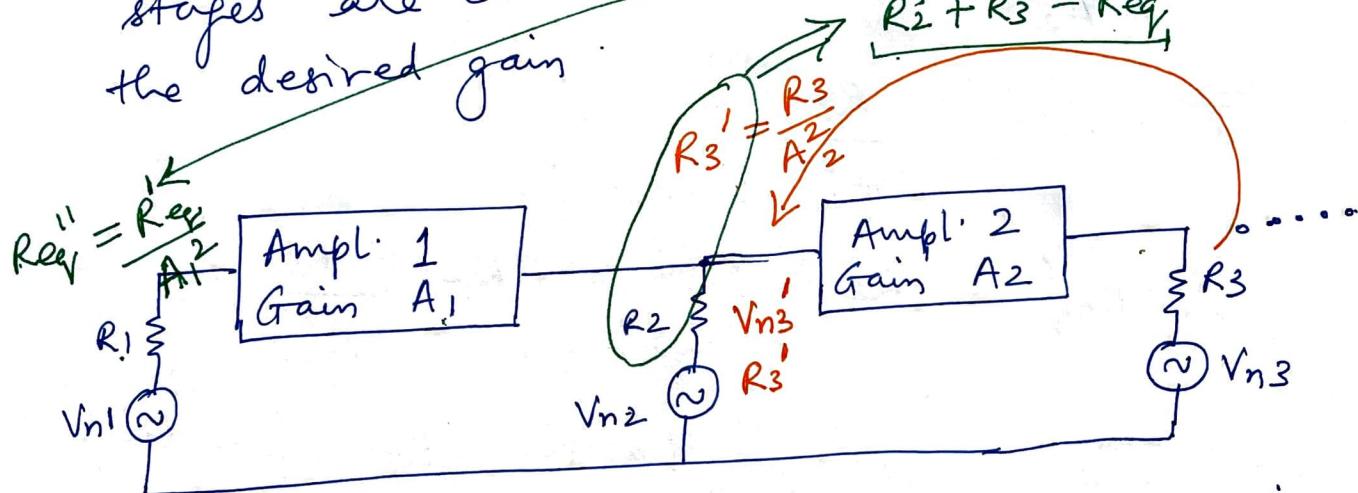
$$I_n = \sqrt{I_{n1}^2 + I_{n2}^2}$$

→ Practical & Noiseless Amplifier



→ NOISE DUE TO AMPLIFIERS IN CASCADE -

→ Typically, in receivers, number of stages are connected in cascade to get the desired gain.



rms noise voltage at output due to R_3 is -

$$V_{n3} = \sqrt{4 k T_B R_3}$$

→ V_{n3} is produced due to rms noise voltage V_{n3}' at input of second stage such that

$$V_{n3} = A_2 \times V_{n3}'$$

$$\Rightarrow V_{n3}' = \frac{V_{n3}}{A_2} = \frac{\sqrt{4 k T_B R_3}}{A_2} \quad \text{--- (1)}$$

→ Now, assume that V_{n3}' is generated by another resistor R_3' at input of second stage with R_3 absent on output side.

$$V_{n3}' = \sqrt{4kTB R_3'} \quad \text{--- (2)}$$

from (1) & (2),

$$\frac{\sqrt{4kTB R_3}}{A_2} = \sqrt{4kTB R_3'}$$

$$\Rightarrow \frac{4kTB R_3}{A_2^2} = 4kTB R_3'$$

$$\Rightarrow R_3' = \frac{R_3}{A_2^2}$$

$$\therefore R_{eq}' = R_2 + R_3' = R_2 + \frac{R_3}{A_2^2}$$

→ similarly, we can transfer R_{eq}' from output of stage 1 to input of stage 1

$$R_{eq}'' = \frac{R_{eq}'}{A_1^2} = \frac{1}{A_1^2} \left[R_2 + \frac{R_3}{A_2^2} \right]$$

$$R_{eq}'' = \frac{R_2}{A_1^2} + \frac{R_3}{A_1^2 A_2^2}$$

\therefore Equivalent noise resistor at input of first stage is -

$$R_{eq} = R_1 + R_{eq}''$$

$$R_{eq} = R_1 + \frac{R_2}{A_1^2} + \frac{R_3}{A_1^2 A_2^2}$$

generalizing —

$$R_{eq} = R_1 + \frac{R_2}{A_1^2} + \frac{R_3}{A_1^2 A_2^2} + \frac{R_4}{A_1^2 A_2^2 A_3^2} + \dots$$

SCROLL DOWN

NUMERICALS

Q1. Calculate the thermal noise power available from any resistor at room temperature of 17°C for a bandwidth of 1 MHz.

Soln. $B = 1 \text{ MHz} = 1 \times 10^6 \text{ Hz}$

$$T = 17^{\circ}\text{C} = 17 + 273 = 290^{\circ}\text{K}$$

$$\begin{aligned} N &= KTB \\ &= 1.38 \times 10^{-23} \times 290 \times 1 \times 10^6 \\ &= 4.002 \times 10^{-15} \text{ watts} \end{aligned}$$

Q2. Calculate the noise voltage for $R = 60\Omega$ for the example given above

$$\begin{aligned}
 \text{Soln: } V_n^2 &= 4kTB R \\
 &= 4 \times 1.38 \times 10^{-23} \times 290 \times 1 \times 10^6 \times 60 \\
 &= 96.048 \times 10^{-14} \\
 V_n &= \sqrt{96.048 \times 10^{-14}} = 9.8 \times 10^{-7} \text{ volts}
 \end{aligned}$$

Q3. An amplifier operating over the frequency range from 18 to 20 MHz has a $10\text{ k}\Omega$ input resistor. What is the rms noise voltage at the input of this amplifier if ambient temperature is 27°C .

$$\begin{aligned}
 \text{Soln: } T &= 27^\circ\text{C} = 27 + 273 = 300^\circ\text{K} \\
 B &= (20 - 18)\text{MHz} = 2\text{MHz} = 2 \times 10^6 \text{ Hz} \\
 R &= 10\text{ k}\Omega = 10 \times 10^3 \Omega = 10^4 \Omega
 \end{aligned}$$

$$\begin{aligned}
 \therefore V_n &= \sqrt{4kTB R} \\
 &= \sqrt{4 \times 1.38 \times 10^{-23} \times 300 \times 2 \times 10^6 \times 10^4} \\
 &= 18.1 \mu\text{volts}
 \end{aligned}$$

Q4. Calculate the noise voltage at the input of a television RF amplifier using a device that has a $200\ \Omega$ equivalent noise resistance and a $300\ \Omega$ input resistance. The bandwidth of the amplifier is 6 MHz and the temperature is 290°K .

Soln: $T = 290^\circ\text{K}$

$$B = 6\text{ MHz} = 6 \times 10^6 \text{ Hz}$$

$$R_n = 200\ \Omega$$

$$R_i = 300\ \Omega$$

$$\begin{aligned} V_n &= \sqrt{4KT B (R_n + R_i)} \\ &= \sqrt{4 \times 1.38 \times 10^{-23} \times 290 \times 6 \times 10^6 \times (200 + 300)} \\ &= 6.93 \text{ microvolts} \end{aligned}$$

Q5. For a bandwidth of 150 kHz , calculate thermal noise voltage generated by two resistors of $30\text{k}\Omega$ and $60\text{k}\Omega$ when they are connected —

(a) individually

(b) in series

(c) in parallel

Given the temperature is 17°C .

soln. $T = 17^\circ C = 17 + 273 = 290^\circ K$

$$B = 150 \text{ kHz} = 150 \times 10^3 \text{ Hz}$$

(a) when $R_1 = 30 \text{ k}\Omega = 30 \times 10^3 \Omega$

$$\begin{aligned} V_{n1} &= \sqrt{4kTB R_1} \\ &= \sqrt{4 \times 1.38 \times 10^{-23} \times 290 \times 150 \times 10^3 \times 30 \times 10^3} \\ &= 8.48 \mu V \end{aligned}$$

when $R_2 = 60 \text{ k}\Omega = 60 \times 10^3 \Omega$

$$\begin{aligned} V_{n2} &= \sqrt{4kTB R_2} \\ &= \sqrt{4 \times 1.38 \times 10^{-23} \times 290 \times 150 \times 10^3 \times 60 \times 10^3} \\ &= 12 \mu V \end{aligned}$$

(b) when in series -

$$\begin{aligned} V_n &= \sqrt{V_{n1}^2 + V_{n2}^2} \\ &= \sqrt{(8.48)^2 + (12)^2} \\ &= 14.7 \mu V \end{aligned}$$

(c) when in parallel -

We know -

$$V_{n1} = 8.48 \mu V$$

$$R_1 = 30 \text{ k}\Omega$$

$$V_{n2} = 12 \mu V$$

$$R_2 = 60 \text{ k}\Omega$$

$$I_{n1} = \frac{V_{n1}}{R_1} = \frac{8.48 \times 10^{-6}}{30 \times 10^3} = 2.82 \times 10^{-10} \text{ A}$$

$$I_{n2} = \frac{V_{n2}}{R_2} = \frac{12 \times 10^{-6}}{60 \times 10^3} = 2 \times 10^{-10} \text{ A}$$

$$\begin{aligned} I_n &= \sqrt{I_{n1}^2 + I_{n2}^2} \\ &= \sqrt{(2.82 \times 10^{-10})^2 + (2 \times 10^{-10})^2} \\ &= 3.46 \times 10^{-10} \text{ A} \end{aligned}$$

$$R = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = \left(\frac{30 \times 60}{30 + 60} \right) \text{ k}\Omega$$

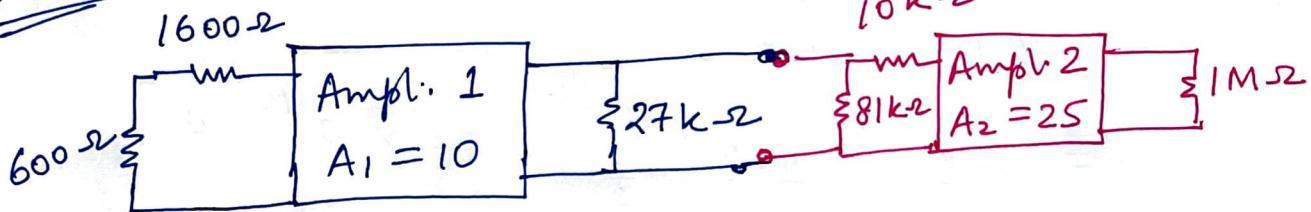
$$= 20 \text{ k}\Omega$$

$$\therefore V_n = I_n R = 3.46 \times 10^{-10} \times 20 \times 10^3$$

$$= 6.92 \mu\text{V}$$

Q6. The first stage of a 2-stage amplifier has a voltage gain of 10, a 600Ω input resistor, a 1600Ω equivalent noise resistance and a $27\text{k}\Omega$ output resistor. For the second stage, these values are 25 , $81\text{k}\Omega$, $10\text{k}\Omega$ and $1\text{M}\Omega$ respectively. Calculate the equivalent input noise resistance of this two stage amplifier.

Sohln.



$$R_{eq} = R_1 + \frac{R_2}{A_1^2} + \frac{R_3}{A_1^2 A_2^2}$$

$$A_1 = 10 \quad ; \quad A_2 = 25$$

$$R_1 = 600 + 1600 = 2200 \Omega$$

$$R_2 = (27 \text{ k}\Omega \parallel 81 \text{ k}\Omega) + 10 \text{ k}\Omega$$

$$= 20 \cdot 2 \text{ k}\Omega + 10 = 30 \cdot 2 \text{ k}\Omega = 30 \cdot 2 \times 10^3 \Omega$$

$$R_3 = 1 \text{ M}\Omega = 1 \times 10^6 \Omega$$

$$\therefore R_{eq} = 2200 + \frac{30 \cdot 2 \times 10^3}{10^2} + \frac{1 \times 10^6}{10^2 \times 25^2}$$

$$= 2200 + 302 + 16$$

$$= 2518 \Omega$$

NOISE PARAMETERS

Signal to Noise Ratio (SNR) -

→ Defined as the ratio of signal power to noise power at the same point

$$\therefore \frac{S}{N} = \text{SNR} = \frac{P_s}{P_n}$$

It is expressed in decibel (dB)

→ Higher the SNR \Rightarrow Better system performance

$$\therefore \text{SNR}_{\text{dB}} = 10 \log_{10} \left(\frac{P_s}{P_n} \right) \text{ dB} = 10 \log_{10} \left(\frac{V_s^2}{V_n^2} \right)$$

$$= 20 \log_{10} \left(\frac{V_s}{V_n} \right) \text{ dB}$$

Noise Factor (F) -

$$F = \frac{\text{SNR at input}}{\text{SNR at output}} = \frac{P_{si}/P_{ni}}{P_{so}/P_{no}}$$

$$F = \frac{P_{si}}{P_{ni}} \times \frac{P_{no}}{P_{so}}$$

→ NOTE -

SNR at input $>$ SNR at output

\therefore Noise is added by the amplifier

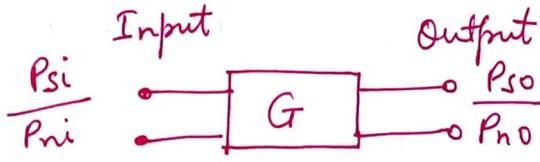
\Rightarrow F is a measure of amount of noise added & its value will be always > 1

→ Ideal value of F is 1.

Noise power in terms of F -

We know -

$$F = \frac{P_{\text{si}}}{P_{\text{ni}}} \times \frac{P_{\text{no}}}{P_{\text{so}}}$$



$$\text{Power gain} = \frac{P_{\text{so}}}{P_{\text{si}}}$$

$$\therefore F = \frac{P_{\text{no}}}{G P_{\text{ni}}}$$

\Rightarrow Noise power at the output is,

$$P_{\text{no}} = F G P_{\text{ni}}$$

We know,

$$P_{\text{ni}} = kTB$$

$$\therefore P_{\text{no}} = FGkTB$$

\Rightarrow With increase in F, noise power at output will increase.

Higher is the F, more is the noise contributed by amplifier.

Noise Figure (NF) -

→ Noise figure is actually noise factor expressed in dB.

$$NF = F_{dB} = 10 \log_{10} F$$

$$NF = 10 \log_{10} \left\{ \frac{SNR \text{ at Input}}{SNR \text{ at Output}} \right\}$$

$$= 10 \log_{10} \left[\frac{SNR_i}{SNR_o} \right]$$

$$= 10 \log_{10} SNR_i - 10 \log_{10} SNR_o$$

$$= \left(\frac{S}{N} \right)_i dB - \left(\frac{S}{N} \right)_o dB$$

→ Ideal value of $NF = 0$

→ Amplifier Noise contribution in terms of F (P_{namp})

$$\begin{matrix} \text{Noise Power} \\ \text{at output} \end{matrix} = P_{no} = FGKTB$$

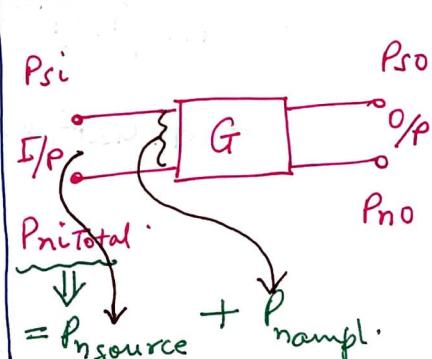
$$\text{Total } P_{ni} = \frac{P_{no}}{G} = \frac{FGKTB}{G} = FKTB$$

Also,

$$\text{Total } P_{ni} = P_{nsource} + P_{namp}.$$

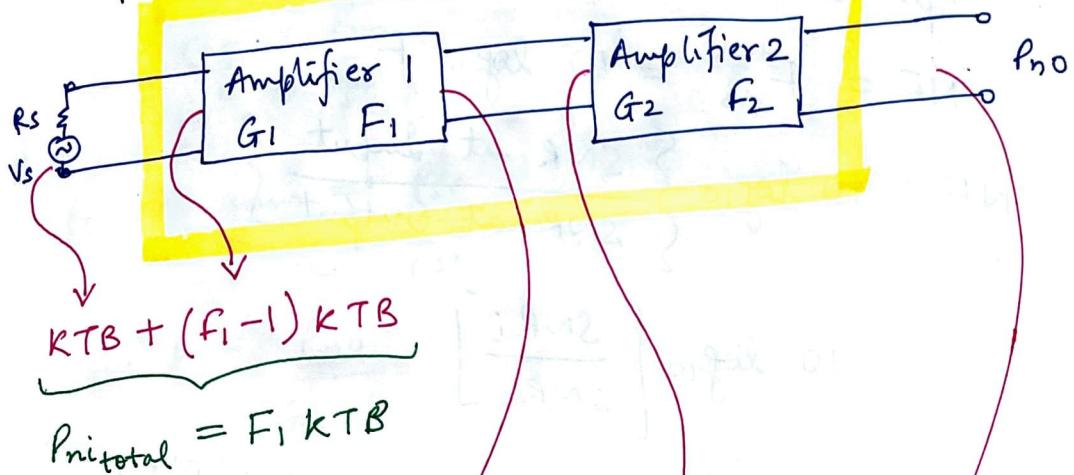
$$FKTB = KTB + P_{namp}.$$

$$\therefore P_{namp} = KTB(F-1)$$



→ Noise factor of Amplifiers in Cascade -

$$F = ? \text{ FRIIS formula } G = G_1 G_2$$



$$G_2 [G_1 F_1 k T_B + (F_2 - 1) k T_B]$$

$$= G_1 G_2 F_1 k T_B + G_2 (F_2 - 1) k T_B$$

→ Noise due to source, $P_{n\text{source}} = KTB$

Noise due to Amplifier 1, $= (F_1 - 1) KTB$

∴ Total noise at input of Amplifier 1,

$$P_{ni\text{total}} = KTB + (F_1 - 1) KTB = F_1 KTB$$

→ ∴ Noise at output of Amplifier 1 is -

$$= G_1(F_1 KTB) = F_1 G_1 KTB$$

→ Noise due to Amplifier 2,

$$= (F_2 - 1) KTB$$

→ Total noise at input of Amplifier 2,

$$= F_1 G_1 KTB + (F_2 - 1) KTB$$

→ ∴ Noise at output of Amplifier 2, P_{no}

$$P_{no} = G_2 \{ F_1 G_1 KTB + (F_2 - 1) KTB \}$$

$$P_{no} = G_1 G_2 F_1 KTB + G_2 (F_2 - 1) KTB \quad \text{--- (A)}$$

→ Now,

$$\text{Overall gain} = G = G_1 G_2$$

$$\text{Overall } F = \frac{P_{no}}{G P_{ni}} \quad [\because P_{no} = FG P_{ni}]$$

$$= \frac{P_{no}}{G_1 G_2 KTB} \quad [\because P_{ni} = P_{\text{source}} = KTB]$$

From (A),

$$\therefore F = \frac{G_1 G_2 F_1 KTB + G_2 (F_2 - 1) KTB}{G_1 G_2 KTB}$$

$$\Rightarrow F = F_1 + \frac{(F_2 - 1)}{G_1}$$

FRIIS
FORMULA

We can extend this as -

$$F = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2} + \dots$$

\Rightarrow 1st stage is the most important stage in deciding overall noise factor

\rightarrow Equivalent Noise Temperature of Amplifiers in cascade -

We know from Friis formula -

$$F = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2} + \dots$$

\rightarrow Noise at the input of amplifier,

$$P_{\text{ampl.}} = (F - 1) K T_B$$

This is the noise contributed by the amplifier.

\rightarrow Let the noise be represented alternatively by some fictitious temperature, T_{eq} , such that -

$$K T_{\text{eq}} B = (F - 1) K T_B$$

$$\Rightarrow T_{\text{eq}} = (F - 1) T \quad \boxed{\text{1}}$$

$\therefore T_{\text{eq}}$ is an alternative measure for F .

NUMERICALS

Q1. An amplifier has a gain of 3 dB. Calculate the power at the output of the amplifier in mW if the power applied at the input ~~is~~ is 0.25 mW

Soln. $10 \log_{10} G = 3 \text{ dB}$

$$\log_{10} G = \frac{3}{10} = 0.3$$

$$G = \text{antilog } 0.3 = 2$$

$$P_i = 0.25 \text{ mW}$$

$$P_o = G P_i = 2 \times 0.25 \text{ mW} = 0.5 \text{ mW}$$

Q2. For an amplifier with output signal power of 10 W and output noise power of 0.01 W, determine SNR in dB.

Soln. $\text{SNR} = 10 \log_{10} \left(\frac{P_s}{P_n} \right) = 10 \log_{10} \left(\frac{10}{0.01} \right)$

$$= 30 \text{ dB}$$

Q3. For an amplifier,

$$P_{si} = 2 \times 10^{-10} \text{ W}$$

$$P_{so} = 2 \times 10^{-4} \text{ W}$$

$$P_{ni} = 2 \times 10^{-18} \text{ W}$$

$$P_{no} = 8 \times 10^{-12} \text{ W}$$

Calculate Noise factor and Noise figure.

We know that -

$$F = f_1 + \frac{(f_2 - 1)}{G_1} + \frac{(f_3 - 1)}{G_1 G_2} + \dots$$

Subtract 1 from both sides,

$$(F - 1) = (f_1 - 1) + \frac{(f_2 - 1)}{G_1} + \frac{(f_3 - 1)}{G_1 G_2} + \dots$$

We know from ①,

$$T(F - 1) = T_{eq}$$

$$\Rightarrow \frac{T_{eq}}{T} = (F - 1)$$

$$\therefore \frac{T_{eq}}{T} = \frac{T_{eq1}}{T} + \frac{T_{eq2}}{TG_1} + \frac{T_{eq3}}{TG_1 G_2} + \dots$$

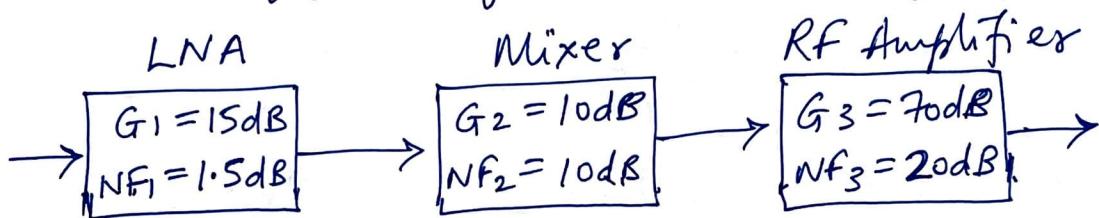
$$\Rightarrow \boxed{\frac{T_{eq}}{T} = T_{eq1} + \frac{T_{eq2}}{G_1} + \frac{T_{eq3}}{G_1 G_2} + \dots}$$

Soln.

$$F = \frac{\frac{P_{si}/P_{ni}}{P_{so}/P_{no}}}{\frac{2 \times 10^{-10}}{2 \times 10^{-18}}} = \frac{2 \times 10^{-10}}{2 \times 10^{-18}} / \frac{2 \times 10^{-4}}{8 \times 10^{-12}} = 4$$

$$NF = 10 \log_{10} F = 10 \log_{10} 4 = 6.02 \text{ dB}$$

Q4. Calculate the overall noise factor and noise figure for the circuit given below -



Soln. $G_1 = 15 \text{ dB} \Rightarrow 10 \log_{10} G_1 = 15 \Rightarrow G_1 = 31.62$

$$NF_1 = 1.5 \text{ dB} \Rightarrow 1.413$$

$$G_2 = 10 \text{ dB} \Rightarrow 10$$

$$NF_2 = 10 \text{ dB} \Rightarrow 10$$

$$G_3 = 70 \text{ dB} \Rightarrow 10^7$$

$$NF_3 = 20 \text{ dB} \Rightarrow 100$$

Using Friis formula -

$$F = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2}$$

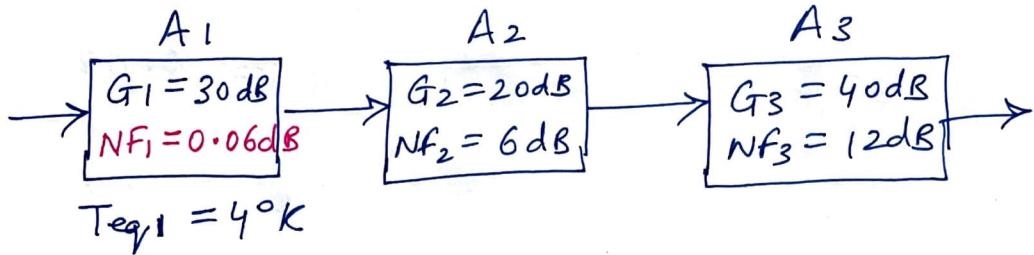
$$= 1.413 + \frac{(10 - 1)}{31.62} + \frac{(100 - 1)}{31.62 \times 10}$$

$$= 2.010$$

$$NF = 10 \log_{10} F = 10 \log_{10} 2.010$$

$$= 3.031 \text{ dB}$$

Q5. Calculate overall noise figure for the given cascaded amplifier.
Assume $T = 290^\circ\text{K}$



Soln: We know —

$$(F - 1) = \frac{T_{eq}}{T}$$

$$\Rightarrow (F_1 - 1) = \frac{T_{eq1}}{T}$$

$$\Rightarrow F_1 = 1 + \frac{T_{eq1}}{T} = 1 + \frac{4}{290} = 1.014 = 0.06 \text{ dB}$$

$$G_1 = 30 \text{ dB} \Rightarrow 1000$$

$$N\!F_1 = 0.06 \text{ dB} \Rightarrow 1.014$$

$$G_2 = 20 \text{ dB} \Rightarrow 100$$

$$N\!F_2 = 6 \text{ dB} \Rightarrow 3.98$$

$$G_3 = 40 \text{ dB} \Rightarrow 10000$$

$$N\!F_3 = 12 \text{ dB} \Rightarrow 15.85$$

We know,

$$F = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2}$$

$$F = 1.014 + \frac{(3.98 - 1)}{1000} + \frac{(15.85 - 1)}{1000 \times 100}$$

$$= 1.017$$

$$NF = 10 \log_{10} F = 10 \log_{10} 1.017 = 0.073 \text{ dB}$$

Homework -

- Q6. Consider a receiver as described -
- The first stage is the RF amplifier which has a gain of 20 dB and noise figure of 7 dB. The second stage is the mixer with the corresponding values of 8 dB and 8 dB respectively. The final stage is the IF amplifier with gain of 60 dB and noise figure of 6 dB. Find the overall noise figure.

Ans → Noise figure = 7.04 dB