Module II

Lecture 1

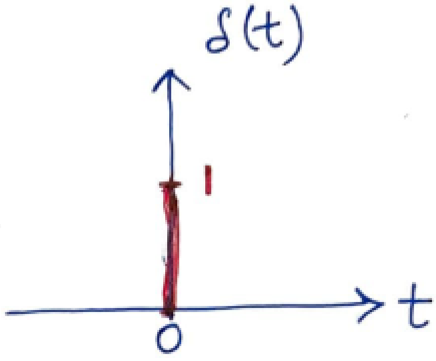
- Basics of signal representation and analyses
- Mathematical representation of signals (TBS)
- Concept of Time shifting, Time scaling, Time reversal (TBS)

Representation of Standard Continuous Time Signals

Impulse (Delta) signal -

Denoted by $\delta(t)$

$$\delta(t) = 1 \; ; \; t = 0$$
$$= 0 \; ; \; t \neq 0$$

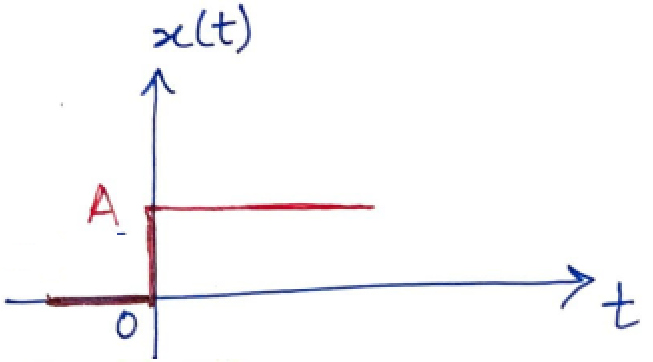


Unit impulse signal is a signal with infinite magnitude and zero duration but with unit area,

$$\int_{-\infty}^{\infty} \delta(t) . dt = 1$$

Step signal -

$$x(t) = A \quad ; \quad t \geq 0$$
$$= 0 \quad ; \quad t < 0$$

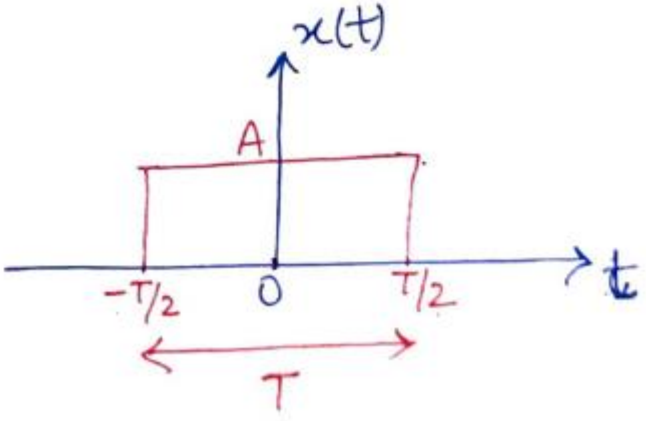


When $A = 1$, unit step signal $(u(t))$

Gate/Rectangular/Pulse signal -

$$x(t) = A \quad ; \quad -\frac{T}{2} \leq t \leq \frac{T}{2}$$
$$= 0 \quad ; \quad \text{otherwise}$$

When $A = 1$, unit gate signal

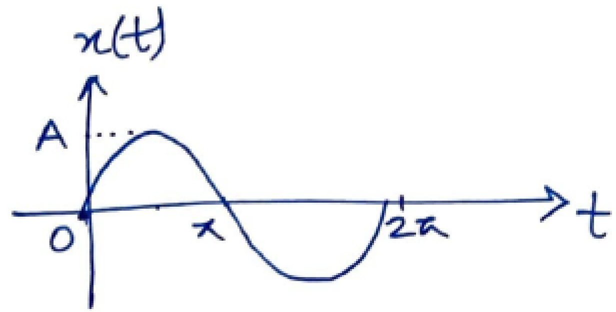


Sine signal -

$x(t) = A \sin(\omega t + \phi)$

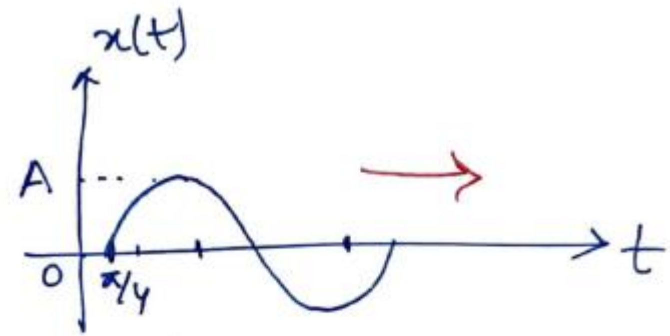
→ when $\phi = 0$

$x(t) = A \sin \omega t$



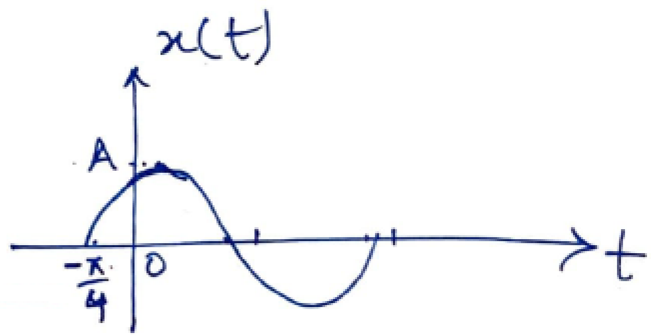
→ when $\phi = 45^\circ$ or $\frac{\pi}{4}$ but lagging

$x(t) = A \sin\left(\omega t - \frac{\pi}{4}\right)$



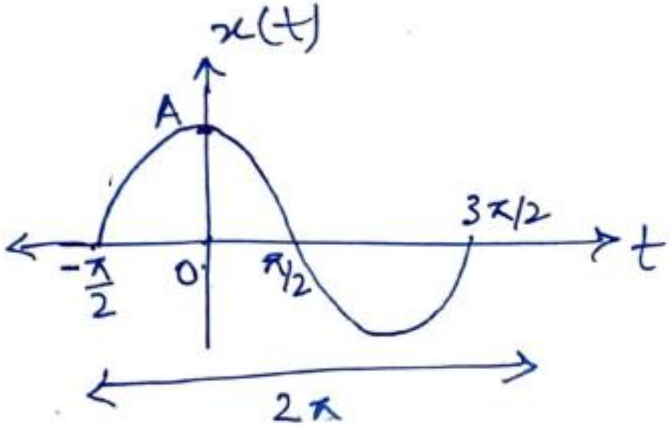
→ when $\phi = 45^\circ$ but leading

$x(t) = A \sin\left(\omega t + \frac{\pi}{4}\right)$



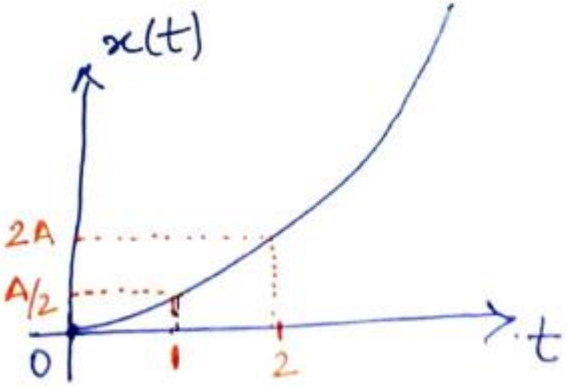
Cosine signal -

$$x(t) = A\cos(\omega t + \phi)$$



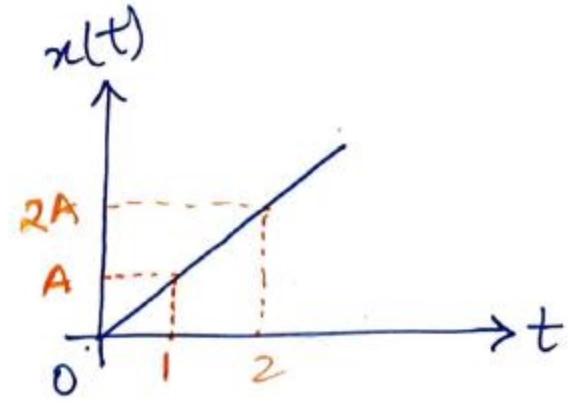
Parabolic signal -

$$x(t) = \frac{At^2}{2} \; ; \quad t \geq 0$$
$$= 0 \; ; \quad t < 0$$



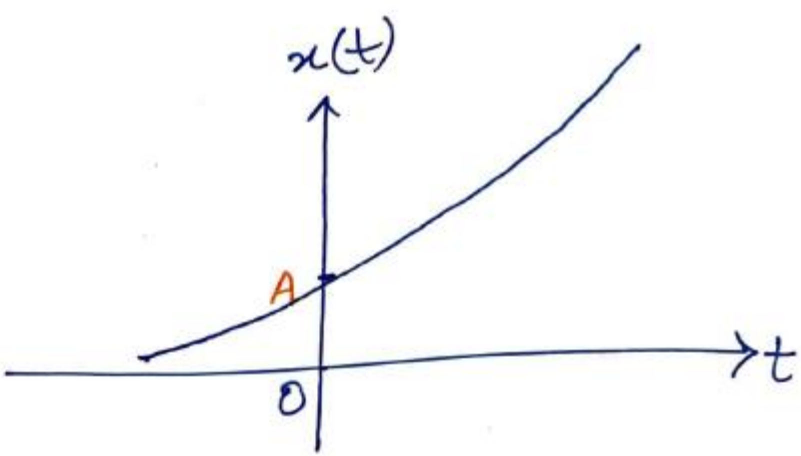
Ramp signal -

$$x(t) = At \; ; \; t \geq 0$$
$$= 0 \; ; \; t < 0$$

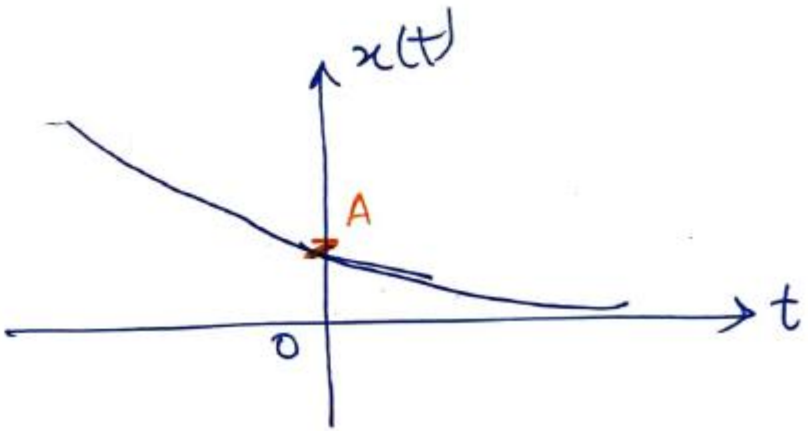


Exponential signal -

$$x(t) = A e^{bt}$$

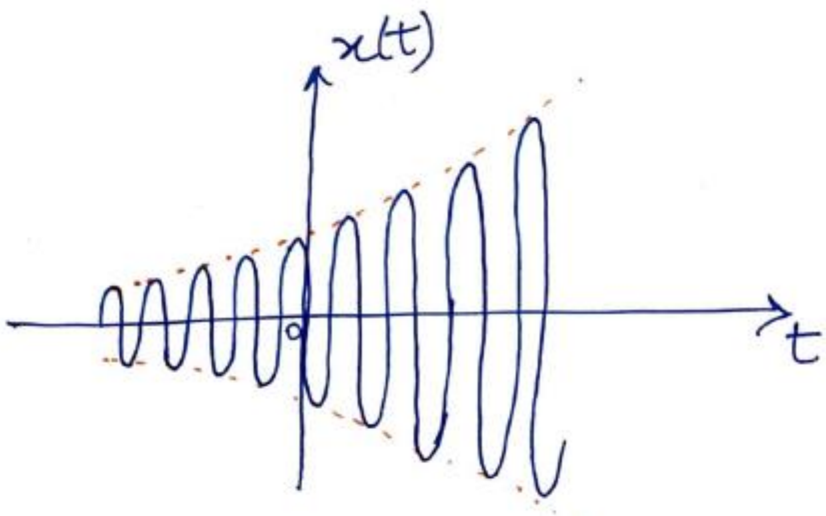


$$x(t) = A e^{-bt}$$

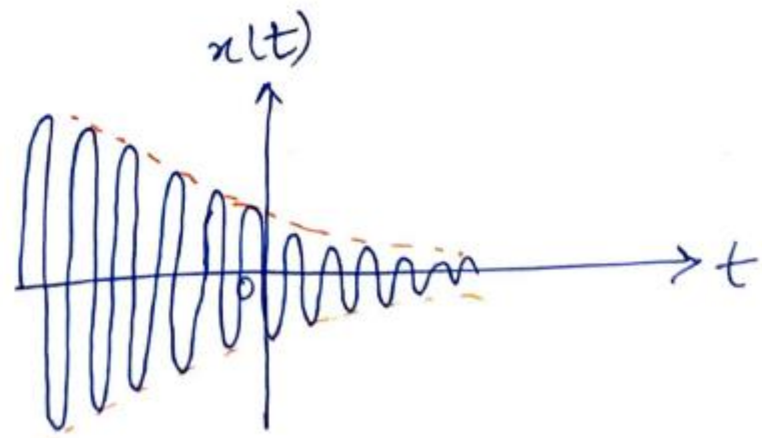


Exponentially rising & Exponentially damping signal -

$$x(t) = A \cdot e^{bt} \cdot \sin \omega t$$

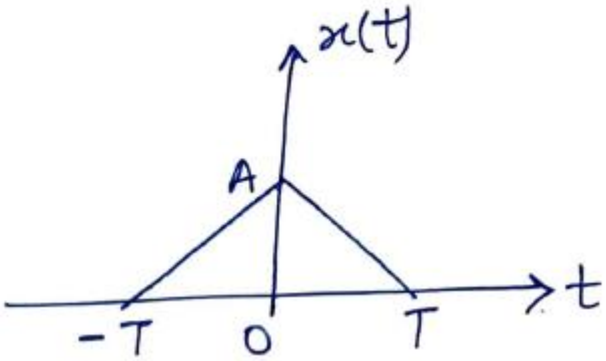


$$x(t) = A \cdot e^{-bt} \cdot \sin \omega t$$



Triangular signal -

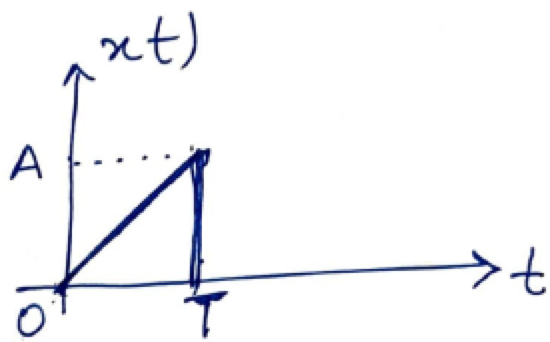
$$x(t) = A\left(1 - \frac{|t|}{T}\right)$$



Sawtooth signal -

$$x(t) = \frac{A}{T}t \ ; \quad 0 \leq t \leq T$$

$$= 0 \ ; \quad \text{otherwise}$$



Gaussian signal -

$$x(t) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(t-\mu)^2}{2\sigma^2}}$$

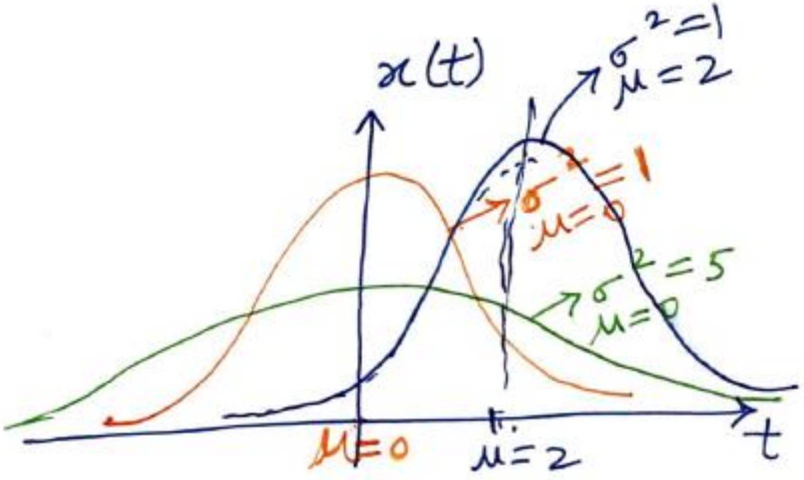
$\mu \rightarrow$ mean (W/F centred around mean)

$\sigma \rightarrow$ std. dev. (spread)

$\rightarrow \mu = 0, \sigma^2 = 1$

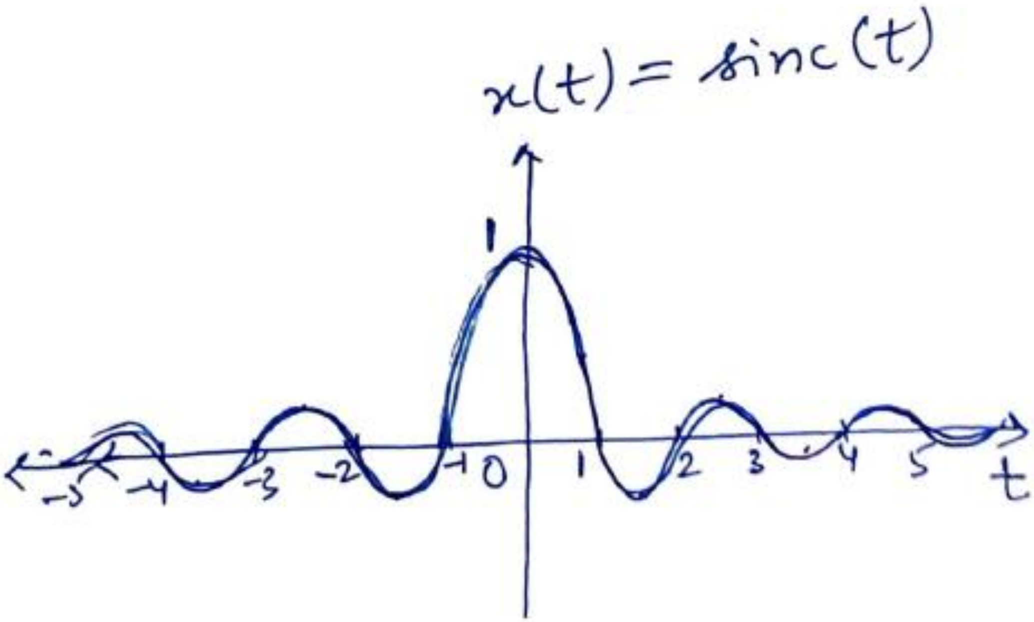
$\rightarrow \mu = 0, \sigma^2 = 5$

$\rightarrow \mu = 2, \sigma^2 = 1$



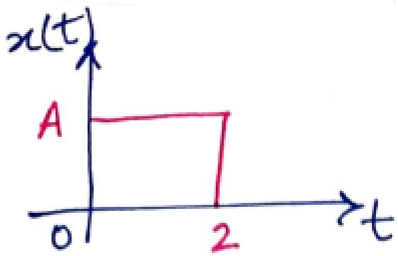
Sinc signal -

$$x(t) = \text{sinc}(t) = \frac{\sin t}{t}$$



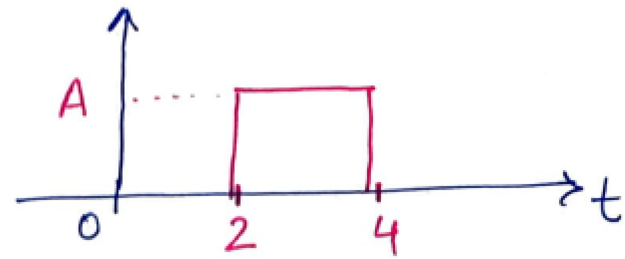
Basic Operations

Time Shifting-



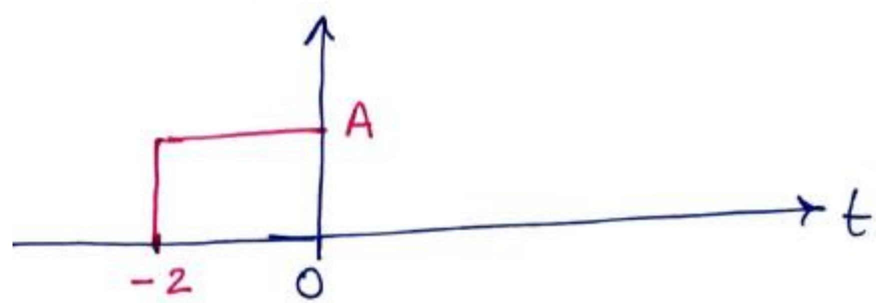
t	$\rightarrow$ New
0	$\rightarrow$ 2
2	$\rightarrow$ 4

$x(t-2) \rightarrow$ Right shift

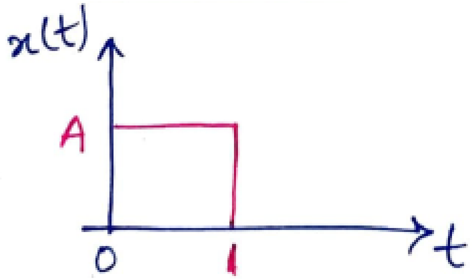


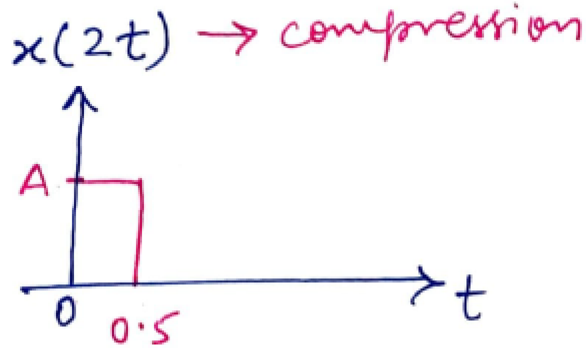
t	$\rightarrow$ New
0	$\rightarrow$ -2
2	$\rightarrow$ 0

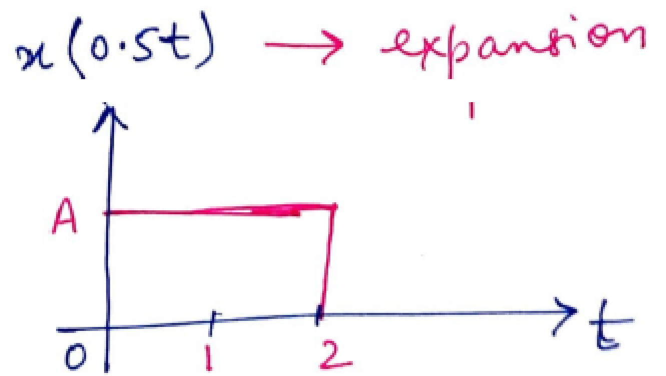
$x(t+2) \rightarrow$ Left shift



Time Scaling -

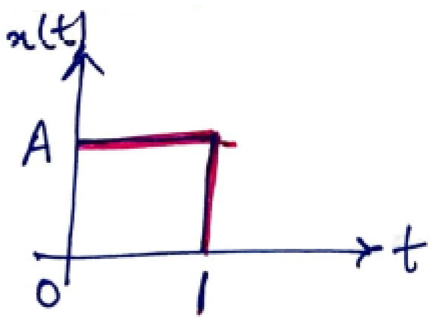


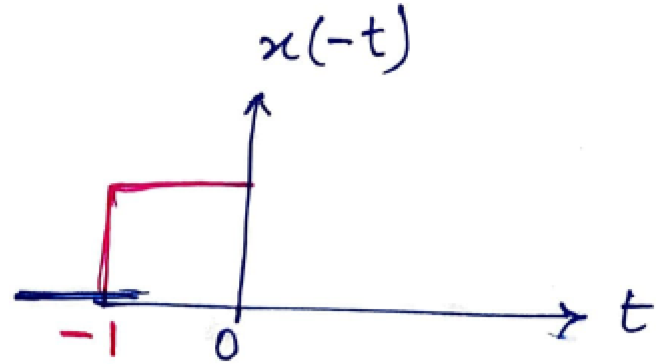




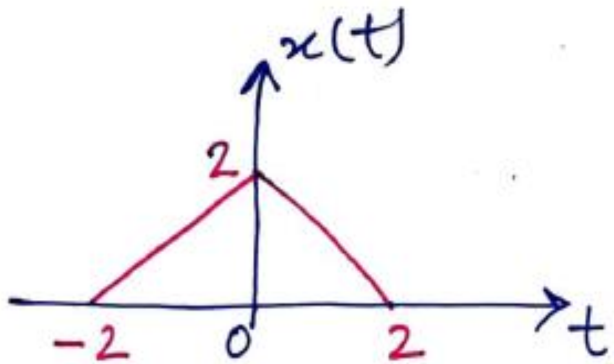
Time Reversal -

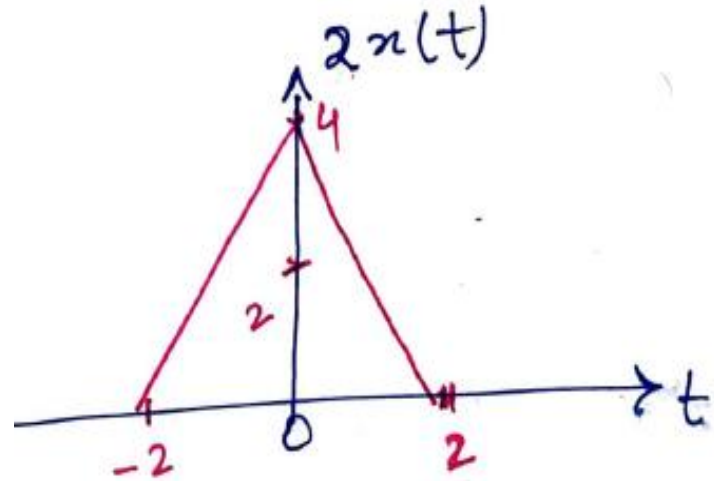
$$\underline{t} \rightarrow \underline{-t}$$
$$1 \rightarrow -1$$

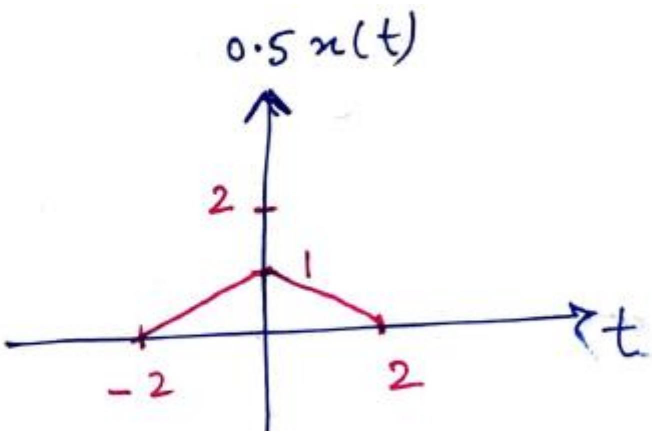


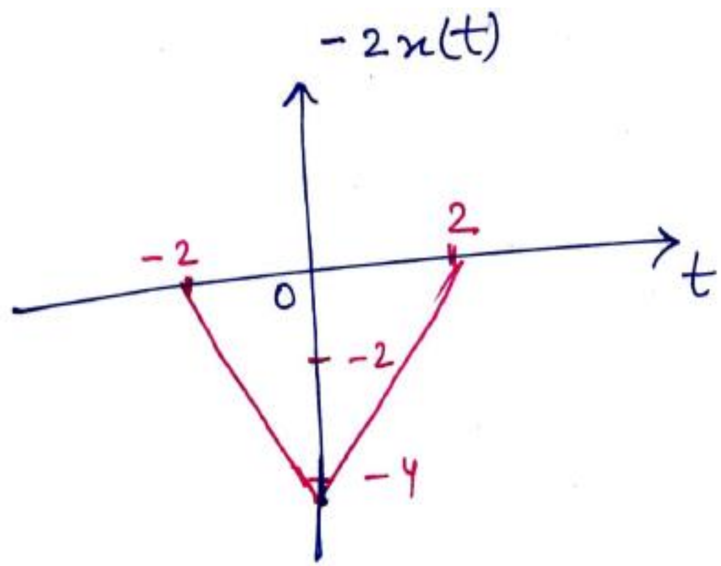


Amplitude Scaling -









Module II

Lecture 2

- Fourier Transform
    - Definition of Fourier Transform
    - Definition of Inverse Fourier Transform
    - Conditions for existence of Fourier Transform
    - Properties of Fourier Transform

Fourier Transform

- **Recap** - Already studied signals and its representation in time domain
- In communication systems, we need to analyze signals in frequency domain as well
- Fourier transform is used for transformation of a signal represented in time domain into a signal represented in frequency domain

Definition of Fourier Transform

- If $x(t)$ represents a continuous signal in time domain, its Fourier transform is given as –

$$F[x(t)] = X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$$

- We can get back the signal from frequency domain to time domain by carrying out Inverse Fourier Transform

Definition of Inverse Fourier Transform

- If $X(f)$ represents the signal in frequency domain, its inverse Fourier transform is given as –

$$F^{-1}[X(f)] = x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df$$

- Therefore, $x(t)$ and $X(f)$ form a Fourier transform pair

Definition of Inverse Fourier Transform

- If $X(f)$ represents the signal in frequency domain, its inverse Fourier transform is given as –

$$F^{-1}[X(f)] = x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}df$$

- Therefore, $x(t)$ and $X(f)$ form a Fourier transform pair

$$x(t) \underset{\text{IFT}}{\overset{\text{FT}}{\rightleftarrows}} X(f)$$

Conditions for existence of Fourier Transform -

- $x(t)$ should be singled valued in any finite time interval, T
- $x(t)$ should have finite number of discontinuities in any finite time interval, T
- $x(t)$ should have a finite number of maxima and minima in any finite time interval, T
- $x(t)$ should be an absolutely integrable function

Properties of Fourier Transform

I. Linearity or Superposition -

$$a_1 x_1(t) + a_2 x_2(t) \overset{F}{\leftrightarrow} a_1 X_1(f) + a_2 X_2(f)$$

where, a_1 and a_2 are constants

II. Time Scaling-

$$x(\alpha t) \overset{F}{\leftrightarrow} \frac{1}{|\alpha|} X\left(\frac{f}{\alpha}\right)$$

where, α is a constant

III. Duality or Symmetry -

If $x(t) \overset{F}{\leftrightarrow} X(f)$, then

$X(t) \overset{F}{\leftrightarrow} x(-f)$

IV. Time Shifting-

$$x(t-d) \overset{F}{\leftrightarrow} e^{-j2\pi f d} X(f)$$

V. Area under $x(t)$ -

$$\int_{-\infty}^{\infty} x(t)dt \overset{F}{\leftrightarrow} X(0)$$

VI. Area under X(f)-

$$\int_{-\infty}^{\infty} X(f)df \overset{F}{\leftrightarrow} x(0)$$

VII. Frequency Shifting-

$$e^{-j2\pi f_c t}x(t) \overset{F}{\leftrightarrow} X(f - f_c)$$

VIII. Differentiation in time domain -

$$\frac{d}{dt}x(t) \overset{F}{\leftrightarrow} j\pi 2f.X(f)$$

IX. Integration in time domain -

$$\int_{-\infty}^{\infty} x(t)dt \overset{F}{\leftrightarrow} \frac{1}{2\pi f} X(f)$$

X. Multiplication in time domain -

$$x_1(t).x_2(t) \overset{F}{\leftrightarrow} \int_{-\infty}^{\infty} X_1(v).X_2(f-v)dv$$

OR

$$x_1(t).x_2(t) \overset{F}{\leftrightarrow} X_1(f) * X_2(f)$$

XI. Convolution in time domain -

$$x_1(t) * x_2(t) \overset{F}{\leftrightarrow} X_1(f).X_2(f)$$

XII. Conjugate Functions

If $x(t) \overset{F}{\leftrightarrow} X(f)$, then

$$x^*(t) \overset{F}{\leftrightarrow} X(-f)$$

In Syllabus…

Time Scaling	$x(\alpha t) \overset{F}{\leftrightarrow} \frac{1}{\lvert\alpha\rvert} X\left(\frac{f}{\alpha}\right)$ where, α is a constant
Time Shifting	$x(t-d) \overset{F}{\leftrightarrow} e^{-j2\pi f d}\, X(f)$
Frequency Shifting	$e^{-j2\pi f_c t} x(t) \overset{F}{\leftrightarrow} X(f-f_c)$
Differentiation in time domain	$\frac{d}{dt} x(t) \overset{F}{\leftrightarrow} j\pi 2f . X(f)$
Convolution in time domain	$x_1(t) * x_2(t) \overset{F}{\leftrightarrow} X_1(f).X_2(f)$

→ TIME SCALING –

The time scaling property states that if $x(t)$ and $X(f)$ form a Fourier transform pair, then –

$$x(\alpha t) \xleftrightarrow{F} \frac{1}{|\alpha|} \cdot X\left(\frac{f}{\alpha}\right)$$

where, α is a constant.

Proof – We know that –

$$F[x(t)] = X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi ft} \cdot dt$$

$$\therefore F[x(\alpha t)] = \int_{-\infty}^{\infty} x(\alpha t) \cdot e^{-j2\pi ft} \cdot dt$$

when α is +ve Limits $-\infty$ to $+\infty$
when α is –ve Limits $+\infty$ to $-\infty$ To change limits $-\infty$ to $+\infty$ introduce –ve sign

Let, $\alpha t = y$

$$\Rightarrow t = \frac{y}{\alpha} \Rightarrow dt = \frac{1}{\alpha} \cdot dy$$

$$\therefore F[x(\alpha t)] = \int_{-\infty}^{\infty} x(y) \cdot e^{-j2\pi \frac{f}{\alpha} y} \cdot \frac{1}{\alpha} \cdot dy$$

$$= \frac{1}{|\alpha|} \int_{-\infty}^{\infty} x(y) \cdot e^{-j2\pi \frac{f}{\alpha} y} \cdot dy$$

$$= \frac{1}{|\alpha|} \cdot X\left(\frac{f}{\alpha}\right)$$

Hence Proved.

→ TIME SHIFTING –

The time shifting property states that if $x(t)$ and $X(f)$ form a fourier transform pair, then –

$$\underbrace{x(t - t_d)}_{\text{Time shifted signal}} \xleftrightarrow{F} e^{-j2\pi f t_d} \cdot X(f)$$

Proof – We know –

$$F[x(t)] = X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi f t} \cdot dt$$

$$\therefore F[x(t - t_d)] = \int_{-\infty}^{\infty} x(t - t_d) \cdot e^{-j2\pi f t} \cdot dt$$

Let, $(t - t_d) = \tau$

$$\Rightarrow t = \tau + t_d \qquad \Rightarrow dt = d\tau$$

$$\therefore F[x(t - t_d)] = \int_{-\infty}^{\infty} x(\tau) \cdot e^{-j2\pi f(\tau + t_d)} \cdot d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau) \cdot e^{-j2\pi f \tau} \cdot e^{-j2\pi f t_d} \cdot d\tau$$

$$= e^{-j2\pi f t_d} \int_{-\infty}^{\infty} x(\tau) \cdot e^{-j2\pi f \tau} \cdot d\tau$$

$$= e^{-j2\pi f t_d} \cdot X(f)$$

Hence proved.

→ FREQUENCY SHIFTING –

It states that if $x(t)$ and $X(f)$ form a Fourier transform pair, then –

$$e^{j2\pi f_c t} \cdot x(t) \xleftrightarrow{F} X(f-f_c)$$

where, f_c is a real constant.

Proof – We know –

$$F[x(t)] = X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi ft} \cdot dt$$

$$\therefore F\left[e^{j2\pi f_c t} \cdot x(t)\right] = \int_{-\infty}^{\infty} e^{j2\pi f_c t} \cdot x(t) \cdot e^{-j2\pi ft} \cdot dt$$

$$= \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi(f-f_c)t} \cdot dt$$

$$= \underbrace{X(f-f_c)}$$ → shifted frequency spectrum

Hence proved.

→ DIFFERENTIATION IN TIME DOMAIN –

Let $x(t) \xleftrightarrow{F} X(f)$ and let derivative of $x(t)$ be fourier transformable, then –

$$\frac{d}{dt} x(t) \xleftrightarrow{F} j2\pi f \cdot X(f)$$

Proof – By definition of IFT, we know –

$$x(t) = F^{-1}[X(f)] = \int_{-\infty}^{\infty} X(f) \cdot e^{j2\pi ft} \cdot df$$

$$\therefore \frac{d}{dt} x(t) = \frac{d}{dt} \int_{-\infty}^{\infty} X(f) \cdot e^{j2\pi ft} \cdot df$$

$$\frac{d}{dt} x(t) = \int_{-\infty}^{\infty} X(f) \cdot \left[\frac{d}{dt} e^{j2\pi ft}\right] \cdot df$$

$$\frac{d}{dt} x(t) = \int_{-\infty}^{\infty} X(f) \cdot e^{j2\pi ft} \cdot j2\pi f \cdot df.$$

$$\frac{d}{dt} x(t) = \int_{-\infty}^{\infty} \underbrace{j2\pi f \, X(f) \cdot}_{\text{signal}} e^{j2\pi ft} \cdot df.$$

$$\frac{d}{dt} x(t) = F^{-1}[j2\pi f \cdot X(f)]$$

Taking fourier transform on both sides –

$$F\left[\frac{d}{dt} x(t)\right] = j2\pi f \cdot X(f)$$

Hence proved.

→ CONVOLUTION IN TIME DOMAIN –

This property states that the convolution of signals in time domain will be transformed into multiplication of their Fourier transform in frequency domain,

This means that –

$$[x_1(t) * x_2(t)] \xleftrightarrow{F} X_1(f) \cdot X_2(f)$$

Proof – The convolution of two signals in time domain is given by –

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\lambda) \cdot x_2(t-\lambda) \cdot d\lambda$$

Taking Fourier Transform, we get

$$F[x_1(t) * x_2(t)] = \int_{-\infty}^{\infty} [x_1(t) * x_2(t)] \cdot e^{-j2\pi ft} \cdot dt$$

$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x_1(\lambda) \cdot x_2(t-\lambda) \cdot d\lambda\right] \cdot e^{-j2\pi ft} \cdot dt$$

multiplying and dividing RHS by $e^{-j2\pi f\lambda}$, we get –

$$= \underbrace{\int_{-\infty}^{\infty} x_1(\lambda) \cdot e^{-j2\pi f\lambda} \cdot d\lambda}_{X_1(f)} \cdot \int_{-\infty}^{\infty} x_2(\overset{m}{t-\lambda}) \cdot e^{-j2\pi f\overset{m}{(t-\lambda)}} \cdot \underset{dm}{dt}$$

Let $t - \lambda = m$

$dt = dm$

$\Rightarrow X_2(f)$

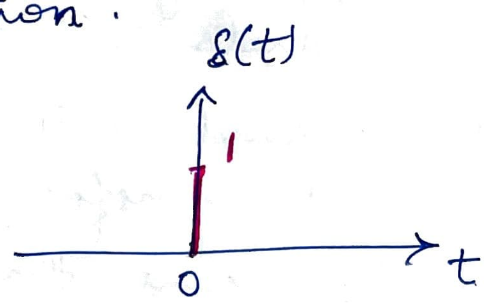
$$= X_1(f) \cdot X_2(f)$$

Hence Proved.

EXAMPLES

Q1. Obtain Fourier Transform of a unit Delta (Impulse) function.

$$\delta(t) = 1 \quad ; \quad t = 0$$
$$= 0 \quad ; \quad \text{otherwise}$$



Soln. We know —

$$F[x(t)] = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi ft} \cdot dt$$

$$= \int_{-\infty}^{\infty} \delta(t) \cdot e^{-j2\pi ft} \cdot dt$$

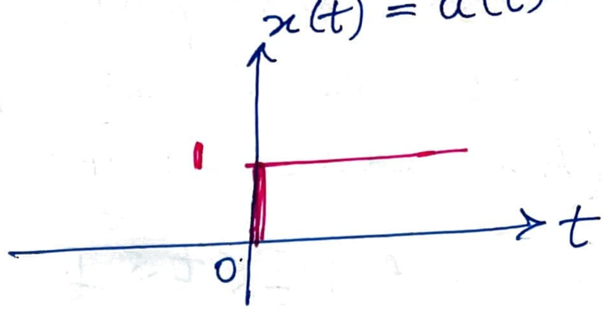
$$= \delta(t) \cdot e^{-j2\pi ft} \Big|_{t=0}$$

$$= \delta(0) \cdot e^{-j2\pi ft \times 0}$$

$$= 1 \cdot 1$$

$$= \underline{\underline{1}}$$ Ans.

Q2. Obtain Fourier Transform of a unit step signal –



$$u(t) = x(t) = 1 ; \quad t \geq 0$$
$$= 0 ; \quad t < 0$$

Soln. We know –

$$F[x(t)] = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi ft} \cdot dt$$

$$F[u(t)] = \int_{-\infty}^{\infty} u(t) \cdot e^{-j2\pi ft} \cdot dt$$

$$= \int_{0}^{\infty} 1 \cdot e^{-j2\pi ft} \cdot dt$$

$$= \frac{1}{-j2\pi f} \left[e^{-j2\pi ft} \right]_0^{\infty}$$

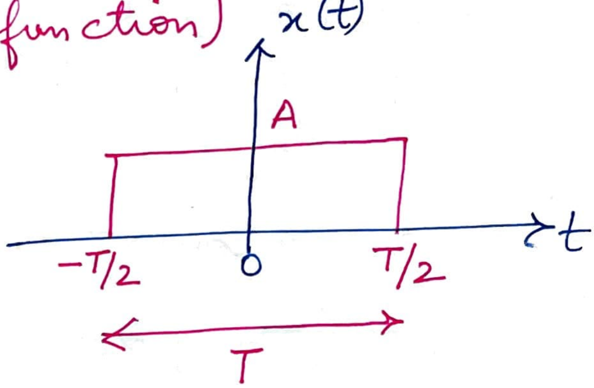
$$= \frac{-1}{j2\pi f} \left[e^{-\infty} - e^{0} \right]$$

$$= \frac{-1}{j2\pi f} \left[0 - 1 \right]$$

$$= \frac{1}{j2\pi f}$$

Ans.

Q3. Obtain the Fourier Transform of a rectangular pulse of duration, T and amplitude, A as shown in the figure.
(Also called as gate function)



$$x(t) = A \;; -\frac{T}{2} \leq t \leq \frac{T}{2}$$
$$= 0 \;; \text{ otherwise}$$

Soln: We know —

$$F[x(t)] = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi ft} \cdot dt$$

$$= \int_{-T/2}^{T/2} A \cdot e^{-j2\pi ft} \cdot dt$$

$$= \frac{A}{-j2\pi f} \left[e^{-j2\pi ft} \right]_{-T/2}^{T/2}$$

$$= \frac{-A}{j2\pi f} \left[e^{-j\cancel{2}\pi f \frac{T}{\cancel{2}}} - e^{j\cancel{2}\pi f \frac{T}{\cancel{2}}} \right]$$

$$= \frac{-A}{j2\pi f} \left[e^{-j\pi fT} - e^{j\pi fT} \right]$$

$$= \frac{A}{j2\pi f} \left[e^{j\pi fT} - e^{-j\pi fT} \right]$$

$\pi f T \rightarrow \theta$

$$= \frac{A}{\pi f}\left[\frac{e^{j\pi f T} - e^{-j\pi f T}}{2j}\right]$$

$\pi f T \rightarrow \theta$

According to Euler's theorem –

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$\therefore F[x(t)] = \frac{A}{\pi f}\left[\sin(\pi f T)\right]$$

$$= \frac{AT}{\pi f T}\left[\sin(\pi f T)\right]$$

$$= AT\left[\frac{\sin(\pi f T)}{\pi f T}\right]$$

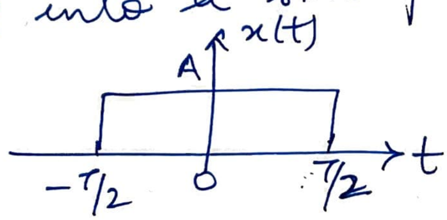
$\pi f T \rightarrow \theta$

We know –

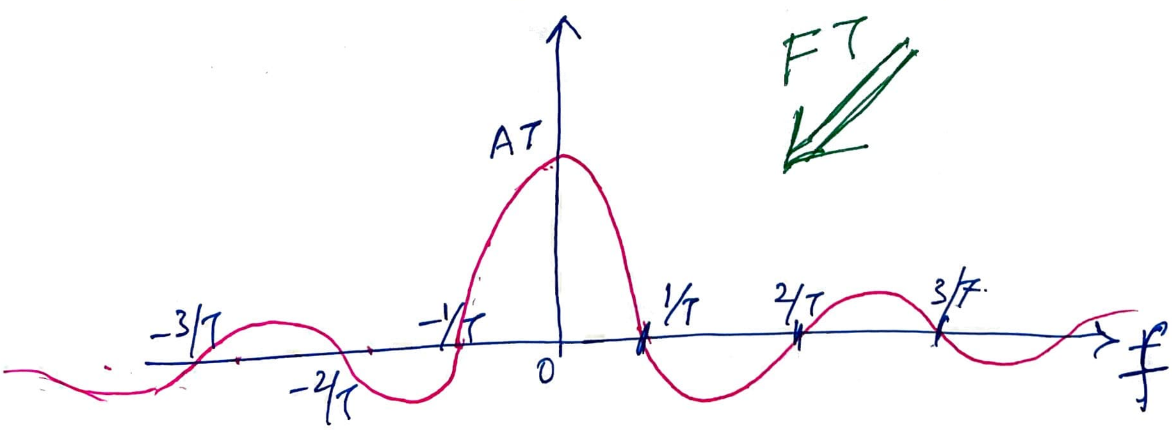
$$\operatorname{sinc}\theta = \frac{\sin\theta}{\theta}$$

$$\boxed{\therefore F[x(t)] = AT\,\operatorname{sinc}(\pi f T)}$$

⇒ Rectangular pulse transforms into a sinc func.



FT



Module II

Lecture 4

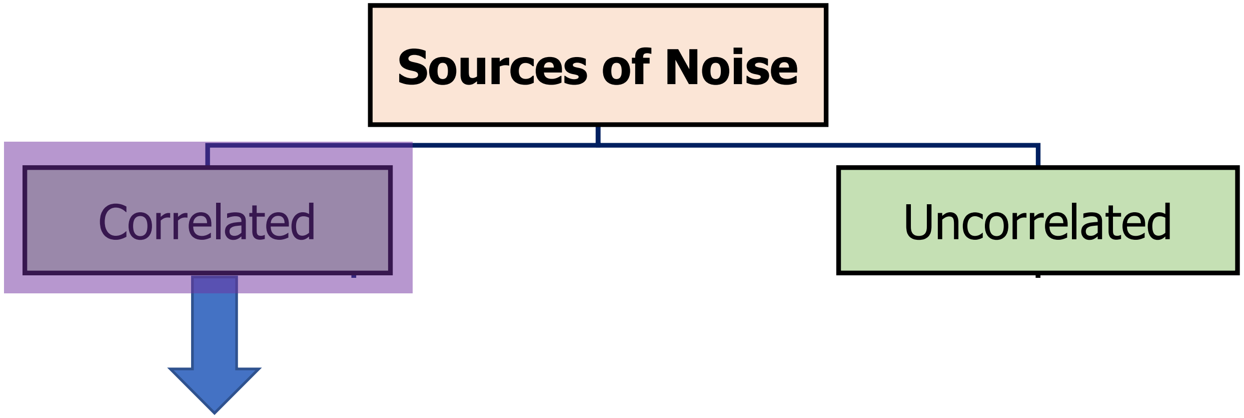
- Noise in Communication Systems
- Types of Noise

Noise

- Important parameter in communication design
- Affects receiver more as signal weak there
- Unwanted, undesired signal that interferes with the desired signal
- Gets superimposed on signal and impossible to separate signal from noise
- Random in nature
- Examples: hiss, crackle, snow

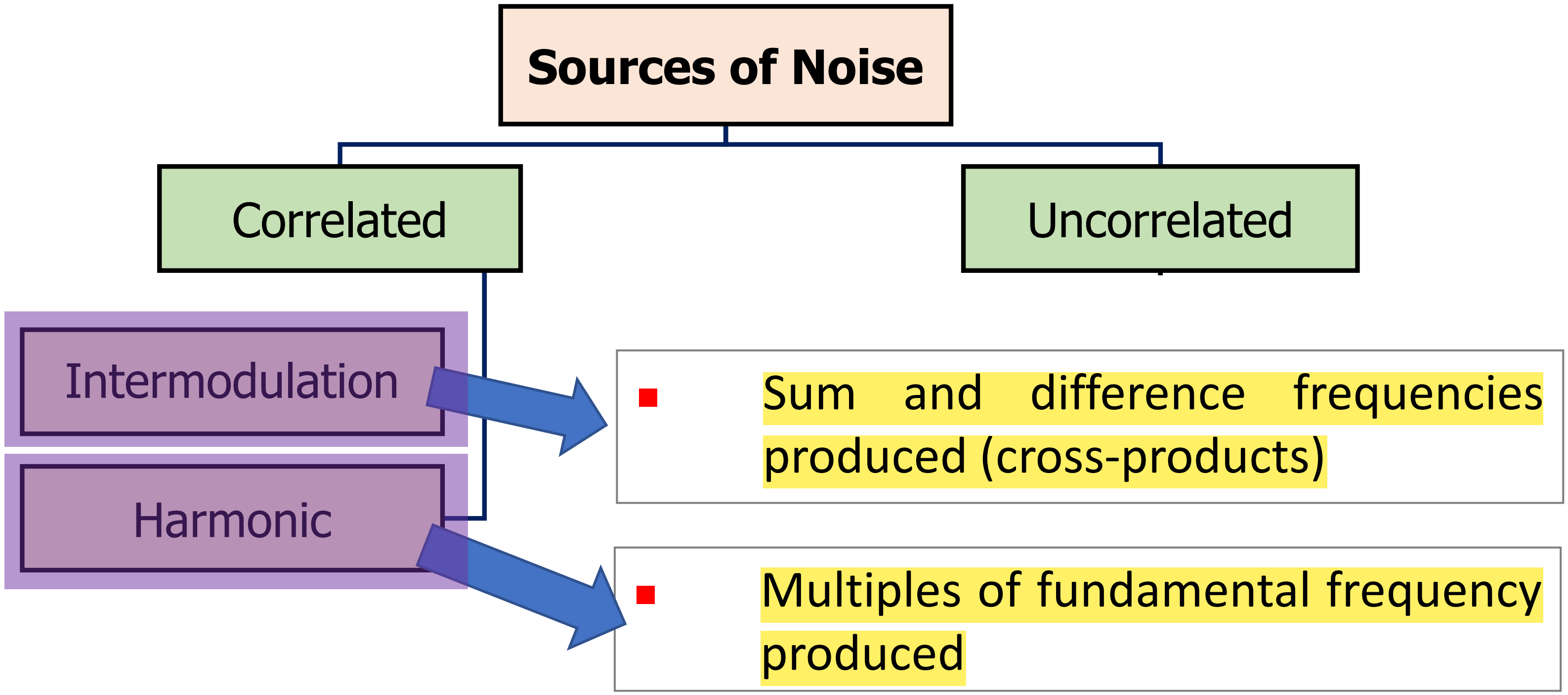
Objective: To produce highest possible SNR

Classification of Noise



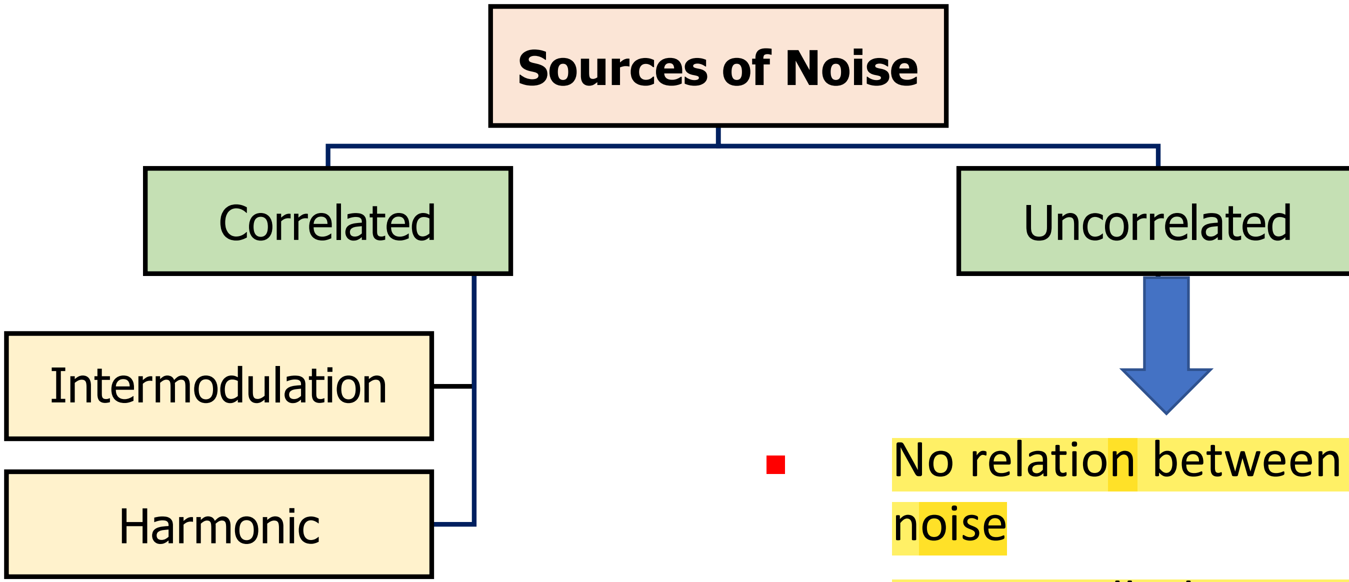
- Exists a relation between signal and noise
- Exists only when signal is present

Classification of Noise



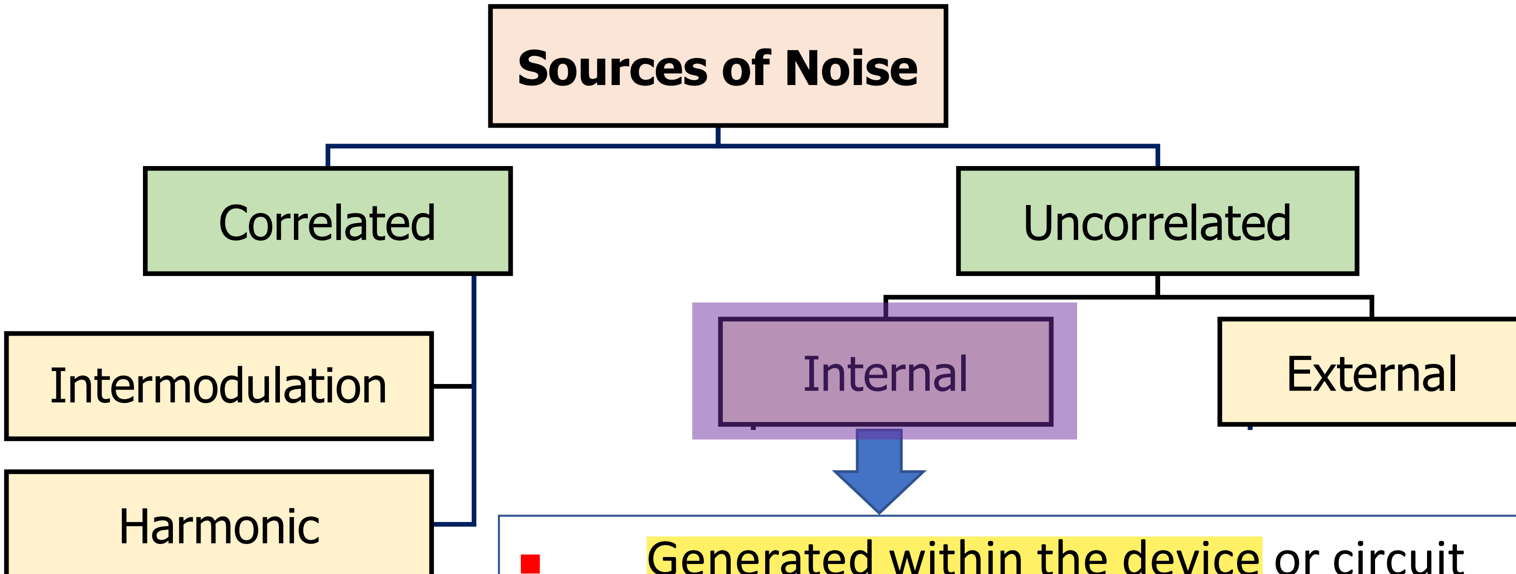
- Produced when signal passes through nonlinear devices like diodes, FET
- Hence, both together also called as nonlinear distortion

Classification of Noise



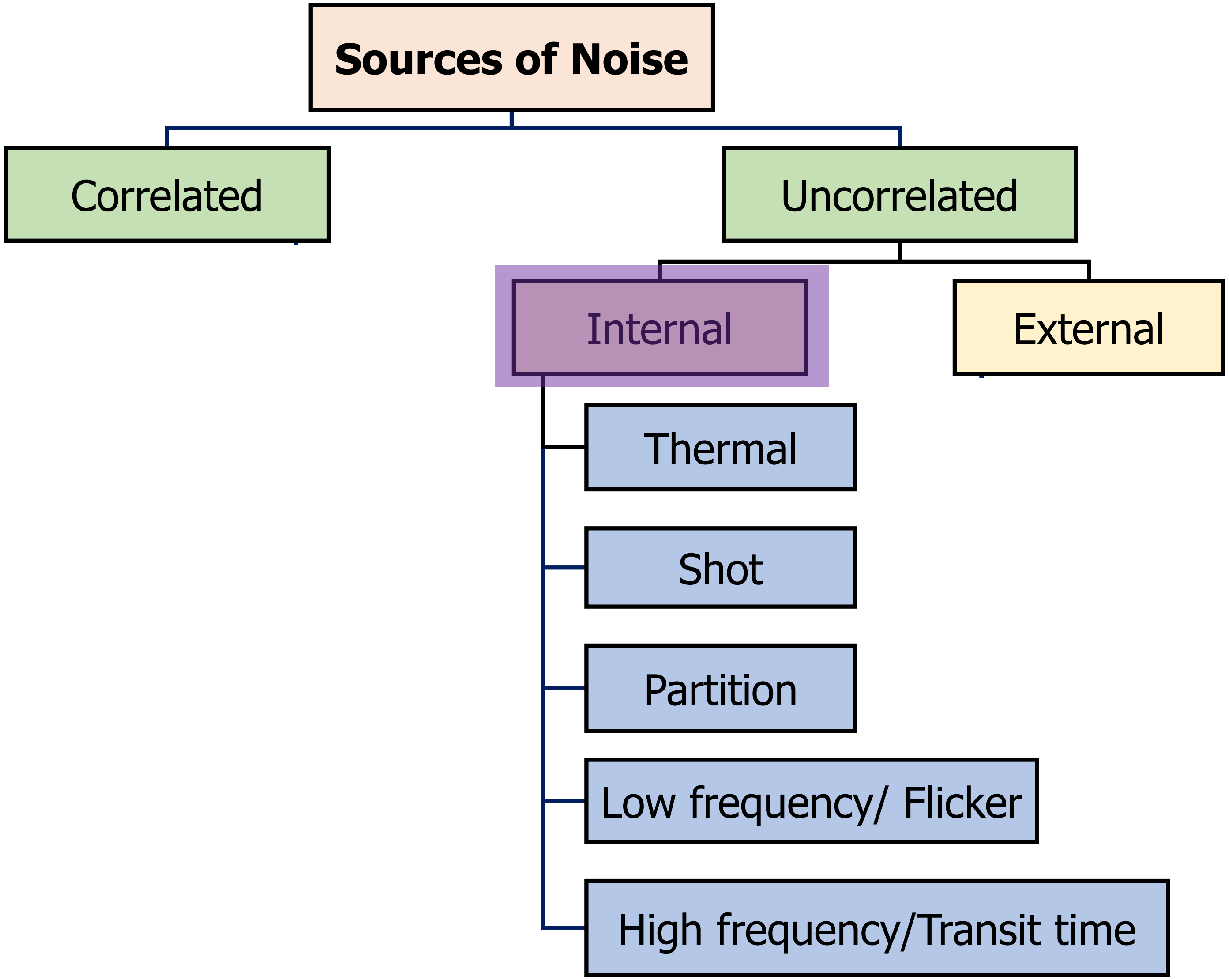
- No relation between signal and noise
- Present all the time whether there is signal or no signal

Classification of Noise



- Generated within the device or circuit
- Integral part of physical nature of material used for manufacturing components
- Produced by resistors, diodes, transistors
- Low level noise
- Affects weak signals more
- Obeys certain laws and hence we can design equipments where it can be minimized

Classification of Noise



Thermal noise/ White noise/ Johnson noise

- Due to **rapid and random movement of free electrons** within a conductor due to thermal agitation
- Thermal noise power, $P \propto$ BW (B) and Temp (T)
- $N = KTB$ Where N = Pn = Noise Power
- Power spectral density, $S_n = N/B = KT$ W/Hz
- Since equally distributed throughout the frequency spectrum, also called White noise `
- To minimize the effect…
  - Noise depends on temperature… So keep temperature low
  - Noise contains a large no. of random frequency components and noise level dependent on BW of circuit…So, use BPF
  - Noise depends on amount of current flowing in a component… So, keep I low

- Consider an equivalent circuit for a thermal noise source where internal resistance R_1 is in series with rms noise voltage, V_N
- For maximum transfer of noise power
  - $R_L = R_1$
  - $V_{R_L} = \frac{V_N}{2}$
- Hence, noise power developed across $R_L = KTB$
  - $N = KTB = \frac{\left[\frac{V_N}{2}\right]^2}{R_L} = \frac{V_N^2}{4R_L}$
  - $\boxed{V_N^2 = 4KTBR_L}$

$P_{max} = \frac{V_n^2}{4R_L^2}$

where, Boltzmann constant,
K = 1.38064852 × 10^{-23} m^2 kg s^{-2} K^{-1}

Shot Noise

- We assume that current in electronic devices under dc condition is constant
- However, its only time average flow of electrons which is constant
- Caused by random arrival of carriers at output element
- Sounds like a shower of lead shots on a metal sheet
- Inversely proportional to g_m of device
- Mean square shot noise current is given by –

$$I_n^2 = 2I_{dc}q_eB \text{ Amp}^2$$

where, I_{dc} is direct current in amperes

q_e is the magnitude of the electron charge (1.6×10^{-19} C)

B is the equivalent noise bandwidth in Hz

Partition Noise

- Occurs when current has to divide between two or more paths and results due to random fluctuations in division of current
- eg. BJT : random motion of carriers crossing junction and random recomb. in base (random division of current between C and B)
- Higher in BJT than diode
- Noise generated depends on Q-pt and source R

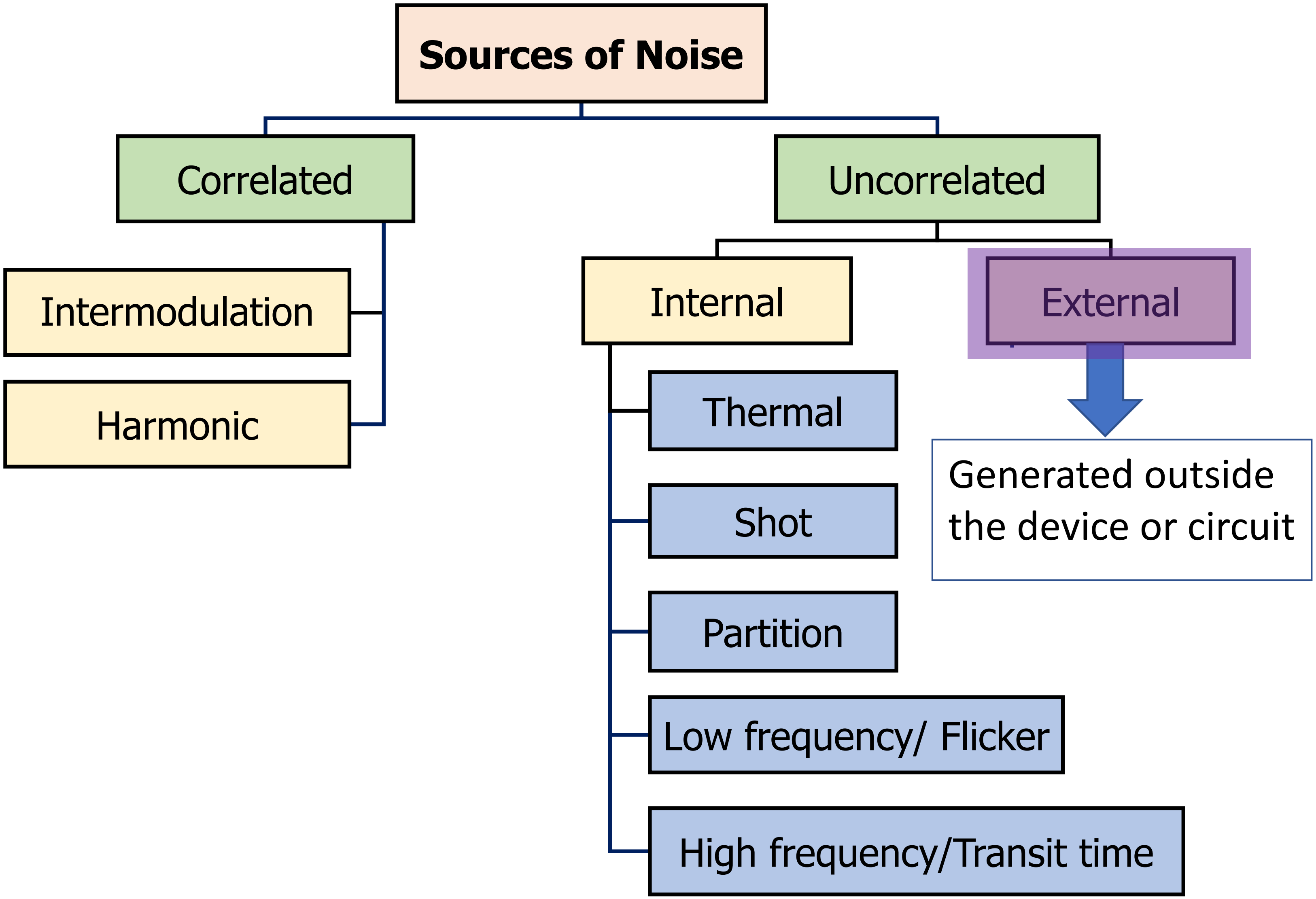
Low frequency/ Flicker Noise

- Appears for frequency < few kHz
- Inversely proportional to frequency
- It is due to fluctuations in carrier density which in turn affects the conductivity... Hence.... fluctuating voltage drop produced
- Proportional to square of direct current flowing through the device

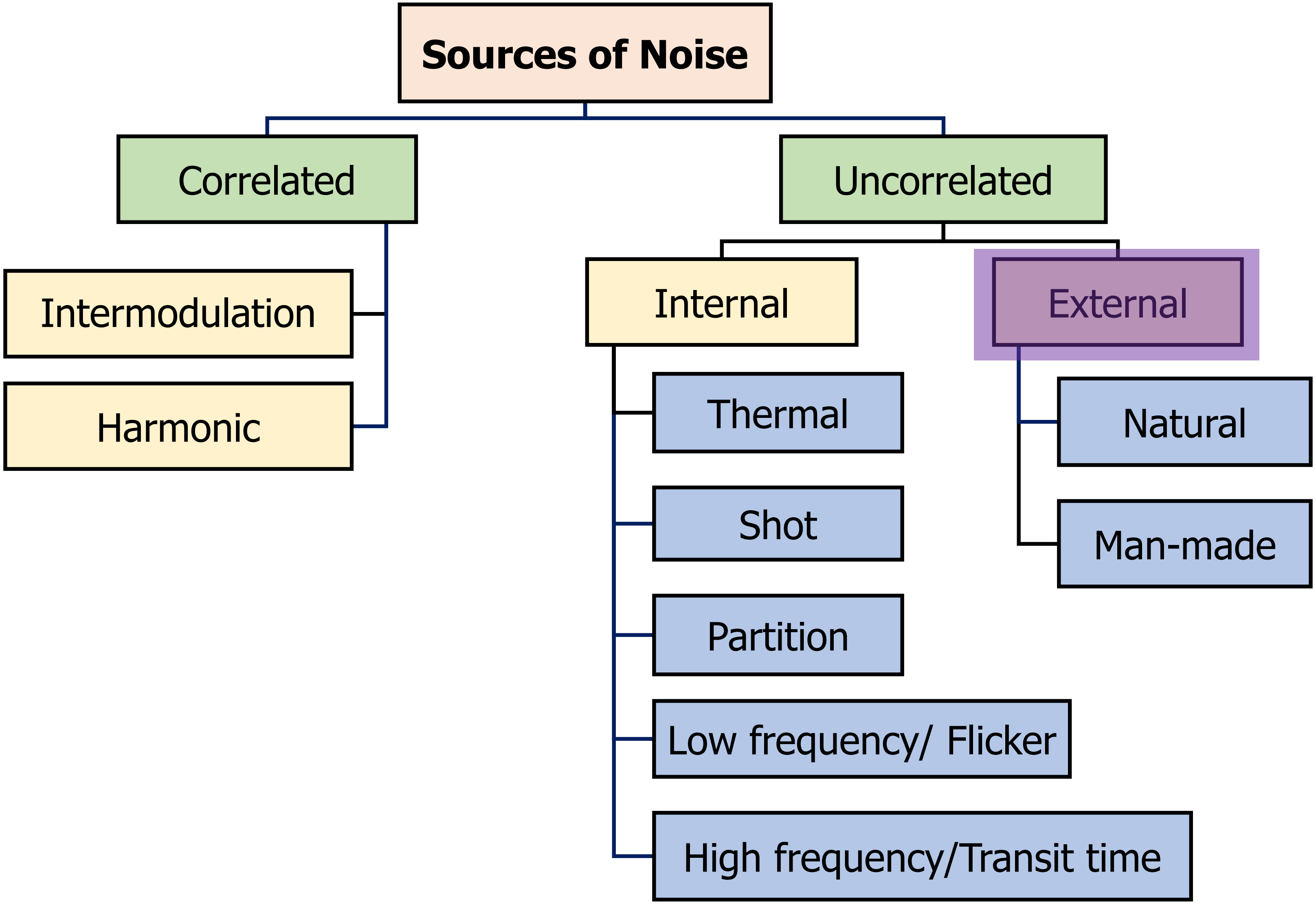
High frequency/ Transit time

- Transit time defined as time taken by carriers to cross junction
- When signal frequency high, periodic time small and maybe comparable to transit time
- Hence, some carriers may diffuse back to the source
- This results in a kind of random noise
- Determined by carrier mobility, bias voltage and transistor construction
- This noise is proportional to frequency of operation

Classification of Noise



Classification of Noise



Types of External Noise

- **Natural**
  - **Atmospheric** – produced within Earth's atmosphere
    - Lightning (electrical disturbance)
    - Electric discharge between clouds and between clouds and earth
  - **Extra-terrestrial** (Source - space) - Two types
    - Solar (Source - Sun)
    - Wide range of signals in a broad noise spectrum
    - Vary with time
  - **Cosmic** (Source - Stars)
    - Impact less because of distance
    - Greatest impact in 15-150MHz

- **Man-made Noise**
  - Industrial – generated due to make and break process in a current carrying circuit
  - Automotive ignition systems
  - Electrical motors
  - Fluorescent lights
  - Welding
  - Switching gears
  - Gas filled tubes
  - Can be minimized by controlling it

[redacted]

NOISE

→ We know –

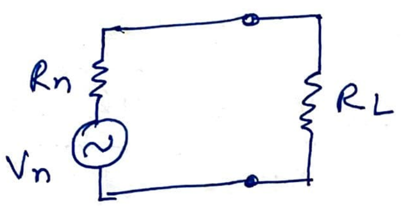
Noise Power $= P_n = N \propto BW$ & Temp

$\Rightarrow N = KTB$ —— (1)

$K \rightarrow$ Boltzmann's constant $= 1.38 \times 10^{-23}$ J/°K

→ EQUIVALENT CIRCUIT FOR THERMAL NOISE –

Consider an equivalent circuit for thermal noise source where internal resistance, R_n is in series with rms noise voltage, V_n.



→ We know –

For max. power transfer –

$R_n = R_L$

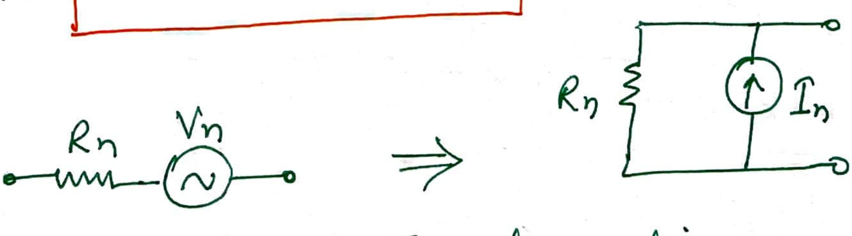
$\therefore P_{max} = N = \dfrac{V_n^2}{4R_L}$ —— (2)

From (1) & (2),

$$\frac{V_n^2}{4R_L} = KTB$$

$$V_n^2 = 4KTB\,R_L$$

$$\therefore \boxed{V_n = \sqrt{4KTBR_L}}$$

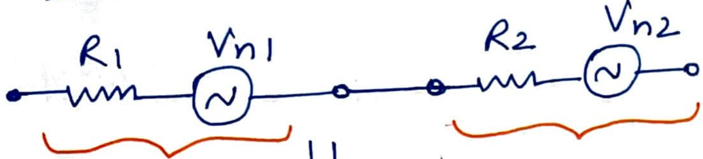


Source Transformation

→ IN SERIES –

$$V_{n1} = \sqrt{4KTBR_1}$$

$$V_{n2} = \sqrt{4KTBR_2}$$



$$\Rightarrow \quad R = R_1 + R_2$$

$$V_n = \sqrt{4KTBR}$$

$$\Rightarrow \quad V_n = \sqrt{4KTB(R_1 + R_2)}$$

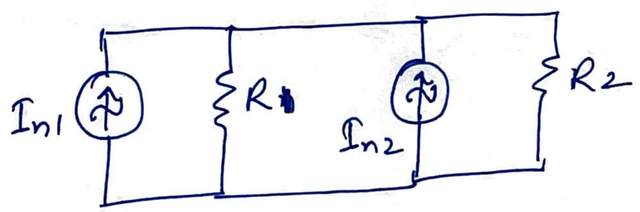
$$V_n = \sqrt{4KTBR_1 + 4KTBR_2}$$

$$\boxed{V_n = \sqrt{V_{n1}^2 + V_{n2}^2}}$$

→ IN PARALLEL –

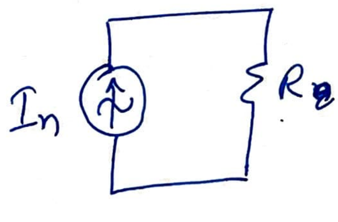
$$I_{n1} = \frac{V_{n1}}{R_1} = \frac{\sqrt{4KTBR_1}}{R_1}$$

$$= \sqrt{\frac{4KTB}{R_1}} = \sqrt{4KTBG_1}$$



$$I_{n2} = \sqrt{4KTBG_2}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$



$$\Rightarrow \quad G = G_1 + G_2$$

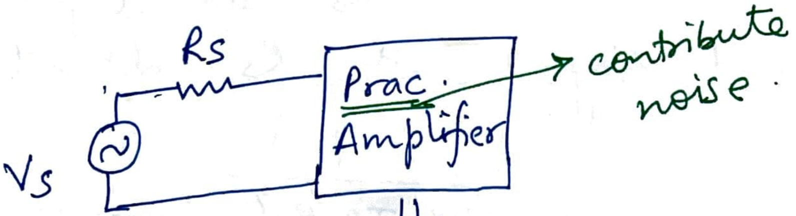
$$I_n = \sqrt{4KTBG}$$

$$\Rightarrow \quad I_n = \sqrt{4KTB(G_1 + G_2)}$$

$$I_n = \sqrt{4KTBG_1 + 4KTBG_2}$$

$$\boxed{I_n = \sqrt{I_{n1}^2 + I_{n2}^2}}$$

→ Practical & Noiseless Amplifier



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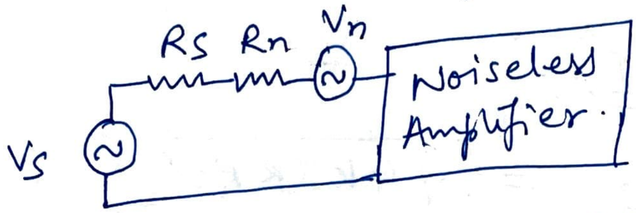
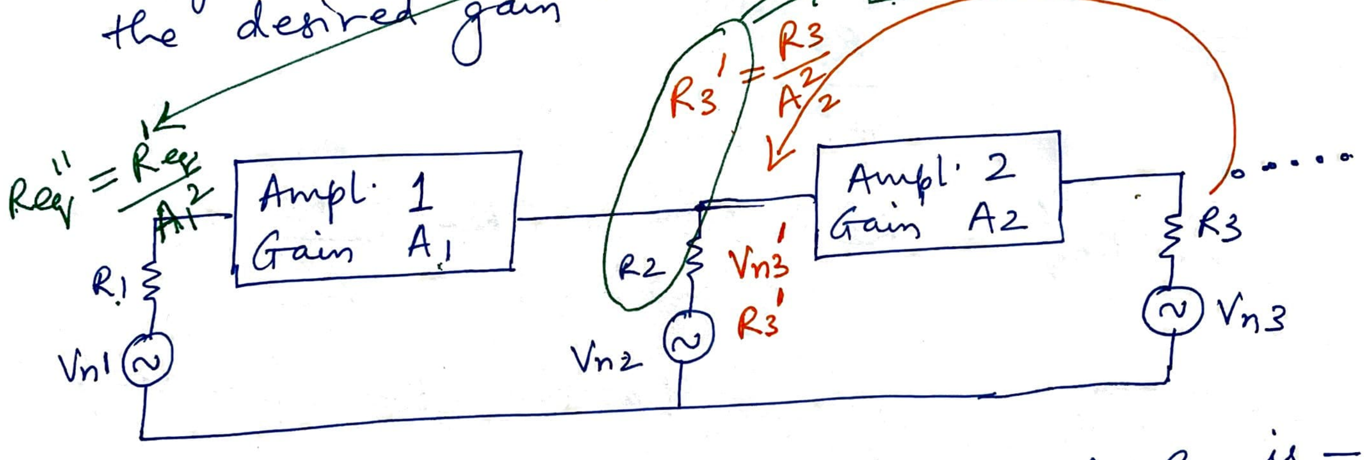


→ NOISE DUE TO AMPLIFIERS IN CASCADE –

→ Typically, in receivers, number of stages are connected in cascade to get the desired gain.

$R_2 + R_3' = R_{eq}'$



rms noise voltage at output due to R_3 is –

$$V_{n3} = \sqrt{4KTBR_3}$$

→ V_{n3} is produced due to rms noise voltage V_{n3}' at input of second stage such that

$$V_{n3} = A_2 \times V_{n3}'$$

$$\Rightarrow V_{n3}' = \frac{V_{n3}}{A_2} = \frac{\sqrt{4KTBR_3}}{A_2} \quad \text{—— (1)}$$

→ Now, assume that V_{n3}' is generated by another resistor R_3' at input of second stage with R_3 absent on output side.

$$V_{n3}' = \sqrt{4KTBR_3'} \quad \text{—— (2)}$$

from (1) & (2),

$$\frac{\sqrt{4KTBR_3}}{A_2} = \sqrt{4KTBR_3'}$$

$$\Rightarrow \frac{4KTBR_3}{A_2^2} = 4KTBR_3'$$

$$\Rightarrow R_3' = \frac{R_3}{A_2^2}$$

$$\therefore R_{eq}' = R_2 + R_3' = R_2 + \frac{R_3}{A_2^2}$$

→ Similarly, we can transfer R_{eq}' from output of stage 1 to input of stage 1

$$R_{eq}'' = \frac{R_{eq}'}{A_1^2} = \frac{1}{A_1^2}\left[R_2 + \frac{R_3}{A_2^2}\right]$$

$$R_{eq}'' = \frac{R_2}{A_1^2} + \frac{R_3}{A_1^2 A_2^2}$$

∴ Equivalent noise resistor at input of first stage is –

$$R_{eq} = R_1 + R_{eq}''$$

$$\boxed{R_{eq} = R_1 + \frac{R_2}{A_1^2} + \frac{R_3}{A_1^2 A_2^2}}$$

Generalizing –

$$R_{eq} = R_1 + \frac{R_2}{A_1^2} + \frac{R_3}{A_1^2 A_2^2} + \frac{R_4}{A_1^2 A_2^2 A_3^2} + \cdots$$

SCROLL DOWN

NUMERICALS

Q1. Calculate the thermal noise power available from any resistor at room temperature of 17°C for a bandwidth of 1 MHZ.

Soln.

$B = 1\ MHZ = 1 \times 10^6\ Hz$

$T = 17°C = 17 + 273 = 290°K$

$N = KTB$

$= 1.38 \times 10^{-23} \times 290 \times 1 \times 10^6$

$= 4.002 \times 10^{-15}$ watts.

Q2. Calculate the noise voltage for $R = 60\Omega$ for the example given above

Soln: $V_n^2 = 4kTBR$

$= 4 \times 1.38 \times 10^{-23} \times 290 \times 1 \times 10^{6} \times 60$

$= 96.048 \times 10^{-14}$

$V_n = \sqrt{96.048 \times 10^{-14}} = 9.8 \times 10^{-7}$ volts

Q3. An amplifier operating over the frequency range from 18 to 20 MHz has a 10 kΩ input resistor. What is the rms noise voltage at the input of this amplifier if ambient temperature is 27°C.

Soln: $T = 27°C = 27 + 273 = 300° K$

$B = (20 - 18) MHz = 2 MHz = 2 \times 10^{6} Hz$

$R = 10 K\Omega = 10 \times 10^{3} \Omega = 10^{4} \Omega$

$\therefore V_n = \sqrt{4kTBR}$

$= \sqrt{4 \times 1.38 \times 10^{-38} \times 300 \times 2 \times 10^{6} \times 10^{4}}$

$= 18.1 \mu$ volts.

Q4. Calculate the noise voltage at the input of a television RF amplifier using a device that has a 200 Ω equivalent noise resistance and a 300 Ω input resistance. The bandwidth of the amplifier is 6 MHz and the temperature is 290° K.

Soln. $T = 290^\circ K$

$B = 6\,MHz = 6 \times 10^6\,Hz$

$R_n = 200\,\Omega$

$R_i = 300\,\Omega$

$$V_n = \sqrt{4KTB(R_n + R_i)}$$

$$= \sqrt{4 \times 1.38 \times 10^{-23} \times 290 \times 6 \times 10^6 \times (200 + 300)}$$

$= 6.93\ \mu volts$

Q5. For a bandwidth of 150 kHz, calculate thermal noise voltage generated by two resistors of 30 kΩ and 60 kΩ when they are connected —

(a) individually

(b) in series

(c) in parallel.

Given the temperature is 17°C.

Soln: $T = 17°C = 17 + 273 = 290°K$

$B = 150\ KHZ = 150 \times 10^3\ Hz$

(a) when $R_1 = 30\,k\Omega = 30 \times 10^3\ \Omega$

$$V_{n1} = \sqrt{4KTBR_1}$$

$$= \sqrt{4 \times 1.38 \times 10^{-23} \times 290 \times 150 \times 10^3 \times 30 \times 10^3}$$

$$= 8.48\ \mu V$$

when $R_2 = 60\,k\Omega = 60 \times 10^3\ \Omega$

$$V_{n2} = \sqrt{4KTBR_2}$$

$$= \sqrt{4 \times 1.38 \times 10^{-23} \times 290 \times 150 \times 10^3 \times 60 \times 10^3}$$

$$= 12\ \mu V$$

(b) when in series -

$$V_n = \sqrt{V_{n1}^2 + V_{n2}^2}$$

$$= \sqrt{(8.48)^2 + (12)^2}$$

$$= 14.7\ \mu V$$

(c) when in parallel -

we know -

$V_{n1} = 8.48\ \mu V$ $\quad R_1 = 30\,k\Omega$

$V_{n2} = 12\ \mu V$ $\quad R_2 = 60\,k\Omega$

$$I_{n1} = \frac{V_{n1}}{R_1} = \frac{8.48 \times 10^{-6}}{30 \times 10^3} = 2.82 \times 10^{-10} \text{ A}$$

$$I_{n2} = \frac{V_{n2}}{R_2} = \frac{12 \times 10^{-6}}{60 \times 10^3} = 2 \times 10^{-10} \text{ A}$$

$$I_n = \sqrt{I_{n1}^2 + I_n^2}$$

$$= \sqrt{(2.82 \times 10^{-10})^2 + (2 \times 10^{-10})^2}$$

$$= 3.46 \times 10^{-10} \text{ A}$$

$$R = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = \left(\frac{30 \times 60}{30 + 60}\right) k\Omega$$

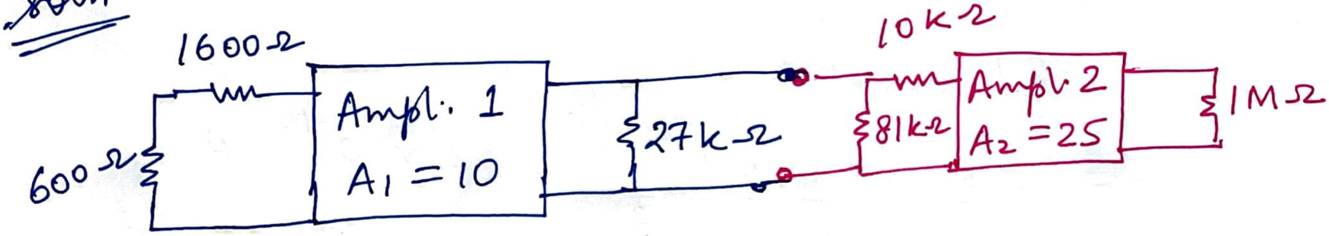
$$= 20 \; k\Omega$$

$$\therefore V_n = I_n R = 3.46 \times 10^{-10} \times 20 \times 10^3$$
$$= 6.92 \; \mu V$$

Q6. The first stage of a 2-stage amplifier has a voltage gain of 10, a 600 Ω input resistor, a 1600 Ω equivalent noise resistance and a 27 kΩ output resistor.

For the second stage, these values are 25, 81 kΩ, 10 kΩ and 1 MΩ respectively. Calculate the equivalent input noise resistance of this two stage amplifier.

Soln:



$$R_{eq} = R_1 + \frac{R_2}{A_1^2} + \frac{R_3}{A_1^2 A_2^2}$$

$A_1 = 10$; $A_2 = 25$

$R_1 = 600 + 1600 = 2200\ \Omega$

$R_2 = (27k\Omega \parallel 81k\Omega) + 10\ k\Omega$

$= 20.2\ k\Omega + 10 = 30.2\ k\Omega = 30.2 \times 10^3\ \Omega$

$R_3 = 1M\Omega = 1 \times 10^6\ \Omega$

$$\therefore R_{eq} = 2200 + \frac{30.2 \times 10^3}{10^2} + \frac{1 \times 10^6}{10^2 \times 25^2}$$

$= 2200 + 302 + 16$

$= 2518\ \Omega$

NOISE PARAMETERS

Signal to Noise Ratio (SNR) –

→ Defined as the ratio of signal power to noise power at the same point

$$\therefore \frac{S}{N} = SNR = \frac{P_s}{P_n}$$

It is expressed in decibel (dB)

→ Higher the SNR ⇒ Better system performance

$$\therefore SNR_{dB} = 10 \log_{10}\left(\frac{P_s}{P_n}\right) dB = 10 \log_{10}\left(\frac{V_s^2}{V_n^2}\right)$$

$$= 20 \log_{10}\left(\frac{V_s}{V_n}\right) dB$$

Noise Factor (F) –

$$F = \frac{SNR \text{ at input}}{SNR \text{ at output}} = \frac{P_{si}/P_{ni}}{P_{so}/P_{no}}$$

$$F = \frac{P_{si}}{P_{ni}} \times \frac{P_{no}}{P_{so}}$$

→ NOTE –

SNR at input > SNR at output

∵ Noise is added by the amplifier

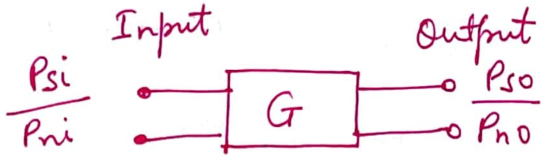
⇒ F is a measure of amount of noise added & its value will be always > 1

→ Ideal value of F is 1.

Noise power in terms of F -

We know -

$$F = \frac{P_{si}}{P_{ni}} \times \frac{P_{no}}{P_{so}}$$



$$\text{Power gain} = \frac{P_{so}}{P_{si}}$$

$$\therefore F = \frac{P_{no}}{G P_{ni}}$$

⇒ Noise power at the output is,

$$P_{no} = F G P_{ni}$$

We know,

$$P_{ni} = kTB$$

$$\therefore \boxed{P_{no} = FGkTB}$$

⇒ with increase in F, noise power at output will increase.
Higher is the F, more is the noise contributed by amplifier.

Noise Figure (NF) –

→ Noise figure is actually noise factor expressed in dB.

$$NF = F_{dB} = 10 \log_{10} F$$

$$NF = 10 \log_{10} \left\{ \frac{\text{SNR at Input}}{\text{SNR at Output}} \right\}$$

$$= 10 \log_{10} \left[\frac{SNR_i}{SNR_o} \right]$$

$$= 10 \log_{10} SNR_i - 10 \log_{10} SNR_o$$

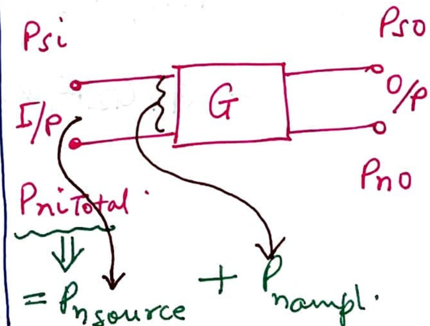
$$= \left(\frac{S}{N}\right)_i dB - \left(\frac{S}{N}\right)_o dB$$

→ Ideal value of $NF = 0$

→ Amplifier Noise contribution in terms of F ($P_{n\,ampl.}$)

$$\left.\begin{matrix}\text{Noise Power}\\ \text{at output}\end{matrix}\right\} = P_{no} = FGKTB$$

$$\text{Total } P_{ni} = \frac{P_{no}}{G} = \frac{FGKTB}{G} = FKTB$$



Also,

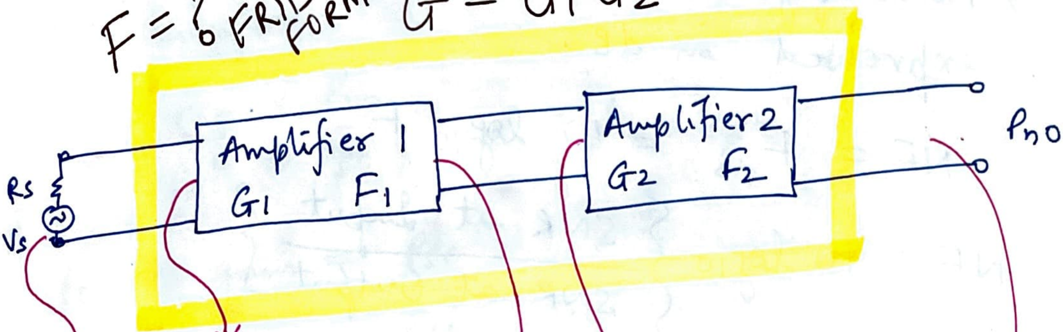
$$\text{Total } P_{ni} = P_{n\,source} + P_{n\,ampl.}$$

$$FKTB = KTB + P_{n\,ampl.}$$

$$\boxed{\therefore P_{n\,ampl.} = KTB(F-1)}$$

→ Noise factor of Amplifiers in Cascade –

$F = ?$ FRIIS FORMULA $\quad G = G_1 G_2$



$$\underbrace{KTB + (F_1 - 1)KTB}$$

$$P_{ni_{total}} = F_1 KTB$$

$$\underbrace{G_1 F_1 KTB + (F_2 - 1)KTB}$$

$$G_2\left[G_1 F_1 KTB + (F_2 - 1)KTB\right]$$

$$= G_1 G_2 F_1 KTB + G_2 (F_2 - 1)KTB$$

→ Noise due to source, $P_{nsource} = KTB$

Noise due to Amplifier 1, $= (F_1 - 1) KTB$

∴ Total noise at input of Amplifier 1,

$$P_{nitotal} = KTB + (F_1 - 1) KTB = F_1 KTB$$

→ ∴ Noise at output of Amplifier 1 is –

$$= G_1 (F_1 KTB) = F_1 G_1 KTB$$

→ Noise due to Amplifier 2,

$$= (F_2 - 1) KTB$$

→ ∴ Total noise at input of Amplifier 2,

$$= F_1 G_1 KTB + (F_2 - 1) KTB$$

→ ∴ Noise at output of Amplifier 2, P_{no}

$$P_{no} = G_2 \{ F_1 G_1 KTB + (F_2 - 1) KTB \}$$

$$P_{no} = G_1 G_2 F_1 KTB + G_2 (F_2 - 1) KTB \quad \text{—— (A)}$$

→ Now,

Overall Gain $= G = G_1 G_2$

Overall $F = \dfrac{P_{no}}{G P_{ni}}$ $\quad [\because P_{no} = F G P_{ni}]$

$= \dfrac{P_{no}}{G_1 G_2 KTB}$ $\quad [\because P_{ni} = P_{nsource} = KTB]$

From (A),

$$\therefore F = \frac{G_1 G_2 F_1 KTB + G_2 (F_2 - 1) KTB}{G_1 G_2 KTB}$$

$$\Rightarrow \boxed{F = F_1 + \frac{(F_2 - 1)}{G_1}}$$

FRIIS FORMULA

We can extend this as –

$$F = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2} + \cdots$$

⇒ I^{st} stage is the most important stage in deciding overall Noise Factor

→ Equivalent Noise Temperature of Amplifiers in cascade –

We know from Friis formula –

$$F = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2} + \cdots$$

→ Noise at the input of amplifier,

$$P_{n\,ampl.} = (F - 1) KTB$$

This is the noise contributed by the amplifier.

→ Let the noise be represented alternatively by some fictitious temperature, T_{eq}, such that –

$$K T_{eq} B = (F - 1) KTB$$

$$\Rightarrow \boxed{T_{eq} = (F - 1) T} \quad \text{——— (1)}$$

∴ T_{eq} is an alternative measure for F.

NUMERICALS

Q1. An amplifier has a gain of 3 dB. Calculate the power at the output of the amplifier in mW if the power applied at the input ~~of~~ is 0.25 mW

Soln:

$$10 \log_{10} G = 3 \text{ dB}$$

$$\log_{10} G = \frac{3}{10} = 0.3$$

$$G = \text{antilog } 0.3 = 2$$

$$P_i = 0.25 \text{ mW}$$

$$P_o = G P_i = 2 \times 0.25 \text{ mW} = 0.5 \text{ mW}$$

Q2. For an amplifier with output signal power of 10 W and output noise power of 0.01 W, determine SNR in dB.

Soln:

$$SNR = 10 \log_{10}\left(\frac{P_s}{P_n}\right) = 10 \log_{10}\left(\frac{10}{0.01}\right)$$

$$= 30 \text{ dB}$$

Q3. For an amplifier,

$P_{si} = 2 \times 10^{-10}$ W $\qquad P_{ni} = 2 \times 10^{-18}$ W

$P_{so} = 2 \times 10^{-4}$ W $\qquad P_{no} = 8 \times 10^{-12}$ W

Calculate Noise factor and Noise figure.

We know that -

$$F = f_1 + \frac{(f_2 - 1)}{G_1} + \frac{(f_3 - 1)}{G_1 G_2} + \cdots$$

Subtract 1 from both sides,

$$(F - 1) = (f_1 - 1) + \frac{(f_2 - 1)}{G_1} + \frac{(f_3 - 1)}{G_1 G_2} + \cdots$$

We know from (1),

$$T(F - 1) = T_{eq}$$

$$\Rightarrow \frac{T_{eq}}{T} = (F - 1)$$

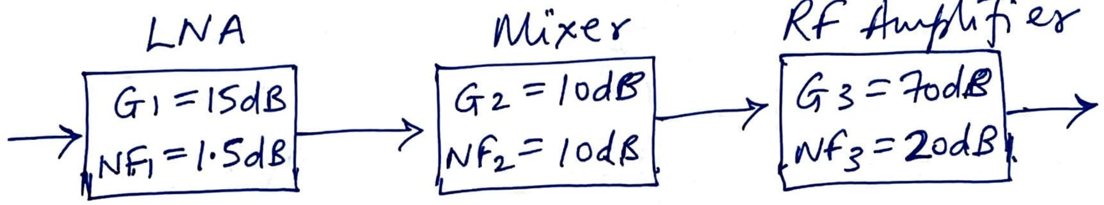
$$\therefore \frac{T_{eq}}{T} = \frac{T_{eq1}}{T} + \frac{T_{eq2}}{T G_1} + \frac{T_{eq3}}{T G_1 G_2} + \cdots$$

$$\Rightarrow \boxed{T_{eq} = T_{eq1} + \frac{T_{eq2}}{G_1} + \frac{T_{eq3}}{G_1 G_2} + \cdots}$$

Soln: $F = \dfrac{P_{si}/P_{ni}}{P_{so}/P_{no}} = \dfrac{2\times10^{-10}/2\times10^{-18}}{2\times10^{-4}/8\times10^{-12}} = 4$

$NF = 10\log_{10} F = 10 \log_{10} 4 = 6.02\ dB$

Q4. Calculate the overall noise factor and noise figure for the circuit given below -



Soln: $G_1 = 15dB \Rightarrow 10\log_{10} G_1 = 15 \Rightarrow G_1 = 31.62$

$NF_1 = 1.5dB \Rightarrow 1.413$

$G_2 = 10dB \Rightarrow 10$

$NF_2 = 10dB \Rightarrow 10$

$G_3 = 70dB \Rightarrow 10^7$

$NF_3 = 20dB \Rightarrow 100$

Using friis formula -

$F = F_1 + \dfrac{(F_2-1)}{G_1} + \dfrac{(F_3-1)}{G_1 G_2}$

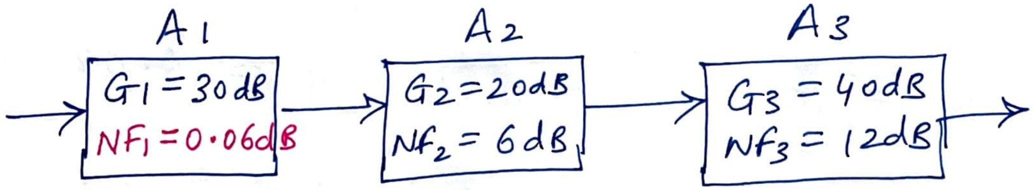
$= 1.413 + \dfrac{(10-1)}{31.62} + \dfrac{(100-1)}{31.62\times10}$

$= 2.010$

$NF = 10\log_{10} F = 10\log_{10} 2.010$

$= 3.031\ dB$

Q5. Calculate overall noise figure for the given cascaded amplifier.
Assume $T = 290^\circ K$



$T_{eq1} = 4^\circ K$

Soln: We know —

$$(F-1) = \frac{T_{eq}}{T}$$

$$\Rightarrow (F_1 - 1) = \frac{T_{eq1}}{T}$$

$$\Rightarrow F_1 = 1 + \frac{T_{eq1}}{T} = 1 + \frac{4}{290} = 1.014 = 0.06\,dB$$

$G_1 = 30\,dB \Rightarrow 1000$

$NF_1 = 0.06\,dB \Rightarrow 1.014$

$G_2 = 20\,dB \Rightarrow 100$

$Nf_2 = 6\,dB \Rightarrow 3.98$

$G_3 = 40\,dB \Rightarrow 10000$

$NF_3 = 12\,dB \Rightarrow 15.85$

We know,

$$F = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2}$$

$$F = 1.014 + \frac{(3.98-1)}{1000} + \frac{(15.85-1)}{1000 \times 100}$$

$$= 1.017$$

$$NF = 10 \log_{10} F = 10 \log_{10} 1.017 = 0.073 \text{ dB}$$

Homework –

Q6. Consider a receiver as described –
The first stage is the RF amplifier which has a gain of 20 dB and noise figure of 7 dB. The second stage is the mixer with the corresponding values of 8 dB and 8 dB respectively. The final stage is the IF amplifier with gain of 60 dB and noise figure of 6 dB.
Find the overall noise figure.

Ans → Noise figure = 7.04 dB