Q. For the plant $G(s) = \dfrac{1}{s(0.4s+1)(s+2)}$. design a controller such that

$K_V = 4$.

Phase margin $= 60^\circ$

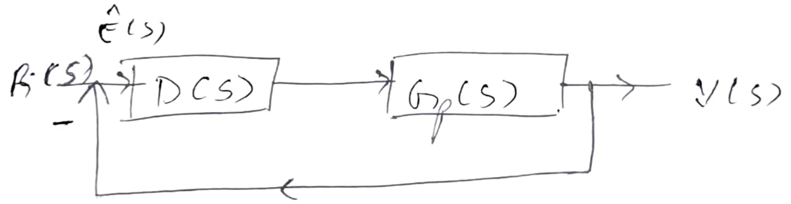
Zero steady state error for step input.

Sol $K_V = 4$; $P.M = 60^\circ$

We consider ϕ_M or P.M and K_V as major design specifications.

We obtain other parameters by using bode plot from matlab.

We have to design cascade phaselead compensator for given specifications.



We are going to design a compensator $D(s)$ with phase lead device as compensator for $G(s)$ (lead compensator).

Primary function of lead compensator is to reshape the frequency domain plot by providing sufficient phaselead angle

The transfer function of a lead compensator is given by, $D(s) = \dfrac{\tau s + 1}{\alpha \tau s + 1}$

The given system is type one hence, for step input, steady state error is zero.

Hence we apply Ramp input and ss is,

$e_{ss} = \frac{1}{K_v}$ in response to unit ramp

$$K_v = \frac{K \prod_i (s+z_i)}{\prod_j (s+p_j)}$$

Hence the $\hat{E}(s)$ is given by

$$\hat{E}(s) = \frac{1}{1 + D(s) \cdot G_p(s)}$$

Then steady state error, $\hat{e_{ss}} = \lim_{s \to 0} \frac{s R(s)}{1 + D(s) G_p(s)}$

$\therefore K_v = \lim_{s \to 0} s D(s) G(s)$

Steady state error to velocity input

$\therefore e_{ss} = \frac{1}{K_v}$

| $r(t) \Rightarrow R(s)$
| $R(s) = \frac{1}{s^2}$ for ramp input $r(t)$.

Hence we can write,

$$K_v = \lim_{s \to 0} s \cdot K \cdot G_p(s).$$

$$4 = \lim_{s \to 0} \cancel{s} \cdot K \cdot 1/3 \times \frac{1}{\cancel{s} \cdot \underset{(0)}{(s+2)} \underset{(0)}{(0.4s+1)}}$$

$$K_v = \frac{K}{2}$$

$$G_p(s) = \frac{K}{(s)(s+2)(0.4s+1)}$$

$\therefore K = 8$

where K is the gain.

Thus $G_p(s) = \dfrac{8}{(s)(0.4s^2+1.8s+2)}$

$$= \frac{8}{0.4s^3+1.8s^2+2s}$$

$$= \frac{20}{s^3+4.5s^2+5s}$$

This is the uncompensated system.
We shall obtain bode plot for this, hence finding the phase margin of uncompensated system

We consider $G(j\omega) = \dfrac{20}{s^3+4.5s^2+5s}$

If cascade lead compensation scheme is employed to increase the margin to the specified value, the additional phase required at new gain crossover frequency ω_g'

$\omega_g = 2.1$ rad/sec

phase margin $= 4°$ | $-176+180° = 4°$

Additional phase req = specified phase margin − Phase margin of uncomp s/s $+ \varepsilon$

$\varepsilon = 3°$

hence we get,

$\phi_m = 60° - 4° + 3° = 59°$.

we consider parameter α, K, τ as a constants, they are found in different ways

$$\alpha = \frac{1 - \sin\phi_m}{1 + \sin\phi_m}$$

$$\alpha = 0.076908$$

To locating the frequency at which uncompensat-ed system of has gain of

$$= -20\log(1/\sqrt{\alpha})$$

$$= -20\log\left(\frac{1}{\sqrt{0.769}}\right)$$

$$= -11.14 \text{ dB}$$

locating this, we have $\omega_g' = 3.87$ rad/sec.

the above parameters can be observed in fig - 1

By setting $\omega_m = \omega_g'$

$$\omega_m = \frac{1}{\sqrt{\alpha}\,\tau}$$

$$\tau = \frac{1}{\sqrt{0.0769} * 3.87 \text{ dB}}$$

$$\tau = \frac{1}{\sqrt{0.0769} \cdot 3.8 r/s}$$

$\omega g' = \omega_m = 3.83 r/sec$

$\therefore \quad \tau = 0.942$

$\tau = .942$

Thus $D(s) = \dfrac{\tau s + 1}{\alpha \tau s + 1}$

$$D(s) = \frac{0.9741 s + 1}{0.07241 s + 1}$$

Thus compensated system will be,

$$G(s)\,D(s) = \frac{0.9419s+1}{0.0794s+1} \cdot \frac{20}{s^3+4.s^2+5s}$$

$$= \frac{18.2s+20}{0.0724s^4+1.3258s^3+4.862s^2+5s}$$

The obtained phase margin of the compensated system is 28°. It was obtained from fig-2

Result:

* Phase margin increased from 4° to 28°
* Bandwidth increased from 3.12 rad/sec to 5.17 rad/sec. (fig-4)
* Resonance peak reduced from 25.4 dB to 7.1 dB
It was found from fig-2.

This was proved in figures 7 and 8 attached at last.

Conclusion:

- The lead compensation increases the gain of crossover frequency of the system.

Steady state error of feedback was not affected by unity zero frequency gain lead compensator

We also observed that the α of the system was not smaller than 0.07 and ϕ_m was more than or equal to 60°.

When we try to increase the value of 'K', it results in amplification of the signals at all frequencies. Thus introduction of lead compensator provides additional gain for higher frequency signals, hence boosting the noise signal level to control signal. This imposes a limit on the improvement of steady state error by raising gain K in a lead compensation.

In our case, the phase margin was increased almost 7 times and the assumption of ϵ to 3° is proved correct as we obtained the phase margin of uncompensated system as 4° which is greater than assumed 'ϵ' hence we dont need to change or increase the gain K