# A Small World Network Model for Energy Efficient Wireless Networks

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***Abstract*—Wireless ad hoc networks can be modeled as small world networks based on the complex network theory. In this letter, we propose a new energy efficient small world network model which considers the battery energy of the wireless nodes, the multi-hop transmission distance and the geographical distance between wireless nodes. In order to quantify the energy efficiency of the proposed model, we propose a new energy efficiency metric which jointly considers: 1) the energy efficiency of the wireless links, and 2) the impact of a wireless node on the entire network. By adopting the proposed metric, we evaluate the energy efficiency of each wireless node and the entire network. Our results show that the proposed model: 1) follows the small world feature, and 2) is more energy efficient compared to a random network.**

***Index Terms*—Ad hoc networks, energy efficiency, small world networks.**

## I. INTRODUCTION

A WIRELESS ad hoc network is a decentralized type of wireless network. The concept of small world can be introduced to wireless networks, typically to reduce the path length, and in turn provide better capacity and end-to-end delay.

Small world networks are one of the two classes of complex networks. Fundamentally, the two main models used in small world network modeling are Watts and Strogatz (WS) [1] and Newmann-Watts (NW) model [2]. In the WS model, a small world network is constructed via rewriting a few links in an existing regular network (such as a ring lattice graph) [1]. In the NW model [2], a small world network is constructed by adding a few new links without rewriting existing links. The newly added links are named as shortcuts.

The construction of small world ad hoc networks using the WS model is typically impractical due to excessive rewriting of existing wireless links [3]. As such, the NW model is the more popular option [3-6]. In 2003, Helmy [4] discussed the influence of shortcuts in wireless networks, e.g., ad hoc or wireless sensor networks. The author in [4] observed that when long-range edges are introduced to a spatial graph, the average path length of the wireless network represented by the spatial graph is dramatically reduced. Indeed, this is a feature of a small world network. Several works have addressed the question of how to construct a wireless network topology in such a way that the small world feature is preserved. In tackling this, [3] added shortcuts to the wireless mesh network taking into account practical constraints, such as radio transmission range of nodes, numbers of radios per mesh router, and available bandwidth of wireless links. Long range shortcuts can be created by either adding wired links [5] or directional beamforming [6]. The authors in [7] proposed a multicast reprogramming protocol for wireless sensor networks based on small world concepts. Despite the promised gains of the small world feature, energy efficiency in small world ad hoc networks has not been investigated and remains an open problem. In designing the ad hoc network, we focus on energy efficiency as the preferred metric, which gives a tradeoff between energy consumption and network performance.

In this letter, we propose an energy efficient small world network model based on the NW model. In doing so, we consider the battery energy of the wireless nodes, the multi-hop transmission distance and geographical distance between the wireless nodes.

Generally speaking, energy efficiency metrics are used for evaluating energy savings or measuring energy efficiency of the networks. However, there is no common accepted energy efficiency metrics [8]. Broadly speaking, the energy efficiency metrics in the literature can be classified into three main categories according to the facility level, equipment level and network level [9-10]. These metrics however do not transparently reveal the interrelation between the energy efficiency of the single node and that of the entire network.

In this letter, we identify the key network parameters that determine the relationship between the energy efficiency of the single node and the energy efficiency of the entire network. To do so, we propose a new energy efficiency metric which jointly considers: 1) the energy efficiency of the wireless links, and 2) the impact of a wireless node on the entire network. The proposed metric conveniently evaluates the energy efficiency of each wireless node and that of the entire network. We show that the network topology plays a key role in determining the energy efficiency. Indeed, networks with same scale but different topologies have different energy efficiencies since each wireless node has a different impact on the entire network.

## II. PROPOSED SMALL WORLD NETWORK MODEL

We propose an energy efficient small world network model for ad hoc networks with the network topology following the small world feature. In doing so we adopt the NW model.

### A. *Small world feature and NW model*

If we model a network by graph, we can use the concepts of average path length and clustering coefficient of the graph to

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characterize the network. A small world network is characterized by a high clustering coefficient similar to that of regular networks and a low average path length, similar to that of random networks [1-2].

In an unweighted graph $\mathbf{G}_1(\mathbf{V}, \mathbf{E})$ with *N* vertices, the *i*th vertex is denoted as $v_i\,(v_i \in \mathbf{V})$, the edge of vertex $v_i$ and $v_j$ is $e_{ij}\,(e_{ij} \in \mathbf{E})$ , and the degree of $v_i$ is $k_i$. The average path length of a graph represents the connection status of the entire network. The average path length $L$ of $\mathbf{G}_1$ is

$$L = \frac{1}{N\,(N-1)} \sum_{i \neq j} l_{ij}, \tag{1}$$

where $l_{ij}$ denotes the shortest path length from vertex $v_i$ to $v_j$. A network with a low average path length would have better connectivity and performance.

Clustering coefficient is the fraction of nodes neighbors that are neighbors of each other. Clustering relates to the structure of the network. Suppose that $v_i$ in the graph has $k_i - 1$ edges, which means $v_i$ connects to $k_i - 1$ other nodes. These nodes are all neighbors of $v_i$ . Clearly, at most edges can exist among these $k_i$ nodes, and this occurs when every node connect to each other, that is, these $k_i$ nodes form a group coupled graph. The clustering coefficient of vertex $v_i$ is

$$C_i = \frac{2\varsigma}{k_i\,(k_i - 1)}, \tag{2}$$

where $\varsigma$ is the number of edges that actually exist among these $k_i$ nodes. The clustering coefficient *C* of $\mathbf{G}_1$ is

$$C = \frac{1}{N} \sum_{i=1}^{N} C_i. \tag{3}$$

The process of modeling a small world network by NW model is as follows.

1) Network initialization: One-dimensional lattice of *N* vertices with periodic boundary conditions (i.e., on a ring). Then join each vertex to its *K* neighbors, where *K* is even and is the initial degree of each vertex.

2) Adding shortcuts: Based on the existing lattice, adding a shortcut between each vertex pair with probability *p* [2]. It is not allowed to couple a node to another node more than once or to couple with itself.

Setting *p*=0, the network reduces to the original nearest-neighbor coupled network, and setting *p*=1 network reduced to a globally coupled network. We note that the condition of *N* vertices on a ring is only for illustration purpose to demonstrate the relationship between the nodes, and is not a necessary requirement in practical networks.

### B. Proposed model

We consider a wireless ad hoc network with N wireless nodes distributed in a geographical area, e.g., the mobile ad hoc networks for inter-vehicle communication constructed by vehicles on the road.

We use a spatial graph $\mathbf{G}_2(\mathbf{V}, \mathbf{E})$ whose nodes have geographical locations to represent the wireless network. The vertex $v_i\,(v_i \in \mathbf{V})$ represents the wireless node and the edge $e_{ij}\,(e_{ij} \in \mathbf{E})$ is the link from the transmitter node $v_i$ to receiver node $v_j$. If $v_i$ and $v_j$ has direct link, $e_{ij} = 1$, otherwise, $e_{ij} = 0$.

In small world network modeling, the design of the connection probability $p$ is the key player for energy efficiency. In the proposed model, the connection probability is defined by considering the battery energy of nodes and the multi-hop transmission distance and geographical distance between nodes, meanwhile, taking the constraints on the radio transmission range between nodes into account.

First, we assign a larger connection probability to the vertex pair with larger multi-hop transmission distance since small world networks have the feature of low average path length. However, considering the limitation of the radio transmission range between nodes, only the vertex pair whose geographical distance is in the radio transmission range of the transmitter can have the shortcut added.

Second, we assign a smaller connection probability to the node with the larger degree. In the NW model, the node with the larger degree has more link connections. However, in practical ad hoc networks, the node with more link connections has a higher chance to get routed as a relay for data transmission. This implies that the node with the larger degree will have higher energy consumption. Considering the battery energy constraint of wireless nodes, the connection probability is inversely proportional to the degree of nodes. As such, the connection probability $p_{ij}$ of vertex $v_i$ and $v_j$ is

$$p_{ij} = \begin{cases} \min\left(1, p_0 \frac{\frac{1}{N}\sum\limits_i k_i}{\sqrt{k_i k_j}} \frac{d_h^{(i,j)}}{d_{ij}}\right), \text{if} d_{ij} < d_s \\ 0, \text{if} d_{ij} \geq d_s \end{cases} \tag{4}$$

where $p_0$ is the initial connection probability, $k_i$ is the degree of $v_i$ , $d_{ij}$ is the geographical distance between node $v_i$ and $v_j$, is the multi-hop transmission distance from $v_i$ to $v_j$ by $h$ hops through the network, and $d_s$ is the radio transmission range of the wireless node with a constraint on maximum transmission power. If $d_{ij} \geq d_s$, then vertex $v_i$ and $v_j$ are not connected, and therefore $p_{ij} = 0$. Note that $p_{ij}$ increases with increasing $d_h^{(i,j)}$, which in turn reduces the average path length and improve the efficiency of data transmission over the network. On the other hand, transmissions with larger geographical distance have lower transmission energy efficiency. In this case, we should reduce $p_{ij}$ for larger $d_{ij}$, which indicates that multi-hop transmission is more energy efficient. As such, the normalized distance $d_h^{(i,j)} \big/ d_{ij}$ can be seen as a tradeoff between energy efficiency and data transmission efficiency in the connection probability design.

As shown in (4), the effect of battery energy constraint on the connection probability is considered such that the connection probability $p_{ij}$ is smaller for larger $k_i k_j$. This is reflected by$(k_i k_j)^{-1/2}$ with the normalization of the average degree of all *N* vertices $\frac{1}{N}\sum\limits_i k_i$.

The process of constructing wireless ad hoc network by proposed model is as follows.

1) Network initialization: Initial network topology with *N* wireless nodes and each node has links with its *K* neighbors within the radio transmission range.

2) Adding shortcuts: Based on the existing network, adding a shortcut for each transmission pair $(v_i, v_j)$ with connection probability $p_{ij}$ calculated using (4).

The process of network construction is executed by a centralized controller which has the information of the average degree of all the nodes and the minimal geographical distance from any source to its destination.

## III. Proposed Energy Efficiency Metric

In this letter, the energy efficiency metric of the wireless nodes considers two main components: 1) the energy efficiency of a single hop link from a transmitter to its next hop receiver, and 2) the impact of a node on the entire network energy efficiency.

Here, we update the spatial graph $\mathbf{G}_2(\mathbf{V}, \mathbf{E})$ with weighted links. The weight of the edge $e_{ij}$ represents the energy efficiency of the single hop link from $v_i$ to $v_j$, denoted as $G_{ij}$. The energy efficiency of the single hop link is defined as the ratio between single hop data rate and the energy cost. The data rate from $v_i$ to $v_j$ is $r_{ij}$ (the unit is bit/s), and the energy cost is $w_{ij}$ (the unit is joule/s), therefore $G_{ij} = r_{ij}/w_{ij}$ and its unit is bit/joule. However, such a simple metric cannot reveal the impact of the network topology on the network energy efficiency, and the interrelation between the energy efficiency of a single wireless node and that of the entire network.

Motivated by the above, we consider the impact of a node on the entire network energy efficiency, by taking into account: 1) the degree of a node, and 2) the geographical distance between a node and its neighbors. The energy efficiency of a specific wireless node should have such feature: 1) proportional to the degree of the node, which means that the node with larger degree is more important to the connectivity of the network, and 2) inversely proportional to the geographical distance between the nodes with minimum distance limitation. In this manner, the definition of the energy efficiency metric follows the Newton's law of universal gravitation.

Borrowing tools from Newton's law of universal gravitation, we present the gravitation of the wireless link between a node and its neighbor. This is used to evaluate the impact of a node on the entire network. The gravitation of the wireless link between node $v_i$ and $v_j$ is defined as

$$F_{ij} = G_{ij}\frac{k_i k_j}{(d_{ij}+d_0)^2}, j \in \mathbf{E}_i, \tag{5}$$

where$k_i$ is the degree of $k_i$, $\mathbf{E}_i$ is the set of the neighbor node of $k_i$, $d_{ij}$ is the geographical distance between $v_i$ and $v_j$, and $d_0$ is the minimum distance between nodes which should be larger than the reference distance of the far-field of antenna.

Then the energy efficiency metric of a single node is defined as the sum of the gravitation of wireless links between the node and all its neighbor nodes. The energy efficiency metric of node $v_i$ is defined as

$$F_i = \sum_j F_{ij}, j \in \mathbf{E}_i. \tag{6}$$

A node with a larger $F_i$ indicates that the node has stronger impact on the energy efficiency of the entire network. Based on (5), we obtain the following observations.

1) The node with larger degree has a stronger impact on the energy efficiency of the entire network.

2) If two nodes have the same degree, the node whose neighbor node has a larger degree has a stronger impact on the energy efficiency of the entire network.

3) For a wireless communication pair $(v_i, v_j)$ and $(v_{i'}, v_{j'})$, if $k_i = k_{i'}$ and $k_j = k_{j'}$, then the pair with the smaller geographical distance has a higher energy efficiency and has a stronger impact on the energy efficiency of the entire network.

4) The edge of the single hop link $F_{ij}$ with larger energy efficiency is a key player to the energy efficiency of the entire network.

5) The gravitation is asymmetric, i.e., $F_{ij} \neq F_{ji}$ when the transmission data rate is $r_{ij} \neq r_{ji}$.

Using the proposed energy efficiency metric, we obtain a relationship for the energy efficiency between the wireless nodes and the entire network.

## IV. Simulation Results

In this section, we present the simulation results of the proposed small world network model and energy efficiency metric. The simulation is realized using MATLAB. The number of the wireless nodes N is equal to 200. We assume that these nodes are distributed randomly in a field of one square kilometers. We set the minimum distance between two nodes $d_0$ equals 40 meters, and maximum transmission range $d_s$ is equal to 400 meters. We use the shortest path routing for the end-to-end path selection [11].

In the network initialization stage, we set initial degree of each node to 6. The initial connection probability $p_0$ is varying in the simulation. We obtain the average value of the results from 100 simulations.

When constructing the network topology, the average path length is $L(p)$ and the clustering coefficient is $C(p)$. Here, we set $L(0)$ and $C(0)$ as the average path length and the clustering coefficient of the initial topology, respectively, for $L(p)$ and $C(p)$ normalization.

The small world properties of the proposed small world model for ad hoc network are verified by simulation firstly. Figure 1 is the normalized average path length and normalized clustering coefficient varying with $p_0$. We see that when $p_0$ is greater than 0.1, the normalized clustering coefficient is greater than 1, which means that the clustering coefficient of proposed model is larger than that of the initial network. At the same time, the normalized average path length is 0.4, which means that average path length of proposed model is smaller than that of the initial network. This results show that, the proposed model has the feature a small average path length along with a large clustering coefficient.

Next, we evaluate the energy efficiency of the proposed model using the proposed energy efficiency metric. For comparison purposes, we define a network which has the same number and distribution of wireless nodes as the proposed model. In the compared network, the links of wireless nodes are added randomly only considering the maximum transmission range constraint (such network is named as random network). For a fair comparison, we set the number of wireless links in the random network as that of the proposed model. The proposed model and the random network both have the same network scale but different network topologies, thus still maintain a fair comparison.

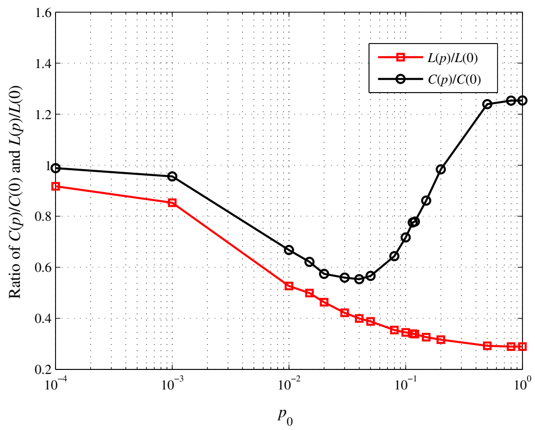


Fig. 1. Normalized average path length and normalized clustering coefficient varying with $p_0$ (Small world properties verification).

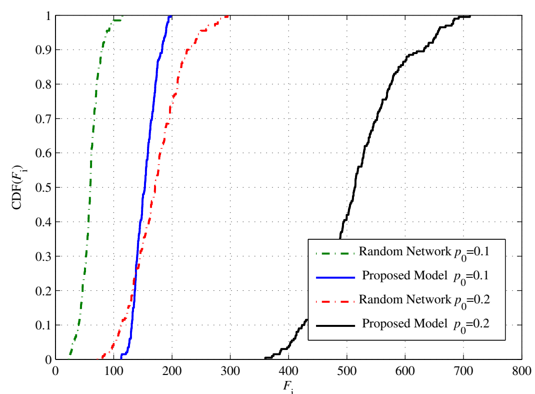


Fig. 2. The CDF of the energy efficiency of wireless nodes (Energy efficiency evaluation).

Figure 2 shows the cumulative distribution function (CDF) of the energy efficiency of wireless nodes. The black line is the proposed model with $p_0 = 0.2$ and the red line is the random network which has the same network scale as the proposed model. The blue line and the green line have the same case with $p_0 = 0.2$ . Our results confirm that networks with different topologies have different energy efficiencies and the proposed model is more energy efficient than that of the random network with the same network scale.

Meanwhile, energy efficiency gains increase as the connection probability increases. This is due to the fact that the network has a better connectivity with a larger initial connection probability.

## V. Conclusion

In this letter, we propose a new small world network model for energy efficient wireless ad hoc network and a new energy efficiency metric of the wireless nodes in the wireless networks. The energy efficiency of the proposed small world network is evaluated by the proposed energy efficiency metric. The proposed model can be used for designing an energy efficient ad hoc network which is consistent with the practical network topology. The proposed metric provides information in order to directly compare and assess the network energy efficiency, especially in term of the impact of wireless nodes on the entire network. We should notice that the proposed energy efficiency metric can be applied to various wireless networks.

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