Green Power Control in Cognitive Wireless Networks

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***Abstract*—A decentralized network of cognitive and noncognitive transmitters where each transmitter aims at maximizing his energy efficiency is considered. The cognitive transmitters are assumed to be able to sense the transmit power of their noncognitive counterparts and the former have a cost for sensing. The Stackelberg equilibrium analysis of this two-level hierarchical game is conducted, which allows us to better understand the effects of cognition on energy efficiency. In particular, it is proven that the network energy efficiency is maximized when only a given fraction of terminals are cognitive. Then, we study a sensing game where all the transmitters are assumed to take the decision of whether to sense (namely to be cognitive) or not. This game is shown to be a weighted potential game, and its set of equilibria is studied. Playing the sensing game in a first phase (e.g., of a time slot) and then playing the power control game is shown to be more efficient individually for all transmitters than playing a game where a transmitter would jointly optimize whether to sense his power level, showing the existence of a kind of Braess paradox. The derived results are illustrated by numerical results and provide some insights on how to deploy cognitive radios in heterogeneous networks in terms of sensing capabilities.**

***Index Terms*—Cognitive wireless networks, game theory, wireless ad hoc networks.**

I. Introduction

In FIXED communication networks, the paradigm of peer-to-peer communications has seen a powerful surge of interest during the past two decades with applications such as the Internet. Remarkably, this paradigm has also been found to be very useful for wireless networks. Wireless ad hoc and cognitive networks are two illustrative examples of this. One important typical feature of these networks is that the terminals have to take some decisions in an autonomous or quasi-autonomous manner. Typically, they can choose their power control (PC) and resource allocation policy. The corresponding framework, which is discussed in this paper, is the one of decentralized or distributed PC or resource allocation. More specifically, the scenario of interest is the case of PC over quasi-static channels in cognitive networks [13]. In such a context, which is broader than that of ad hoc and cognitive wireless networks, we assume that some (possibly all) transmitters are able to sense the power levels of noncognitive transmitters and adapt their power level accordingly. The considered model of multiuser networks is a multiple access channel (MAC) with time-selective nonfrequency selective links, but the methodology can be applied to other types of interference networks. Technical issues related to spectrum usage are not considered in this paper, leaving this aspect as a relevant extension of this paper. Rather, we want to study the effect of cognition in terms of energy usage, the potential benefits in terms of spectral efficiency having been well investigated. The selected performance metric for a transmitter is derived from the energy efficiency definition of [15]. The authors of [15] define energy efficiency as the number of bits successfully decoded by the receiver per joule consumed at the transmitter (in [15] the radiated power is concerned). More specifically, the authors analyze the problem of decentralized PC in flat-fading MACs. The problem is formulated as a noncooperative one-shot game where the players are the transmitters, the action of a given player is his transmit power, and his payoff/reward/utility function is the energy efficiency of his communication with the receiver; we will not provide here the motivations for using game theory to study distributed PC problems, but some of them can be found, e.g., in [22]. The results reported in [15] have been extended to the case of multicarrier systems in [10].

The framework of the present paper is close in spirit to [10] and [15] but differs from them in several aspects. The most important one is that there can be a hierarchy among the transmitters in terms of observation capabilities, i.e., some transmitters can be cognitive and observe others, whereas the latter cannot observe the actions played by the former. Technically, this leads to a Stackelberg-type formulation of the problem [33]. The closest work to the one reported here is [21], where a Stackelberg model of energy-efficient PC problems is introduced for the first time. The present paper reports a significant extension of the framework introduced in [21]. Two games are studied in detail. The PC game corresponds to a generalization of the game addressed in [21]. The sensing costs are taken into account (observing/sensing the others has a cost), and more importantly, our analysis is not limited to one noncognitive transmitter (i.e., a single game leader). Then, we introduce a new game where the transmitters decide whether to sense or not. A third game, which is a hybrid control game and includes the two mentioned games as special cases, is shown to be not worth being studied because of the existence of a Braess paradox [7].

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This paper is organized as follows. In Section II, the assumed signal model to describe the distributed PC problem over time-selective nonfrequency-selective MACs is provided. Known results concerning the case where the transmitters tune their power levels from block to block in a distributed way and without observing the other transmitters (i.e., they cannot sense the powers chosen by the others) are provided. In Section III, we assume that some transmitters have sensing capabilities, which creates a hierarchy in terms of observation capabilities between the transmitters. The effect of this is that choosing rational PC policies in this setting leads to a more efficient network outcome [a Stackelberg equilibrium (SE)] provided that the sensing cost for a cognitive transmitter is not too high. While in Section III a transmitter was imposed to sense or not, it is assumed in Section IV that this choice is left to the transmitter itself. It is shown that there exists an optimal number of cognitive transmitters in terms of network utility, and therefore, having too many advanced terminals can be detrimental to global performance. It is shown that leaving the choice to a transmitter to choose in a joint manner its power level and whether to sense is in fact less energy efficient than imposing that the transmitters choose these two quantities separately. This shows the interest in studying the PC game (as in Section III) and the sensing game separately. The sensing game is a new game we introduce and is shown to possess attractive properties for distributed optimization and learning algorithms. Finally, in Section V, numerical illustrations are provided, and this paper is concluded in Section VI.

II. Problem Statement

A. System Model

We consider a decentralized MAC with a finite number of transmitters, which is denoted by K. The network is said to be decentralized in the sense that the receiver (e.g., a base/mobile station) does not dictate to the transmitters (e.g., mobile/base stations) their PC policy. Rather, all the transmitters choose their policy by themselves and want to selfishly maximize their energy efficiency. In particular, they can ignore some specified centralized policies. We assume that the users transmit their data over time-selective nonfrequency-selective channels at the same time and on the same frequency band; channels are considered to be constant over each block of data. Note that a block is defined as a sequence of M consecutive symbols that comprise a training sequence that is a certain number of consecutive symbols used to estimate the channel (or other related quantities) associated with a given block. A block has therefore a duration less than the channel coherence time. The equivalent baseband signal at the receiver can be written as

$$y(t) = \sum_{k=1}^{K} h_k x_k(t) + z(t) \tag{1}$$

where $k \in \mathcal{K}$, $\mathcal{K} = \{1, \ldots, K\}$, $x_k(t)$ represents the symbol transmitted by transmitter k at time $t \in \mathbb{N}$, $\mathbb{E}|x_k|^2 = p_i$, the noise z is assumed to be decentralized according to a zero-mean Gaussian random variable with variance σ^2, and each channel gain h_k varies over time but is assumed to be constant over each block; the symbol index t will be omitted in this paper. In terms of channel state information (CSI), the receiver is assumed to know all the channel gains (global CSI), whereas each transmitter only knows his own channel (local CSI). For each block, the expression of the receive signal-to-interference-plus-noise ratio (SINR) of user k is given by

$$\gamma_k = \frac{g_k p_k}{\sigma^2 + \sum\limits_{j \neq k}^{K} \theta_j g_j p_j} \tag{2}$$

where for all $j \in \mathcal{K}$, $g_j = |h_j|^2$, and θ_j represents a parameter depending on the interference scenario. For example, in a random code-division multiple-access (CDMA) system with single-user decoding, we would have $\theta_j = (1/N)$, where N is the spreading factor [10]. This is the choice we will do. Nonetheless, note that the present paper is not restricted to CDMA systems. Indeed, by choosing $\theta_j = 1$ (i.e., $N = 1$), the foregoing signal model corresponds to the information-theoretic channel model used for studying MACs [8], [34]; in this setup, good channel codes are assumed (see, e.g., [5] for more comments on the multiple access technique involved). Indeed, what matters the most in the model is that it captures the different aspects of the problem (especially the SINR structure). At last, the case where successive interference cancelation is used at the receiver ($\theta_j \in \{0, 1\}$, depending on the decoding order) is left as an extension of the present paper.

B. Performance Metric

Assuming the foregoing signal model, we assume that each transmitter wants to selfishly maximize the energy efficiency of his communication with the receiver. The used performance metric is the one originally proposed in [15]. For a given block, transmitter k wants to maximize the following quantity:

$$v_k(p_k, \mathbf{p}_{-k}) = \frac{R_k \mathsf{f}(\gamma_k)}{p_k} \text{ [bit/J]} \tag{3}$$

where R_k is the transmission rate (in bits per second), f is an efficiency function (representing the block success rate), and the subscript $-k$ on vector $\mathbf{p}$ stands for "all the transmitters except transmitter k," i.e., $\mathbf{p}_{-k} = (p_1, \ldots, p_{k-1}, p_{k+1}, \ldots, p_K)$. Note that, as a standard assumption, R_k is assumed to be independent of γ_k or p_k, which may correspond in practice to a given choice of modulation coding scheme. As motivated in [4] and [30], the efficiency function is assumed to be an increasing and sigmoidal (or S-shaped) function verifying $0 \leq \mathsf{f}(.) \leq 1$ with $\mathsf{f}(0) = 0$ and $\lim_{x \to +\infty} \mathsf{f}(x) = 1$. The fact that f is sigmoidal has at least two important consequences: 1) the utility function v_k is quasi-concave w.r.t. p_k, and 2) the derivative of v_k vanishes at only one point that is different from 0. We see that R_k might be chosen to be SINR dependent without affecting the problem analysis provided that the product $R_k \mathsf{f}$ be a sigmoidal function. For the sake of clarity, we assume that the players have the same efficiency function f. In [15] and related works, p_k represents the power radiated by the transmitter. Interestingly, the preceding utility can also model situations where the power consumed by the whole transmitting device has to be accounted for. Indeed, by replacing the denominator of u_k by $ap_k + b$, with (a, b) being a pair of nonnegative constants, one obtains a first-order model of the device power consumption, which includes both the consumption part that does not depend on the radiated power and the one due to the transmit power [12]. This does not change significantly the mathematical analysis of the PC problem. To focus our attention on the most important points of our analysis and make the exposition as clear as possible, the original model of [15] has been selected (i.e., $(a, b) = (1, 0)$).

C. Game-Theoretic Modeling: Review of the Noncooperative Game of [15]

An appropriate model for the PC problem described above is given by a strategic form game [15]. A strategic form game consists of an ordered triplet comprising the set of players, their action or strategy sets, and their preference orders (or their utilities when they exist, which is the case here). The set of players is the set of transmitters $\mathcal{K}$, the action set is $\mathcal{P}_k = [0, P_k^{\max}]$, $k \in \mathcal{K}$, and the utility functions are defined by (3). This describes the model introduced by Goodman and Mandayam [15]. As in [15] and related references such as [10], the power levels are chosen to be continuous. This allows us to conduct a complete comparison analysis in terms of performance. However, this assumption is not always suitable, and the case of discrete power levels is therefore left as a complementary way of tackling the problems under investigation. An important solution concept for this game is the Nash equilibrium[1] (NE) [27], which is a power profile/vector that is robust against unilateral deviations (no player has interest in deviating if the others keep the equilibrium strategy). The unique NE of this PC game is

$$p_k^{\mathrm{NE}} = \frac{\sigma^2}{g_k} \frac{\beta^\star}{1 - \frac{K-1}{N}\beta^\star}, \; k \in \mathcal{K} \tag{4}$$

where $\beta^\star$ denotes the best SINR choice for user k at the NE; as explained in [10] and [15], a necessary condition for this equilibrium to be defined is that the system load is not too high ($(K - 1/N)\beta^\star < 1$). Note that the equilibrium SINR is common to all users. It is easy to verify that $\beta^\star$ is the positive solution of the differential equation $x\mathsf{f}'(x) - \mathsf{f}(x) = 0$, which is obtained by solving $\max_{x \neq 0}(\mathsf{f}(x)/x)$, i.e., an equivalent problem of $\max_{p_k} v_k$ (following from the assumption that R_k is independent of γ_k and p_k).

The equilibrium solution holds if the power constraint $p_k \leq P_k^{\max}$ is satisfied, which is what we will assume throughout this paper (see, e.g., [15] for further details about the case where the constraint is active). This game model, although leading to a decentralized solution in terms of decisions and CSI (see [15]), has one main drawback: the equilibrium solution can be inefficient. Interestingly, introducing some hierarchy between the players in terms of observation can improve the game outcome, as shown in [21]. It turns out that hierarchy is naturally present in networks where some transmitters are equipped with a cognitive radio while the others are not. This is one of our motivations for formulating the problem in decentralized cognitive networks as a two-level Stackelberg game, with arbitrary numbers of cognitive radios, generalizing the one-leader $K-1$-follower game of [15]. Compared to the latter reference [15], a second interesting feature of the game described below is that the cost induced by sensing is accounted for in the utility function of the cognitive transmitters. The proposed approach may be relevant in most applications where cognitive radio is useful. Indeed, one of the messages of this paper is that if the fraction of transmitters who can observe their environment is too high, this may degrade the global performance. To mention an existing scenario where this type of approach might be applied in the future, the case of WiFi systems can be mentioned. In France, operators provide more and more advanced access points (APs). Typically, they want to optimize channel selection (which is a special case of power allocation) in a more and more efficient manner. Assuming that some APs are optimized according to Nash strategies while others implement Stackelberg strategies allows one to provide a simplified model to account for the fact that advanced APs coexist with less advanced APs. Interestingly, as shown in this paper, as far as PC is concerned, having too many advanced APs might not be as good as common sense would indicate.

III. Two-Level Power Control Game With Sensing Costs

The set of transmitters $\mathcal{K} = \{1, 2, \ldots, K\}$ comprises F terminals equipped with a cognitive radio, whereas the $L = K - F$ other terminals have no sensing capabilities. The pair (F, L) is assumed to be fixed throughout the whole section; it will be optimized in a centralized (respectively, decentralized) manner in Section IV-A (respectively, Section IV-B). Without loss of generality, the set of noncognitive (respectively, cognitive) terminals will be $\mathcal{L} = \{1, 2, \ldots, L\}$ (respectively, $\mathcal{F} = \{L+1, L+2, \ldots, K\}$). This two-level hierarchical game is played as follows. For each block, the noncognitive transmitters (called the leaders) choose their power level rationally knowing that their decisions are going to be observed by the cognitive transmitters. The cognitive transmitters (called the followers) react to these decisions rationally. A choice is said to be rational in the sense that the transmitter maximizes his utility. To this end, we denote by $\mathbf{p}_{\mathcal{L}} \triangleq (p_1, \ldots, p_L)$ and $\mathbf{p}_{\mathcal{F}} \triangleq (p_{L+1}, \ldots, p_K)$ the vectors of actions (transmit powers) of the leaders and followers, respectively. Moreover, denote by $\mathcal{U}^\star(\mathbf{p}_{\mathcal{L}})$ the set of NE for the group of followers when the leaders play $\mathbf{p}_{\mathcal{L}}$. The resulting outcome of this interaction is an SE, which is defined as follows.

Definition 3.1 (SE): A vector of actions $\mathbf{p}^{\mathrm{SE}} = (\mathbf{p}_{\mathcal{L}}^{\mathrm{SE}}, \mathbf{p}_{\mathcal{F}}^{\mathrm{SE}})$ is called an SE if $\mathbf{p}_{\mathcal{F}}^{\mathrm{SE}} \in \mathcal{U}^\star(\mathbf{p}_{\mathcal{L}}^{\mathrm{SE}})$ and the action $\mathbf{p}_{\mathcal{L}}^{\mathrm{SE}}$ is an NE for the leaders.[2]

[1] Appendix A reviews several game-theoretic notions. Note that the unconditional existence of a pure NE is, in part, a consequence of quasi-concavity for the utility functions.

[2] We assume, w.l.o.g., two players in a noncooperative game with one leader and one follower. If the leader plays the NE action, then as the follower observes this action and plays the best response against it, the follower will play the NE strategy. Then, the NE strategy profile, if it exists, can be an SE. The behavior is the same when there are several leaders and followers. If the NE between the leaders corresponds to the NE of the game between leaders and followers, then the followers respond by playing the NE.

By looking at the mathematical expression of the SE defined above, we can see that if the NE exists, then the SE also exists. However, the SE is not included in the set of NEs of a noncooperative game. There exist several examples like the Cournot game, in which the action chosen by the leader at the SE is different compared to the action chosen at the SE [14]. As the best response of each player is a scalar-valued function (see [21]), the determination of the Stackelberg equilibria of the game amounts to solving the following bilevel optimization problem:

$$p_\ell^{\mathsf{SE}} \in \arg\max_{p_\ell} u_\ell\left(p_\ell, \mathbf{p}_{-\ell}^{\mathsf{SE}}, p_{L+1}^{\mathsf{SE}}\left(p_\ell, \mathbf{p}_{-\ell}^{\mathsf{SE}}\right), \ldots, p_K^{\mathsf{SE}}\left(p_\ell, \mathbf{p}_{-\ell}^{\mathsf{SE}}\right)\right), \ \forall \ell \in \mathcal{L} \quad (5)$$

where for all $\mathbf{p}_\mathcal{L}$

$$p_f^{\mathsf{SE}}(\mathbf{p}_\mathcal{L}) = \arg\max_{p_f} u_{\mathsf{f}}\left(\mathbf{p}_\mathcal{L}, p_{L+1}^{\mathsf{SE}}(\mathbf{p}_\mathcal{L}), \ldots, p_{f-1}^{\mathsf{SE}}(\mathbf{p}_\mathcal{L}), p_f, \ldots, p_{f+1}^{\mathsf{SE}}(\mathbf{p}_\mathcal{L}), \ldots, p_K^{\mathsf{SE}}(\mathbf{p}_\mathcal{L})\right), \ \forall f \in \mathcal{F} \quad (6)$$

where the utility functions are given by

$$u_k(\mathbf{p}) = \left| \begin{array}{ll} v_k(\mathbf{p}) & \text{if } k \in \mathcal{L}, \\ (1-\alpha_k)v_k(\mathbf{p}) & \text{if } k \in \mathcal{F}. \end{array} \right. \quad (7)$$

The parameter $\alpha_k \in [0, 1]$, $k \in \mathcal{F}$, is a constant w.r.t. the power levels that account for the sensing cost (to be illustrative, we will choose $\alpha_k = \alpha$ in some places). This constant has no effect on the equilibrium strategies. However, when it will come to knowing whether being a follower or not, this constant will play a role. To elaborate further on this constant, it can be interpreted as the fraction of time a cognitive user $k \in \mathcal{F}$ spends for sensing. To have good sensing capabilities,[3] we assume that there exists a certain energy threshold $\xi_{\min}$ (see, e.g., [17]) expressed in joule

$$\alpha_k T \min_{\ell \in \mathcal{L}}(g_{k\ell} p_\ell) \geq \xi_{\min}$$

where T is the block duration in second and p_ℓ in watts, whereas α_k and $f(.)$ are unitless; $g_{k\ell}$ is the channel gain between any leader $\ell \in \mathcal{L}$ and the considered follower $k \in \mathcal{F}$. If the foregoing inequality holds, it means that the cognitive user $k \in \mathcal{F}$ is able to sense the presence of the primary ones. Apparently, we assume that the sensing constraint is feasible in the sense that there exists a minimum fraction $\alpha_k \leq 1$, $k \in \mathcal{F}$, above which the minimum energy threshold for sensing is attained. A necessary condition for this is that $T \ \min_{\ell \in \mathcal{L}}(g_{k\ell} p_\ell) \geq \xi_{\min}$. For the sake of clarity, we suppose that the sensing cost is the same for every player $\alpha_k = \alpha$, $k \in \mathcal{F}$. At this point, the two-level hierarchical PC game is completely defined: the players are the L noncognitive transmitters and the F cognitive transmitters, their action sets are $[0, P_k^{\max}]$, and their utilities are defined by (7).

[3]Under the assumption of single-user decoding (each useful signal is detected by considering the other signals as noise), a good sensing capability means that a follower can detect the existence of all the leaders. In particular, the leader whose link with the follower is the worst is detected.

Following the standard methodology of equilibrium analysis (see, e.g., [20], [22]), three important issues to be dealt with are the existence, uniqueness, and efficiency issues for the SE. The next theorem provides an element of response to the first two issues.

Proposition 3.1 ([21]): There always exists an SE $\mathbf{p}^{\mathsf{SE}}$ in the two-level hierarchical game with $L \geq 1$ leaders and $F \geq 1$ followers. The power profile defined by

$$\forall \ell \in \mathcal{L}, \ p_\ell^{\mathsf{SE}} = \frac{\sigma^2}{g_\ell} \frac{N\gamma_L^\star(N+\beta^\star)}{N^2 - N(F-1)\beta^\star - [(N+\beta^\star)(L-1) + F\beta^\star]\gamma_L^\star}$$

$$\forall f \in \mathcal{F}, \ p_f^{\mathsf{SE}} = \frac{\sigma^2}{g_f} \frac{N\beta^\star(N+\gamma_L^\star)}{N^2 - N(F-1)\beta^\star - [(N+\beta^\star)(L-1) + F\beta^\star]\gamma_L^\star}$$

is an SE, $\beta^\star$ is the positive root of $xf'(x) = \mathsf{f}(x)$, and $\gamma_L^\star$ is the positive root of $x(1-\epsilon_L x)f'(x) = \mathsf{f}(x)$, with

$$\epsilon_L = \frac{F\beta^\star}{N^2 - N(F-1)\beta^\star}. \quad (8)$$

Moreover, the equilibrium $\mathbf{p}^{\mathsf{SE}}$ is unique if the following two conditions hold: 1) $\lim_{x\to 0^+}(f''(x)/f'(x)) > 2\epsilon_L$; and 2) equation $x(1-\epsilon_L x)f'(x) - \mathsf{f}(x) = 0$ has a single root in $(0, \beta^\star)$.

Note that the existence of such an equilibrium is ensured from the properties of the Stackelberg game, especially the sigmoidness of f [21]. In [21], it is also explained that the best response of the players are scalar-valued functions, which facilitate the SE analysis. Interestingly, the provided sufficient conditions for uniqueness can be checked to be satisfied for two typical efficiency functions used in the related literature, namely, $\mathsf{f}(x) = (1-e^x)^M$ and $\mathsf{f}(x) = e^{-(c/x)}$, $c \geq 0$, used in [15] and [4], respectively.

Last but not least, we address the issue of efficiency for the derived SE. The key point at stake is whether cognition helps to obtain a better decentralized network in terms of global energy efficiency. For this purpose, we first compare the utility a player would get in a system where no cognitive transmitters exist with the one he would obtain in a system with cognitive transmitters (i.e., at the NE corresponding to [15]). Our main results are summarized by the following proposition.

Proposition 3.2 (SE Versus NE): The utility at the SE of the two-level hierarchical game with sensing cost of any leader is always greater or equal to the one obtained at the NE.

If the cost for sensing is negligible, the next corollary follows.

Corollary 3.3 (SE Versus NE With No Sensing Cost): The power profile at the SE Pareto dominates the power profile at the NE.

The proof of Proposition 3.2 and Corollary 3.3 is given in [21]. Another relevant question, initially raised in a context with a single leader and no sensing cost [21], is whether it is better to follow or lead the PC game. Said otherwise, is it beneficial for a transmitter to be equipped with a cognitive radio when sensing costs are accounted for? The answer is provided below.

Proposition 3.4 (Following Versus Leading): At the SE of the two-level PC game with sensing cost, a transmitter prefers to be a follower (that is to say, to sense) if the minimum energy threshold for sensing verifies

$$\xi_{\min} \leq \left[1 - \frac{\frac{\mathsf{f}(\gamma_L^\star)}{\gamma_L^\star}}{\frac{\mathsf{f}(\beta^\star)}{\beta^\star}} \frac{(N+\gamma_L^\star)}{(N+\beta^\star)}\right] T \min_{\ell \in \mathcal{L}} g_f p_\ell^{\mathsf{SE}}. \tag{9}$$

The proof of this result is given in Appendix B. Interestingly, it is possible to provide an explicit lower bound on the energy threshold for a cognitive radio for being energy efficient. For a transmitter, this bound mathematically translates the tradeoff between the benefit (in terms of energy efficiency) of being informed about the actions played by the others and the cost induced by acquiring this knowledge. If the sensing cost is negligible, then following becomes always better than leading, giving an incentive to equip a transmitter with a cognitive radio. However, if all the transmitters of the network are cognitive, the network energy efficiency is not maximized, which is what is proved in the next section.

IV. Sensing Game

In the preceding section, the pair (F, L) and the identities of cognitive and noncognitive transmitters were fixed. A quite natural question is to ask whether the transmitters would effectively sense or not in a fully decentralized network, where the decision to sense is left to them. Providing answers to this question is the purpose of this section. As a first step, we show the existence of an optimal number of cognitive transmitters in terms of social welfare (sum utility). The corresponding upper bound can be used to assess the price of anarchy of the network [18] and therefore measuring the cost of having this decision decentralized. As a second step, we consider a sensing game in which each transmitter has two actions (sense/not sense). It is shown that each transmitter can learn his best decision provided that the number of blocks is sufficiently large.

A. On the Optimal Number of Cognitive Transmitters

The global energy efficiency of the network is measured in terms of social welfare [1] or sum utility at the equilibrium, which is defined by

$$w_L^e = \sum_{k=1}^{K} u_k = \sum_{k=1}^{L} u_k + \sum_{k=L+1}^{K} u_k \tag{10}$$

where $e \in \{\text{SE}, \text{NE}\}$. Note that an NE is obtained when $L = K$; indeed, in this context, there are no followers, and the game is no more hierarchical. The subscript L has been added to the equilibrium profiles to clearly indicate that it is related to the number of leaders whenever $L \neq K$ [see (10)]. As the parameter L, which is the number of leaders or noncognitive transmitters, belongs to a discrete set $\mathcal{K}$, the function $w_L : \mathcal{K} \to \mathbb{R}^+$ has necessarily a maximum.[4] Is this maximum reached at the nontrivial points $L = 1$ or $L = \text{K}?\ \ >$. From Proposition 3.2, we know that this is not the case. Indeed, if the sensing cost is small enough, then the power profile at any SE Pareto dominates the NE obtained with $L = 1$ or $L = K$. Thus, the latter points are not the maximizer candidates for the sum utility. However, when the sensing cost is arbitrary, answering the question analytically does not seem to be trivial. This is why we solve this maximization problem numerically in Section V. To still get some insights into the problem, we study a very special case of it. The interest in doing so is that it clearly shows that the sum utility maximizer is nontrivial, and a little more about the connection with the network load can be learned.

[4] Note that, however, we do not try to optimize the identity of the followers or leaders w.r.t. their channel quality. This type of issue, which is relevant in centralized scenarios, is addressed in [21] in a special case.

We now consider the special case defined by the following four assumptions:

- Assumption 1: $g_k = g$, $R_k = R$ for all $k \in \mathcal{K}$.
- Assumption 2: $N \gg (K-1)\beta^\star$.
- Assumption 3: $\mathsf{f}(x) = e^{-(c/x)}$, $c \geq 0$.
- Assumption 4: $1 \leq L \leq K-1$.

The social welfare at the SE is given by

$$\begin{aligned} w_L^{\mathsf{SE}} &= \sum_{\ell \in \mathcal{L}} R_\ell \frac{\mathsf{f}(\gamma_L^\star)}{p_\ell^{\mathsf{SE}}} + \sum_{f \in \mathcal{F}} R_f \frac{\mathsf{f}(\beta^\star)}{p_f^{\mathsf{SE}}} \\ &= R \frac{g a_L}{\sigma^2} \left[\frac{L \mathsf{f}(\gamma_L^\star)}{\gamma_L^\star (N+\beta^\star)} + \frac{(K-L)\mathsf{f}(\beta^\star)}{\beta^\star(N+\gamma_L^\star)} \right] \end{aligned} \tag{11}$$

where

$$a_L \triangleq N - (F-1)\beta^\star - \frac{[(N+\beta^\star)(L-1) + F\beta^\star]\,\gamma_L^\star}{N}. \tag{12}$$

Note that in [4], $c = 2^r - 1$, where r is the spectral efficiency of the used channel coding scheme (in bit/s per Hz). A possible choice is $r = (R/B)$, where R is the data transmission rate, and B is the required bandwidth to transmit. While Assumptions 2 and 3 are reasonable (they respectively correspond to a small system load and the case where the efficiency function is derived from the outage probability on the mutual information [4]), the first assumption is very strong and may happen in practice in specific scenarios, e.g., in virtual multiple-input–multiple–output (MIMO) networks with clusters of transmitters [16] with similar flow types (a voice service typically). Indeed, if fast PC is considered, namely, g_k represents the fast fading, this symmetry assumption will almost never be verified in practice. Now, if slow PC is considered, g_k may represent shadowing and path loss effects, and the g_k's will be almost equal for all users being in a given neighborhood (as explained in [16], where the notion of clusters of users is shown to be relevant). In any case, note that the symmetry assumption is very local and is only exploited to reinforce the existence of a nontrivial optimal number of followers/leaders and provide insights on this number. Again, the goal is not to claim a general expression for the optimal number of leaders. Rather, we just want to derive it in a special case to better understand the general optimization problem. Assumption 4 is not restrictive since the cases $K = 0$ and $K = L$ are ready. Under Assumption 2, we have that $a_L \approx N$. From (8), we have $\epsilon_L \approx 0$, which implies that $\gamma_L^\star \approx \beta^\star, \mathsf{f}(\gamma_L^\star) \approx \mathsf{f}(\beta^\star)$, and

$$\frac{\mathsf{f}(\gamma_L^\star)}{N+\beta^\star} \approx \frac{\mathsf{f}(\beta^\star)}{N+\gamma_L^\star}. \tag{13}$$

This allows one to approximate the social welfare at the equilibrium (11) as

$$\widetilde{w}_L^{\mathsf{SE}} \approx R\frac{g\mathsf{f}(\beta^\star)N}{(N+\beta^\star)\sigma^2}\left(\frac{L}{\gamma_L^\star}+\frac{K-L}{\beta^\star}\right). \tag{14}$$

Note that the term $(g\mathsf{f}(\beta^\star)N/(N+\beta^\star)\sigma^2)$ is independent of L, and the optimal solution $L^\star$ can be approximated by

$$\tilde{L}^\star = \arg\max_L \left(\frac{L}{\gamma_L^\star}+\frac{K-L}{\beta^\star}\right). \tag{15}$$

The main point in the above equation is that $\widetilde{L}^\star$ is related to $\gamma_L^\star$, which is the positive root of the equation $x(1-\epsilon_L x)\mathsf{f}'(x) - \mathsf{f}(x) = 0$. It turns out that, under Assumption 3, a very simple expression for $\gamma_L^\star$ can be obtained. One can easily check that

$$\gamma_L^\star = \frac{c}{1+\epsilon_L c}. \tag{16}$$

Using the above, the optimization problem given by (15) boils down to

$$\widetilde{L}^\star = \arg\max_L \left(\frac{L}{\frac{c}{1+\epsilon_L c}}+\frac{K-L}{c}\right) = \arg\max_L(\epsilon_L L).$$

Replacing the discrete variable L with a nonnegative real λ, the optimal solution of the corresponding concave function can be checked to be

$$\widetilde{\lambda}^\star = \left| \begin{array}{ll} \left(1+\frac{N}{c}\right)\left(1-\sqrt{1-\frac{K}{N}\frac{c}{\frac{c}{N}+1}}\right) & \text{if } \frac{K-1}{N} \leq \frac{1}{c} \\ K-\kappa & \text{if } \frac{K-1}{N} > \frac{1}{c} \end{array} \right. \tag{17}$$

where $\kappa > 0$ is arbitrary small (this constraint is added to meet the constraint $L \leq K-1$). Therefore, the optimal number of noncognitive transmitters can be approximated by $\widetilde{L}^\star = \lfloor \widetilde{\lambda}^\star \rfloor$ or $\widetilde{L}^\star = \lceil \widetilde{\lambda}^\star \rceil$, depending on which number gives the maximal social welfare. From this expression of $\widetilde{L}^\star$, some interesting insights can be easily extracted. First of all, note that if the load is sufficiently small compared to $1/\beta^\star$ but at the same time greater than $1/c$, the social welfare is maximized for $\widetilde{L}^\star = K-1$. Indeed, when the load is small, the interaction between players is not strong, and the impact of a hierarchy on the overall system is small (at least for a reasonably large system). Thus, the social welfare is maximized at the NE point, which corresponds to the case where all users are leaders and play at the same time. A second type of insights can be obtained by considering the spectral efficiency related to channel coding, namely, $c = 2^r - 1$. As already mentioned, it is assumed that the spectral efficiency involved in the multiple access technique is sufficiently small (Assumption 2). If we go further by assuming that both the multiple access technique spectral efficiency $((K/N) \to 0)$ and the channel coding spectral efficiency $(c \to 0)$ are small, we obtain that

$$\widetilde{\lambda}^\star \approx \left(1+\frac{N}{c}\right)\frac{1}{2}\frac{K}{N}\frac{c}{\frac{c}{N}+1} \approx \frac{K}{2} \tag{18}$$

which means that in the low spectral efficiency regime, the global energy efficiency of the network is maximized when half of the transmitters are cognitive. If c is large but the load is still small (meeting the constraint $(K-1/N) \leq (1/c)$), the same conclusion is obtained. Conducting a deep analysis to discuss the connections between spectral efficiency and energy efficiency in decentralized cognitive networks in the general case (arbitrary load, with cost of sensing) seems to be a relevant and nontrivial extension of the simplified analysis provided here.

B. Sensing Game: Description and Key Property

In the two-level hierarchical PC described in Section III, the transmitter is, by construction, either a cognitive transmitter or a noncognitive one, and the action of a player consists of choosing his transmit power level. In fully decentralized networks, it is legitimate to ask about what a transmitter would decide between sensing (being cognitive or not) or not sensing. To analyze such a problem, we assume that two games occur sequentially for each block (this separation assumption will be fully justified in Section IV-C). First, the transmitters decide to sense (S) or not to sense (NS). Then, they choose their power level based on their status (follower or leader) and therefore play the two-level PC game described in Section III. The sensing game can therefore be described by a static game whose representation is given by the following triplet:

$$\mathcal{G} = (\mathcal{K}, (\mathcal{A}_k)_{k\in\mathcal{K}}, (U_k)_{k\in\mathcal{K}}) \tag{19}$$

where the actions sets are $\mathcal{A}_k = \mathcal{A} = \{\mathsf{S}, \mathsf{NS}\}$, and the utility functions are those obtained at the SE of the PC game played in the second phase. If transmitter k is a follower (i.e., he senses) and there were F followers during the sensing phase of the block, then his utility is

$$U_k\left(\mathsf{S}, \mathbf{s}_{-k}^{(F,L)}\right) = \frac{(1-\alpha_k)g_k R_k}{\sigma^2}\frac{\mathsf{f}(\beta^*)}{N\beta^\star\left(N+\gamma_{L+1}^\star\right)} \times \left\{N^2 - N\beta^\star - \left[(N+\beta^\star)L+(F+1)\beta^\star\right]\gamma_{L+1}^\star\right\}$$

where the notation $\mathbf{s}_{-k}^{(F,L)}$ means that there are $F-1$ followers and L leaders in the set of players $\mathcal{K}\setminus k$. On the other hand, if transmitter k chooses the action NS, he obtains

$$U_k\left(\mathsf{NS}, \mathbf{s}_{-k}^{(F,L)}\right) = \frac{g_k R_k}{\sigma^2}\frac{\mathsf{f}\left(\gamma_L^*\right)}{N\gamma_{L+1}^\star(N+\beta^\star)} \times \left\{N^2 - N\beta^\star - \left[(N+\beta^\star)L+(F+1)\beta^\star\right]\gamma_{L+1}^\star\right\}$$

in terms of utility. The considered sensing game is a congestion game [29] (and therefore a potential game) under strong conditions but is always a weighted potential game. The latter property is known to be very useful for studying the existence of pure NE and convergence of learning algorithms or distributed iterative algorithms toward NE. For instance, in [24] and [25], Monderer and Shapley proved that every weighted potential game has the Fictitious Play[5] property. This guarantees that every learning algorithm that is a Fictitious Play process converges in belief to equilibrium. All of this is the purpose of the remainder of this section. For making this paper sufficiently self-containing, we review several useful definitions concerning potential games [24].

Definition 4.1 (Monderer and Shapley 1996 [24]): The strategic form game $\mathcal{G}$ is a potential game if there is a potential function $V : \mathcal{A} \longrightarrow \mathbb{R}$ such that

$$U_k(s_k, \boldsymbol{s}_{-k}) - U_k\left(s'_k, \boldsymbol{s}_{-k}\right) = V(s_k, \mathbf{s}_{-k}) - V\left(s'_k, \boldsymbol{s}_{-k}\right), \quad \forall\, k \in \mathcal{K}, s_k, s'_k \in \mathcal{A}_k.$$

Theorem 4.2: The sensing game $\mathcal{G} = (\mathcal{K}, (\mathcal{A})_{k\in\mathcal{K}}, (U_k)_{k\in\mathcal{K}})$ is an exact potential game if and only if one of the two following conditions is satisfied:

$$\forall\, i, j \in \mathcal{K} \quad R_i g_i = R_j g_j \tag{20}$$

$$U_L(F+1, L+1) - U_L(F, L+2) = (1-\alpha)\left(U_F(F+2, L) - U_F(F+1, L+1)\right) \tag{21}$$

where $U_L(F+1, L+1)$ is defined by $(\sigma^2 U_i(F+1, L+1)/R_i g_i)$ when player $i \in \mathcal{K}$ is one of the $F+1$ followers, and $U_F(F+1, L+1)$ is defined by $(\sigma^2 U_i(F+1, L+1)/R_i g_i)$ when player $i \in \mathcal{K}$ is one of the $L+1$ leaders.

Condition (20) is a (strong) symmetry condition and is obtained under Assumption 1 (see Section IV-A), which would be reasonable for a cluster of transmitters in a virtual MIMO network with a common service (e.g., voice). In fact, it is more realistic not to make these assumptions and claim for a potential property that is sufficient for key issues such as convergence of some important learning dynamics.

Definition 4.3 (Monderer and Shapley 1996 [24]): The strategic form game $\mathcal{G}$ is a weighted potential game if there is a vector $(\mu_i)_{i\in\mathcal{K}}$ and a potential function $V : \mathcal{A} \longrightarrow \mathbb{R}$ such that

$$\forall\, i \in \mathcal{K}, (s_i, s'_i) \in \mathcal{A}_i^2,$$
$$U_i(s_i, s_{-i}) - U_i\left(s'_i, s_{-i}\right) = \mu_i\left(V(s_i, s_{-i}) - V\left(s'_i, s_{-i}\right)\right).$$

It turns out that such a vector can be found.

Theorem 4.4: The sensing game $\mathcal{G} = (\mathcal{K}, (\mathcal{A}_i)_{i\in\mathcal{K}}, (U_i)_{i\in\mathcal{K}})$ is a weighted potential game with the weight vector

$$\forall\, i \in \mathcal{K}, \quad \mu_i = \frac{R_i g_i}{\sigma^2}. \tag{22}$$

The proof is given in Appendix F.

C. Equilibrium Analysis

1) Existence: First of all, note that since the sensing game is finite (i.e., both the number of players and the sets of actions are finite), the existence of at least one mixed NE is guaranteed [28]. Now, since the game is a weighted potential game, the existence of at least one pure NE is guaranteed [24]. We might restrict our attention to pure and mixed Nash equilibria. However, as will be clearly seen in the two-player case study (Section IV-C3), this may pose a problem of fairness and efficiency. This is the main reason why we also study the set of correlated equilibria (CE; Appendix A-1) of the sensing game. The concept of CE [2] allows one to enlarge the set of equilibrium utilities. Every utility vector inside the convex hull of the NE utilities is an CE, which guarantees the existence of CE in general.

2) Uniqueness: Here, we provide a brief analysis of uniqueness for the pure NE. This matters since pure NEs are attractors of important dynamics, such as the replicator dynamics (which corresponds to the limit of important learning schemes) [6]. One obvious advantage of having uniqueness of the game outcome is to make the game predictable, which may be useful from a designer standpoint. As mentioned above, by contrast, the number of CE is generally greater than one and more typically infinite. The following proposition provides sufficient condition under which the sensing game (always with costs) has a unique pure NE.

Proposition 4.5: Assume the following two conditions are satisfied:

$$\alpha > 1 - \frac{\left(N + \gamma^\star_{K-1}\right)\left(N - (K-1)\beta^\star\right)}{N^2 - N\beta^\star - \left[(N+\beta^\star)(K-1) + 2\beta^\star\right]\gamma^\star_{K-1}} \tag{23}$$

$$\alpha > 1 - \frac{\gamma^\star_{K-1}(N+\beta^\star)\mathsf{f}(\beta^\star)}{\beta^\star(N+\gamma^\star_{K-1})\mathsf{f}(\gamma^\star_{K-1})}. \tag{24}$$

Then, the unique NE of the game is $(s_1^*, s_2^*, \ldots, s_K^*) = (\mathsf{NS}, \mathsf{NS}, \ldots, \mathsf{NS})$.

Condition (23) insures that the nonsensing strategy NS dominates the sensing strategy S when none of the other players sense. Condition (24) insures that the nonsensing strategy NS dominates the sensing strategy S when some of the other player sense. Both conditions together imply that the sensing strategy S is always a dominated strategy for each player. The unique NE of the game is $(s_1^*, s_2^*, \ldots, s_K^*) = (\mathsf{NS}, \mathsf{NS}, \ldots, \mathsf{NS})$.

3) Efficiency: In a decentralized network, since no or little coordination between terminals is available, an important issue is the efficiency of the network at the equilibrium state. Are the mixed or pure NEs of the sensing game efficient in terms of utility? To be illustrative and to understand in a deep manner the problem under investigation, our choice, in this section, is to mainly focus on the two-transmitter case, but most of the provided results can be extended to the general case $K \geq 2$.

Theorem 4.6 (Number of NE): The matrix game has the following NE:

1) a unique NE if and only if $(C1) : \alpha > (\beta^* - \gamma^*/1 - \beta^*\gamma^*)$;
2) three NEs if and only if $(C2) : \alpha < (\beta^* - \gamma^*/1 - \beta^*\gamma^*)$;
3) an infinite number of NEs if $(C3) : \alpha = (\beta^* - \gamma^*/1 - \beta^*\gamma^*)$.

The proof of this result is provided in Appendix C. There is also a strictly mixed equilibrium that can be found using the

[5]The FP learning algorithm can be found in the cited references or [35], but essentially, it consists of assuming that each player observes the actions of the others and maximizes his average utility based on the empirical frequencies of use of actions of the others.

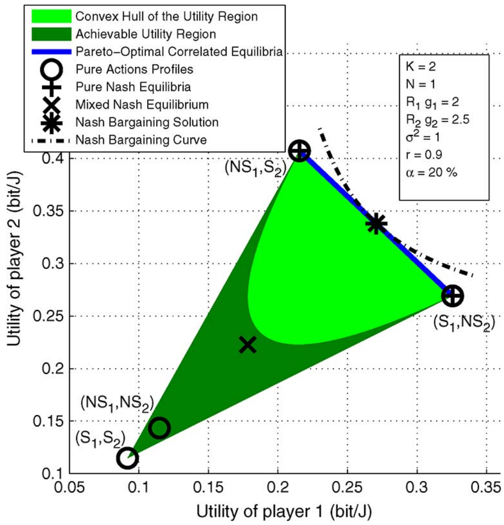

Fig. 1. Depiction of, for a two-transmitter scenario, the region of achievable utilities of the sensing game, where each transmitter decides whether to sense or not to sense. The figure also shows the different equilibrium points of the game. One of the messages of this figure is the interest in terms of fairness in stimulating a CE instead of an NE. In particular, a Nash bargaining solution can be obtained.

indifference principle. Let $(x, 1-x)$ be the mixed strategy for player 1 and $(y, 1-y)$ be the mixed strategy for player 2. As proven in Appendix D, there is a unique pair (x^*, y^*) satisfying the indifference principle. The corresponding distribution is given by

$$x^* = y^* = \frac{(1-\alpha)\frac{\mathsf{f}(\beta^*)}{\beta^*}(1-\beta^*) - \frac{\mathsf{f}(\gamma^*)}{\gamma^*}\frac{1-\gamma^*\beta^*}{1+\beta^*}}{X} \tag{25}$$

with $X = (1-\alpha)(\mathsf{f}(\beta^*)/\beta^*)(1-\beta^*) - (\mathsf{f}(\gamma^*)/\gamma^*)(1-\gamma^*\beta^*/1+\beta^*) + (\mathsf{f}(\beta^*)/\beta^*)(1-\beta^*) - (1-\alpha)(\mathsf{f}(\beta^*)/\beta^*)(1-\gamma^*\beta^*/1+\gamma^*)$, and the corresponding equilibrium utilities are

$$U_1(x^*, y^*) = \frac{R_1 g_1}{\sigma^2}\upsilon$$
$$U_2(x^*, y^*) = \frac{R_2 g_2}{\sigma^2}\upsilon$$

with

$$\upsilon = \frac{(1-\alpha)\frac{\mathsf{f}(\beta^*)}{\beta^*}(1-\beta^*)\frac{\mathsf{f}(\beta^*)}{\beta^*}(1-\beta^*)}{X} - \frac{\frac{\mathsf{f}(\gamma^*)}{\gamma^*}\frac{1-\gamma^*\beta^*}{1+\beta^*}(1-\alpha)\frac{\mathsf{f}(\beta^*)}{\beta^*}\frac{1-\gamma^*\beta^*}{1+\gamma^*}}{X}.$$

Fig. 1 represents the three equilibrium utility points for a typical scenario. The shaded area represents the region of feasible utilities for a given scenario (described under Fig. 1). Operating at one of the pure NEs can be unfair for one of the transmitters and therefore inefficient for a certain fairness criterion [11]. Operating at the mixed NE is clearly suboptimal since it is Pareto dominated by some feasible pairs of utilities. A way of dealing with fairness or/and Pareto inefficiencies is to induce CE in the game.

In practice, having a CE means that the players have no interest in ignoring (public or private) signals that would recommend them to play according to certain joint distribution over the action profiles of the game. In wireless networks, a CE can be induced by a common signaling from a source that is exogenous to the game. It may be a signal generated by the receiver itself but also a frequency modulation signal or a Global Positioning System signal, meaning that the additional cost for adding this signal may be zero if the terminals are already able to decode such a signal. Last, note that such a coordination mechanism is scalable in the sense that it can accommodate a high number of transmitters; in practice, physical limitations may arise, e.g., if the signal is sampled into a finite number of bits. If $\alpha > (\beta^* - \gamma^*/1 - \beta^*\gamma^*)$, as there is only one NE, the convex hull of NE boils down to a point, and there does not exist any other CE other than this NE. Rather, we assume that the sensing cost verifies condition (C1), which is the case of interest since several NEs exist (see Theorem 4.6). In this case, the following result holds.

Theorem 4.7: Any convex combination of NE is a CE. In particular, if there exists a utility vector $\nu = (\nu_1, \nu_2)$ and a parameter $\lambda \in [0, 1]$ such that

$$\nu_1 = \lambda U_1(\mathsf{S}_1, \mathsf{NS}_2) + (1-\lambda)U_1(\mathsf{NS}_1, \mathsf{S}_2) \tag{26}$$
$$\nu_2 = \lambda U_2(\mathsf{S}_1, \mathsf{NS}_2) + (1-\lambda)U_2(\mathsf{NS}_1, \mathsf{S}_2) \tag{27}$$

then ν is a CE.

Clearly, a signal recommending the transmitters to play the action profile $(\mathsf{S}_1, \mathsf{NS}_2)$ (respectively $(\mathsf{NS}_1, \mathsf{S}_2)$) for a fraction of the time equal to λ (resp., to $1-\lambda$) induces a CE. This specific signaling structure leads to the set of equilibria represented by the bold segment in Fig. 1. The figure illustrates the potential gains that can be obtained by implementing a simple coordination mechanism in the sensing game with costs.

We would like to end this section dedicated to the efficiency of the equilibria of the game by mentioning the potential suboptimality induced by playing the sensing game and PC game separately (in two consecutive phases). Indeed, it would be legitimate to ask what would happen if a transmitter were deciding jointly whether to sense or not and his power level. In such a case, the action set of a transmitter would be

$$\widetilde{\mathcal{A}}_k = \{\mathsf{S}_k, \mathsf{NS}_k\} \times [0, P_k^{\max}]. \tag{28}$$

An action $a_k = (s_k, p_k)$ has therefore two components. The first component is discrete, whereas the second component is continuous. This framework is referred to as hybrid control in control theory [9], [26]. While the control theory literature is rich concerning hybrid control, this is not the case for hybrid control games. In particular, general existence theorems for Nash equilibria seem to be unavailable. This is one of the reasons we will only consider the special case of two transmitters. In the two-player hybrid control game, it can easily be seen that the two pure NEs of the sensing game are no longer equilibria in this new game. Instead, we have the following result.

Proposition 4.8: The unique NE of the two-player hybrid control game is given by

$$(a_1^*, a_2^*) = \left(\mathsf{NS}_1, p_1^{\mathtt{NE}}, \mathsf{NS}_2, p_1^{\mathtt{NE}}\right) \tag{29}$$

where $p_k^{\mathtt{NE}}$ is given by (4).

This result immediately follows from the fact that action every action under the form (S, p_k) is dominated by the action (NS, p_k). Although the proof of this result is trivial, the interpretation is nonetheless interesting. It shows the existence of a Braess paradox in the hybrid control game. Although the players have more options in the hybrid game, the equilibrium utilities are less than those obtained in the separated case, where they first decide to sense or not and then adapt their power level. In addition to implementation considerations, this gives us another reason to perform the decision process in two consecutive phases.

V. Numerical Results

In this section, numerical results are provided to validate our theoretical claims. Note that, although simple scenarios are considered, the authors believe that most of messages and insights conveyed by the present numerical analysis hold in more advanced simulation setups, e.g., considering standardized channel modulation and coding schemes (MCS), real frequency-selective channel impulse responses, imperfect CSI, and sensing techniques accounting for estimation noise. Indeed, as explained in [15], the choice of a specific MCS will generally lead to a packet success rate having the assumed properties. As shown in [10], the case of frequency selective channels is treatable once the frequency-flat case has been treated. Therefore, only the impact of channel estimation noise seems to be more uncertain and would call for a more challenging extension of the results provided here. We consider a random CDMA scenario with a spreading factor equal to N, and the efficiency function is chosen to be $f(x) = e^{-(2^r - 1/x)}$ with different parameters r [4]. We consider two scenarios. The first scenario is provided in Fig. 1. This scenario provides a clear understanding of the variety of equilibria in the sensing game. The pure, mixed, and correlated equilibria are represented on the utility region. The utility region of the sensing game with two players $K = 2$, no spreading $N = 1$, the sensing cost $\alpha = 20\%$, the sigmoidal function $f(x) = e^{-(2^r - 1/x)}$ with $r = 0.9$, and the following parameters: $R_1\, g_1 = 2$, $R_2\, g_2 = 2.5$, $\sigma^2 = 1$. The pure actions lead to the utilities marked by circles, the dark green region corresponds to the pair of utilities that is achievable with mixed actions, whereas the light green region corresponds to the utilities that are achievable only with correlated actions. The two pure equilibrium utilities, denoted by $+$, correspond to both upper left extremal pure utilities, and the completely mixed equilibrium utility, denoted by $\times$, is located in the interior of the dark green region. The blue line between the two pure equilibria represents a subset of CE utilities that corresponds to the Pareto-optimal frontier. The Nash bargaining solution, denoted by $*$, corresponds to the intersection of the hyperbolic curve with the set of CE utilities. It provides a fair and optimal equilibrium solution for the sensing game.

The second scenario considers a sensing game with 17 players, $N = 128$ subcarriers, the sensing cost $\alpha = 5\%$, and the sigmoidal function $f(x) = e^{-(2^r - 1/x)}$. For simplicity, we assume a homogeneous scenario in terms of transmission rate $R_k = R$ and $r = (R/B)$ 3 bit/s/Hz for different numbers of leaders. Note that the value for R will not matter since only normalized/relative performance gains will be considered. The seemingly nontypical choice for K results from typical choices on the other parameters. Indeed, when fixing the spreading factor to $N = 128$ (typical e.g., in cellular systems), the spectral efficiency to $r = 3$ bit/s/Hz (also typical in cellular systems), one finds that the maximum number of admissible users for the NE to be implemented is 18 [see the denominator of (4)]: $r = 3 \Rightarrow c = 7 \Rightarrow \beta^* = 7 \Rightarrow (K - 1/N) < (1/7) \Rightarrow K < 18$, where $f(x) = e^{-(c/x)}$, $c = 2^r - 1$, and β^* is the unique solution of $xf'(x) - f(x) = 0$. Figs. 2 and 3 allow one to evaluate the improvement brought by the Stackelberg approach compared to the NE approach for the utility of one leader, one follower, and the social utility. The sensing cost

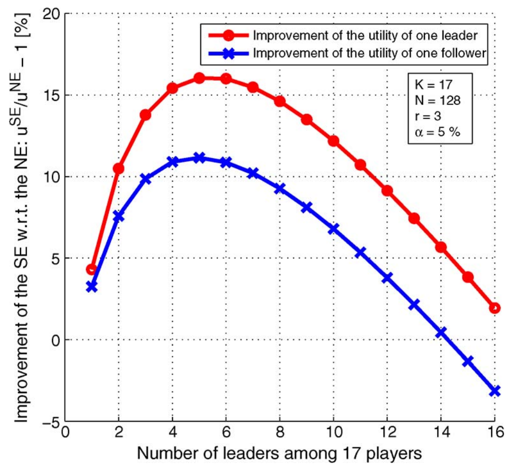

Fig. 2. This figure represents the relative gain (in percent) in terms of individual energy efficiency obtained by equipping $F = K - L$ transmitters with a cognitive radio. For typical scenarios, we see that maximizing the number of cognitive transmitters is not optimal. On the other hand, if there is only one cognitive radio ($F = 1$ or $L = 16$, as assumed in [21]), one can degrade the individual performance for a typical value for the sensing cost (5 of the time slot is spent for sensing).

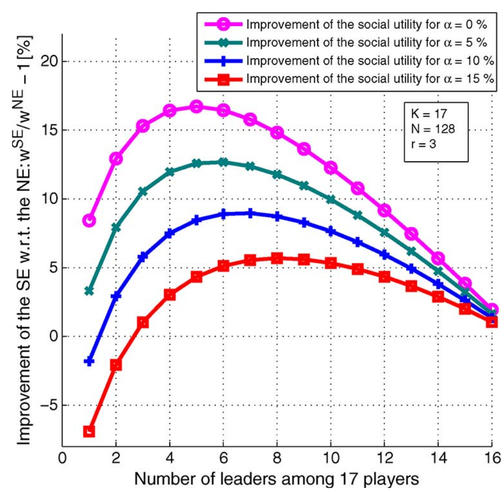

Fig. 3. This figure represents the improvement in terms of utility sum or network energy efficiency at the SE compared to the case of NE (i.e., no transmitter is equipped cognitive radio) with $K = 17$ players, $N = 128$, the efficiency function is $f(x) = e^{-(2^r - 1/x)}$ with $r = 3$ bit/s/Hz, and for different numbers of leaders. The different curves correspond to different values for the sensing cost: $\alpha \in \{0\%, 5\%, 10\%, 15\%\}$. When 5 of the time has to be spent for sensing, the network energy efficiency can be improved by 13, whereas it is 17 when this cost is close to zero.

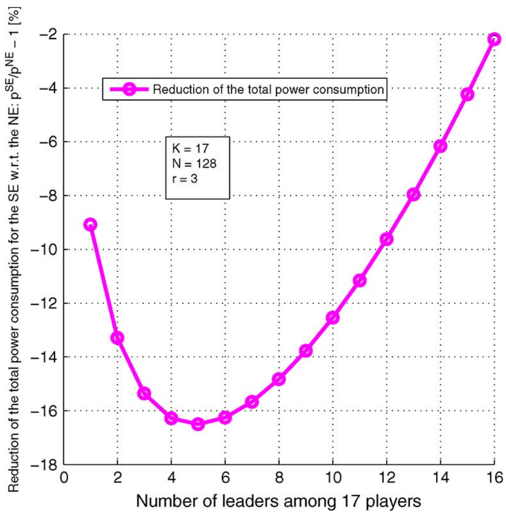

Fig. 4. From previous figures, we know that equipping the network with an adequate number of cognitive radios can significantly improve the network energy efficiency. It turns out that not only efficiency is maximized by doing so but that the total consumed power is also reduced. Scenario illustrated: $K = 17$ players, $N = 128$, the efficiency function is $f(x) = e^{-(2^r - 1/x)}$ with $r = 3$ for different numbers of leaders.

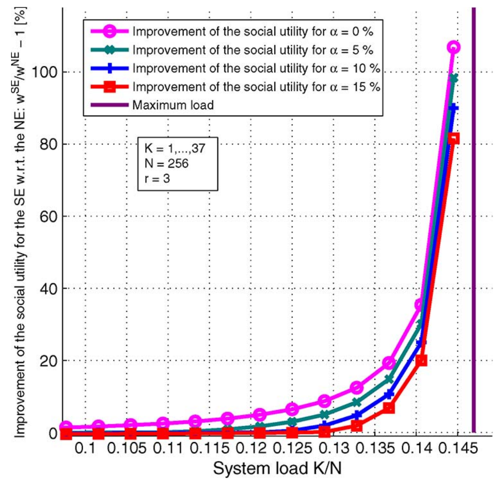

Fig. 5. The objective of this figure is to show the relationship between the maximum gain obtained in terms of network energy efficiency and the spectral efficiency in terms of user load (K/N) (the spectral efficiency in terms of channel coding is always fixed and set to $r = 3$ bit/s/Hz). From this figure, it is seen that there is a strong interest in operating at a load level close to the maximum limit tolerated by the system (ensuring the existence of an equilibrium, see Section II-C).

influences the results in two ways. First, the sensing cost affects the gain obtained by the follower compared with the leader. Indeed, in this figure, the improvement of a leader is always larger than the improvement of one follower, which is not true in general. Second, the improvement of the utility of one follower compared to the NE utility is negative when the number of leaders is strictly more than 14. In that case, the sensing cost is compensated by the improvement due to the Stackelberg approach compared to the NE approach. The optimal number of leaders is 5 when considering either the improvement of the utility of one leader or the improvement of the utility of one follower. The improvement of the sum utility of the SE compared to the social utility at the NE for the sensing game is given in Fig. 3. The sensing cost decreases the improvement of the social utility. This is especially the case when the number of followers is large and the number of leaders is small. The Stackelberg approach provides up to 16.5% of improvement compared to the NE approach.

Fig. 4 illustrates how much the total power consumption can be reduced for the sensing game with $K = 17$ players, $N = 128$ subcarriers, and the sigmoidal function $f(x) = e^{-(2^r - 1/x)}$ with $r = 3$ for different numbers of leaders. Note that at the same time, the energy efficiency is optimized. The best reduction of power consumption is achieved when the number of leaders is 5, and the reduction of the power consumption is more than 16%. We observe that the total power reduction is maximum when the number of cognitive users maximizes the social welfare of the system. Finally, Fig. 5 represents the improvement of the maximal social welfare (depending on the number of cognitive users) of the sensing game compared to the social welfare at the NE solution, depending on the load (K/N) of the system. The four curves correspond to different sensing costs $\alpha \in \{0\%, 5\%, 10\%, 15\%\}$. When the load approaches its maximal value $1/\beta^\star + 1/N$, the improvement of the social utility is greater than 100%. Then, we can conclude that our hierarchical framework with optimal number of cognitive equipments becomes more efficient in terms of social utility when a cognitive wireless network is high loaded.

VI. Conclusion

In this paper, we introduced a new PC game where the action of a player is hybrid. One component of the action is discrete, whereas the other is continuous. The first component is discrete since it corresponds to deciding whether to sense the radio environment or not; the second component is continuous because it corresponds to choosing the transmit power level in an interval. Whereas the general study of hybrid games is of independent and game-theoretic interest and remains to be done, it turns out that, in our case, we can prove the existence of a kind of Braess paradox, which allows us to restrict our attention to two separate games played consecutively. Choosing the discrete and continuous actions jointly is less efficient than choosing them separately over time. The PC game is studied in detail, and it is shown that there exists an optimal number of cognitive transmitters that maximizes the network utility, meaning that introducing too much cognition is not globally energy efficient. This holds whether the cost of sensing is set to zero or not. From an individual point of view, the intuition that consists of saying that sensing is beneficial only if the sensing cost is acceptable can be proved. As distributed networks are considered, global efficiency of the network is generally not guaranteed. Equilibria are indeed less energy efficient (say in terms of sum utility) than the centralized solution. The (hierarchical) approach we propose can therefore be seen as a tradeoff in terms of global performance and required signaling. Conducting a refined analysis in terms of signaling for the PC problem would be relevant. On the other hand, the sensing game can be shown to have desirable properties like being weighted potential. This is a key property since many learning algorithms are known to converge in such games, proving that this decision can be learned over time with partial information only. Additionally, this game is shown to have a nontrivial set of CEs. These equilibria are very useful since they allow one to introduce some fairness among the transmitters and can be stimulated by a public signal incurring no cost in terms of extra signaling from the receiver; in this respect, the famous Nash bargaining solution (used in the wireless literature for having both a fair and cooperative solution; see, e.g., [19] and [23]) can be reached. This paper therefore provides several new results of practical interest for cognitive wireless networks, but, of course, the proposed concepts would need to be developed further to make them more appealing in terms of implementability. In particular, technical issues related to spectrum usage might be considered by introducing frequency-selective channels and the corresponding power allocation problem. Considering a more general structure of interference networks, the relevance of successive interference cancelation in terms of energy efficiency might be assessed. Of course, classical issues such as the impact of channel estimation is also of practical interest, particularly regarding the fact that some learning algorithms are known to be robust against this type of errors.

Appendix A
Review of Some Game-Theoretic Concepts

A. CE

In this section, we provide the definition of CE. This equilibrium concept was introduced by Aumann [3] and extends the concept of NE. CEs are used in Section IV-C.3 to provide fairer equilibrium solutions.

Definition A.1: A probability distribution $Q \in \Delta(\mathcal{A})$ is a canonical CE if for each player i, for each action $a_i \in \mathcal{A}_i$, that satisfies $Q(a_i) > 0$ we have

$$\sum_{a_{-i}\in\mathcal{A}_{-i}} Q(a_{-i}|a_i)u_i(a_i,a_{-i}) \geq \sum_{a-i\in\mathcal{A}_{-i}} Q(a_{-i}|a_i)u_i(b_i,a_{-i}) \quad \forall\, b_i \in \mathcal{A}_i. \tag{30}$$

The result of Aumann [3] states that for any CE, it corresponds to a canonical CE.

Theorem A.2 ([3]): The utility vector u is a CE utility if and only if there exists a distribution $Q \in \Delta(\mathcal{A})$ satisfying the linear inequality constraint (30) with $u = E_Q U$.

B. Potential of the Sensing Game

In this section, we provide the potential function of the sensing game presented in Section IV.

Theorem A.3: The equilibria of the foregoing potential game are the maximizers of the Rosenthal potential function [31]

$$\begin{aligned}
&\{S=(\mathcal{A}_1,\ldots,\mathcal{A}_K)|S\in NE\}\\
&=\arg\max_{(F,L)} \Phi(F,L)\\
&=\arg\max_{(F,L)}\left[(1-\alpha)\sum_{i=1}^{F} U(S,i,K-i)+\sum_{j=1}^{L} U(NS,K-j,j)\right].
\end{aligned}$$

The proof follows directly from one of Rosenthal's theorem [31]. Let us simplify the expression of the potential function, which gives

$$\begin{aligned}
&\Phi(F,L)\\
&=\left[(1-\alpha)\sum_{i=1}^{F} U(S,i,K-i)+\sum_{j=1}^{L} U(NS,K-j,j)\right]\\
&=(1-\alpha)\sum_{i=1}^{F}\frac{g_iR_i}{\sigma^2}\frac{\mathsf{f}(\beta^\star)}{N\beta^\star\left(N+\gamma^\star_{(K-i)}\right)}\\
&\quad\times\left(N^2-N\beta^\star-[(N+\beta^\star)(K-i)+(i+1)\beta^\star]\,\gamma^\star_{(K-i)}\right)\\
&\quad+\sum_{j=1}^{L}\frac{g_iR_i}{\sigma^2}\frac{\mathsf{f}\left(\gamma^\star_j\right)}{N\gamma^\star_j(N+\beta^\star)}\\
&\quad\times\left(N^2-N\beta^\star-[(N+\beta^\star)j+(K-j+1)\beta^\star]\,\gamma^\star_j\right).
\end{aligned}$$

Appendix B
Proof of Proposition 3.4

In this section, we prove Proposition 3.4. To have good sensing capabilities, there exists a certain energy threshold $\xi_{\min}$ such that

$$\alpha\, T\, \min_{\ell\in\mathcal{L}}(g_{f\ell}p_\ell) \geq \xi_{\min} \Longleftrightarrow \alpha \geq \frac{\xi_{\min}}{T\,\min_{\ell\in\mathcal{L}}(g_{f\ell}p_\ell)}$$

where T is the block duration, $g_{f\ell}$ is the channel gain between any leader $\ell \in \mathcal{L}$ and the considered follower $f \in \mathcal{F}$, p_ℓ is the power level of the leader $\ell \in \mathcal{L}$, and α is the sensing cost. The utility of the follower is maximized when the sensing cost α is minimal, that is, $\alpha^\star = (\xi_{\min}/T\min_{\ell\in\mathcal{L}}(g_{f\ell}p_\ell))$

$$\begin{aligned}
&\frac{\max_{\alpha\in[0,1]} U^{\mathsf{SE}}_f(\alpha)}{U^{\mathsf{SE}}_l} \geq 1\\
&\Longleftrightarrow \frac{(1-\alpha^\star)\frac{R_k\mathsf{f}(\beta^\star)}{p^{\mathsf{SE}}_f}}{\frac{R_k\mathsf{f}(\beta^\star)}{p^{\mathsf{SE}}_l}} \geq 1\\
&\Longleftrightarrow \left(1-\frac{\xi_{\min}}{T\,\min_{\ell\in\mathcal{L}}(g_{f\ell}p_\ell)}\right)\frac{\frac{\mathsf{f}(\beta^\star)}{\beta^\star}(N+\beta^\star)}{\frac{\mathsf{f}(\gamma^\star_L)}{\gamma^\star_L}(N+\gamma^\star_L)} \geq 1\\
&\Longleftrightarrow \frac{\frac{\mathsf{f}(\beta^\star)}{\beta^\star}(N+\beta^\star)}{\frac{\mathsf{f}(\gamma^\star_L)}{\gamma^\star_L}(N+\gamma^\star_L)}-1 \geq \frac{\xi_{\min}}{T\,\min_{\ell\in\mathcal{L}}(g_{f\ell}p_\ell)}\frac{\frac{\mathsf{f}(\beta^\star)}{\beta^\star}(N+\beta^\star)}{\frac{\mathsf{f}(\gamma^\star_L)}{\gamma^\star_L}(N+\gamma^\star_L)}\\
&\Longleftrightarrow \left(1-\frac{\frac{\mathsf{f}(\gamma^\star_L)}{\gamma^\star_L}(N+\gamma^\star_L)}{\frac{\mathsf{f}(\beta^\star)}{\beta^\star}(N+\beta^\star)}\right) \geq \frac{\xi_{\min}}{T\,\min_{\ell\in\mathcal{L}}(g_{f\ell}p_\ell)}\\
&\Longleftrightarrow T\,\min_{\ell\in\mathcal{L}}(g_{f\ell}p_\ell)\left(1-\frac{\frac{\mathsf{f}(\gamma^\star_L)}{\gamma^\star_L}(N+\gamma^\star_L)}{\frac{\mathsf{f}(\beta^\star)}{\beta^\star}(N+\beta^\star)}\right) \geq \xi_{\min}.
\end{aligned}$$

This concludes the proof of Proposition 3.4.

APPENDIX C
PROOF OF THEOREM 4.6

In this section, we characterize the equilibria of the two-player sensing game. The first important remark is that the NE utilities are always dominated by the SE utilities. This implies that the following equation holds for any parameters $\alpha \geq 0$:

$$U_1(\mathsf{NS}_1, \mathsf{S}_2) \leq U_1(\mathsf{S}_1, \mathsf{S}_2)$$
$$U_2(\mathsf{S}_1, \mathsf{NS}_2) \leq U_2(\mathsf{S}_1, \mathsf{S}_2).$$

Thus, the action $(\mathsf{S}_1, \mathsf{S}_2)$ is not an equilibrium of the game. To compute the equilibria of this game, it remains to compute the following differences:

$$U_1(\mathsf{NS}_1, \mathsf{NS}_2) - U_1(\mathsf{S}_1, \mathsf{NS}_2)$$
$$U_2(\mathsf{NS}_1, \mathsf{NS}_2) - U_2(\mathsf{NS}_1, \mathsf{S}_2).$$

The foregoing differences are equal and do not depend on a particular player. We provide the proof of Theorem 4.6. Suppose first that condition (C2) is met, i.e.,

$$\begin{aligned}
&\alpha < \frac{\beta^\star - \gamma^\star}{1 - \beta^\star\gamma^\star}\\
&\iff \frac{1 - \gamma^\star\beta^\star - \beta^\star - \gamma^\star}{(1 - \gamma^\star\beta^\star)} < 1 - \alpha\\
&\iff \frac{\mathsf{f}(\beta^\star)}{\beta^\star}(1 - \beta^\star) < (1 - \alpha)\frac{\mathsf{f}(\beta^\star)}{\beta^\star}\frac{1 - \beta^\star\gamma^\star}{1 + \gamma^\star}\\
&\iff \frac{R_i g_i \mathsf{f}(\beta^\star)(1 - \beta^\star)}{\sigma^2\beta^\star} < (1 - \alpha)\frac{R_i g_i \mathsf{f}(\beta^\star)(1 - \gamma^\star\beta^\star)}{\sigma^2\beta^\star(1 + \gamma^\star)}.
\end{aligned}$$

The last inequality implies that the games have two pure equilibria $(\mathsf{NS}_1, \mathsf{S}_2)$, $(\mathsf{S}_1, \mathsf{NS}_2)$ and one strictly mixed equilibrium (x^*, y^*) defined by (25). If condition (C1) is satisfied, then the strategies (S_1) and (S_2) are dominated, and then the game has one pure equilibrium $(\mathsf{NS}_1, \mathsf{NS}_2)$, and if condition (C3) is met, the game has an infinite number of NEs.

APPENDIX D
MIXED NASH EQUILIBRIA

If condition (C2) is met, the sensing game has a strictly mixed equilibrium. In this section, we provide a characterization of the mixed equilibrium strategies (x^*, y^*) using the indifference principle

$$\begin{aligned}
&\frac{R_1 g_1 \mathsf{f}(\beta^\star)(1 - \beta^\star)}{\sigma^2\beta^\star} \cdot y_2 + \frac{R_1 g_1 \mathsf{f}(\gamma^\star)(1 - \gamma^\star\beta^\star)}{\sigma^2\gamma^\star(1 + \beta^\star)} \cdot (1 - y_2)\\
&\quad = (1 - \alpha)\frac{R_1 g_1 \mathsf{f}(\beta^\star)(1 - \gamma^\star\beta^\star)}{\sigma^2\beta^\star(1 + \gamma^\star)} \cdot y_2\\
&\qquad + (1 - \alpha)\frac{R_1 g_1 \mathsf{f}(\beta^\star)(1 - \beta^\star)}{\sigma^2\beta^\star} \cdot (1 - y_2)
\end{aligned}$$

which is equivalent to

$$\begin{aligned}
y_2 \cdot \Bigg[&\frac{R_1 g_1 \mathsf{f}(\beta^\star)(1 - \beta^\star)}{\sigma^2\beta^\star} - (1 - \alpha)\frac{R_1 g_1 \mathsf{f}(\beta^\star)(1 - \gamma^\star\beta^\star)}{\sigma^2\beta^\star(1 + \gamma^\star)}\\
&+(1 - \alpha)\frac{R_1 g_1 \mathsf{f}(\beta^\star)(1 - \beta^\star)}{\sigma^2\beta^\star} - \frac{R_1 g_1 \mathsf{f}(\gamma^\star)(1 - \gamma^\star\beta^\star)}{\sigma^2\gamma^\star(1 + \beta^\star)}\Bigg]\\
&= (1 - \alpha)\frac{R_1 g_1 \mathsf{f}(\beta^\star)(1 - \beta^\star)}{\sigma^2\beta^\star} - \frac{R_1 g_1 \mathsf{f}(\gamma^\star)(1 - \gamma^\star\beta^\star)}{\sigma^2\gamma^\star(1 + \beta^\star)}.
\end{aligned}$$

Then, we obtain

$$y_2 = \frac{(1 - \alpha)\frac{\mathsf{f}(\beta^\star)}{\beta^\star}(1 - \beta^\star) - \frac{\mathsf{f}(\gamma^\star)}{\gamma^\star}\frac{1 - \gamma^\star\beta^\star}{1 + \beta^\star}}{X}$$

with $X = (1-\alpha)(\mathsf{f}(\beta^\star)/\beta^\star)(1-\beta^\star) - (\mathsf{f}(\gamma^\star)/\gamma^\star)(1 - \gamma^\star\beta^\star/1+\beta^\star) + (\mathsf{f}(\beta^\star)/\beta^\star)(1 - \beta^\star) - (1 - \alpha)(\mathsf{f}(\beta^\star)/\beta^\star)(1-\gamma^\star\beta^\star/1 + \gamma^\star)$. Replacing the above y_2 into the indifference equation, we obtain the utility of player 1 at the mixed equilibrium

$$U_1(x_2, y_2) = \frac{R_1 g_1}{\sigma^2}\left(\frac{(1 - \alpha)\frac{\mathsf{f}(\beta^\star)}{\beta^\star}(1 - \beta^\star)\frac{\mathsf{f}(\beta^\star)}{\beta^\star}(1 - \beta^\star)}{X} - \frac{\frac{\mathsf{f}(\gamma^\star)}{\gamma^\star}\frac{1-\gamma^\star\beta^\star}{1+\beta^\star}(1 - \alpha)\frac{\mathsf{f}(\beta^\star)}{\beta^\star}\frac{1-\gamma^\star\beta^\star}{1+\gamma^\star}}{X}\right)$$

with $X = (1-\alpha)(\mathsf{f}(\beta^\star)/\beta^\star)(1-\beta^\star) - (\mathsf{f}(\gamma^\star)/\gamma^\star)(1-\gamma^\star\beta^\star/1 + \beta^\star) + (\mathsf{f}(\beta^\star)/\beta^\star)(1-\beta^\star) - (1-\alpha)(\mathsf{f}(\beta^\star)/\beta^\star)(1-\gamma^*\beta^\star/1 + \gamma^\star)$. The same argument applies

$$U_2(x_2, y_2) = \frac{R_2 g_2}{\sigma^2}\left(\frac{(1 - \alpha)\frac{\mathsf{f}(\beta^\star)}{\beta^\star}(1 - \beta^\star)\frac{\mathsf{f}(\beta^\star)}{\beta^\star}(1 - \beta^\star)}{X} - \frac{\frac{\mathsf{f}(\gamma^\star)}{\gamma^\star}\frac{1-\gamma^\star\beta^\star}{1+\beta 2}(1 - \alpha)\frac{\mathsf{f}(\beta^\star)}{\beta^\star}\frac{1-\gamma^\star\beta^\star}{1+\gamma^\star}}{X}\right).$$

APPENDIX E
PROOF OF THEOREM 4.2

The proof of Theorem 4.2 uses [24] (see also [32]).

Theorem E.1: The game G is a potential game if and only if for every player $i, j \in \mathcal{K}$, every pair of actions $s_i, t_i \in \mathcal{A}_i$ and $s_j, t_j \in \mathcal{A}_j$, and every joint action $s_k \in \mathcal{A}_{K\backslash\{i,j\}}$, we have that

$$\begin{aligned}
&U_i(t_i, s_j, s_k) - U_i(s_i, s_j, s_k) + U_i(s_i, t_j, s_k) - U_i(t_i, t_j, s_k)\\
&+U_j(t_i, t_j, s_k) - U_j(t_i, s_j, s_k) + U_j(s_i, s_j, s_k) - U_j(s_i, t_j, s_k) = 0.
\end{aligned}$$

Let us prove that the two conditions provided by our theorem are equivalent to the one in [24]. We introduce the following notation defined for each player $i \in \mathcal{K}$ and each action $T \in \mathcal{A}$:

$$\mu_i = \frac{R_i g_i}{\sigma^2} \quad \text{and} \quad U^T(t_i, t_j, s_k) = \frac{U_i^T(t_i, t_j, s_k)}{\mu_i}.$$

For every player $i, j \in \mathcal{K}$, every pair of actions $s_i, t_i \in \mathcal{A}_i$ and $s_j, t_j \in \mathcal{A}_j$, and every joint action $s_k \in S_{K\setminus\{i,j\}}$, we have the following equivalences:

$$\begin{aligned} &U_i(\mathsf{NS}_i, \mathsf{S}_j, \mathsf{S}_k) - U_i(\mathsf{S}_i, \mathsf{S}_j, \mathsf{S}_k) \\ &\quad + U_i(\mathsf{S}_i, \mathsf{NS}_j, \mathsf{S}_k) - U_i(\mathsf{NS}_i, \mathsf{NS}_j, \mathsf{S}_k) \\ &= U_j(\mathsf{NS}_i, \mathsf{S}_j, \mathsf{S}_k) - U_j(\mathsf{NS}_i, \mathsf{NS}_j, \mathsf{S}_k) \\ &\quad + U_j(\mathsf{S}_i, \mathsf{NS}_j, \mathsf{S}_k) - U_j(\mathsf{S}_i, \mathsf{S}_j, \mathsf{S}_k) \\ &\iff \begin{cases} \mu_i = \mu_j \\ U_L(F+1, L+1) - U_L(F, L+2) + (1-\alpha) \\ \times (U_F(F+1, L+1) - U_F(F+2, L)) = 0. \end{cases} \end{aligned}$$

Thus, the sensing game is a potential game if and only if one of the two following conditions is satisfied:

$$\begin{aligned} &\forall (i,j) \in \mathcal{K}, \quad R_i g_i = R_j g_j \\ &U_L(F+1, L+1) - U_L(F, L+2) \\ &\quad = (1-\alpha)\left[U_F(F+2, L) - U_F(F+1, L+1)\right]. \end{aligned}$$

Appendix F
Proof of Theorem 4.4

The proof of Theorem 4.4 follows the same line of the proof in Appendix E. It suffices to show that the auxiliary game defined as follows is a potential game:

$$\widetilde{G} = \left(K, (\mathcal{A})_{i\in\mathcal{K}}, (\tilde{U}_i)_{i\in\mathcal{K}}\right) \tag{31}$$

where the utility is defined by the following equations with $\mu_i = (R_i g_i/\sigma^2)$:

$$\tilde{U}_i(s_i, s_{-i}) = \frac{U_i(s_i, s_{-i})}{\mu_i}. \tag{32}$$

From the above demonstration, it is easy to show that, for every player $i, j \in \mathcal{K}$, every pair of actions $s_i, t_i \in \mathcal{A}_i$ and $s_j, t_j \in \mathcal{A}_j$, and every joint action $s_k \in \mathcal{A}_{K\setminus\{i,j\}}$, we have the following equality:

$$\begin{aligned} &\tilde{U}_i(t_i, s_j, s_k) - \tilde{U}_i(s_i, s_j, s_k) + \tilde{U}_i(s_i, t_j, s_k) - \tilde{U}_i(t_i, t_j, s_k) \\ &= \tilde{U}_j(t_i, s_j, s_k) - \tilde{U}_j(t_i, t_j, s_k) + \tilde{U}_j(s_i, t_j, s_k) - \tilde{U}_j(s_i, s_j, s_k). \end{aligned}$$

Using [24], we conclude that the sensing game is a weighted potential game.

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