National Institute of Technology, Calicut

Machine Learning Assignment 1

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Topic - Vector

Contents

1 Introduction

Vectors are geometrical entities that have magnitude and direction. A vector can be represented by a line with an arrow pointing towards its direction and its length represents the magnitude of the vector.

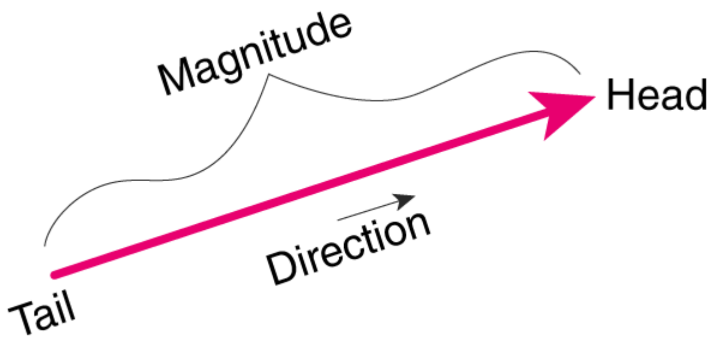


Figure 1 - Vector

2 Some Important Terms in Vector

2.1 Magnitude of a Vector

The magnitude of a vector can be calculated by taking the square root of the sum of the squares of its components. If (x,y,z) are the components of a vector A, then the magnitude formula of **A** is given by,

$$|A| =$$

$$\sqrt{x^2 + y^2 + z^2}$$

The magnitude of a vector is a scalar value.

2.2 Angel Between Two Vectors

The angle between two vectors can be calculated using the dot product formula. Let us consider two vectors **a** and **b** and the angle between them to be

θ. Then, the dot product of two vectors is given by $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}| \cos\theta$. The angle between two vectors also indicates the directions of the two vectors. θ can be evaluated using the following formula:

$$\theta = cos^{-1}(\frac{\mathbf{u.v}}{|\mathbf{u}||\mathbf{v}|})$$

3 Types Of Vectors

The vectors are termed as different types based on their magnitude, direction, and their relationship with other vectors. Let us explore a few types of vectors and their properties:

3.1 Zero Vectors

Vectors that have 0 magnitude are called zero vectors, denoted by $\overrightarrow{0} = (0, 0, 0)$. The zero vector has zero magnitudes and no direction. It is also called the additive identity of vectors.

3.2 Unit Vectors

Vectors that have magnitude equals to 1 are called unit vectors, denoted by $\hat{a}$. It is also called the multiplicative identity of vectors. The magnitude of a unit vectors is 1. It is generally used to denote the direction of a vector.

3.3 Position Vectors

Position vectors are used to determine the position and direction of movement of the vectors in a three-dimensional space. The magnitude and direction of position vectors can be changed relative to other bodies. It is also called the location vector.

3.4 Equal Vectors

Two or more vectors are said to be equal if their corresponding components are equal. Equal vectors have the same magnitude as well as direction. They may have different initial and terminal points but the magnitude and direction must be equal.

3.5 Negative Vector

A vector is said to be the negative of another vector if they have the same magnitudes but opposite directions. If vectors A and B have equal magnitude but opposite directions, then vector A is said to be the **negative of vector** B or vice versa.

3.6 Parallel Vectors

Two or more vectors are said to be parallel vectors if they have the same direction but not necessarily the same magnitude. The angles of the direction of parallel vectors differ by zero degrees. The vectors whose angle of direction differs by 180 ° are called antiparallel vectors, that is, antiparallel vectors have opposite directions.

3.7 Orthogonals Vectors

Two or more vectors in space are said to be orthogonal if the angle between them is 90°. In other words, the dot product of orthogonal vectors is always 0.

$$a.b = |a|.|b| \cos 90° = 0$$

3.8 Co-initial Vectors

Vectors that have the same initial point are called co-initial vectors.

4 Properties Of Vectors

The following properties of vectors help in better understanding of vectors and are useful in performing numerous arithmetic operations involving vectors.

The addition of vectors is commutative and associative.

1)$\overrightarrow{A}.\overrightarrow{B} = \overrightarrow{B}.\overrightarrow{A}$

2)$\overrightarrow{A} * \overrightarrow{B} = \overrightarrow{B} * \overrightarrow{A}$

3)$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

4)$\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

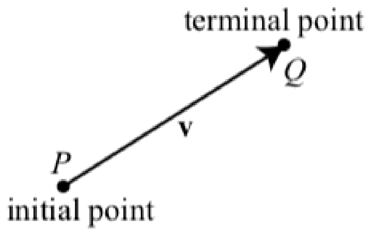
5)$\hat{i} * \hat{i} = \hat{j} * \hat{j} = \hat{k} * \hat{k} = 0$

$$6)\hat{i} * \hat{j} = \hat{k}; \hat{j} * \hat{k} = \hat{i}; \hat{k} * \hat{i} = \hat{j}$$
$$7)\hat{j} * \hat{i} = \hat{-k}; \hat{k} * \hat{j} = \hat{-i}; \hat{i} * \hat{k} = \hat{-j}$$

5 Notations Of A Vector

5.1 Geometric

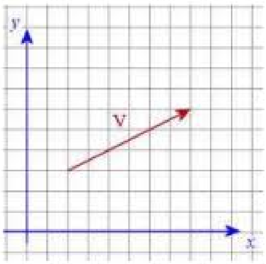
Here we use an arrow to represent a vector. Its length is its **magnitude**, and its direction is indicated by the direction of the arrow.



The vector here can be written **PQ** or $\overrightarrow{\mathbf{PQ}}$ with an arrow above it. Its magnitude (or length) is written $|\mathbf{PQ}|$.

5.2 Rectangular Notation $\left\langle \mathbf{a, b} \right\rangle$

A vector may be located in a rectangular coordinate system, as is illustrated here.



The rectangular coordinate notation for this vector is $\mathbf{v} = \left\langle 6, 3 \right\rangle or \overrightarrow{\mathbf{v}}$ $\left\langle 6, 3 \right\rangle$. Note the use of **angle brackets** here.

An alternate notation is the use of two **unit vectors** $\hat{\imath} = \left\langle 1, 0 \right\rangle$ and $\hat{\jmath} = \left\langle 0, 1 \right\rangle$ so that

$$\mathbf{v} = 6\hat{\imath} + 3\hat{\jmath}$$

5.3 Polar Notation $\left\langle r\angle\theta \right\rangle$

In this notation we specify a vector's magnitude r, $r \geq 0$, and its angle θ with the positive x-axis, $0° \leq \theta < 360°$. In the illustration above, $r \approx 6.7$ and $\theta \approx 27°$ so that we can write

$$\overrightarrow{\mathbf{v}} = \left\langle 6.7 \angle 27° \right\rangle$$

6 Operations On Vectors

6.1 Scalar Multiplication

Geometrically, a scalar multiplier $k < 0$ can change the length of the vector but not its direction. If $k > 0$, then the scalar product will "reverse" the direction by $180°$.
In rectangular form, if k is a scalar then

$$k\left\langle a, b\right\rangle = \left\langle ka, kb\right\rangle$$

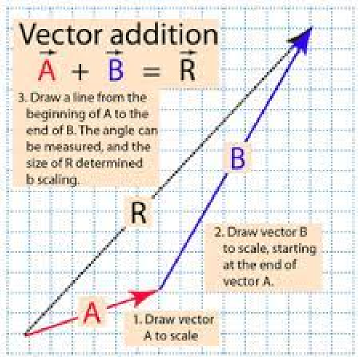
In the case of a polar form vector

$$k\left\langle r\angle\theta\right\rangle = \begin{cases} \left\langle kr\angle\theta\right\rangle & if k \geq 0 \\ \\ \left\langle |\mathbf{kr}|\angle\theta \pm 180° \right. & if k < 0 \end{cases} \tag{1}$$

In the case where $k < 0, choose\, \theta + 180°\, if\, 0° \leq \theta < 180°. Choose\, \theta - 180°\, if\, 180° \leq \theta < 360°$

6.2 Vector Addition

In geometric form, vectors are added by the **tip-to-tail** or **parallelogram** method



In rectangular form, if $\mathbf{u} = \left\langle a, b \right\rangle and\ \mathbf{v} = \left\langle c, d \right\rangle then$

$$\mathbf{u} + \mathbf{v} = \left\langle a + c, b + d \right\rangle$$

It's easy in rectangular coordinates. The sum of two vectors is called the **resultant**. In polar coordinates there are two approaches, depending on the information given.
1. Convert polar form vectors to rectangular coordinates, add, and then convert back to polar coordinates.
2. If the magnitudes of the two vectors and the angle between is given (but not the directions of each vector), then a triangle sketch with a Law of Cosines solution is used.

6.3 Vector Dot Product

If if $\mathbf{u} = \left\langle a, b \right\rangle and\ \mathbf{v} = \left\langle c, d \right\rangle then$ the dot product of $\mathbf{u}$ and $\mathbf{v}$ is

$$\mathbf{u.v} = \text{ac + bd}$$

The dot product may be positive real number, 0, or a negative real number. If the magnitudes of the two vectors are known and the angle θ between them is known, then

$$\mathbf{u.v} = |\mathbf{u}||\mathbf{v}| \cos \theta$$

This last formula can be used to find the angle between two vectors whose rectangular forms are given

$$\cos \theta = \frac{\mathbf{u.v}}{|\mathbf{u}||\mathbf{v}|}$$

6.4 Cross Product Of Vectors

If A and B are two independent vectors, then the result of the cross product of these two vectors (Ax B) is perpendicular to both the vectors and normal to the plane that contains both the vectors. It is represented by:

$$AxB = |A||B| \sin \theta$$

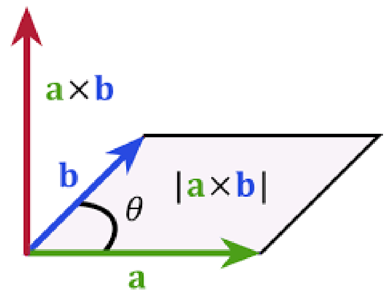


figure 2 - Cross Product Of Two Vectors

7 Examples

Example 1 : If a = 3i +2j + 5k and b = i + 4j +6k, find **a x b** and **b x a** .
solution :

a x b = (3)(1)(i x i) + (3)(4)(i x j) + (3)(6)(i x k) +
(2)(1)(j x i) + (2)(4)(j x j) + (2)(6)(j x k) +
(5)(1)(k x i) + (5)(4)(k x j) + (5)(6)(k x k)
= 12k – 18j – 2k + 12i +5j – 20i = –8i – 13j + 10k

b x a = 2k – 5j – 12k + 20i + 18j – 12i = 8i + 13j – 10k

Example 2 : Compute the angle between two vectors 3i + 4j – k and 2i – j + k.
solution :

Let $\overrightarrow{a}$= 3i + 4j – k and $\overrightarrow{b}$= 2i – j + k The dot product is defined as = (3i

+ 4j – k).(2i – j + k)
= (3)(2) + (4)(-1) + (-1)(1)
= 6-4-1
= 1
thus , $\overrightarrow{a}.\overrightarrow{b} = 1$
The Magnitude of vectors is given by
$|\overrightarrow{a}| =$

$$\sqrt{3^2 + 4^2 + (-1)^2} = \sqrt{26} = 5.09$$

$$|\overrightarrow{b}| =$$

$$\sqrt{2^2 + (-1)^2 + (1)^2} = \sqrt{6} = 2.45$$

The Magnitude of vectors is given by
$\cos\theta = \frac{\mathbf{a.b}}{|\mathbf{a}||\mathbf{b}|}$

$$\theta = cos^{-1}(\frac{\mathbf{1}}{\mathbf{5.09*2.45}})$$

$$\theta = 85.39°$$