# INDR 460 | Homework 4

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Part a) Formulate a mathematical model to find the optimal investment plan that maximizes the expected return without exceeding the VTR limit of V (dollar square).

Solution:

Let the A be the decision variable associated with amount (in $) invested in asset 1 and B be the decesion variable associated with amount (in $) invested in asset 2. Then according to given constrained of total amount of 6M $: B = $6*10^6 - A$

Let the random variable for return in asset 1 be R1, and for asset 2 be R2.
Let E[x] denote the expectation of random variable x, and Var(x) denote its variance.

E[R1] = 0.12, E[R2] = 0.06, Var(R1) = 0.01, Var(R2) = 4/30000

Let total available amount be $L = 6*10^6$
Mathematical model.

$$MAXIMIZE: E[R1 * A + R2 * (L - A)]$$

$$Constraint\ 1: Var\big(R1 * A + R2 * (L - A)\big) \leq V$$

$$Constraint\ 2: L \geq A \geq 0$$

After simplification:

$$MAXIMIZE: 0.12 * A + 0.06 * (L - A)$$

$$Constraint\ 1: 0.01 * A^2 + \frac{4}{30000}(L - A)^2 \leq V$$

$$Constraint\ 2: L \geq A \geq 0$$

Part b) Find the optimal solution for V={$10^{11}$, $2*10^{11}$, $4*10^{11}$}. Discuss whether the solution you have found is a local or global optimum.

Solution:

| Case | Fraction amount invested in Asset 1 | Fraction amount invested in Asset 2 | Expected Return(Objective function) | Expected Risk |
|---|---|---|---|---|
| V = $10^{11}$ | 0.524174 | 0.475825 | $548702 | $10^{11}$ |
| V = $2*10^{11}$ | 0.744773 | 0.255226 | $628118 | $2*10^{11}$ |
| V = $4*10^{11}$ | 1 | 0 | $720000 | $3.6*10^{11}$ |

Since the objective function is convex (since it is a linear function) and convex set is convex (since it is a quadratic function) the above solutions are global optimums.

Part c) Now assume that it is not possible to invest more than $1M in asset-2 and solve the problem for V=$4x10^{11}$. How does your solution in part b changes? What are the binding constraints?

Solution:
Additional constraint is added in the above formulation:

$$Constraint\ 3\ :\ L - A \leq \frac{L}{6}$$

Fraction amount invested in Asset 1: 1
Fraction amount invested in Asset 1: 0
Expected Return(Objective function): $720000

The solution does not change in this case since this constraint is already satisfied in Part b for V=$4*10^{11}$. This can be observed in the above table where Percentage amount invested in Asset 1 for V = $4*10^{11}$ is 0, i.e. it is already less than $1M.

The only binding constrains is:

$$L \geq A$$

Which is the total invested amount.

Part d) Modify your formulation in part a to implement the regulation that total amount invested in asset-1 cannot be more than two times of the total amount invested in asset-2. Solve the problem with this new restriction and V=4x1011. What is the maximum return in this case? Comment on how is this solution different from the one you get in part b.

Solution:

Additional constraint is added in the part a formulation:

$$Constraint\ 3\ :\ A \leq 2 * (L - A)$$

Fraction amount invested in Asset 1: 0.6666666
Fraction amount invested in Asset 1: 0.3333333
Expected Return (Objective function): $600000

The maximum value of objective function in part b for case V=4*$10^{11}$ is more than Expected return in this part. That means this additional constraint is reducing feasible region and this constraint is new binding constraint.