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How Students' Everyday Situations Modify Classroom Mathematical Activity: The Case of Water Consumption

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Our aim is to discuss how school mathematical activity is modified when students' everyday situations are brought into the classroom. One illustrative sequence—7th grade classes solving problems that required proportional reasoning—is characterized as a system of interconnected activities within the theoretical perspective of activity theory. We discuss the tensions and contradictions that evolve when a generic school procedure emphasized by the teacher meets the specific procedures applicable to everyday situations proposed by the students. We evaluate the modifications that we perceived in the power relationships and other components of the school activity and the expansion of the meaning of those procedures as positive outcomes of how everyday situations were dealt with in school mathematics.

Key words: Everyday situations; Classroom mathematical activity; Proportional reasoning; Tensions and contradictions

In this article, we aim to deepen the discussion about the relationship between what we call school mathematics and everyday mathematics. Although research has not yet fully explained this relationship and sometimes even points to difficulties regarding the articulation between the two (e.g., Brenner & Moschkovich, 2002), we believe this is an important aim to be pursued. There is a current tendency in many school curricula and also in large-scale assessments to stress the connections between school mathematics and everyday mathematics. This is the case in Brazil and also the United States. For example, in *Principles and Standards for School Mathematics,* the National Council of Teachers of Mathematics (2000) stated that, in addition to the connections that should be explored and developed within the mathematical content, from prekindergarten through Grade 12, instructional programs should also enable all students to "recognize and apply mathematics in contexts outside of mathematics" (p. 65). Likewise, the Program for International Student Assessment is said to focus on "real-world problems, moving beyond the kinds of situations and problems typically encountered in school classrooms" (Organisation for Economic Co-operation and Development, 2009, p. 84).

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Our research is founded on the following two assumptions. First, *school mathematics* comprises the set of practices and knowledge associated with the development of the process of basic school education in mathematics, which is not restricted to what is taught in school because it also includes, for example, the teachers' knowledge originating from this process. Second, *everyday mathematics* comprises ideas, procedures, and knowledge—matched with school mathematics—that are normally used in out-of-school contexts. By taking a cultural historical perspective of activity, grounded in the ideas of Leont'ev (1978), Engeström (1987, 1996, 2001), and Engeström and Sannino (2010), we look at school mathematics as a coordinated system of shorter term activities.

A vast body of literature considers the relationship between school and everyday mathematics from a variety of perspectives. In Brenner and Moschkovich (2002), the practices of mathematicians (academic mathematics) were also considered in a discussion of this relationship in order "to provide a theoretical basis for evaluating the advantages and disadvantages of everyday mathematical activities as resources for student activity in mathematics classrooms" (p. v). Several other studies have emphasized out-of-school practices, stressing the mathematical ideas involved in them to legitimize the mathematics of different cultural groups (D'Ambrósio, 1985; Ferreira, 2006; Knijnik, 2006; Lave, 1988).

Taking a different standpoint by pointing out the differences between out-of-school and school mathematics, de Abreu (2008) analyzed the relationship between home and school mathematics and discussed how both are reflected in multicultural classrooms. In Brazil, researchers have also examined the differences between school and everyday mathematics; the work of Carraher, Carraher, and Schliemann (1985) is a major landmark in this area. They examined the mathematics used by children selling coconuts in the streets of Recife and how it differed from the mathematics the children experienced at school.

Although these studies did not necessarily deepen the discussion of mathematics learning in formal teaching situations, they did stress some of the differences between school and out-of-school mathematics. Thus they serve as an important background for our research, which focuses on school mathematics when real-world problems and situations are brought into the classroom.

Other researchers, such as Skovsmose (1994), have placed greater emphasis on school mathematics. Following the direction of the work of Powell and Frankenstein (1997), who reflected on what "counts" in mathematics education, Skovsmose underscored the political aspects of mathematics education. He highlighted the relevance of reflecting on which type of mathematics is prevalent in school curricula, and he advocated for the inclusion of classroom discussions related to the social role of mathematics based in real-world situations as a means to ensure democratic access to powerful mathematical ideas.

Moreover, there is extensive literature aligned with the perspectives of *realistic mathematics education* (RME; e.g., Doorman et al., 2007) and the Mathematics in Context curriculum (e.g., Fennema & Romberg, 1999), which describes learning trajectories, different kinds of everyday mathematics tasks, and the effectiveness of the real world as a source for the development of mathematical concepts. The literature also contains descriptions of how instruction changes when everyday mathematical practices are brought into the classroom (e.g., Bonotto, 2007; Brenner, 2002; Planas & Civil, 2002). Bonotto (2007), in particular, stressed the importance of "encouraging students to analyze '*mathematical facts*' embedded in appropriate '*cultural artefacts*'" (pp. 187–188). Cobb, Zhao, and Visnovska (2008) introduced and discussed three adaptations to the theory of RME after conducting a sequence of classroom design experiments. They have taken a broader perspective on the means of supporting teachers in achieving their instructional agendas, rather than directly supporting students' learning.

Although our work has a lot in common with the previously mentioned studies, we have taken a different perspective in discussing how classroom activity changes when everyday mathematics is brought in. Whereas the other studies emphasized different types of problems, artifacts, and instructional designs, we focused on the tensions and contradictions perceived within classroom activity and the modifications they triggered. In the literature, we found very few studies addressing issues related to possible tensions or difficulties with the articulation between school and everyday mathematics. Moreover, the tensions, conflicts, or difficulties identified by researchers tend to be described in a very generic manner and do not have the leading role we ascribe to them in our analysis. By way of illustration, we contrast each of those studies with the focus of our research.

In contrast to our research, which is circumscribed to school activity, Rangnes (2011) reported a study in which a group of eighth graders were engaged in two different mathematical practices (i.e., school practice and a building company practice) with different goals for the use of mathematics. The analysis, based on Bakhtin's (1981) idea of dialogicity, illustrated how the encounter with the two practices and goals created tensions that influenced the pupils' positioning and decision making within the school activity.

The work by Civil (2002) characterized three kinds of mathematics—*everyday mathematics, mathematicians' mathematics,* and *school mathematics*—and examined some relationships between them. Brenner and Moschkovich (2002) described Civil's work as a teaching experiment conducted in a fifth-grade classroom that took "children's everyday activities as starting points for exploring mathematics from a mathematician's perspective" (p. viii–ix). Civil discussed very broad differences between those forms of mathematics and, at a more specific level, analyzed how some tensions were reflected in changes in the patterns of the students' classroom participation as the situations moved to more formal mathematics. In our work, the concepts of tensions and contradictions took a leading role, deepening the discussion of the changes in classroom activity beyond the changes in the students' participation.

Taking a closer theoretical perspective to the one adopted in our study, Rønning (2010) used the framework of Leont'ev's (1981) activity theory to examine the discursive practices of pupils when presenting everyday and school solutions for a measuring problem. He analyzed the teacher's and the pupils' goals and actions in the lesson and postulated that "there is some tension between the teacher's and the pupils' goals, and that this tension is due to the fact that the lesson is operating on the border between a school practice and an everyday practice" (Rønning, 2010, p. 1020). However, Rønning did not extend the discussion about changes in the activity related to these tensions, as we propose.

Jurdak (2006) also discussed differences between school mathematics and everyday mathematics. Although the author did not mention possible tensions or conflicts between these forms of knowing mathematics, we consider his work important because he also used activity theory (Engeström, 1987; Leont'ev, 1981) to analyze and compare problem solving in the real world with solving *situated problems* (problem tasks conducted in the school setting that simulate real-world situations). Three situated problems, to be solved in a laboratory context, were given to a group of high school students. They were later interviewed about the task and asked to compare the problems with school and real-world tasks. Jurdak found that the students themselves perceived substantial differences between problem-solving activities in real-world and school settings.

The studies mentioned above were developed in experimental contexts and examined tensions and changes in students' participation and in school mathematical practices, at times emphasizing the students' participation, learning strategies, communication, or the teaching styles or types of problems. In contrast to those studies, we took a natural classroom context and analyzed, in a more comprehensive manner, how school activity unfolded when the teacher brought everyday situations into the classroom in order to show that school mathematics can be applied to solve real-world problems.

Thus, our main objectives were to:

1. Analyze the tensions that may emerge in school mathematical activity when everyday situations are brought into the classroom.
2. Discuss the modifications in the school mathematical activity triggered by the tensions that emerge when students' everyday situations are brought into the classroom.

Theoretical Framework

As stated before, our work is based on the theoretical framework of the cultural-historical activity theory (Engeström, 1987, 1996, 2001; Leont'ev, 1978, 1981), which elects *activity* as the unit of analysis. According to Engeström (2001), activity theory was developed through the contributions of three generations of researchers. The first generation included Vygotsky (1978), with his concept of activity mediated by artifacts. The second generation comprised the innovations proposed by Leont'ev (1978) regarding the levels of the activity. According to Leont'ev, an activity consists of a group of people (*subjects*) engaged in the same goal, with a direction for their work (*object* or *motive* of the activity). The activity emerges from a necessity, which directs the motives towards a related object. To satisfy motives, *actions* are necessary. The actions, in turn, are performed according to the *conditions* of the activity, which determine the *operations* related to each action. Therefore, in the structure proposed by Leont'ev, the activity, directed to a motive or object, is located on the first level. On the second level, there are actions directed to specific objectives. The third level consists of the operations or routines that keep the system functioning and are dependent upon the conditions of the activity.

The third and current generation of researchers, represented by Engeström and Sannino (2010), have developed new concepts to expand the unit of analysis to a system of a minimum of two interacting activities and to give prominence to the concept of contradiction as a driving force for transformations in the activity system. Engeström (1987) started from Leont'ev's idea that activity is a set of mediated subject–object interactions (Leont'ev, 1978) and introduced the community into these interactions as another component. For Engeström, an activity is always understood as a collective phenomenon in a community, and individuals can only perform actions inside a larger system of collective activities. Therefore, according to Engeström, there is a close relation between an activity and an activity system because every activity can be seen as a system of at least two interconnected activities, and every system of activities with a shared object can be seen as an activity.

Thus, the researcher may train the lenses of analysis towards a specific activity, sometimes considering it an activity by itself, sometimes zooming in or out to gain different understandings about the activity. By zooming in, it is possible to see the activity as a system of two or more interconnected activities, and by zooming out, to see it as one of the activities composing a wider activity system.

According to Engeström (1987), the object of an activity can be expressed as concern, motivation, effort, and meaning. For him, the object of an activity is predominantly related to producing an outcome and is not necessarily its real motive. Because individual needs and motives must also be considered, this opens the possibility of making a distinction between the object of the collective activity and the subjects' motives, for example, between the motive of the teacher and of the students.

Through participation in multiple activities, people constantly change and create new objects for their activity. These new objects may originate from a single activity, or from a variety of interconnected activities. Accordingly, multiple activity systems interacting with each other may be directed to a potentially shared object. The collective activity object is an invitation to interpretation—the making of personal sense and social transformation. As Engeström and Sannino (2010) asserted,

> One needs to distinguish between the generalized object of the historically evolving activity system and the specific object as it appears to a particular subject, at a given moment, in a given action. The generalized object is connected to societal meaning, [sic] the specific object is connected to personal sense. (p. 6)

The authors also concluded that the tensions between the generalized and the specific object can evolve into a deterioration of the situation, which can be changed by an "expansive learning process in which the two parties together generate a new shared object and concept for their shared activity" (Engeström & Sannino, 2010, p. 6).

To depict a collective activity system, we offer a model composed of triangles, adapted from Engeström and Sannino (2010), that includes elements from Vygotsky's (1978) and Leont'ev's (1978) models of activity (see Figure 1). In the nodes of the activity system, we place the following components: subject, object, mediating artifacts, community, division of labor, and rules. In this model, *subject* consists of an individual or group engaged in a common purpose whose agency is the focus of the analysis; *object* is the "problem space" toward which the activity is directed and which is molded and transformed into outcomes; *mediating artifacts* are instruments, tools, and signs; *community* refers to people who share the same object; *division of labor* is the division of tasks, power, and status between the members of the community; and *rules* "refer to the explicit or implicit regulations, norms, conventions and standards that constrain actions [and interactions] within the activity system" (Engeström & Sannino, 2010, p. 6). The circle around the *object* is meant to draw attention to the fact that, explicitly or implicitly, object-oriented actions are always characterized by ambiguity, surprise, interpretation, sense making, and potential for change and are also related to an outcome.

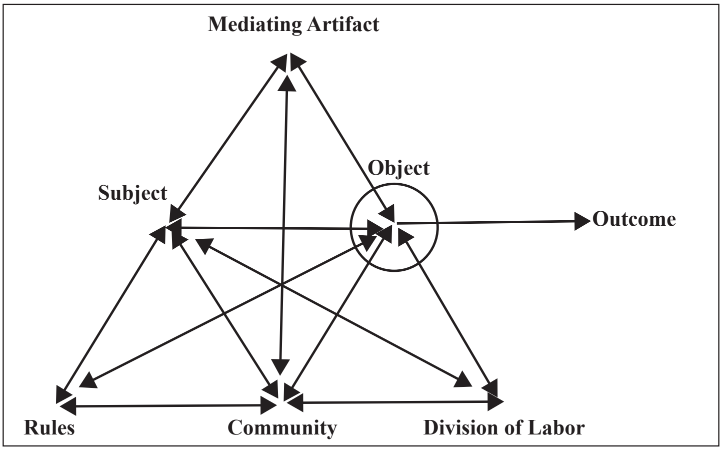


Figure 1. Collective activity system. From "Studies of expansive learning: Foundations, findings and future challenges," by Y. Engeström and A. Sannino, 2010, *Educational Research Review, 5*(1), p. 6. Copyright by Elsevier. Adapted with permission.

Like Engeström (1990), we consider *mediating artifacts* to be transitional, fluid entities (p. 189), and we maintain that deciding if something is an artifact or another component of the activity depends on the focus of the analysis. For example, in David and Tomaz (2012), we have shown how, depending on the level of analysis, an artifact (a drawing of a geometrical figure) can also be seen as the object or subject of an activity system.

According to Engeström (1996), when using activity theory to analyze an activity system, one should consider that it is always connected to other activity systems through some of its components (p. 72). If one component of the activity system changes, other changes must take place in order to adjust the whole system; that is, every component is in a close relationship with all the others. In doing this sort of analysis, one should also consider three basic principles. First, the unit of analysis should be the activity system itself, giving context and meaning to individual events. In our special case of classroom activity, we consider that the unit of analysis is the activity system of the students and teacher in the classroom. Second, according to the historicity principle, "activity systems take shape and get transformed over lengthy periods of time. Their problems and potentials can only be understood against their own history" (Engeström, 2001, p. 136). The analysis of an activity and its constituent components should be done both locally, within its inner dynamic relationships and historical changes, and taking a more global historical perspective, in relation with other activities composing a bigger activity system. Thus, to analyze a particular classroom activity one should not only discuss if and how the object, rules, and other components of this activity changed over time but also if and how they have changed in relation to other selected activities. Third, the inner contradictions inherent in human activity may be analyzed as a source of perturbations, innovations, changes, and development in the activity and in its individual participants. For Engeström (1987), "the basic inner contradiction of human activity is its dual existence as the total societal production and one specific production among many. Any specific production is at the same time independent of and subordinated to the total societal production" (p. 82).

Contradictions, according to Engeström (2001), are not the same as problems or conflicts; they are historically accumulated structural tensions within and between activity systems. Contradictions generate the questioning of practices by the subjects and cause ruptures that can produce fragmentation or *expansive transformations* of the activity when tensions and contradictions are overcome.

> An expansive transformation is accomplished when the object and motive of the activity are reconceptualized to embrace a radically wider horizon of possibilities than in the previous mode of the activity. (Engeström, 2001, p. 137)

Engeström (1987) developed the expansive learning theory, which emerged as a result of the multidimensional treatment of the learner as an individual and as a community. Engeström and Sannino (2010) explained that the core idea of expansive learning is qualitatively different from the ideas related to the metaphors of acquisition and participation; it relies on the metaphor of expansion, according to which learners learn something that is not yet there.

Engeström (1987) based this learning theory on the ideas of Russian cultural-historical school representatives such as Vygotsky (1978), Leont'ev (1978), and Davydov (1990) as well as the work of Bateson (1972) and Bakhtin (1981). According to the theory of expansive learning, contradictions are the necessary but insufficient engine of expansive learning in an activity system, and the process of expansive learning should be understood as the construction and resolution of successively evolving contradictions (Engeström & Sannino, 2010). In expansive learning theory, learning is manifested primarily as changes in the object of the collective activity, in contrast to the traditional perspective wherein learning is manifested as changes in the subject. Thus, contradictions are a key concept in this theory because they may produce changes in the object, expansive transformations of the activity, and new cultural patterns of the collective activity.

Although the cultural-historical perspective of activity was initially developed while considering long-term human activity, Engeström and Sannino (2010) indicated the possibility of using it for short-term analysis. Thus, we have used it to conduct a microanalysis of a school mathematical activity by examining tensions, contradictions, changes, and expansions in the activity when everyday situations are brought into the classroom.

In summary, following the orientation of the third generation of activity theory, we focused our analysis on one system of interconnected activities, which was formed by a constellation of classroom problem-solving activities involving proportional reasoning. Thus, we aimed to contribute to the field of mathematics education by examining problem-solving activities when everyday situations were brought into classroom tasks and by deepening the theoretical awareness of mathematical classroom activity.

Methods

Origin of the Study

In this article, we present a reinterpretation of a sequence of classes analyzed in a larger study (Tomaz, 2007) that examined what and how students learn when they participate in interdisciplinary school practices. The study involved three teachers from a public school in Brazil who taught mathematics, Portuguese, and geography. The data for the larger study were collected by one of the authors through participant observations in four classrooms (two seventh-grade and two eighth-grade classrooms, each with 35 students) for a period of 6 months. Data were also collected through interviews with students and teachers. We produced field notes and transcriptions of audio and video recordings for all empirical data. As described below, the study reported in this article utilized only a subset of data[1] from the larger study.

Participants

The participants in this study were the mathematics teacher, Telma,[2] and the students in her two seventh-grade classes (Class 1 and Class 2). The two groups

[1] The data used in this article are part of a database, MediAÇÃO, collected by a research group we belong to that investigates what mathematics is being taught, and how it is being taught, at different school levels.

[2] Pseudonyms are used throughout the article.

of students were very similar in many respects. Both had mixed socioeconomic backgrounds, and the students' ages varied from 13 to 15 years. There was also no significant disparity in the students' performance on mathematics tests or in the level of participation of the students observed in both classes. Telma had more than 23 years of experience as a teacher at the middle and secondary school levels. In the interviews, the students stated that Telma was a good mathematics teacher, and during our observations of her classes, we noticed that she usually succeeded in establishing open and amicable relationships with the students. For the most part, students participated actively in her mathematics lessons.

In the larger study, we observed 50 lessons of Telma's Grade 7 mathematics classes. Each of those lessons followed the same regular pattern: explanation of the subject matter, discussion with the students about different ways of solving the proposed problems, and manipulation and production of artifacts by the students, such as origami, protractors, and measuring tools. At times, the students were encouraged to summarize the subject matter covered. In all of Telma's classes, there were many opportunities for the students to participate, thus, in this sense, Telma's classes were very dynamic. During most of our period of observation, she proposed *typical school problems* to the students, that is, those with a structure such that there was a school procedure perfectly adjustable to them. These could be classified as of *symbolic control* (Walkerdine, 1990) because even if the storyline of the problems sometimes presented elements that could be part of everyday life, they were not directly relevant to these students' lives. Thus, the problems became application exercises of routine calculations.

However, when Telma taught the topic of proportionality, she introduced a sequence of tasks that were different from the problems proposed during the larger period of observation. She and the other teachers in the larger study had decided to develop a collection of lessons related to the theme "Water," which adhered to the main agenda promoted by several religious and civic entities for extensive discussion in the local community. They each developed different activities, within their respective disciplines, to make their students aware of the need to reduce water consumption. The tasks Telma developed carried embedded *cultural artifacts,* similar to the ones stressed by Bonotto (2007), and could be classified as situated problems (Jurdak, 2006) because they were school problems that simulated real-world situations. These lessons provided the main source of data for this study.

Data Selection

Although we recognize the importance of mathematical activity in the work of the other disciplines and the discussions held in other environments and social groups in which the students participated, this study focused only on the work done within the mathematics classroom. Data were drawn from the mathematics lessons in which Telma developed tasks based on the students' actual water bills, which were intended to show the application of a mathematical procedure to real-life situations.

We analyzed a total of six 50-minute lessons. The lessons we analyzed included the three lessons in which Telma introduced the water bill tasks as well as the lessons just before and just after the water bill lessons, which were included because we followed the historicity principle (Engeström, 1996, 2001) in our methodological approach. In the two lessons before the water bill tasks, Telma had introduced the "rule of three" to solve typical school problems involving proportions, and in the lesson after the tasks, she proposed a related problem on water consumption reduction. See the Appendix for a brief history of the rule of three.

Also, data were drawn from interviews conducted with Telma and her students. From the six interviews conducted with the teachers in the larger study, there was only one interview with Telma about the water bill lessons. In addition, 12 students were interviewed about the water bill activity, and to illustrate our analysis, we selected five of them in which we found elements to complement and clarify our records. These five students exhibited, either during class or in the work done, alternative interpretations and procedures. Three students reproduced exactly the same procedure as the teacher and said that they had used a rule of three but that it was a different rule of three; one student did not use the teacher's procedure correctly; and another one used a different (arithmetical) procedure.

Data Analysis

We adopted the perspective of activity theory to structure the observed practices within the six lessons we analyzed. We also used it as an analytical tool to examine the ways in which school mathematical activity was modified when the everyday situation involving the water bill was brought in to Telma's classroom. According to Russell (2009), activity theory provides a level of analysis that goes beyond a microanalysis of phenomena (including discourse) but does not seek to attain macrolevel generalizations, which are common in cultural studies and sociological analysis. He added that many studies that take this intermediate level of analysis use ethnographic and case study methods (Russell, 2009, p. 41).

Aligned with the methods used in other studies, the methodological approach used in this study was also grounded in ethnography as the logic of inquiry in education (Green, Dixon, & Zaharlick, 2003). According to this logic, the focus is on the process, and there are no strict, previously defined protocols for the observations and interviews. Ethnography assumes that it is not possible to avoid a certain degree of subjectivity in the data collection and in the analysis, which is essentially interpretive. However, it is possible for the researcher to achieve the required scientific rigor by carefully describing all the research procedures and by contrasting his or her interpretations with the others subjects' perspectives, for example, through interviews and discussion of the video records.

Accordingly, we assumed in this study that Engeström's (1987) structure of an activity system is an abstraction made by the researcher that is not supposed to rely solely on empirical data. When analyzing a school activity, it is not possible to grasp all its complexity at once in a direct and objective manner. Thus, we initially discussed the field of related research using Leont'ev's three-level structure of an activity (Leont'ev, 1978, 1981). We first proceeded to gather clues, perceptions, and information about the operations and actions of the subjects involved in the activity, which seemed more directly observable than the more comprehensive level of the collective activity itself. We started by looking for the operations or routines that kept the mathematical activity functioning (e.g., solving problems on proportionality using the rule of three in an automatic manner), the conditions they were dependent upon (e.g., being in a mathematics classroom), and the actions performed by the subjects in a more conscious way (e.g., solving problems on proportionality without using the rule of three). Actions are directed to specific objectives (like choosing to solve mathematical problems using your own procedures), and the subjects' objectives are sometimes declared by the subjects themselves but, more commonly, are perceived or inferred by the observer. All the elements and information gathered through this preliminary procedure helped in our initial discussion of the research field and constituted the basis for a more comprehensive and systematic analysis at the level of the collective activity. In this article, such analysis is made in accordance with Engeström's six-component activity structure, which is an expansion of Leont'ev's three-level structure. We structured our unit of analysis as a collective activity system by contrasting empirical evidence extracted from the transcripts[3] of the lessons and interviews with more holistic perceptions and interpretations about what was going on in the school mathematical activity.

According to Engeström (1987), the object of an activity is what gives form and direction to the activity, and each activity has its own object. If the object of an activity changes, the activity is transformed into a different activity. This idea supported our methodological option of first prioritizing the identification of the activities' objects in order to characterize our activity system. We first characterized the observed mathematics lessons as a system of three interconnected activities that composed our unit of analysis: Rule of Three Problems, Family Water Consumption Problems, and Reduction of Water Consumption Problem. These were characterized as activities by (a) reexamining the classroom observations and interviews in order to extract what was giving form and direction to the classroom activity as it was developing to meet specific necessities and (b) identifying three different corresponding objects: the rule of three, applications of the rule of three in solving problems about the students' families' water consumption, and alternatives for reducing water consumption and applying the rule of three in everyday situations.

We then zoomed in on the Family Water Consumption Problems activity by describing the trajectories of actions triggered by the questions proposed by the teacher, which resulted in another subsystem composed of two activities involving the students' water bills. Finally, we made the complementary move of zooming out to the other two activities in this system—Rule of Three Problems and

[3] The transcript's rules are based on Cameron (2001) and Kock (1997). In these references, the "…" represents a pause, "(xxx)" completed imagined text, "(…)" unintelligible text, and "(( ))" comments by the transcriber.

Reduction of Water Consumption Problem—in order to identify inner dynamic relations and historical changes (Engeström, 1996). By zooming in and zooming out to different activities, it was possible to identify the components of each activity and reveal moments when different perspectives came into contact, giving rise to tensions and contradictions in the activity system.

In the narrative that follows, we describe this process of analyzing and interpreting the data through the lens of activity theory. We first present a brief description of the activity system that comprised our unit of analysis, and we describe the procedures followed to identify the components of the activities involved. Then, we examine the tensions and contradictions between the components of the Family Water Consumption Problems activity, and finally we discuss modifications in the school mathematical activity.

Water Theme Activity System

In this study, we analyzed an activity system that pertained to solving proportional reasoning problems about water. It was composed of three interconnected activities: Rule of Three Problems (A_0), Family Water Consumption Problems (A_1), and Reduction of Water Consumption Problem (A_2).

Rule of Three Problems

The first activity comprised two lessons that Telma taught on the rule of three, a conventional procedure to solve typical school problems involving proportions.[4] In these problems, the values of some magnitudes were given, and the aim was to find an unknown value while maintaining the relationships of proportionality. Telma's procedure fits into the *cross-multiply and solve for x* strategy (Post, Behr, & Lesh, 1988) and was associated with a visual display, or scheme, that helped students organize the data to facilitate the application of the procedures to find the unknown value.In Figure 2, this procedure is illustrated with an example involving a hypothetical water consumption problem that Telma used in one of her lessons.

Family Water Consumption Problems

The second activity was the main focus of our analyses. It pertained to a set of three lessons that involved the use of actual water bills that students were asked to bring to class. In the first lesson, Telma assigned a homework problem in which students used their water bills to answer five questions (see Figure 3). Using her own water bill as an example, she explained how to find the necessary information and showed the students how to make the calculations, emphasizing the use of the rule of three as the appropriate procedure. Because several students raised questions about the problem, Telma was compelled to take it up again in the second

[4] The version of the rule of three used by Telma is the "double rule of three direct." This rule provides a way of finding one unknown when five related quantities are known and the pairs of known quantities show directly proportional relationships.

Telma's Resolution of a Problem Using the Rule of Three

In a household where 7 people live, 42 m^3 of water are consumed in 30 days. If, in addition to the 7 people, another 8 people come to live in this household, in 18 days how many m^3 of water would be consumed?

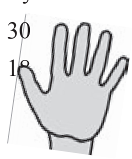
First a visual scheme is used to organize the values of the magnitudes (number of people, volume and time) in columns, identified with the units considered, with the unknown value in the bottom line.

People	m^3	Days
7	42	30
15	x	18

Afterwards, this scheme is used to define whether the proportionality is direct or inverse by means of gestures with the hands (ex. each time, one column is compared with the one that has the unknown and the other one is covered with the hands). Thus, the column with the unknown value (always placed in the bottom line) is successively compared with one of the others, and a sign (+ or –) is made below those other columns according to the variation of the quantities (increase or decrease) and the type of proportionality relation (direct or inverse).

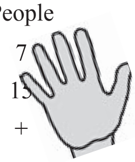
Comparison of People and volume (m^3): the two magnitudes are directly proportional and, as one of the quantities is going to increase (from 7 people to 15) the amount of water should also increase. Thus, we place the + (plus) sign under the column of number of people.

People	m^3	Days
7	42	30
15	x	18
+		



Comparison of volume (m^3) and Days: the two magnitudes are also directly proportional and, as one of the quantities is going to decrease (from 30 to 18 days) the amount of water should also decrease. Thus we place the – (minus) sign under the column of number of days.

People	m^3	Days
7	42	30
15	x	18
+		–



Finally, the two comparisons are put together and the unknown value is calculated according to the following rule:

Since number of people and number of days are both directly proportional to the volume of water and one of the quantities is going to increase (from 7 people to 15) the higher value (15) goes to the numerator of the fraction that is going to represent the unknown value (x), while the lower value (7) goes to the denominator. On the other hand, as the other quantity is going to decrease (from 30 to 18 days), the higher value (30) goes to the denominator of the fraction, while the lower value (18) goes to the numerator. The given value (42) in the column of the unknown value (m^3 - volume), goes to the numerator of the fraction, where the values should be multiplied.

People	m^3	Days
7	42	30
15	x	18
+		–

$$x = \frac{15 \,.\, \cancel{42}^{6} \,.\, \cancel{18}^{3}}{\cancel{7} \,.\, \cancel{30}_{5}} = 54\ m^3$$

If the two magnitudes being compared are inversely proportional we just switch the + (plus) and – (minus) signs when placing them under the columns being compared with the column of the unknown value: If the value of one magnitude increases from top to bottom line, we place the – (minus) sign under the last line. If the value of one magnitude decreases from top to bottom line, we place the + (plus) sign under the bottom line. However, the rule for the positioning of the values in the fraction remains the same: In a column with a + sign, the higher value goes to the numerator of the fraction and the lower value goes to the denominator; in a column with a – sign, the smaller value goes to the numerator.

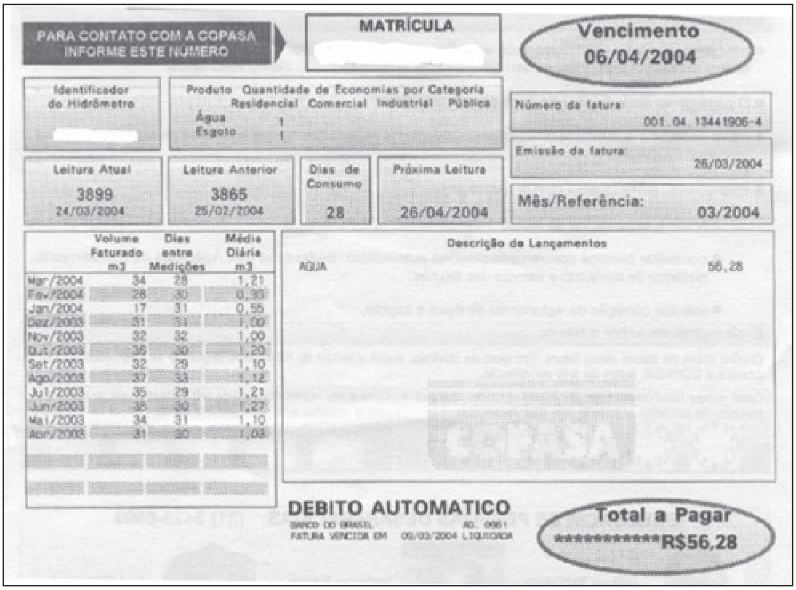
Figure 2. Telma's explanation of the rule of three procedure.

lesson, during which all the calculations related to the first four questions were discussed and reviewed. During the third lesson, the students presented their tips for saving water at home, resuming the work with the water bill.

Reduction of Water Consumption Problem

The third activity of the system was based on a lesson planned to make the students more aware of the need to reduce water consumption and to reinforce the application of the rule of three in everyday situations. The teacher expected that students would apply the procedures they had been taught and use the information shown in a magazine table to calculate the reduction of water consumption when adopting economical alternatives for daily water use.

The magazine table presented information for five activities: brushing teeth, shaving, washing dishes, watering plants, and washing the car. Data included the average time (in minutes) usually spent on each activity as well as the corresponding number of liters of water used in each activity if the tap was running all the time or if the tap was closed at certain times during the activity. For example,



PARA CONTATO COM A COPASA INFORME ESTE NÚMERO

MATRÍCULA

Vencimento 06/04/2004

Identificador do Hidrômetro

Produto Quantidade de Economias por Categoria

Produto	Residencial	Comercial	Industrial	Pública
Água	1			
Esgoto	1			

Número da fatura: 001.04.13441906-4

Emissão da fatura: 26/03/2004

Leitura Atual	Leitura Anterior	Dias de Consumo	Próxima Leitura
3899 24/03/2004	3865 25/02/2004	28	26/04/2004

Mês/Referência: 03/2004

	Volume Faturado m3	Dias entre Medições	Média Diária m3
Mar/2004	34	28	1,21
Fev/2004	28	30	0,93
Jan/2004	17	31	0,55
Dez/2003	31	31	1,00
Nov/2003	32	32	1,00
Out/2003	36	30	1,20
Set/2003	32	29	1,10
Ago/2003	37	33	1,12
Jul/2003	35	29	1,21
Jun/2003	38	30	1,27
Mai/2003	34	31	1,10
Abr/2003	31	30	1,03

Descrição de Lançamentos

AGUA 56,28

DEBITO AUTOMATICO

BANCO DO BRASIL AG. 0061

FATURA VENCIDA EM 09/03/2004 LIQUIDADA

Total a Pagar ************R$56,28

Q1) How many days of consumption?

Q2) What is the average daily consumption of the family?

Q3) What is the average consumption per person?

Q4) What is the average consumption per person, per day?

Q5) What will you do to save water in your home?

Figure 3. Example of the water bill issued by COPASA, the basic sanitation company of Minas Gerais, and the teacher's questions.

when brushing teeth for 5 minutes with the tap running all the time, one would use 12 liters of water; however, if the tap were closed during brushing, only 1 liter would be used.

To start the discussion, Telma asked the students the following question: "Based on this table, if a person who usually brushes his teeth with the tap open begins to close the tap while brushing teeth, how much water would this person save during brushing?" Before solving this problem, she reminded the students about what they had already studied, not only in the mathematics lessons but also in other disciplines, regarding the shortage of water much publicized by the media. Then Telma focused on the data in the magazine table and solved the problem on the blackboard with the participation of the class as a whole.

Tensions

Our main reference for analysis was the Family Water Consumption Problems activity (A_1) whereby the activities Rule of Three Problems (A_0) and Reduction of Water Consumption Problem (A_2) served mainly to situate the discussion of A_1 historically. Although we have characterized A_0, A_1, and A_2 as individual activities, for the sake of concision and clarity, we present only selected parts of the complete characterization to justify, at least in part, how we interpreted the modifications in the school mathematical activity.

Family Water Consumption Problems Activity System

Family Water Consumption Problems (A_1) was an activity system of two interconnected activities: Water Consumption Average ($A_{1.1}$) and Tips for Saving Water at Home ($A_{1.2}$). To analyze the tensions in this system (Figure 4) and the resulting modifications, we first identified the object of A_1 as the applications of the rule of three in solving problems about the students' families' water consumption. In turn, this object related to multiple motives: show applications of the rule of three in everyday situations, develop an awareness of family water consumption, and propose water-saving tips. We further analyzed the sequence of activities within A_1 by moving downward and inward (zooming in) to focus on $A_{1.1}$ and $A_{1.2}$. We describe the structure of these two activities, highlighting the tensions perceived in their components that, according to Engeström (1996), are a source of perturbations, innovations, changes, and development in the activity.

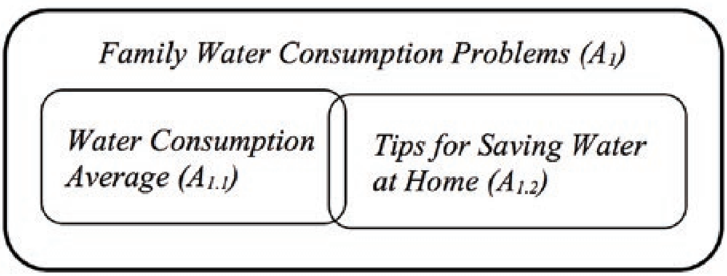


Figure 4. Family Water Consumption Problems (A_1) activity system.

Water Consumption Average ($A_{1.1}$) activity. In the discussion that follows, we present the structure of activity $A_{1.1}$ (see Figure 5) and describe the tensions that evolved when students calculated the water consumption averages for Questions 1–4 (see Figure 3). In this activity, the *object*—the calculation of the family's average water consumption—is related to two motives: showing applications for the rule of three (teacher's motive) and analyzing the family's water consumption (motive common to the teacher and students). The object-oriented actions of these two motives, as Engeström (2001) indicated, are implicitly characterized by ambiguities. Initially, when Telma taught the calculations for the rule

Activity System: Water Consumption Average ($A_{1.1}$)	
Object	The calculation of the family's average water consumption
Subject	Students and teacher
Community	Other students, other teachers of mathematics and of other disciplines, mathematicians, textbook authors, curriculum developers, COPASA, family, media, religious and civic organizations, and researchers
Division of Labor	Starts with teacher and students sharing the authority, and moves toward greater autonomy for the students
Artifacts	Water bills of teacher and students, the four questions (Q1, Q2, Q3, Q4) related to the water bill, other teachers' questions, calculation procedures used by the students (rule of three, arithmetic calculations), and calculator
Rules	Use the water bills to extract the data (water volume, number of days considered, and average daily consumption); apply a mathematical procedure to do the calculation (rule of three, simple division, and/or intermediate calculations); apply the procedures that the teacher demonstrated on her water bill to your own calculations; compare the averages calculated by COPASA and your own; and determine the number of water consumers in the homes.

Figure 5. Components of the activity system: water consumption average ($A_{1.1}$).

of three, her motive of showing applications for the rule of three was explicit. At that time, the water consumption of the families, which was the topic of the more general discussion related to the water bill artifact, remained implicit. However, when actions were explicitly directed to the water consumption analysis so that students could devise ways of saving water, the averages calculated using the rule of three (the use of which was still being reinforced by the teacher) became the implicit motive. In spite of the ambiguity resulting from multiple motives, the actions of the individuals were generally directed toward the same object, the calculation of the family's average water consumption.

In addition to the activities' objects, some components gained a more prominent role than others in our analysis, namely the artifacts and the rules. Furthermore, we had to decide, based on our judgment, which artifacts and rules appeared to be more relevant to our analysis. By way of illustration, there are innumerable rules that regulate the teacher's and students' actions in a school mathematical activity; many of these rules go far beyond the mathematical activity itself. For example, a common rule in many schools in Brazil is that students should not arrive late and should never leave the room without the teacher's authorization. The rules that interested us were more specifically related to school mathematical procedures and could vary according to the development of the activity. In our analysis, we focused attention on the sort of rules—regulations, norms, conventions, and standards—observed or inferred from our observations, which we thought were most influential in regulating the actions of the teacher and of the students.

On several occasions while developing this activity, Telma reinforced some general guidelines on how the students should use their water bills to make the required calculations, such as using the rule of three to calculate their family's average water consumption based on the calculation of the teacher's average. That is, Telma was trying to treat her own calculation (specific object) as a generalized or socially shared object (Engeström & Sannino, 2010), while the students were focused on their own bills. They were asking for particular guidelines in calculating average consumption using each individual bill, which hindered them from making a connection between the general guidelines of the teacher and their own calculations. Because the students did not seem to consider the calculations of the teacher as a generalized object, we can say that at this point, the teacher and the students did not share the same object. In this manner, two different perspectives regarding the object of this activity came into contact, thereby creating tension (T1) and triggering discoordinations in the actions of the teacher and students.

In addition to tension in the object of the activity, we also perceived other tensions associated with the artifacts of $A_{1.1.}$ More specifically, those tensions were connected to the first four questions related to the water bill, which played a key role as an artifact in the development of this activity. The first question required the students to find the number of days of consumption on their bills and did not generate any tension. In some cases, however, there was a discrepancy between the students' answers to Question 2 and the average posted by COPASA on their bills. The COPASA[5] average was in cubic meters (m^3), and some of the students' averages, calculated according to the instruction of the teacher, were in liters; also, the rounding procedures used by the students and the water company were not necessarily in agreement. This mismatch, originated by Question 2, gave rise to a tension (T2) between two artifacts, the water bill and the calculation procedures used by the students (see Figure 6). This tension may have led students to doubt the validity of using school procedures to solve everyday problems. In this case, T2 momentarily redirected the classroom discussion to the procedure used in school mathematics in rounding numerical results and the one used by COPASA, wherein the everyday artifact (water bill) seemed to hold more authority than the school calculation.

Tensions related to the artifacts accumulated as the students calculated the average water consumption per person (Question 3). The intervention of one of the students prompted a tension about the number of people to be considered in calculating the average (T3). This student proposed computing the water consumption per person according to the time each spent at home. Other students

[5] At that time, the artifact (water bill) issued by COPASA, the basic sanitation company of Minas Gerais, showed some numerical information in a format that is more common in a school environment (m^3) than in everyday situations. Because the liter is the most common unit of measurement for water in Brazil, the teacher seemed to assume that expressing the volume of water in liters would facilitate the students' understanding of their water consumption.

Question	Tension	Excerpts	Comments
Q2 – What is the average daily consumption of the family?	T2 – The discrepancy between the students' results and the average posted by COPASA	012. Telma: Just a minute and I'm going there…remark… COPASA rounded off my daily average consumption to 8401… why is that? because there ((in the water bill)) it is like that…oh…0,840 m^3…but one m^3 is not the same as 1000 liters?… by what number do I have to multiply?... it's equal to 0,840 times 1000 liters… this is equivalent to…equals to 840?... liters…on my calculation it is 839, and in the COPASA bill it was rounded off to 840…just look…839 is very close to 840… 013. Neusa: But mine is not so close (…)... 014. Joaquim: Nor is mine. 015. Telma: There's no problem. ((Several simultaneous comments of the students about the numbers in their bills)) 016. Telma: There's no problem...this is only a consumption average that COPASA calculates…using your global consumption and making the division…so you will find the cubic meters…	To calculate this average, the teacher guided the students. First, she instructed them to express the volume of water in liters instead of the cubic meters reflected in the bill. Then, she suggested the use of the artifact rule of three, illustrating the calculation with her own bill. The students had to perform various actions: find the monthly consumption and the consumption period on the form; calculate the average consumption in liters, as guided by the teacher; check the average consumption (m^3) posted on the form (when it was given); and round off the average that the student calculated when it did not match the one that was given. When the teacher yielded to the COPASA result so that the school activity could go on as planned, it seemed to imbue the everyday artifact (water bill) with *more authority* than the school calculation.

Figure 6. Classroom excerpts that illustrate the emergence of Tension 2.

joined the discussion and considered other ways to determine the number of water consumers in a home (see Figure 7).

Sônia and Cássia did not follow the general guidance of the teacher about the rule—count the family members—for determining the number of people that should be used to calculate this average. Sônia, in particular, introduced a new rule for setting this number when she asked how to account for family members who did not stay home for equal amounts of time and other people who did not belong to the family but regularly spent time in the house. This is another instance of how a tension evolved from an artifact (Question 3) and resonated in the rules for determining the number of water consumers in the homes, showing the close relation between the activity components.

Question 4 required the calculation of the average daily water consumption per person. This question gave rise to a new tension, which was perceived when some students found different ways for making this calculation (T4) that were contrary to the teacher's expectation that they would do it by using the preceding result found with Question 3. This tension between the rules used to make the calculation arose in the activity related to a combination of several artifacts (questions, rule of three, and water bill). However, the teacher eventually accepted the students' generating different ways to use the intermediate calculations (see Figure 8, Lines 98–100, on pages 476–477). As the rule of applying the teacher's procedures to the students' own calculations seemed to lose its power because each participant could choose a unique path to follow, both Telma and the students began sharing the authority in the classroom, moving towards greater autonomy for the students.

In order to clarify which calculations were to be made, the teacher asked, "If I am talking about consumption…I am talking about...?" (Figure 8, Line 81). Once again, the students did not answer as she expected because they were using different procedures and comparing different magnitudes to make the calculation (T5). The subsequent interventions of the teacher—for example, "What? In…what unit?" (Line 82)—impelled the students to think about the magnitudes involved. Telma's questions in Lines 81 and 82 gave rise to a tension between artifacts (Question 4 and Telma's other questions) and resonated on the object of this activity when students started to question the meaning of water consumption and the magnitudes used to express it. Prompted by the teacher, the students wondered which magnitudes to consider or which previous results they should rely on in order to answer the question. We believe that the intensification of the tension at this moment may have been caused by the fact that Question 4 involved more magnitudes (volume, time, and people) than the previous questions.

Tips for Saving Water at Home ($A_{1.2}$) activity. For the last task with the water bill, which was related to Question 5, "What will you do to save water in your home?" Telma asked the students to individually write tips on how to save water in their houses. She then called on those who had the highest and the lowest water consumption, according to the averages previously calculated, and invited them

Question	Tension	Excerpts	Comments
Q3 – What is the average consumption per person?	T3 – The criteria that should be used to determine the number of people: a choice between counting only the family members or including all the people who are present at the home and consume water	012. Sônia: Madam, look here...in my house... hmm…my brother...he hardly ever stays there…because he just comes for the weekend. 013. Telma: But…is there anyone who works in your house? 014. Sônia: Yes…there is…Cleusa works on Mondays… 015. Joaquim/Neusa: The two become one person. 016. Telma: When does she work in your house? 017. Sônia: On Mondays. 018. Cássia: Madam!...see this here…I counted...in my house... ((After Sônia raised her doubt, the other students, all talking at the same time, began to count the total number of people, including family members, people who worked in their houses, and even the number of pets.)) 019 Cássia: See this...there is a woman...she works in my house...but at night she...so I counted her… 020. Telma: Very good!...she is using water from your apartment building… 021. Cássia: And there is another one that…she cleans the apartment building…but not every day...so I counted (both) as another person…	The teacher and students used different criteria to determine the number of people for the calculation: The teacher considered only the number of family members who resided in the house, regardless of water consumption; some students made an estimate of the average consumption of each person according to the amount of time spent in the house, and included in the calculation the family members and all other people who were regularly present in the house and consumed water.

Figure 7. Classroom excerpts that illustrate the emergence of Tension 3.

Questions	Tensions	Excerpts	Comments
Q4 – What is the average consumption per person, per day?	T4 – The sequence of calculations that the student should adopt: a choice between the preceding result, another intermediate result, or the consumption information found in the water bill	081 Telma: Daily water consumption average… per person…so if I am talking about consumption…I am talking about?... ((All students speak at the same time.)) 082 Telma: what? In…what unit? 082 Armando: Money… 084 Telma: No… 085 Dayse: Day…day… 086 Neusa: Per day… 087 Telma: No…consumption…is consumption a day? 088 Sônia: No…consumption is liters… 089 Telma: Liters… 090 Sônia: The other one is day 091 Armando: What? Madam…isn't it all people?... ((Several students talk at the same time to each other and with the teacher.))	The questions opened several possibilities for doing the calculation, according to the previous results to be considered by each student. They could answer by using several of the preceding results or taking the total volume of water consumed that was posted in the water bill.

Questions	Tensions	Excerpts	Comments
		092 Telma: (…) which other magnitude… 093 Neusa: Per day…	
If I am talking about consumption…I am talking about? *What? In what unit?*	T5 – The magnitudes that should be compared to do the calculation: a choice between comparing the consumption per day with the number of people or the consumption per person with the number of days	094 Telma: No… 095 Neusa: DAY… 096 Armando: People… 097 Sônia: But how?...this can be "in days"? ((The debate goes on, with many students talking at the same time.)) 098 Neusa: Madam! Madam! You just take the 4,000, which is divided by people and…it's already the per-day amount… 099 Neusa: Madam…I am doing per day… 0100 Telma: I am going to take…she ((Neusa)) said that she could take the monthly consumption…and divide by the number of days… mine was 31…there it is (everyone makes his own)…I am going to do it with this average from COPASA.	The question "*In what unit?*" (Line 082) arose when the teacher tried to guide the students to assemble the layout of the rule of three, requiring a clear organization of the magnitudes involved in the calculation. In response, the students considered various possibilities for the meanings that could be attributed to the word *consumption* (money, day, by day, and liters), generating ambiguities and tension. In fact, the connection between consumption and money is common in daily life. This seems to have led the teacher to go beyond the definition of the unit of measurement, bringing the discussion to the identification of the magnitudes involved in the calculation. When, in the sequence of class events (Line 100), the teacher validated both Neusa's and her own procedure of using COPASA's average to proceed with her calculation, this freed the students to make their own choices about the calculations, and the tension seemed to have been attenuated.

Figure 8. Classroom excerpts that illustrate the emergence of Tensions 4 and 5.

to talk about their consumption habits, hoping that they would make a connection between the two. In our interview, Telma said, "(The) second thing…I wanted to make that final comparison…(between) the attitudes in the homes where the average was low and in the others where the average was high." Following this discussion, she requested that all of the students write down steps they could take to save water from that date onward (Question 5). However, most students just submitted a list of tips (e.g., "Take a quick shower," "Do not play with water," "Close the tap while you are brushing teeth"), which was contrary to Telma's expectation that they would mention the average water consumption at home and make a comparison between those averages and the attitudes at home.

The structure and components of this activity ($A_{1.2}$) are presented in Figure 9. The object of this activity was the elaboration of a list of water-saving measures. Because Telma used the comparison of averages as a starting point in the discussion of these measures, the calculation of the family's average water consumption (the object of $A_{1.1}$) was introduced as an artifact in the present activity ($A_{1.2}$), and the related rule to use as reference the averages previously found was incorporated into the rules of this activity. On the other hand, through their growing autonomy, the students introduced additional artifacts related to the water consumption habits of their families. In turn, these artifacts brought about additional rules, such as referring to the families' daily water consumption habits and the debate about water consumption in other disciplines and social spaces in which students participated. The students' use of artifacts other than the previously calculated averages to compare water consumption and support their water-saving measures generated another tension (T6)—a choice between water consumption habits

Activity System: Tips for Saving Water at Home ($A_{1.2}$)	
Object	The elaboration of a list of water-saving measures
Subject	Students and teacher
Community	Others students, other teachers of mathematics and of other disciplines, mathematicians, textbook authors, curriculum developers, COPASA, family, media, religious and civic organizations, and researchers
Division of Labor	Greater autonomy and leadership for the students
Artifacts	Question (Q5), which is related to the list of tips; the calculation of the family's average water consumption; information presented in the water bill; and water consumption habits of the family
Rules	Use as reference the averages previously found, the consumers' daily habits, and the debate in other disciplines and social spaces in which students participate.

Figure 9. Components of the activity system: Tips for saving water at home ($A_{1.2}$).

(prioritized by the students) and calculations and water consumption habits (emphasized by the teacher).

Contradictions and Modifications

In Figure 10, we summarize the tensions perceived in the Family Water Consumption Problems activity system (A_1). We identified these tensions by zooming in on its two subsystems, $A_{1.1}$ and $A_{1.2}$, and zooming out to the entire A_1 system. We also identified the components (objects and artifacts) of the activity system that prompted the tensions.

We noticed that the artifact rule of three was directly or indirectly involved in Tensions 2, 4, and 5 because Telma insisted on showing its applicability in the situations proposed. In T2, the students' use of the rule of three resulted in a discrepancy between the water bill average expressed by COPASA and the one they found through their own calculations. In T4, the rule of three was not the

Tensions	Components where the tensions originated
T1– Discoordination of the teacher's and the students' perspectives regarding the teacher's calculations: seen as a generalized object by the teacher and as a specific object by the students.	Object A1.1 (the calculation of the family's average water consumption)
T2 – The discrepancy between the students' results and the average posted by COPASA.	Artifact Q2 (What is the average daily consumption of the family?)
T3 – Which rule should be used to determine the number of people: count only the family members or calculate all the people who are present and consume water in the home?	Artifact Q3 (What is the average consumption per person?)
T4 – What sequence of calculations should the student adopt: using the preceding result or other intermediate result or the consumption information given in the water bill?	Artifact Q4 (What is the average consumption per person, per day?)
T5 – What magnitudes should be compared to do the calculation: compare the consumption per day with the number of people or the consumption per person with the number of days?	Between the object of the A1.1 and artifact Q4 and teacher's questions (*If I am talking about consumption...I am talking about? What? In...what unit*)
T6 – What artifacts should be used as reference to elaborate the list of water-saving measures: the habits of consumption (prioritized by the students) or calculations and the habits of consumption (emphasized by the teacher)?	Artifact Q5 (What will you do to save water in your home?)

Figure 10. Tensions associated with the teacher's questions in A_1.

only viable procedure, and sometimes it was not the best procedure. In T5, the rule of three was not an appropriate procedure for answering Telma's questions about which magnitudes or units were to be considered, but indirectly those questions were provoked by the students' struggle to apply her scheme for the rule of three to a previous specific situation. These tensions provided a context for a broader discussion of the contradictions and modifications that occurred in the structure of school mathematical activity when everyday situations were brought into the proportional reasoning problem-solving lessons in Telma's class.

Contradictions in Classroom Events Involving Proportional Reasoning

Our analysis of the tensions in A_1 indicated a contradiction between the rule of three as a generic school procedure and the specific procedures applicable to singular situations. This contradiction was amplified when we examined other mathematics lessons in which problem-solving situations involving proportional reasoning were discussed. In accordance with the idea that the analysis of an activity system should be done historically (Engeström, 1996), we examined the lessons preceding and following the lessons in A_1. Thus, we considered the constellation of three interconnected systems (A_0, A_1, and A_2), which comprised the Water Theme activity system (see Figure 11, on pages 482–483).

In A_0, chronologically the first group of lessons, Telma introduced the rule of three using typical school problems. Although the students were initially allowed to solve the problems using their own strategies, they were subsequently instructed to use the conventional strategy taught by the teacher. In A_1, the second group of lessons chronologically, the students solved problems based on the consumption of water registered in Telma's water bill and in their own water bills. In A_2, the third group, the teacher expected the students to apply the procedures they had previously been taught using generic data from a magazine to calculate the reduction of water consumption when adopting economical alternatives for daily water use. As represented in Figure 12 (see page 483), situations involving proportional reasoning and the rule of three were common among systems A_0, A_1, and A_2, whereas only systems A_1 and A_2 involved situated problems with embedded cultural artifacts regarding water consumption.

In the Rule of Three Problems activity (A_0), the aim of the teacher was to introduce the procedure of the rule of three and then have the students apply the procedure to solve typical school problems; thus, the rule of three was the object of the classroom activity. Although one of the problems involved a water consumption situation, the students did not question the feasibility of the problem, as they did with the problems related to their water bills. No one wondered if the algebraic procedure was actually necessary for its solution. The emphasis on the application of the rule of three as an algorithmic procedure made the students ascribe to it a central role in the solution of problems involving proportional reasoning.

In the Family Water Consumption Problems system (A_1), the aim of the teacher was to show the students that the rule of three was applicable in everyday situations as well as to analyze their families' water consumption. As Telma explained in the interview:

> First of all, I wanted to ascertain if the rule of three…if they would succeed applying the rule of three in their lives…(The) second thing…I wanted to make that final comparison…(between) the attitudes in the homes where the average was low and in the others where the average was high…What was happening in a specific home that was spending a lot of water? ((I wanted that)) besides the mathematical view ((developed with)) the rule of three…(that) they would observe if they were increasing the consumption or not…((I wanted to see)) if they (would) take action (informed by) what is not strictly mathematics…how they could apply that in their lives to improve the standard of living…also reduce water consumption since water wastage is alarming in all texts we are reading.

The problems in A_1 were different from the ones previously solved by the students because these required personal information, making a direct application of the rule of three more difficult. Only when the students had their own water bills in hand, and after the teacher used her water bill to explain how to apply the rule of three, did students begin to question the procedure. They exhibited greater autonomy, getting involved in the solution of the problem and even modifying it. The students' agency grew as they understood that the generic algorithm, formerly perfectly suited to the solution of problems involving proportional magnitudes, was not directly applicable to their specific situations. These changes in the students' actions and position of authority in the A_1 activity system clarified for us how the tensions that we perceived to emerge in the artifacts of this system evolved historically into a contradiction between the rule of three as a generic school procedure and the specific procedures applicable to everyday situations.

Going forward to the Reduction of Water Consumption Problem system (A_2), it was possible to perceive other changes in the way the students were solving problems and how the same contradiction was also reflected therein. The object in this system was the application of the rule of three in everyday situations and alternatives for reducing water consumption. As in the problems of A_1, there was a cultural artifact: a table with ways to reduce water consumption while performing daily routines (e.g., toothbrushing, washing dishes) by keeping the tap closed at certain times. The alternatives were the same for everybody, and the question formulated did not require any other additional information. Telma expected the students to simply apply the rule of three to make the calculations, as they did with the typical school problems of system A_0. However, the students apparently made a connection with the discussion that occurred in A_1, possibly because this problem also referred to everyday situations. The excerpt presented in Figure 13 (on page 484) was taken from a transcript of the class discussion during which the toothbrushing problem was discussed.

We can see that the students raised alternatives not indicated in the magazine table to save even more water when brushing teeth (e.g., leave the tap running just a little and use only a small glass of water), indicating that they had connected this table with their own everyday situations and modified the initial conditions of the problem. However, the algebraic procedure was not so easily applicable to the alternatives they proposed, which highlights the contradiction already mentioned in this system of activities. For the rule of three to be applicable in this case, it would be

	Rule of Three Problems (A_0) activity system	**Family Water Consumption Problems (A_1) activity system**	**Reduction of Water Consumption Problem (A_2) activity system**
Object	**O_0** The rule of three	**O_1** The applications of the rule of three in solving problems about the students' families' water consumption	**O_2** The calculation of the reduction of water consumption in everyday situations and applications of the rule of three
Subject	**S_0** Students and teacher	**S_1** Students and teacher	**S_2** Students and teacher
Mediating Artifacts	**T_0** Algebraic calculations, calculator, visual scheme; typical school problems	**T_1** Water bills of teacher and students; questions (Q1, Q2, Q3, Q4, Q5); other teacher's questions; students' calculation procedures (rule of three, arithmetic calculations); calculator; the calculation of the family's average water consumption; and the family's water consumption habits	**T_2** Table with ways to reduce water consumption while performing daily routines; calculation procedures (rule of three, arithmetic calculations); calculator; school problem with embedded everyday situations
Community	**C_0** Other students; other mathematics teachers; mathematicians; textbook authors; curriculum developers, researcher	**C_1** Other students; other teachers of mathematics and of other disciplines; mathematicians; textbook authors; curriculum developers; COPASA; family; media; religious and civic organizations; researcher	**C_2** Other students; other teachers of mathematics; mathematicians; textbook authors; curriculum developers; family; media; researcher
Division of labor	**D_0** The teacher is the authority	**D_1** Starts with teacher and students sharing the authority and moves toward greater autonomy for the students	**D_2** Starts with teacher and students sharing the authority, moves toward greater autonomy for the students, and then the teacher recovers more authority

Rules	R_0	R_1	R_2
	Use the information given in the problem; compare the magnitudes using the visual scheme; apply the rule of three as an algorithmic procedure; solve an algebraic equation; show the solution in an adequate unit	Use the water bill to extract the data; apply a mathematical calculation (rule of three, simple division); follow the procedures demonstrated by the teacher in your calculations; compare your averages with the one shown by COPASA; determine the number of water consumers at home; use the averages previously found, the consumers' daily habits, and the debate in other disciplines and social spaces	Apply a mathematical procedure to make a calculation (rule of three, simple division); use the consumers' daily habits (open the tap just a little bit, take just a glass of water); and the debate in other disciplines and social spaces in which students participate

Figure 11. Water Theme activity system.

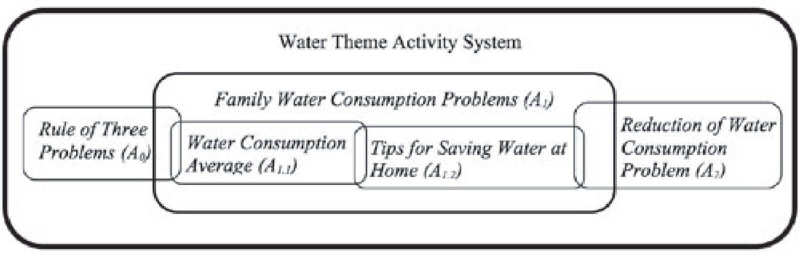


Figure 12. Water Theme activity system representing our main focus in A_1 and the changes in the components through A_0, A_1, and A_2. Thin borders enclose what we considered to be the elementary activities; thick borders enclose activity systems composed of two or more activities. Intersecting borders represent some common components.

005.	Telma:	When someone is brushing teeth...you…I….we…all of us brush the teeth every day…the table says that a person spends five minutes brushing teeth…if he brushes with the tap running…he uses 12 liters of water…if he closes the tap while brushing…what would happen? He uses…
006.	Joaquim:	One liter…
007.	Telma:	He is going to spend one liter…
008.	Cássia:	But…if he leaves the tap running just a little…
009.	Telma:	But… the person….has he wasted water while brushing his teeth…or not?
((The students start discussing how much water they would save if one were to do the tasks listed in the table with the tap running just a little.))		
012.	Sônia:	What…if we change the tap running a little for a glass of water?
013.	Telma:	This way…we would save more water…wouldn't we?
014.	Sônia:	No…
015.	Telma:	Yes…if you have a glass of water…you use less than one liter…if you are going to brush your teeth using a glass of water…you'll have less water…do you remember the table we studied? If a person spends five minutes brushing her teeth with the tap running…she'll use 12 liters of water… if she closes the tap…how much water would she save?
016.	Joaquim:	Eleven liters.

Figure 13. Other alternatives for economical water consumption.

necessary to remake the table, calculating the amount of water that would be used if the new alternatives suggested by the students were implemented. Unlike with the water bill problems, Telma did not explore the alternatives proposed by the students; they did not insist on them, and she moved on to another task.

Observing the development of the three activity systems, we noticed a transformation of school mathematical activity permeating all activity components. As the students acquired more autonomy, we contend that the contradiction that evolved from the calculation artifacts was resolved: The rule of three as a generic algebraic procedure became one of the procedures applicable to specific everyday situations involving proportional reasoning. Problems involving the comparison of proportional magnitudes that had previously (in A_0) been seen by the students as problems to be solved using the rule of three came to be seen, from A_1 onwards, as problems for which the rule of three was just one possible way to solve them.

Everyday Situations and the Modification of School Mathematical Activity

We observed several changes in the structure of school activity that were prompted by the growing influence of students' everyday situations. We claim that these changes can be seen as a transformation in the activity system because the object of A_0—the rule of three—gradually became an artifact in A_1 and A_2. These changes were discussed by several students, as illustrated in the excerpts from the interviews with Cássia, Romero, and Rubens (see Figure 14).

Another change pertained to Telma's relation with the students, which became more horizontal in terms of power and leadership in the classroom as she allowed the new elements that they brought in to be discussed and socialized. Moreover, although she did not abandon her initial motive of showing that the rule of three is applicable to the students' everyday situations, she admitted that the rule of three that is taught in school is different from the procedure they use for solving everyday situations. This can be seen in the interview excerpt in Figure 15.

As it has already been said, the object of system A_0 was transformed into an artifact in the other two systems when the students' everyday situations were introduced into the activity through the use of cultural artifacts. Moreover, the objects of systems A_1 and A_2[6] were directed to multiple motives, which caused ambiguities in the objects. This occurred because the actions of the students and teacher were simultaneously directed to a broader discussion of water consumption and the application of the rule of three and their focus of attention was on one or the other.

In the two systems (A_1 and A_2) bearing this ambiguity of object, there was an expansion of the artifacts in relation to the previous system (A_0) because new artifacts were incorporated therein. Further, some of them were brought into the activity by everyday situations. This expansion did not occur as a simple addition of new artifacts because one did not play its role independently of the others. In

Interviewer:	Did anything change from what you already knew about the rule of three when you used it to solve the water bill problems?
Cássia:	Ah!…more or less...because...you have to pay attention to everything… you have to measure…exactly what we spend…so…I think it changes a little bit…because it is real…from ourselves…(and) before it was not…it was a normal problem ((school problem))…I think that it is different…because here ((water bill problems)) you need to be more connected (with your own life).
Romero:	No...ah!...yes...because…we had to make more calculations...liters per day...liters per person...water per person...monthly average...
Rubens:	Yes...I think that it is a bit different because...there ((school problems))…if you just look…you already see what you have to put (there)….but here ((water bill problem)) no...here… you have to…. analyze it well for you to know ((how to solve)).

Figure 14. Interview excerpt with Cássia, Romero, and Rubens.

[6] Object of A_1: The applications of the rule of three in solving problems about the students' families' water consumption. Object of A_2: The application of the rule of three in everyday situations and alternatives for reduction of water consumption.

Interviewer:	Why do you think the students told me that the water bill rule of three is different?
Telma:	Maybe this is because the data are personal...so these here ((water bill problems)) are things from their lives...and the rule of three ((problems)) is the type of problem that they do at school and not in their lives…it can be…or not.
Interviewer:	All students agreed that it is different...
Telma:	(He) uses the rule of three in everyday life...but it cannot be the same rule of three...because the rule of three he does at school…I think that this distancing may be taking place…the school is the school…my home is my home…the street is the street…then what I can do in a supermarket is not the same thing I do at school…there is a rule of three for the supermarket, but it is not what I do at school…because school is school.
Interviewer:	Did you really want to teach mathematics when you gave ((to them)) this water bill activity?
Telma:	It was just a way for them to see (the application in their life)…not teaching...I had already taught…before…I taught what the rule of three was…before the water bill problems…this objective ((of the water bill activity)) was to make them see…what they had failed to see ((before))…that is…that this rule of three…(can) be applied to their lives… what they had failed ((with the other problems)).

Figure 15. Interview with Telma: "The school's rule of three *is different*."

A_0, it was possible to consider only the following artifacts: school problems, visual display to compare the magnitudes, algebraic equation, calculator, and data expressed in the problem. From system A_1 onwards, the following artifacts were incorporated: arithmetic calculations, water bill, consumption habits of families and other people, school problems with elements from everyday situations, and the table from the magazine.

Associated with the quantitative aspect of the incorporation of new artifacts, a qualitative transformation of the artifact rule of three occurred. Initially, it was an algorithm perfectly adaptable to the problems proposed. However, in A_1 and A_2, the problems did not fit this procedure as perfectly, and the students sought other ways of solving them. They either adjusted the problem to the procedure or used intuitive procedures (Lamon, 2007) that did not necessarily involve a visual display or an algebraic equation and that were not emphasized by Telma. Thus, regarding the use of the rule of three, the students moved from the level of operations to the level of actions, which can be considered an expansion of this rule's meaning for them. The students themselves expressed this expansion, saying that it is the rule of three, "but it is different."

The expansion of artifacts resulting from the growing presence of everyday situations brought changes to the community members of the system. In A_1, the water bill introduced COPASA, and the families' water consumption habits introduced family members and teachers of other subjects.[7] In A_2, the magazine table with alternatives to reduce water consumption introduced other people from outside school. Moreover, the transformation of the object and expansion of the community and artifacts led to changes in the rules. In A_0, there was a single procedure for solving the problems. In the other two systems, however, this procedure became merely one of the possibilities for the solution when other strategies coming from everyday experiences or created by the students themselves began to be accepted. This sequence of changes was also reflected in the division of labor; the teacher and students shared the authority, thus moving towards greater autonomy for the students.

Discussion

Our analysis indicated that the insertion of everyday situations into school tasks modified the school mathematical activity in Telma's classroom in several different ways, compared to the usual activities in which no such insertion was made. At first, students were given typical school problems that could be classified as symbolic control (Walkerdine, 1990). The solution of these problems was an exercise in applying a calculational procedure. In the subsequent tasks of A_1 and A_2, aspects of everyday life were taken into account in the resolution of situated problems with embedded cultural artifacts, similar to *problems of practical and material necessity* (Walkerdine, 1990). In such situations, students related the calculations they performed to a practical necessity in their own lives, although this was in the context of a theoretical exercise.

However, there are some differences between the tasks involved in systems A_1 and A_2 that are worth highlighting. The water bill tasks of A_1 seemed to help students recall many more aspects associated with their everyday practices than did the magazine table tasks of A_2. In A_1, the students had to make a considerable effort to adapt the school task in order to deal with their real-life situations, which were different from one another. In A_2, the everyday habits mentioned by the students did not seem to have the same strength as in A_1 because the students did not resist the teacher's arguments to concentrate on the data provided by the table, which was the same for everybody. We claim that this discussion about types of situated problems with embedded cultural artifacts provides new insights with respect to the related literature because it shows not only how different cultural artifacts brought forth the students' everyday practices but also how they modified the classroom activity in different ways.

Beswick (2011) highlighted the importance of discussing "contextualized" tasks and said that we need more convincing arguments to support the common belief that linking mathematics as closely as possible with authentic, relevant, and real-life tasks is good practice. She also emphasized the importance of unraveling

[7] Because the theme "Water" was also being discussed in other disciplines, some teachers had asked the students to write about or elaborate proposals based on their consumer habits, which may have influenced the activity in the mathematics discipline.

the extent to which the introduction of real-life problems into school mathematical activity is able to show students the applicability of school procedures to everyday situations. We believe that our work presents the kind of contribution that Beswick said was needed, as our analysis revealed that some students seemed to have perceived that school mathematics can be applied to solve everyday situations. However, because we did not interview all the students, we do not know whether this was the case for all students in the class. It is possible that for some students this activity reinforced the idea that school mathematics is not good for anything, which is a worthwhile issue to pursue in future research. Thus, because we used activity theory to analyze the classroom activity in a comprehensive manner, our work not only extends the literature that has emphasized the use of real-world problems (e.g., Beswick, 2011; Doorman et al., 2007) but also contributes to the literature that has focused on student participation (e.g., Civil, 2002).

In fact, our analysis also revealed how the changes introduced in the school activity by everyday situations might be reflected in the levels of students' participation in the activity (Leont'ev, 1978). At first, the students in this study saw a given problem merely as a data supplier for the application of a standardized mathematical procedure (acting at the operations level). Then, they came to see it as a particular situation, introduced by the storyline, and often acted as if they were the very subjects of the problem (acting at the level of actions or of the activity itself). Because of these changes in the level of participation, the students acquired greater power of action (agency) and developed a more productive relationship with mathematics (cf. Boaler, 2002).

The increased agency of the students modified the power relationships, making them more horizontal, because the students' actions were no longer restricted to the artifacts and rules defined by the teacher. This increased autonomy allowed the students to experiment with alternatives to the standardized school procedure (rule of three) when applied to the resolution of school problems (involving proportional reasoning) into which everyday situations were introduced. In this process, the school procedure (rule of three) took different meanings according to the situation, alternating between being a generic object with direct application to typical school problems and one of the artifacts available for solving everyday situations (involving the proportionality of magnitudes).

However, given the more horizontal power relationships thus established, the teacher relinquished some of her agency. As Telma admitted in the interview, in decreasing her power of action, she encountered difficulties—"when data are too different...it is too complicated...I was in trouble at first with all the different data"—because this was not a common practice for her. In contrast to the cases discussed by Planas and Civil (2002), Telma succeeded in giving a voice to the students and making their alternative interpretations visible, and we have learned from her how it is possible to create a more democratic environment in a classroom without completely giving up one's teaching goals.

At a different level, the transformation of the school activity object and, therefore, the activity itself put students in a situation of having to deal with a greater number of relationships, as they revealed in the interviews. Such a change could have directed the actions to a multiplicity of motives that could fragment the activity in such a way that the desired outcome would not have been achieved. Instead, because of the teacher's openness to sharing power with the students, the change afforded an expansion of various components of the activity and of the meaning of the rule of three. The students started considering it as merely one of many procedures for solving problems involving proportional reasoning, and the teacher recognized that the school's rule of three is different from the "rule of three for the supermarket" (see Figure 15). To some extent, this met the teacher's purpose of expanding the students' understanding of the rule of three, as she stated in the interview.

At some point, there was a negotiation in which the teacher sought to incorporate and legitimize the way the students began to freely explore different possibilities for solving the problems. This changed what she apparently anticipated to be the way some calculations ought to have been done. According to Knijnik (2006), this sort of negotiation between formalized knowledge and common knowledge is not free from conflicts and tensions. To overcome the tensions, Telma had to act differently from the way we have observed her acting in the activities preceding the discussion of water consumption in the classroom. In Telma's interview, she said that showing students that mathematical concepts are applicable to everyday situations was not as simple as she expected. Moreover, she voiced agreement with the students that the rule of three they were using was not the same as the one she taught them (see Figure 15).

In summary, we claim that everyday situations should not be seen as merely a source of motivation, scope of application for school knowledge, or justification for the relevance of studying some school mathematics concepts. Rather, after such situations are brought into the classroom, they become part of the school activity, and it is important for the teacher to be aware of the role they are likely to play and of the changes they may introduce into the activity.

Final Remarks

The theoretical perspective adopted in our analysis compelled us to take a comprehensive view of the classroom mathematical activity by considering the role that each single component and the union of all components played in the development of the activity. Considering a constellation of interconnected activity systems, this perspective facilitated the description of a movement both up and down and outward and inward in these systems. This highlighted the tensions, contradictions, and transformations, thereby illuminating possibilities and constraints for the teacher's actions when students' everyday situations are brought into the classroom.

Our comprehensive perspective of analysis allowed us to capture specificities of the school mathematical activity that have not been discussed in previous works (Tomaz, 2007, 2009; Tomaz & David, 2008). One such specificity deserving special mention is that a contradiction may arise in a school mathematical activity when everyday situations are incorporated. In this study, the contradiction was between a generic school procedure (such as the rule of three) and specific procedures that are applicable to everyday situations. In the case of the rule of three, when the procedures of the students and the teacher came into contact, this contradiction prompted an expansion of the meaning of this rule, both for the students and for the teacher. This expansion happened when the students started questioning the mechanical use of the rule of three and considered it to be one of the possible procedures to solve problems involving proportional reasoning.

Depending on how the activity unfolds, there is a risk that a different objective from the one initially desired by the teacher will be reached—for example, if the aim of the teacher is to show that school mathematics applies to out-of-school situations, but the students conclude the opposite because there are simpler ways to solve everyday problems. In our study, we saw instead that some students understood both the similarities and differences between school and everyday procedures, what we have considered as an expansion of meanings for school procedures. Despite the results achieved in this study, we believe that the similarities and differences between school and everyday mathematics still need to be investigated in more depth, particularly what they may or may not contribute to the *expansion of learning* (Engeström, 1987) in school mathematics.

When everyday situations were brought into the classroom, we identified perturbations and changes in school activity from which we discerned changes in rules and power relationships. These demanded modifications in the teacher's regular practice and created opportunities for expansive learning, both for the students and the teacher. It is worth noting that in our study several opportunities for teacher learning emerged. That is, the teacher recognized that there are similarities and differences between school procedures and everyday procedures, and she became aware that showing students that mathematical concepts were applicable to everyday situations was not as simple as she had anticipated. These opportunities for the teacher's learning, in practice, have not been deeply explored in the literature and seem to be a promising issue for future research.

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APPENDIX

Rule of Three

For centuries, the rule of three was taught in many countries as an algorithm for solving different types of problems involving proportional relationships. Throughout the 18th century and during the first half of the 19th century, arithmetic books devoted a number of pages to teach different variations of the rule of three (direct, inverse, compound) according to the sort of problem to be solved. The procedure used to solve these problems was essentially numerical, and the operations to be performed were explained by words, as illustrated in a book by Grout (1809, pp. 51, 59–61):

RULE OF THREE DIRECT.

The Rule of three direct conſiſts of three numbers given to find a fourth, which ſhall have the ſame proportion to the third, as the ſecond has to the firſt.

Two of the three given numbers are always of one name or quality, viz. money, weight, meaſure, &c. of which one is the demand. The other is of the ſame name or quality with the anſwer required. In ſtating the queſtion, theſe numbers muſt be placed in the following order.

RULE. 1. That number which asks the queſtion* muſt be the third term; that which is of the ſame name, or quality, the firſt; and the other, which is of the ſame name or quality with the anſwer required, the ſecond.

2. If the numbers have ſeveral denominations, reduce them to the loweſt mentioned, and the firſt and third to one name.

3. Multiply the ſecond and third numbers together, divide the product by the firſt, and the quotient will be the anſwer to the queſtion in the ſame name you left the middle number, which, if in a ſmall denomination, muſt be brought to the higheſt poſſible.

OBSERVATION 1. If the product of the ſecond and third numbers, when multiplied together, be leſs than the firſt term, reduce the ſecond to a leſs denomination.

* *The number which asks the queſtion has commonly theſe words before it, viz. How much? How many? What will? What coſt? &c.*

RULE OF THREE INVERSE.

The Rule of three Inverſe conſiſts of three numbers given to find a fourth, which ſhall have the ſame proportion to the ſecond, as the firſt has to the third.

If a greater number require a greater, or a leſs, require a leſs, the queſtion belongs to the rule of three direct.

But if a greater number require a leſs, or a leſs require a greater, it belongs to the rule of three Inverſe.

RULE. 1. State and reduce the numbers, as in the rule of three direct.

2. Multiply the firſt and ſecond terms together, divide their product by the third, and the quotient will be the anſwer.

EXAMPLES.

1. What number of men will it take to finiſh in 6 days, what 36 men would be 9 days about?

d. men d.
As 9 : 36 :: 6
9

6) 324
Ans. 54 men.

2. What principal will gain as much in 8 months as £ 100 will in twelve months? Ans. £ 150.

3. What length of a board, that is 8 inches wide, will make a foot ſquare?

Ans. 18 inches.

4. If a paſture will feed 12 horſes 16 weeks; how long will it feed 48 horſes?

Ans. 4 weeks.

60 DOUBLE RULE OF THREE.

5. How many yards of carpet, that is 3 quarters wide, will cover a floor, that is 30 feet long, and 24 feet wide? 106⅔ yd. Ans.

6. When an acre of land is 4 rods wide, it muſt be 40 in length: what muſt be the length, when it is 10 rods wide? Ans. 16 rods.

7. Suppoſe 275 yards of cloth, which is 5 quarters wide, make coats for 130 men; how many yards of ſhalloon of 3 quarters wide will line the coats? Ans. 458⅓

8 A garriſon, conſiſting of 1300 men, and being beſieged, have proviſions only for 3 months, but it being neceſſary they ſhould hold out 5 months; how many men muſt depart that the ſaid proviſions may ſerve that time? Ans. 600.

COMPOUND PROPORTION

OR, DOUBLE RULE OF THREE,

Conſiſts of five numbers given to find a ſixth, which, if the proportion be direct, muſt have the ſame proportion to the fourth and fifth, as the third has to the firſt and ſecond. But if the proportion be inverſe, then the ſixth number muſt have the ſame proportion to the fourth and fifth, as the firſt has to the ſecond and third.

Three of the given numbers contain a ſuppoſition, and two a demand.

RULE. 1. Place the numbers of ſuppoſition as follows; that number which is the cauſe of gain, loſs, or action, muſt be the firſt term; that which denotes time or diſtance, the ſecond; and the other, which is the gain, loſs, or action the third; and the other two terms,

DOUBLE RULE OF THREE. 61

which contain the demand, muſt be placed under thoſe of the ſame name.

2. If the blank fall under the third term, multiply the two firſt terms together for a diviſor, and the three laſt for a dividend, and the quotient will be the anſwer.

3. If the blank fall under the firſt or ſecond term, multiply the third and fourth terms together for a diviſor, and the other three for a dividend, and the quotient will be the anſwer.

EXAMPLES.

1. If £100 in 12 months gain £6 at intereſt; what will £600 gain in 4 months?

£ M 12.
100 : 12 : 6
600 : 4

100×12=1200 diviſor.
6×4×600=14400 dividend.
12|00) 144|00

£ 12 Anſwer.

2. If 6 men mow 72 acres of graſs in 12 days; how many men will it take to mow 120 acres in 4 days?

men. da. acres.
6 : 12 : 72
4 : : 120 Anſwer 30 men.

3. If a family of 9 people ſpend 40 dollars in 8 months, how much will ſerve a family of 24 people 16 months?

peo. mo. D.
9 : : 8 : 40 D. cts. m.
24 : 16 Ans. 213 33 3

F

Although these procedures could be explained through equality of ratios and properties of proportions, many authors, like Grout (1809), did not emphasize these explanations. The rules were taught essentially through a brief explanation of the steps to be performed, followed by examples, which the learner was supposed to reproduce in similar situations. Other authors (e.g., Bézout, 1795), who addressed their writing to engineers or military cadres, more clearly connected these rules with the properties of proportions. However, they still presented the different variations of the rule of three as algorithms to be reproduced exactly the same way as in the examples solved in the text.

In many countries, during the second half of the 19th century, the use of algorithms for the various rules of three fell out of favor. Thereafter, in Brazil, and in other parts of the world, proportional relationships were solved by resolution of equations (Ávila, 1986). As noted in Shield and Dole (2002):

> The standard algorithm for proportional situations is the representation of equal ratios, that is *a/b* = *c/d* (Touriniare & Pulos, 1985), or *a/b* = *c/x* where *a, b* and *c* are given, and *x* is the unknown. The standard solution procedure for solving proportion equations is via algebraic means: "cross-multiply and solve for *x*" (Post, Behr & Lesh, 1988, p. 81) or through rule application: "multiply the two numbers across from one another and divide by the other number" (Robinson, 1981, p. 6). (p. 609)

In Brazil, many teachers like Telma refer to this procedure as the "rule of three" because in the simplest form of the rule, three given values are used to find a missing value.