

Progress in Particle Physics and Modern Cosmology

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This is the translation into English of my paper published in "Einshteinovskij Sbornik" 1980-1981 [1]. The content of the book is presented in the Appendix. This is the first paper where the dynamical mechanism of "phoenix universe" was worked out. The notion of phoenix universe was first mentioned in the paper by Lemaitre in 1933 [2], where he assumed that a repetition of successive phases of expansion and contraction was possible. Lemaitre called such model of the universe, that is born, dies and is reborn, the phoenix model, named after the mythical bird able to reborn from the ashes. According to the author, there could be an infinite number of such cycles in the past and future. In this model, however, a possible solution to the fundamental question was unclear, what was the mechanism for the "rebirth" of the universe? This mechanism was worked out in ref. [1], published in 1980 in Russian. Moreover, in our recent paper [3] a mechanism of dynamical cancellation of vacuum energy was proposed, that permits to eliminate vacuum energy locally down to zero and permits universe to jump to a lower hot level, leading to rebirth of a hot universe.

Introduction

Modern cosmology was born as a result of Einstein's formulation [4] of the general theory of relativity and Friedmann's discovery [5] of non-stationary solutions of Einstein's equations. Einstein, when he first expressed the idea of applying the equations he had discovered to cosmology was, however, discouraged by the fact that these equations do not have stationary solutions in a cosmological situation. To eliminate this "deficiency," Einstein proposed generalizing the equations of general relativity by adding the so-called cosmological term [6], which could stabilize the Universe. However, it soon became clear that the Universe is, after all, expanding in full accordance with Friedmann's predictions. This discovery was made by Hubble [7], who saw that distant astronomical objects are moving away from us at a speed proportional to distance to them. The natural next step was the formulation of the hot model of the Universe by Gamow [8], based on the theory of synthesis of elements, which became the generally accepted cosmological model after the discovery of the relic electromagnetic radiation by Penzias and Wilson [9]. The status of the hot model was further strengthened after detailed calculations of abundances of light elements produced in a hot Universe that were performed by Wagoner, Fowler, and Hoyle [10]. The results of these calculations were in excellent agreement with astro-

nomical observations. Particularly successful was the coincidence with observations of the calculated 4He abundance, which had not been achieved in other models.

The impressive achievements of Friedmann's cosmology further highlight the fact that the assumptions underlying it are truly mysterious. Of course, we are not talking about the theoretical basement which is very simple and beautiful; the question concerns the choice of model parameters, the initial conditions that determine the development of the universe. It is precisely the realization of the maximally symmetrical, homogeneous, isotropic initial state, very similar in its properties to vacuum. This requires a precise matching of the model parameters with precision, which has no analogues in physics.

Otherwise, the world would be completely different, unsuitable for life, at least in its current form. One might think that the Creator took special care to prepare comfortable conditions for us, taking care of each one from 10^{80} particle of the visible part of the world to make it suitable for our life.

This circumstance is the basis for the so-called anthropic principle in its strong formulation: the fact of our existence is the answer to the question of why the universe is the way it is, and why we are in the universe, since in the universe not adapted for life such a question cannot be asked, because there simply would be no one to ask it. In such a situation, physicists have nothing to do. I would therefore like to find some kind of explanation for these "fundamental" problems of cosmology, to construct a model in which the universe developed to its current state more or less independently of the initial conditions, adhering to the fundamental laws of physics. Later, thanks to revolutionary advances in elementary particle theory, it became possible to do this. In this sense, cosmology is entering a new level: fundamental cosmological parameters, considered as given values, as a result of the choice of initial conditions, may turn out to be calculable quantities. However, it should not be assumed that all these cosmological problems have already been solved; there are still many difficulties ahead, and, possibly, the final answer will differ significantly from the variants currently under consideration, but in any case, there is a fundamental possibility of answering the question about the selection of initial conditions in the "best of all possible worlds" scenario.

The order of presentation of the material in the article is the following: in section 1, the basic observational facts about the universe, that will be relevant for the future content, are briefly discussed. Section 2 examines important cosmological problems and indicates possible ways of solving them within the framework of the so-called model of an expanding universe, the inflationary model. Section 3 provides a more detailed description of the inflationary model, its difficulties, and possible ways to overcome them. The conclusion summarizes the results.

Universe today (observational data)

1. The fact of the expansion of the universe is not disputed by anyone. Apparently, it does not cause any objections to the Hubble's law of the proportionality of the speed of an object to the distance of that object

$$v = Hr \quad (1)$$

this is true of course on the average, with exclusion of the chaotic motion of individual galaxies in their clusters. So far, however, there is no consensus on the value of the coefficient of proportionality, known as the Hubble constant H . Most astronomers currently give values of H close to 100km/sec/Mps [11] but there are works [12] in which a twice smaller value is presented. The criticism of the latter paper by the supporters of the large value of H seems convincing but the data on the universe age (see section 6 below) force us to conclude that $H = 100$ km/sec/Mpc remains a viable possibility.

2. Whether the expansion will stop or continue indefinitely is determined by the ratio of the average energy density in the universe to the so-called critical density ¹

$$\Omega = \varrho/\varrho_c, \quad \varrho_c = 3H^2/(8\pi G) = 1.86 \cdot 10^{-29} h_{100}^2 \text{ g/cm}^3 \quad (2)$$

where G is the gravitational constant: $G \equiv m_P^{-2} = (1.22 \cdot 10^{19} \text{ GeV})^{-1}$ and

$$h_{100} = H/(100 \text{ km/sec/Mpc}). \quad (3)$$

If $\Omega > 1$, then the universe is closed and a period of contraction will eventually take place; if $\Omega \leq 1$, then expansion will continue indefinitely. The case $\Omega = 1$ corresponds to the spatially flat, Euclidean universe,

Modern estimates of the average energy density [13], based on measurements of its gravitational effects, are close to $\Omega = 0.3$, which supports the open universe model.

It is interesting that the determination of the matter density by direct estimate of the amount of matter contained in visible objects and interstellar space, gives the result approximately an order of magnitude smaller, $\Omega_B \sim 0.03$. The sub-index B indicates that this quantity is the usual proton-neutron or baryonic matter. The discrepancy between Ω found from dynamics of galaxies and Ω_B creates the problem of dark matter of the Universe [14].²

One might think that by some not yet known reasons we do not see a significant part of the usual proton-neutron-electron matter. However, given the abundance of deuterium and helium-4 in the Universe, on the one hand, and the theory of galaxy formation, on the other, this possibility is unlikely. The most popular view is that the invisible matter is either massive neutrinos or some kind of weakly interacting particles, such as axions or photinos. A discussion of these issues in the literature can be found in ref. [16]. A modifications of gravitational interaction cannot be ruled out as well.

In principle, an invisible matter could lift Ω up to unity but only if it is distributed homogeneously throughout all the space [17], though is seems quite unlikely, but strictly speaking not excluded. This may be of interest for the model of inflationary universe cosmology discussed below.

3. As is well known, the General Relativity equations allow for a generalization by introducing the so-called cosmological term, Λ [6]:

$$R_{\mu\nu} - (1/2)g_{\mu\nu}R = 8\pi GT_{\mu\nu} - \Lambda g_{\mu\nu}, \quad (4)$$

¹We use here the system of units used in particle physics, where speed of light, Boltzmann constant, and reduced Planck constant are all equal to unity: $C = k = \hbar = 1$. For example the proton mass is equal to $m_p = 940 \text{ MeV} = 10^{13} \text{ K} = 5 \cdot 10^{13} \text{ cm}^{-1} = 1.5 \cdot 10^{24} \text{ sec}^{-1}$.

²Fritz Zwicky who discovered dark matter in the 30th and was badly criticized by the community referred contemptuously to “the useless trash in the bulging astronomical journals”, saying “Astronomers are spherical bastards. No matter how you look at them they are just bastards.”

where $T_{\mu\nu}$ is the energy-momentum tensor of matter, and the term proportional to Λ describes gravitating vacuum and can be written in the form with ϱ_{vac} being the vacuum energy density:

$$\Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}^{vac} = -8\pi G \varrho_{vac} g_{\mu\nu}. \quad (5)$$

In the standard scenario of the universe evolution it is assumed that $\Lambda = 0$, though the observational bounds evaluated in units of the critical energy density are not very strong:

$$|\varrho_{vac}| < 5 \cdot 10^{-47} m_N^4 \approx 10^{-29} g/cm^3 \sim \varrho_c. \quad (6)$$

On the other hand, the vacuum energy is very small compared to the characteristic values of ϱ at the time of the universe creation and in view of that it looks natural to assume that ϱ_{vac} identically vanishes. We will return to this issue below.

4. The averaged over large cosmological scales distribution of matter in the universe is highly uniform. Of course, the inhomogeneities are huge at the galactic scales but at distances greater than about 100 Mpc, with the variations of density over directions are quite small

$$\Delta\varrho/\varrho < 10^{-3}. \quad (7)$$

The homogeneity and isotropy of the universe is also supported by the observations of cosmic microwave background radiation which directional variations do not exceed 10^{-4}

5. There are compelling observational reasons to believe that antimatter is practically absent in the universe, i.e. positrons, antiprotons, antineutrons, but it follows from theory that the amount of antimatter is most likely not insignificant. Strictly speaking, it cannot be ruled out that distant galaxies consist of antimatter, but in all known cases of colliding galaxies or galaxies surrounded by common clouds of interstellar gas, it is clear that these regions contain matter of the same type. This circumstance, as well as the fact that there are few antiprotons in cosmic rays, makes the hypothesis of the existence of antimatter in significant amount highly unlikely. The presence only of matter in the surrounding universe is called the charge or baryon asymmetry of the universe.

An important constant in cosmology is the ratio of the average density of baryons N_B to the density of relic photons N_γ :

$$N_\gamma = 550(T/3K)^3 \text{cm}^{-3}. \quad (8)$$

According to current data, this ratio is in the range

$$\beta = N_B/N_\gamma = 10^{-9} - 10^{-10}. \quad (9)$$

6. The time elapsed from the hot singularity to the present time, is called the age of the Universe t_U . The value of t_U is surely greater than the estimated age of the Earth: $5 \cdot 10^9$ years. Nuclear chronology, as well as the theory of stellar evolution, together with the fact of the observations of old star clusters, leads to a much larger value [18]

$$t_u \approx 15 \cdot 10^9 \text{ years}. \quad (10)$$

Theory permits to express the age of the universe through the current values of Hubble's constant and parameter Ω . Since, according to the standard scenario, the universe spent

most of its life in a state of dominance of non-relativistic matter the following expression is valid:

$$t_u = 10.8 \cdot 10^9 \text{ years} [h_{100}(1 + \sqrt{\Omega}/2)]^{-1} \quad (11)$$

(under assumption that the cosmological constant vanishes).

So we have to choose between the following possibilities:

- a) the Hubble parameter is smaller than that presented in majority of papers, $h_{100} < 0.6$;
- b) nuclear chronometry and stellar evolution theory suggest a significantly larger value: $t_u \approx 15 \cdot 10^9$ years;
- c) the cosmological constant is non-zero and close to its upper limit on ϱ_{vac} (6), permitted by astronomical observations.

Let us note, anticipating what is written below, that in the inflationary universe model these contradictions become even more profound, since in this model the value $\Omega = 1$ is predicted and the universe age t_u should be smaller than that in the case $\Omega = 0.3$.

Fundamental Cosmological Problems.

The Einstein equations which make the basis of the modern cosmology have the following very simple form for homogeneous and isotropic distribution of matter:

$$\ddot{a} = \frac{4\pi G}{3} a (3p + \varrho), \quad (12)$$

$$\dot{a}^2 = \frac{8\pi G}{3} \varrho a^2 - k. \quad (13)$$

where dots denote differentiation over time, ϱ and P are respectively energy density and pressure of matter (with possible inclusion of the vacuum term); a is the scale factor, the value of which is not determined. If $k \neq 0$, the value of a can be normalized by the conditions $k = +1$ or $k = -1$.

Equation (13) can be conveniently rewritten as

$$\varrho = \varrho_c - \frac{k}{a^2}, \quad \varrho_c = \frac{\dot{a}^2}{8\pi G a^2}, \quad (14)$$

from which it follows that $k < 0$ corresponds to a closed universe, and $k > 0$ corresponds to an open one.

There are different expansion regimes depending on the equation of state. Assuming $k = 0$ we find the following.

Relativistic gas: $\varrho \sim a^{-4}, a \sim t^{1/2}$:

$$p = \varrho/3, \quad \varrho(a) \sim a^{-4}, \quad a \sim t^{1/2}; \quad (15)$$

Non-relativistic gas: $\varrho \sim a^{-3}, a \sim t^{2/3}$:

$$p = 0, \quad \varrho(a) \sim a^{-3}, \quad a \sim t^{2/3}; \quad (16)$$

Cosmic strings:

$$p = -\varrho/3, \quad \varrho(a) \sim a^{-2}, \quad a \sim t; \quad (17)$$

Domain walls:

$$p = -2\varrho/3, \quad \varrho(a) \sim a^{-1}, \quad a \sim t^2; \quad (18)$$

Gravitating vacuum:

$$p = -\varrho, \quad \varrho(a) = \text{const}, \quad a \sim \exp \left[\sqrt{\frac{8\pi G\varrho}{3}} t \right]. \quad (19)$$

At the present time the universe is dominated by nonrelativistic matter and the expansion regime is close to (16), if ϱ is not too much different from ϱ_c . The transition from relativistic regime to non-relativistic one takes place at the redshift $z = 4 \cdot 10^4 h_{100}^2$. It is unknown if there were regimes dominated by cosmic strings or domain walls, but it is often assumed that there was period of cosmological term dominance,

The cosmological impact of domain walls which appears at spontaneous discrete symmetry breaking was considered in ref. [19], where it was pointed out that these walls, if they existed, would destroy the homogeneity of the universe.

The role of cosmic strings was studied in papers [20], where it was shown that the inhomogeneities created by strings are safely small and, moreover, strings that appear in unified theories of strong and electroweak interactions with characteristic energy scale $10^{14} - 10^{15}$ GeV could explain the observed the large scale structure of the universe in the forms of galaxies or their clusters.

How far can we travel backward in time depends upon our knowledge of the particle interactions at high energies and densities. One may be sure that the simple relativistic expansion regime (15) was realized up to energies of several tens or even hundreds MeV. Somewhere close to these energies the equation of state of the primeval plasma might be changed because of the QCD phase transition from free quarks and gluons down to hadrons. Prior to this phase transition the equation of state of the primeval plasma is also close to that of the ideal gas. According to our present day knowledge, confirmed by laboratory experiments, this is possibly true starting down from the temperatures about 100 GeV. Advancing into the region of higher temperatures is not so reliable, since experimental data on the particle properties at $E > 100$ GeV are practically absent.

On the other hand, there is a theory that describes strong and electroweak interactions of elementary particles in a unified way and has a number of other attractive features, which asserts that nothing revolutionary happens up to the Planck energy $E \approx 10^{19}$ GeV when, apparently, the effects of quantum gravity become significant. It is not yet clear how to deal with this.

Within the framework of such theories, it can be concluded that the dynamics of the universe is in principle known up energies $E_{PL} = 10^{19}$ GeV. If we assume that all phase transitions occurring during the cooling of the Universe are phase transitions of the second type or weakly delayed transitions of type I, then the influence of these transitions on the nature of expansion will not be particularly noticeable and the expansion regime (15) would be approximately valid up to the Planck energy.

The statement about phase transition in theories with spontaneously broken symmetry was first made in ref. [21] and since then it has been used in cosmological models.

Returning to equation (12), we rewrite it in the form:

$$\Omega_2^{-1} - 1 = (\Omega_1^{-1} - 1) \frac{\varrho_1 a_1^2}{\varrho_2 a_2^2}. \quad (20)$$

where the indices 1 and 2 refer to the values of the quantities at times t_1 and t_2 . Taking t_1 as the present moment and t_2 as the moment of transition from relativistic to non-relativistic expansion law we find:

$$\Omega_2^{-1} - 1 = (\Omega_1^{-1} - 1) z_2^{-1} \approx 10^{-4}. \quad (21)$$

Choosing t_2 as the beginning of the primordial nucleosynthesis: $t_2 = 1$ sec and $T_2 = 1$ MeV, and in this state the universe surely was, as is proven by the abundance of light elements, we find $\Omega_2^{-1} - 1 \approx 10^{-16}$. If we move further deeper to the beginning up to $T_2 = T_{Pl} = 10^{19}$ GeV, we find that $|\Omega_2 - 1| = 10^{-59}$. In other words, for the Universe to reach the present day state the initial state should be fine-tuned with fantastic accuracy $|\Omega_2 - 1| \approx 10^{-59}$. The result looks natural enough since for $|\Omega_2 - 1| \sim 1$, the characteristic time when the universe expansion for a closed universe would turn into contraction during time close to the Planck one, while an open universe would expand so fast that neither galaxies nor stars or planets would be formed. Thus our existence demonstrates the extremely fine-tuned initial state of the universe close to the flat 3D one ($k = 0$).

This mysterious fact is the subject of one of the most pressing problems in cosmology: how did such favorable extremely fine tuned initial conditions could arise? This problem is called **the problem of initial conditions** (the first problem).

There are several other cosmological problems, without solving which we cannot be sure that we understand how our world was created.

The second problem: Isotropy and homogeneity of the universe also imply very specific initial conditions, noticeable deviations from which are not allowed even by the anthropic principle. This makes it all the more desirable to find a natural explanation for this fact.

The isotropy of the cosmic background radiation presents a new problem (the third), which is the so called the horizon problem. It is related to the fact that the scale factor in the regimes (15,16) rises slower than the size of the causally connected region $r_h \approx t$.

Relic radiation became free, i.e., it ceased to interact with anything at the hydrogen recombination temperature $T_{rec} \approx 3000$ K. This corresponds to the change of the scale factor by about the factor of 1000. For the standard model of the universe evolution this corresponds to the horizon size at this moment equal to $\sim 10^{13}$ sec. Correspondingly the size of the region of the physical processes which could create background radiation cannot exceed this value.

The attempt to explain this phenomenon, based on the hypothesis of the existence of a maximum density of energy, was made in [22], but the question of the existence of a maximum density of energy was not resolved. An attempt of this kind, based on the hypothesis of the existence of maximum energy density, was made in ref. [22], but the question of entropy growth in this model remains unclear.

The problem of charge asymmetry in the universe (i.e. the presence in the universe only of particles (i.e., protons, neutrinos, electrons) and practically absence of

antiparticles, which has been a problem for cosmologists for many years, now can be considered as solved according to ref. [23].

Within the frameworks of the model, there is not only qualitative but also quantitative agreement with astronomical data for the value of N_b/N_γ . To avoid returning to this question later, let us briefly review main features of the mechanism of generation of the excess of particles over antiparticles in the universe. A more detailed discussion can be found in review [24] and in the popular paper [25].

The basic hypothesis is that the baryon charge B is not conserved. Such conservation is indeed predicted by models of grand unification. In these models, there are superheavy particles with masses of the order of $10^{14} - 10^{15}$ GeV decaying into the states with different values of the baryonic number. Two other essential ingredients of the model are the violation of charge invariance (i.e., the difference in interactions between particles and antiparticles) and deviation from thermodynamic equilibrium in the expanding universe. It can be shown that under such conditions, the decay of the aforementioned superheavy particles (X-, or H-bosons) will lead to an excess of particles over antiparticles which because of the violation of thermal equilibrium will not be compensated for by other processes. Although the magnitude of this excess cannot be precisely calculated, since the details of the interaction between X-f-bosons are unknown, order-of-magnitude estimates give an answer that is reasonable and consistent with observations. For a number of the models the result does not depend on the initial conditions, i.e. on whether there was initially an excess of baryons or antibaryons, or whether the plasma was charge neutral. An essential feature of this scheme in its classical version is the presence of a large number of X- or Z-bosons in the initial state of the plasma, for which it is necessary to reach the temperatures about $T \approx 10^{15}$ GeV.

The seventh problem, the problem of magnetic monopoles, is somewhat different from those listed above, since it is not a specifically cosmological problem, but is related to the prediction of the existence of magnetic monopoles in models of grand unification. Born in the phase transition from symmetric to asymmetric states during the cooling of the universe, these monopoles would survive until the present day, and their current concentration, calculated within the framework of the standard model, turns out to be unacceptably large [26].

Problems 1-3 and 7 are uniquely and beautifully solved by the **inflationary universe model** [27, 28], but at the same time, problem 4 only gets worse. The main idea of the inflationary universe model is that at some stage the energy-momentum spectrum was dominated by a vacuum energy term, $T_{\mu\nu}^{(vac)} = \varrho_{vac}g_{\mu\nu}$. According to eq. (19) the scale factor rises exponentially and the energy density quickly approaches the critical one, i.e. $\Omega \rightarrow 1$, see eq. (20).

In this model, the universe at a given time moment looks as an expanding empty space (density of the usual matter exponentially tends to zero). At the same time, all initial conditions are "forgotten" and the world becomes as uniform as emptiness can be. Then the vacuum "matter" explodes, giving birth to elementary particles [29], which are thermalized and the regime of expansion turns into the Friedman one. The required duration of the exponential (De Sitter) period τ depends on the temperature of the created particles. To ensure $\Omega \sim 1$ at the present time it is necessary that $\exp(H\tau) > 10^{30}(T/M_{Pl})$,

that is

$$H\tau > 70 - \ln(M_{pl}/T) \quad (22)$$

This does not look unreasonably large.

If the de Sitter stage actually took place, then at the "zero" moment, parameter Ω could have had practically any value. If the universe is open and $\Omega \leq 1$, there no problem arose as the density of matter decreases, the vacuum energy ϱ_{vac} begins to dominate, which does not change in the course of expansion. If however, $\Omega > 1$ the universe might start to contract when still $\varrho_m > \varrho_{vac}$ and the exponential stage would not be there. However, (this comment belongs to L.B. Okun) it is known that in the oscillating universe the amplitude of its oscillations should increase due to rise of entropy and respectively ϱ_m would be diminished at the the point of maximum expansion. Hence sooner or later a closed universe should come to the exponential expansion, of course, if $\varrho_{vac} > 0$.

Thus, the problem of proximity of ϱ to ϱ_c at the present time can be solved without hypothesis of the fine tuning of the "initial" parameters. However, if we take such point of view, then for the explanation of the proximity ϱ to ϱ_c , inflationary scenario is unnecessary since the swinging universe will gradually come to then observed present day state and we just happened to be there when the conditions for life became suitable.

The problem of the horizon in inflationary universe is also naturally solved because the scale factor $a \sim \exp Ht$ grows faster than horizon.

Note that inflationary model, parameter Ω should be extremely close to unity because even a very small excess of the exponential period duration over its necessary value (22) and deviation of Ω from unity is actually determined by the fluctuations of density in this model. Astronomical data, however, rather contradicts this, giving a value of Ω close to 0.3, unless the universe is filled with homogeneously distributed massive particles, as for example, neutrinos [30].

In this regard, it is very important to clarify the value of the Hubble's parameter, since for $\Omega = 1$: $h_{100} = 7.2 \times 10^9 \text{ years}/t_u$. This may be a realistic way to test inflationary models.

The problem of monopole can be also solved in this scenario if, after the phase transition which leads to monopole creation, there is still a noticeable exponential expansion. Note that this is required to solve the problem of homogeneity (see above). In this case, there would be no more than one monopole [31] in the visible universe, and a discovery of a second one would require a strong modification of the standard model.

Let us note right away that in supersymmetric inflation models [32] the problem of the universe revives, and there are possibilities when the number of monopoles in the universe is non-vanishing but still does not contradict observations

Return now to the problem of homogeneity. In the first proposed model [27] it was assumed that the exponential expansion occurred in a symmetric state before transitioning to a non-symmetric phase. Subsequently, the bubbles of the new phase were formed that filled al the space. Generally in such a model the inhomogeneities created by the bubble walls should be very large (in ref. [34] a mechanism of vacuum burning is discussed for which it is probably not so). Much smaller inhomogeneities arise in the new inflationary scenario [28], according to which considerable exponential expansion took place not only before but also after phase transition, since in such models the vacuum average of the

scalar field (order parameter) very slowly in comparison with the universe expansion rate tended to its limiting value.

The problem of inhomogeneities was studied in a series of papers [35], where it was shown that in the standard approach based on Coleman-Weinberg $SU(5)$ model (see below) the inhomogeneities would be acceptably small only for a very unnatural choice of model parameters. This forced us to turn to supersymmetric theories [36], in which such a disadvantage could be overcome. However, in the supersymmetric approach, other difficulties may arise, related to magnetic monopoles or to particle production, and generation of the baryon asymmetry, which should be treated with more complicated models

Summarizing, it can be said that, in principle, the inflationary scenario offers a beautiful way to solve a number of cosmological problems, but there is hardly a natural concrete model that is completely free from all shortcomings.

Concluding this section, I would like to note that, without solving the problems of cosmological inflationary models, in which exponential growth is caused by the nonzero vacuum energy, at least psychologically, does not seem entirely satisfactory, since it is one thing to assume that ϱ_{vac} is always zero, while it is completely different to assume that at the "beginning" there was $\varrho_{vac} = -\delta\varrho$ where $\delta\varrho$ is the change of vacuum energy at the phase transition³. (However, defenders of these models may reasonably argue that the cosmological vacuum energy problem exists independently of inflation.)

An exception is presented by the model suggested in ref. [37], where inflation occurs in the pre-Planck era and is caused by non-linear quantum corrections from vacuum polarization to the Einstein equations. It is also possible that the inflationary model is not the only way to solve the cosmological problems under discussion; in fact, there is an alternative attempt [38] based on the hypothesis of a large number of phase transitions in the early universe with a huge increase of entropy.

However, one way or another, inflationary model is the first cosmological model in which a natural solution of several "eternal" cosmological problems have been realized which were previously considered as peculiarities of specific initial conditions. In more details, with some technicalities and analysis of the difficulties encountered, these issues are discussed in the following section.

Returning to the problem of cosmological constant, we note that, as is mentioned above, that the data on t_u , H , and Ω strongly indicate that $\varrho_{vac} \neq 0$. If inflationary model is valid, and $\Omega = 1$ then for $h_{100} = 1$, and $t_u = 15 \cdot 10^9$ years ϱ_{vac} should be positive and contribute today approximately $0.95\varrho_c$.

Since ϱ_c varies with time (roughly speaking, as m_{Pl}^2/t^2), and $\varrho_{vac} = const$ it means that only at the present stage is the role of ϱ_{vac} is noticeable, while at earlier it could be neglected, which is also one of the mysterious coincidences.

Nowadays there is no satisfactory model explaining the smallness of the cosmological constant, but if the presented above values of H and t_u are confirmed, it would be natural to demand not a complete compensation of ϱ_{vac} but only down to the terms of order m_{Pl}^2/t^2 .

This, in turn, means that the non-compensated part of ϱ_{vac} , which, strictly speaking, is not proportional to $g_{\mu\nu}$ could be noticeable during all the history of the universe and to

³Recently A.D. Linde suggested a model of exponential expansion that occurred without strong first order phase transition. It is discussed in more detail in the next section.

make an impact on the big bang nucleosynthesis, galaxy formation, and to give a contribution to the hidden mass of the universe. In ref. [39], a model is suggested in which vacuum energy is cancelled down by the condensate of a scalar field with non-minimal coupling to gravity. The magnitude of the condensate increases by the impact of the cosmological term and this negative back reaction effectively eliminates vacuum energy. The vacuum energy decreases rather slowly, with the remaining value always being of the order of $\varrho_c(t)0$.

This is achieved due to the very strict and unnatural requirements imposed by the quantum field theory used in the model. It can be said that the problem of cosmological constant is the central problem of cosmology, without whose solution no cosmological model can be considered satisfactory.

As for the problems of singularity and the "creation of the world," there are several "crazy" ideas on this subject in the literature. I would like to add one more to them. Suppose there is, say, a scalar field ϕ with an effective potential that has an infinite number of local minima, separated from each other by potential barriers, with each minimum becoming deeper and deeper as ϕ increases. As simplest examples of such potentials one can take $m^2\phi^2 \cos(\phi/\sigma)$ or $m\phi^3[1-\epsilon \cos(\phi/\sigma)]$. The universe once created, maybe infinitely long ago, stuck in one of those local minima for some time, generally quite long. Then after quantum tunneling this universe or a part of it undergoes into another vacuum state with lower energy and so on, so fourth infinitely many times.

The energy released during this phase transition ends up in the form of elementary particles, which, as a result of the expansion of the universe, especially in exponential period, "dissolves" in the universe. The cosmological constant (or, rather, cosmological constants) inherent in this model can be compensated for by a mechanism of the type described in ref. [39].

Such a model describes the eternally expanding universe, infinitely many times cooling down practically to a state of vacuum and exploding again. A suitable name for such a universe would be "Phoenix-Universe." Unfortunately, or perhaps fortunately, this concept cannot be verified, since in the event of another Big Bang, a possible observer will disappear before noticing anything.

Models of inflating universe.

Most models of inflating universe are based on the assumption that the phase transition from a symmetric to an asymmetric phase is a strongly delayed first order phase transition. In the symmetric phase the scalar field condensate (order parameter) is absent, $\langle\phi\rangle = 0$, while vacuum energy is non-zero:

$$T_{\mu\nu}^{vac} = g_{\mu\nu}V(\sigma), \quad (23)$$

where V is the effective potential of the scalar field and σ is its vacuum average value after the end of the phase transition. Assumption (23) does not have any natural physical basis but it is only a demand that cosmological term disappeared after the phase transition in agreement with observations.

Initially in the universe there could be some matter with energy density ϱ_m but $\varrho_m \rightarrow 0$, while $\varrho_{vac} = const$. If ϱ_{vac} would be larger than ϱ_m earlier than the phase transition took

place, then the universe would exponentially expand for a while:

$$a \sim \exp(Ht), \quad H = \left[\frac{8\pi}{3m_{Pl}^2 V(\sigma)} \right] \quad (24)$$

Typical value of $V(\sigma)$ in Grand Unification Models is about $(10^{15}\text{GeV})^4$, and hence $H \approx 10^{11} \text{ GeV}$.

For description of the phase transition let us consider a simple scalar field model with the Lagrangian

$$L = \frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} m_0^2 \phi^2 - \frac{\lambda}{4} \phi^4 + \dots \quad (25)$$

where multidots stand for the terms describing interactions with other fields: gauge bosons, fermions, and other scalar fields. This Lagrangian is symmetric with respect to the transformation $\phi \rightarrow -\phi$ and possibly some other higher symmetry, if e.g. ϕ is a multi-component field. In the standard scheme of spontaneous symmetry violation it is assumed that the mass squared of the scalar field is negative, $m_0^2 < 0$ and hence the point of the stable potential extremum is $\phi_0^2 = -m_0^2 > 0$. At non-zero temperature the potential acquires an additional term $\alpha \phi^2 T^2$ which could shift the equilibrium point to $\phi = 0$ [40]. So it is clear how field ϕ would behave in the course of the universe cooling down it this simplest model. At high temperature the average value of ϕ vanishes, $\langle \phi \rangle = 0$. This corresponds to the state with unbroken symmetry. With decreasing temperature the sum $m_0^2 + \alpha T^2$ becomes negative and $\langle \phi^2 \rangle = -(m_0^2 + \alpha T^2) \neq 0$ and the particles interacting with ϕ acquire non-zero masses proportional to $\langle \phi \rangle$. It is easy to see that the phase transition in this model is the second order phase transition, which is not what we need. However, in more complex models, for example, those based on the $SU(5)$ group, quantum corrections could lead to the first order phase transition. For the details and references to the original literature one can address reviews [41, 42].

One can show that in certain class of theories the effective potential calculated in one loop approximation, has indeed the desired form [40, 43]:

$$\begin{aligned} V(\phi, T) = & \frac{1}{2} (m_0^2 + \alpha T^2) \phi^2 + \frac{\lambda}{4} \ln \frac{\phi^2}{\sigma^2 \sqrt{e}} \\ & + a T^4 \int_0^\infty dx x^2 \ln \left(\frac{1 - e^{\sqrt{x^2 + b \phi^2 / T^2}}}{1 - e^{-x}} \right). \end{aligned} \quad (26)$$

where $\sigma = 10^{14} - 10^{15} \text{ GeV}$, α , λ , and β are some numbers which depend upon the coupling constant; the value of a does not depend upon the interaction; the contribution of the corresponding term at $\phi = 0$ gives simply thermal energy of particles. In the limit of low temperature we may neglect the last term. This does not change qualitatively our conclusion.

Potential (26) has a minimum at $\phi = 0$ if $m_0^2 + \alpha T^2 > 0$. If $m_0^2 + \alpha T^2 < \lambda \sigma^2 / e$, then there is another minimum at $\phi \approx \sigma$. This minimum will be deeper than the first one and will therefore be stable if $m_0^2 + \alpha T^2 < \lambda \sigma^2 / 4$. Thus the stable at high temperatures minimum of $V(\phi, T)$ at $\phi = 0$ transforms at lower temperatures into a quasistable one (the true vacuum of this theory) separated from the original stable minimum by some potential barrier.

As a rule, the probability of passage through the potential barrier is exponentially suppressed, so the system can remain in a quasi-stable state for a very long time. Tunneling in quantum theory was first considered in paper [44]. A more elegant method was proposed in [45] According to the results of these works, the probability of tunneling is determined by the value of the action calculated on the solution of the classical equation of motion in imaginary time. An approximate answer can be obtained in a simpler way by finding the extremum of the action using the variational method. In particular, the probability of tunneling per unit volume per unit time in potential (26) at zero temperature is equal to

$$\frac{dW}{dVdt} \approx M^4 \exp \left[-\frac{8\pi^2}{3\lambda} \left(\ln \frac{\lambda\sigma^2}{m^2} \right)^{-1} \right], \quad (27)$$

where M is some unknown factor with dimension of mass. Most likely, M is of the order of the inverse size of the bubble of the new phase, i.e. $M \approx m$. Since $\lambda < 1$, the tunneling time is monstrously long and the universe will indeed have time to expand so much that no traces of matter, that was originally in it, would remain. In reality, however, one should not think that the temperature is equal to zero, because due to the presence of the horizon, the temperature in the de Sitter universe cannot be lower than $H/(2\pi)$ [46] (in comoving coordinates).

Since the state with $\langle \phi \rangle = 0$ is unstable, then, sooner or later the transition to the new phase with $\langle \phi \rangle \neq 0$ should take place. Immediately after formation of the bubble of the new phase, the magnitude of the classical field inside it should be of the order $m/\sqrt{\lambda}$. This can be seen from the form of the dependence of the effective effective potential (26) on ϕ . For natural values of parameters of the scalar field potential the value of ϕ quickly, in comparison with H , tends to its limiting value σ . Indeed after the quantum jump creating the phase transition the evolution of ϕ is governed by the classical equation of motion:

$$\ddot{\phi} + 3H\dot{\phi} = -\partial V/\partial\phi \quad (28)$$

with the initial conditions $\phi(0) = m/\sqrt{\lambda}$ and $\dot{\phi}(0) = 0$. Here we neglected the term containing spatial derivatives since their contribution is divided by the scale factor and quickly decreases with time. The value of $\partial V/\partial\phi$ at $\phi = m/\sqrt{\lambda}$ can be estimated using Eq. (26) as $const \cdot m^2\phi$, where it is natural to expect that $m \gg H$. Hence ϕ rises as $\exp(mt)$ and after the bubble formation the phase transition proceeds in time much shorter than H^{-1} and thus an exponential bloating of the bubble does not take place. In this case, there would be many bubbles in the visible today part of the universe, which in turn would lead to too large density inhomogeneities. More detailed discussion and references to the relevant literature can be found, for example, in ref [41].

In the modified version of the inflationary model [33] a strong condition is imposed on potential $V(\phi)$, namely $m^2T \equiv \partial^2V/\partial\phi^2 \ll H^2$ which does not naturally follow from the theory. This condition was specially invented to ensure a slow growth of the classical field $\phi(t)$ after the phase transition, $\phi(t) \sim \exp(m^2t/3H)$. For a solution of standing in front of us problems it is sufficient that $m^2 < H^2/25$. In this case, the bubble's walls quickly disappear at infinity, and the entire universe is contained inside the interior of a single bubble, the characteristic size of which at the moment of formation was of the order of

m^{-1} and which, as a result of the universe expansion, reached the size up to

$$r \gtrsim \frac{1}{r} \exp(3H^2/m^2) \cdot (T/(3K)). \quad (29)$$

Here, T is the temperature of the primary plasma after the phase transition. The total universe expansion in this version of the model is by far larger and differs from that given by eq. (29) by the exponential factor presented in eq. (27). Naturally in this variant of the model, the inhomogeneities will be significantly smaller, but as shown by calculations in the frameworks of the standard $SU(5)$ theory, their magnitude happened to be larger than in reality approximately by two orders of magnitude [47].

Let us note that in contrast to the previous case the inhomogeneities are not connected with the bubble walls, but with rising quantum fluctuations in de Sitter universe. It is interesting that in the standard scenario of the universe evolution, i.e. without de Sitter stage, an estimate of inhomogeneities generated by quantum fluctuations leads to the result approximately two orders of magnitude smaller than it is necessary for galaxy formation.

The condition of small inhomogeneities demands extremely slow variation of the effective potential $V(\phi)$ in rather large interval of variation of ϕ , including the region of small ϕ . However, since the rate of particle production by external field $\phi(t)$ is proportional to the speed of the field variation, then in such a model, the particles that subsequently should fill the expanding void, forming out world, are created too slowly and and their density and temperature, if they manage to thermalize, turn out to be too small and it would be impossible to explain the observed baryon asymmetry of the universe. Detailed discussion of these problem and the gravitational and temperature effects is presented in paper [42]. We note only that usually the role of gravity at the energies much smaller than the Planck one is not essential. However in the model under consideration where a new hierarchy of masses is introduced, namely $m^2 \ll H^2$ or $m^2 \ll R$, where R is the four dimensional curvature of space-time. Just because of that the role of gravitational corrections to the effective potential happens to be non-negligible. In particular scalar field theory allows an addition to the Lagrangian the term $\xi\phi^2R$ that essentially changes the effective mass in de Sitter space,

The smallness of the effective mass m^2 in comparison with H^2 mentioned above implies smallness or cancellation of several terms each making a contribution to the coefficient in front of $\phi^2/2$ in the effective Lagrangian:

$$m^2 = m_0^2 + \alpha T^2 + \xi R + \lambda \langle \phi^2 \rangle + \dots \quad (30)$$

Here m_0 it the field mass in symmetric state in flat space-time with zero temperature. The natural value of m_0 in Grand Unification models is 10^{14} GeV. An exception is presented by the Coleman-Weinberg model [48] which is actually defined by the condition $m_0 = 0$, imposed on $V(\phi)$. Perhaps there is some beauty in that, but strictly speaking, we do not have any basis for this assumption, as e.g. symmetry arguments, to impose $m_0 = 0$. In addition, it should be noted that the Coleman-Weinberg model has been formulated in the flat space, where the condition $m_0 = 0$ means

$$\frac{\partial^2 V}{\partial^2 \phi} \Big|_{\phi=0, R=0} = 0, \quad (31)$$

where R is the curvature of space-time. However, at $\phi = 0$ the condition $R = 0$ in inflationary model is not fulfilled but instead of (32) the relation $R = -8\pi GT_\mu^\mu = 32\pi GV(\sigma)$, was suggested in ref. [42] to change it to

$$\frac{\partial^2 V}{\partial^2 \phi} \Big|_{\phi=0, R=32\pi GV(\sigma)} = 0, \quad (32)$$

Since we do not understand the origin of the cosmological term, the justification of the above condition looks mysterious. In particular in the model of ref [39] the relation $R = 32\pi GV(\sigma)$ generally speaking is not fulfilled. Hence the self-consistent formulation of the Coleman-Weinberg condition should be modified.

The second term in eq. (30) arises due to interaction with thermal bath in which field ϕ is situated. Then constant α is expected to be of the order of Ng^2 where $g \sim 0.5$ is the gauge coupling constant and n is the number of vector fields in the underlying symmetry group. Due to existence of the lower limit on the temperature in De Sitter world $T_H = H/(2\pi)$ [46] this term itself may break the necessary condition $m^2 < H^2/25$. To prevent this from happening, in works [36] where super-symmetric inflation was considered, it was assumed that ϕ is a gauge singlet and hence it does not interact with vector fields. Interactions with other fields may be made arbitrarily weak and so constant α may be very small. This field which only role is to ensure inflation is called the inflaton. If we impose the condition of the conformal invariance at zero temperature we have to take $\xi = 1/6$. In this case the third term in eq. (30) gives too large contribution $\xi R = 12\xi H^2 = 2H^2$. However we know the conformal invariance is, as a rule, broken, hence there is no necessity in this condition. In particular it can be shown that for Goldstone bosons $\xi = 0$ [49].

The last term in eq. (30) is induced by quantum fluctuations in curved space-time [50]. It may be not essential if the self-interaction constant λ of field ϕ is taken sufficiently small. So, leaving aside questions about the naturalness we see that it is possible to ensure sufficient duration of inflation after the phase transition. It is somewhat more difficult to solve the problem of inhomogeneities and generation of the baryon asymmetry. For that to be true the effective potential should be very smooth at $\phi < H$, so that $V''(\phi) \ll H^2$ but very abruptly falling down for a large ϕ . Models leading to potentials of such a type exist but it is still premature to say that the final version of the mechanism of exponential expansion is indeed found.

Let us briefly dwell on particle production in inflationary model. Immediately after phase transition the universe was formless and void, and darkness was upon it. There was no matter in the form of elementary particles. The amplitude of ϕ rose in accordance with eq. (28). The energy-momentum tensor in the r.h.s. of the equations of the General Relativity was given by the expressions:

$$\varrho = \varrho_{vac} + V(\phi) + \dot{\phi}^2/2; \quad p = -\varrho_{vac} - V(\phi) + \dot{\phi}^2/2 \quad (33)$$

with $V(0) = 0$ and $V(\sigma)$ satisfying equation (23), when ϕ reaches the value σ which corresponds to the stable minimum of the potential.

In the new inflationary model [33] the tunneling, as it was already noted, goes to small $\dot{\phi} = \dot{\phi}_0$, such that $V(\phi_0) \ll \varrho_{vac}$. It is also assumed the field varies very slowly, so that $\dot{\phi}/\phi \ll H$. Hence at the first stage the character of expansion practically would not be

changed. Particle production at this stage practically would not take place because of slow variation of ϕ . Later when ϕ reaches sufficiently large values $V(\phi)$ becomes steeper and damped oscillations of ϕ around equilibrium point σ begin. Damping of oscillations is induced by the two reasons: firstly by the universe expansion which is described by the friction term $3H\dot{\phi}$ in eq. (28) and secondly by the particle production which is not explicitly taken into account in (28). Since at large ϕ the oscillation frequency is quite high, $\omega = m(\phi = \sigma) = (10^{14} - 10^{15})$ GeV, particle production at this period becomes quite essential. The expansion regime is drastically changed going from the exponential (19) to the power law one (15). Indeed for harmonic oscillations the pressure given by eqs. (33) is zero. In our case deviations from harmonicity are not essential. The estimates made for several concrete models show that the rate of particle production \dot{N}/N (where N is particle density in unit of space) is, as a rule, higher than the universe expansion rate, $H = 2/(3t)$. It is assumed in the standard model that ϕ is the Higgs field, so its couplings to other particles are proportional to the masses of the latter. Thus predominantly heaviest particles are created under condition that their mass is not too much larger than the oscillations frequency ω . Hence the universe would be filled by superheavy boson in strongly out of equilibrium state. The last condition is favorable for generation of the baryon asymmetry of the universe. After superheavy boson decays light particles such as leptons and quarks are created and the primeval plasma was at last thermalized and acquires some temperature T_1 and the expansion law became relativistic (15). The fact that the energy-momentum tensor was for a while dominated by heavy particles and the particle number density was smaller than the equilibrium one leads to some diminishing of the baryon asymmetry after thermalization:

$$\beta = 3\beta_0 \frac{T_1}{m(\phi = \sigma)}, \quad (34)$$

where $m(\phi = \sigma)$ is the scalar field mass at the equilibrium point σ and β_0 is the baryon asymmetry originally created via decays of heavy bosons. Thus the models in which the primeval plasma is cooled down too much to the moment of thermalization are excluded.

In addition to the difficulties described above, which are more technical than fundamental, the inflating universe theory faces a number of problems related to quantum tunneling in a gravitational field. The tunneling theory proposed in refs. [44, 45] for flat space was generalized to the case of tunneling in de Sitter space [51].

However, the results of these works cannot be directly applied to the case of interest, since the transition to the imaginary time, which lies at the heart of the method used, was performed for the exact de Sitter space, while the real universe is not such, and deviations of the exact solution from the approximate one may be significant.

Here it might make sense⁴ to use the Hamilton formalism and to try to solve the functional Schrödinger equation in quasi classical approximation (there is no other known way anyhow), that describes the considered quantum field theory in the external curved metric. The neglect of the back reaction of the field on gravity is justified by fact that the condition of a long inflationary phase after formation of the bubble of the new phase is equivalent, as one can easily see, to the demand that the vacuum energy changes very little after the quantum jump.

⁴Analogous arguments have been used by A. Goncharov and A. Linde, private communication.

Assuming the metric has the form $ds^2 = dt^2 - e^{2Ht}dr^2$ we obtain for the wave functional $\Psi(\phi)$ the the following equation:

$$\left[2i\frac{\partial}{\partial\tau} + \frac{\delta^2}{\delta\phi^2} - \frac{2}{9H^2\tau^2} \int d^3x \left[\frac{1}{2}(3H\tau)^{2/3}(\nabla\phi)^2 + V(\phi) \right] \right] \Psi = 0, \quad (35)$$

where $\tau = e^{-3Ht}/(3H)$, and $V(\phi)$ is the effective potential of field ϕ :

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4 \ln\frac{\phi^2}{\sigma^2}. \quad (36)$$

Thus there arises a problem of tunneling in time dependent potential.

An example of significantly simpler one dimensional (and not infinitely dimensional) quantum mechanical problem as (35) with the potential $U = \tau^{-n}v(x)$ shows that for $\tau \rightarrow 0$ the usual expression for the tunneling probability

$$\Gamma \sim \exp \left[- \int dx \sqrt{2mU} \right] \quad (37)$$

is applicable if $n > 2$ but is not if $n < 2$. In the case $n = 2$ the result depends upon the parameter of the potential $V(\phi)$ and eq. (37) is valid if the coefficient in front of x^2 is sufficiently large. If this result is directly applied to eq. (35), then one can see that for the validity of the improved inflationary model the situation is opposite, $m^2 \ll H^2$. But in this case quasiclassical approximation is not applicable. Thus finally we do not have an adequate formalism for description of tunneling in gravitational field.

For realization of inflationary model the value ϕ_0 , which takes field ϕ after tunneling, is of primary importance. For potential (36) in flat space-time the value $\phi_0 = m/\sqrt{\lambda}$ is sufficiently small to lead to slow motion of ϕ to the limiting value σ , not destroying long exponential expansion. On the opposite if ϕ_0 , is large, the equilibrium state is reached quickly and exponential expansion turns into the power law one. One can show that if the size of the created bubble of the new phase is r , the magnitude of the field in this bubble is $(\sqrt{\lambda}r)^{-1}$. Thus for successful inflation large bubbles are necessary.

However in the theory described by the non-stationary equation (35), the bubble size is unknown. In the case considered in ref [51] it is shown that the bubble size does not exceed H^{-1} that is natural, because this is the horizon size in De Sitter space. Still it is unclear if this result is a consequence of the thin wall approximation used in the quoted papers or, which is probably more important, that the universe is not exactly the De Sitter one. The point is that the transition to imaginary time leads to the transformation of the De Sitter space into four dimensional sphere of radius H^{-1} so the size of the bubble in three dimensional space in the moment of its formation cannot exceed that. If this result survives in the real situation, the inflationary model could be in serious danger, because in this case ϕ_0 would be big and hence a large expansion of the bubble is impossible.

Another point could be serious is that we use effective Lagrangian assuming that the fields are slowly changing but their variation in the expanding world is not so small, generally speaking it is $\dot{\phi}/\phi \approx H$. So we need to take into account loop corrections (but how?) not assuming $\phi = const$. It is not clear how all that may influence on the tunneling and the value of ϕ_0 .

The super-small size of the region from which our Universe began to inflate is also often a source of concern. According to equations (29) and (27) even for a rather modest value $\lambda = 0.1$ the size of the region which now makes all the visible universe was surely smaller than, say, 10^{-100} cm. It is difficult to agree that at so small distances, even in vacuum (which as we now know is quite complicated) a serious modifications of the known to us physical laws have not took place. In my opinion there is no reasons for anxiety, because one can always speak about exponential expansion of a sufficiently large regions where no surprise in the vacuum structure happens. Even if in the course of inflation of very small regions some unknown phenomena appear, they should disappear when the size of these regions becomes sufficiently large,

Recently an interesting version of inflationary model for which no phase transition is necessary was suggested by Linde [52]. The starting point of this model is the assumption that at some initial moment scalar field ϕ might take very large values $\phi \gg m_{Pl} \approx 10^{19}$ GeV, with spatial variation of ϕ being sufficiently low. Such a situation can be realized in the case of chaotic initial conditions if the self-interaction coupling constant is small, so that $V = \lambda\phi^4/4 < m_{Pl}^4$. In this case at the right hand side of evolutionary equation (28) the term $H\dot{\phi}$ starts to dominate, where $H \approx (8\pi\varrho m_{Pl}^{-2}/3)^{1/2} \approx (2\pi\lambda/3)^{1/2}\phi^2 m_{Pl}^{-1}$ and the solution of this equation takes the form

$$\phi = \phi_i \exp \left[-\frac{\sqrt{\lambda}}{\sqrt{6\pi}} m_{Pl} t \right]. \quad (38)$$

Hence the expansion rate of the universe $\dot{a}/a = H$ happens to be larger than the rate of ϕ decrease due to the factor $\phi_i/m_{Pl} \gg 1$ and the universe region, where the conditions mentioned above were accidentally, has expanded exponentially:

$$\frac{a}{a_0} = \exp \left(2\pi \frac{\phi_i^2}{m_{Pl}^2} \right). \quad (39)$$

This expansion could provide a solution of the problems discussed above if the ratio ϕ_i^2/m_{Pl}^2 is sufficiently large, namely $\phi_i^2/m_{Pl}^2 \gtrsim 10$. So with chaotic initial conditions in infinite universe there always could found a region that strongly exponentially expanded and as a result reach the state suitable for our existence. Other uncomfortable regions of the universe would be outside of possibilities of our observations

For realization of this model it is not necessary to impose many special condition on the field theory, it is enough to use a simple hypothesis on existence of weakly interacting and self-interacting field ϕ , i.e the assumption of a small λ and weak coupling to other fields. Not yet worked out is the problem of quantum gravity corrections to the classical equations of motion for a large ϕ .

Of course there is still a question about naturalness of the initial conditions. In contrast to the classical Friedman cosmology, where a very precise fine tuning of the initial state is demanded, here we have the stochastically distributed field ϕ in chaotic universe near singularity. In my opinion the hypothesis about initial chaos is much more attractive and this variant possibly corresponds to the real case, though the question about the origin of the initial chaotic state remains open.

Conclusion

So presently there are two principally different approaches to the problem of the origin and evolution of the universe. The first one, to one or other degree, is based on the anthropic principle, according to which the fact that life existence in the universe makes senseless the question why the universe is such but not other. This approach cannot be denied the right to exist, especially if there are an infinite set of different universes is realized. Then out of this chaotic set only a few universes with very specific conditions could be available for us. However from the point of view of the anthropic principle the colossal redundancy of other galaxies is mysterious.

In other approach is assumed that the universe is one and only one but initial conditions there is arbitrary. However, and this lies in the basement of all theoretical models, the laws of physics are such that practically from any initial state we arrive to our very non-trivial world.

Inflationary model discussed above responds positively to the latter demand. However, the version with chaotic universe is an intermediate one between those two approaches. It goes without saying that this model cannot be considered as the final theory. From one side there are some unsolved problems inside the model, such as e.g. that concerning tunneling in the expanding world. On the other hand it is not established on which field theory this model is based. It's difficult to expect to find the solution to the last problem until the elementary particle theory is not worked out that is applicable up to the Planck energies. Sooner it is another way around, if it is assumed that inflationary model based on strongly delayed first order phase transition, does indeed correctly describes reality, one could derive conditions that the elementary particle theory must satisfy.

What ground we have to believe that the model of inflationary universe is really true? First, it is beautiful, since it is based on one very simple condition on existence of De Sitter stage some time in the past. This allows to solve in a uniform way the problems of homogeneity, isotropy, horizon, flatness, and relic magnetic monopoles. These facts are surely in favor of this scenario. Against the the model, though indirectly, is the problem of the cosmological constant and to a smaller (up to the present time) extent and an absence of the detailed theoretical scheme. The status of inflationary model would be very much stronger if it confirmed that the cosmological parameter $\Omega = \varrho/\varrho_c$ is equal to 1. Unfortunately at the present time it is not seen if this can be established with sufficient accuracy On the opposite it seems it is easier to reject the model obtaining an upper bound on Ω .

Still even if inflationary model obtains convincing evidence in its favor (most probably they will be theoretical but not observational) still complete happiness will be far away until the approaches to solving the two remaining critical problems have been found: the problem of cosmological constant and the problem of the universe creation and singularity. However, we mustn't forget that "appetite comes with eating" - after all, quite recently, those fundamental problems that we now, thanks to the inflationary model, consider already solved, or we say (those who are more cautious) that there appears a possibility of their solution. A few years ago, they seemed completely impregnable, and the importance of this achievement should not be underestimated.

Appendix - translation of the content of Einshteinovskij Sbornik

- A. Einstein. How the Theory of Relativity Was Created.
- B.E. Yavelov, V. Ya. Frenkel. On Some Historical and Physical Aspects of the Einstein-de Haas Experiments.
- V.Ya. Frenkel, B.E. Yavelov. "This is What Can Happen to a Person Who Thinks a Lot but Reads Little."
- B.G. Kuznetsov. The Einstein-Bohr Collision, the Einstein-Bergson Collision, and Science in the Second Half of the Twentieth Century.
- V.P. Vizgin. Einstein, Hilbert, Weyl: The Genesis of the Program of Unified Geometrized Field Theories.
- A. Salam. Einstein's Final Vision: Unifying Fundamental Interactions and the Properties of Space-Time.
- A.D. Dolgov. Progress in Particle Physics and Modern Cosmology.
- B. M. Bolotovsky. On the Apparent Form of Rapidly Moving Bodies.
- G. A. Lorentz. On Einstein's Theory of Gravitation.
- T. Levi-Civita. On an Analytical Expression for the Gravitational Tensor in Einstein's Theory.
- E. Schrödinger. Components of the Gravitational Field Energy.
- G. Bauer. On the Components of the Gravitational Field Energy.
- G. Nordström. On the Gravitational Field Energy in Einstein's Theory.
- F. Klein. On the Integral Form of the Conservation Laws of the Theory of a Spatially Closed World.
- L. Rosenfeld. On the Gravitational Actions of Light.
- M.P. Bronstein. Quantum Theory of Weak Gravitational Fields.
- M.P. Bronstein. On the Possibility of Spontaneous Photon Splitting.
- G.E. Gor'lik, V.Ya. Frenkel. M.P. Bronstein and His Role in the Development of the Quantum Theory of Gravity.
- I.Yu. Kobzarev. Review of W. Rindler's book "Foundations of the Theory of Relativity (WHAT, GTR, and Cosmology)"

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