Weighting Techniques For Single Point Positioning

Ajay Ragh[a], Ohm Panyawat[a], MSc. Ansgar Dreier[a]

[a] *University of Bonn, Institut für Geodäsie und Geoinformation,*

Abstract

Absolute positioning is a widely employed operational method, extensively utilized in various fields such as automotive, aerospace, and maritime navigation. The functional framework for absolute positioning, which establishes a connection between pseudorange measurements and unknown variables, is clearly established. However, the stochastic model that characterizes the behavior of measurement errors is presently being researched and constitutes the focal point of this study. In this work we look through the various techniques proposed that tries to improve upon the traditional single point positioning using weighting based approaches. We investigated Carrier to Noise ratio (C/N_0) and satellite elevation angle based weighting techniques and further usage of a redundancy matrix (Angrisano et al., 2018) aimed at better handling of leverage measurements. The benefits of using these techniques were evaluated on GNSS data sequences collected as part of this study.

Keywords: GNSS, weighting schemes, elevation angle, SNR, redundancy matrix

1. Introduction

In the context of GNSS, absolute positioning also called as single point positioning (SPP) is one of the most common operational modes used widely for various applications like the commonly used positioning in our smartphones (Farzaneh and Yang, 2021). Environments with poor signal quality and unfavorable satellite positioning, often caused by natural or man-made obstructions, are categorized as challenging settings. Examples of such environments include mountainous regions and densely urbanized areas. In urban settings, issues like multipath interference and non-line-of-sight (NLOS) conditions commonly impact GNSS signals. Multipath interference occurs when a signal arrives via multiple paths, leading to signal distortion and resulting in range errors that can span several tens of meters (Hegarty and Kaplan, 2005). NLOS reception occurs when the direct signal is obstructed, and only reflected signals are received. This occurrence is highly prevalent in urban environments and can result in range errors extending to the order of kilometers (Groves and Jiang, 2013).

In this work we look through various existing weighting techniques that are aimed at coping with these sources of errors, mainly C/N_0 (Kuusniemi et al., 2004),(Kuusniemi et al., 2007) and elevation based (Hartinger and Brunner, 1999),(Collins and Langley, 1999) weighting techniques and also usage of redundancy matrix (Angrisano et al., 2018) to further improve upon these techniques.

We will begin by providing a brief overview of the SPP operational mode, followed by an explanation of weighting schemes that rely on C/N_0 (Carrier to Noise Ratio) and elevation angles. Subsequently, we will explore the influence of the redundancy matrix on the outcomes produced by these techniques. Finally, we will delve into the results derived from our experimentation with a GNSS raw data sequence and offer our analysis of these methodologies.

2. Single Point Positioning

In SPP, the idea is to solve the functional model that defines the relationship between the observations and unknowns. The solution is the estimated values for the unknowns. This gives us the receiver position coordinates. The measurement geometry can be seen in figure 1. The functional model that defines the relationship between the measurements and unknowns is:

$$\boldsymbol{z} = \boldsymbol{H}\boldsymbol{\Delta x} + \boldsymbol{\varepsilon} \tag{1}$$

Here $\boldsymbol{z}$ represents the measurement vector computed from the pseudorange, $\boldsymbol{H}$ is the design matrix, $\boldsymbol{\Delta X}$ is the state vector which contains the corrections to update the receiver coordinates and clock offset (unknowns), $\boldsymbol{\varepsilon}$ is the measurement error vector.

The method we use to estimate the unknown receiver position and clock correction is Least Squares Adjustment. To perform least squares adjustment, the equation (1) above is a linearized representation of the measurement model. The variance-covariance matrix that represents the uncertatinty of the measurement model is

$$\boldsymbol{C} = \begin{bmatrix} \sigma_1^2 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sigma_n^2 \end{bmatrix} \tag{2}$$

The variance-covariance matrix $\boldsymbol{C}$ is diagonal , this is based on the ususal assumption that the measurement errors are independent. $\boldsymbol{\sigma^2_{i=1,...n}}$ are the covariances of each individual measurements with $\boldsymbol{n}$ number of measurements.

To perform least squares adjustment we require an overdetermined system. This means that the number of measurements

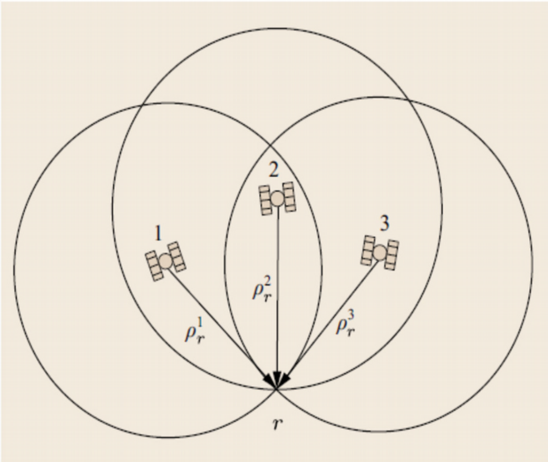

Figure 1: measurement geometry

is higher than the number of observations. In this situations we have 4 unknowns (3 position and 1 clock offset), so we need 4 or more measurements to perform least squares adjustment. Now equation (3) is solved using the weighted least squares method(WLS):

$$\hat{\Delta x} = (H^T W H)^{-1} H^T W z \tag{3}$$

where W is the weight matrix that is usually set to inverse of C. In next section we will look into the weighting techniques currently investigated as part of this study.

3. Weighting Techniques

Most weighting schemes are based on Carrier to Noise ratio (C/N_0), satellite elevation angle (EL) or a combination of both these parameters. In this study we worked with all these three types of techniques.

3.1. C/N_0 Based Weighting

In the work done in Hartinger and Brunner (1999) they proposed a variance model for carrier phase observations. This was then reused and adapted for pseudorange measurements by Groves and Jiang (2013). The model which is exclusively based on C/N_0 is:

$$\sigma_{PR}^2 = c \cdot 10^{-\frac{C/N_0}{10}} \tag{4}$$

here the value c is an empirically defined constant.

Wieser and Brunner (1999) have made modifications to the Hartinger and Brunner (1999) model, incorporating an additional additive component. These alterations are aimed at facilitating parameter estimation for the model's practical utilization in carrier phase observations. In the works Kuusniemi et al. (2004) and Kuusniemi et al. (2007) they further adapted the Wieser and Brunner (1999) model to pseudorange observations, fine-tuning the model's parameters based on real measurement errors and specific equipment used. The resulting variance model is as follows:

$$\sigma_{PR}^2 = a + b \cdot 10^{-\frac{C/N_0}{10}} \tag{5}$$

In the above model a and b are model parameters. The values for these parameters were estimated in (Kuusniemi, 2005) for lightly and heavily degraded signal environments. In our work we use the above model in equation (5) and experimented with the parameter values for lightly and heavily degraded conditions. The parameter values for heavily degraded case are $a = 0.001\frac{m^2}{s^2}$, $b = 40\frac{m^2}{s^2}Hz$. For Lightly degraded case the parameter values are $a = 0.01\frac{m^2}{s^2}$, $b = 25\frac{m^2}{s^2}Hz$.

3.2. Elevation Based Weighting

The most prevalent variance models for GPS carrier phase, which are based on satellite elevation, utilize sine functions like $\sin(El)$ or $\sin^2(El)$ (Hartinger and Brunner, 1999), (Collins and Langley, 1999). Despite their initial design for carrier phase measurements, they were subsequently adapted for pseudorange observations (Petovello, 2003), (Petovello), as illustrated below.

$$\sigma_{PR}^2 = \frac{\sigma_0^2}{\sin(El)} \tag{6}$$

$$\sigma_{PR}^2 = \frac{\sigma_0^2}{\sin^2(El)} \tag{7}$$

here σ_0^2 is the pseudorange error variance when elevation angle $El = 90^o$. In our work we follow the $\sin^2(El)$ model with equation as:

$$\sigma_{PR}^2 = \frac{1}{\sin^2(El)} \tag{8}$$

3.3. Combined C/N_0 And Elevation Based Weighting

There are few weighting schemes that combine both satellite elevation angle and C/N_0 available in literature's. In our current work we follow the model which was developed in (Realini and Reguzzoni, 2013) which combines the C/N_0 and elevation angle information. The model used is given by the following equation:

$$\sigma_{PR}^2 = \begin{cases} \frac{1}{\sin^2(El)} \cdot r & \text{if } C/N_0 < s_1 \\ 1 & \text{if } C/N_0 \geq s_1 \end{cases} \tag{9}$$

where

$$r = 10^{-\frac{C/N_0 - s_1}{B}} \left[\left(\frac{A}{10^{-\frac{s_0 - s_1}{B}}} - 1 \right) \frac{C/N_0 - s_1}{s_0 - s_1} + 1 \right] \tag{10}$$

In the above equation the term s_1 is a threshold value for the measured C/N_0. So the measurement is considered to be accurate if the measured signal power is greater than the threshold value and hence the weight is set to 1. But if this consdition is not satisfied we calculate a weight value according to the above equation. The other parameter values are determined empirically as $s_0 = 10, A = 30, B = 30$ in (Angrisano et al., 2018).

4. Redundancy Matrix And Contribution To Weighting Schemes

In least squares (LS) estimation, the residuals v is the difference between the actual measurements and the estimated measurements (Hintz, 1991).

$$v = z - \hat{z} = z - \boldsymbol{H}\hat{\Delta \boldsymbol{x}} \tag{11}$$

In (Angrisano et al., 2018) it is demonstrated that the relationship between the measurement errors ϵ and LS residuals on considering equally weighted measurements is given by,

$$\boldsymbol{v} = \boldsymbol{R}\boldsymbol{\epsilon} \tag{12}$$

The R here is the Redundancy Matrix (Angrisano et al., 2018) and can be calculated from the design matrix H in the LS estimation as,

$$\boldsymbol{R} = \boldsymbol{I} - \boldsymbol{H}(\boldsymbol{H}^T\boldsymbol{H})^{-1}\boldsymbol{H}^T \tag{13}$$

with m rows and n columns. The i^{th} diagonal element of R (r_i) with i=1,...,m is the redundancy number. The r_i is the contribution of the i^{th} measurement to the total redundancy (Schaffrin, 1997).

This concept of redundancy matrix can be used in combination with the weighting schemes to calculate a modified weight for each measurement. The diagonal elements $\boldsymbol{r_i}$ of the redundancy matrix describe the local reliability of the measurement model (Schaffrin, 1997). Measurements with low $\boldsymbol{r_i}$ are critical for the solution as they can strongly influence it. So an effective weighting scheme should be able to take the redundancy number into consideration. In (Angrisano et al., 2018) the diagonal elements of the redundancy matrix, that is, the redundancy numbers of the measurements are used to correct the original weight matrix obtained from any of the previously mentioned weighting schemes. The corrected weight $\boldsymbol{w_i}$ for the i^{th} measurement is obtained by multiplying $\boldsymbol{r_i}$ with inverse of $\boldsymbol{\sigma^2_{PRi}}$.

$$\boldsymbol{w_i} = \begin{cases} \frac{r_i}{\sigma^2_{PRi}} & \text{,in case of redundant measurements} \\ \frac{1}{\sigma^2_{PRi}} & \text{,in case of lack of redundancy} \end{cases} \tag{14}$$

The above scheme was used in this study to observe the impact of redundancy matrix.

5. Experiments

In this section we look at the experiments done as part of this study. The data sequence we are using is collected for approximately 24 hours at the Institute for Geodesy and Geoinformation (IGG) building of the University of Bonn, seen in figure 2. There is a total of 9841 epochs in the complete data sequence each having varying amount of satellite visibility as seen in figure 3. From this big data sequence an arbitrary small data sequence of 75 epochs as extracted, whose satellite visibility can be seen in figure 4. Additionally we extracted two more data sequences one with high satellite visibility ranging for 1000 epochs from epoch 1500 to epoch 2500 and another

Figure 2: Measurement Site

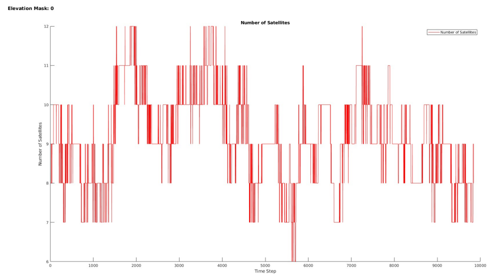

Figure 3: Satellite Visibility (full sequence)

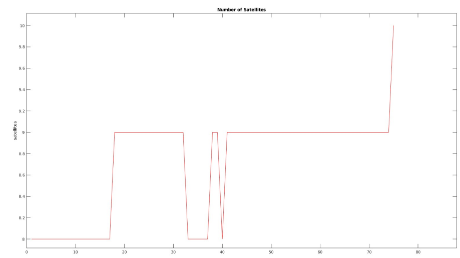

Figure 4: Satellite Visibility (short sequence)

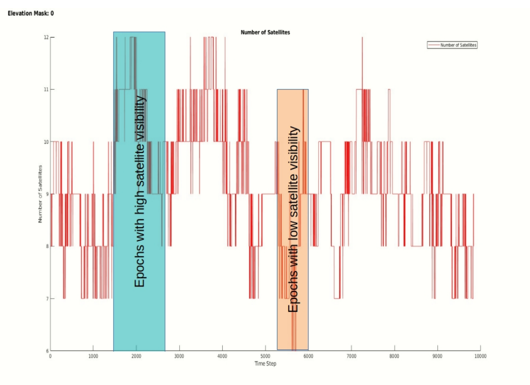

Figure 5: High and Low satellite visibility regions

Abbreviation	Technique
EQW	Equal Weights
EQW+RDM	EQW With Redundancy Matrix
ELV	Elevation Weights (see eq(8))
ELV+RDM	ELV With Redundancy Matrix
CN	C/N_0 Weights (see eq(5))
CN+RDM	CN With Redundancy Matrix
ELVCN	Combined ELV and CN weights (see eq(9))
ELVCN+RDM	ELVCN With Redundancy Matrix

Table 1: Table of Abbreviations used for the techniques

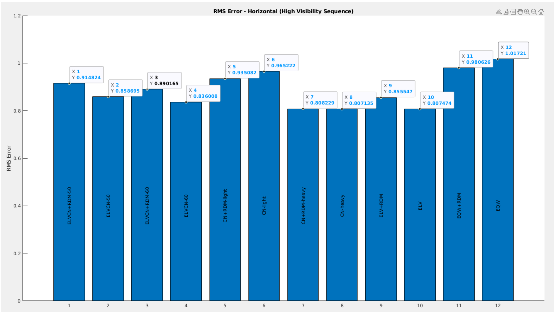

Figure 6: RMS error in horizontal component of high satellite visibility data

with low satellite visibility of 700 epochs from epoch 5300 to 6000, as seen in figure 5.

At the beginning an unweighted least squares estimation was done ton calculate the unknowns (receiver position and clock offset). Then we followed up with weighted least squares (WLS) techniques. Refer to table 2 for information regarding the abbreviations used for the various techniques.

We performed the WLS with and without using redundancy matrix for all the previously mentioned techniques. For the combined elevation and C/N_0 technique we further experimented with varying signal thresholds of 50,60 dBHz. We looked at the maximum error, mean error and rms error in both the horizontal and vertical components of the position.

6. Results

We implemented the WLS estimation for SPP in MATLAB. For each data sequence we generated plots to visualize information. The figures 6-9 show the bar graphs for the root mean squared (RMS) error of the different techniques on the horizontal.

The tables 2 and 3 shows the final results of the experiment with the weighting techniques that performed the best for the various data sequences.

7. Conclusion

In GNSS applications, especially in the widely used technique of SPP, the user can be faced with varying environmental conditions that can drastically impact the accuracy of positioning. Depending on the type of application field, the degree of

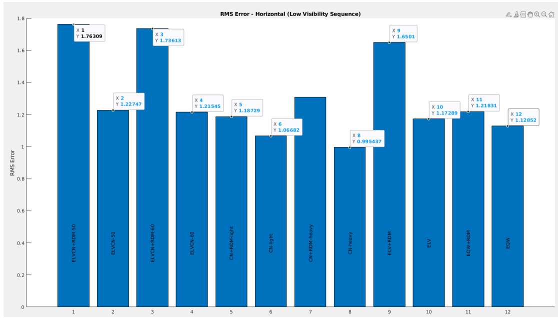

Figure 7: RMS error in horizontal component of low satellite visibility data

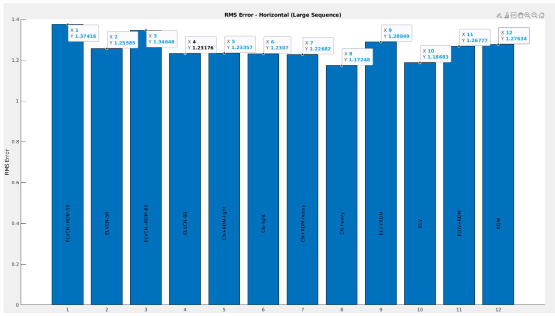

Figure 8: RMS error in horizontal component of large data sequence

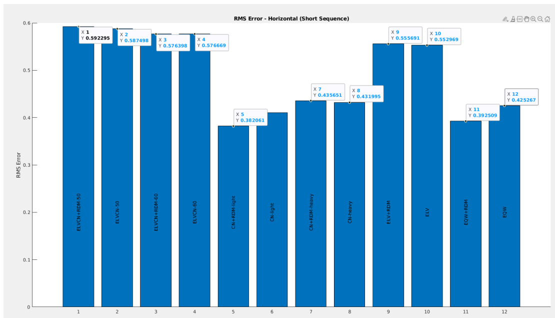

Figure 9: RMS error in horizontal component of short data sequence

Data Sequence	Max Error	Mean Error	RMS Error
Short	CN+RDM(L)	CN+RDM(L)	CN+RDM(L)
Large	CN(H)	ELV	CN(H)
Low Sat. Visibility	CN(H)	CN(H)	CN(H)
High Sat. Visibility	ELVCN60	CN(H)	CN(H)

Table 2: Best Performing Techniques in Horizontal Component

Data Sequence	Max Error	Mean Error	RMS Error
Short	ELVCN50	ELVCN50	ELVCN50
Large	CN(H)	CN(H)	CN(H)
Low Satellite Visibility	CN(L)	CN(L)	CN(L)
High Satellite Visibility	CN+RDM(H)	CN+RDM(H)	CN+RDM(H)

Table 3: Best Performing Techniques in Vertical Component

this inaccuracy may or may not be acceptable. So it is important to look at advanced techniques to improve the quality of GNSS positioning in SPP.

In this study we looked into the various existing techniques that look into the concept of weighted least squares (WLS) estimation. The techniques explored here mainly looked into satellite elevation angle and carrier to noise ratio and a combination of both as indicators of measurement quality to calculate an uncertainty factor for each measurement. We also looked into the usage of redundancy matrix to de-weight leverage measurement to reduce the possibilities of them affecting measurement quality.

At the end of the study we found that, as expected the usage of weighting techniques improved the accuracy of positioning. Another important observation is that carrier to noise ratio $\boldsymbol{C/N_0}$ based techniques performed the best in most experiments, this was followed by combination technique. This result can be observed from the results displayed in tables 2 and 3.

With this we can conclude that the weighting techniques are effective in improving the quality of single point positioning and that carrier to noise ratio is an important indicator of the quality of measurements as the weighting techniques based on C/N_0 performed the best in our experiments.

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