NE 250, F17

November 9, 2017

Monte Carlo for neutral particle transport

I'd like to spend the few lectures talking about Monte Carlo methods, variance reduction, and how we can use the adjoint with variance reduction.
Monte Carlo is used extremely widely. I think it's useful for all of you to have a sense of how it works, even if we don't go into it too deeply.

What is Monte Carlo?

- The use of *random processes* to determine a *statistically-expected* solution to a problem
- Random processes can fulfill two roles:
  - Statistical approximation to *mathematical equations*
  - Statistical approximations to *physical processes*
- Construct a random process for a problem,
- Carry out a numerical simulation by N-fold sampling from a (pseudo-)random # sequence

For MC in radiation transport, we simulate many independent particles in a system:

- Treat each physical process as a *probabilistic process*
- *Randomly sample* each process using an independent stream of pseudo-random numbers
- Follow each particle from birth until it no longer matters
- Accumulate the contributions of each particle to find the statistically-expected mean behavior and variance

WHY use Monte Carlo?
Monte Carlo, when done properly, can be highly accurate and can be considered a "gold standard" answer. Table 1 compares MC and deterministic methods. Figure 1 shows the algorithm that is basically what happens in MC.
We will go through the very basics of the ideas about probability distributions, sampling, scoring, and statistics.

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Probability Distributions

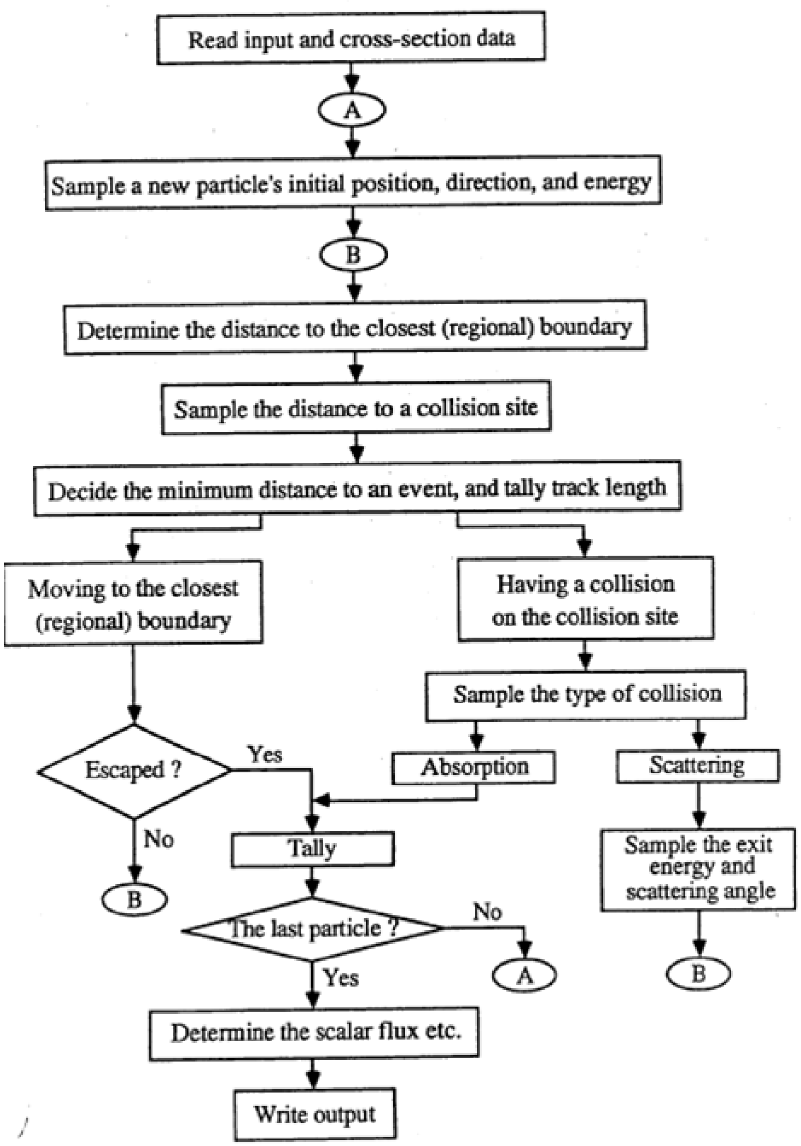

Figure 1: Monte Carlo neutral particle transport algorithm

We need to figure out all of these things about how particles are moving around in our system, but how do we do it?
We get functional expressions of the probability that various things will happen and try to take enough samples to effectively capture those expressions.

- For a random variable, x, the probability that x will have a value between a and b is $P\{a \leq x \leq b\}$.

- The *probability density fuction* expresses the likelihood that x' will take on a value between

Monte Carlo	Deterministic
* General geometry * Continuous Energy * Continuous in Angle * Number of particles governs solution accuracy * Must adequately sample phase space * Solutions have statistical error * **Local** solutions only; variable quality	* Discretized geometry * Multigroup in energy * Angular Quadrature * Discretization and solver methods govern solution accuracy * Must adequately discretize phase space * Solution contains truncation error * **Global** solutions; equal quality
* Easy to parallelize on CPUs * Slow * Might be memory intensive * Need efficient Variance Reduction	* Can be complicated to parallelize on CPUs * Can be quite fast * Might be memory intensive * Need acceleration methods

Table 1: Comparison of Monte Carlo and Deterministic Methods

x and $x + \Delta x$:

$$\lim_{\Delta x \to 0} f(x)\Delta x = P\{x \le x' \le x + \Delta x\}$$
$$\int_a^b f(x)dx = P\{a \le x \le b\}$$

- We often normalize this PDF to integrate to one, using one of

$$\int_{-\infty}^{\infty} f(x)dx = 1 \quad \text{or} \quad \int_{x^-}^{x^+} f(x)dx = 1$$

- To get the probability that our random variable x' is less than or equal to some value x, we use a *cumulative distribution function*:

$$F(x) = P\{x' \le x\}$$
$$F(x) = \int_{-\infty}^{x} f(x')dx'$$
$$\lim_{x \to \infty} F(x) \equiv F(\infty) = 1$$
$$\lim_{x \to -\infty} F(x) \equiv F(-\infty) = 0$$
$$P\{a \le x' \le b\} = F(b) - F(a)$$

Various physical phenomena can be represented by probability distributions

- Photon emission energy: Each possible energy has a different probability (intensity)
- Scattering cross-sections: Each possible scattering angle has a different probability as a function of the energy
- Transmission through a medium: Probability of reaching a particular position depends on the cross-section

We in one way or another get these PDFs and/or CDFs and use random numbers to select values for use in simulation.