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Foundations of Data Science
Lecture 8
Language Modeling

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Language Modeling

Introduction to N-grams

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Probabilistic Language Models

- Today's goal: assign a probability to a sentence
- Machine Translation:
- P(**high** winds tonite) > P(**large** winds tonite)
- Spell Correction
- The office is about fifteen **minuets** from my house
- P(about fifteen **minutes** from) > P(about fifteen **minuets** from)
- Speech Recognition
- P(I saw a van) >> P(eyes awe of an)
- + Summarization, question-answering, etc., etc.!!

1 , 3/22/2016

Probabilistic Language Models

- Goal: compute the probability of a sentence or sequence of words:
  $P(W) = P(w_1,w_2,w_3,w_4,w_5...w_n)$
- Related task: probability of an upcoming word:
  $P(w_5|w_1,w_2,w_3,w_4)$
- A model that computes either of these:
  $P(W)$ or $P(w_n|w_1,w_2...w_{n-1})$ is called a **language model**.
- Better: **the grammar** But **language model** or **LM** is standard

How to compute P(W)

- How to compute this joint probability:
- P(its, water, is, so, transparent, that)
- Intuition: let's rely on the Chain Rule of Probability

Reminder: The Chain Rule

- Recall the definition of conditional probabilities

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad P(A \cap B) = P(A|B)P(B)$$

- More variables:
  $P(A,B,C,D) = P(A)P(B|A)P(C|A,B)P(D|A,B,C)$

- The Chain Rule in General

$$P(x_1,x_2,x_3,\ldots,x_n) = P(x_1)P(x_2|x_1)P(x_3|x_1,x_2)\ldots P(x_n|x_1,\ldots,x_{n-1})$$

The Chain Rule applied to compute joint probability of words in sentence

$$P(w_1 w_2 \ldots w_n) = \prod_i P(w_i \mid w_1 w_2 \ldots w_{i-1})$$

P("its water is so transparent") =
P(its) × P(water|its) × P(is|its water)
× P(so|its water is) × P(transparent|its water is so)

How to estimate these probabilities

- Could we just count and divide?

$$P(\text{the} \mid \text{its water is so transparent that}) = \frac{Count(\text{its water is so transparent that the})}{Count(\text{its water is so transparent that})}$$

- No! Too many possible sentences!
- We'll never see enough data for estimating these

Markov Assumption

Andrei Markov

- Simplifying assumption:

$$P(\text{the} \mid \text{its water is so transparent that}) \approx P(\text{the} \mid \text{that})$$

- Or maybe

$$P(\text{the} \mid \text{its water is so transparent that}) \approx P(\text{the} \mid \text{transparent that})$$

Markov Assumption

$$P(w_1 w_2 \ldots w_n) \approx \prod_i P(w_i \mid w_{i-k} \ldots w_{i-1})$$

- In other words, we approximate each component in the product

$$P(w_i \mid w_1 w_2 \ldots w_{i-1}) \approx P(w_i \mid w_{i-k} \ldots w_{i-1})$$

Simplest case: Unigram model

$$P(w_1 w_2 \square \ w_n) \approx \prod_i P(w_i)$$

Some automatically generated sentences from a unigram model

fifth, an, of, futures, the, an, incorporated, a, a, the, inflation, most, dollars, quarter, in, is, mass

thrift, did, eighty, said, hard, 'm, july, bullish

that, or, limited, the

Bigram model

Condition on the previous word:

$$P(w_i \mid w_1 w_2 \square \ w_{i-1}) \approx P(w_i \mid w_{i-1})$$

texaco, rose, one, in, this, issue, is, pursuing, growth, in, a, boiler, house, said, mr., gurria, mexico, 's, motion, control, proposal, without, permission, from, five, hundred, fifty, five, yen

outside, new, car, parking, lot, of, the, agreement, reached

this, would, be, a, record, november

N-gram models

- We can extend to trigrams, 4-grams, 5-grams
- In general this is an insufficient model of language
- because language has **long-distance dependencies**:

"The computer which I had just put into the machine room on the fifth floor crashed."

- But we can often get away with N-gram models

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Language Modeling

Estimating N-gram Probabilities

Estimating bigram probabilities

- The Maximum Likelihood Estimate

$$P(w_i \mid w_{i-1}) = \frac{count(w_{i-1}, w_i)}{count(w_{i-1})}$$

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

An example

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

<s> I am Sam </s>
<s> Sam I am </s>
<s> I do not like green eggs and ham </s>

$$P(\texttt{I} \mid \texttt{<s>}) = \tfrac{2}{3} = .67 \qquad P(\texttt{Sam} \mid \texttt{<s>}) = \tfrac{1}{3} = .33 \qquad P(\texttt{am} \mid \texttt{I}) = \tfrac{2}{3} = .67$$
$$P(\texttt{</s>} \mid \texttt{Sam}) = \tfrac{1}{2} = 0.5 \qquad P(\texttt{Sam} \mid \texttt{am}) = \tfrac{1}{2} = .5 \qquad P(\texttt{do} \mid \texttt{I}) = \tfrac{1}{3} = .33$$

More examples:
Berkeley Restaurant Project sentences

- can you tell me about any good cantonese restaurants close by
- mid priced thai food is what i'm looking for
- tell me about chez panisse
- can you give me a listing of the kinds of food that are available
- i'm looking for a good place to eat breakfast
- when is caffe venezia open during the day

Raw bigram counts

- Out of 9222 sentences

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Raw bigram probabilities

$$P(w_i \mid w_{i-1}) = \frac{count(w_{i-1}, w_i)}{count(w_{i-1})}$$

- Normalize by unigrams:

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

- Result:

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Bigram estimates of sentence probabilities

P(<s> I want english food </s>) =

P(I|<s>)
× P(want|I)
× P(english|want)
× P(food|english)
× P(</s>|food)

= .000031

What kinds of knowledge?

- P(english|want) = .0011
- P(chinese|want) = .0065
- P(to|want) = .66
- P(eat | to) = .28
- P(food | to) = 0
- P(want | spend) = 0
- P (i | <s>) = .25

Practical Issues

- We do everything in log space
- Avoid underflow
- (also adding is faster than multiplying)

$$\log(p_1 \times p_2 \times p_3 \times p_4) = \log p_1 + \log p_2 + \log p_3 + \log p_4$$

Language Modeling Toolkits

- IRSTLM

  https://hlt-mt.fbk.eu/technologies/irstlm

Google N-Gram Viewer

https://books.google.com/ngrams

The Google Ngram Viewer or Google Books Ngram Viewer is an online search engine that charts frequencies of any set of comma-delimited search strings using a yearly count of n-grams found in sources printed between 1500 and 2008 in Google's text corpora in American English, British English, French, German, Spanish, Russian, Hebrew, or Chinese.

The program can search for a single word or a phrase, including misspellings or gibberish.

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Language Modeling

Evaluation and Perplexity

Evaluation: How good is our model?

- Does our language model prefer good sentences to bad ones?
- Assign higher probability to “real” or “frequently observed” sentences
- Than “ungrammatical” or “rarely observed” sentences?
- We train parameters of our model on a **training set**.
- We test the model’s performance on data we haven’t seen.
- A **test set** is an unseen dataset that is different from our training set, totally unused.
- An **evaluation metric** tells us how well our model does on the test set.

Extrinsic evaluation of N-gram models

- Best evaluation for comparing models A and B
- Put each model in a task
- spelling corrector, speech recognizer, MT system
- Run the task, get an accuracy for A and for B
- How many misspelled words corrected properly
- How many words translated correctly
- Compare accuracy for A and B

Difficulty of extrinsic (in-vivo) evaluation of N-gram models

- Extrinsic evaluation
- Time-consuming; can take days or weeks
- So
- Sometimes use **intrinsic** evaluation: **perplexity**
- Bad approximation
- unless the test data looks **just** like the training data
- So **generally only useful in pilot experiments**
- But is helpful to think about.

Intuition of Perplexity

- The Shannon Game:
- How well can we predict the next word?
- Unigrams are terrible at this game. (Why?)

I always order pizza with cheese and _____

The 33rd President of the US was _____

I saw a _____

mushrooms 0.1
pepperoni 0.1
anchovies 0.01
....
fried rice 0.0001
....
and 1e-100

- A better model of a text
- is one which assigns a higher probability to the word that actually occurs

Perplexity

The best language model is one that best predicts an unseen test set

Gives the highest P(sentence)

Perplexity is the inverse probability of the test set, normalized by the number of words:

$$PP(W) = P(w_1 w_2 ... w_N)^{-\frac{1}{N}}$$

$$= \sqrt[N]{\frac{1}{P(w_1 w_2 ... w_N)}}$$

Minimizing perplexity is the same as maximizing probability

Perplexity

$$PP(W) = P(w_1 w_2 ... w_N)^{-\frac{1}{N}}$$

$$= \sqrt[N]{\frac{1}{P(w_1 w_2 ... w_N)}}$$

Chain rule:

$$\mathrm{PP}(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i | w_1 \ldots w_{i-1})}}$$

For bigrams:

$$\mathrm{PP}(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i | w_{i-1})}}$$

The Shannon Game intuition for perplexity

- From Josh Goodman
- How hard is the task of recognizing digits ‘0,1,2,3,4,5,6,7,8,9’
- Perplexity 10
- How hard is recognizing (30,000) names at Microsoft.
- Perplexity = 30,000
- Perplexity is weighted equivalent branching factor

Perplexity as branching factor

- Let's suppose a sentence consisting of random digits
- What is the perplexity of this sentence according to a model that assign P=1/10 to each digit?

$$\begin{aligned} \mathrm{PP}(W) &= P(w_1 w_2 \ldots w_N)^{-\frac{1}{N}} \\ &= \left(\frac{1}{10}^{N}\right)^{-\frac{1}{N}} \\ &= \frac{1}{10}^{-1} \\ &= 10 \end{aligned}$$

Lower perplexity = better model

- Training 38 million words, test 1.5 million words, WSJ

N-gram Order	Unigram	Bigram	Trigram
Perplexity	962	170	109

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Language Modeling

Generalization and zeros

The Shannon Visualization Method

- Choose a random bigram (<s>, w) according to its probability
- Now choose a random bigram (w, x) according to its probability
- And so on until we choose </s>
- Then string the words together

<s> I
I want
want to
to eat
eat Chinese
Chinese food
food </s>
I want to eat Chinese food

Approximating Shakespeare

Unigram

To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have
Every enter now severally so, let
Hill he late speaks; or! a more to leg less first you enter
Are where exeunt and sighs have rise excellency took of.. Sleep knave we. near; vile like

Bigram

What means, sir. I confess she? then all sorts, he is trim, captain.
Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow.
What we, hath got so she that I rest and sent to scold and nature bankrupt, nor the first gentleman?

Trigram

Sweet prince, Falstaff shall die. Harry of Monmouth's grave.
This shall forbid it should be branded, if renown made it empty.
Indeed the duke; and had a very good friend.
Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, 'tis done.

Quadrigram

King Henry.What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv'd in;
Will you not tell me who I am?
It cannot be but so.
Indeed the short and the long. Marry, 'tis a noble Lepidus.

Shakespeare as corpus

- N=884,647 tokens, V=29,066
- Shakespeare produced 300,000 bigram types out of V^2= 844 million possible bigrams.
- So 99.96% of the possible bigrams were never seen (have zero entries in the table)
- Quadrigrams worse: What's coming out looks like Shakespeare because it ***is*** Shakespeare

The wall street journal is not shakespeare (no offense)

Unigram

Months the my and issue of year foreign new exchange's september were recession exchange new endorsed a acquire to six executives

Bigram

Last December through the way to preserve the Hudson corporation N. B. E. C. Taylor would seem to complete the major central planners one point five percent of U. S. E. has already old M. X. corporation of living on information such as more frequently fishing to keep her

Trigram

They also point to ninety nine point six billion dollars from two hundred four oh six three percent of the rates of interest stores as Mexico and Brazil on market conditions

The perils of overfitting

- N-grams only work well for word prediction if the test corpus looks like the training corpus
- In real life, it often doesn't
- We need to train robust models that generalize!
- One kind of generalization: Zeros!
- Things that don't ever occur in the training set
- But occur in the test set

Zeros

- Training set:

... denied the allegations
... denied the reports
... denied the claims
... denied the request

Test set
... denied the offer
... denied the loan

P("offer" | denied the) = 0

Zero probability bigrams

- Bigrams with zero probability
- mean that we will assign 0 probability to the test set!
- And hence we cannot compute perplexity (can't divide by 0)!

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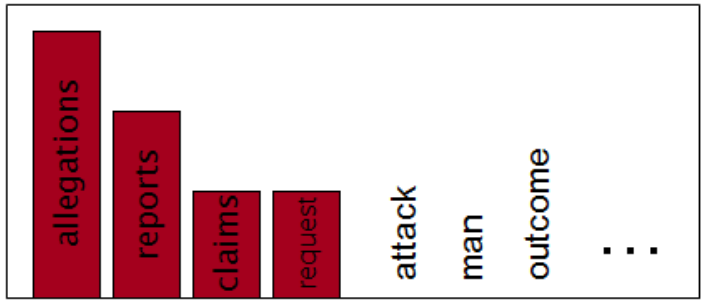
Language Modeling

Smoothing: Add-one (Laplace) smoothing

The intuition of smoothing (from Dan Klein)

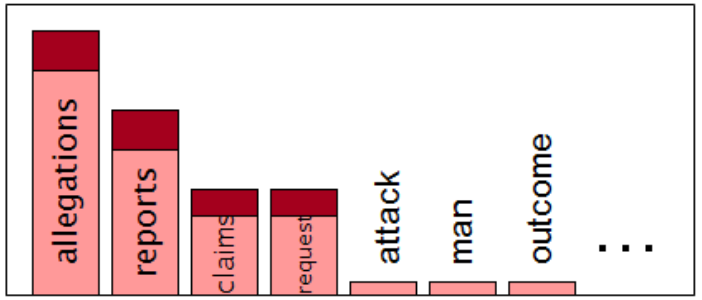
- When we have sparse statistics:

  P(w | denied the) 7 total
  3 allegations
  2 reports
  1 claims
  1 request



- Steal probability mass to generalize better

  P(w | denied the) 7total
  2.5 allegations
  1.5 reports
  0.5 claims
  0.5 request
  2 other



Add-one estimation

- Also called Laplace smoothing
- Pretend we saw each word one more time than we did
- Just add one to all the counts!

- MLE estimate:

$$P_{MLE}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

- Add-1 estimate:

$$P_{Add-1}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)+1}{c(w_{i-1})+V}$$

Maximum Likelihood Estimates

- The maximum likelihood estimate
- of some parameter of a model M from a training set T
- maximizes the likelihood of the training set T given the model M
- Suppose the word “bagel” occurs 400 times in a corpus of a million words
- What is the probability that a random word from some other text will be “bagel”?
- MLE estimate is 400/1,000,000 = .0004
- This may be a bad estimate for some other corpus
- But it is the **estimate** that makes it **most likely** that “bagel” will occur 400 times in a million word corpus.

Berkeley Restaurant Corpus: Laplace smoothed bigram counts

	i	want	to	eat	chinese	food	lunch	spend
i	6	828	1	10	1	1	1	3
want	3	1	609	2	7	7	6	2
to	3	1	5	687	3	1	7	212
eat	1	1	3	1	17	3	43	1
chinese	2	1	1	1	1	83	2	1
food	16	1	16	1	2	5	1	1
lunch	3	1	1	1	1	2	1	1
spend	2	1	2	1	1	1	1	1

Reconstituted counts

$$P^*(w_n|w_{n-1}) = \frac{C(w_{n-1}w_n)+1}{C(w_{n-1})+V}$$

	i	want	to	eat	chinese	food	lunch	spend
i	0.0015	0.21	0.00025	0.0025	0.00025	0.00025	0.00025	0.00075
want	0.0013	0.00042	0.26	0.00084	0.0029	0.0029	0.0025	0.00084
to	0.00078	0.00026	0.0013	0.18	0.00078	0.00026	0.0018	0.055
eat	0.00046	0.00046	0.0014	0.00046	0.0078	0.0014	0.02	0.00046
chinese	0.0012	0.00062	0.00062	0.00062	0.00062	0.052	0.0012	0.00062
food	0.0063	0.00039	0.0063	0.00039	0.00079	0.002	0.00039	0.00039
lunch	0.0017	0.00056	0.00056	0.00056	0.00056	0.0011	0.00056	0.00056
spend	0.0012	0.00058	0.0012	0.00058	0.00058	0.00058	0.00058	0.00058

Laplace-smoothed bigrams

$$c^*(w_{n-1}w_n) = \frac{[C(w_{n-1}w_n)+1] \times C(w_{n-1})}{C(w_{n-1})+V}$$

	i	want	to	eat	chinese	food	lunch	spend
i	3.8	527	0.64	6.4	0.64	0.64	0.64	1.9
want	1.2	0.39	238	0.78	2.7	2.7	2.3	0.78
to	1.9	0.63	3.1	430	1.9	0.63	4.4	133
eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

Compare with raw bigram counts

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

	i	want	to	eat	chinese	food	lunch	spend
i	3.8	527	0.64	6.4	0.64	0.64	0.64	1.9
want	1.2	0.39	238	0.78	2.7	2.7	2.3	0.78
to	1.9	0.63	3.1	430	1.9	0.63	4.4	133
eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

Add-1 estimation is a blunt instrument

- So add-1 isn't used for N-grams:
- We'll see better methods

- But add-1 is used to smooth other NLP models
- For text classification
- In domains where the number of zeros isn't so huge.

Language Modeling

Interpolation, Backoff, and Web-Scale LMs

Backoff and Interpolation

- Sometimes it helps to use **less** context
- Condition on less context for contexts you haven't learned much about
- **Backoff:**
- use trigram if you have good evidence,
- otherwise bigram, otherwise unigram
- **Interpolation:**
- mix unigram, bigram, trigram

- Interpolation works better

Linear Interpolation

- Simple interpolation

$$\hat{P}(w_n|w_{n-1}w_{n-2}) = \lambda_1 P(w_n|w_{n-1}w_{n-2}) + \lambda_2 P(w_n|w_{n-1}) + \lambda_3 P(w_n)$$

$$\sum_i \lambda_i = 1$$

- Lambdas conditional on context:

$$\hat{P}(w_n|w_{n-2}w_{n-1}) = \lambda_1(w_{n-2}^{n-1})P(w_n|w_{n-2}w_{n-1}) + \lambda_2(w_{n-2}^{n-1})P(w_n|w_{n-1}) + \lambda_3(w_{n-2}^{n-1})P(w_n)$$

Linear Interpolation

- Simple interpolation

$$\hat{P}(w_n|w_{n-1}w_{n-2}) = \lambda_1 P(w_n|w_{n-1}w_{n-2}) + \lambda_2 P(w_n|w_{n-1}) + \lambda_3 P(w_n)$$

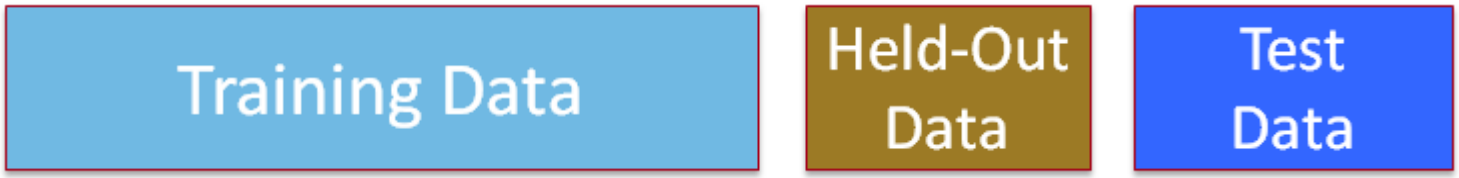
$$\sum_i \lambda_i = 1$$

- Lambdas conditional on context:

$$\hat{P}(w_n|w_{n-2}w_{n-1}) = \lambda_1(w_{n-2}^{n-1})P(w_n|w_{n-2}w_{n-1}) + \lambda_2(w_{n-2}^{n-1})P(w_n|w_{n-1}) + \lambda_3(w_{n-2}^{n-1})P(w_n)$$

How to set the lambdas?

- Use a **held-out** corpus



- Choose λs to maximize the probability of held-out data:
- Fix the N-gram probabilities (on the training data)
- Then search for λs that give largest probability to held-out set:

$$\log P(w_1 \ldots w_n \mid M(\lambda_1 \ldots \lambda_k)) = \sum_i \log P_{M(\lambda_1 \ldots \lambda_k)}(w_i \mid w_{i-1})$$

Unknown words: Open versus closed vocabulary tasks

- If we know all the words in advanced
- Vocabulary V is fixed
- Closed vocabulary task
- Often we don't know this
- **Out Of Vocabulary** = OOV words
- Open vocabulary task
- Instead: create an unknown word token <UNK>
- Training of <UNK> probabilities
- Create a fixed lexicon L of size V
- At text normalization phase, any training word not in L changed to <UNK>
- Now we train its probabilities like a normal word
- At decoding time
- If text input: Use UNK probabilities for any word not in training

Huge web-scale n-grams

- How to deal with, e.g., Google N-gram corpus
- Pruning
- Only store N-grams with count > threshold.
- Remove singletons of higher-order n-grams
- Entropy-based pruning
- Efficiency
- Efficient data structures like tries
- Bloom filters: approximate language models
- Store words as indexes, not strings
- Use Huffman coding to fit large numbers of words into two bytes
- Quantize probabilities (4-8 bits instead of 8-byte float)

Smoothing for Web-scale N-grams

- "Stupid backoff" (Brants *et al*. 2007)
- No discounting, just use relative frequencies

$$S(w_i \mid w_{i-k+1}^{i-1}) = \begin{cases} \dfrac{\text{count}(w_{i-k+1}^{i})}{\text{count}(w_{i-k+1}^{i-1})} & \text{if } \text{count}(w_{i-k+1}^{i}) > 0 \\ 0.4\,S(w_i \mid w_{i-k+2}^{i-1}) & \text{otherwise} \end{cases}$$

$$S(w_i) = \frac{\text{count}(w_i)}{N}$$

N-gram Smoothing Summary

- Add-1 smoothing:
- OK for text categorization, not for language modeling

- The most commonly used method:
- Extended Interpolated Kneser-Ney

- For very large N-grams like the Web:
- Stupid backoff

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Language Modeling

Advanced: Good Turing Smoothing

Reminder: Add-1 (Laplace) Smoothing

$$P_{Add-1}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + 1}{c(w_{i-1}) + V}$$

More general formulations: Add-k

$$P_{Add-k}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + k}{c(w_{i-1}) + kV}$$

$$P_{Add-k}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + m(\frac{1}{V})}{c(w_{i-1}) + m}$$

Unigram prior smoothing

$$P_{Add-k}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + m(\frac{1}{V})}{c(w_{i-1}) + m}$$

$$P_{\text{UnigramPrior}}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + mP(w_i)}{c(w_{i-1}) + m}$$

Advanced Language Modeling

- Intuition used by many smoothing algorithms
- Good-Turing
- Kneser-Ney
- Witten-Bell
- Use the count of things we've **seen once**
- to help estimate the count of things we've **never seen**

Notation: N_c = Frequency of frequency c

- N_c = the count of things we've seen c times
- Sam I am I am Sam I do not eat

I 3
sam 2
am 2
do 1
not 1
eat 1

$N_1 = 3$
$N_2 = 2$
$N_3 = 1$

Good-Turing smoothing intuition

- You are fishing (a scenario from Josh Goodman), and caught:
- 10 carp, 3 perch, 2 whitefish, 1 trout, 1 salmon, 1 eel = 18 fish
- How likely is it that next species is trout?
- 1/18
- How likely is it that next species is new (i.e. catfish or bass)
- Let's use our estimate of things-we-saw-once to estimate the new things.
- 3/18 (because N_1=3)
- Assuming so, how likely is it that next species is trout?
- Must be less than 1/18
- How to estimate?

Good Turing calculations

$$P^*_{GT}(\text{things with zero frequency}) = \frac{N_1}{N} \qquad c^* = \frac{(c+1)N_{c+1}}{N_c}$$

Unseen (bass or catfish)

- c = 0:
- MLE p = 0/18 = 0

P^*_{GT} (unseen) = N_1/N = 3/18

Seen once (trout)
c = 1
MLE p = 1/18

C*(trout) = 2 * N2/N1
= 2 * 1/3
= 2/3
P^*_{GT}(trout) = 2/3 / 18 = 1/27

Good-Turing smoothing intuition

- You are fishing (a scenario from Josh Goodman), and caught:
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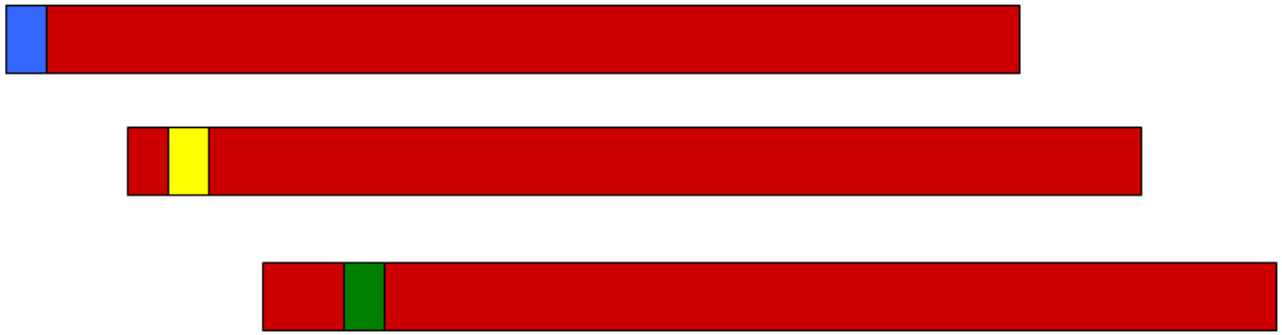
Ney et al.'s Good Turing Intuition

H. Ney, U. Essen, and R. Kneser, 1995. On the estimation of 'small' probabilities by leaving-one-out. IEEE Trans. PAMI. 17:12,1202-1212



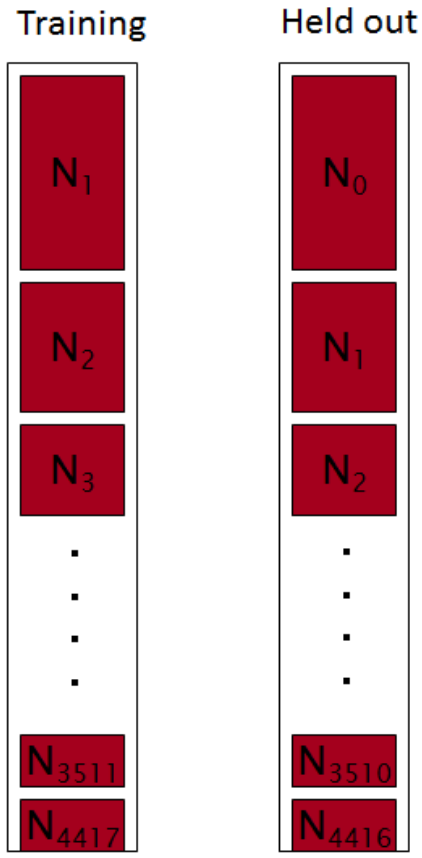
Held-out words

Ney *et al.* Good Turing Intuition

- Intuition from leave-one-out validation
- Take each of the c training words out in turn
- c training sets of size c–1, held-out of size 1
- What fraction of held-out words are unseen in training?
- N_1/c
- What fraction of held-out words are seen k times in training?
- $(k+1)N_{k+1}/c$
- So in the future we expect $(k+1)N_{k+1}/c$ of the words to be those with training count k
- There are N_k words with training count k
- Each should occur with probability:
- $(k+1)N_{k+1}/c/N_k$
- ...or expected count:

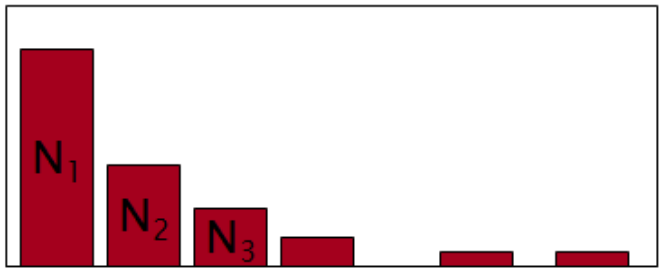
$$k^* = \frac{(k+1)N_{k+1}}{N_k}$$

Ney *et al.* Good Turing Intuition

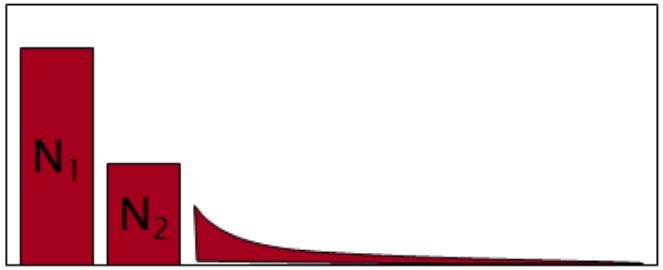


Good-Turing complications

- Problem: what about “the”? (say c=4417)
- For small k, $N_k > N_{k+1}$
- For large k, too jumpy, zeros wreck estimates



- Simple Good-Turing [Gale and Sampson]: replace empirical N_k with a best-fit power law once counts get unreliable



Resulting Good-Turing numbers

- Numbers from Church and Gale (1991)
- 22 million words of AP Newswire

$$c^* = \frac{(c+1)N_{c+1}}{N_c}$$

Count c	Good Turing c*
0	.0000270
1	0.446
2	1.26
3	2.24
4	3.24
5	4.22
6	5.19
7	6.21
8	7.24
9	8.25

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Language Modeling

Advanced: Kneser-Ney Smoothing

Resulting Good-Turing numbers

- Numbers from Church and Gale (1991)
- 22 million words of AP Newswire

$$c^* = \frac{(c+1)N_{c+1}}{N_c}$$

- It sure looks like c* = (c - .75)

Count c	Good Turing c*
0	.0000270
1	0.446
2	1.26
3	2.24
4	3.24
5	4.22
6	5.19
7	6.21
8	7.24
9	8.25

Absolute Discounting Interpolation

- Save ourselves some time and just subtract 0.75 (or some d)!

$$P_{\text{AbsoluteDiscounting}}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) - d}{c(w_{i-1})} + \lambda(w_{i-1})P(w)$$

discounted bigram

Interpolation weight

unigram

- (Maybe keeping a couple extra values of d for counts 1 and 2)
- But should we really just use the regular unigram P(w)?

Kneser-Ney Smoothing I

- Better estimate for probabilities of lower-order unigrams!
- Shannon game: *I can't see without my readi* _____ ?
- "Francisco" is more common than "glasses"
- ... but "Francisco" always follows "San"
- The unigram is useful exactly when we haven't seen this bigram!
- Instead of P(w): "How likely is w"
- $P_{continuation}(w)$: "How likely is w to appear as a novel continuation?
- For each word, count the number of bigram types it completes
- Every bigram type was a novel continuation the first time it was seen

Kneser-Ney Smoothing II

- How many times does w appear as a novel continuation:

$$P_{CONTINUATION}(w) \propto \left|\{w_{i-1} : c(w_{i-1}, w) > 0\}\right|$$

- Normalized by the total number of word bigram types

$$\left|\{(w_{j-1}, w_j) : c(w_{j-1}, w_j) > 0\}\right|$$

$$P_{CONTINUATION}(w) = \frac{\left|\{w_{i-1} : c(w_{i-1}, w) > 0\}\right|}{\left|\{(w_{j-1}, w_j) : c(w_{j-1}, w_j) > 0\}\right|}$$

Kneser-Ney Smoothing III

- Alternative metaphor: The number of # of word types seen to precede w

$$|\{w_{i-1} : c(w_{i-1}, w) > 0\}|$$

- normalized by the # of words preceding all words:

$$P_{CONTINUATION}(w) = \frac{|\{w_{i-1} : c(w_{i-1}, w) > 0\}|}{\sum_{w'} |\{w'_{i-1} : c(w'_{i-1}, w') > 0\}|}$$

- A frequen San) will have a low continuation probability

Kneser-Ney Smoothing IV

$$P_{KN}(w_i \mid w_{i-1}) = \frac{\max(c(w_{i-1}, w_i) - d, 0)}{c(w_{i-1})} + \lambda(w_{i-1}) P_{CONTINUATION}(w_i)$$

λ is a normalizing constant; the probability mass we've discounted

$$\lambda(w_{i-1}) = \frac{d}{c(w_{i-1})} \left| \{w : c(w_{i-1}, w) > 0\} \right|$$

the normalized discount

The number of word types that can follow w_{i-1}
= # of word types we discounted
= # of times we applied normalized discount

Kneser-Ney Smoothing: Recursive formulation

$$P_{KN}(w_i \mid w_{i-n+1}^{i-1}) = \frac{\max(c_{KN}(w_{i-n+1}^{i}) - d, 0)}{c_{KN}(w_{i-n+1}^{i-1})} + \lambda(w_{i-n+1}^{i-1}) P_{KN}(w_i \mid w_{i-n+2}^{i-1})$$

$$c_{KN}(\bullet) = \begin{cases} count(\bullet) & \text{for the highest order} \\ continuationcount(\bullet) & \text{for lower order} \end{cases}$$

Continuation count = Number of unique single word contexts for •