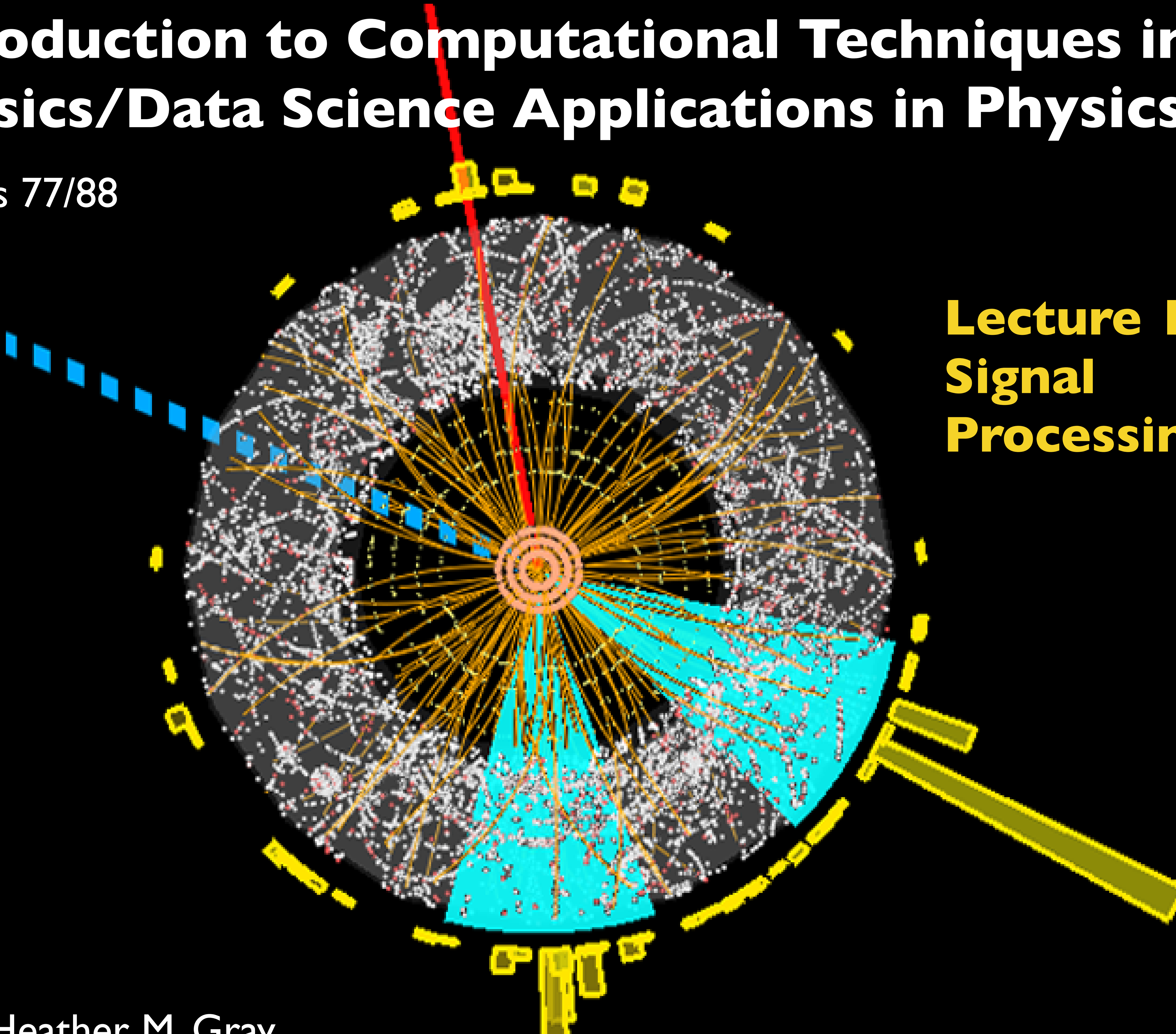
# Introduction to Computational Techniques in Physics/Data Science Applications in Physics

Physics 77/88



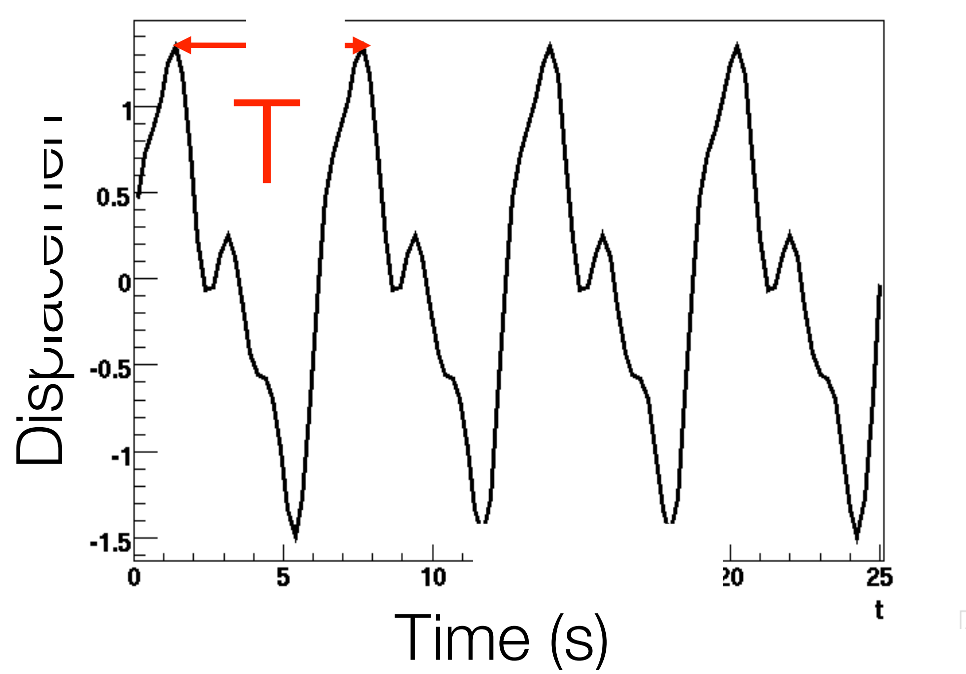
## Lecture 11: Signal Processing

Prof. Heather M. Gray

Many thanks to Yury Kolomensky for material used in this course

# Definitions

- Suppose is some function that we at some frequency
  - Measure at times $t_n =$
- Goal is to analyze and properties of
  -
  -
  -



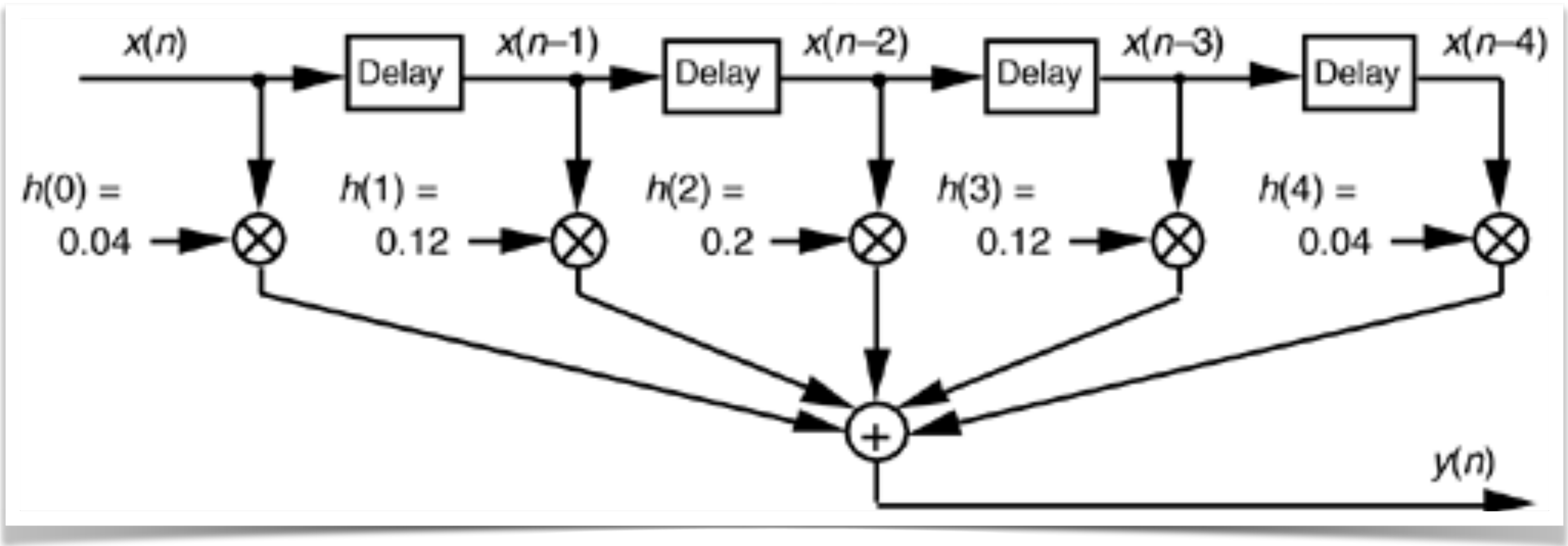
# Acronyms

- FFT =
  - DFT =
- DSP =
- SNR =
  - Often expressed in ,
    - i.e. SNR = where S and N are and amplitudes
    - For power, SNR =
- ASD =
  - PSD =
  - NPS =

# Acronyms

- FIR =
- IIR =

# Schematic Notation




- Represent , , operations
- Most can be this way
    - i.e.

# Fourier Transform

- Most common way to a is by means of a
    - Represent as a of
        - $x(t) = \int$
            - Here $x(\omega)$ is a : represents the of the at a
                - Also known as the

# Fourier Transform

- Inverse transform
  - $x(\omega) = C \int$
  - $x(f) = C \int$
  - =

# Discrete Fourier Transform

- For a , replace the with a
  - $x(m) = \sum$
- Define
  - $x_{mag}(m) =$
  - $\Delta\phi(m) =$
  - $P(m) =$

# Aliasing

- For real signals , can show that
  - Exercise for the reader
    - $x(m) =$
  - Moreover (obvious)
    - $x(m) =$
- This is called
- Spectrum for is with
  - theorem:
    - can be reconstructed from
      - iff is limited to the and
    - is often called the frequency

# Fast Fourier Transform

- Brute-force transforms can be
  - Require calculations
  - However, are and obey , which is used in the
    - Scale as
    - Multiple algorithms exist, and are in
    - E.g. from

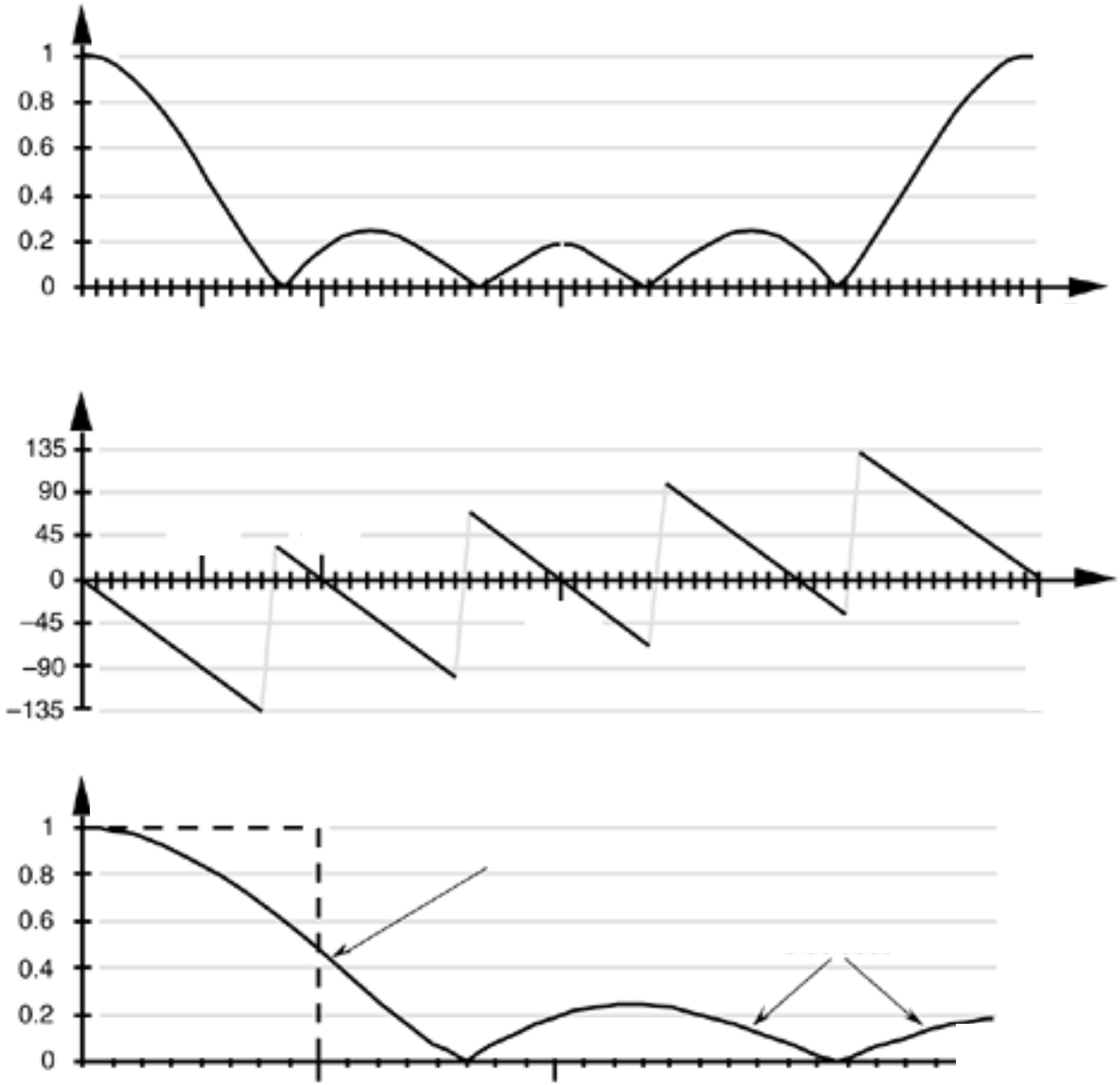
# Applications of FFT

- FFT is useful to
  - Look at the                                 of a
    - e.g.
  - Look at the                             of
    - 
    - 
    - 
  - Signal processing
    - Measure                          of the                 in
    - Design                            to

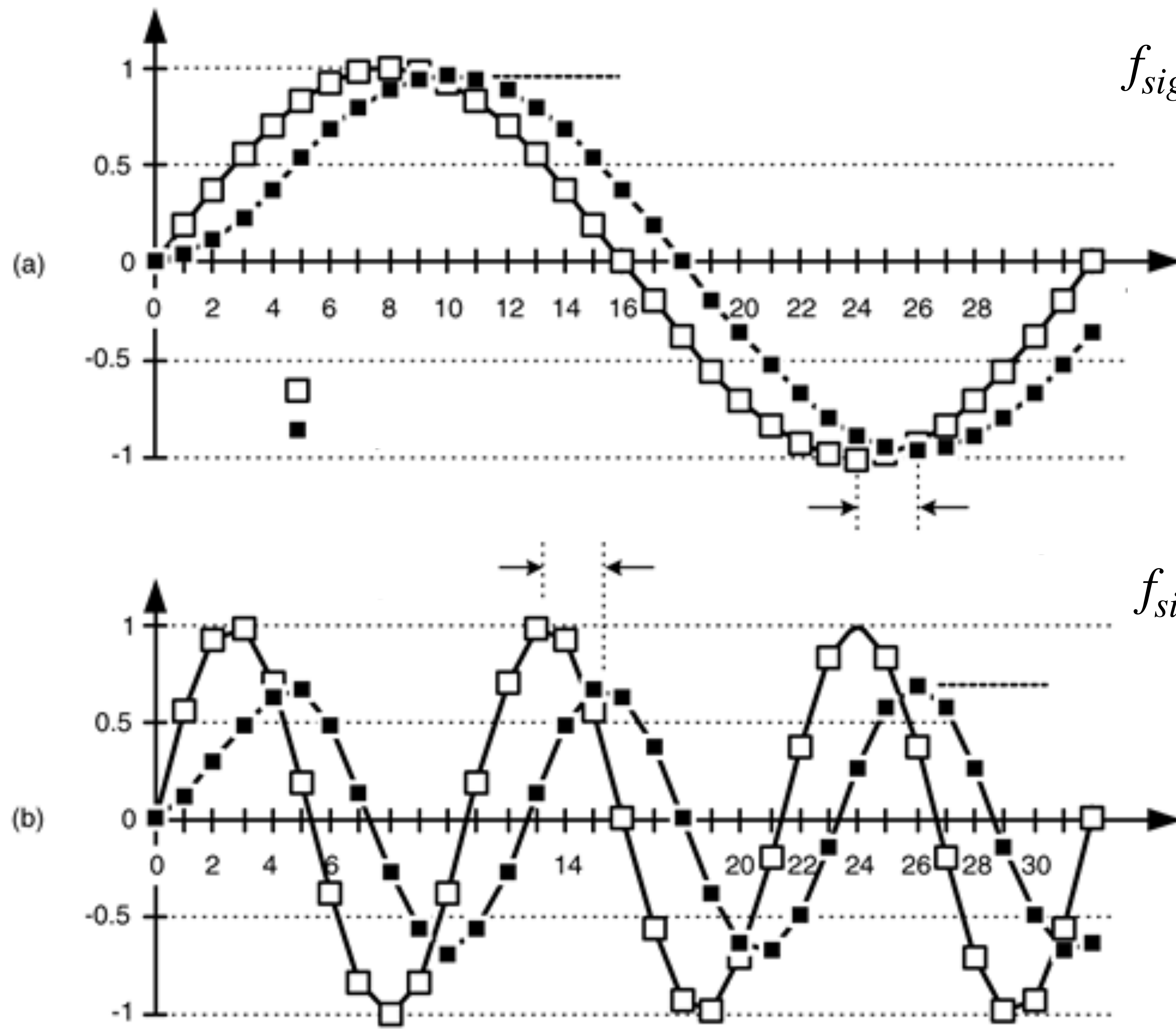
# FIR filters

- is a way to the contributions of (random) to the of the
- Usually by with some (predetermined)
  - Equivalent to

# Box-car Filter Response



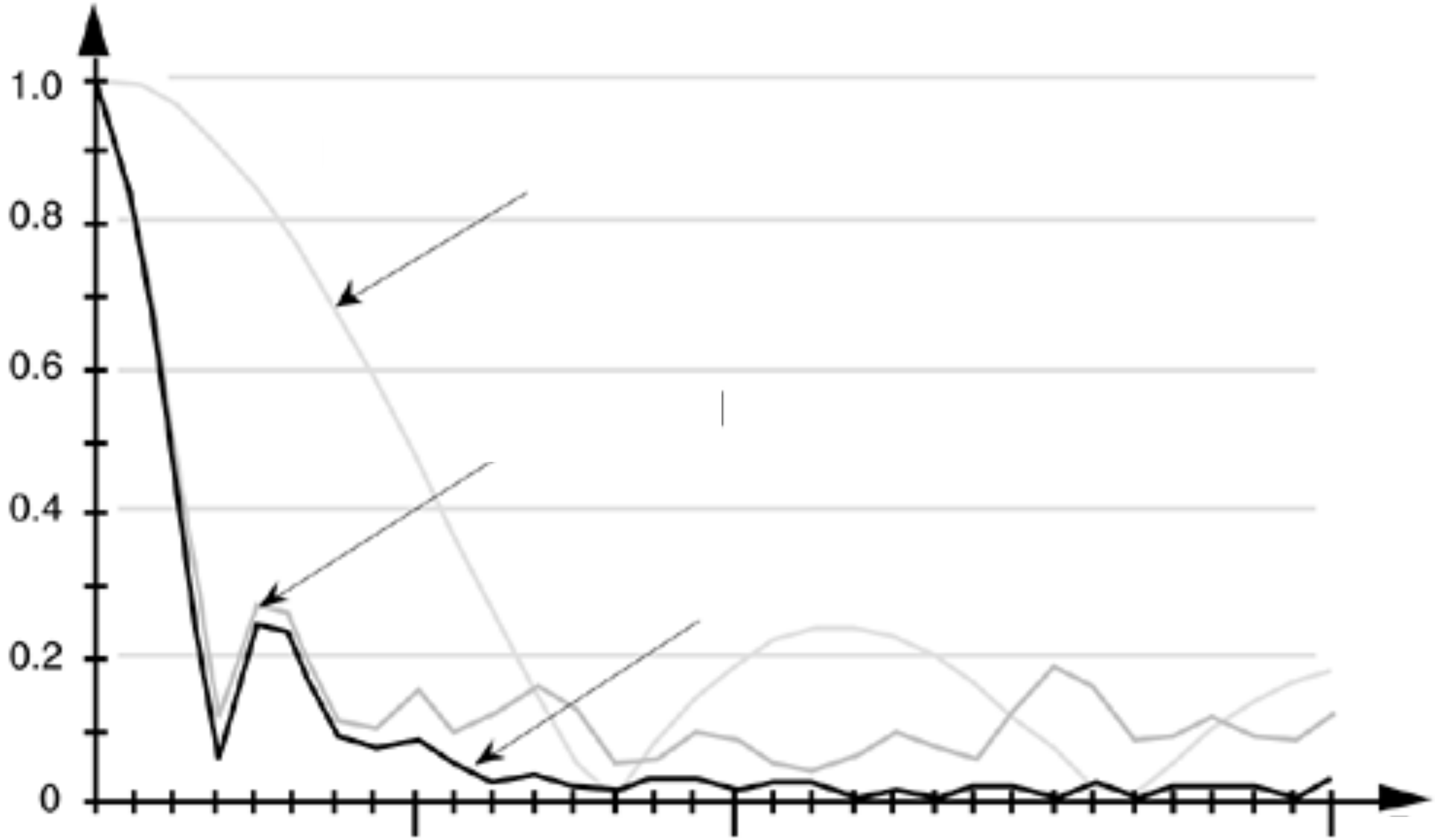
# Time-Domain Signal After Filtering



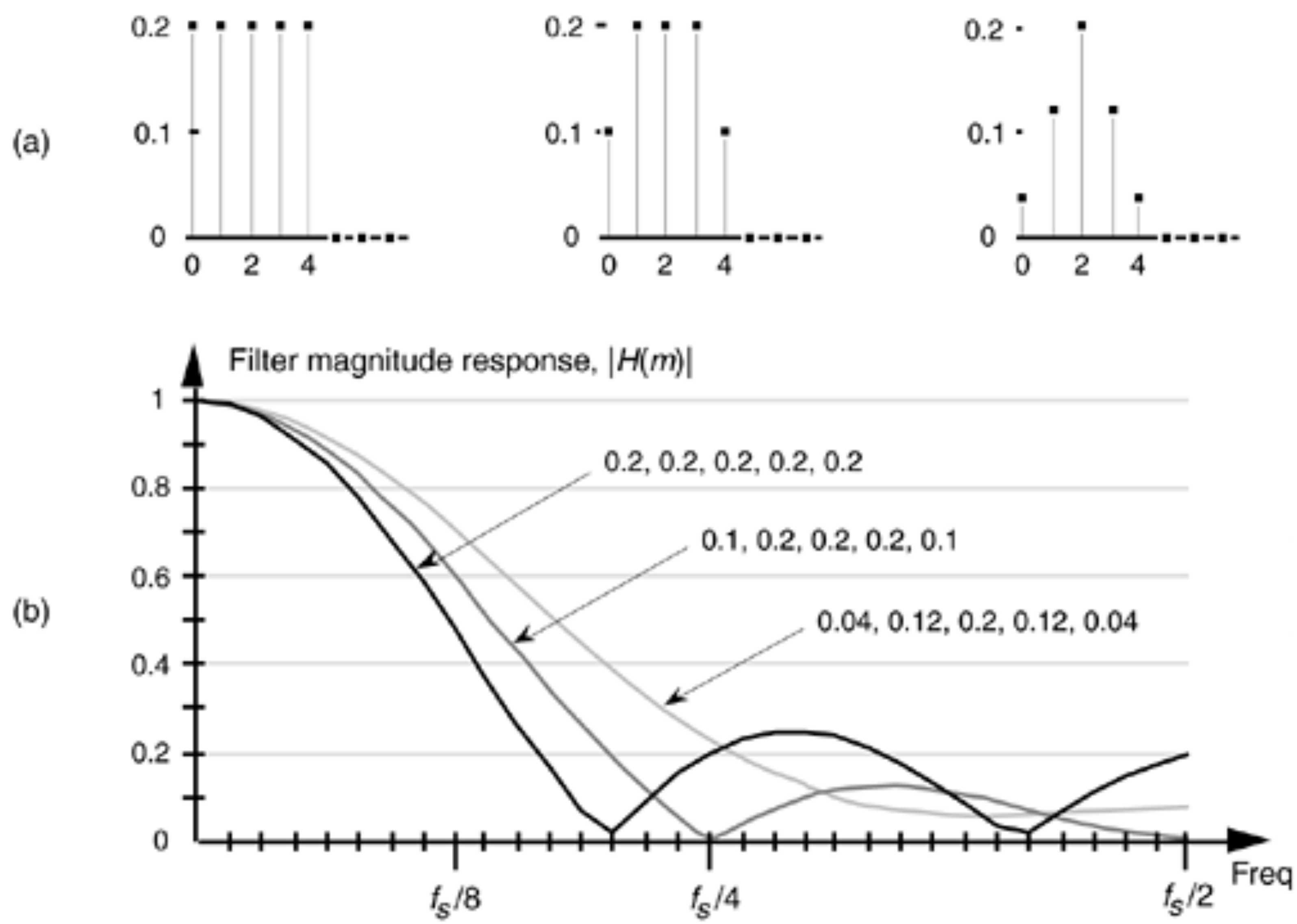

$f_{signal} =$

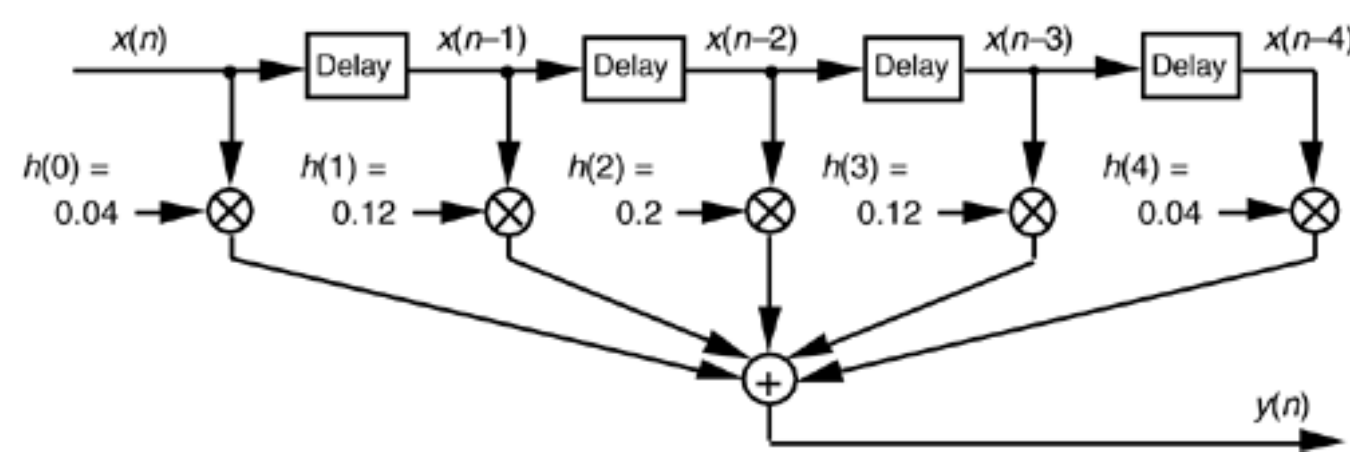
$f_{signal} =$

# Frequency-Domain Response

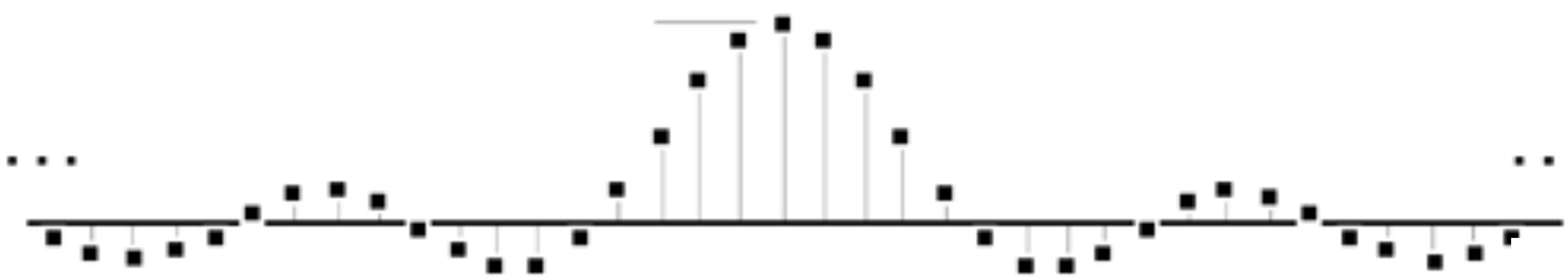


# More FIR Filters



# Ideal Low-Pass FIR Filter

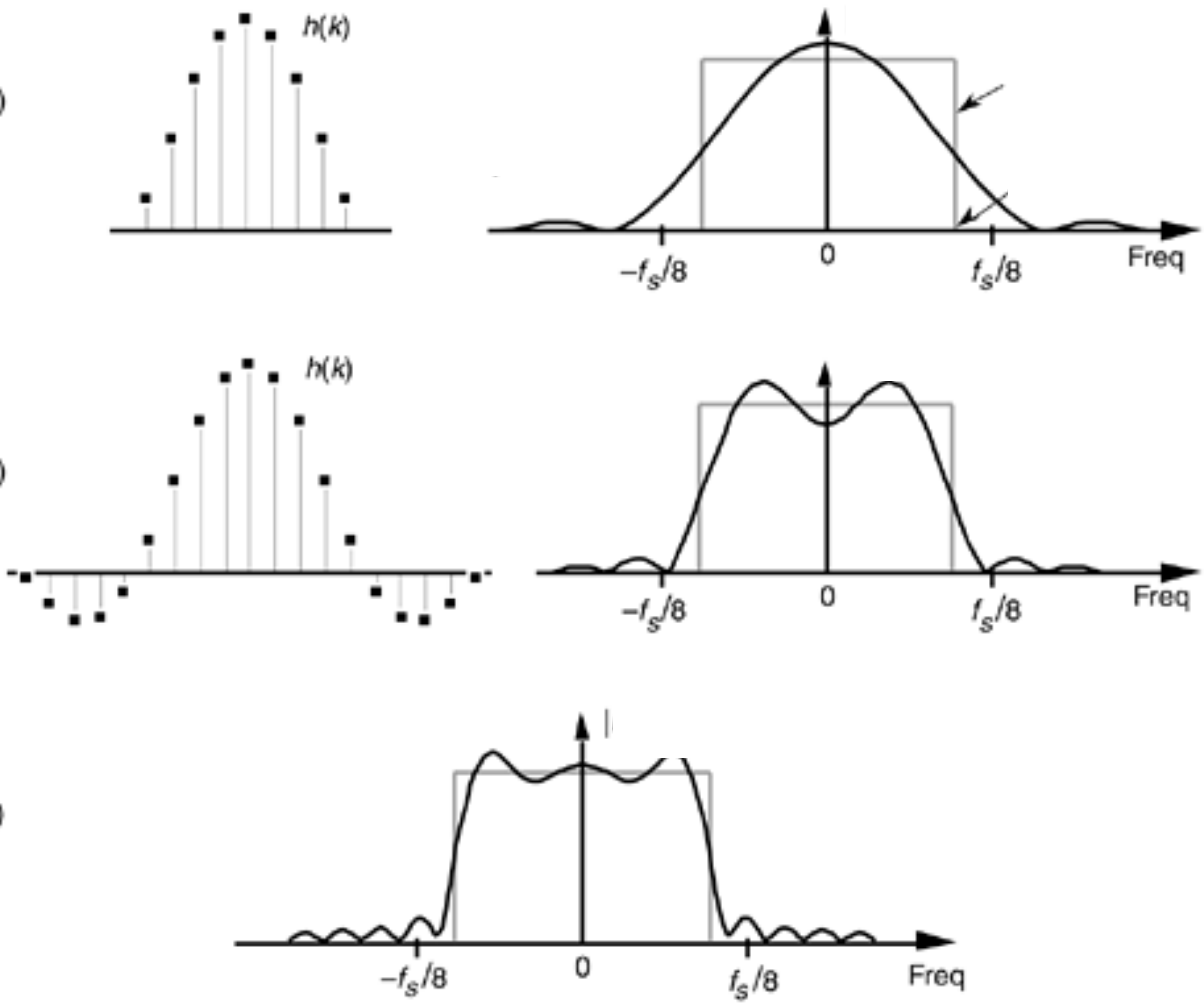



# Convoluted Low-Pass Filter

(a)

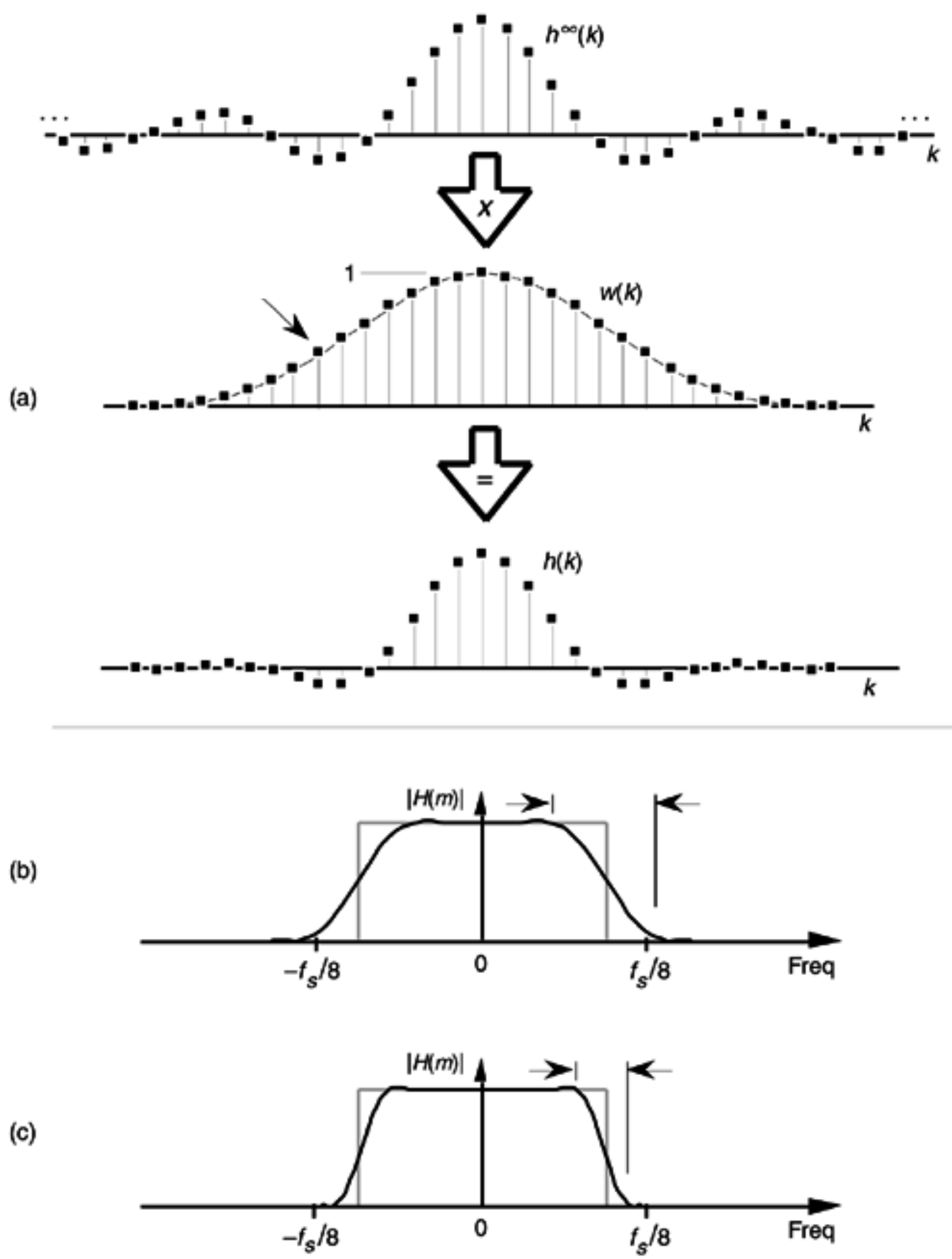
(b)

(c)

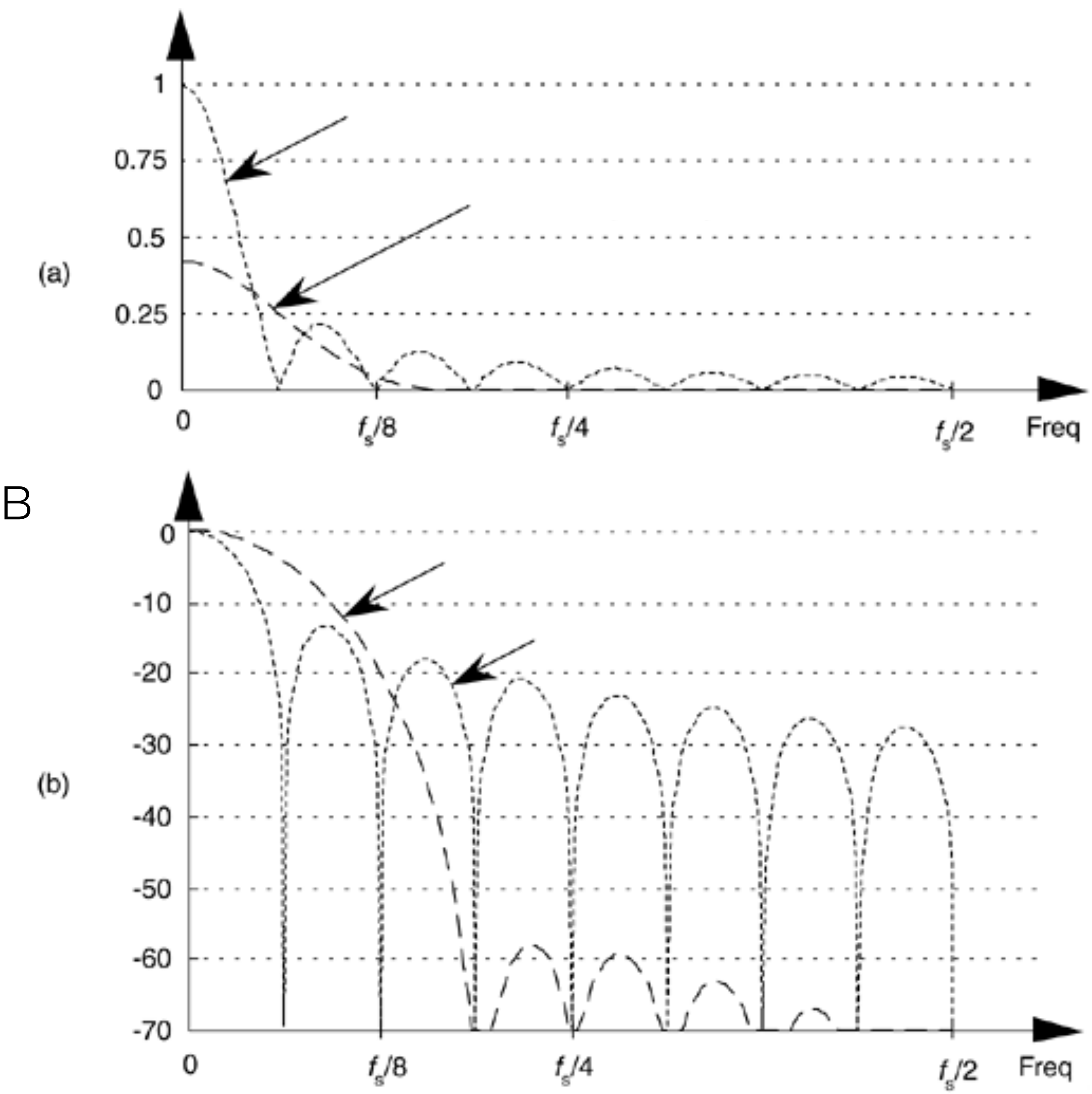


# Example: Blackman Window

$$w(k) =$$




dB



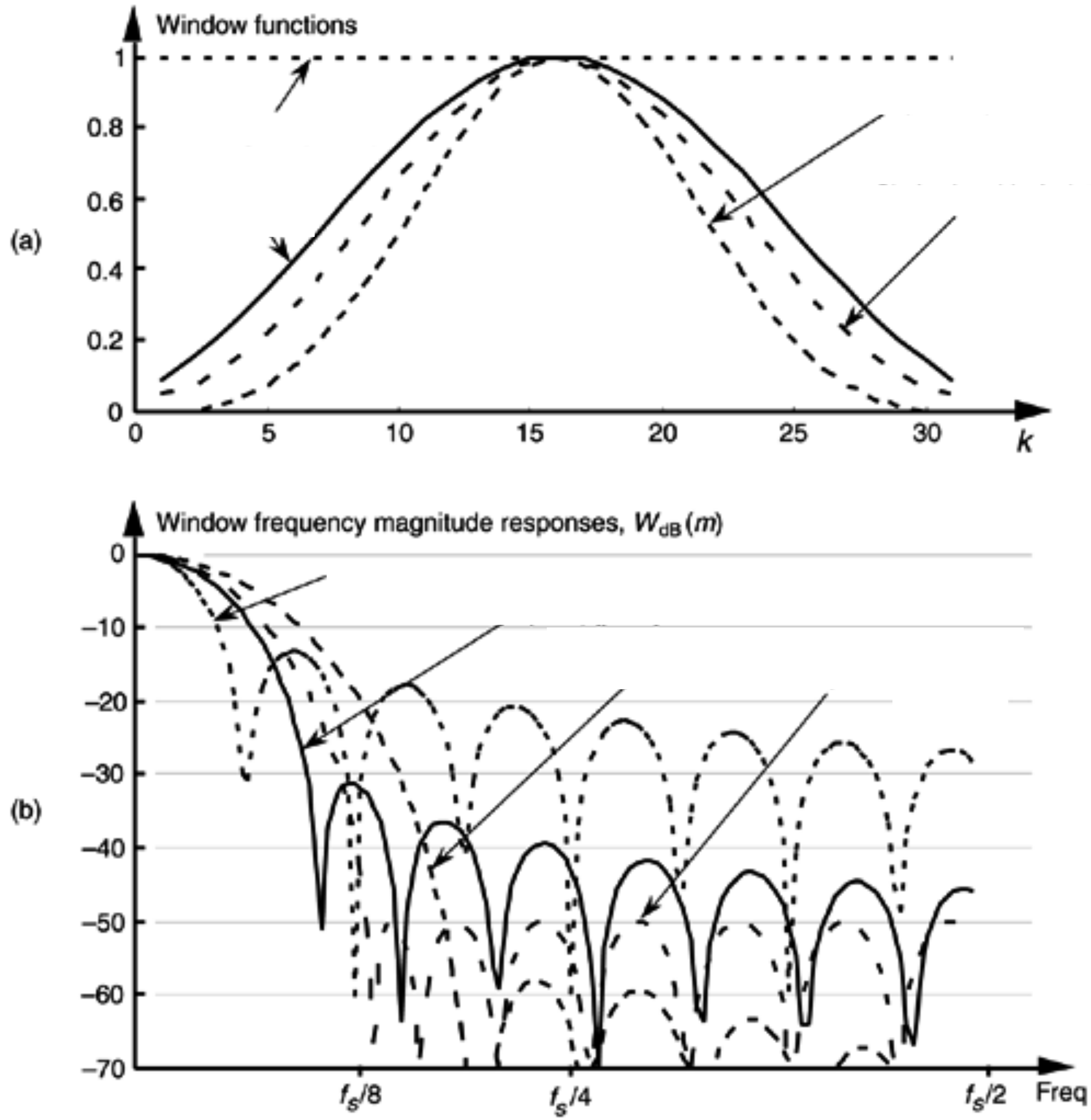
# More (Tunable) Filters

- $w(k) =$

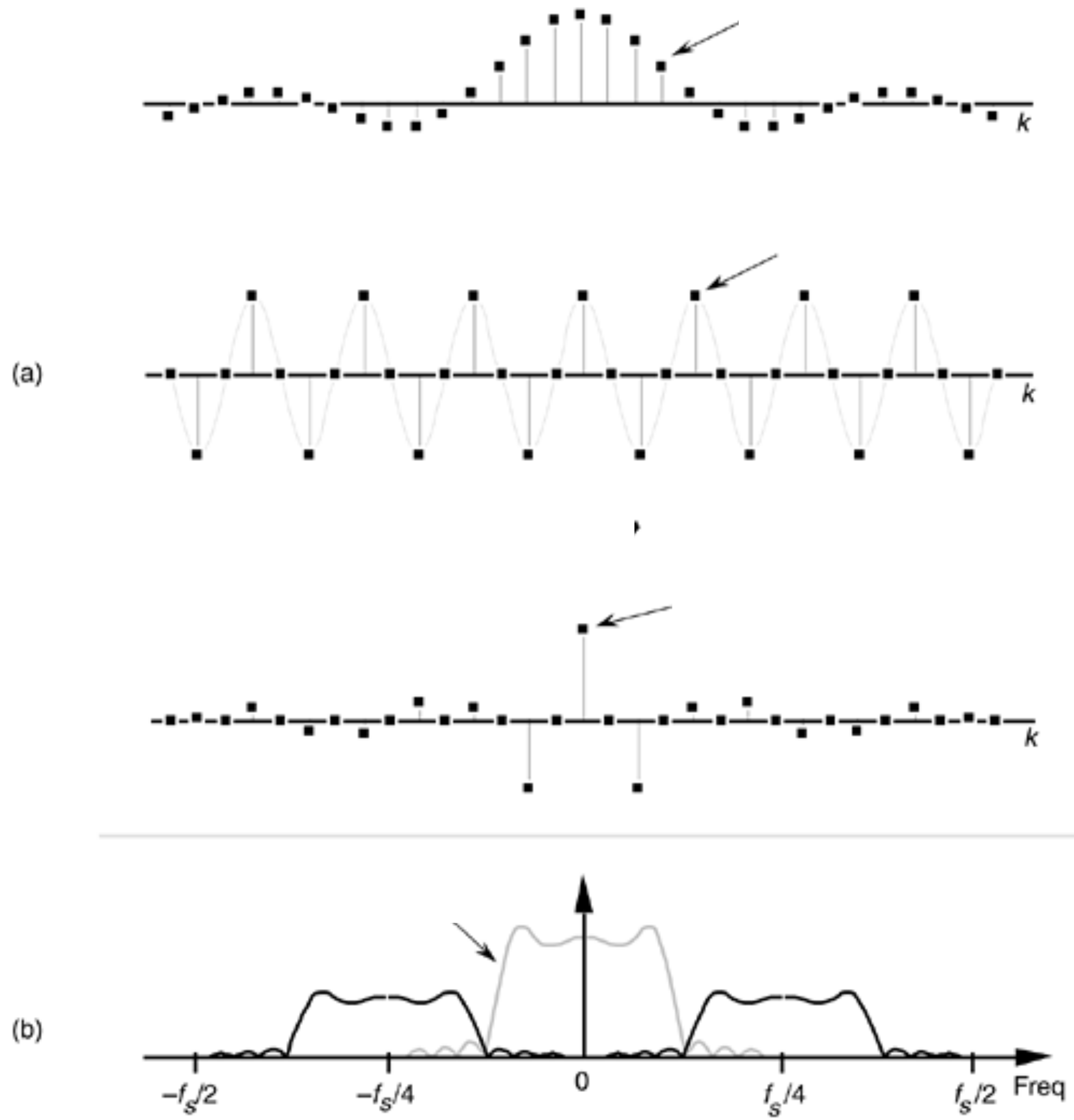
- where $\alpha =$

- $\omega(k) =$

# More Tunable Filters

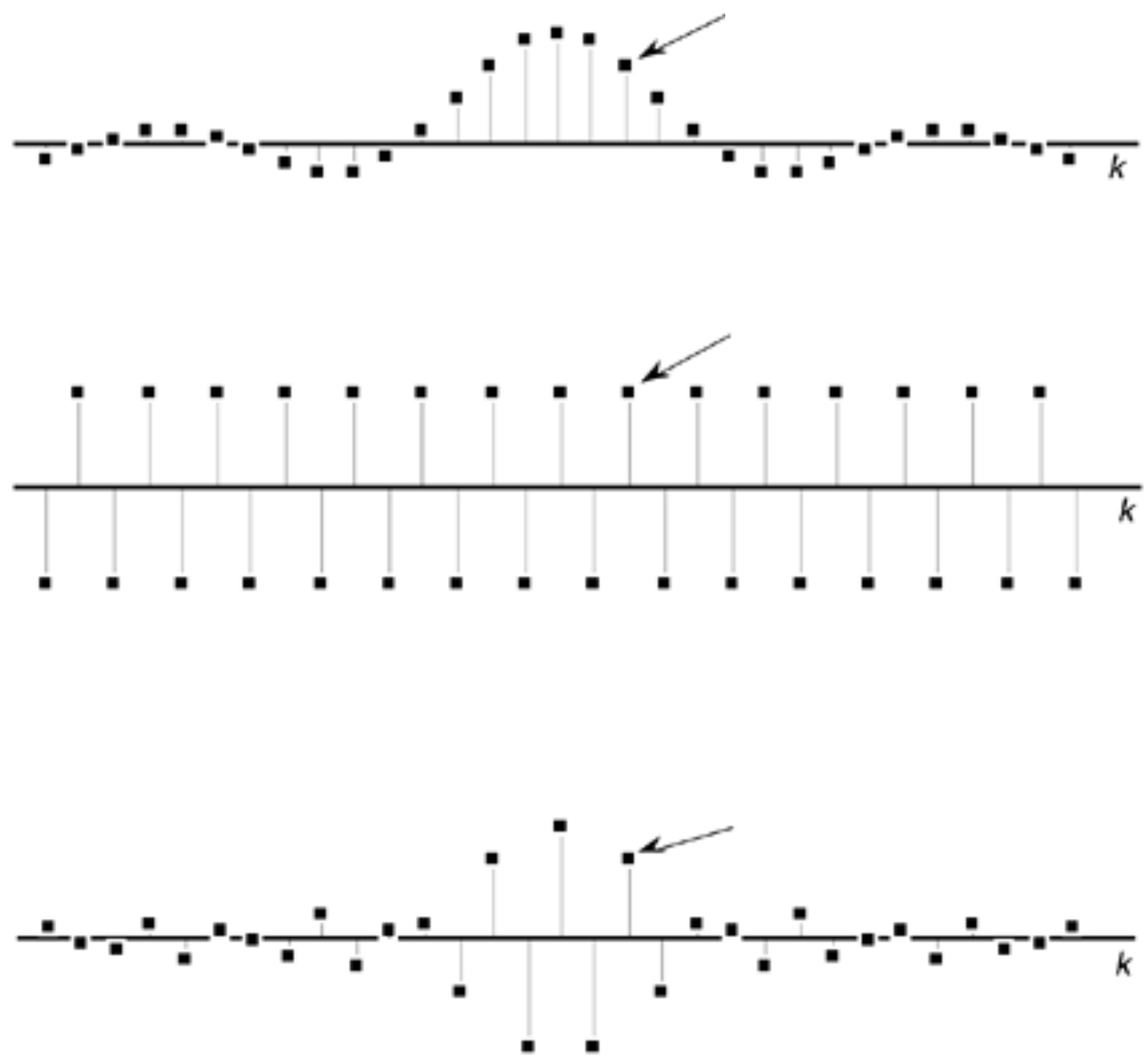


# Bandpass Filter

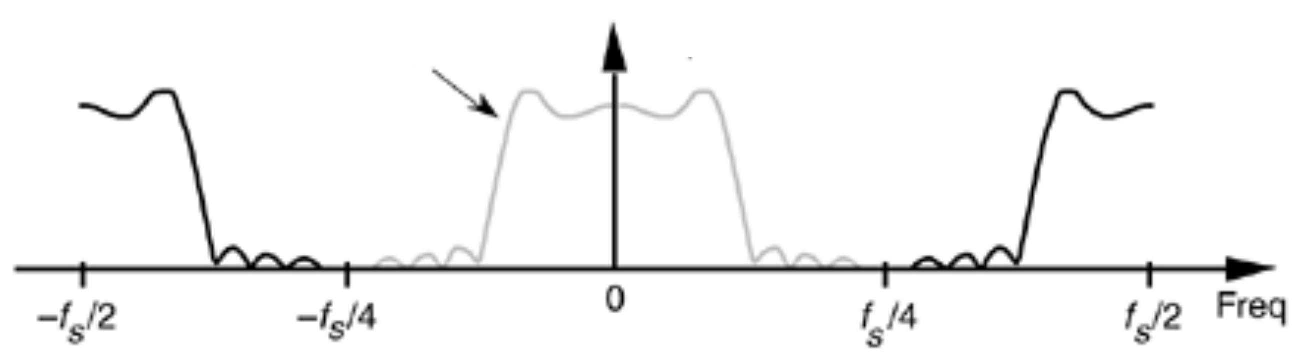


# Highpass Filter

(a)



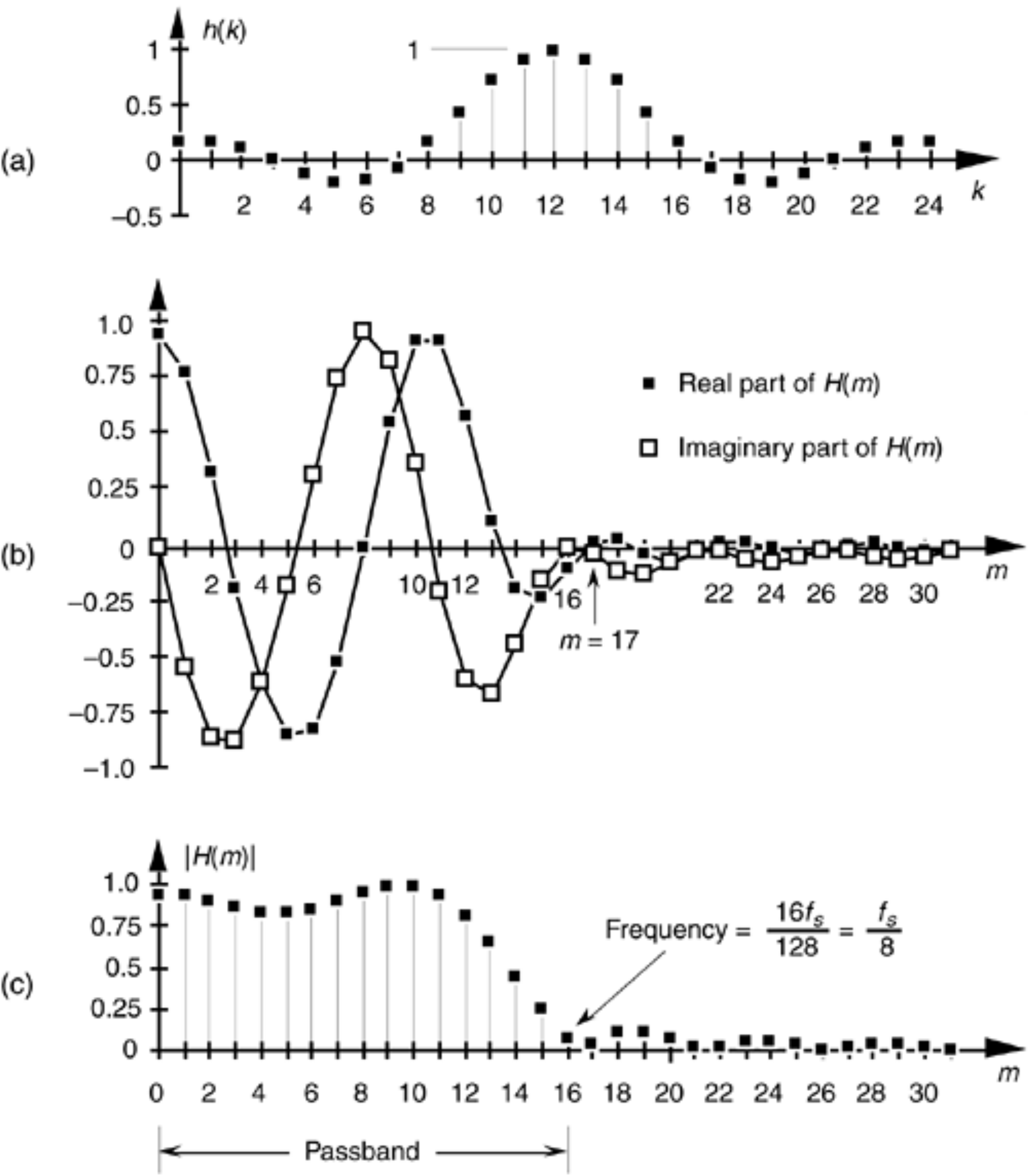

(b)

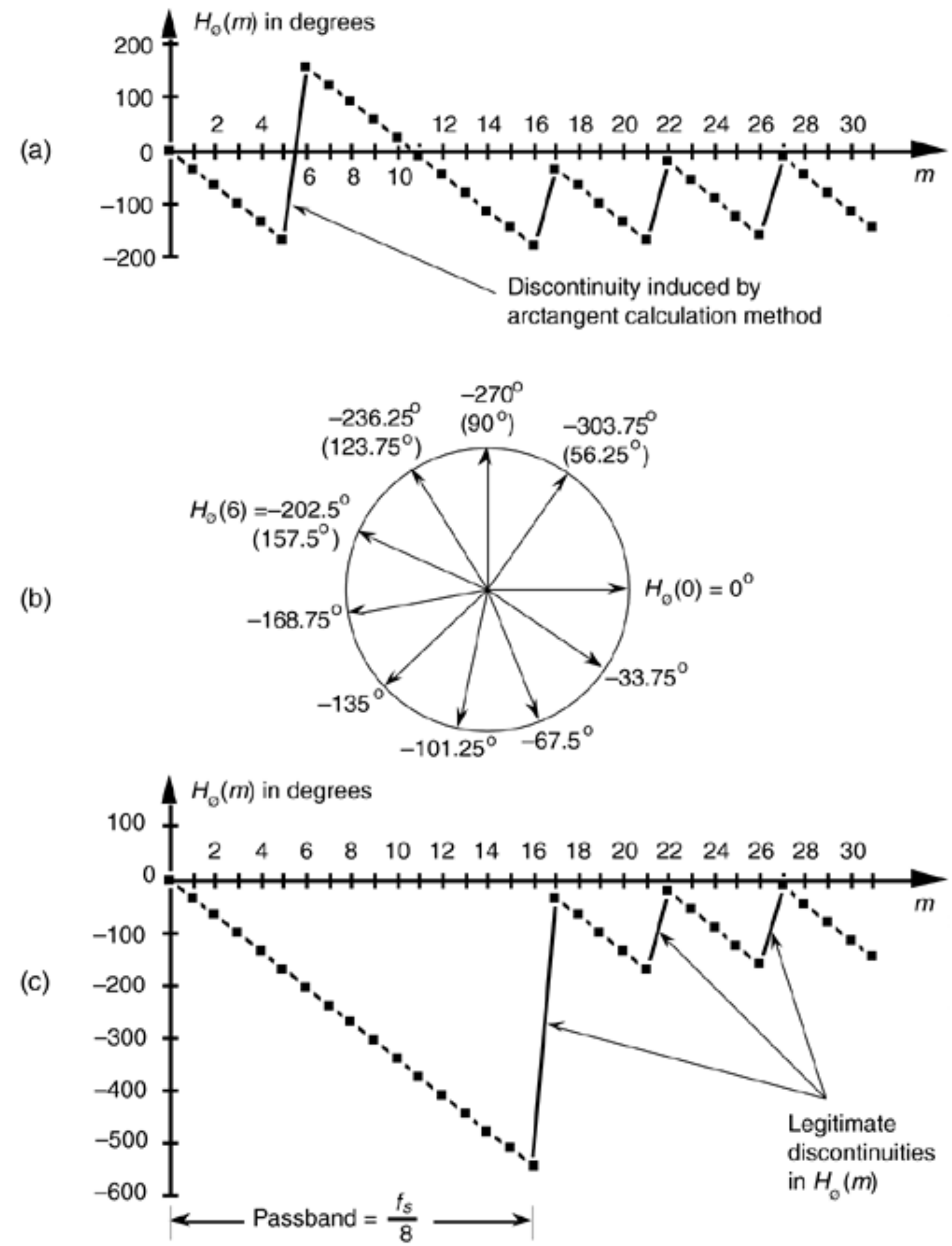


# Phase Response in FIR Filters

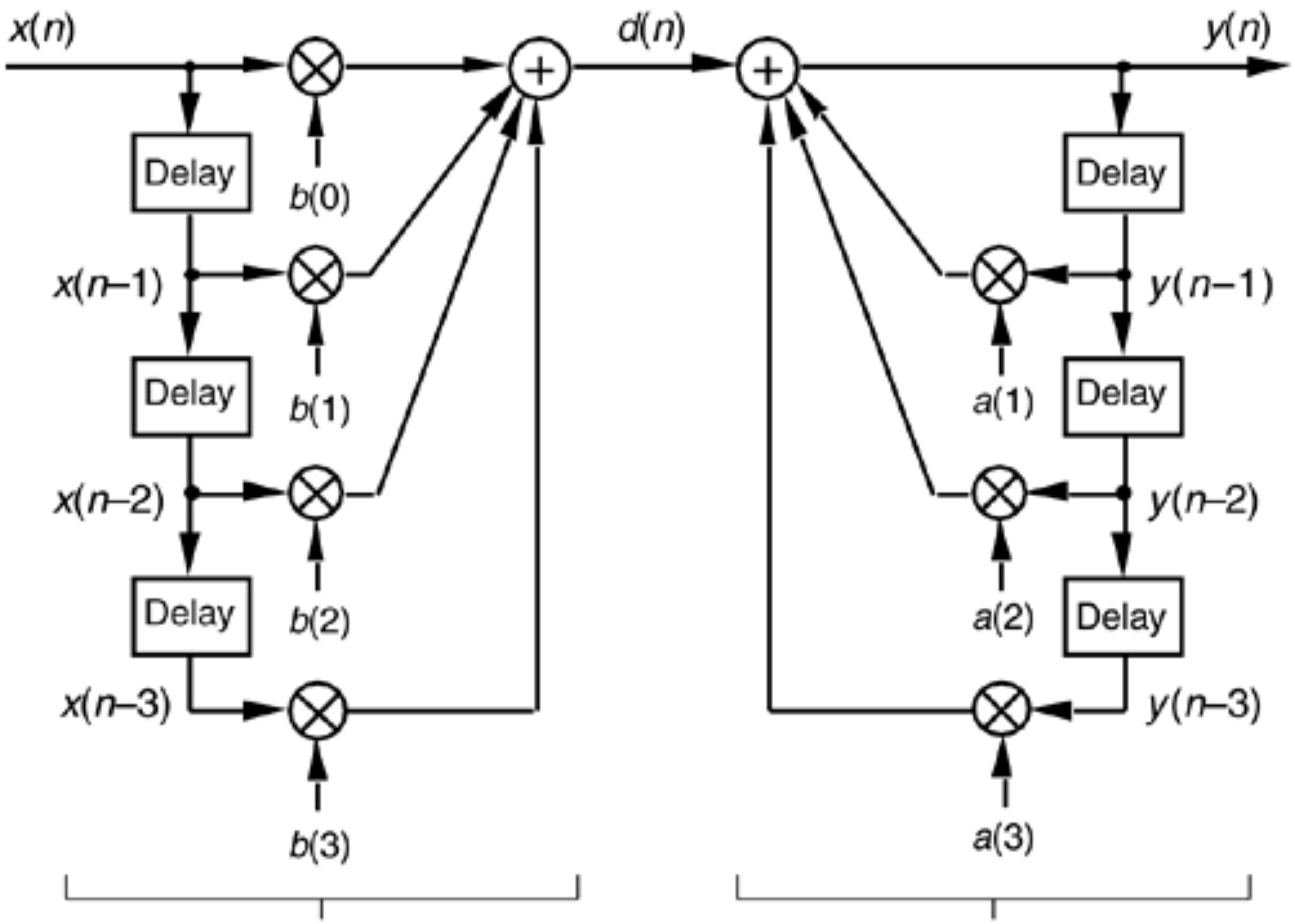
phase shift in :

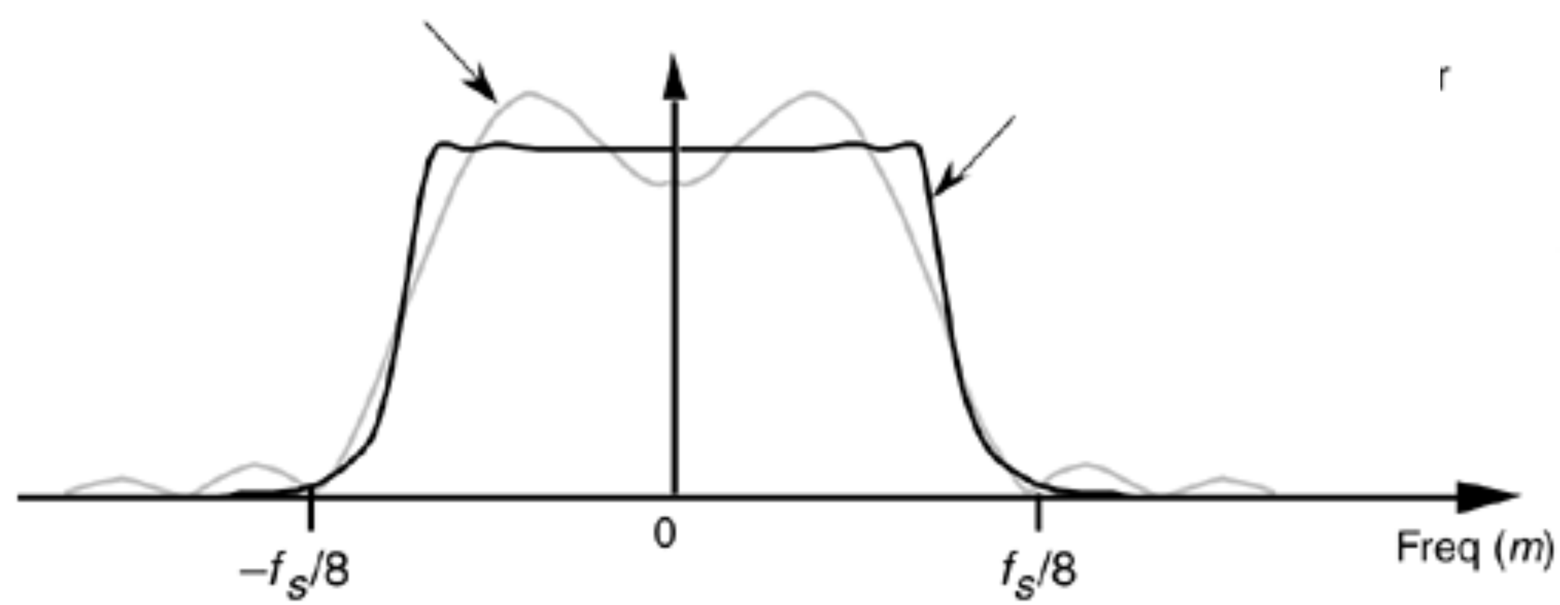
constant group delay (no ):



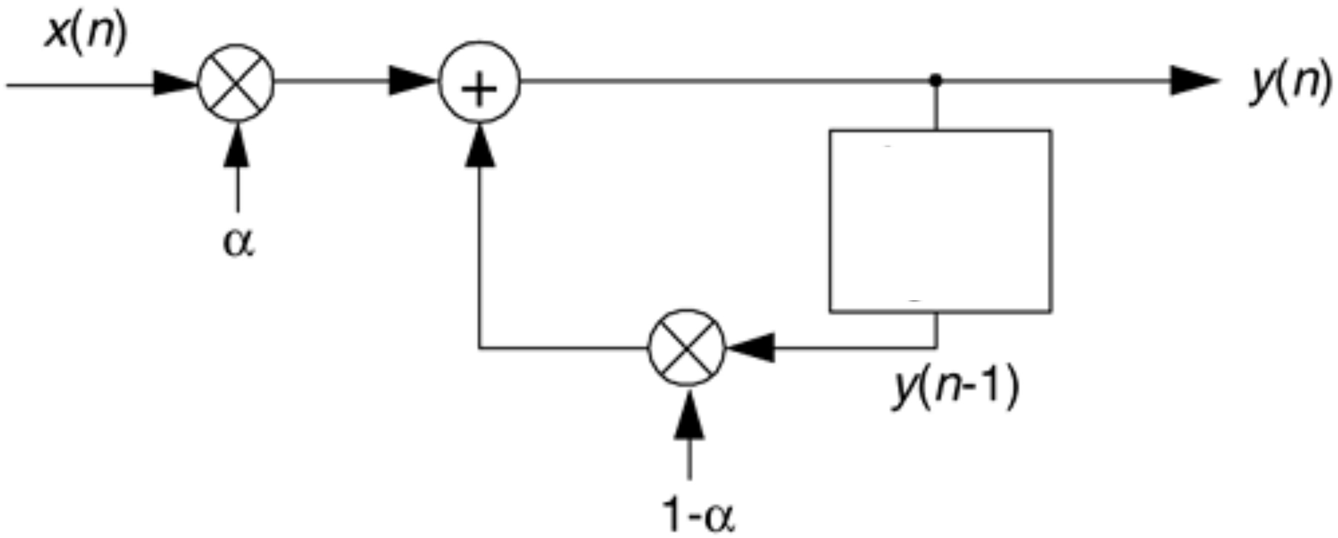
# IIR Filters

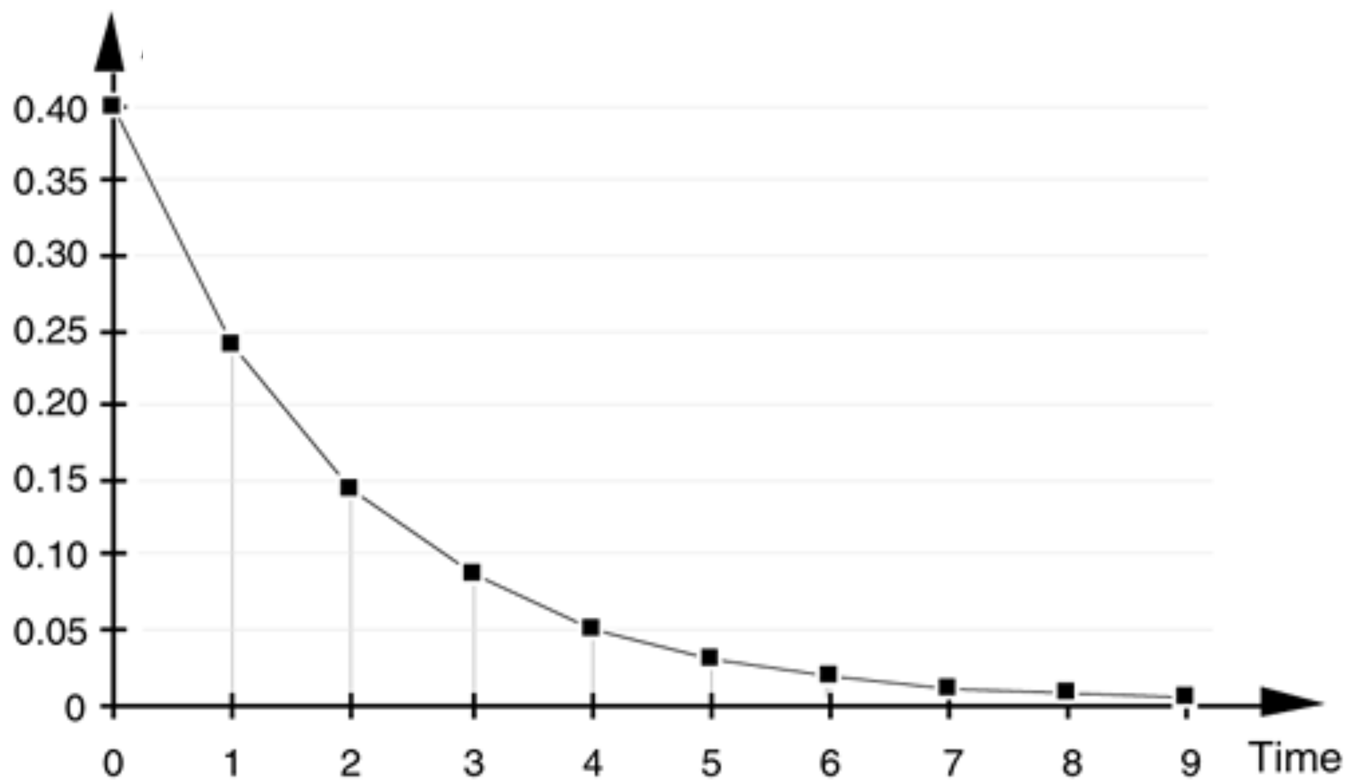

x(n)
d(n)
y(n)
Delay
Delay
Delay
Delay
Delay
Delay
b(0)
b(1)
b(2)
b(3)
a(1)
a(2)
a(3)
x(n–1)
x(n–2)
x(n–3)
y(n–1)
y(n–2)
y(n–3)


0
–fs/8
fs/8
Freq (m)

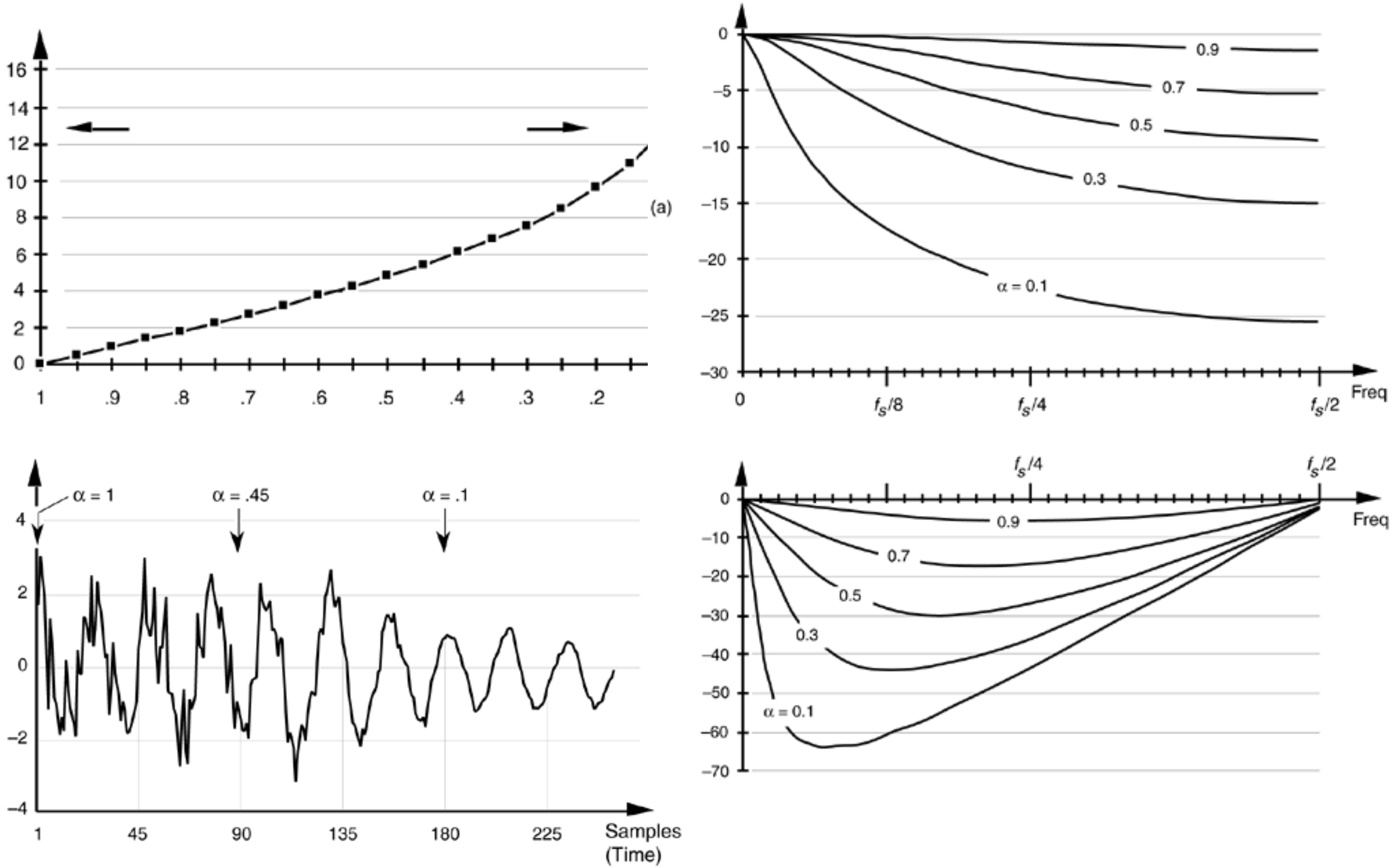
# Example: Exponential Averaging Filter

$y(n) =$



# Exponential IIR Filter



# What We Have Not Covered

- Many to cover, so I so far focused on most immediately
    - There are dedicated to
- Other possible topics of interest (at your leisure):
    - Digital Signal Processing
        - Digital mixing
        - Modulation/demodulation
    - Smoothing, windowing
        - Often useful for image processing
    - Down-sampling (decimating), re-sampling