Leinster - Basic Category Theory - Selected problem solutions for Chapter 3

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3.1.1

There are bijections

$$(A + B, C) \leftrightarrow ((A, B), \Delta C)$$
$$f \leftrightarrow \overline{f}$$

where $\overline{f} = (f, f)$

$$(\Delta A, (B, C)) \leftrightarrow (A, B \times C)$$
$$g = (p, q) \leftrightarrow \overline{g}$$

where $\overline{g}(x) = (p(x), q(x))$

So the sum is left adjoint to Δ, and the product is its right adjoint.

3.1.2

We are given the definition of a sequence, where there is a unique function x such that the square below commutes.

We have $x_0 = a$, and $x_{n+1} = r(x_n)$.

$$\begin{array}{ccc} \mathbb{N} & \xrightarrow{s} & \mathbb{N} \\ \downarrow{\scriptstyle x} & & \downarrow{\scriptstyle x} \\ X & \xrightarrow{r} & X \end{array}$$

This is precisely the definition of the comma category $(\mathbb{N} \Rightarrow X)$, where objects are $(n \in \mathbb{N}, x, t \in X)$.