Leinster - Basic Category Theory - Selected problem solutions for Chapter 3

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January 10, 2022

4.1.27

Take maps $f : A \to A'$, and $g : A' \to A$.

The component of the natural transformation from H_A to $H_{A'}$ can be expressed $p \mapsto f \circ p$, where p is $H_A(B),\ B \in \mathcal{A}$. The component of the natural transformation from $H_{A'}$ to H_A can be expressed $q \mapsto g \circ q$, where $q = H_{A'}(B),\ B \in \mathcal{A}$.

Composing these maps gives $q \mapsto f \circ g \circ q$. Since we are given $H_A \cong H_{A'}$, this mapping is the identity, for all B in $\mathcal{A}$. Applying any element of B to both sides yields $fgA' = A'$. The other direction can be checked analogously. Hence $A \cong A'$.