Test 4

1) $\int_{-2}^{\emptyset} (2x+5)\,dx =$

$\int_{-2}^{\emptyset} 2x\,dx + \int_{-2}^{\emptyset} 5\,dx$

$= \cancel{2}\frac{x^2}{\cancel{2}} + 5x \Big]_{-2}^{\emptyset}$

$= [(0)^2 + 5(\emptyset)] - [(-2)^2 + 5(-2)]$

$= \emptyset - [4 - 10]$

$= \emptyset + 6$

$\boxed{= 6}$

2) $\int_0^{\pi} \sin x\,dx = -\cos x \Big]_{\emptyset}^{\pi}$

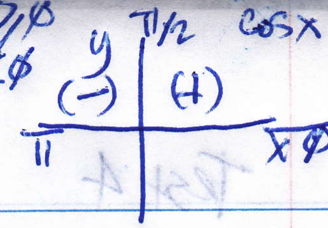
$= -\cos \pi - [-\cos(\emptyset)]$

$= -(-1) + 1$

$\boxed{= + +1 \quad = 2}$

Test 4 (Page 2)

$|x| = \begin{cases} x \text{ if } x \geq \emptyset \\ -x \text{ if } x < \emptyset \end{cases}$



③ $\int_0^{\pi} \frac{1}{2}(\cos x + |\cos x|)\,dx$

$= \frac{1}{2}\left[\int_\emptyset^{\pi} \cos x\,dx + \int_\emptyset^{\pi/2} \cos x\,dx + \int_{\pi/2}^{\pi} -\cos x\,dx\right]$

$= \frac{1}{2}\left[\left[\sin x\right]_\emptyset^{\pi} + \sin x\Big]_\emptyset^{\pi/2} + -\sin x\Big]_{\pi/2}^{\pi}\right]$

$= \frac{1}{2}\Big([\sin(\pi) - \sin(\emptyset)] + [\sin(\pi/2) - \sin(\emptyset)]$
$+ [-\sin(\pi) - (-\sin(\pi/2))]\Big]$

$= \frac{1}{2}\big(\emptyset + ((1) - \emptyset) + (\emptyset + 1)\big]$

$\boxed{= \frac{1}{2}(2) = 1}$

④ $\int \frac{9r^2\,dr}{(1-r^3)^{1/2}} = \int (1-r^3)^{-1/2} \cdot (9r^2)\,dr$ $\quad 3 \cdot 3$

$u = (1-r^3)$
$du = -3r^2\,dr$

$= -3\int u^{-1/2}\,du$

$= -3\,\frac{u^{1/2}}{1/2} = \boxed{(-3)(2)(1-r^3)^{1/2} + C}$

Test 4 (Page 3)

(5) $\int \frac{dx}{\sqrt{5x+8}} = \int \frac{1/5\,du}{\sqrt{u}} = \frac{1}{5}\int (u)^{-1/2}\,du$

$= \frac{1}{5}\,\frac{u^{1/2}}{1/2}$

$u = 5x+8$
$du = 5\,dx$
$1/5\,du = dx$

$= 2/5\,u^{1/2}$

$\boxed{= 2/5\,(5x+8)^{1/2} + C}$

(6) $\int \tan^7\left(\frac{x}{2}\right) \sec^2\left(\frac{x}{2}\right) dx$

$u = \tan x/2$
$du = \sec^2 x/2\,dx\ (1/2)$ $\quad 2du = \sec^2 \frac{x}{2}\,dx$

$= \int 2u^7\,du \qquad = (2)\frac{(u)^8}{8} = 1/4\,u^8$

$\boxed{= 1/4\,(\tan x/2)^8 + C}$

(7) $\int x e^{2x}\,dx = \int u\,dv = uv - \int v\,du$

$= x\,e^{2x} - \int 1/2\,e^{2x}$

$u = x$
$dv = e^{2x}\,dx \quad \int dv = v = 1/2\,e^{2x}$

$$\boxed{= \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} + C}$$

⑧ $\displaystyle\int \frac{x^2}{(x+1)}\,dx =$

$$\int \frac{(x^2-1)+1}{(x+1)}\,dx = \int \frac{(x+1)(x-1)+1}{(x+1)}\,dx$$

$$= \int \left[(x-1) + \frac{1}{(x+1)}\right] dx = \boxed{\frac{x^2}{2} - x + \ln|(x+1)| + C}$$

⑨ $\displaystyle\int \frac{1}{\sqrt{16-x^2}}\,dx$ $\qquad \cos^2\theta = 1 - \sin^2\theta$

$$= \int \frac{1}{\sqrt{4^2 - x^2}}\,dx$$

$x = 4\sin\theta$ $\qquad \sqrt{4^2 - x^2} = \sqrt{16 - 16\sin^2\theta}$

$dx = 4\cos\theta\,d\theta$ $\qquad = \sqrt{16(1-\sin^2\theta)}$

$\theta = \cancel{4} \operatorname{Arcsin}\left(\frac{x}{4}\right)$

$$= \int \frac{\cancel{4}\cos\theta\,d\theta}{\cancel{4}\sqrt{1-\sin^2\theta}} = \int \frac{\cos\theta\,d\theta}{\sqrt{\cos^2\theta}} = \int \frac{\cancel{\cos\theta}\,d\theta}{\cancel{\cos\theta}}$$

Test 4

$$= \int d\theta = \theta + C$$

$$\boxed{= \text{Arcsin}\left(\frac{x}{4}\right) + C}$$

(10) $\int_2^{\infty} x^{-3/2}\, dx$

$-3/2 + 2/2$

$$= \frac{x^{-1/2}}{-1/2}\Bigg]_2^{\infty} = \frac{-2}{\sqrt{x}}\Bigg]_2^{\infty}$$

$$\left[\frac{-2}{\sqrt{\infty}} - \frac{-2}{\sqrt{2}}\right]_2^{\infty} = \emptyset + \frac{2}{\sqrt{2}}$$

$$\boxed{= \frac{2^1}{2^{1/2}} = \sqrt{2}}$$