Test 1

1

$$f(x) = \left(\frac{1}{x} + \frac{1}{x^2}\right)\left(3x^3 + 27\right)$$

$$= \left(x^{-1} + x^{-2}\right)\left(3x^3 + 27\right)$$

$$\cancel{= 3x^2 + 27x^{-1} + 3x + 27x^{-2}}$$

$$\boxed{f'(x) = 6x - 27x^{-2} + 3 - 54x^{-3}}$$

2

$$y = \left(2x^4 - x^2\right) \cdot \frac{(x-1)}{(x+1)}$$

$$= \frac{2x^8 - x^3 - 2x^7 + x^2}{(x+1)}$$

$\cancel{f}$ $y'(1) =$

$$\frac{\left(16x^7 - 3x^2 - 14x^6 + 2x\right)(x+1) + \left(2x^8 - x^3 - 2x^7 + x^2\right)}{(x+1)^2}$$

$$\boxed{= \frac{2 + 0}{4} = 1/2}$$

3

$$f(x) = \left(x^2 + 1\right)\sec x$$

$$\boxed{f'(x) = 2x\sec x + \left(x^2 + 1\right)\sec x \tan x}$$

(4) $f(x) = \frac{1}{\cot x}$

$f'(x) = \frac{-(1)(-\csc^2 x)}{\cot^2 x}$

$= \frac{\csc^2 x}{\cot^2 x} = \frac{1/\sin^2 x}{\frac{\cos^2 x}{\sin^2 x}}$

$= \frac{1}{\cancel{\sin^2 x}} \cdot \frac{\cancel{\sin^2 x}}{\cos^2 x}$

$= \frac{1}{\cos^2 x} = \boxed{\sec^2 x}$

(5) $y = \tan x$

$\frac{dy}{dx} = \sec^2 x$

$\frac{d^2y}{dx^2} = 2 \sec x \sec x \tan x (1)$

$\boxed{= 2\sec^2 x \tan x}$

(6) $f(x) = \sin^3 x$

$\boxed{= 3\sin^2 x \cos x}$

(7) $f(x) = \cos^3 \left(\frac{x}{x+1}\right)$

$$f'(x) = 3\cos^2\left(\frac{x}{x+1}\right)\left(-\sin\frac{x}{(x+1)}\right)\left(\frac{(1)(x+1)-x(1)}{(x+1)^2}\right)$$

$$= -3\cos^2\left(\frac{x}{x+1}\right)\sin\left(\frac{x}{x+1}\right)\left(\frac{1}{(x+1)^2}\right)$$

(8) $f(x) = [x^4 - \sec(4x^2-2)]^{-4}$

$$f'(x) = (-4)(x^4-\sec(4x^2-2))^{-5}$$
$$(4x^3 - \sec(4x^2-2)\tan(4x^2-2))$$
$$(8x)$$

$$\boxed{= -32x\left(x^4-\sec(4x^2-2)\right)^{-5} \cdot \left(4x^3-\sec(4x^2-2)\tan(4x^2-2)\right)}$$

(9) $y = \dfrac{\sin x}{\sec(3x+1)}$

$= \sin x \cos(3x+1)$

$y' = \cos x \cos(3x+1) + 3\sin x \cdot -(\sin(3x+1))$

$$\boxed{= \cos x\cos(3x+1) - 3\sin x\sin(3x+1)}$$

10)

$$y = \left(\frac{1+x^2}{1-x^2}\right)^{17}$$

$$y' = 17\left(\frac{1+x^2}{1-x^2}\right)^{16} \cdot \frac{2x(1-x^2)-(1+x^2)-2x}{(1-x^2)^2}$$

$$= 17\left(\frac{1+x^2}{1-x^2}\right)^{16} \cdot \frac{\cancel{2x}-\cancel{2x^3}+\cancel{2x}+\cancel{2x^3}}{(1-x^2)^2}$$

$$= 17\left(\frac{1+x^2}{1-x^2}\right)^{16} \frac{2x(1-\cancel{x^2}+1+\cancel{x^2})}{(1-x^2)^2}$$

$$= 17\left(\frac{1+x^2}{1-x^2}\right)^{16} \frac{4x}{(1-x^2)^2}$$

$$= \frac{68x(1+x^2)^{16}}{(1-x^2)^{18}}$$

Page 1

Test 2

① $y = \ln \left(\frac{(x^2+1)^5}{\sqrt{1-x}} \right)$

$y = 5\ln(x^2+1) - \ln\sqrt{1-x}$

$y' = 5\left(\frac{1}{(x^2+1)}\right)(2x)$

$- \frac{1}{(1-x)^{1/2}} \, \frac{1}{2}(1-x)^{-1/2}(-1)$

$$\boxed{= \frac{10x}{(x^2+1)} + \frac{1}{2(1-x)}}$$

② $y = \ln(\ln x)$

$y' = \frac{1}{\ln x} \cdot \frac{1}{x} \cdot (1)$

$$\boxed{= \frac{1}{x \ln x}}$$

(3) $y = \dfrac{x\ln x}{1+\ln x}$

$y' = \dfrac{\left(\ln x + x\frac{1}{x}\right)(1+\ln x) - (x\ln x)\cdot\left(\frac{1}{x}\right)}{(1+\ln x)^2}$

$= \dfrac{(\ln x + \cancel{1})(1+\ln x) - \frac{\cancel{x}\ln x}{\cancel{x}}}{(1+\ln x)^2}$

$= \dfrac{\overset{(\ln x+1)^2}{\cancel{\ln x + \ln^2 x + x + x\ln x}} - \overset{\ln x}{\cancel{\ln x}}}{(1+\ln x)^2}$

$\boxed{= 1 - \dfrac{\ln x}{(1+\ln x)^2}}$

(4) $y = \dfrac{1+\ln t}{t}$

$y' = \dfrac{\left(\frac{1}{t}\right)t - (1+\ln t)}{t^2}$

$= \dfrac{1-(1+\ln t)}{t^2} = \boxed{\dfrac{-\ln t}{t^2}}$

(5) $y = t\sqrt{\ln t}$

$= t(\ln t)^{1/2}$

$y' = (\ln t)^{1/2} + \cancel{t}\,\frac{1}{2}(\ln t)^{-1/2}\,\frac{1}{\cancel{t}}$

$$\boxed{= (\ln t)^{1/2} + \frac{1}{2(\ln t)^{1/2}}}$$

(6) $y = \ln 10 - \ln x$

$$\boxed{y' = \cancel{0} - \frac{1}{x}}$$

(7) $f(x) = x^{1-x}$

$\cancel{f'(x)} = y = x^{1-x}$

$\ln y = \ln x^{1-x} = (1-x)\ln x$

$y' = (-1)\ln x + (1-x)\frac{1}{x}$

$= -\ln x + \frac{(1-x)}{x}$

$= -\ln x + \frac{1}{x} - 1$

$$\boxed{= x^{1-x}\left(-\ln x + \frac{1}{x} - 1\right)}$$

Test 2

(8) $f(x) = e^{3x-1}$

$f'(x) = e^{3x-1} \cdot (3)$

$\boxed{= 3e^{3x-1}}$

(9) $f(x) = \dfrac{1-x}{e^x}$

$f' = \dfrac{(-1)e^x - (1-x)e^x}{(e^x)^2}$

$= \dfrac{-e^x - e^x + xe^x}{(e^{x})^2}$

$= \dfrac{\cancel{e^x}(-1-1+x)}{(\cancel{e^x})^2}$

$\boxed{= \dfrac{x-2}{e^x}}$

(10) $f(x) = e^{\sqrt{x}} + e^{-\sqrt{x}}$

$$= e^{x^{1/2}} \cdot \tfrac{1}{2} x^{-1/2}$$

$$\boxed{+ e^{x^{-1/2}} \cdot -\tfrac{1}{2} x^{-3/2}}$$

$$= \frac{e^{x^{1/2}}}{2x^{1/2}} - \frac{e^{x^{-1/2}}}{2\,x^{3/2}}$$

$$= \frac{1}{2x^{1/2}} \cdot e^{x^{1/2}} - e^{x^{-1/2}} \left(1 - x^{2/2}\right)$$

(11) $f(x) = x\left(3^{-5x}\right)$

$$\ln y = \ln x \cdot 3^{-5x}$$

$$= \ln x + \ln 3^{-5x}$$

$$= \ln x - 5x \ln 3$$

$$y' = \frac{1}{x} - 5(\ln 3)$$

$$= x \cdot 3^{-5x}\left(\frac{1}{x} - 5 \ln 3\right)$$

$$\boxed{= 3^{-5x}\left(1 - 5x \ln 3\right)}$$

Test 2

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(11) $f(x) = \sin(2e^x)$

$f'(x) = \cos(2e^x)\ 2e^x$

$\boxed{= 2\ e^x \cos(2e^x)}$

Test 3

① $\lim_{\theta \to 0} \frac{\tan\theta}{\theta} = \lim_{\theta \to 0} \frac{\sin\theta}{\cos\theta \cdot \theta} = \lim_{\theta \to 0} \left(\frac{1}{\cos\theta}\right)\left(\frac{\sin\theta}{\theta}\right)$

$$\boxed{= (1)(1) = 1}$$

② $\lim_{x \to \infty^+} \frac{x^{100}}{e^x} =$

using L'Hopitals Rule

$$= \lim_{x \to \infty^+} \frac{f'(x^{100})}{g'(e^x)} = \lim_{x \to \infty^+} \frac{100x^{99}}{e^x}$$

$$= \lim_{x \to \infty^+} \frac{100 \cdot 99 \cdot 98 \cdots 1}{e^x}$$

$$\boxed{= \frac{1}{\infty^+} = \phi^+}$$

③ $\lim_{x \to \phi} \frac{\sin^{-1} 2x}{x}$

(4) $\lim_{x \to \pi/2^-} \sec 3x \cos 5x = \lim_{x \to \pi/2^-} \frac{\cos 5x}{\cos 3x}$

Applying L'Hospital Formula

$\lim_{x \to \pi/2^-} \frac{f'(\cos 5x)}{g'(\cos 3x)} = \lim_{x \to \pi/2} \frac{\cos 5x}{\cos 3x} =$

$\lim_{x \to \pi/2^-} \frac{(-5)\sin 5/2\pi}{(-3)\sin 3/2\pi} = \frac{(-5)(1)}{(-3)(-1)}$

$\boxed{= \frac{-5}{3}}$

(5) $\lim_{x \to \infty} (1 + a/x)^{bx} =$

$\ln y = \ln (1 + a/x)^{bx}$

$= bx \ln (1 + a/x) = \frac{\ln(1 + a/x)}{(bx)^{-1}}$

Applying L'Hospital Formula

$\lim_{x \to \infty} \frac{f'(\ln(1 + a/x))}{g(bx)^{-1}} = \lim_{x \to \infty} \frac{\left(\frac{1}{1+a/x}\right)\left(\frac{-a}{x^2}\right)}{-bx^{-2} \cdot b} = \frac{-a}{-b^{-1}}$

Test 3

$\boxed{= a/b}$ $\lim_{x \to \infty} \ln y = \ln (1 + a/x)^{bx}$ Page 3

$\boxed{= ab}$

(6) $\lim_{x \to 1} (2-x)^{\tan (\pi/2) x}$

$\ln y = \tan(\pi/2) x \cdot \ln (2-x)$

Applying Le Hopitan Rule

$\lim_{x \to 1} = \frac{f'(\ln(2-x))}{g'(\cot \pi/2 x)} = \lim_{x \to 1} \frac{\frac{1}{2-x}(-1)}{-\csc^2 \pi/2 x \cdot \pi/2}$

$\lim_{x \to 1} = \frac{\cancel{(-1)} \sin^2 \pi/2 x}{\cancel{(-)} 2-x} \cdot \frac{2}{\pi}$

$= \frac{(1)}{(2-1)} \cdot \frac{2}{\pi} = \boxed{\frac{2}{\pi}}$

(7) $\lim_{x \to 0} (\csc x - 1/x)$

$= \lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right) = \lim_{x \to 0} \frac{(x - \sin x)}{(x \sin x)}$

Applying Le Hopital Rule

$= \lim_{x \to 0} \frac{f'(x - \sin x)}{g'(x \sin x)}$

(8) $$\lim_{x\to\infty} \frac{\ln(\ln x)}{\sqrt{x}}$$

Applying le Hopital's Rule

$$\lim_{x\to\infty} \frac{f'(\ln(\ln x))}{g'(x^{1/2})} = \lim_{x\to\infty} \frac{\frac{1}{\ln x}\cdot\frac{1}{x}\cdot(1)}{1/2\, x^{-1/2}}$$

$$\lim_{x\to\infty} = \frac{2\cancel{x}^{1/2}}{\cancel{x}\ln x} = \lim_{x\to\infty} \frac{2}{x^{1/2}\ln x} = \frac{2}{\infty} = \boxed{0}$$