Solution to the problems in https://piazza.com/class/llb4bhwh59g9n/post/1039
(I'm assuming reductions are made in polynomial time.)

1. ShortestPath, Primality

2. All of them

3. (a) false. To be NP-complete, the problem must be in NP-hard and in NP.
   (b) true. By definition (see above).

4. (a) true (it's NP-complete)
   (b) false
   (c) true (all NP-complete problems can be reduced to all other NP-complete problems)
   (d) true (by same logic above)

5. (a) yes, SAT can be reduced to ILP(0/1), which can then be reduced to A
   (b) no, we have no guarantee that a reduction exists in the opposite direction

6. (a) true. Use the reduction to transform the certificate into an instance of B, which is NP. Therefore, A is also NP.
   (b) true. Reducing A to B doesn't tell if it is in P or NP-hard. (However, if we reduced B to A, we'd know A was NP-hard.)
   (c) false. As explained above, this isn't necessarily true.

7. Create a graph $G' = G$. Let $k' = |V| - k$. Run independent set on G' and k'. Let S' be the returned indpendent set. Then, the solution to edge-monitoring is $S = V - S'$. (Clearly, this reduction is polynomial-time.)

   Now, we prove that G' has a size-k' independent set S' IFF G has a size-k edge-monitoring set S.

   Assume, G' has a size-k' indpendent set S'.
   For every edge $e \in E$, e cannot have both vertices in S' since S' is an independent set.
   Therefore, at least one of the vertices of e must be in S.
   Therefore, e is monitored.
   Therefore, every edge is monitored. (i.e. S is a "vertex cover")

   Assume G has a size-k solution to edge-monitoring (a "vertex cover") S.
   For every edge $e \in E$, e must have at least one vertex in S since it is covered by S.
   Therefore, e does not have both vertices in S'.
   Therefore, no edge connects 2 vertices in S'.
   Therefore, S' is an independent set.