Problem 1

Pick the gas station that is farthest away from you that you can still reach given you gas tank.

Problem 2

Swap the two stations o_1 and g_1. Since the greedy picks the farthest station away, then either o_1 *is* g_1 or o_1 is closer than g_1, so we can still make it there.

Problem 3

We can continually swap o_i for g_i until we run out of gas stations. The greedy must have the same number of stops, otherwise the optimal would not have been optimal.

Note that the greedy cannot have more stops than the optimal since we always choose the farthest away gas station.

Problem 4

No, counter example: $S = 4$, pairs: $(4, 3), (2, 2), (1, 2)$. Greedy chooses $(4, 3)$, but optimally we would choose $(2, 2), (1, 2)$.

Problem 5

No, counter example: $S = 2$, pairs: $(1, 4), (2, 7)$. Greedy chooses $(1, 4)$ but optimally we want $(2, 7)$.