Solution to the problems in https://piazza.com/class/llb4bhwh59g9n/post/1039
(I'm assuming reductions are made in polynomial time.)

1. ShortestPath, Primality

2. All of them

3. (a) false. To be NP-complete, the problem must be in NP-hard and in NP.
   (b) true. By definition (see above).

4. (a) true (it's NP-complete)
   (b) false
   (c) true (all NP-complete problems can be reduced to all other NP-complete problems)
   (d) true (by same logic above)

5. (a) yes, A must be NP-complete, and all NP-complete problems can be reduced to all other NP-complete problems.
   (b) yes by above logic.

6. (a) true. Use the reduction to transform the certificate into an instance of B, which is NP. Therefore, A is also NP.
   (b) true. Reducing A to B doesn't tell if it is in P or NP-hard. (However, if we reduced B to A, we'd know A was NP-complete.)
   (c) false. As explained above, this isn't necessarily true.

7. Create a graph $G' = G$. Let $k' = |V| - k$. Run independent set on $G'$ and $k'$. Let $S'$ be the returned indpendent set. Then, the solution to edge-monitoring is $S = V - S'$. (Clearly, this reduction is polynomial-time.)

   Now, we prove that G' has a size-k' independent set S' IFF G has a size-k edge-monitoring set S.

   Assume, G' has a size-k' indpendent set S'.
   For every edge $e \in E$, $e$ cannot have both vertices in $S'$ since $S'$ is an independent set.
   Therefore, at least one of the vertices of $e$ must be in $S$.
   Therefore, $e$ is monitored.
   Therefore, every edge is monitored. (i.e. $S$ is a "vertex cover")

   Assume $G$ has a size-k solution to edge-monitoring (a "vertex cover") $S$.
   For every edge $e \in E$, $e$ must have at least one vertex in $S$ since it is covered by $S$.
   Therefore, $e$ does not have both vertices in $S'$.
   Therefore, no edge connects 2 vertices in $S'$.
   Therefore, $S'$ is an independent set.