Problem 1

Need to show that $(a_1 \cdot b_j) + (a_k \cdot b_n) > (a_1 \cdot b_n) + (a_k \cdot b_j)$

Note: since a1 is the smallest element and $b_j < b_n$ then swapping b_j for b_n must result in a smaller term.

Also note: since $a_k \cdot b_j < a_k \cdot b_n$ then we really only need to show that $a_1 \cdot b_j > a_1 \cdot b_n$, but if we divide out a_1 then we are left with $b_j > b_n$ which cannot happen since b_j is supposed to be the smaller term. Hence contradiction to the idea that the vectors are in ascending order.

Problem 2

Sort both vectors, one in ascending and one in descending order, $O(n \log n)$. Then do the dot product of the two vectors, $O(n)$. Total: $O(n \log n)$

Problem 3

Use Union find / disjoint sets data structure

(a) Initialize each guest to a separate table

(b) For each pair of friends, union their tables together

(c) For each pair of enemies, find their tables and make sure they are different

Problem 4

(a) $O(n)$

(b) $O(|F| \log n)$

(c) $O(|E| \log n)$

Since $|E| + |F| = m$, total: $O(n + m \log n)$