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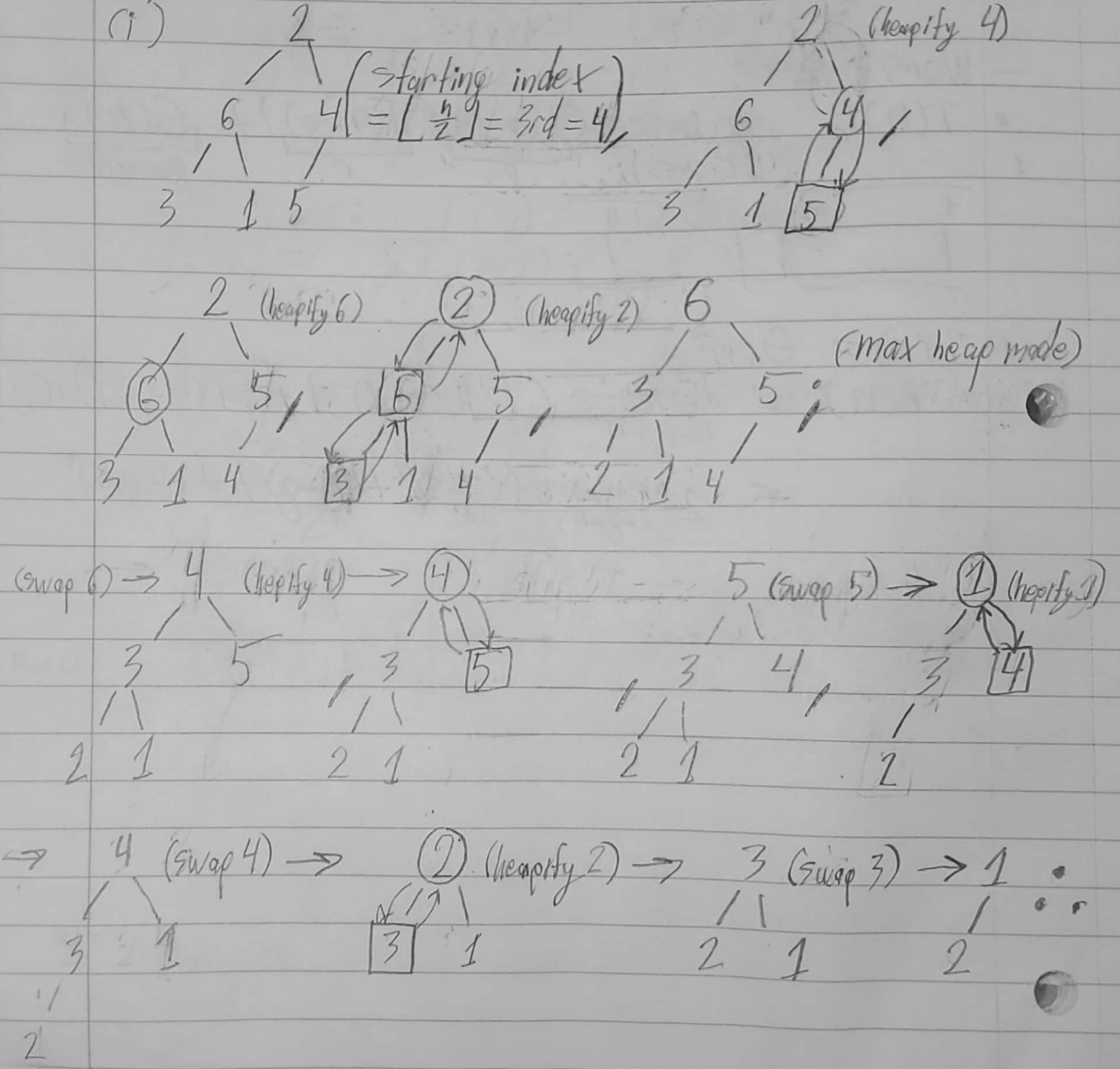
Homework 2

1)

(i)



Homework 2

1

(ii) $p = pivot = 0, i = p-1 = -1, j = p = 0, x = A[p]$

	i	j p									
(start)		1	5	2	3	0	2	2	1	4	5
		p,i,j									
(A[j]≤x)		1	5	2	3	0	2	2	1	4	5
		p	i++			j					
(j+=4;A[j]≤x)		1	5	2	3	0	2	2	1	4	5
		p	i			j					
(swap)		1	0	2	3	5	2	2	1	4	5
		p		i++					j		
(j+=3;A[j]≤x)		1	0	2	3	5	2	2	1	4	5
		p		i					j		
(swap)		1	0	1	3	1	2	2	2	4	5
		p		i							j
(j+=2;end for)		1	0	1	3	1	2	2	2	4	5
		p		i							j
(swap)		1	0	1	3	1	2	2	2	4	5

	i	p j	
(left)		1	0
		p,i,j	
(A[j]≤x)		1	0

Homework

(j++; A[j]≤x)

p	i++, j
1	0

(end for)

p	i, j
1	0

(swap)

0	1

(min size of 2)

(right)

i	p, j						
	3	1	2	2	2	4	5

(A[j]≤x)

p, i++, j						
3	1	2	2	2	4	5

(j++, A[j]≤x)

p	i++, j					
3	1	2	2	2	4	5

(j++, A[j]≤x)·4

p				i++, j		
3	1	2	2	2	4	5

(j+=2, end for)

p				i		j
3	1	2	2	2	4	5

(swap)

p				i		j
2	1	2	2	3	4	5

↓

(l right)

i	p, j			
	2	1	2	2

(A[j]≤x)

p, i++, j			
2	1	2	2

(j++, A[j]≤x)

p	i++, j		
2	1	2	2

(j+=2, end for)

p	i		j
②	①	2	2

(swap)

p	i		j
①	2	②	②

↓

(r of l of right)

p, i++, j	
②	2

,

p, i	j
②	2

,

p, i	j
2	2

;

(r of right)

i

p, j	
4	5

(A[j] ≤ x)

p, i++, j	
④	5

(j++, end for)

p, i	j
④	5

,

p, i	j
4	5

.

(combined)

0	1	1	2	2	2	3	4	5	5

∴

Homework 2

3) n – range of vals, m – size of value array

(1)

C_n	Partition(A, low, high):	T(n)
C_6	initial_low = A[low]	1
C_7	initial_high = A[high]	1
C_8	l_pivot = A[low]	1
C_9	h_pivot = A[high]	1
C_{10}	if ~~l_pivot > h_pivot~~	1
C_{11}	~~swap A[low] with [high]~~	1
C_{12}	~~l_pivot = A[low]~~	1
C_{13}	~~h_pivot = A[high]~~	1
C_{14}	low = low + 1	1
C_{15}	high = high + 1	1
C_{16}	current = low	1
C_{17}	while current <= high (<= low)	n + (high – current)
C_{18}	if A[current] <= l_pivot	1
C_{19}	swap A[current] with A[low]	1.
C_{20}	low += 1	1
C_{21}	current += 1	1

Homework 2

3)

```
C_n   | :
C_22  | if A[current] >= h_pivot (>=high)             1
C_23  |     swap A[high] with A[current]              1
C_24  |     high -= 1                                 1
C_25  | else (>low, <high)                            1
C_26  |     current += 1                              1
      | (end while)

C_27  | low = low - 1                                 1
C_28  | high = high - 1                               1

C_29  | swap A[initial_low] with A[low]               1
C_30  | swap A[initial_high] with A[high]             1
C_31  | return (low, high)                            1
```

(2)

```
                                                                T(n)
C_n  | Quicksort(Arr, low, high):                                1
C_1  |     if low < high:                                        f(n)
C_2  |         p_low, p_high = partition(A, low, high)
C_3  | (<=low) Quicksort(A, low, p_low - 1)
C_4  | (>low, <high) Quicksort(A, p_low+1, p_high-1)
C_5  | (>=high) Quicksort(A, p_high+1, high)
```

$T(p\_low-1)$

$T(p\_high - p\_low)$

$T(n - p\_high+1)$

Homework 2

3) (3)

```
# assume every element has a duplicate
def dual_pivot_duplicate (arr):                          | T(n)
C32  | n = len(arr)                                       | 1
C33  | for i = 1 to n                                     | n-1
C34  | for j = i+1 to n                                   | n-2
C35  |       if arr[i] == arr[j]                          | 1
C36  |           swap arr[j] w/ arr[n 1]                  | 1
C37  |           break                                    | 1
     |
C38  | call Quicksort (arr, low, high)                    | Q(n)
```

$$T(n) = (C_{32} + C_{37} + C_{35} + C_{36})(1) + C_{33}(n-1)$$
$$C_{34}(n-2) + C_{38}\,Q(n)$$

$$= \Theta(1) + \Theta(n)^2 + Q(n)$$
$$= \Theta(n^2) + Q(n),$$

$$Q(n) = C_1(1) + C_2\,f(n) + C_3\,T(g\_low - 1)$$
$$+ C_4\,T(g\_high - g\_low) + C_5\,T(n - g\_high + 1)$$

$$= \Theta(1) + \Theta(f(n)) + T(g\_low - 1)$$
$$+ T(g\_high - g\_low) + T(n - g\_high + 1);$$

Homework 2

3)

(4)

$$f(n) = (C_6 \Rightarrow C_{16} + C_{18} \Rightarrow C_{31})(1) + C_{17}(n)$$

$$= \Theta(1) + \Theta(n)$$

$$= \Theta(n),$$

(sub)

$$Q(n) = \Theta(1) + \Theta(\Theta(n)) + T(g\_low-1) + T(g\_high-g\_low) + T(n-g\_high+1)$$

$$= \Theta(n) + T(g\_low-1) + T(g\_high-g\_low) + T(n-g\_high+1);$$

(sub)

$$T(n) = \Theta(n) + \Theta(\Theta(n) + T(g\_low-1) + T(g\_high-g\_low) + T(n-g\_high+1))$$

$$= \Theta(n) + T(g\_low-1) + T(g\_high-g\_low) + T(n-g\_high+1);$$

~~Best : $g\_high = n$, $g\_low = 1$~~ (all are same #)

~~$T(n) = \Theta(n) + T(1-1) + T(n-1) + T(n-n+1)$~~

~~$= \Theta(n) + T(n-1) + 0$~~

— Worst Case:

$p1 \cong p2 \rightarrow p$, only $A[i] == A[j]$ when $i = n-1, j = n$

$$T(n) = \text{maximize } (T(p-1) + T(p-p) + T(n-p) + \Theta(n) + \Theta(ij))$$

$$= \text{maximize } [T(p-1) + T(n-p) + \Theta(n^2)]$$

- Worst case partitioning:

$$T(n) = T(n-2) + T(0) + T(0) + \Theta(n^2)$$
$$= T(n-2) + \Theta(n^2)$$

- $T(n) = T(n-2) + cn^2$

- $T(n-2)$, n must be $2k$ (even)

- $T(2k) = T(2k-2) + 2k$, $T(2k) = T(2(k-1)) + 2k$

- $T'(k) = T'(k-1) + (2k)^2 = \frac{k(k+1)}{2} + (2k)^2$

- $T(2k) = 2 \cdot \frac{k(k+1)}{2} + (2k)^2$

- $T(n) = \frac{n}{2}\left(\frac{n}{2} + 1\right) + n^2$

- $T(n) = \Theta(n^2)$;

Homework 2

- substitute $\Theta(n^2)$

- $T(n) = \text{maximize}\,[\,\Theta(p-1)^2 + \Theta(n-p)^2\,] + \Theta(n)$
  $\leq c \cdot \text{max}\,((p-1)^2 + (n-p)^2) + \Theta(n^2)$

- $(p-1)^2 + (n-p)^2 \leq (n-2)^2$

- $f(\varphi) = (p-1)^2 + (n-\varphi)^2$

- $f'(p) = 2(\varphi-1) - 2(n-\varphi)$
  $= 2p - 2 - 2\varphi + 2\varphi$
  $= 4\varphi - 2n - 2$

- $f'(\varphi) = 4,\ p = n-1;$

- $T(n) \geq \text{max}(\ c_1 (q-1)^2 + c_1 (n-n)^2) + \Theta(n),$

- $T(an^2 + bn + c) \geq \text{max}\ c_1 (an^2 + bn + c) + \Theta(n)$
  $= \Omega(n^2),$

- $\Omega(n^2) \leq T(n) \leq O(n^2),\ \boxed{T(n) = \Theta(n^2)};$

Homework 2

3)

— Best Case: (unnecessary)

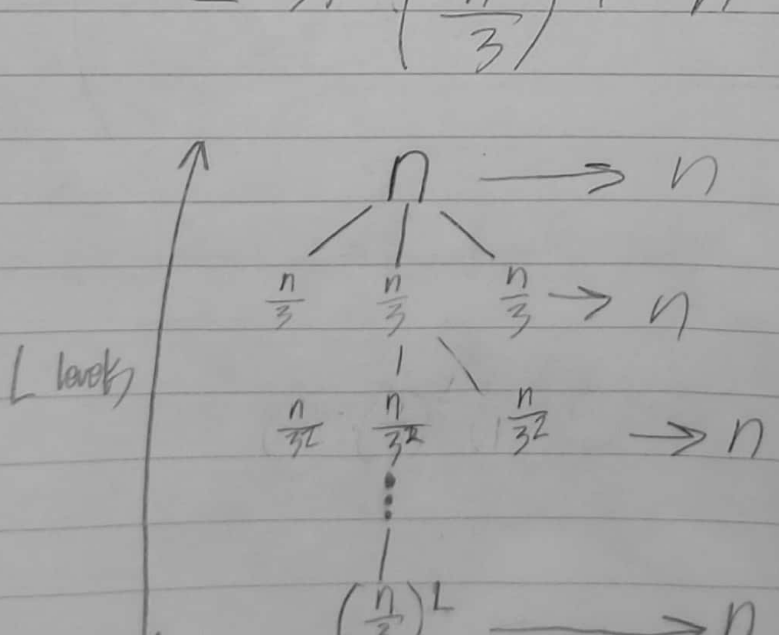
— $A[1] =\neq A[2]$, all 3 partitions have same size (partition $A[1:p-1]$ length $= A[p_2+1:n]$ length $= A[p_1, p_2]$ length

- $T(n) = 3T\left(\frac{n-2}{3}\right) + \Theta(n) + \Theta(1 \cdot (2-1))$

$$= 3T\left(\frac{n-2}{3}\right) + \Theta(n)$$

$$= 3T\left(\frac{n-2}{3}\right) + \Theta(n)$$

$$= 3T\left(\frac{n}{3}\right) + n$$



$n \rightarrow n$
$\frac{n}{3}$ $\frac{n}{3}$ $\frac{n}{3} \rightarrow n$
L levels
$\frac{n}{3^2}$ $\frac{n}{3^2}$ $\frac{n}{3^2} \rightarrow n$
$\left(\frac{n}{3}\right)^L \rightarrow n$

Homework 2

- $\frac{n}{3^i} = 1, \quad n = 3^i, \quad L = \log_3 n = \frac{\log n}{\log 3}$

- $T(n) = \sum_{i=0}^{\log_3 n} n = \frac{n \log n}{\log 3},$

$\boxed{T(n) = \Theta(n \log n);}$