Missouri University of Science & Technology — Department of Computer Science
Spring 2023 — **CS 2500: Algorithms**

Homework 1a: Foundations (Analysis)

Instructor: *Sid Nadendla* — **Due:** *February 3, 2023*

Problem 1 Asymptotic Analysis *30 points*

1. Prove the following statements:

   (i) For any $0 < x < 1$, $e^x = \sum_{i=0}^{m-1} \frac{x^i}{i!} + \Theta(x^m)$, for all $m = 1, 2, \cdots$. **(10pts)**

2. Solve the following problems in *CLRS*.

   (a) Problems 3.1(a,b,c) (Page 71 in 4th edition). **(9pts)**

   (b) Problems 4.1(g) (Page 119 in 4th edition) and 4.4(j) (Page 121 in 4th edition) using recursion trees method, and use Master's theorem to verify your result. State your base case clearly and articulate your solution. **(6pts)**

   (c) Problems 4.1(h) (Page 119 in 4th edition) and 4.4(h) (Page 121 in 4th edition) using substitution method. State your base case clearly and articulate your solution. **(5pts)**

Problem 2 Validation: Theory and Experiments *20 points*

In the class, we studied different kinds of algorithms. This problem is about the theoretical analysis of the algorithms presented in the class and their validation using simulation experiments.

1. Compute the average running time of *Insertion Sort*, *Merge Sort* and the *maximum element* algorithms formally. **(10pts)**

2. Implement *Insertion Sort*, *Merge Sort* and the *maximum element* algorithms in Python, validate their correctness and simulate their average run-time performance empirically, for different input sizes. **(10pts)**
   (Hint: Random inputs can be generated using `random.sample()` function.)