HW #1

2)

(d)

(a) If $k \geq d$, then $p(n) = O(n^k)$

- $k \geq d$, $n^k \geq n^d \quad \forall n \geq 1$

- $p(n) = \sum_{i=0}^{d} a_i n^i = a_0 + a_1 n + \dots + a_n n^d$

$\leq |a_0| \underbrace{n^k}_{>1} + |a_1| \underbrace{n^k}_{>n} + \dots + |a_d| \underbrace{n^k}_{\geq n^d} = \sum_{i=0}^{d} |a_i| n^k = k_0 n^k$,

$\forall n \geq 1$: $p(n) \leq k_0 n^k$, $k_0 = \sum_{i=0}^{d} |a_i| \rightarrow p(n) = O(n^k)$

(b) If $k \leq d$, then $p(n) = \Omega(n^k)$

- $k \leq d$, $\exists\, c, n_0$ s.t. $n_0 \geq 0$, $0 \leq c \cdot n^k \leq p(n)$ for $n \geq n_0$

- $p(n) = \sum_{i=0}^{d} a_i n_i = a_0 + a_1 n + \dots + a_n n^d \geq a_k n^k$,

- so, $0 \leq c \cdot n^k \leq p(n)$, $c = a_k$, $n^0 = 1$

- Thus, $p(n) = \Omega(n^k)$

(c) $k = d$, $p(n) = \Theta(n^k)$

- $k = d$, $k \geq d$, $p(n) = O(n^k)$
- $k = d$, $k \leq d$, $p(n) = \Omega(n^k)$