L. Bowman
Due Tuesday July 10, 2018

Math 314 Homework 2

Name:________________________

Written Problems

Instructions. (342 points) After reading the sections and completing the MyMathLab assignments, write out the solutions to the following problems (or if you prefer, you can use LaTex to type your solutions). This assignment covers sections 3.1-4.6 of the text. Work should be shown to support your answers and to receive partial credit. Please review the written assignment requirements, and ask questions about problems before submitting your work.

(6^{pts}) **1.** Compute the determinant of each of the following using a cofactor expansion across the first row.

(a) (2 pts) $\begin{vmatrix} 1 & 3 & 5 \\ 2 & 1 & 1 \\ 3 & 4 & 2 \end{vmatrix}$

(b) (2 pts) $\begin{vmatrix} 5 & -2 & 4 \\ 0 & 3 & -5 \\ 2 & -4 & 7 \end{vmatrix}$

(3^{pts}) **2.** Compute the determinant of the matrix using a cofactor expansion by choosing a row or column that will require the least amount of computations.

$$\begin{vmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -6 & -7 & 5 \\ 5 & 0 & 4 & 4 \end{vmatrix}$$

(3^{pts}) **3.** Compute the determinant of the matrix using the diagonalization method.

$$\begin{vmatrix} 1 & 3 & 5 \\ 2 & 1 & 1 \\ 3 & 4 & 2 \end{vmatrix}$$

(3^{pts}) **4.** Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. State the row operation needed to obtain the matrix $A' = \begin{bmatrix} a+kc & b+kd \\ c & d \end{bmatrix}$. Describe how this operation affects the determinant.

(3^{pts}) **5.** Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $E = \begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix}$. Verify that $detEA = (detE)(detA)$.

(3^{pts}) **6.** Let $A = \begin{bmatrix} 1 & 5 & -3 \\ 3 & -3 & 3 \\ 2 & 13 & -7 \end{bmatrix}$. Find the determinant by row reduction to echelon form.

(3^{pts}) **7.** Let $A = \begin{bmatrix} -1 & 2 & 3 & 0 \\ 3 & 4 & 3 & 0 \\ 5 & 4 & 6 & 6 \\ 4 & 2 & 4 & 3 \end{bmatrix}$. Combine the methods of row reduction and cofactor expansion to compute the determinant of A.

(3pts) **8.** Let $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7$. Evaluate $\begin{vmatrix} g & h & i \\ a & b & c \\ d & e & f \end{vmatrix}$

(3pts) **9.** Use determinants to determine if the vectors are linearly independent.
$\begin{bmatrix} 4 \\ 6 \\ -7 \end{bmatrix}, \begin{bmatrix} -7 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ -5 \\ 6 \end{bmatrix}$

(15pts) **10.** Let A and B be 4×4 matrices with $detA = -1$ and $detB = 2$. Compute the following:

(a) (2 pts) $detAB$

(b) (2 pts) $detB^5$

(c) (2 pts) $det2A$

(d) (2 pts) $detA^TA$

(e) (2 pts) $detB^{-1}AB$

(6pts) **11.** Use Cramer's Rule to find the solutions of the systems:

(a) (2 pts) $\begin{aligned} 4x_1 + x_2 &= 6 \\ 5x_1 + 2x_2 &= 7 \end{aligned}$

(b) (2 pts) $\begin{aligned} 2x_1 + x_2 + x_3 &= 4 \\ -x_1 + 2x_3 &= 2 \\ 3x_1 + x_2 + 3x_3 &= -2 \end{aligned}$

(4pts) **12.** Compute the adjugate of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 2 & -2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. Then find the inverse of the matrix.

(3pts) **13.** Find the area of the parallelogram whose vertices are (0,0), (-1,3), (4,-5), and (3,-2).

(3pts) **14.** Let $T : \mathbb{R}^m \to \mathbb{R}^n$ be a linear transformation, let $\mathbf{p}$ be a vector, and S a set in $\mathbb{R}^m$. Show that the image of $\mathbf{p} + S$ under T is the translated set $T(\mathbf{p}) + T(S)$ in $\mathbb{R}^n$.

(3pts) **15.** Let S be the parallelogram determined by
$\mathbf{b}_1 = \begin{bmatrix} 4 \\ -7 \end{bmatrix}$, and $\mathbf{b}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
Let $A = \begin{bmatrix} 7 & 2 \\ 1 & 1 \end{bmatrix}$. Compute the area of the image of S under the mapping $\mathbf{x} \mapsto A\mathbf{x}$.

(9pts) **16.** Let W be the union of the first and third quadrants in the xy-plane. That is, let $W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : xy \geq 0 \right\}$.

(a) (2 pts) If $\mathbf{u}$ is in W and c is any scalar, is $c\mathbf{u}$ in W? Why?

(b) (3 pts) Find specific vectors $\mathbf{u}$ and $\mathbf{v}$ in W such that $\mathbf{u} + \mathbf{v}$ is not in W.

(c) (2 pts) Explain why your solutions to parts a and b are enough to show that W is *not* a vector space.

(3pts) **17.** Determine if $\{\mathbf{p} : \mathbf{p} \in \mathcal{P}_n \quad and \quad \mathbf{p}(0) = 0\}$ is a subspace for an appropriate value of n.

(3pts) **18.** Let H be the set of all vectors of the form $\begin{bmatrix} 3t \\ 0 \\ -7t \end{bmatrix}$, where t is any real number. Show that H is a subspace of $\mathbb{R}^3$

(3pts) **19.** Let W be the set of all vectors of the form $\begin{bmatrix} 2x + 4t \\ 2s \\ 2s - 3t \\ 5t \end{bmatrix}$, where s, t are any real numbers. Show that W is a subspace of $\mathbb{R}^4$

(3pts) **20.** Let W be the set of all vectors of the form $\begin{bmatrix} 4a+3b \\ 0 \\ a+3b+c \\ 3b-2c \end{bmatrix}$, where a, b, and c are any real numbers. Find a set S of vectors that spans W or give an example to show that W is nota a vector space.

(3pts) **21.** Let F be a fixed 3×2 matrix, and let H be the set of all matrices A in $M_{2\times 4}$ with the property that $FA = 0$. Determine if H is a subspace of $M_{2\times 4}$.

(3pts) **22.** Let $A = \begin{bmatrix} 1 & 3 & -4 & -3 & 1 \\ 0 & 1 & -3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
Find an explicit description of $NulA$, by listing vectors that span the null space.

(4pts) **23.** Given $W = \left\{ \begin{bmatrix} r \\ s \\ t \end{bmatrix} : 3r - 2 = 3s + t \right\}$
Show that W is a vector space or give a specific example that shows it is not.

(6pts) **24.** Given $A = \begin{bmatrix} 1 & -3 & 2 & 0 & -5 \end{bmatrix}$

(a) (2 pts) Find the value of k such that $NulA$ is a subspace of $\mathbb{R}^k$.

(b) (2 pts) Find the value of k such that $ColA$ is a subspace of $\mathbb{R}^k$.

(3pts) **25.** Let $A = \begin{bmatrix} 5 & -2 & 3 \\ -1 & 0 & -1 \\ 0 & -2 & -2 \\ -5 & 7 & 2 \end{bmatrix}$. Find a nonzero vector in $NulA$ and a nonzero vector in $ColA$.

(6pts) **26.** Let $A = \begin{bmatrix} 10 & -8 & -2 & -2 \\ 0 & 2 & 2 & -2 \\ 1 & -1 & 6 & 0 \\ 1 & 1 & 0 & -2 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 2 \\ 2 \\ 0 \\ 2 \end{bmatrix}$.

(a) (2 pts) Determine if $\mathbf{w}$ is in $ColA$.

(b) (2 pts) Determine if $\mathbf{w}$ is in $NulA$.

(4pts) **27.** Define a linear transformation $T : \mathcal{P}_2 \mapsto \mathbb{R}^2$ by $T(\mathbf{p}) = \begin{bmatrix} \mathbf{p}(0) \\ \mathbf{p}(0) \end{bmatrix}$. Find polynomials $\mathbf{p}_1$ and $\mathbf{p}_2$ in $\mathcal{P}_2$ that span the kernel of T and describe the range of T.

(4pts) **28.** Determine whether the set $\left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} -8 \\ 5 \\ 4 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^3$. If the set is not a basis, determine if the set is linearly independent or if the set spans $\mathbb{R}^3$

(3pts) **29.** Find a basis for the set of vectors in $\mathbb{R}^2$ on the line $y = -3x$.

(10pts) **30.** For each of the following determine whether the statement is true or false.

(a) (2 pts) ______ A linearly independent set in a subspace H is a basis for H.

(b) (2 pts) ______ If a finite set S of nonzero vectors spans a vector space V, then some subset of S is a basis for V.

(c) (2 pts) ______ A basis is a linearly independent set that is as large as possible.

(d) (2 pts) ______ The standard method for producing a spanning set for $NulA$, sometimes fails to produce a basis for $NulA$.

(e) (2 pts) ______ If B is an echelon form of a matrix A, then the pivot columns of B for a basis for $ColA$.

(3^{pts}) **31.** In the vector space of all real-valuedfunctions, find a basis for the subspace spanned by $\{\sin t, \sin 2t, \sin t \cos t\}$.

(5^{pts}) **32.** Suppose that T is a one-to-one transformation, so that an equation $T(\mathbf{u}) = T(\mathbf{v})$ always implies $\mathbf{u} = \mathbf{v}$. Show that if the set of images $\{T(\mathbf{v}_1), ..., T(\mathbf{v}_p)\}$ is linearly dependent, then $\{\mathbf{v}_1, ..., \mathbf{v}_p\}$ is linearly dependent.

(5^{pts}) **33.** Find the vector $\mathbf{x}$, detemined by $\left[\mathbf{x}\right]_B = \begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix}$ and the basis $B = \left\{ \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix} \right\}$.

(4^{pts}) **34.** Find the coordinate vector $\left[\mathbf{x}\right]_B$ of $\mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$ relative to the basis $B = \left\{ \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} \right\}$.

(3^{pts}) **35.** Find the change of coordinate matrix from $B = \left\{ \begin{bmatrix} 3 \\ 0 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} \right\}$ to the standard basis in $\mathbb{R}^n$.

(3^{pts}) **36.** Use coordinate vectors to test the linear independence of the set $\{1 - 2t^2 - t^3, t + 2t^3, 1 + t - 2t^2\}$

(6^{pts}) **37.** Let $\mathbf{p}_1(t) = 1 + t^2, \mathbf{p}_2(t) = t - 3t^2, \mathbf{p}_3(t) = 1 + t - 3t^2$.

(a) (3 pts) Use coordinate vectors to show that these polynomials form a basis for $\mathcal{P}_2$.

(b) (3 pts) Consider the basis $B = \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$ for $\mathcal{P}_2$. Find q in $\mathcal{P}_2$, given that $\left[\mathbf{q}\right]_B = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$

(6^{pts}) **38.** For the subspace $\left\{ \begin{bmatrix} 3a - c \\ -b - 3c \\ -7a + 6b + 5c \\ -3a + c \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$

(a) (3 pts) Find a basis for the subspace.

(b) (3 pts) State the dimension of the subspace.

(6^{pts}) **39.** Find the dimension of the subspace H of $\mathbb{R}^2$ spanned by $\begin{bmatrix} 1 \\ -5 \end{bmatrix}, \begin{bmatrix} -2 \\ 10 \end{bmatrix}, \begin{bmatrix} -3 \\ 15 \end{bmatrix}$.

(4^{pts}) **40.** Given $A = \begin{bmatrix} 1 & 2 & -4 & 3 & -2 & 6 & 0 \\ 0 & 0 & 0 & 1 & 0 & -3 & 7 \\ 0 & 0 & 0 & 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

(a) (2 pts) Find the dimensions of $NulA$

(b) (2 pts) Find the dimensions of $ColA$

(6^{pts}) **41.** The first four Laguerre p;ynomials are $1, 1 - t, 2 - 4t + t^2$, and $6 - 18t + 9t^2 - t^3$. Show that these polynomials form a basis for $\mathcal{P}_3$.

(16^{pts}) **42.** Given the matrix $A = \begin{bmatrix} 1 & 3 & 4 & -1 & 2 \\ 2 & 6 & 6 & 0 & -3 \\ 3 & 9 & 3 & 6 & -3 \\ 3 & 9 & 0 & 9 & 0 \end{bmatrix}$ which is row equivalent to $B = \begin{bmatrix} 1 & 3 & 4 & -1 & 2 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(a) (2 pts) Find the $rankA$

(b) (2 pts) Find the $dimNulA$

(c) (4 pts) Find the basis for $ColA$

(d) (4 pts) Find the basis for $RowA$

(e) (4 pts) Find the basis for $NulA$

(4^{pts}) **43.** Suppose a 6×8 matrix A has four pivot columns. What is $dimNulA$? Is $ColA = \mathbb{R}^4$? Why or why not?

(10^{pts}) **44.** For each of the following determine whether the statement is true or false.

(a) (2 pts) ______ If B is any echelon form of A, then the pivot columns of B form a basis for the column space of A.

(b) (2 pts) ______ Row operations preserve the linear dependence relations among the rows of A.

(c) (2 pts) ______ The dimension of the null space of A is the number of columns of A that are not pivot volumns.

(d) (2 pts) ______ The row space of A^T is the same as the column space of A.

(e) (2 pts) ______ If A and B are row equivalent, then their row spaces are the same.

(6^{pts}) **45.** Let $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Find $\mathbf{v}$ in $\mathbb{R}^3$ such that $\begin{bmatrix} 1 & -3 & 4 \\ 2 & -6 & 8 \end{bmatrix} = \mathbf{u}\mathbf{v}^T$.

MatLab Problems

For this part of the assignment, you will need to use Matlab to complete these problems. This assignment covers sections 3.1-4.6 of the text. Work should be shown to support your answers and to receive partial credit. You should attach the printout of your solutions to the written assignment and copies of your MatLab files should be uploaded into Canvas.

(6^{pts}) **46.** Is it true that $detAB = (detA)(detB)$? Experiment with four pairs of random matrices and make a conjecture.

(6^{pts}) **47.** How is $detA^{-1}$ related to $detA$? Experiment with random $n \times n$ integer matrices for $n = 4, 5,$ and 6, and make a conjecture. If you get a matrix with a zero determinant, reduce it to echelon form and discuss what you find.

(6^{pts}) **48.** Compute $detA^TA$ and $detAA^T$ for several random 4×5 matrices and several random 5×6 matrices. What can you say about A^TA and AA^T when A has more columns than rows?

(6^{pts}) **49.** Test the inverse formula of Theorem 8 for a random 4×4 matrix A. Use MatLab to compute the cofactors of the 3×3 submatrices, construct the adjugate, and set $B = \dfrac{adj A}{detA}$. Then compute $B - invA$, where $invA$ is the inverse of matrix A as computed by MatLab. use floating point arithmetic with the maximum number of decimal places and report your results.

(6^{pts}) **50.** Test Cramer's rule for a random 4×4 matrix A and a random 4×1 vector $\mathbf{b}$. Compute each entry in the soution of $A\mathbf{x} = \mathbf{b}$, and compare these entries with the entries of $A^{-1}\mathbf{b}$. Write the commands for your program that uses Cramer's rule to prduce the second entry of $\mathbf{x}$.

(5^{pts}) **51.** Show that $\mathbf{w}$ is in the subspace of $\mathbb{R}^4$ spanned by $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, where

$$\mathbf{w} = \begin{bmatrix} 9 \\ -4 \\ -4 \\ 7 \end{bmatrix}, \mathbf{v}_1 = \begin{bmatrix} 8 \\ -4 \\ -3 \\ 9 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -4 \\ 3 \\ -2 \\ -8 \end{bmatrix}, \text{ and } \mathbf{v}_3 = \begin{bmatrix} -7 \\ 6 \\ -5 \\ -18 \end{bmatrix}$$

(5^{pts}) **52.** Determine if $\mathbf{y}$ is in the subspace of $\mathbb{R}^4$ spanned by the columns of A, where

$$\mathbf{y} = \begin{bmatrix} -4 \\ -8 \\ 6 \\ -5 \end{bmatrix} \text{ and } A = \begin{bmatrix} 3 & -5 & -9 \\ 8 & 7 & -6 \\ -5 & -8 & 3 \\ 2 & -2 & -9 \end{bmatrix}$$

(5^{pts}) **53.** Detemine whether $\mathbf{w}$ is in the column space of A, the null space of A, or both, where $\mathbf{w} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}$,

$$A = \begin{bmatrix} -8 & 5 & -2 & 0 \\ -5 & 2 & 1 & -2 \\ 10 & -8 & 6 & -3 \\ 3 & -2 & 1 & 0 \end{bmatrix}$$

(4^{pts}) **54.** Find a basis for the space spanned by the vectors $\begin{bmatrix} -3 \\ 2 \\ 6 \\ 0 \\ -7 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -9 \\ 0 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -4 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 6 \\ -2 \\ -14 \\ 0 \\ 13 \end{bmatrix}, \begin{bmatrix} -6 \\ 3 \\ 0 \\ -1 \\ 0 \end{bmatrix}$

(9^{pts}) **55.** Let $H = Span \mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$ and $K = Span \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ where $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} 0 \\ 2 \\ -1 \\ 1 \end{bmatrix}, \mathbf{u}_3 = \begin{bmatrix} 3 \\ 4 \\ 1 \\ -4 \end{bmatrix}$

$\mathbf{v}_1 = \begin{bmatrix} -2 \\ -2 \\ -1 \\ 3 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ 3 \\ 2 \\ -6 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} -1 \\ 4 \\ 6 \\ -2 \end{bmatrix}$

(a) (3 pts) Find a basis for H.

(b) (3 pts) Find a basis for K.

(c) (3 pts) Find a basis for $H + K$

(4^{pts}) **56.** Show that $1, \cos t, \cos^2 t, ..., \cos^6 t$ is a linearly independent set of functions defined on $\mathbb{R}$

(0^{pts}) **57.** Let $H = Span \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ and $B = \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$. Show that B is a basis for H and $\mathbf{x}$ is in H, and find the B-coordinate vector of $\mathbf{x}$, for $\mathbf{v}_1 = \begin{bmatrix} -6 \\ 4 \\ -9 \\ 4 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 8 \\ -3 \\ 7 \\ -3 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} -9 \\ 5 \\ -8 \\ 3 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 4 \\ 7 \\ -8 \\ 3 \end{bmatrix}$

(9^{pts}) **58.** Let $B = 1, \cos t, \cos^2 t, ..., \cos^6 t$ and $C = 1, \cos t, \cos 2t, ..., \cos 6t$. Assume the following trigonometric identities.
$\cos 2t = -1 + 2\cos^2 t$
$\cos 3t = -3\cos t + 4\cos^3 t$
$\cos 4t = 1 - 8\cos^2 t + 8\cos^4 t$
$\cos 5t = 5\cos t - 20\cos^3 t + 16\cos^5 t$
$\cos 6t = -1 + 18\cos^2 t - 48\cos^4 t + 32\cos^6 t$
Let H be thesubspaceof functions spanned by the functions in B. Then B is a basis for H.

(a) (6 pts) Write the B-coordinate vectors of the vectors in C, and use them to show that C is a linearly independent set in H.

(b) (3 pts) Explain why C is a basis H.

(22^{pts}) **59.** Let $A = \begin{bmatrix} 7 & -9 & -4 & 5 & 3 & -3 & -7 \\ -4 & 6 & 7 & -2 & -6 & -5 & 5 \\ 5 & -7 & -6 & 5 & -6 & 2 & 8 \\ -3 & 5 & 8 & -1 & -7 & -4 & 8 \\ 6 & -8 & -5 & 4 & 4 & 9 & 3 \end{bmatrix}$

(a) (6 pts) Construct matrices C and N whose columns are bases for $Col A$ and $Nul A$ respectively.

(b) (3 pts) Construct a matrix R whose rows from a basis for $Row A$.

(c) (3 pts) Construct a matrix M whose columns from a basis for $Nul A^T$.

(d) (4 pts) Form the matrices $S = \begin{bmatrix} R^T & N \end{bmatrix}$ and $T = \begin{bmatrix} C & M \end{bmatrix}$.

(e) (3 pts) Explain why S and T should be square.

(f) (3 pts) Verify that both S and T are invertible.