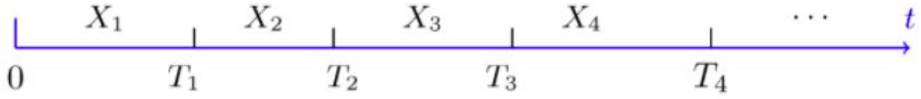
Multivariate Hawkes Processes

Modeling Market Dynamics

Aric Cutuli
Mentored by Xia Li

Defining a Point Process



- Intensity representation
  - The rate of a Poisson process
- Duration representation
  - Interarrival times of a Poisson process

$$X_i \sim Exponential(\lambda)$$

- Counting representation
  - Number of arrivals in an interval of a Poisson process

$$N(t+s) - N(t) \sim Poisson(\lambda s)$$

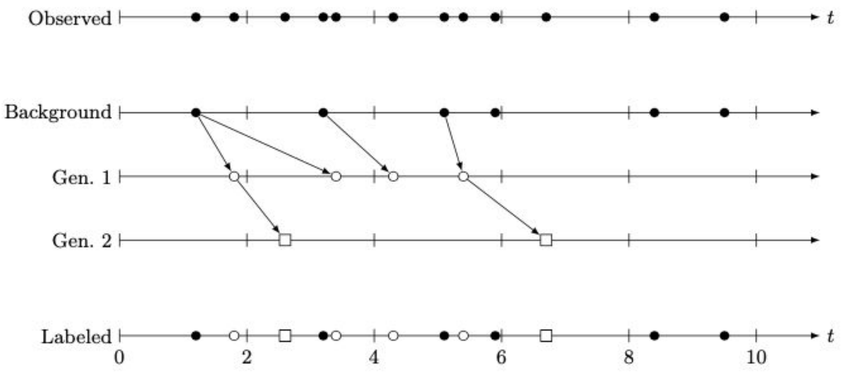
High-Frequency Econometrics

- Oddities of financial transaction data
  - Irregular spacing of events
  - Discrete price jumps
- Fitting a point process to transaction data is thus not so simple
  - Models need to be “dynamic”
- Dynamic duration models are the most widely used in high-frequency econometrics
  - Engle and Russel’s ACD model
- What about dynamic intensity models?

Univariate Hawkes Process

- Simplest form of the self-exciting temporal point process
- Whenever a point event occurs, the process intensity temporarily increases
- Current intensity depends on past events
- Conditional intensity function of the form:

$$\lambda(t|\mathcal{H}_t) = \underbrace{\mu}_{\text{Constant arrival rate of “immigrant” events}} + \underbrace{\int_0^t \omega(t-s)dN(s)}_{\text{Variable self-exciting term: determines the arrival rate of “offspring” events}}$$



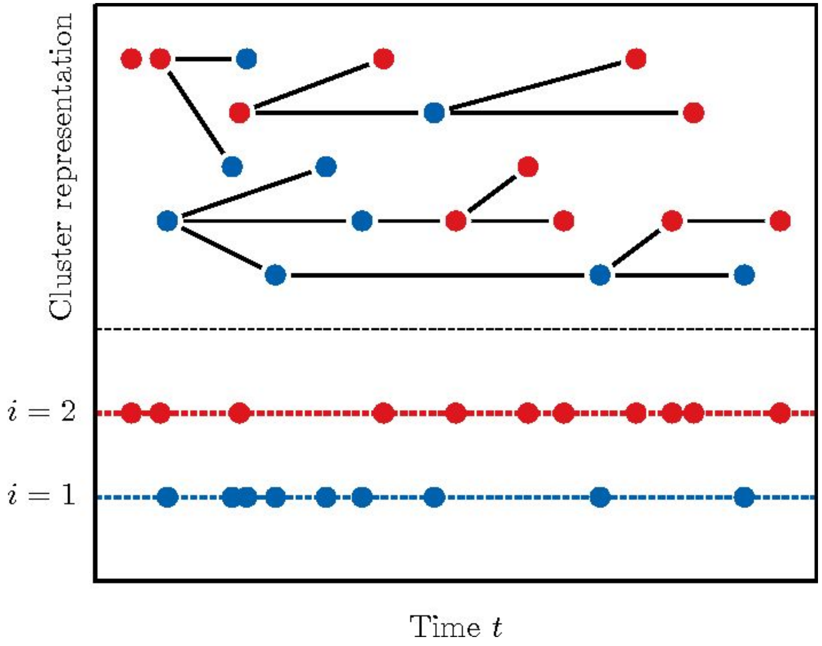
Extending to the Multivariate Case

- Multivariate means the process consists of different types of events
- Accounts for the interactions between different event types
- Conditional intensity function for a p-variate process:

$$\lambda_i(t|\mathcal{H}_t) = \mu_i + \sum_{j=1}^{p} \int_0^t \omega_{i,j}(t-s)dN_j(s)$$

- Each component has its own background rate and vector of triggering functions
    - Each vector element is a function illustrating intra-component excitation

Clustering Property

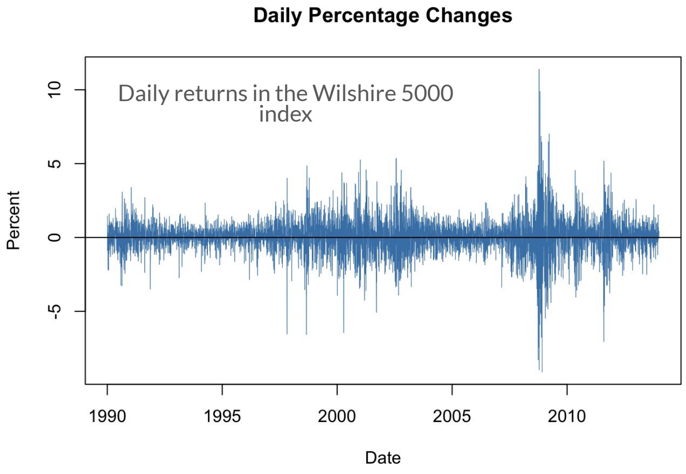


Modeling the Volatility Clustering Phenomenon

- Large changes in price tend to cluster together
- The self-exciting nature of Hawkes processes provides a simple way to illustrate this
- Hawkes models calibrated to high-frequency financial data tend to best be characterized by a slowly decaying power-law function

$$\omega(t) = \alpha(t+\gamma)^{\beta}$$

- Shown to be efficacious in low-frequency modeling as well



Measuring Market Reflexivity

- Price moves cannot be completely explained by the introduction of new information
- Market reflexivity is the measure of how much change is due to endogenous feedback
- Suppose exogenous events are the immigrants in a Hawkes model
    - The decay function drives the amplitude of endogenous feedback
- Market reflexivity can be measured as the fraction of endogenous events within the population of price changes

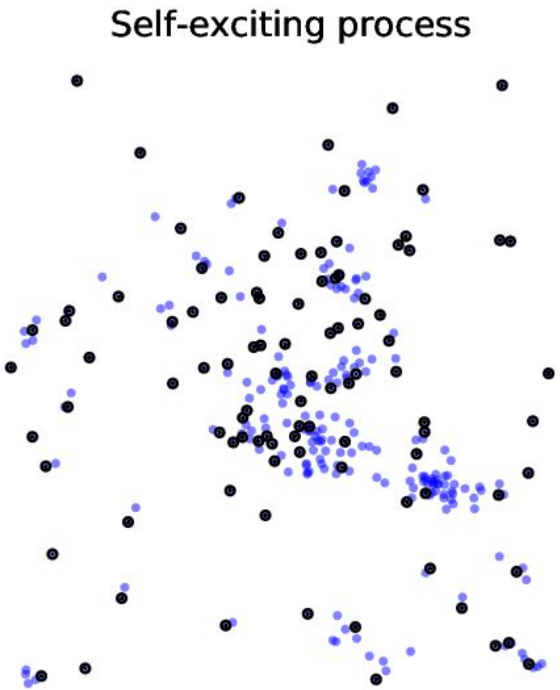
$$||\Omega|| = 1 - \frac{\mu}{\Lambda}$$

- Further study could help build tools to anticipate flash crashes

Other Applications

- Initially introduced to help predict seismic activity
- Criminology
- Epidemiology
- Spatio-temporal extension

$$\lambda(t, x|\mathcal{H}_t) = \mu + \int_{(0,t)\times\mathbb{R}} \omega(t-s, x-y)dN(s \times y)$$



References

Bacry, E., K. Dayri and J.F. Muzy. (2011). Non-parametric kernel estimation for symmetric Hawkes processes. Application to high frequency financial data. *The European Physical Journal B* **85**.

Bacry, E., I. Mastromatteo and J.F. Muzy. (2015). Hawkes Processes in Finance. *Market Microstructure and Liquidity* **01** 1550005.

Bauwens, L. and N. Hautsch. (2007). Modelling financial high frequency data using point processes. *LIDAM Discussion Papers CORE* 2006080.

Cont, R. (2007). Volatility Clustering in Financial Markets: Empirical Facts and Agent-Based Models. *Long Memory in Economics* 289-309.

Embrechts, P., T. Liniger and L. Lin. (2011). Multivariate Hawkes Processes: an Application to Financial Data. *Journal of Applied Probability* **48A** 367-378.

Filiminov, V. and D. Sornette. (2012). Quantifying reflexivity in financial markets: towards a prediction of flash crashes. *Physical Review E* **85** 856108.

Reinhart, A. (2018). A Review of Self-Exciting Temporal Point Processes and Their Applications. *Statistical Science* **33** 299-318.