CS 215 Assignment 3 Report

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Problem 1

0.1 Maximum Likelihood Estimate

Given that the data $x_1, x_2, \cdots x_N$ are drawn from $G(\mu, \sigma^2)$ where σ is known and μ is unknown. We know that,

$$\widehat{\mu}^{\mathrm{ML}} = \frac{\sum_{i=0}^{N} x_i}{N}$$

0.2 Gaussian Prior

Given that the data $x_1, x_2, \cdots x_N$ are drawn from $G(\mu, \sigma^2)$ where σ is known and μ is unknown. Also, μ is known to be drawn from $G(\mu_0, \sigma_0^2)$ where $\sigma = 4$, $\mu_0 = 10.5$ and $\sigma_0 = 1$. The mean of the data is $\overline{x} = \frac{\sum_{i=0}^{N} x_i}{N}$

$$\begin{aligned}\widehat{\mu}^{\mathrm{MAP1}} &= \frac{\overline{x}\sigma_0^2 + \mu_0\sigma^2/N}{\sigma_0^2 + \sigma^2/N} \\ &= \frac{\overline{x} \times 1 + 10.5 \times \frac{16}{N}}{1 + \frac{16}{N}}\end{aligned}$$

0.3 Uniform Prior

Given that the data $x_1, x_2, \cdots x_N$ are drawn from $G(\mu, \sigma^2)$ where σ is known and μ is unknown. Also, μ is known to be drawn from $U(9.5, 11, 5)$ where $\sigma = 4$.

$$\text{posterior PDF} = \begin{cases} \frac{P(x_1,\cdots,x_n|\mu)P(\mu)}{\int_{9.5}^{11.5} P(x_1,\cdots,x_n|\mu)P(\mu)d\mu} & \text{for } \mu \in (9.5, 11.5) \\ 0 & \text{otherwise} \end{cases}$$

Since $P(\mu) = 1$ for $\mu \in (9.5, 11.5)$,

$$\text{Posterior PDF} = \frac{P\left(x_1, \cdots, x_n|\mu\right)}{\int_{9.5}^{11.5} P\left(x_1, \cdots, x_n|\mu\right) d\mu}, \text{ for } \mu \in (9.5, 11.5)$$

Maximum of the posterior within $(9.5, 11.5)$ = maximum of $P\left(x_1, \cdots, x_n|\mu\right)$ within $(9.5, 11.5)$
If the mode of the likelihood function lied within this then the mode of the posterior $\equiv$ ML estimate
Therefore, we have that

$$\widehat{\mu}^{\mathrm{MAP2}} = \min\left(\max\left(9.5, \widehat{\mu}^{\mathrm{ML}}\right), 11.5\right)$$

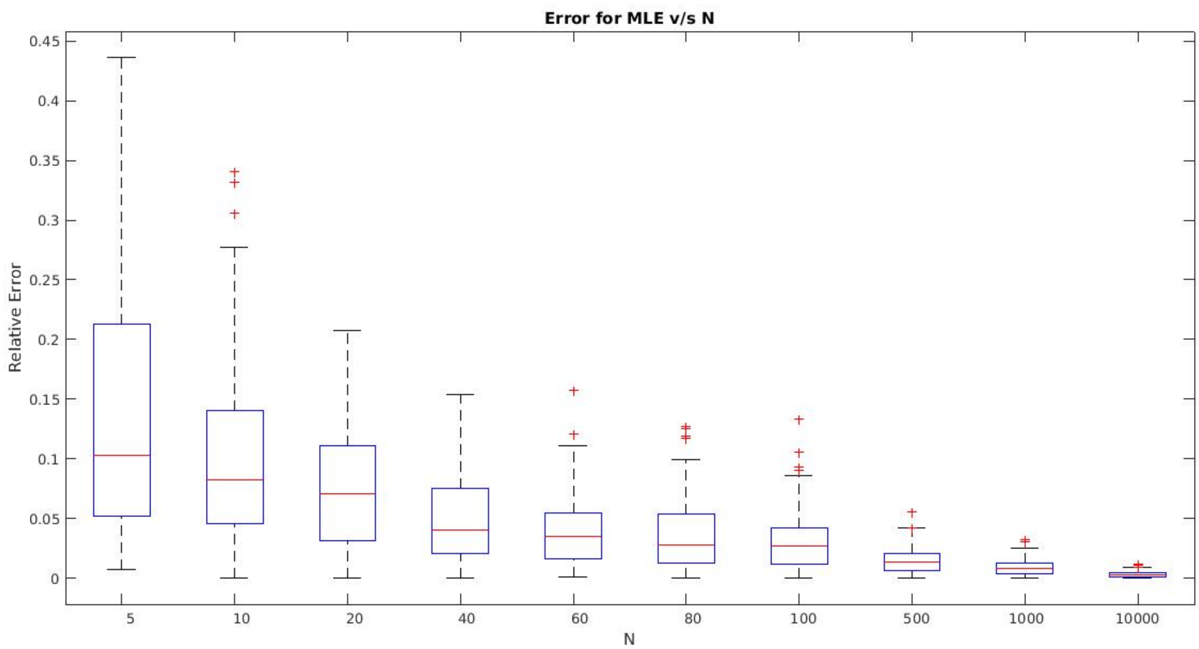
Where

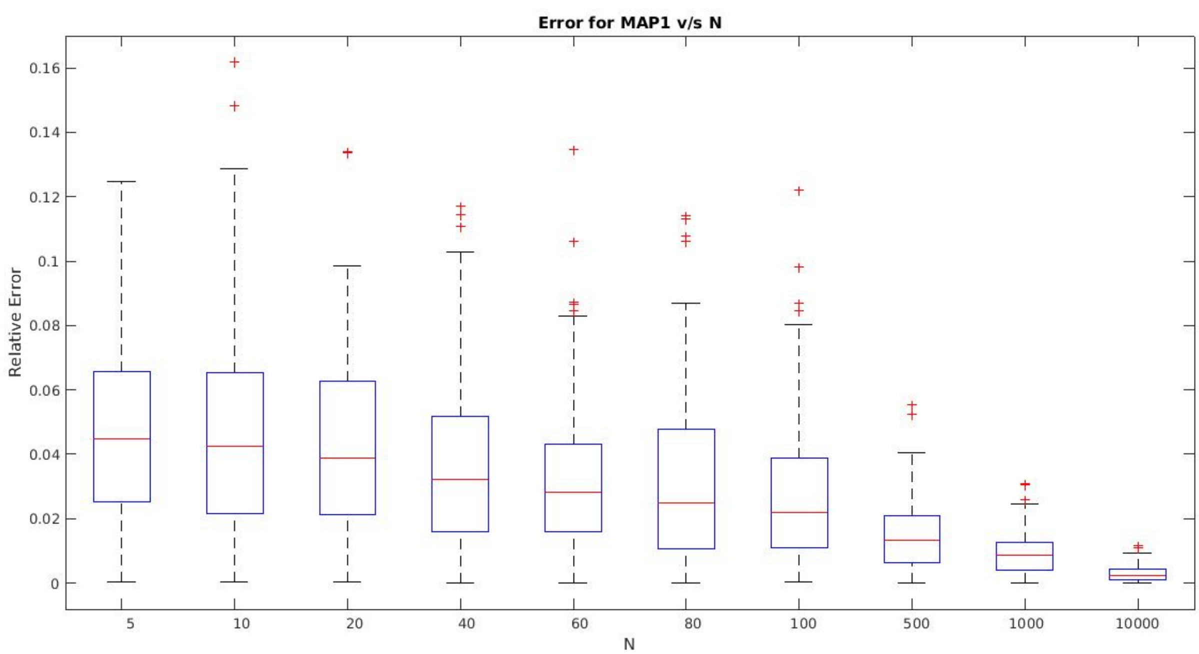
$$\widehat{\mu}^{\mathrm{ML}} = \frac{\sum_{i=0}^{N} x_i}{N}$$

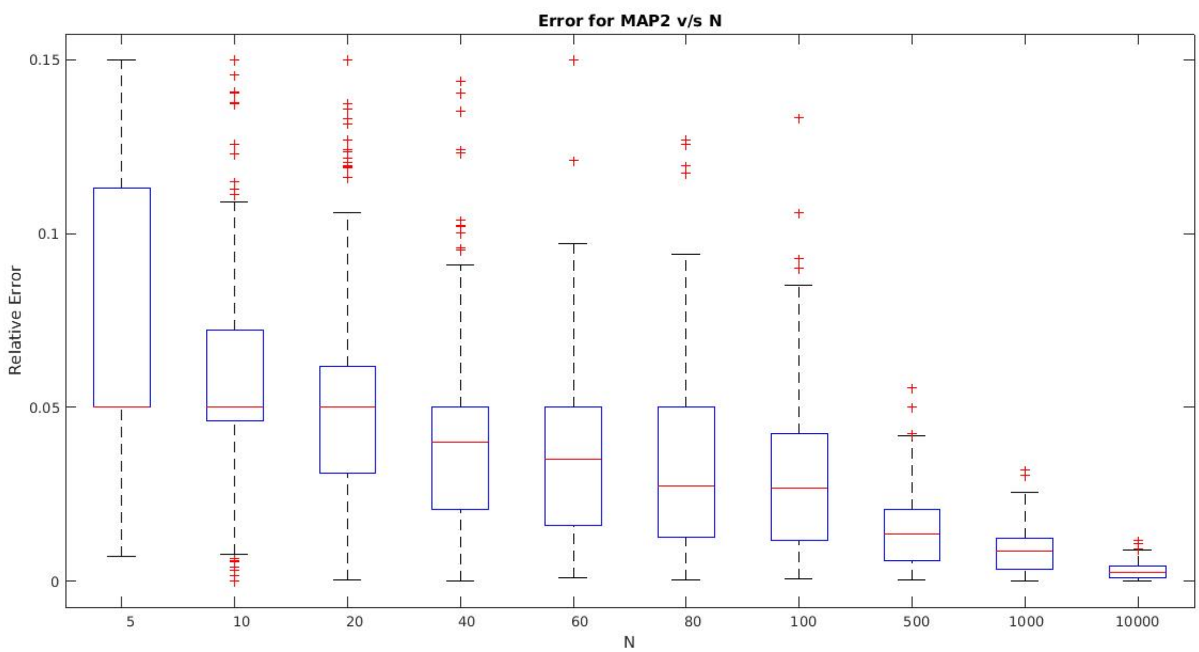
0.4 Boxplots and Interpretation

1. As the value of N increases the error decreases and approaches zero in all the three cases

2. The second estimate, ie. the MAP estimate with a Gaussian Prior is the most preferable since it has the smallest error values even for small values of N. Since in practice, we only have finite data, this estimate should be a good choice. Also, for large values of N it converges to the ML estimate and hence possesses all the desired asymptotic properties(consistency, asymptotic normality, efficiency) of the ML estimate.






Error for MAP2 v/s N
Relative Error
0.15
0.1
0.05
0
5
10
20
40
60
80
100
500
1000
10000
N