CS 215 Assignment 4 Report

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Problem 4

Part (i)

The statement is false. If the bivariate Gaussian $\mathbf{Y} = (Y_1, Y_2)$ has a diagonal covariance matrix C, then $C_{12} = \text{Cov}(Y_1, Y_2) = 0$. Therefore by definition of uncorrelation, Y_1 and Y_2 are uncorrelated.

Part (ii)

The statement is true. If in $\mathbf{X} = (X_1, X_2)$, X_1 and X_2 are uncorrelated, then by definition, $\text{Cov}(X_1, X_2) = 0$. Denoting variances as ${\sigma_1}^2$ and ${\sigma_2}^2$, we have:

$$C = \begin{bmatrix} {\sigma_1}^2 & 0 \\ 0 & {\sigma_2}^2 \end{bmatrix}$$

Thus,

$$\begin{aligned}
f_{(X_1,X_2)}((x_1,x_2)) &= \frac{\exp\left(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^{\mathrm{T}}\boldsymbol{C}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right)}{\sqrt{(2\pi)^2|\boldsymbol{C}|}} \\
&= \frac{\exp\left(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^{\mathrm{T}}\begin{bmatrix} \frac{1}{{\sigma_1}^2} & 0 \\ 0 & \frac{1}{{\sigma_2}^2} \end{bmatrix}(\mathbf{x}-\boldsymbol{\mu})\right)}{\sqrt{(2\pi)^2{\sigma_1}^2{\sigma_2}^2}} \\
&= \frac{\exp\left(-\frac{1}{2}\begin{bmatrix} (x_1-\mu_1)\frac{1}{{\sigma_1}^2} & (x_2-\mu_2)\frac{1}{{\sigma_2}^2} \end{bmatrix}(\mathbf{x}-\boldsymbol{\mu})\right)}{\sqrt{(2\pi)^2{\sigma_1}^2{\sigma_2}^2}} \\
&= \frac{\exp\left(-\frac{1}{2}((x_1-\mu_1)^2\frac{1}{{\sigma_1}^2} + (x_2-\mu_2)^2\frac{1}{{\sigma_2}^2})\right)}{\sqrt{(2\pi)^2{\sigma_1}^2{\sigma_2}^2}} \\
&= \frac{\exp\left(-\frac{(x_1-\mu_1)^2}{2{\sigma_1}^2}\right)}{\sqrt{2\pi}\sigma_1}\frac{\exp\left(-\frac{(x_2-\mu_2)^2}{2{\sigma_2}^2}\right)}{\sqrt{2\pi}\sigma_2} \\
&= f_{X_1}(x_1)f_{X_2}(x_2)
\end{aligned}$$

Thus since the joint pdf splits into its marginals, the 2 random variables are independent.