

Requirements for w_steadystate_reweight.py

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- Overall scheme:
- For each input simulation, load flux matrix.
 - "flux matrix": A square, nonnegative, N*N matrix F where each element
 represents probability flow between two "bins". In this
 context, each element $F_{i,j}$ (where "i" is the column
 index, and "j" is the row index) represents flux calculated
 from bin i, into bin j.
 >>>DOUBLE CHECK THIS

- Load flux matrix from output of w_postanalysis_matrix.py
- Flux matrices are stored for each iteration in
 flux_matrix_h5['iterations/iter_%08d/']
 in a sparse matrix format. Output files from w_postanalysis_matrix
 store data in a manner based on the coordinate matrix format from the
 Scipy library. Each iteration group contains four data sets:
 1) cols: a data set storing a vector of column indices
 2) rows: a data set storing a vector of row indices
 3) flux: a data set storing a vector of flux values
 4) obs: a data set storing a vector of transition counts
 These data sets may be combined to form either the flux matrix
 (described above), or a "count matrix"
 - Count matrix: A square, integer-valued, nonnegative, N*N matrix C
 where each element represents the COUNT of transition
 events between two bins. In this context, each element
 $C_{i,j}$ represents the number of transitions observed
 from bin i, into bin j, during a given iteration.
 For example, given some integer index k , with $0 \leq k < \text{len}(\text{cols})$, we
 have:
 flux_matrix[rows[k], cols[k]] = flux[k]
 count_matrix[rows[k], cols[k]] = obs[k]

- For each input, build a transition matrix (one per input).
 - "transition matrix": For this purpose, a transition matrix is a square,
 nonnegative, N*N matrix T where each row sums to
 one. More formally, it is a RIGHT STOCHASTIC matrix.
 Here, each element $T_{i,j}$ represents the
 probability that a trajectory walker in bin i will
 next transition to bin j during the lag time
 τ .
 >>>WHAT IS THE DEFAULT LAG TIME?

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[illegible]

- In the case of a simulation without recycling conditions, a transition matrix may be constructed from a flux matrix by row-normalizing. Such a transition matrix would depend only upon the events observed within a single iteration. The user may desire that multiple flux matrices are represented in a single transition matrix. Such average should tend to stabilize the estimate of the transition probabilities, as observations from a single iteration may be insufficient to obtain reasonable

statistics. One may imagine multiple methods for averaging.

1) Average the count matrices. Given a collection of  $N \times N$  count matrices

$$\{C^1, C^2, \dots, C^n\}$$

the following is an estimator for the flux per iteration:

$$\langle C \rangle = \frac{\sum_{k=1}^n C^k}{n}$$

The transition matrix may then be estimated by

$$T_{i,j} = \frac{\langle C \rangle_{i,j}}{\sum_{m=1}^N \langle C \rangle_{i,m}}$$

2) Average the transition matrices. Given a collection of  $N \times N$

count matrices

$$\{C^1, C^2, \dots, C^n\}$$

we first row-normalize each count matrix as

$$T_{i,j}^k = \frac{C_{i,j}^k}{\sum_{m=1}^N C_{i,m}^k}$$

for  $k=1,2,\dots,n$ . We may then obtain an estimate of the transition

matrix  $\langle T \rangle$  as:

$$\langle T \rangle = \sum_{k=1}^n T^k$$

Either formulation may be valid. Method (1) places more "emphasis" on high-weight transitions, in that large flux values contribute more to the estimate  $\langle T \rangle$  than do low-weight transitions. Method (2) places equal emphasis on each transition observation, regardless of the associated weight.