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5:03-5:55

1 a. if mean is known + constant

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x^{(i)}-\mu)^2$$

$$\max_{\sigma^2} \sum_{i=1}^{N} -\log(2\pi\sigma^2)^{1/2} - \frac{(x^{(i)}-\mu)^2}{2\sigma^2}$$

$$\frac{d}{d\sigma^2} = \sum_{i=1}^{N}\frac{x^{(i)}-\mu}{2\sigma^2} - \frac{1}{2x^{(i)}\sqrt{\ln(2\pi x)}} = 0 \Rightarrow \sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x^{(i)}-\mu)^2$$

b. if mean isn't known

$$\mu = \frac{1}{N}\sum_{i=1}^{N}x^{(i)} \quad \text{then} \quad \sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x^{(i)}-\mu)^2$$

$$\frac{d}{d\mu} = \sum_{i=1}^{N}\frac{-2(x^{(i)}-\mu)}{2\sigma^2} = 0 \Rightarrow \mu = \frac{1}{N}\sum_{i=1}^{N}x^{(i)}$$

2. $E(x) = \int_{-\infty}^{\infty} x\,p(x)\,dx$ $\qquad$ $E(X|y) = \int_{-\infty}^{\infty} x\,p(x|y)\,dy$

y \ x	1	2
1		
2		

$$E[E(X|y)] = \int_{-\infty}^{\infty} p(y)\,E(X|y)\,dx$$

$$= \int_{-\infty}^{\infty} p(y)\int_{-\infty}^{\infty} x\,p(x|y)\,dy\,dx$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\,\underbrace{p(y)\,p(x|y)}_{\text{product}}\,dy\,dx$$

$$= \int_{-\infty}^{\infty} x \underbrace{\int_{-\infty}^{\infty} p(y,x)\,dy}_{\text{sum}}\,dx$$

$$\int_{-\infty}^{\infty} x\,p(x)\,dx = E[X]$$

3. a. $\theta_{ML} = \max_{\theta} \sum_{i=1}^{N} \log \theta^2 x - \theta x$

b i $\frac{d\theta_{ML}}{d\theta} = \frac{2}{\theta} - x = 0$

$\theta = \frac{x}{2}$ when $x \geq 0$

ii $W(k) = W(k-1) - \rho \nabla p(x)$

where θ + x start as random values s.t. $x \geq 0$, $\theta > 0$

$\nabla p(x) = \begin{bmatrix} \frac{1}{x} - \theta \\ \frac{2}{\theta} - x \end{bmatrix}$