$X \sim N(0, \alpha)$

$Y = X + N(0, \beta)$

$Z = Y + N(0, \gamma)$

$$X, Y, Z \sim N\left(0, \begin{pmatrix} \alpha & \alpha & \alpha \\ \alpha & \alpha+\beta & \alpha+\beta \\ \alpha & \alpha+\beta & \alpha+\beta+\gamma \end{pmatrix}\right)$$

$$\boxed{Y \sim \frac{\alpha+\beta}{\alpha+\beta+\gamma} W}$$

$$\mathbb{E}[Y \mid X, Z] = \Sigma_{Y, XZ} \Sigma_{XZ}^{-1} \begin{pmatrix} X \\ Z \end{pmatrix}$$

$$= \begin{pmatrix} \alpha & \alpha+\beta \end{pmatrix} \begin{pmatrix} \alpha & \alpha \\ \alpha & \alpha+\beta+\gamma \end{pmatrix}^{-1}$$

$$= \frac{\begin{pmatrix} \alpha & \alpha+\beta \end{pmatrix} \begin{pmatrix} \alpha+\beta+\gamma & -\alpha \\ -\alpha & \alpha \end{pmatrix}}{\alpha(\alpha+\beta+\gamma) - \alpha^2}$$

$$= \frac{\begin{pmatrix} \alpha(\alpha+\beta+\gamma) - \alpha(\alpha+\beta) & -\alpha^2 + \alpha(\alpha+\beta) \end{pmatrix}}{\alpha(\beta+\gamma)}$$

$$= \frac{\begin{pmatrix} \gamma & \beta \end{pmatrix}}{\beta+\gamma} = \begin{pmatrix} \frac{\gamma}{\beta+\gamma} & \frac{\beta}{\beta+\gamma} \end{pmatrix}$$

OLS coefficients don't depend on $\alpha \Rightarrow$ Noise intervention on X fails to eliminate Z

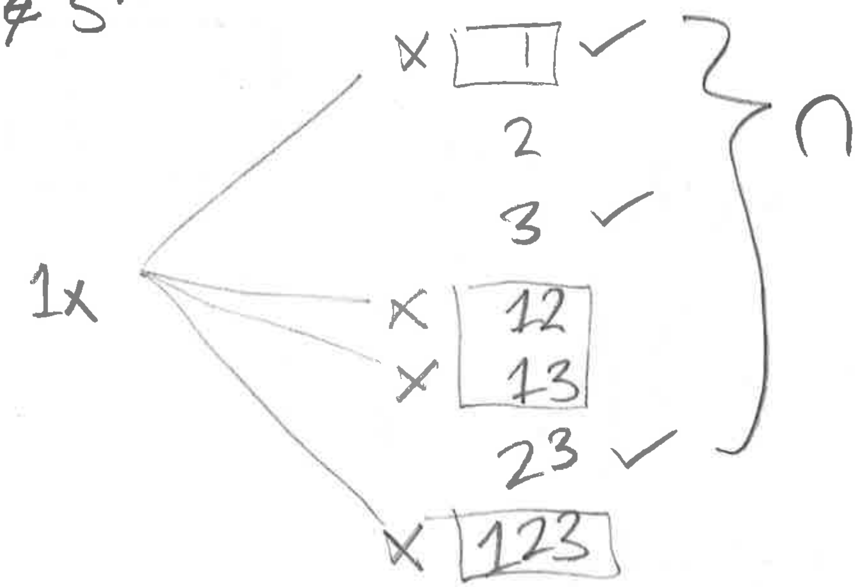
Our approach

Find

$x^i \notin S^*$

PMB approach

Find $\tilde{S} \subseteq S^*$



qingy zhao