Causal Inference and Invariance

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(Part 2/2)

From Last Week: Causal Graph

Causal relationships in a system represented by a graph. The graph tells you:

I. which variables are affected by an intervention.
II. what conditional independence relationships exist in the joint distribution (*d-separation*.)
III. which sets of predictors and responses will have "invariant" optimal predictive rules.

This talk is restricted to directed acyclic graph (DAG), i.e. no feedback!

From Last Week: Three *Causal* Questions

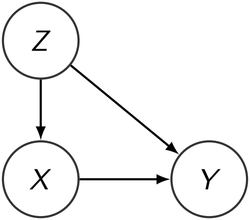
- Given a number of variables, which pairs are causally related?
  - Infer the *graph*.
- Given a number of variables and a fixed Y, which variables causally affect Y?
  - Infer the *invariance set*.
- Given a fixed X and a fixed Y, what is the causal effect of X on Y?
  - Infer the *causal effect*.

Why different languages? Convenience!

Section 1

Overview of Previous methods

Known Causal Structure



For example, suppose we want to estimate the causal effect of X on Y with known confounders Z.

- Graphical approach: the backdoor formula

$$\mathrm{P}(y|do(x)) = \sum_{z} \mathrm{P}(y|x,z)\mathrm{P}(z).$$

- Functional approach: outcome regression $Y \sim X + Z$.
- Potential outcome approach: estimate the propensity score.

Unknown Causal Structure

Conventional approach:

1. Estimate the Markov equivalence class of causal graphs via conditional independence relationships.
2. Infer or bound the identifiable causal effects.

More recent approach: impose additional functional/distributional assumptions to the structural equation model: for any variable Y,

$$Y = f(\mathrm{parents}(Y); \epsilon_Y).$$

How should we think about the assumptions?

One thing for sure: They are no monsters!

How should we think about the assumptions?

- In statistics we make assumptions all the time: parametric, independence, function form, etc.
  - George Box: "All models are wrong but some are useful".
- To infer causation, we need to make different kinds of assumptions.
  - Problem statement: Can what we learned from this environment be generalized to another environment?
  - The ancient wisdom: "Correlation does not imply causation" (observational $\not\Rightarrow$ interventional).
  - Causal assumptions: causal graph, structural equation model, or invariant prediction.

What if we are willing to make both kinds of assumptions?

Section 2

Invariance

Assumed invariance

Focus: Given a number of variables and a fixed Y, which variables causally affect Y?

Data: i.i.d. samples of (X^e, Y^e) from different environments $e \in \mathcal{E}$.

Assumption (Invariant prediction)

There exists a vector of coefficients γ^* with support S^* such that for all $e \in \mathcal{E}$, X^e has an arbitrary distribution and

$$Y^e = \mu + X^e \gamma^* + \epsilon^e, \ \epsilon^e \sim F_\epsilon, \ \epsilon^e \perp\!\!\!\perp X^e_{S^*}.$$

Important:

- F_ϵ does not depend on e.
- ϵ is always independent of X.

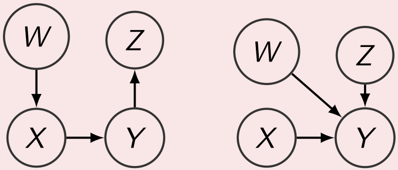
This is essentially a single structural equation with $\mathrm{parents}(Y) = S^*$.

Building block

Testing the null hypothesis that (γ, S) satisfies the assumption.

$$H_{0,\gamma,S}(\mathcal{E}): \ \gamma_k = 0 \text{ if } k \in S, \ and \ \exists F_\epsilon \text{ such that for all } e \in \mathcal{E},$$
$$Y^e = X^e\gamma + \epsilon^e, \ \epsilon^e \sim F_\epsilon, \ \epsilon^e \perp\!\!\!\perp X^e_S.$$
$$H_{0,S}(\mathcal{E}): \ \exists\gamma \text{ such that } H_{0,\gamma,S}(\mathcal{E}) \text{ is true.}$$

Difficulty



Statistically, we may end up accepting both $Y^e = X^e + \epsilon^e$ and $Y^e = X^e + 0.01W^e + 0.01Z^e + \epsilon^e$, for both causal structures.

Generic procedure

1. For each $S \subseteq \{1, \ldots, p\}$, test $H_{0,S}(\mathcal{E})$ at level α.
2. Set $\hat{S}(\mathcal{E}) = \bigcap_{S:H_{0,S}(\mathcal{E}) \text{ not rejected}} S$.
3. For the confidence sets, set $\hat{\Gamma}(\mathcal{E}) = \bigcup_{S\subseteq\{1,\ldots,p\}} \hat{\Gamma}_S(\mathcal{E})$, where

$$\hat{\Gamma}_S(\mathcal{E}) = \begin{cases} \emptyset & H_{0,S}(\mathcal{E}) \text{ is rejected at level } \alpha, \\ \hat{S} & \text{otherwise.} \end{cases}$$

$\hat{C}(S)$ is a $(1-\alpha)$-confidence set for γ obtained by pooling the data.

Theorem (Peters et al.)

$$\mathrm{P}(\hat{S}(\mathcal{E}) \subseteq S^*) \geq 1-\alpha, \ \mathrm{P}(\gamma^* \in \hat{\Gamma}(\mathcal{E})) \geq 1-2\alpha.$$

The Statistical Challenge

Depending on the modeling assumption, this hypothesis can be:

$$\begin{aligned}
H_{0,S,\mathrm{lin}}(\mathcal{E}): &\ \exists\gamma \text{ s.t. } \gamma_k = 0 \text{ if } k \in S, \textit{and} \\
&\ \exists F_\epsilon \text{ s.t. } Y^e = X^e\gamma + \epsilon^e,\ \epsilon^e \sim F_\epsilon,\ \epsilon^e \perp\!\!\!\perp X_S^e,\ \forall e \in \mathcal{E}. \\
H_{0,S,\text{lin-gauss}}(\mathcal{E}): &\ H_{0,S,\mathrm{lin}}(\mathcal{E}) \text{ and } F_\epsilon = \mathrm{N}(0,\sigma^2). \\
H_{0,S,\mathrm{nonlin}}(\mathcal{E}): &\ \exists g(X_S,\epsilon),\ F_\epsilon \text{ s.t. } Y^e = g(X_S^e,\epsilon^e),\ \epsilon^e \ldots. \\
H_{0,S,\mathrm{additive}}(\mathcal{E}): &\ H_{0,S,\mathrm{nonlin}}(\mathcal{E}) \text{ and } g(X_S,\epsilon) \text{ is additive.} \\
H_{0,S,\mathrm{hidden}}(\mathcal{E}): &\ \epsilon^e \sim F_\epsilon,\ \forall e \in \mathcal{E},\ \text{ but } F_\epsilon \text{ can have nonzero mean.}
\end{aligned}$$

How to test $H_{0,S}(\mathcal{E})$?

Peters et al. give concrete proposals for $H_{0,S,\text{lin-gauss}}$ and $H_{0,S,\text{lin-gauss-hidden}}$. They are implemented in their `InvariantCausalPrediction` package.

Robustness of the invariance approach

In Meinshausen's talk: we don't make false discoveries, even under a misspecified model!
Truth (at least what we believe in):

Things can go wrong	ICP's behavior
Intervene on Y (or a missing cause)	$\bigcap\limits_{\emptyset}$
Non-linear, non-additive	$\bigcap\limits_{\emptyset}$
Not enough interventions	False positives
Small sample size	$\emptyset$
Left out a confounder	$\emptyset$
Left out an unconfounding predictor	okay

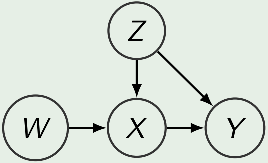
Splitting purely observational data

A big bonus: we can "create" an environment by conditioning on a variable U that we know precedes Y. This is valid because

$$Y|X_{S^*} \stackrel{d}{=} Y|X_{S^*}, U = u.$$

Note: this statement is true only in the region that both conditional distributions are well defined.

Creating environment by instrumental variable



If there is a hidden confounder Z, we can condition on the instrumental variable W.

Back to the three causal questions

Can ICP help to answer the other two questions?

Infer the graph

We can run ICP for every node with caution. Returns a partially identified graph.

Infer the causal effect of X on Y

Two options:

- Treat X as the target variable: *propensity score*.
- Treat Y as the target variable: *outcome regression*.

Okay if $\hat{S}$ itself is invariant. Otherwise ICP may miss important causes, resulting in biased causal effect estimate.
Idea: maybe we can just use (many) "minimal" S.