C1 Derivation

a) $Z = Ae^{jP}$

$Z^{1/n} = (Ae^{jP})^{1/n} = Re^{jB}$

$Z^{1/n} = A^{1/n} \cdot (e^{jP})^{1/n} = Re^{jB}$

$Z^{1/n} = A^{1/n} \cdot e^{j(P/n)} = Re^{jB}$

$\therefore R = A^{1/n} \; ; \; B = P/n$ — ①

$Re^{jB} = X + jY$

$\therefore R[\cos(B) + j\sin(B)] = X + jY$

$\therefore R\cos(B) + jR\sin(B) = X + jY$

$\therefore X = R\cos(B) \; ; \; Y = R\sin(B)$

$Z_2 = Ae^{j(P+360^\circ)}$

$\therefore Z_2^{1/n} = (Ae^{j(P+360^\circ)})^{1/n} = Re^{jB_2}$

$\therefore Z_2^{1/n} = A^{1/n} \cdot (e^{j(P+360^\circ)})^{1/n} = Re^{jB_2}$

$\therefore Z_2^{1/n} = A^{1/n} \cdot e^{j(P/n + 360^\circ/n)} = Re^{jB_2}$

$\therefore R = A^{1/n} ; B_2 = \frac{P}{n} + \frac{360^\circ}{n}$

$\therefore B_2 = B + \frac{360^\circ}{n}$ — from ①

$dB = B_2 - B$

$\therefore dB = \left(B + \frac{360^\circ}{n}\right) - B$

$= B + \frac{360^\circ}{n} - B$

$= \frac{360^\circ}{n}$

$\therefore dB = \frac{360^\circ}{n}$

e) $Z = -1 = e^{j180°}$ — (from d3)

$Z^{1/3} = (-1)^{1/3} = (e^{j180°})^{1/3} = e^{j\frac{180°}{3}} = e^{j60°}$

$\therefore$ Principal cube root of -1 is $e^{j60°}$ since the phase angle is between $180°$ and $-180°$ not including $-180°$

$e^{j180°} \neq e^{j60°}$

$\therefore$ Principal cube root of -1 cannot be -1

f) from e, $Z = -1 = e^{j180°}$ and $Z^{1/3} = e^{j60°}$

from d4, $Z_2 = -1 - j0.0001 = e^{-j179.9943°}$

$\therefore Z_2^{1/3} = (-1-j0.0001)^{1/3} = (e^{-j179.9943°})^{1/3} = e^{-j59.9981°}$

Thus, although the amplitude of the cube roots is same in d3 and d4, the phase angles differ significantly for both the cube roots.

Therefore, the principal cube root in d3 is different from the principal cube root in d4