ECE201 L2 Hand Calculation

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

$t = M_{11} + M_{22}$ (Trace - Sum of diagonals)

$d = M_{11} M_{22} - M_{21} M_{12}$ (Determinant)

$$|M - \lambda I| = 0$$

$$\left| \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{bmatrix} M_{11} - \lambda & M_{12} \\ M_{21} & M_{22} - \lambda \end{bmatrix} = 0$$

$$(M_{11} - \lambda)(M_{22} - \lambda) - M_{21} M_{12} = 0$$

$$\lambda^2 - \lambda M_{11} - \lambda M_{22} + M_{11} M_{22} - M_{21} M_{12} = 0$$

$$\lambda^2 - (M_{11} + M_{22})\lambda + (M_{11} M_{22} - M_{21} M_{12}) = 0$$

$$\lambda^2 - t\lambda + d = 0$$

$$\lambda_{1,2} = \frac{t \pm \sqrt{t^2 - 4d}}{2}$$

$$\lambda_1 = \frac{t + \sqrt{t^2 - 4d}}{2} \qquad \lambda_2 = \frac{t - \sqrt{t^2 - 4d}}{2}$$

$$\text{Ratio} = \frac{a}{b}$$

~~For top row,~~

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \lambda \begin{bmatrix} a \\ b \end{bmatrix}$$

For top row,

$$M_{11} a + M_{12} b = \lambda a$$

$$M_{11} a - \lambda a = -M_{12} b$$

$$(M_{11} - \lambda) a = -M_{12} b$$

$$a = \frac{-M_{12} b}{(M_{11} - \lambda)}$$

$$\frac{a}{b} = \frac{-M_{12}}{(M_{11} - \lambda)}$$

For Bottom Row,

$$M_{21} a + M_{22} b = \lambda b$$

$$-M_{21} a = M_{22} b - \lambda b$$

$$-M_{21} a = (M_{22} - \lambda) b$$

$$a = \frac{(M_{22} - \lambda)}{-M_{21}} b$$

$$\frac{a}{b} = \frac{M_{22} - \lambda}{-M_{21}}$$