Endogenous Rents Example

Suppose we have two cities 1 & 2. Between the two, total population is seven. ($L_1 + L_2 = 7$)

Each individual is identical & has the following utility function:

$$u(w_j, r_j(L_j), a_j) = w_j - \frac{1}{2} r_j(L_j) + a_j$$

where

Wages: $w_1 = 12, \quad w_2 = 7$

Rents: $r_j(L_j) = 2 \times L_j$

Amenities: $a_1 = a_2 = 0$

(1) How many people live in each city?

Our Locational Equilibrium Condition is

$$u_1(w_1, r_1(L_1), a_1) = u_2(w_2, r_2(L_2), a_2)$$

$$\Rightarrow \quad w_1 - \frac{1}{2} r_1(L_1) - a_1 = w_2 - \frac{1}{2} r_2(L_2) - a_2$$

$$\Rightarrow \quad 12 - \frac{1}{2}(2L_1) = 7 - \frac{1}{2}(2L_2)$$

$$\Rightarrow \quad -L_1 = -5 - L_2 \quad \Rightarrow \quad \boxed{L_1 = 5 + L_2}$$

Recall: $L_1 + L_2 = 7$

$$L_1 + L_2 = 7$$

$$(5 + L_2) + L_2 = 7$$

$$2L_2 = 2 \Rightarrow \boxed{L_2^* = 1}$$

$$L_1 + 1 = 1 \Rightarrow \boxed{L_1 = 6}$$

(ii) What are rents in each city?

$$r_1 = 2L_1 = 2 \times 6 = 12 \Rightarrow \boxed{r_1^* = 12}$$

$$r_2 = 2L_2 = 2 \times 1 = 2 \quad \boxed{r_2^* = 2}$$

(iii) What would change if the government implemented an flat income tax of 10%?