Econ 330: Urban Economics

Lecture 05

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Lecture 05: Rents

Schedule

Today:

(i). Intro to rents

(ii). City "shape"

Upcoming:

- **Reading** (Chapter 4)
- **Problem set 01 due on TBD***

Introduction: City shape

First Week: philosophicalish questions

- What is a city?
- Why do cities exist?
- What determines city size?
- How do cities grow?

Moving forward:

- What economic forces determine **city shape**?
    - Why does the price of land change?
    - Why are buildings taller in city centers?

Questions?

Introduction: City shape

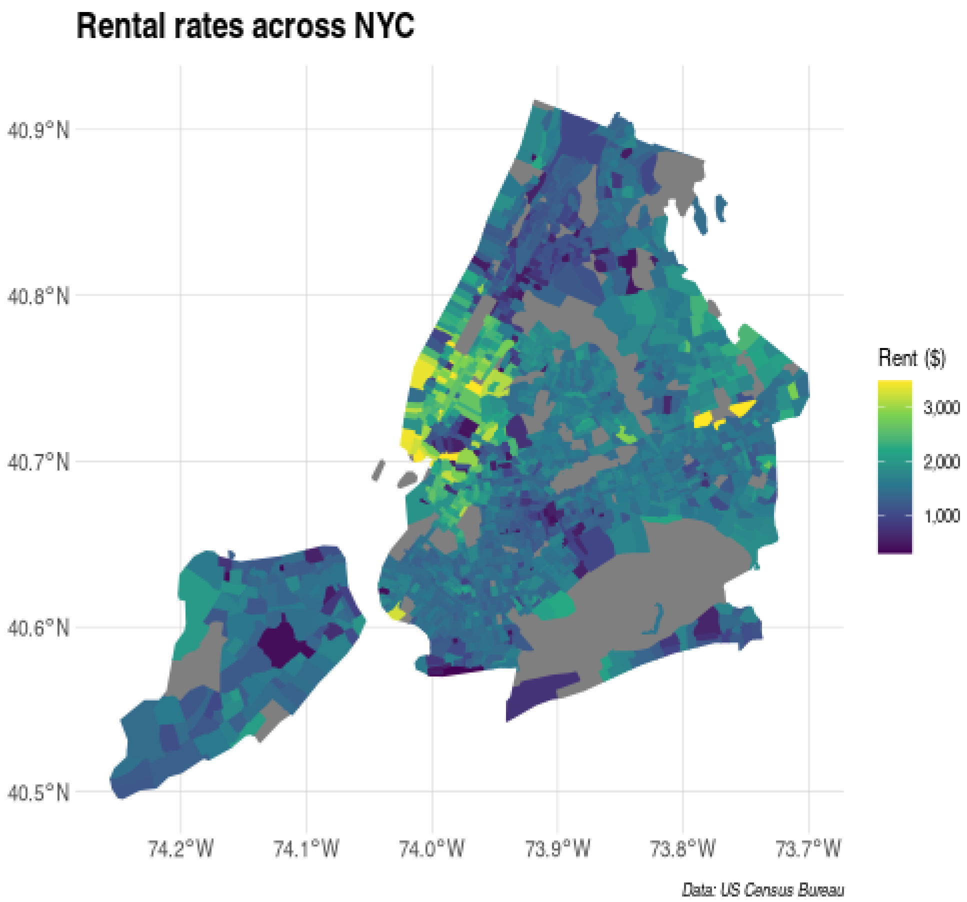
Why do people and firms choose a particular location?

What influences these choices?

Can we explain the current *and* historical "shape" of cities?

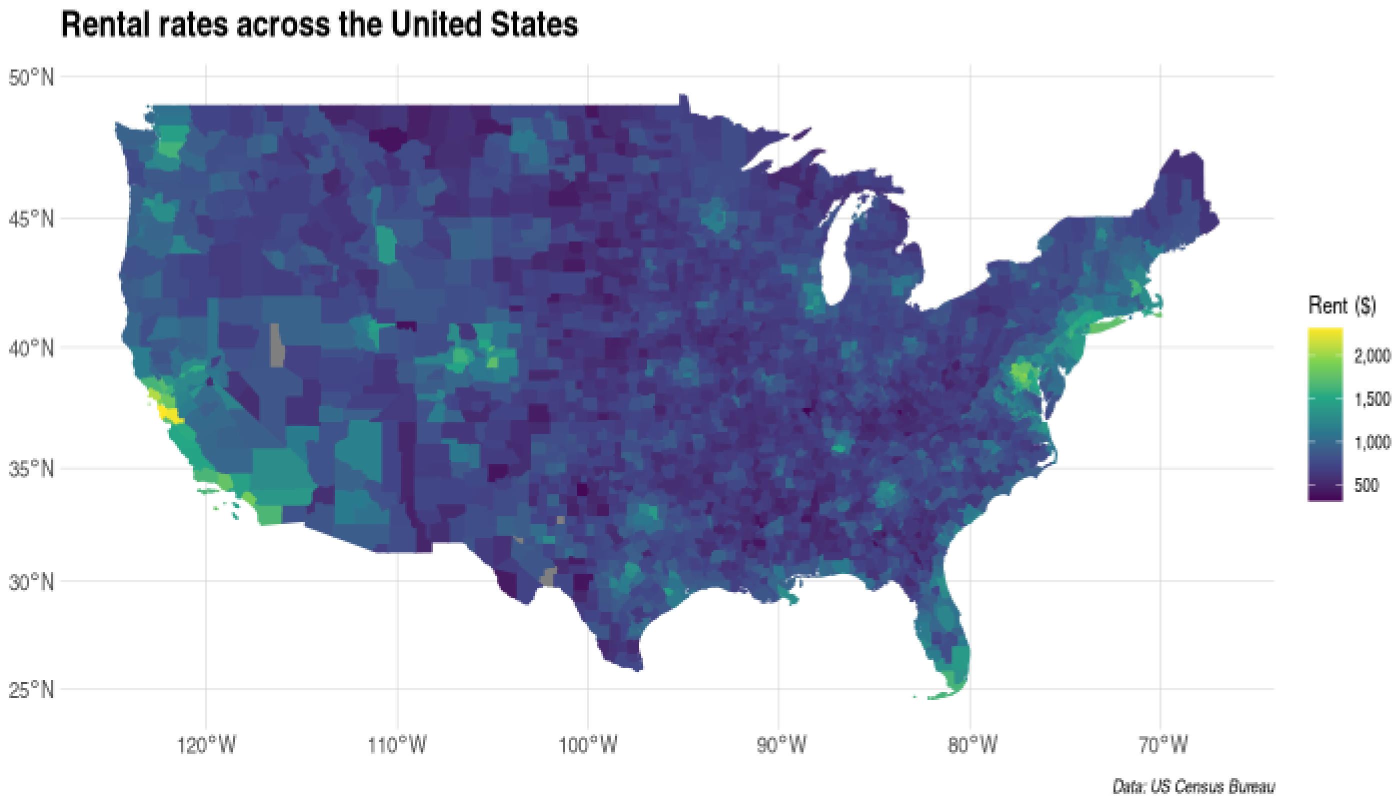
- **Today:** How do these choices impact rental prices within cities
- **Later:** How do these choices impact rental prices across cities
  - Basic introduction into discrete choice theory*

Introduction to rents: NYC

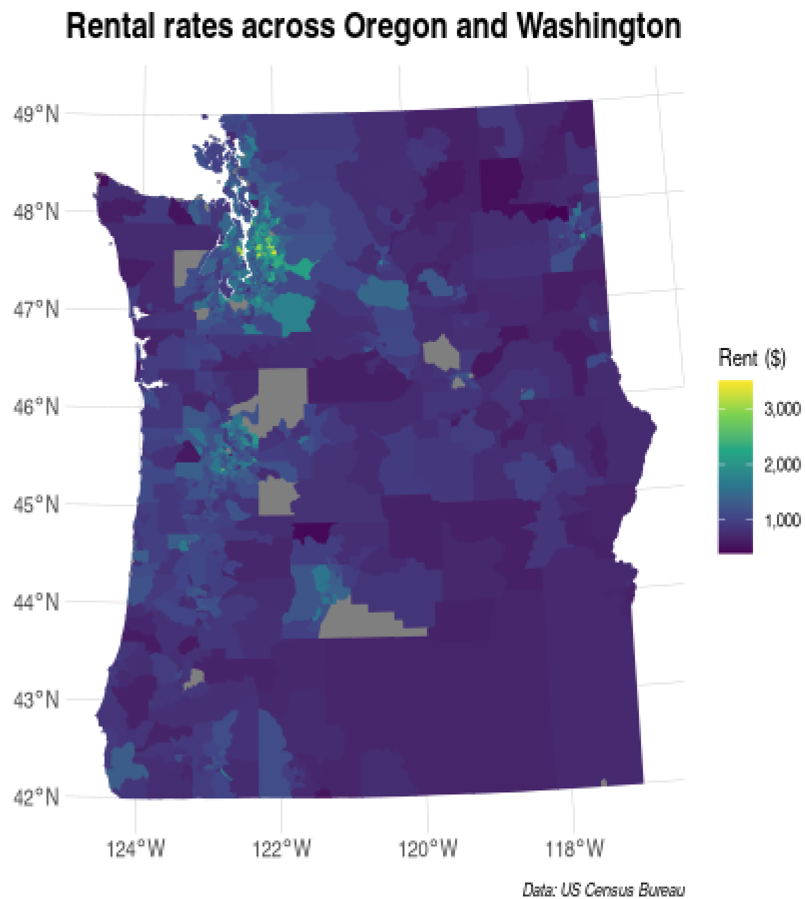


Introduction to rents: NYC

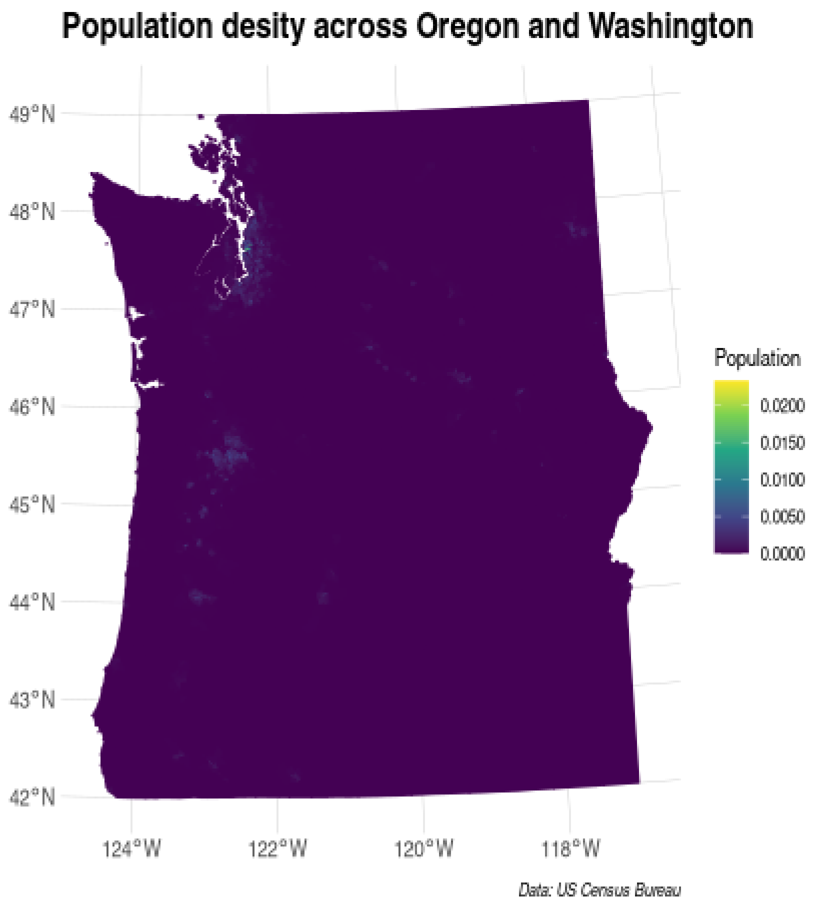
Introduction to rents: US

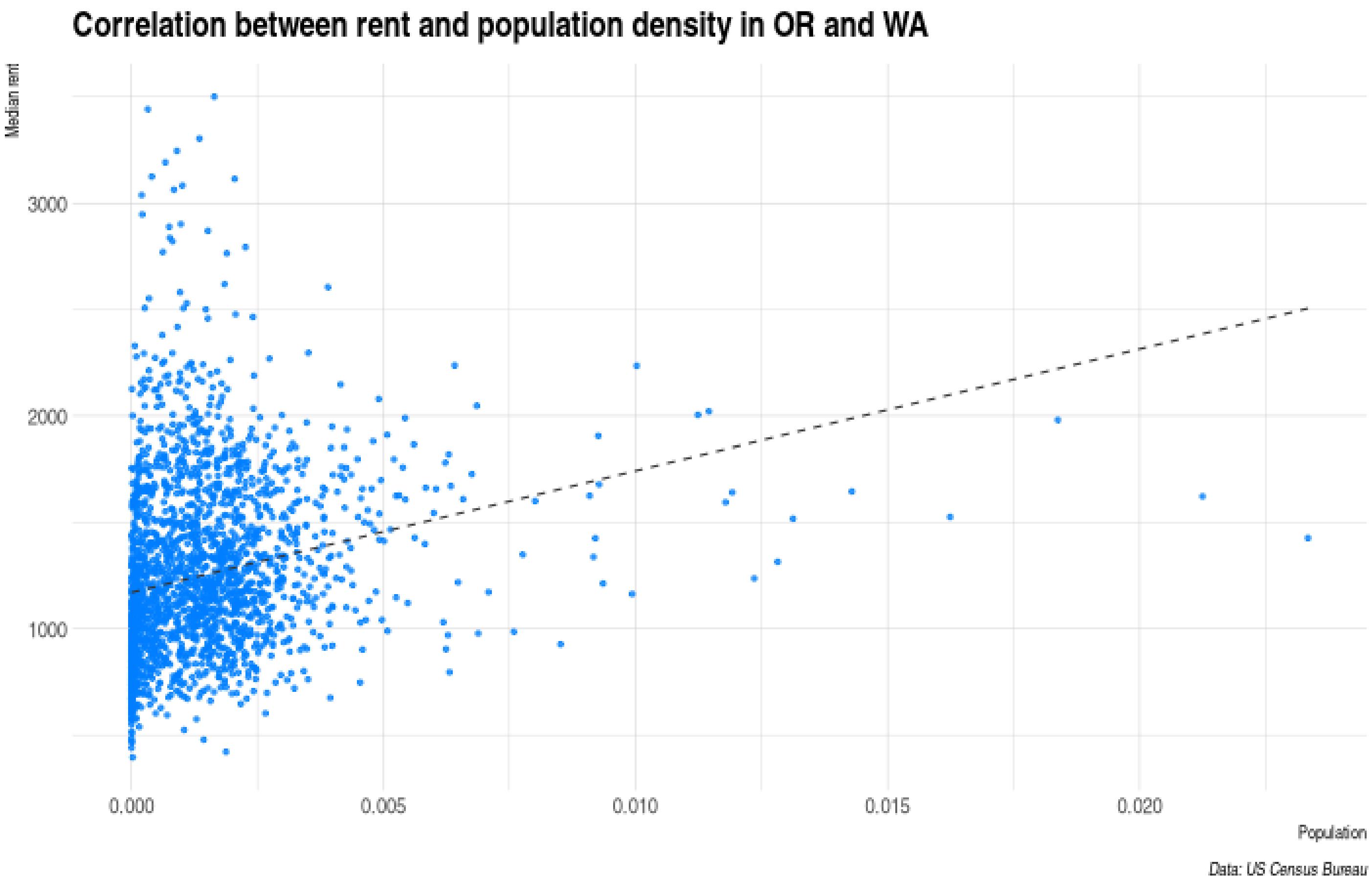


Introduction to rents: OR, WA



Introduction to rents: OR, WA




Correlation between rent and population density in OR and WA
Median rent
3000
2000
1000
0.000
0.005
0.010
0.015
0.020
Population
Data: US Census Bureau

Leaflet

Bid-Rent Curves

The Bid-Rent Curve

A **Bid - Rent Curve:** The relationship between rental prices and the distance of land from the city center †

These curves vary across sectors:

- **Housing:** Accessibility to employment (low commuting costs)
- **Industrial Space:** Accessibility to consumers and suppliers
- **Tech/Office Space:** Accessibility to information

But first a super simple agricultural land rent model

† It actually does not have to be the city center -- can be a point of attraction. In this class we will always use the city center though.

Agricultural land rent model

Definitions:

- **Land rent:** Periodic payment by a land consumer to a landowner
- **Market value:** The amount paid to become the landowner

Setup:

Rent on a plot of land is determined by how productive the plot is

- **Agriculture:** Price of plot is determined by fertility

Consider a setting where farmers grow corn on two types of land

- High fertility (HF): Produces 4 units of corn
- Low fertility (LF): Produces 2 unites of corn

Agricultural land rent model

Assumptions:

- **(i).** Farmers rent from landowners
- **(ii).** No barriers to the corn market
- **(iii).** Perfect competition

$TC = 15$ (excluding rent)

$P_{corn} = 10$

How much will farmers bid for land?

Revenue: $TR = P_{corn} \cdot Q_{corn}$

- HF: $TR_{HF} = 10 \cdot 4 = 40$
- LF: $TR_{LF} = 10 \cdot 2 = 20$

Profit: $\Pi = TR - TC$

- $\Pi_{HF} = TR_{HF} - TC = 40 - 15 - r$
- $\Pi_{LF} = TR_{LF} - TC = 20 - 15 - r$

Recall A05: Competition drives economic profit to zero

Agricultural land rent model

The following table computes maximum WTP for rent:

TABLE 6–1 Fertility and Land Rent

	Price of Corn	Quantity Produced	Total Revenue	Nonland Cost	WTP for Land	Bid Rent for Land
Low fertility	$10	2	$20	$15	$ 5	$ 5
High fertility	$10	4	$40	$15	$25	$25

Since there are no barriers to entry, more firms will enter

- $\Pi \to 0$
- $\Pi_{HF} = TR_{HF} - TC = 40 - 15 - r = 0 \Rightarrow r = 15$
- $\Pi_{LF} = TR_{LF} - TC = 20 - 15 - r == \Rightarrow r = 5$

(i) Housing prices model

Extend the bid-rent model to the housing sector within a city

In cities WTP for land depends on **accessibility** rather than productivity

Assumptions:

(i). Commuting costs are the **only** location factor in decision making

(ii). Only one member of household commutes to employment area

(iii). They only consider the monetary cost of commuting (no time cost)

(iv). Noncommuting travel is insignificant

(v) Public services, taxes, amenities are the same everywhere

Assumptions ensure the employment area is the focal point of the city

(i) Housing prices model: Indifference

A1: *Housing prices adjust until there is locational indifference*

- Locational Eq
- IE: A marginal increase in rent just offsets the lower commuting costs

We call this the locational equilibrium condition. In math:

$$\Delta P \cdot h + \Delta x \cdot t = 0$$

- P: **Price** of housing (per ft^2)
- h: **Housing quantity** (ft^2)
- x: **Distance** of commute (miles)
- t: **Commuting costs** (per mile)

(i) Housing prices model: Bid-Rent

With locational indifference, we can derive the **slope** of the **bid-rent** curve:

$$\underbrace{\Delta P \cdot h}_{\text{Marginal change in housing cost}} + \underbrace{\Delta x \cdot t}_{\text{Marginal change in commuting cost}} = 0$$

(i) Housing prices model: Bid-Rent

With locational indifference, we can derive the **slope** of the **bid-rent** curve:

$$
\begin{aligned}
\Delta P \cdot h + \Delta x \cdot t &= 0 \\
\Delta P \cdot h &= -\Delta x \cdot t
\end{aligned}
$$

(i) Housing prices model: Bid-Rent

With locational indifference, we can derive the **slope** of the **bid-rent** curve:

$$\begin{aligned}\Delta P \cdot h + \Delta x \cdot t = 0 \\ \Delta P \cdot h = -\Delta x \cdot t \\ \frac{\Delta P}{\Delta x} = -\frac{t}{h}\end{aligned}$$

Notice: $\frac{\Delta P}{\Delta x}$ is the slope of the bid-rent curve

Note: Price on the verticle axis, distance on the horizontal. Rise over run

$\Delta P \cdot h = -\Delta x \cdot t$: Another way of putting this: $MC = MB$!

(i) Housing prices model: Bid-Rent

Alternatively: Suppose you have decided that the optimal amount of money to spend on housing and commuting per month is M^*

- You can allocate this as

$$P \cdot h + x \cdot t = M^*$$

- Since we graph the bid rent curve in the (x, P) space, we solve for P:

$$\begin{aligned} P \cdot h + x \cdot t &= M^* \\ P \cdot h &= M^* - x \cdot t \end{aligned}$$

(i) Housing prices model: Bid-Rent

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- You can allocate this as

$$P \cdot h + x \cdot t = M^*$$

- Since we graph the bid rent curve in the (x, P) space, we solve for P:

$$\begin{aligned} P \cdot h + x \cdot t &= M^* \\ P \cdot h &= M^* - x \cdot t \\ P &= \frac{M^*}{h} - \frac{t}{h} \cdot x \end{aligned}$$

- Slope: $\Delta P = 0 - \frac{t}{h} \cdot \Delta x \implies \frac{\Delta P}{\Delta x} = -\frac{t}{h}$

We can use calculus and take derivative if P w.r.t x and get the same thing

(i) Housing prices model: Substitution

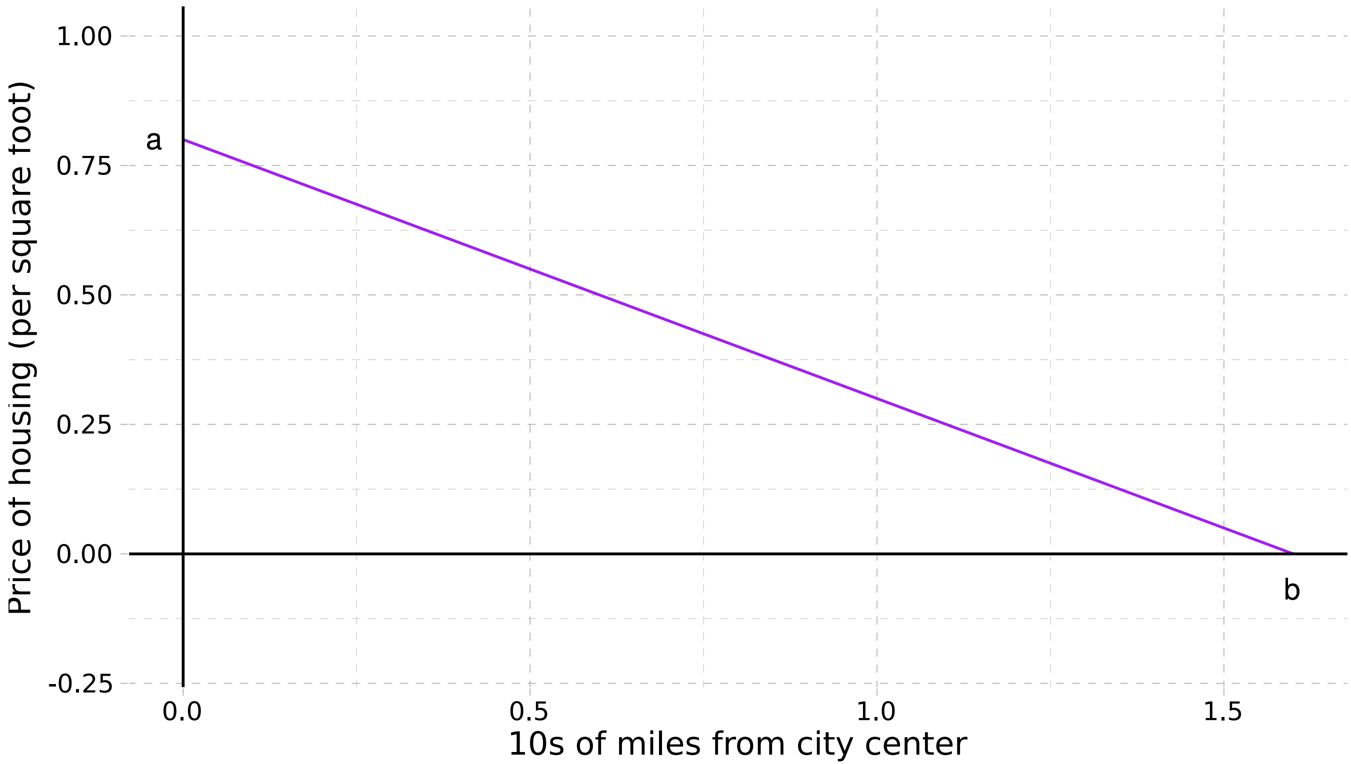
Suppose the following:

- Each household has \$800 a month to spend on housing and commuting
- All rental units are the same size (*1000 sq/ft*), one HH occupying each
- Monthly commuting cost is \$50 dollars per mile from city center

Task: Draw the housing - price curve.

- Put miles from city center on **x axis** and price per square foot on **y axis**

Example: The housing price curve



a: Max WTP for a square foot (at center of city) (80 c per square foot)

b: Furthest away from center HH is willing to live (16 miles)

(i) Housing prices model: Substitution

Q1: If you really wanted to live closer to campus -- or an exciting downtown in a big city -- would you be willing to live in a smaller apartment to do so?

A1: Most people: Yes. You are willing to substitute

Q2: What do I mean by substitute? Substitute what?

A2: Substitute housing consumption for lower commuting cost

(or anything else being close to the center of the city gets you)

(i) Housing prices model: Substitution

Let's formalize the mechanism for substitution a bit:

higher prices $\implies$ **higher oppurtunity cost** per square foot of housing

- As rent $\uparrow$, consumers are likely to substitute towards other goods
  - decreasing the square footage of housing demanded

Housing units closer to city centers are thus likely to be smaller in size

Adding substitution to the model

Q3: Did our model of locational indifference accomdate for substitution?

$$\Delta P \cdot h + \Delta x \cdot t = 0$$

A3: No because h (*quantity of housing consumed*) is **independent of distance** from center, x

- h is exogenous in the model

If consumers can substitute, our locational indifference condition becomes:

$$\Delta P \cdot h(x) + \Delta x \cdot t = 0$$

Where $h(x)$ is an *increasing* function of x

Ex: $h(10) > h(5)$

- Quantity of housing demanded 10 miles away exceeds that of 5 miles

Manufacturing Bid-Rent

Manufacturing Bid Rent

WTP for land from manufacturing firms is a function of accessibility

Fact: Urban manufacturing employment is largely decentralized

Most firms locate close to the highway. **Why**?

Firms are balancing **freight** and **labor costs**

- Further from labor $\implies$ higher wage
  - Compensating for increased commuting cost
- Further from shipping center $\implies$ higher freight cost

Manufacturing Bid Rent

Let's start with a simple model. **Assumptions:**

(i). Input & output **prices** & **quantities** are fixed s.t. firms only decides location

(ii). Firms import intermediate goods and export output to other cities via a **central terminal** (train)

(iii). Wage are a function of commute time.

- Wage is highest at center

(iv). Firms use horse drawn carts to transport inputs and output to the **central terminal**

Firm's Bid Rent

What do we use to get the firm's bid - rent equation?

A5: Competition generates zero economic profit

Recall the profit equation:

$$\pi = TR - TC$$

In this model:

- $TR = P * Q$ (fixed, exogenous)
- TC is a function of freight cost, labor cost, and intermediate goods cost

$$TC(x) = \text{Freight Cost}(x) + \text{Labor Cost}(x) + \text{Land Cost}(x) + \text{Intermediate Input Cost}$$

Firm Bid Rent

From here on out, let's call $\textbf{Intermediate Input Cost} = \bar{I}$

- Invoking zero economic profit, from the last slide we can write:

$$TR - (\textbf{Freight Cost}(x) + \textbf{Labor Cost}(x) + \textbf{Land Cost}(x) + \bar{I}\,) = 0$$

In words: The most a firm would be willing to pay for land then is revenue net of non land cost

Rearranging:

$$\textbf{Land Cost}(x) = TR - \textbf{Freight Cost}(x) - \textbf{Labor Cost}(x) - \bar{I}$$

Note: Land Cost = $P(x) * L_m$, where:

- $P(x)$ is the *price of land at x miles away from the center*
- L_m is the *amount of land the manufacturer uses in production*

Firm Bid Rent: Equation

We can replace land cost with $P(x) * L_m$ to get the equation for the **manufacturing bid rent** curve

$$P(x) * L_m = TR - \text{Freight Cost}(x) - \text{Labor Cost}(x) - \bar{I}$$

Firm Bid Rent: Equation

We can replace land cost with $P(x) * L_m$ to get the equation for the **manufacturing bid rent** curve

$$P(x) * L_m = TR - \text{Freight Cost}(x) - \text{Labor Cost}(x) - \bar{I}$$
$$P(x) = \frac{TR - \text{Freight Cost}(x) - \text{Labor Cost}(x) - \bar{I}}{L_m}$$

Comparative statics:

In words, this equation says:

- Higher revenues $\implies$ higher land prices for every distance x
- An increase in freight costs, labor costs, or intermediate input costs will **decrease** the price for every distance x

Example

Suppose:

$P = 5$, $Q = 2$, $FC(x) = 4x$, $\texttt{Labor}(x) = 1 - 3x$, $L_m = 1$, $\bar{I} = 0$

(i). Derive the firm's bid rent curve. Carefully write down your steps

(ii). What is the price the firm is willing to pay for land at $x = 1$?

(iii). Is the WTP higher or lower when we move away from the center?

(iv). What distance away from the center is the WTP zero?

Example

(i). Start with zero profit condition:

$$\pi(x) = 0 \implies TR - FC(x) - LC(x) - P(x) = 0$$

Plugging in:

$$\begin{aligned} 5 \times 2 - 4x - (1 - 3x) - P(x) &= 0 \\ 9 - x &= P(x) \end{aligned}$$

(ii). $P(1) = 8$

(iii). Lower (if $x_2 > x_1$, $P(x_2) < P(x_1)$)

(iv). $P(x) = 0 \implies x = 9$

Back to Reality

How can a model like this help us understand the industrial revolution?

- What happened to freight costs? **They fell** A few innovations:

Transportation Innovations:

- Omnibus (1827)
- Cable Cars (1873)
- Electric Trolley (1886)
- Subways (1895)

In our model, what do these innovations do?

Decrease labor costs relative to freight

More History

The *intracity* truck (1910): twice as fast and half as costly as the horse-drawn wagon†

- Decreased the cost of moving output **relative** to the cost of moving workers
- Manufacturing Firms moved closer to low-wage suburbs

Intercity truck (1930): alternative to ships and rail††

- **Highways**: Industry shifted from **ports** and **railroad terminals** to **roads**
- **Modern cities**: Industry oriented toward highways and beltways
    - Freight costs decreased relative to labor

† Intra = Within †† Inter = Across

(iii) Office space bid-rent

(iii) Office space bid-rent

Final rent bidders we will consider - **offices**

Same as the other bidders, WTP for land depends on accessibility

Why?

Office firms use high skilled labor. Need *face to face* interaction for production

- Proximity to other office firms is an important input

Oppurtunity cost of high skilled labor is greater than other types of labor

Office Bid Rent

So as office firms get further from center their "transit" cost goes up. So what must happen to WTP?

City Organization

So how do we put all of this together? And why are these called **bid** rent curves anyways?

- **Land will be allocated to highest bidder**
- This will vary by location in the city

Example: Assume profit for office and manufacturing is given by

$$\pi_{\text{office}} = 105 - P(x_{\text{Office}}) - (5 + 4 \times x_{\text{office}})$$

$$\pi_{\text{manufact}} = 75 - P(x_{\text{manufact}}) - (5 + 2 \times x_{\text{manufact}})$$

For consumers, they can allocate money between housing and commuting:

$$r(x_{\text{commuter}}) = \frac{50}{2} - \frac{1}{2} \times x_{\text{commuter}}$$

Example

$$\pi_{\text{office}} = 105 - r(x_{\text{Office}}) - (5 + 4 \times x_{\text{office}})$$

$$\pi_{\text{manufact}} = 75 - r(x_{\text{manufact}}) - (5 + 2 \times x_{\text{manufact}})$$

$$r(x_{\text{commuter}}) = \frac{100}{4} - \frac{2}{4} \times x_{\text{commuter}}$$

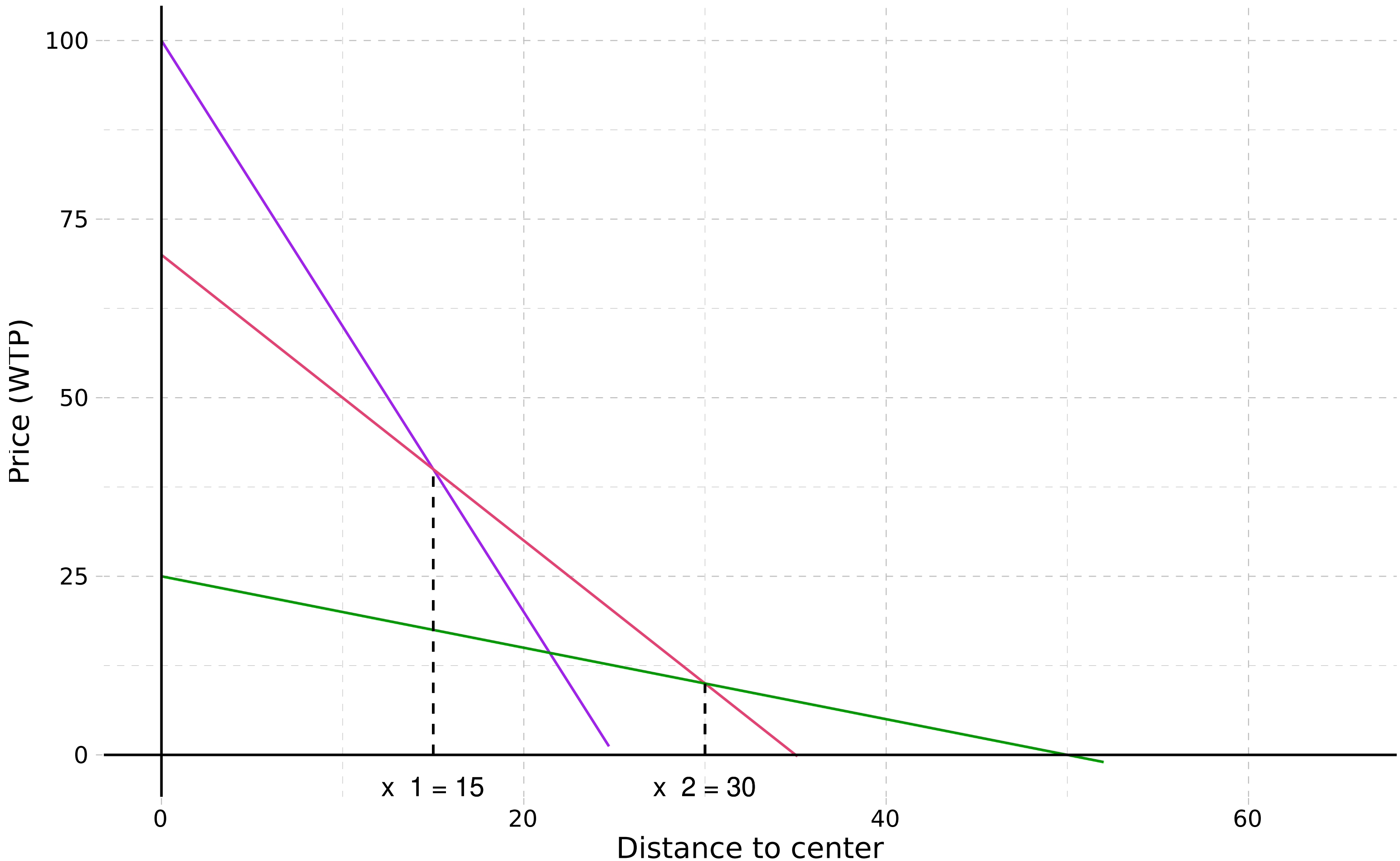
Task:

(i). Derive the bid rent curve for office space, manufacturing, and commuters. Plot all of them.

(ii). Find how land is allocated. What range from the center is:

- Office space
- Manufacturing space
- Housing space?

Example



Example

Bid rent curves for office and manufacturing come from zero profit. Commuters curve was given.

- Office: $r(x_{\text{office}}) = 105 - (5 + 4 \times x_{\text{office}})$
- Manufacturing: $r(x_{\text{manufact}}) = 75 - (5 + 2 \times x_{\text{manufact}})$
- Commuters: (given) $r(x_{\text{commuter}}) = \frac{100}{4} - \frac{2}{4} \times x_{\text{commuter}}$
- Office firms locate in the range of x in $[0, 15]$
- Manufacturing firms locate in the range of x in $[15, 30]$
- Commuters locate in the range of x in $[30, 50]$

Bonus: COVID19 and Cities research

Questions:

Q1) How does COVID19 impact housing/rental prices?

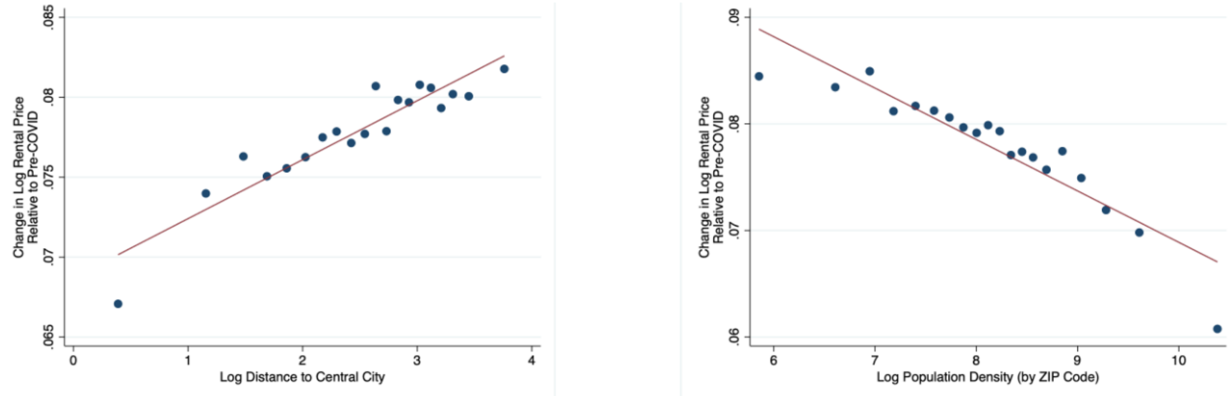
- Is the effect the same everywhere? Why or why not?

Q2) How many jobs can be done remotely? Does this vary systematically across sectors? Cities?

Q3) What do we think will happen to city structure as a result of increased (potentially permanent) WFH

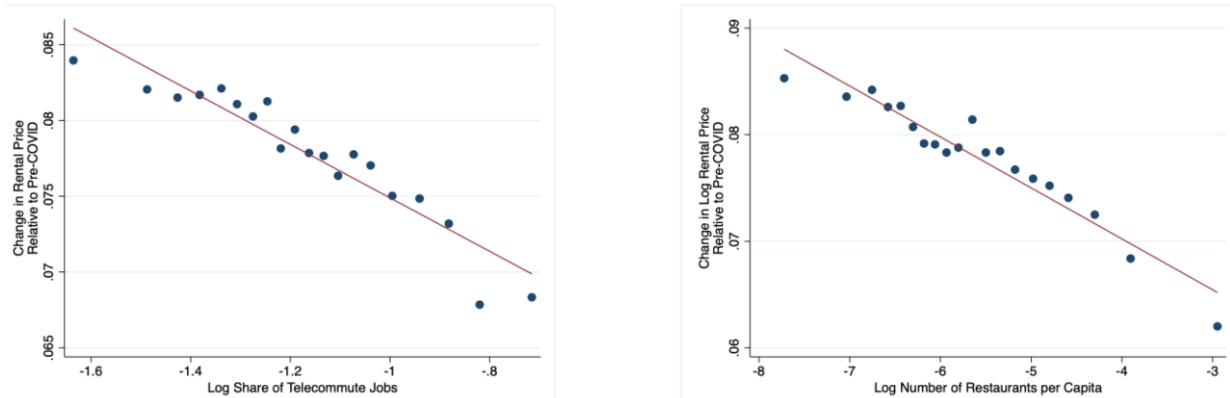
Bonus: COVID19 and Cities Research

- **A1:** *The Impact of the COVID-19 Pandemic on the Demand for Density: Evidence from the U.S. Housing Market* (Liu & Su, 2020)



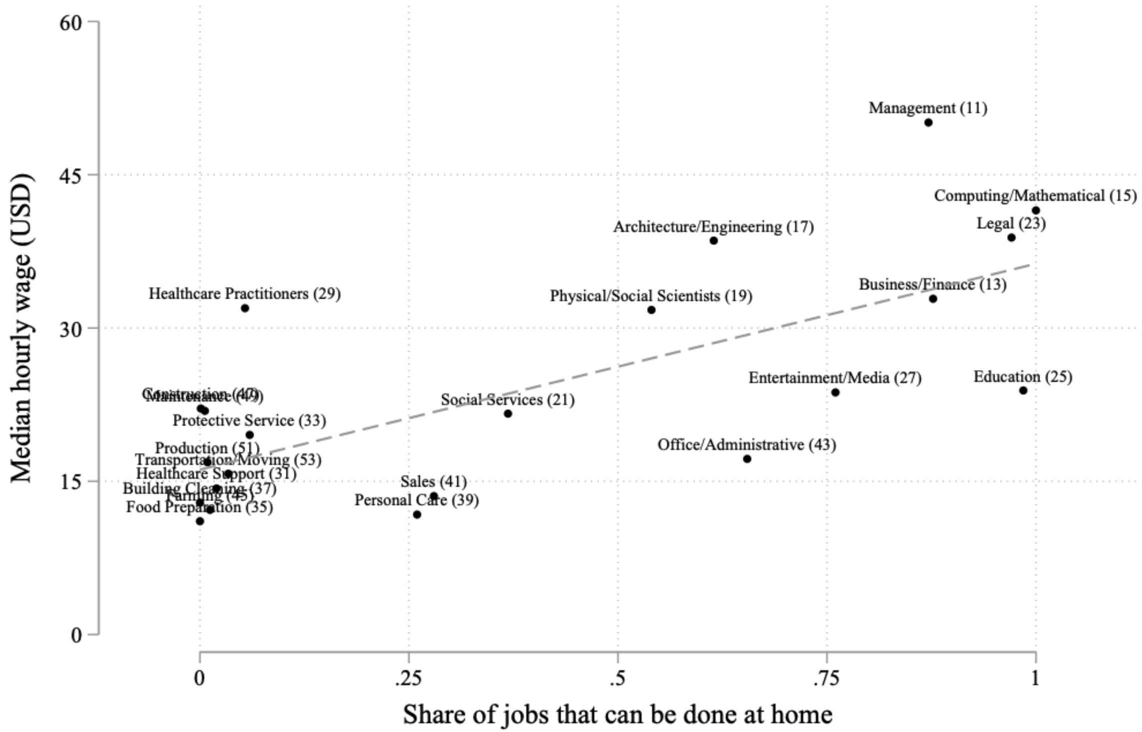
(a) Distance to the Central City

(b) Population Density



Bonus: COVID19 and Cities research

- **A2:** *How many jobs can be done at home?* (Dingel & Nieman, 2020)

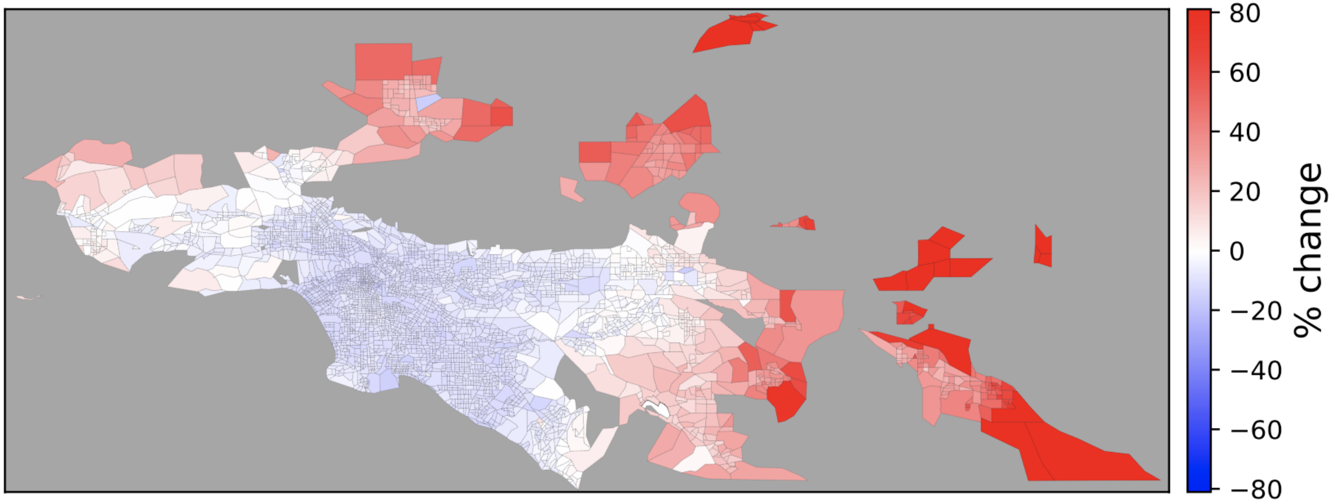


Bonus: COVID19 and Cities research

Q3) What do we think will happen to city structure as a result of increased (potentially permanent) WFH?

- **A3:** *How Do Cities Change When We Work from Home?* (Delventhal et. al, 2020)

Figure 2: House prices



Note: Percentage change relative to benchmark economy in counterfactual with $\psi = 0.33$. See main text for details.

Checklist

1) ✅

1.5) ✅

2) ✅

3) ✅

4) ✅