Multiscale Entropy-Based Weighted Distortion Measure for ECG Coding

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***Abstract*—In this letter, a novel objective distortion measure is proposed for compressed electrocardiogram (ECG) signals. The measure is a weighted percentage root-mean-square difference (WPRD) between the subband coefficients of the original and compressed signals with weights equal to the multiscale entropies of the corresponding subbands. The measure appears to be a correct representation of the amount of signal distortion at all the subbands. Experiments show that the measure works better than the other existing measures and correlates well with the subjective assessments.**

***Index Terms*—Electrocardiogram, multiscale entropy, PRD, wavelet energy, wavelet energy-based diagnostic distortion (WEDD), weighted distortion measure, WWPRD.**

I. INTRODUCTION

In recent years, wavelet subband coding approach has emerged as an efficient method for electrocardiogram (ECG) compression [1]–[4]. Since signal quality is essential for diagnosis, an integrated design of compression and automatic quality control based on the PRD criterion is reported in the literature [2], [4]. However, choice of distortion measure is of critical importance when simultaneous noise suppression and signal compression are established by the wavelet-based methods [1]–[4]. Subjective and objective error or distortion measures are used to quantify the distortions in the compressed signal [5]–[7]. Subjective test is an accepted way of assessing the clinical quality [5]. However, subjective tests are not only cumbersome and expensive, but they also cannot be incorporated into closed-loop quality control [5]–[7]. On the other hand, objective measure is repeatable and simple, while it does not always match with the subjective one.

The most widely used objective distortion measure is the PRD [1]–[4]. Although PRD does not exactly correspond to the subjective test and significantly reflect the actual behavior of the compression method, it is simple to calculate [1], [4]–[6]. In wavelet-based coding schemes, a large number of small coefficients, which are thresholded or quantized to zero value, result in insignificant error. The magnitude of this error may not be of much relevance from the point of view of clinical quality of the compressed signal. The weighted diagnostic distortion (WDD) measure [5] correlates well with subjective test, but it suffers from high computational complexity [1], [6], [7]. Another simple measure is the wavelet-based weighted PRD (WWPRD) [6] with weights corresponding to the wavelet subband normalized area (WSNA) criterion. The WWPRD measure provides a local error estimation. However, insignificant errors in higher subbands dominate the global error while significant errors in other bands may not reflect any contribution to the global error [7]. Moreover, WWPRD/PRD criterion-based target algorithm may not reflect the best compression ratio (CR) of the method when data contain noise [1], [4]. The wavelet energy-based diagnostic distortion (WEDD) measure minimizes the influence of insignificant errors, and thus, it outperforms other measures [7]. Its weighting criterion emphasizes the errors in the lower subbands, but it does not always correspond to a better clinical quality at low CR.

The above facts have motivated a great deal of research on objective measures with clinical relevance. In this letter, we present a multiscale entropy-based weighted distortion measure which consists of four stages: 1) decomposition of the ECG signal into subbands; 2) estimation of the error between the original and the compressed subband coefficients; 3) estimation of multiscale entropy-based weights which discriminate different frequency subbands, particularly bands corresponding to noise; and 4) calculation of global distortion by adding all weighted local distortions.

II. MULTISCALE ENTROPY-BASED DISTORTION MEASURE

Results shown in the previous studies [6], [7] on weighted PRD measures establish the fact that wavelet-based weights can capture the clinical distortions. However, there is no clear ideas on how to choose an optimal set of weights. The purpose of this letter is to show that a well-chosen set of weights can lead to a better evaluation of subband distortions, and to illustrate the advantages of the measure in quality control.

A. Multiresolution Signal Decomposition (MSD)

Let A_L denote the lowest order approximation and $\mathrm{D}_1, \mathrm{D}_2, \ldots, \mathrm{D}_L$ denote the details in a L-level DWT of the ECG signal. The frequency range of subbands A_j and D_j are given by $[0, 2^{-j-1}\mathrm{F}_s]$, and $[2^{-j-1}\mathrm{F}_s, 2^{-j}\mathrm{F}_s]$, where F_s is the sampling frequency [6]. The discrete signal $x[n]$ can be expressed as the summation of approximation $\mathrm{A}_L[n]$ signals and detail $\{\mathrm{D}_j[n]\}_{1\leq j\leq L}$ signals, that is: $\sum_{j=1}^{L} \mathrm{D}_j[n] + \mathrm{A}_L[n]$. Fig. 1 illustrates five-level MSD analysis of a test ECG signal which contains different PQRST morphologies, noise, and artifacts. It is observed that the noise is captured by a few subbands that contain fine detail and its effect disappears at the lower subbands. The effects of baseline artifact appear at lower subbands. The DWT of most ECG signals are compact,

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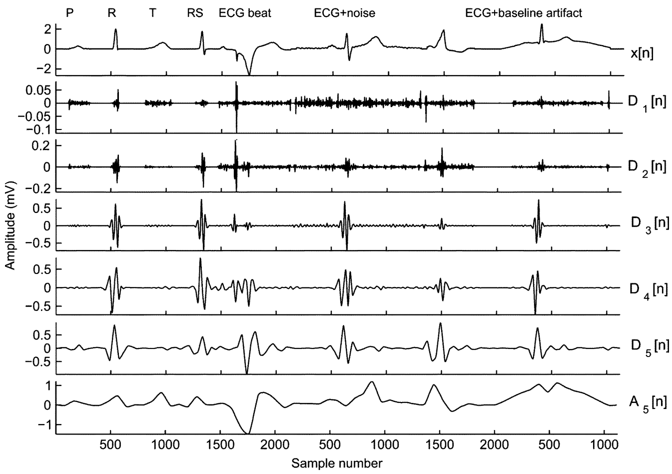

Fig. 1. Test ECG signal $x[n]$ contains different PQRST complexes, noise, and baseline artifacts. Multiresolution ECG signal decomposition using a five-level 9/7 wavelet filters WT structure. Approximation $\mathrm{A}_5[n]$ signal and detail $\mathrm{D}_j[n]$ signals at resolution $j = 1, 2, \ldots. 5$. Noise components and fine details of ECG at D_1 and D_2. Components are localized in bands $\mathrm{D}_j,\ j = 1, 2, 3, 4$.

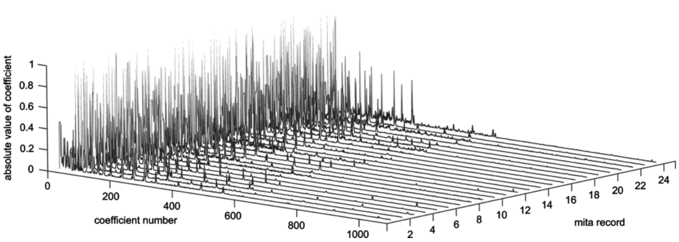

Fig. 2. Absolute amplitude distribution of wavelet coefficients for first 2.84 s of signal from 48 records of the MIT-BIH arrythmia (mita) database. Large number of small coefficients in higher bands.

resulting in a large number of small coefficients and a small number of large coefficients as shown in Fig. 2. We represent the subband coefficients using a compact set of features that capture the actual contribution of each subband signal.

B. Relative Wavelet Subband Energies

The wavelet subband energy gives a good measurement of information of the signal contents [7], [8]. The mean wavelet subband energy (MWSE) and the wavelet subband energy (WSE) estimators are exploited for discriminating different subbands particularly subbands corresponding to noise. The MWSE of the Lth approximation band and the jth detail bands are given by $\bar{\mathrm{E}}_{\mathrm{A_L}} = 1/N_{\mathrm{A_L}} \sum_{k=1}^{N_{\mathrm{A}_L}} |\mathrm{A}_L(k)|^2$ and $\bar{\mathrm{E}}_{\mathrm{D_j}} = 1/N_{D_j} \sum_{k=1}^{N_{\mathrm{D}_j}} |\mathrm{D}_j(k)|^2$, respectively. The N_{A_L} and N_{D_j} are the lengths of the approximation and jth level detail band, respectively. Then, the total mean subband energy is given by $\bar{\mathrm{E}}_{\mathrm{t}} = \bar{\mathrm{E}}_{\mathrm{A_L}} + \sum_{j=1}^{L} \bar{\mathrm{E}}_{\mathrm{D_j}}$. Similarly, the energy of the coefficients in the jth detail band and the Lth approximation band are given by $\sum_{k=1}^{N_{\mathrm{D}_j}} |\mathrm{D}_j(k)|^2$ and $\sum_{k=1}^{N_{\mathrm{A}_L}} |\mathrm{A}_L(k)|^2$, respectively, and then the total subband energy is given by $\mathrm{E_t} = \sum_{k=1}^{N_{\mathrm{A}_L}} |\mathrm{A}_L(k)|^2 + \sum_{j=1}^{L} \sum_{k=1}^{N_{\mathrm{D}_j}} |\mathrm{D}_j(k)|^2$. In this work, the approximation band, A_L, is referred as to as the $(L+1)$th subband. Two normalized subband energy values, the relative MWSE (RMWSE) and the relative WSE (RWSE) values, are given as $\bar{\Upsilon}_j = \bar{\mathrm{E}}_j/\bar{\mathrm{E}}_{\mathrm{t}}$ and $\Upsilon_j = \mathrm{E}_j/\mathrm{E}_{\mathrm{t}}$, $j = 1, 2, \ldots. L+1$. Energy normalization is done to ensure that the energy vector is invariant to the subband signal strength. Since $\sum_{j=1}^{L+1} \bar{\Upsilon}_j = \sum_{j=1}^{L+1} \Upsilon_j = 1$, the distributions of $\{\bar{\Upsilon}_j\}_{(1 \le j \le L+1)}$ and $\{\Upsilon_j\}_{(1 \le j \le L+1)}$ can be considered as time-scale energy probability densities. This provides local information associated to the different frequency bands present in the ECG segment and their corresponding degree of importance [8]. The RWSE is our first weight function [7] which tends to favor lower subbands due to their larger energy values, but the band error values do not always correspond to a better clinical quality at low CR. Similarly, the RMWSE-based weight function emphasizes lower subbands and deemphasizes higher subbands since the definition depends on the subband dimensions. This weight function may inhibit significant errors in higher bands. The measure of local distortions can be further improved using multiscale entropy-based features.

C. Multiscale Entropy and Subband Entropic Distortions

With the definition of information measure, we adopt the distribution of the energy sequence $\{\Upsilon_j\}$ or $\{\bar{\Upsilon}_j\}$ of all subbands to substitute for the probability distribution of signals. The multiscale self-information $\mathrm{I}_j = -\log(\bar{\Upsilon}_j)$ emphasizes the quantity of noise information at higher subbands since its relative energies are less. Thus, the weight function refined in multiscale entropy information measure at subband j is defined as $\mathrm{H}_j = -\bar{\Upsilon}_j \log(\bar{\Upsilon}_j)$, $j = 1, 2, \ldots..L+1$. These entropic information are varying over different scales depending on the signals. The degree of similarity between two energy distributions is measured using the relative wavelet entropy. By using the multiscale information measures, the subband entropic distortion (d_j^h) at scale j is measured. The average d_j^h as a function of the percent retained energy (RE) is shown in Fig. 3(b). The average d_1^h of 0.0129 is obtained for RE value of 99.4% and below. It is observed that d_1^h captures the noise (insignificant) information. The average d_6^h of the band A_5 is zero for RE values above 98.6% while the average d_j^h of the other bands are larger. The clinical distortions are introduced severely. This test is carried out to study the characteristics of the subband distortion by compression, and to select the optimal weight for subband A_5. One can argue that information criterion can be used as a quality measure. This approach is suitable to compare the energy distributions for two subbands of a signal or of two different signals. However, this is not always a relevant measure for subband distortions based on the distributions of the original and compressed signals. The reason lies in the fact that the subband changes or disorders are relatively small since large coefficients in the MSD tree are usually kept and small coefficients are zeroed in compression [see Fig. 3(b)]. Moreover, since the data contain noise, the entropic distortion measure is influenced by noise.

D. Selection of Optimal Weight Function

The selection of optimal weights is based on the weight functions shown in Fig. 3(a), the multiscale signal contents shown in Figs. 1 and 2, and the reported works in [6] and [7]. In the MSD, it is noticed that the bands D_1 and D_2 contain most of the energy attributed to the noise and that the noise energy is

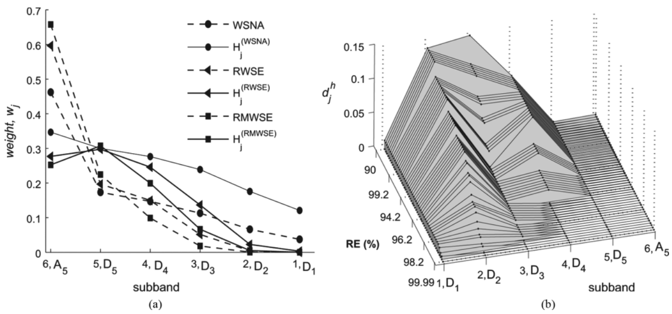

Fig. 3. (a) Characteristics of different weight functions. (b) Average subband entropic distortion d_j^h as a function of RE (%): 2.84 s of signal taken from mita records in [1]. Subbands are obtained using the 9/7 wavelet filters. H_j is the wavelet entropy at scale j. Noise information is seen in D_1 and D_2. d_1^h is linear up to 99.4%. d_2^h is higher for removal of noise structure.

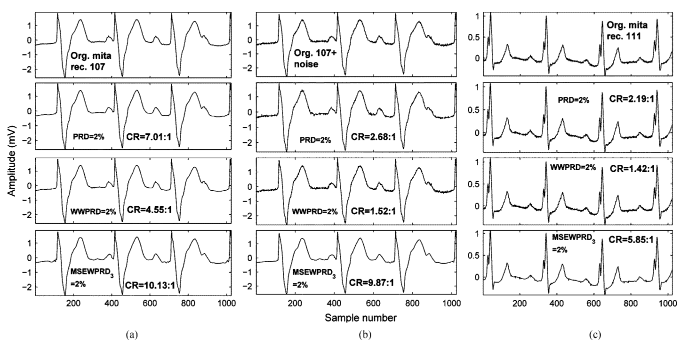

Fig. 4. Compression results of the quality controlled ECG compression method for a desired global error value of 2%. (a) Rec. 107. (b) Rec. 107 plus noise $(\mathrm{SNR} = 30\ \mathrm{dB})$. (c) Rec. 111. Results are summarized in Table III.

TABLE I
PERFORMANCE OF THE WEIGHTED LOCAL AND GLOBAL MEASURES

Rec.	Measure	Local Errors A_5	D_5	D_4	D_3	D_2	D_1	Global Error
mita 107	PRD_{ws}	0.863	3.85	14.13	21.58	45.51	100	**3.904**
	WWPRD	**0.553**	0.635	1.183	1.286	**1.542**	**1.628**	**6.828**
	WEDD	**0.758**	0.356	0.270	0.173	0.053	0.010	1.620
	$\mathrm{MSEWPRD}_1$	0.246	1.144	2.935	3.626	**5.220**	**6.705**	19.875
	$\mathrm{MSEWPRD}_2$	0.098	0.848	1.069	0.833	**0.357**	**0.088**	3.293
	$\mathrm{MSEWPRD}_3$	**0.088**	0.855	0.690	0.313	**0.071**	**0.009**	**2.026**
mita 111	PRD_{ws}	0.415	1.542	3.860	16.259	66.716	100	**5.071**
	WWPRD	**0.182**	0.292	0.575	1.407	**5.871**	**4.823**	**13.151**
	WEDD	**0.269**	0.356	0.386	0.293	0.207	0.052	1.562
	$\mathrm{MSEWPRD}_1$	0.150	0.486	1.095	3.444	**14.269**	**14.622**	34.066
	$\mathrm{MSEWPRD}_2$	0.117	0.522	0.889	1.176	**1.195**	**0.392**	4.291
	$\mathrm{MSEWPRD}_3$	**0.107**	0.532	0.645	0.477	**0.255**	**0.043**	**2.059**

TABLE II
VALIDATION OF THE DISTORTION MEASURES AGAINST $\mathrm{MOS_e}$. VALIDATION METRICS: CORRELATION COEFFICIENT (CC), ROOT-MEAN-SQUARED ERROR (RMSE), MEAN-ABSOLUTE ERROR (MAE), SPEARMAN RANK-ORDER CORRELATION COEFFICIENT (SROCC)

Metrics	Values are quoted from ref. [7]			multiscale-Entropic WPRD_w		
	PRD1	WWPRD	WEDD	WSNA	RWSE	RMWSE
CC	0.8424	0.8916	0.9690	0.6814	0.9764	**0.9852**
RMSE	12.250	10.296	5.6198	23.492	4.072	**3.4612**
MAE	9.2793	8.004	4.5321	16.314	3.2496	**2.7538**
SROCC	0.8504	0.8865	0.9624	0.6782	0.9715	**0.9837**

practically nonexistent at lower subbands. The band D_2 contain high-frequency parts of the QRS complex (see Fig. 2). The thresholding of the bands D_1 and D_2 leads to noise suppression and amplitude reduction of the QRS components. Band D_3 contains some portions of the QRS complex. Band D_4 contains most part of the QRS complex and a small part of the P-wave. Band D_5 contains most part of the P-wave and small parts of the T-wave and QRS complex. Band A_5 contains most part of the T-wave and small part of the P-wave.

In wavelet-based methods, small coefficients are set to zero and large coefficients in the MSD tree are kept with better resolution [1], [4]. Large energy value of the A_5 indicates that it has large coefficients since its dimension is always smaller, and thus, A_5 coefficients are preserved for clinically acceptable level. The bands D_1 and D_2 may contain sharp and small local waves which are usually distorted. Thresholding of band D_5 introduces severe local wave distortions since the significant small coefficients are distributed. Based on the observations, the distortion of the subbands are ordered as $\mathrm{D}_5 > \mathrm{A}_5 > \mathrm{D}_4 > \mathrm{D}_3 > \mathrm{D}_2 > \mathrm{D}_1$. Among the weight functions shown in Fig. 3(a), the function based on the multiscale entropy of the RMWSE estimate fulfills the weight requirements for the higher and lower subbands. Advantages and drawbacks of these functions are further investigated with squared-error criterion in Section III.

E. Multiscale Entropy-Based Weighted PRD Measure

The error between the samples is found to play an important role in clinical evaluation of distortions in PQRST features viz. amplitudes, durations, and shapes [7]. The squared-error distortion criterion is more suitable for rate-distortion optimization. The multiscale entropy-based weighted PRD (MSEWPRD) measure is defined as

$$\mathrm{MSEWPRD} = w_{A_L} \times \left(\sqrt{\frac{\sum_{k=1}^{N_{A_L}} [\mathrm{A}_L(k) - \tilde{\mathrm{A}}_L(k)]^2}{\sum_{k=1}^{N_{A_L}} [\mathrm{A}_L(k)]^2}} \times 100 \right) + \sum_{j=1}^{L} w_{D_j} \times \left(\sqrt{\frac{\sum_{k=1}^{N_{D_j}} [\mathrm{D}_j(k) - \tilde{\mathrm{D}}_j(k)]^2}{\sum_{k=1}^{N_{D_j}} [\mathrm{D}_j(k)]^2}} \times 100 \right) \quad (1)$$

where w_{A_L} denotes the weight for the Lth approximation band; w_{D_j} denotes the weight for the jth level detail subband; A_L and $\tilde{\mathrm{A}}_L$ denotes the Lth approximation band coefficients of the original and the compressed signals, respectively; and D_j and $\tilde{\mathrm{D}}_j$ denotes the jth detail band coefficients of the original and the compressed signals, respectively. The weights are: $w_{A_L} = \mathrm{H}_{L+1}$ and $w_{D_j} = \mathrm{H}_j,\ j = 1, 2, \ldots, L$. The energy normalization employed in the wavelet subband PRD in (1) ensures that the local distortion is invariant to the strength of the coefficients in the band. This property is highly desirable for a desired error percentage-based quality control.

III. EXPERIMENTAL RESULTS AND DISCUSSION

In this section, the performances of three variants of MSEWPRD measure viz. $\mathrm{MSEWPRD}_1$, $\mathrm{MSEWPRD}_2$,

TABLE III
PERFORMANCE OF THE PRD, WWPRD, AND $\mathrm{MSEWPRD}_3$ CRITERIA-BASED QUALITY CONTROLLED ECG COMPRESSION METHOD. HERE, ERROR $= 2\%$

mita record	PRD	WWPRD	$\mathrm{MSEWPRD}_3$	CR	MOS_e	PRD	WWPRD	$\mathrm{MSEWPRD}_3$	CR	MOS_e	PRD	WWPRD	$\mathrm{MSEWPRD}_3$	CR	MOS_e
Orginal 107	2.096	3.7154	0.7731	**7.01**	0.836	1.178	2.0571	0.2482	**4.55**	0	3.904	6.8277	2.0257	**10.13**	2.827
Org. 107+noise	2.081	5.3083	0.4896	**2.68**	0.362	0.784	2.0278	0.1615	**1.52**	0	5.112	12.6257	2.0654	**9.87**	2.364
Orginal 111	2.012	5.5426	0.6728	**2.19**	0.528	0.781	2.0417	0.1752	**1.42**	0	5.071	13.1508	2.0592	**5.85**	2.01

and $\mathrm{MSEWPRD}_3$ are compared with WEDD [7], WWPRD [6], and PRD. These variants are based on WSNA, RWSE, and RMWSE distributions. A block of 1024 samples (each) taken from mita record 107 and 111 are compressed for an $\mathrm{MSEWPRD}_3$ value of 2% using the wavelet-based target distortion level compression algorithm [4]. The behavior of local and global errors of the distortion measures is shown in Table I. The original and the compressed signals are shown in Fig. 4 for clinical evaluation. For WWPRD and $\mathrm{MSEWPRD}_1$ measures, the insignificant errors in D_1 and D_2 bands due to the noise suppression affects the global error. The WEDD value is influenced by A_5 band error due to its large RWSE value which does not always correspond to a better visual quality, whereas $\mathrm{MSEWPRD}_2$ provides better results for A_5 but the amount of errors of the D_2 and D_1 is proportional to noise level of those bands. The WWPRD, $\mathrm{MSEWPRD}_1$, WEDD, and $\mathrm{MSEWPRD}_2$ measures provide the amount of subband distortion but do not appear to be a correct representation of the clinical distortion in some subbands. The $\mathrm{MSEWPRD}_3$ measure is well adapted to the significant errors and is robust to the insignificant errors [see the D_2 and D_1 columns in Table I and Fig. 4(a) and (c)].

Any objective measure must ultimately be validated by comparisons with subjective assessments. An objective measure is considered to be effective if it can reliably predict the score of a subjective quality rating scheme [7]. The mean opinion score (MOS) is used for validating our objective measures. The MOS score is a mapping of clinical levels of distortion on the ECG features P-wave, PR-segment, PR-interval, QRS complex, T-Wave, ST-segment, and QT-interval to either the descriptive terms "excellent, very good, good, not bad, bad," or to an equivalent numerical rating in a scale of 5–1 [7]. The raw scores are then converted to difference scores called as MOS_e which is the relative percentage error.

The validation of the objective measures against $\mathrm{MOS}_\mathrm{error}$ is performed with subjective study involving 210 compressed signals [7]. A monotonic nonlinear mapping (logistic function) between the objective and the subjective scores is used, and the performance validation metrics are computed and are shown in Table II. The PRD, WWPRD, $\mathrm{MSEWPRD}_1$, and $\mathrm{MSEWPRD}_2$ have poor correlations with the subjective scores obtained for the compression of mostly used mita noisy records. Experiments show that the $\mathrm{MSEWPRD}_3$ based on RMWSE estimator correlates well to the subjective assessment, and it has good prediction accuracy and better prediction monotonicity (see last column in Table II). The $\mathrm{MSEWPRD}_3$ measure is much more suitable for evaluating compressed signals than the other measures, and it leads naturally to a new method for quality control in ECG signal compression.

In the literature, many ECG coding schemes are proposed to control the quality based on a desired PRD criterion [2], [4]. The quality controlled compression algorithm reported in [4] is used with $\mathrm{MSEWPRD}_3$ criterion in order to illustrate its advantages. In this experiment, three signal blocks are considered and the results are shown in Fig. 4 and in Table III for a desired global error value of 2% based on three target criteria viz. PRD, WWPRD, and MSEWPRD. Among these, the $\mathrm{MSEWPRD}_3$ criterion-based algorithm reaches an optimal threshold and reflects the best CR of the compression method. It also shows a better rate-distortion performance for the noisy signal. The original and the compressed signals are shown in Fig. 4, and the MOS_e values are given in Table III. The $\mathrm{MSEWPRD}_3$ is superior over the other measures in the sense that it is subjectively meaningful since the small and large values correspond to good and bad quality, respectively.

IV. CONCLUSION

In this letter, we proposed a novel objective distortion measure for compressed and denoised ECG signals. The properties of the weight functions based on the multiscale entropies of the probability distributions are studied. Experiments show that the $\mathrm{MSEWPRD}_3$ measure is a correct representation of the amount of signal distortion, and it correlates well with the subjective assessments. The measure is more suitable for the compression of noisy signal where the noise level is difficult to estimate or avoid.

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