The cutting stock problem formulation that we learned in class is due to P. C. Gilmore and Ralph E. Gomory. They published this formulation in their 1961 paper "A linear programming approach to the cutting-stock problem", *Operations Research*, 8 (1961), 849-859. We call this formulation the Gilmore-Gomory formulation. A little history: Ralph E. Gomory is a renowned mathematician and a key figure in the development of theoretical understanding and computational methods for integer programming in the early days. He has also been very influential in bringing OR to the real world. He first worked at IBM as a research mathematician and later became IBM's Senior Vice President for Science and Technology. He helped IBM attain the best minds for research for over 20 years. He still maintains a blog in the Huffington Post on interesting topics such as technology development and industry research[1].

An alternative optimization formulation for the cutting stock problem was proposed by the Soviet mathematician and economist Leonid V. Kantorovich in 1939. His paper was published in English in 1960 ("Mathematical Methods of Planning and Organising Production" *Management Science*, 6 (1960), 366-422. You can get the paper online through Tech's library.) Later, Kantorovich won the Nobel Prize in Economics in 1975, shared with another pioneer of operations research Tjalling Koopmans, "for their contributions to the theory of optimal allocation of resources."

Kantorovich's formulation for the cutting stock problem is very different from the Gilmore-Gomory formulation. In this problem, we lead you through steps to formulate the Kantorovich formulation and implement in Xpress. Then, we ask you to implement Column Generation on the Gilmore-Gomory formulation. The purpose is to compare these two different formulations and solution strategies for solving the same cutting stock problem. Through this exercise, you will learn that the a more compact formulation of an optimization problem might not always be computationally easier to solve than a larger formulation. You will also see the power of column generation as a solution strategy to solve large-scale linear programs.

Problem 2.1: The Kantorovich Formulation: A Modelling Exercise

The Kantorovich formulation uses the following data and decision variables:

1. Data:
   - w_i, $i = 1, \ldots, m$: the width of small roll item i.
   - b_i, $i = 1, \ldots, m$: the demand for item i.
   - W: the width of the large rolls.
   - K: the total number of large rolls. (Note that this data is not required in the Gilmore-Gomory formulation.)
2. Decision variables:
   - y^k: if large roll k is cut then $y^k = 1$, otherwise $y^k = 0$, for $k = 1, \ldots, K$.
   - x_i^k: number of times that item i of width w_i is cut on the large roll k.

Now we lead you through the steps to formulate the Kantorovich model. Write down the following three constraints.

- The objective is given as to minimize the number of large rolls cut to satisfy demand:

$$\min \quad \sum_{k=1}^{K} y^k$$

[1] http://www.huffingtonpost.com/ralph-gomory/