Measuring the Period of a Simple Pendulum and Analysing the Decomposition of the Small Angle Approximation

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In this paper we investigate the relationship between the period of a simple pendulum and its length, as well as the impact of the initial angle of displacement on the period. Measurements were conducted for pendulum lengths ranging from 0.284 m to 0.675 m, and the observed data showed excellent agreement with the theoretical predictions of $T = 2\pi\sqrt{\frac{L}{g}}$, with relative errors typically within 2%. The experiment also revealed that at larger angles of displacement, deviations from the theoretical period increased. These deviations were quantified as increasing with the angle, demonstrating the breakdown of the small-angle approximation beyond 15 degrees. By quantifying experimental uncertainties, including those arising from human reaction time and instrument precision, the experiment highlights the importance of accurate measurements in validating theoretical models.

I. INTRODUCTION

The simple pendulum is a fundamental system in classical mechanics, widely used to study periodic motion. The period of a simple pendulum is given by the equation:

$$T = 2\pi\sqrt{\frac{L}{g}}, \tag{1}$$

From reference [1], where T is the period, L is the length of the pendulum, and g is the acceleration due to gravity (9.8 m/s^2). This relationship assumes small angles of displacement, where the restoring force is proportional to the sine of the angle but approximated as linear. Understanding deviations at larger angles has implications for applications where small-angle approximations do not suffice, such as in pendulum clocks or dynamic systems analysis.

The goals of this experiment were to:

- Verify the period-length relationship of a pendulum.
- Assess the impact of the initial angle on the period.
- Quantify experimental uncertainties and their effect on measurements.

II. EXPERIMENT

A. Construction of the Pendulum

For the construction of our pendulum, we attached an inextensible string to a stand by one end and to a mass on the other end (Figure 1). For the various configurations of length used to verify the period-length relationship of the pendulum, a tape measure was used to record the length. All length measurements contain an uncertainty up to half of the smallest division on the tape measure (0.1cm), giving all length measurements are to be considered L ± 0.0005m. As previously stated, equation [1] assumes small angles of displacement. Because the small angle approximation holds for all small angles, there is no need to note the initial angle of the pendulum in the verification of equation [1] provided they are small. However, the analysis of the impact of a pendulum's initial angle on its period certainly requires measurements of angles. Similar to measurements of length, measurements of angle were taken using a protractor with divisions up to one degree, resulting in an error of ± 0.5 degrees.

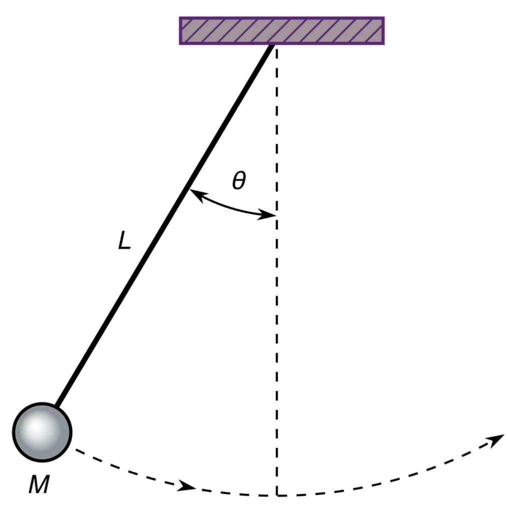

FIG. 1. Simple Pendulum Design. The pendulum consists of a mass M suspended from a string of length L. Figure from ref [2].

B. Measurements of Period

All measurements of the pendulum's period were taken with a stopwatch and recorded to a precision of 0.01 s. To reduce human error, time measurements were taken over ten complete oscillations and then divided by 10 to obtain the period.

The average human reaction time is approximately 0.25 s [3], which contributes to error both at the start and stop of the stopwatch. Thus, the total human error over ten oscillations is estimated to be ± 0.5 s, or ± 0.05 s per oscillation.

Although the stopwatch precision of ± 0.01 s over ten oscillations contributes an uncertainty of ± 0.001 s to the period, this is negligible compared to the error introduced by human reaction time. Therefore, the total uncertainty in each measured period is conservatively taken to be $\delta T \approx \pm 0.05\,\text{s}$. All reported period measurements carry this uncertainty.

III. RESULTS

Five observations were conducted for the verification of Equation 1. The results are displayed below. With an average percent difference on the order of 2%, the recorded data agrees well with the theoretical prediction, as illustrated in Figure 3. As noted earlier, all length values contain uncertainty according to the precision of the yard stick used (± 0.0005 m), and all $T_{observed}$ values carry a time uncertainty of ± 0.5 s, according to the precision of the stop watch and estimation of human error.

Since $T \propto \sqrt{L}$, the percentage uncertainty in $T_{expected}$ is half the percentage uncertainty in L, according to error propagation procedure given by [4]:

$$\frac{\delta T}{T} = \frac{1}{2} \cdot \frac{\delta L}{L}.$$

Each $T_{expected}$ in Table I includes the corresponding absolute uncertainty calculated from this relationship. The uncertainty assigned to $\%\Delta$ is then determined by summing the percent uncertainties of both $T_{observed}$ and $T_{expected}$.

Expected vs Observed Period for Pendulum Lengths

Trial	L (m)	$T_{expected}$ (s)	$T_{observed}$ (s)	$\%\Delta$
1	0.284	1.0700 ± 0.0019	1.091	1.94 ± 9.21
2	0.376	1.2310 ± 0.0016	1.252	1.69 ± 8.15
3	0.447	1.3420 ± 0.0015	1.359	1.26 ± 7.48
4	0.573	1.5190 ± 0.0013	1.520	0.07 ± 6.69
5	0.675	1.6490 ± 0.0012	1.696	2.81 ± 5.99

TABLE I. Comparison between the theoretical period computed using Equation 1 and the experimentally observed period. Percentage error is calculated as $\%\Delta = 100 \times \frac{T_{observed} - T_{expected}}{T_{expected}}$.

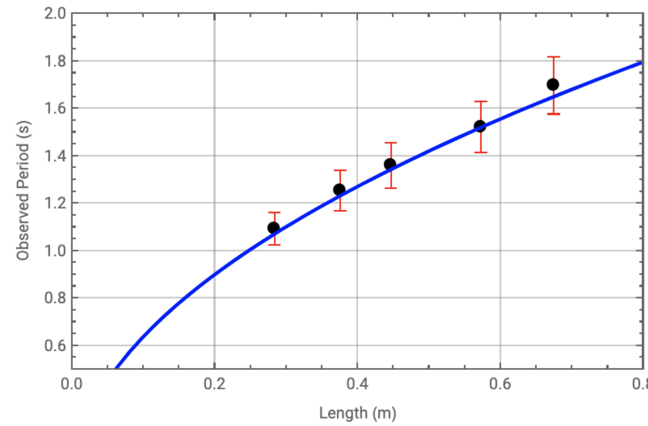

FIG. 2. Predicted(s) vs Observed Periods(s) over length(m). Expected period is displayed in solid blue and experimental values are displayed as points with their corresponding error bars.

As expected, the second part of the experiment demonstrated that larger angles prove Equation [1] increasingly inaccurate, illustrated by Figure [3]. All measurements of θ are to be considered $\pm$ 0.5 degrees and $T_{observed}$ $\pm$ 0.5s, according to the precision of the protractor used to measure θ and our estimation of human error in stop watch operation. Recorded values of $\%\Delta$ were attained through comparison of the recorded values of $T_{observed}$ with the period indicated by Equation [1] for the pendulum configuration used in all six trials ($L = 0.488m$). Using the error in L indicated above ($\pm$ 0.0005m) and the same percentage error calculation discussed earlier, the $T_{expected}$ for the second part of the experiment is calculated to be 1.4020 ± 0.0014 (s). As before, the addition of the percentage error in $T_{expected}$ with the percentage error calculated for each $T_{observed}$ using the aforementioned error in measurements of time ($\pm 0.5s$) gives the uncertainty attached to the values of $\%\Delta$.

Observed Periods for Varying Initial Angles

Trial	θ (degrees)	$T_{observed}$ (s)	$\%\Delta$
1	90	1.631	16.33 ± 6.26
2	60	1.506	7.41 ± 6.77
3	30	1.442	2.85 ± 7.07
4	15	1.437	2.49 ± 7.09
5	10	1.429	1.92 ± 7.13
6	5	1.436	2.42 ± 7.10

TABLE II. Observed pendulum periods for various initial angles using a fixed length of $L = 0.488$ m. All observed periods are compared to the expected value from Equation 1, which assumes the small-angle approximation. The increasing deviation in $\%\Delta$ at higher angles reflects the breakdown of this approximation.

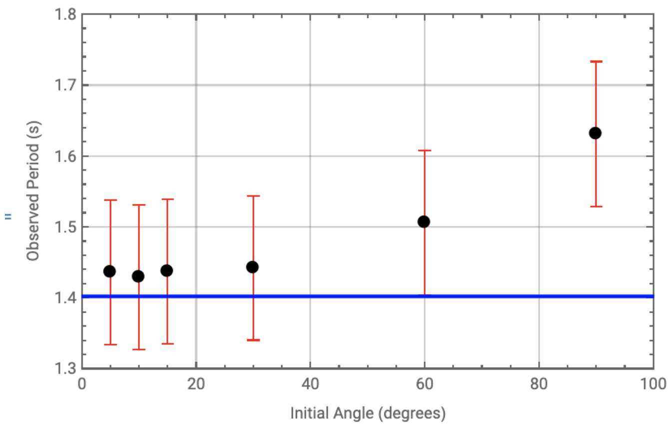

FIG. 3. Impact of Initial Angle on the Pendulum's Period. The expected period (from the small-angle approximation) is shown in solid blue, and experimental values are shown as points with error bars. Based on the data in Table II, the approximation holds reasonably well up to around 30°, with deviations under 3%. Beyond this point—especially at 60° and 90°—the discrepancy becomes significant, highlighting the breakdown of the small-angle approximation in these regimes.

The significant deviation at larger angles indicates the increasing contribution of non-linear terms in the restoring force, suggesting that approximations should be revisited for these configurations.

IV. CONCLUSIONS

Our results were consistent with the theoretical relationship between the period of a simple pendulum and its length. The observed data for pendulum lengths ranging from 0.284 m to 0.675 m showed good agreement with the expected values derived from the equation $T = 2\pi\sqrt{\frac{L}{g}}$, with relative errors typically under 2%. These results validate the assumptions of the small-angle approximation for angles up to approximately 15 degrees.

The experimental uncertainties, including human reaction time during timing and precision limitations of the measuring instruments, were carefully quantified and contributed to the overall error margins. Future experiments could mitigate these sources of error by employing automated timing mechanisms and more precise angle-measuring devices.

Overall, the experiment provided valuable insights into the dynamics of simple harmonic motion and highlighted the limitations of common approximations in classical mechanics.

[1] LibreTexts Physics, Pendulums - university physics i (2025), accessed: 2025-03-25.

[2] I. Encyclopaedia Britannica, Simple pendulum diagram (n.d.), accessed January 19, 2025.

[3] PubNub, How fast is realtime? human perception and technology (2019), accessed January 19, 2025.

[4] J. Caldwell and A. Vahidsafa, Propagation of error (2023), accessed: 2025-04-13.