How to Solve the Traveling Salesman Problem Using Quantum Annealing

To solve the traveling salesman problem, researchers encode it as a QUBO task and process it through quantum annealing hardware while managing system constraints.

Abstract

Quantum annealing addresses the traveling salesman problem by reformulating it as a Quadratic Unconstrained Binary Optimization (QUBO) task. Eight of ten studies applied quantum annealing—with several comparing or combining it with classical methods—to find optimal or near-optimal routes. D-Wave machines (Advantage, 2000Q, and related systems) appear in most implementations; one study used an NMR simulator, and one simulation extended to a 1002-city instance. Reported outcomes include:

1. Optimal or near-optimal tours in simplified or small (6–10 city) problems.
2. Occasional subpar performance when measured against classical solvers.
3. Variable success in constraint management, with techniques such as efficient Hamiltonian formulation and minor embedding used to address subtour constraints.

These studies underscore a practical procedure: encode TSP as a QUBO, apply quantum annealing on hardware limited in qubit count and connectivity, and manage constraints carefully to obtain valid tour solutions.

Paper search

Using your research question "How to Solve the Traveling Salesman Problem Using Quantum Annealing", we searched across over 126 million academic papers from the Semantic Scholar corpus. We retrieved the 50 papers most relevant to the query.

Screening

We screened in papers that met these criteria:

- **Quantum Annealing Focus**: Does the study focus on quantum annealing approaches for solving the Traveling Salesman Problem (TSP)?
- **Implementation Method**: Does the study implement quantum annealing solutions using either actual quantum hardware or validated simulation methods?
- **Methodological Completeness**: Does the study present complete methodology and results that can be evaluated and potentially reproduced?
- **Practical Application**: Does the study address practical methods for solving TSP through quantum annealing, including problem encoding or implementation approaches?
- **Research Contribution**: Does the study present original empirical data or theoretical contributions beyond opinion or commentary?
- **Comparative Analysis**: Does the study provide analysis or comparison of quantum annealing performance against other approaches or benchmarks?
- **Solution Approach**: Does the study include quantum annealing methods (not exclusively classical solutions)?

- **Specific Focus**: Does the study specifically address quantum annealing and TSP (not just general quantum computing)?

We considered all screening questions together and made a holistic judgement about whether to screen in each paper.

Data extraction

We asked a large language model to extract each data column below from each paper. We gave the model the extraction instructions shown below for each column.

- **Quantum Computing Platform**:

  Identify and record the specific quantum computing platform or hardware used in the study:

- Name of quantum computer/annealer (e.g., D-Wave 2000Q, D-Wave Advantage_system4.1)
- Specify the type of quantum computing approach (quantum annealing, hybrid solver, etc.)
- If multiple platforms were used, list all of them
- If no specific platform is mentioned, write "Not specified"

Be precise about the exact model and version of the quantum computing system.

- **Optimization Problem Variant**:

  Specify the exact variant of the Traveling Salesman Problem (TSP) studied:

- Standard TSP
- Selective TSP (sTSP)
- Generalized TSP
- Job Selection Problem (JSP)
- Other specific variant

If multiple variants are discussed, list all of them. Include any unique characteristics or constraints of the problem variant.

- **Quantum Encoding Method**:

  Identify the method used to encode the optimization problem for quantum processing:

- Quadratic Unconstrained Binary Optimization (QUBO) form
- Specific Hamiltonian approach
- Number of qubits used
- Any unique encoding strategies

If multiple encoding methods are discussed, list all of them. Be as specific as possible about the technical approach.

- **Solution Approach Characteristics**:

  Describe the key characteristics of the quantum solution approach:

- Whether the solution is near-optimal or exact
- Time complexity of the approach
- Any specific algorithmic innovations

- Handling of hardware limitations (e.g., for NISQ devices)

Extract quantitative details if available, such as solution quality, computational complexity, or performance metrics.

- **Comparative Performance**:

  Record the performance of the quantum approach compared to classical methods:

- Comparative solution quality
- Computational efficiency
- Any advantages or limitations demonstrated
- Specific performance metrics (if provided)

If no direct comparison is made, note ”No comparative analysis reported”. Be careful to distinguish between claimed advantages and empirically demonstrated results.

Results

Characteristics of Included Studies

Study	Implementation Approach	Hardware Platform	Problem Size	Study Type	Full text retrieved
Chen et al., 2011	Quantum annealing	Nuclear Magnetic Resonance (NMR) quantum simulator	Simplified version of Traveling Salesman Problem (TSP)	Experimental demonstration	No
Heim et al., ”Designing Adiabatic Quantum Optimization”	Adiabatic quantum optimization	No mention found	No mention found	Theoretical analysis	No
Jain, 2021	Quantum annealing	D-Wave Advantage 1.1	8 nodes or less	Experimental study	Yes
Kadowaki, 2002	Quantum annealing	No mention found	No mention found	Theoretical study	No
Le et al., 2023	Quantum annealing, Hybrid Quantum-Classical solver	D-Wave 2000Q	No mention found	Experimental study	Yes
Martoňák et al., 2004	Quantum annealing	D-Wave Advantage, D-Wave 2000Q	1002-city instance	Monte Carlo simulation	Yes
Pérez Delgado et al., 2022	Quantum annealing	D-Wave Advantage_system4.1	9 jobs	Experimental implementation	Yes

Study	Implementation Approach	Hardware Platform	Problem Size	Study Type	Full text retrieved
Villar-Rodriguez et al., 2022	Quantum annealing, hybrid solver	D-Wave Advantage_system6.1	7-node instance	Experimental study	Yes
Warren, 2017	Quantum annealing	D-Wave Systems (2048 qubits)	6, 8, and 10 cities	Theoretical analysis	Yes
Warren, 2021	Hybrid solvers (Kerberos and LeapHybridSampler)	D-Wave DW_2000Q_6	Smallest symmetric TSPs	Experimental study	No

Analysis of the included studies:

- Implementation Approaches :
  - 8 out of 10 studies reported using quantum annealing as their implementation approach
  - 3 studies reported using hybrid approaches
  - 1 study reported using adiabatic quantum optimization
- Hardware Platforms :
  - D-Wave hardware platforms were predominant:
    * 4 studies reported using D-Wave Advantage
    * 3 studies reported using D-Wave 2000Q
    * 1 study reported using D-Wave Systems
  - 1 study reported using an NMR quantum simulator
  - 2 studies didn't provide specific hardware information in their abstracts
- Study Types :
  - 6 out of 10 studies were experimental in nature
  - 3 studies were theoretical
  - 1 study was a simulation study
- Problem Sizes : Problem sizes were reported inconsistently across studies

Effects Analysis

Performance Metrics

Study	Solution Quality	Convergence Time	Success Rate	Hardware Requirements
Chen et al., 2011	Optimal traveling route	No mention found	No mention found	Nuclear Magnetic Resonance (NMR) quantum simulator

Study	Solution Quality	Convergence Time	Success Rate	Hardware Requirements
Heim et al., ”Designing Adiabatic Quantum Optimization”	No mention found	No mention found	No mention found	No mention found
Jain, 2021	Subpar compared to classical solver	2 ms (excluding minor embedding time)	No mention found	D-Wave Advantage 1.1 (5,000 qubits)
Kadowaki, 2002	Higher probability of reaching ground state	Short relaxation time	No mention found	No mention found
Le et al., 2023	Optimal solutions for several instances	No mention found	No mention found	D-Wave 2000Q
Martoňák et al., 2004	Superior to classical methods for certain instances	No mention found	No mention found	D-Wave Advantage, D-Wave 2000Q
Pérez Delgado et al., 2022	Comparable to classical random guess	No mention found	1 optimal solution out of 30,000 runs	D-Wave Advantage_system4.1
Villar-Rodriguez et al., 2022	Not directly compared to classical methods	No mention found	No mention found	D-Wave Advantage_system6.1 (5627 qubits)
Warren, 2017	Expected to find optimal tours quickly	Wall time for 1000 10-city TSPs: about 2 minutes	High probability of optimal answers	D-Wave Systems (2048 qubits)
Warren, 2021	Favorable approximations for small symmetric TSPs	No mention found	No mention found	D-Wave DW_2000Q_6

Analysis of performance metrics:

- Solution Quality : 9 out of 10 studies reported information on solution quality:
  - 3 studies reported optimal or near-optimal solutions
  - 3 studies reported better performance than classical methods
  - 1 study reported subpar performance compared to classical solvers
  - 1 study reported performance comparable to classical random guess
  - 1 study did not directly compare to classical methods
- Convergence Time : 3 out of 10 studies reported convergence time information:
  - 2 studies provided specific time measurements

- 1 study reported a ”short relaxation time”

- Success Rate : 2 out of 10 studies reported success rate information:
  - 1 study reported a high probability of optimal answers
  - 1 study reported a low success rate (1 optimal solution out of 30,000 runs)

- Hardware Information : 8 out of 10 studies reported hardware information:
  - 7 studies reported using various D-Wave quantum annealing systems
  - 1 study reported using an NMR quantum simulator

We didn't find complete information for all categories across all studies, particularly for convergence time and success rate.

Implementation Effectiveness

Study	QUBO Formulation	Constraint Handling	Scalability	Limitations
Chen et al., 2011	No mention found	No mention found	Limited to simplified TSP	Simplified version of TSP
Heim et al., ”Designing Adiabatic Quantum Optimization”	No mention found	Inequality constraints are a major hurdle	No mention found	Challenges with inequality constraints
Jain, 2021	Quadratic Unconstrained Binary Optimization (QUBO) formulation used	Constraints may be violated	Limited to 8 nodes or less	Limited problem size, subpar performance
Kadowaki, 2002	No mention found	No mention found	No mention found	Theoretical study, lacks empirical validation
Le et al., 2023	QUBO formulation used	Efficient Hamiltonian formulation	No mention found	Limited to selective TSP
Martoňák et al., 2004	No mention found	No mention found	Tested on 1002-city instance	Monte Carlo simulation, not actual quantum hardware
Pérez Delgado et al., 2022	QUBO formulation used	O(N) qubits used	Limited by current hardware	No overall advantage over classical methods

Study	QUBO Formulation	Constraint Handling	Scalability	Limitations
Villar-Rodriguez et al., 2022	Two QUBO formulations compared (r-QUBO and h-QUBO)	Minor-embedding for graph embedding	Limited to 7-node instance	Limited by noise and qubit count
Warren, 2017	No mention found	Subtour constraints captured in variable encoding	Limited to small TSPs (up to 10 cities)	Limited by number of qubits and couplings
Warren, 2021	No mention found	No mention found	Limited to smallest symmetric TSPs	Problems too large for current hardware

Analysis of implementation effectiveness:

- QUBO Formulation :
  - 3 studies explicitly mentioned using QUBO formulation
  - 1 study reported comparing multiple QUBO formulations
  - 6 studies didn't mention QUBO formulation in their abstracts or available text
- Constraint Handling : Studies reported various constraint handling approaches:
  - 1 study noted inequality constraints as a challenge
  - 1 study mentioned potential constraint violation
  - 1 study reported using efficient Hamiltonian formulation
  - 1 study reported using O(N) qubits
  - 1 study reported using minor-embedding for graph embedding
  - 1 study reported capturing subtour constraints in variable encoding
  - 4 studies didn't mention constraint handling in their abstracts or available text
- Scalability :
  - 6 studies mentioned scalability limitations
  - 1 study reported testing on a large instance (1002-city)
  - 3 studies didn't explicitly mention scalability information in their abstracts or available text

The studies show diverse approaches to QUBO formulation and constraint handling, with most reporting scalability limitations. The lack of consistent reporting across studies for these aspects suggests a need for more standardized reporting in quantum annealing TSP research.

Thematic Analysis

Quantum Annealing Approaches

- The majority of studies reported using quantum annealing as the primary approach for solving TSP using quantum computing

- Several studies (Jain, 2021; Le et al., 2023; Pérez Delgado et al., 2022) reported utilizing the QUBO formulation to encode TSP for quantum annealing
- A notable trend is the exploration of hybrid quantum-classical approaches, as reported in Le et al. (2023) and Warren (2021)
- The studies highlighted the importance of problem encoding and parameter tuning:
  - Villar-Rodriguez et al. (2022) specifically focused on fine-tuning parameterization and comparing different QUBO formulations

Hardware Considerations

- The majority of experimental studies reported utilizing D-Wave quantum annealing systems, ranging from the D-Wave 2000Q to the more recent Advantage series
- A recurring theme across studies is the limitation imposed by current hardware capabilities:
  - Many implementations reported being restricted to small problem instances due to the limited number of qubits and their connectivity
  - This constraint is particularly evident in studies like Jain (2021) and Warren (2017), which focused on TSP instances with 8 or fewer cities
- The studies also highlighted challenges associated with mapping the TSP onto the physical qubit structure of quantum annealers:
  - Techniques such as minor embedding were reported to address this issue, as mentioned by Villar-Rodriguez et al. (2022)

Practical Implementation Challenges

Several key challenges emerged across the studies in the practical implementation of quantum annealing for TSP:

1. Problem Size : Most studies reported being limited to small TSP instances due to hardware constraints. Scaling to larger, more practically relevant problem sizes remains a significant challenge.
2. Solution Quality : While some studies reported achieving optimal or near-optimal solutions, others found that quantum annealing performs comparably or worse than classical methods. This inconsistency highlights the need for further refinement of quantum annealing techniques for TSP.
3. Constraint Handling : Studies reported difficulties in efficiently encoding problem constraints, particularly inequality constraints, in quantum annealing implementations.
4. Hardware Limitations : Studies reported that current quantum annealing hardware imposes restrictions on problem size and complexity. Issues such as noise, limited qubit count, and connectivity constraints were reported to affect the practical implementation of TSP solutions.
5. Performance Metrics : We found a lack of consistent reporting of performance metrics across studies, making it challenging to compare different approaches and assess progress in the field.

These challenges underscore the need for continued research and development in quantum annealing hardware and algorithms to realize the potential advantages of quantum computing for solving TSP and similar optimization problems.

References

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