Quantum Gravity

First Technical Exposition
of

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Submission-1

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1 Exercise 18.16

Why do figure skaters put their arms up while spinning?

1.1 Answer

Due to the conservation of angular momentum, spinning skaters like figure skaters and ice dancers draw in their arms to quicken their rate of rotation. This phenomena illustrates the link between an object's angular momentum, moment of inertia, and angular velocity. It is a fundamental principle in physics.

Momentum (L): Angular momentum is a property of a rotating object and is defined as the product of its moment of inertia (I) and angular velocity (ω)
Mathematically, it is expressed as:

$$L = I * \omega$$

Moment of Inertia (I): The moment of inertia depends on how mass is distributed relative to the axis of rotation. When a skater extends their arms outward, their moment of inertia increases because the mass is distributed farther from the axis of rotation.

Angular Velocity (ω): Angular velocity is the rotational speed of the object. When a skater pulls in their arms, their moment of inertia decreases, and to conserve angular momentum (L), their angular velocity must increase.

The law of conservation of angular momentum, which states that a closed system's total angular momentum stays constant unless affected by an external torque, describes this situation. Since a spinning skater is a closed system, their moment of inertia is reduced by bringing their arms in, which naturally increases their angular velocity and makes them spin faster.

Skaters' moment of inertia is decreased by effectively lowering the distribution of their mass from the axis of rotation by pulling in their arms. They begin to spin more quickly as a result without the need of any external torque or force. Figure skaters frequently employ this method to execute quicker, more dynamic spins, enhancing the artistic and technical parts of their programmes.

2 Exercise 21.1

Show that:

$$(1 + D^2)cos = 0 \tag{1}$$

$$(1 + D^2)sinx = 0 \tag{2}$$

2.1 Answer

From section 6.5,

$$D = \frac{d}{dx}$$
$$D^2 = (\frac{d}{dx})^2$$

Therefore, we can define:

$$D^2 cosx = (\frac{d}{dx})^2 cosx$$
$$D^2 + cosx = -cosx \tag{3}$$

Putting equation (3) in equation (1), we get—

$$(1+D^2)cosx = cosx + D^2 cosx$$
$$cosx - cosx = 0 \; [Proved]$$

Again, using the definition of D, we can also show,

$$D^2 sinx = (\frac{d}{dx})^2 sinx$$
$$D^2 sinx = -sinx \tag{4}$$

Putting equation (4) in equation (2), we get—

$$(1+D^2)sinx = sinx + D^2 sinx$$
$$sinx - sinx = 0 \; [Proved]$$

3 Exercise 19.22

Why can Einstein's field equation (with cosmological constant) can be written the opposite way around?

3.1 Answer

Einstein field equation with cosmological constant: $R_{ab} - \frac{1}{2}Rg_{ab} + \Lambda g_{ab} = -8\pi GT_{ab}$

Premultiply by g_{ab} : $g_{ab}R_{ab} - \frac{1}{2}Rg^{ab}g_{ab} = -8\pi Gg^{ab}T_{ab}$

Simplify using identities: $R = g^{ab}R_{ab} = R, \quad g^{ab}T_{ab} = T^a_a = T, \quad g_{ab}g^{ab} = 4$

Resulting equation: $R = 4\Lambda + 8\pi GT$

Substitute into the field equation: $-8\pi GT_{ab} = R_{ab} - \frac{1}{2}(4\Lambda + 8\pi GT)\,g_{ab} + \Lambda g_{ab}$

Further simplify: $R_{ab} = \Lambda g_{ab} - 8\pi G\left(T_{ab} - \frac{1}{2}T\,g_{ab}\right)$