Reg. No: ....
Name : ....

K20P 9101

FIRST SEMESTER MSc.STATISTICS DEGREE CCSS-REGULAR EXAMINATION NOVEMBER 2020

(2020 Admission onwards)

MSSTA01C03:DISTRIBUTION THEORY

Time: 3Hours **Maximum Marks: 60**

PART A *(Answer any **five** questions. Each question carries **3** marks)*

1. **The number of floods in a local river during rainy season is known to follow a Poisson distribution with the expected number of floods 3. What is the probability that during one reainy season (i) there are exactly 5 floods, (ii) there is no flood, (iii) at least one flood.**
2. **Define joint probability density function. State its properties.**
3. **Obtain the distribution function of the largest order statistic.**
4. **State and prove lack of memory property for exponential distribuion.**
5. **At Thekkady wildlife reserve, suppose that on any given day the probability of finding a tourist from Kerala is 0.4, from Tamilnadu is 0.3, from other states in India is 0.2 and foreigners is 0.1. On a particular day, there are 20 tourists. What is the probability that out of these 20, 10 are from Tamilnadu and 10 are from other states in India?**
6. **Give the properties of a probability density function. Compute the normalizing constant c when**

$$f(x) = cx^3(1-x)^2;\ 0 \le x \le 1$$

and $f(x) = 0$ otherwise.

PART B *(Answer any **three** questions. Each question carries **5** marks)*

7. **Give the pdf of t-distribution. Obtain the limiting distribution as $n \to \infty$.**
8. **If X_1 and X_2 are independent chi-square variables with n_1 and n_2 degrees of freedom respectively, then obtain the distribution of $U = \frac{X_1}{X_1+X_2}$ follows beta distribution.**
9. **Let $X_1, \ldots, X_n$ be i.i.d $N(1, 1)$ random variables. Let $S_n = X_1^2 + \ldots + X_n^2$ for $n \ge 1$. Find $\lim$.**
10. **A random sample of size n is taken from a population with probability density function**

\nonumber f(x) = \frac{1}{a^\lambda \Gamma(\lambda)} x^{\lambda -1} e^{-\frac{x}{a}}; x \ge 0 and $f(x) = 0$ elsewhere. Find the distribtuion of $\overline{X}$.

11. **Define moment generating function. Give an example in which moment generating function does not exist.**

PART C *(Answer any **three** questions. Each question carries **10** marks)*

12. a. **If X has a geometric distribution with parameter p, then give the p.m.f of X. Also show that for any two positive integers, m and n, $Pr\{X > m+n|X>m\} = Pr\{X \ge n\}$ and conversally.** **5-marks**

    b. **Prove that Poisson distribution as a limiting case of the negative binomial distribution.** **5-marks**

13. a. **Find c such that the following is a joint probability function.** x=-1 x=0 y=1 $\frac{c}{20}$ $\frac{2}{20}$ y=2 $\frac{1}{20}$ $\frac{3}{20}$ y=3 $\frac{2}{20}$ $\frac{8}{20}$ **(b) Compute the following: (i) $Pr\{X = -1, Y = 1\}$; (ii) $Pr\{X < 0, Y > 2\}$; (iii) $Pr\{X \le -2, Y \ge 0\}$.** **7-marks**

    b. **Compute $Corr(X, Y)$.** **3-marks**

14. a. **Obtain the joint distribution of rth and sth order statistics. Use it to obtain the distribution of mid-range.** **7-marks**

    b. **Obtain the density function of the r^{th} order statistics.** **3-marks**

15. a. **If $X \sim U(0,10)$, obtain $Pr\{1 \le X \le 6\}$.** **3-marks**

    b. **Define continuous uniform distribution over the interval $[a,b]$ and obtain the moment generating function. Obtain the mean and variance by differentiating as well as by expanding the moment generating function.** **7-marks**

16. a. **Let X_{1}, X_{2}, X_{3} be i.i.d random variables with common density** $f(x) = \left\{ \begin{array}{l} e^{-x}, 0\le x < \infty \\ 0, {\rm elsewhere} \end{array} \right.$ **Let** $Y_{1} = X_{1} + X_{2} + X_{3}$, $Y_{2} = \frac{X_{1} + X_{2}}{X_{1} + X_{2} + X_{3}}$ **and** $Y_{3} = \frac{X_{1}}{X_{1} + X_{2}}$**. Show that Y_{1}, Y_{2} and Y_{3} are independent and obtain their densities.** **5-marks**

    b. **Let X_{1}, X_{2}, X_{3} be i.i.d random variables with common density** $f(x) = \frac{1}{\sqrt{2 \pi}} e^{\frac{-x^{2}}{2}}, -\infty < x < \infty$ **Let** $Y_{1} = \frac{X_{1} - X_{2}}{\sqrt{2}}$, $Y_{2} = \frac{X_{1} + X_{2} - 2X_{3}}{\sqrt{6}}$ **and** $Y_{3} = \frac{X_{1} + X_{2} + X_{3}}{\sqrt{3}}$**. Show that Y_{1}, Y_{2} and Y_{3} are independent and obtain their densities.** **5-marks**

Processing math: 22%