IIT Madras
BSc Degree

Home Week-9 Week-11

PDSA - Week 10

String matching

Searching for a pattern is a fundamental problem when dealing with text

- Editing a document
- Answering an internet search query
- Looking for a match in a gene sequence

Example

- `an` occurs in `banana` at two positions

Formally

- A text string `t` of length `n` A pattern string `p` of length `m`
- Both `t` and `p` are drawn from an alphabet of valid letters, denoted `Σ`
- Find every position `i` in t such that `t[i:i+m] == p`

Brute force approach

Nested scan from left to right in `t`

```python
def stringmatch(t,p):
    poslist = []
    for i in range(len(t)-len(p)+1):
        matched = True
        j = 0
        while j < len(p) and matched:
            if t[i+j] != p[j]:
                matched = False
            j = j+1
        if matched:
            poslist.append(i)
    return(poslist)
print(stringmatch('abababbababbbbababab','abab'))
```

Output

```
[0, 2, 7, 14, 16]
```

Complexity

$O(nm)$

Nested scan from right to left

```python
def stringmatchrev(t,p):
    poslist = []
    for i in range(len(t)-len(p)+1):
        matched = True
        j = len(p)-1
        while j >= 0 and matched:
            if t[i+j] != p[j]:
                matched = False
            j = j-1
        if matched:
            poslist.append(i)
    return(poslist)
print(stringmatchrev('abababbababbbbababab','abab'))
```

Output

```
[0, 2, 7, 14, 16]
```

Complexity

$O(nm)$

Speeding up the brute force algorithm

- Text `t`, pattern `p` of of lengths `n`, `m`
- For each starting position `i` in `t`, compare `t[i:i+m]` with `p`

    - Scan `t[i:i+m]` right to left
- While matching, we find a letter in `t` that does not appear in `p`
    - `t = bananamania`, `p = bulk`
- Shift the next scan to position after mismatched letter
- What if the mismatched letter does appear in `p`?

Boyer-Moore Algorithm

Algorithm

- Initialize `last[c]` for each `c` in `p`
    - Single scan, rightmost value is recorded
- Nested loop, compare each segment `t[i:i+len(p)]` with `p`
- If `p` matches, record and shift by `1`
- We find a mismatch at `t[i+j]`
    - If `j > last[t[i+j]]`, shift by `j - last[t[i+j]]`
    - If `last[t[i+j]] > j`, shift by `1`
        - Should not shift `p` to left!
    - If `t[i+j]` not in `p`, shift by `j+1`

Implementation

```python
def boyermoore(t,p):
    last = {} # Preprocess
    for i in range(len(p)):
        last[p[i]] = i
    poslist=[]
    i = 0
    while i <= (len(t)-len(p)):
        matched,j = True,len(p)-1
        while j >= 0 and matched:
            if t[i+j] != p[j]:
                matched = False
            j = j - 1
        if matched:
            poslist.append(i)
            i = i + 1
        else:
            j = j + 1
            if t[i+j] in last.keys():
                i = i + max(j-last[t[i+j]],1)
            else:
                i = i + j + 1
    return(poslist)
print(boyermoore('abcaaacabc','abc'))
```

Output

```
[0, 7]
```

Complexity

Worst case remains $O(nm)$

If `t = aaa...a, p = baaa`

Rabin-Karp Algorithm

- Suppose Σ = {0, 1, . . . , 9}
- Any string over Σ can be thought of as a number in base 10
- Pattern p is an m-digit number n_p
- Each substring of length m in the text t is again an m-digit number
- Scan t and compare the number n_b generated by each block of m letters with the pattern number n_p

Implementation

```python
def rabinkarp(t,p):
    poslist = []
    numt,nump = 0,0
    for i in range(len(p)):
        numt = 10*numt + int(t[i])
        nump = 10*nump + int(p[i])
    if numt == nump:
        poslist.append(0)
    for i in range(1,len(t)-len(p)+1):
        numt = numt - int(t[i-1])*(10**(len(p)-1))
        numt = 10*numt + int(t[i+len(p)-1])
        if numt == nump:
            poslist.append(i)
    return(poslist)
print(rabinkarp('233323233454323','23'))
```

Output

```
[0, 4, 6, 13]
```

Analysis

- Preprocessing time is $O(m)$
  - To convert `t[0:m]`, `p` to numbers
- Worst case for general alphabets is $O(nm)$
  - Every block `t[i:i+m]` may have same remainder modulo `q` as the pattern `p`
  - Must validate each block explicitly, like brute force
- In practice number of spurious matches will be small
- If |Σ| is small enough to not require modulo arithmetic, overall time is $O(n+m)$, or $O(n)$, since $m << n$
  - Also if we can choose `q` carefully to ensure $O(1)$ spurious matches

Knuth-Morris-Pratt algorithm

- Compute the automaton for `p` efficiently
- Match `p` against itself
  - `match[j] = k` if suffix of `p[:j+1]` matches prefix `p[:k]`
- Suppose suffix of `p[:j+1]` matches prefix `p[:k]`
  - If `p[j+1]` == `p[k]`, extend the match
  - Otherwise try to find a shorter prefix that can be extended by `p[j+1]`
- Usually refer to match as failure function fail
  - Where to fall back if match fails

Computing the fail function

- Initialize `fail[j] = 0` for all `j`
- `k` keeps track of length of current match
- `j` is next position to update fail
- If `p[j] == p[k]` extend the match, set `fail[j] = k+1`
- If `p[j] != p[k]` find a shorter prefix that matches suffix of `p[:j]`
  - Step back to `fail[k-1]`
- If we don't find a nontrivial prefix to extend, retain `fail[j] = 0`, move to next position

Implementation of fail function

```python
def kmp_fail(p):
    m = len(p)
    fail = [0 for i in range(m)]
    j,k = 1,0
    while j < m:
        if p[j] == p[k]:
            fail[j] = k+1
            j,k = j+1,k+1
        elif k > 0:
            k = fail[k-1]
        else:
            j = j+1
    return(fail)
print(kmp_fail('abcaabca'))
```

Output

```
[0, 0, 0, 1, 1, 2, 3, 4]
```

Complexity

$O(n)$

Implementation of KMP algorithm

- Scan `t` from beginning
- `j` is next position in `t`
- `k` is currently matched position in `p`
- If `t[j] == p[k]` extend the match
- If `t[j] != p[k]`, update match prefix
- If we reach the end of the while loop, no match

```python
def kmp_fail(p):
    m = len(p)
    fail = [0 for i in range(m)]
    j,k = 1,0
    while j < m:
        if p[j] == p[k]:
            fail[j] = k+1
            j,k = j+1,k+1
        elif k > 0:
            k = fail[k-1]
        else:
            j = j+1
    return(fail)

def find_kmp(t, p):
    match =[]
    n,m = len(t),len(p)
    if m == 0:
        match.append(0)
    fail = kmp_fail(p)
    j = 0
    k = 0
    while j < n:
        if t[j] == p[k]:
            if k == m - 1:
                match.append(j - m + 1)
                k = 0
                j = j - m + 2
            else:
                j,k = j+1,k+1
        elif k > 0:
            k = fail[k-1]
        else:
            j = j+1
    return(match)
print(find_kmp('abababbaba','aba'))
```

Output

```
[0, 2, 8]
```

Analysis

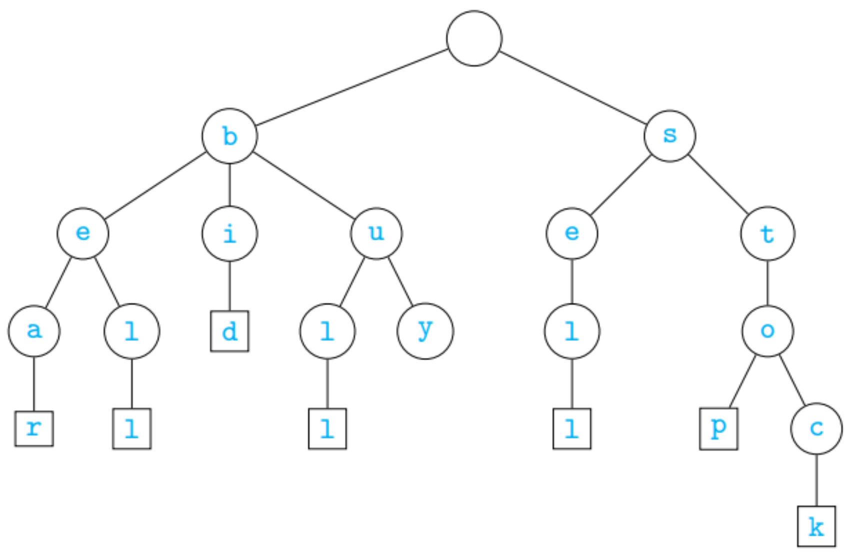
- The Knuth, Morris, Pratt algorithm efficiently computes the automaton describing prefix matches in the pattern `p`
- Complexity of preprocessing the fail function is $O(m)$

- After preprocessing, can check matches in the text t in $O(n)$
- Overall, KMP algorithm works in time $O(m+n)$

Tries

- A trie is a special kind of tree
  - From “information retrieval”
  - Pronounced try, distinguish from tree
- Rooted tree
  - Other than root, each node labelled by a letter from Σ
  - Children of a node have distinct labels
- Each maximal path is a word
  - One word should not be a prefix of another
  - Add special end of word symbol `$`

$\{\texttt{bear,bell,bid,bull,buy,sell,stop,stock}\}$



- Build a trie `T` from a set of words `S` with `s` words and `n` total symbols
- To search for a word `w`, follow its path
  - If the node we reach has `$` as a successor represent $w \in S$
  - $w \notin S-$ if path cannot be completed, or `w` is a prefix of some $w` \in S$
- Build a trie `T` from a set of words `S` with `s` words and `n` total symbols
- **Basic properties for T built from S**
  - Height of `T` is $max_{w \in S} len(w)$
  - A node has at most |Σ| children
  - The number of leaves in `T` is `s`
  - The number of nodes in `T` is `n + 1`, plus s nodes labelled `$`

Implementation of Tries

```
class Trie:
    def __init__(self,S=[]):
        self.root = {}
```

```
        for s in S:
            self.add(s)
    def add(self,s):
        curr = self.root
        s = s + "$"
        for c in s:
            if c not in curr.keys():
                curr[c] = {}
            curr = curr[c]
    def query(self,s):
        curr = self.root
        for c in s:
            if c not in curr.keys():
                return(False)
            curr = curr[c]
        if "$" in curr.keys():
            return(True)
        else:
            return(False)

T = Trie()
T.add('car')
T.add('card')
T.add('care')
T.add('dog')
T.add('done')
print(T.query('dog'))
print(T.query('cat'))
```

Output

```
True
False
```

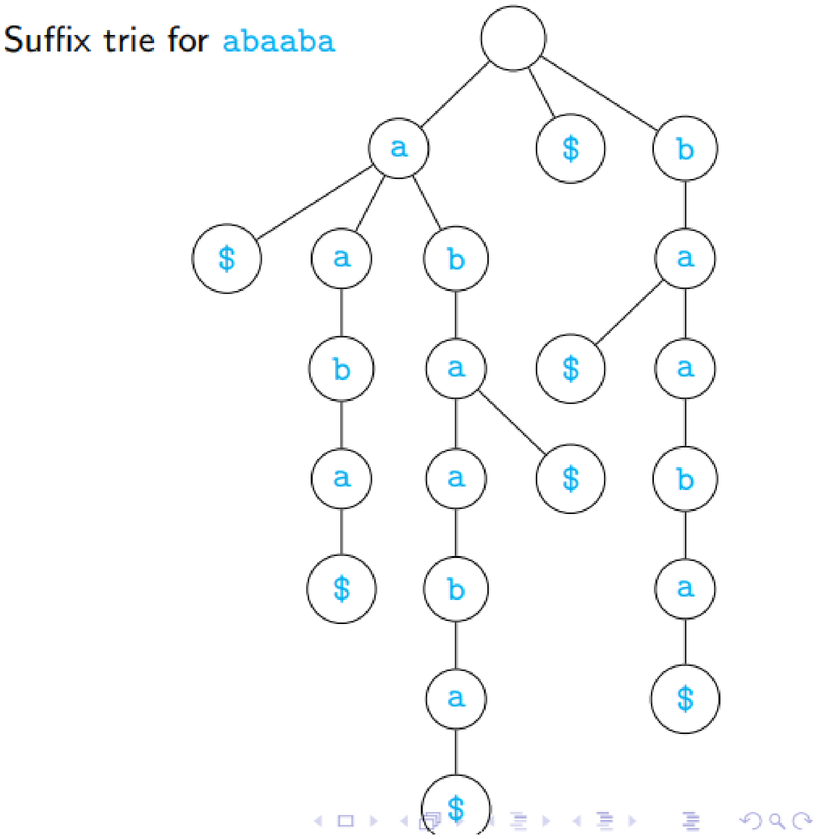
Analysis

- Tries are useful to preprocess fixed text for multiple searches
- Searching for `p` is proportional to length of `p`
- Main drawback of a trie is size

Suffix Tries

- Expand `s` to include all suffixes
  - For simplicity, assume `S = {s}`
  - `suffix(S) = {w | ∃v, vw = s}`
- Build a trie for `suffix(S)`
  - Use `$` to mark end of word
  - Suffix trie for `S`
- Using a suffix trie we can answer the following
  - Is `w` a substring of `s`?
  - How many times does `w` occur as a substring in `s` ?

- What is the longest repeated substring in `s` ?



Implementation of suffix tries

```
class SuffixTrie:
    def __init__(self,s):
        self.root = {}
        s = s + "$"
        for i in range(len(s)):
            curr = self.root
            for c in s[i:]:
                if c not in curr.keys():
                    curr[c] = {}
                curr = curr[c]
    def followPath(self,s):
        curr = self.root
        for c in s:
            if c not in curr.keys():
                return(None)
            curr = curr[c]
        return(curr)
    def hasSuffix(self,s):
        node = self.followPath(s)
        return(node is not None and "$" in node.keys())
ST = SuffixTrie('card')
print(ST.root)
print(ST.followPath('a'))
print(ST.hasSuffix('aa'))
```

Output

```
{'r': {'d': {'$': {}}}}
False
```

Regular expression

use lecture's slides